The Price Adjustment Hazard Function
Evidence from High Inflation Periods

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Real effects of monetary policy depend on nominal rigidities
Nature of price adjustment frictions is crucial
Caballero and Engel (1993, 2007): think about sticky prices with hazard function

\[ H(x) = P(\Delta p_t | x \equiv p^*_t - p_{t-1}) \]

Does not require specifying/solving model
Complementary with price adjustment models
This paper
  - Use price micro data to estimate hazard function
  - Show implications for monetary non-neutrality
Important determinant of aggregate flexibility: selection effect

Caballero and Engel (2007) show flexibility can be derived from hazard function:

\[
\Delta p_t'(\Delta m = 0) = \int H(x)f_t(x)dx + \int xH'(x)f_t(x)dx.
\]

- EM = 0 in Calvo, high in Ss case
- Slope is crucial
Approach

- **Challenge:** do not observe price misalignment
- **Method:**
  - Specify process for optimal price
  - Hazard function yields distribution of price changes
  - Look for hazard function to match empirical moments
  - Use differences between high and low inflation periods
- **Data:** prices underlying U.S. CPI from 1977 onwards
- **Existing estimates**
  - Berger and Vavra (2018), Petrella et al. (2019): period-by-period estimates
  - We use data on **high inflation periods** and co-movement between inflation and price change moments
Set-up

- Idiosyncratic and aggregate shocks to desired/optimal price:
  \[ p_{it}^* = z_{it} + m_t \]

- Specify processes for shocks:
  \[ z_{it} = \begin{cases} 
  \rho z_{it-1} + \epsilon_t, & P = p_\epsilon \\
  z_{it-1}, & P = 1 - p_\epsilon 
  \end{cases}, \quad \epsilon_t \sim iid \ N(0, \sigma^2_\epsilon) \]

  \[ m_t = \mu + m_{t-1} + \eta_t, \quad \eta_t \sim iid \ N(0, \sigma^2_\eta) \]

- Hazard function determines price adjustment probability:
  \[ H(x) = P(\Delta p_t \mid x \equiv p_{t}^* - p_{t-1}) \]

- \( p^* \) observed only when price changes
- Must also estimate parameters of underlying shock process
Flexible quadratic functional form:

\[
H(x)^{\text{quad}} = \begin{cases} 
1, & \text{if } x < c^- \\
p_0 + a^- \cdot x + b^- \cdot x^2, & \text{if } c^- \leq x < 0 \\
p_0, & \text{if } x = 0 \\
p_0 + a^+ \cdot x + b^+ \cdot x^2, & \text{if } 0 < x \leq c^+ \\
1, & \text{if } x > c^+
\end{cases}
\]

- Allows for asymmetry around zero
- Nests simple functions with threshold parameter
- Also use logistic and non-parametric form
- All are flexible, yield similar results
Moments Used

- **Standard unconditional moments:**
  - Average frequency of price change
  - Frequency of increases and decreases
  - Average absolute price change (increases and decreases)

- **Additional unconditional moments:**
  - Average fraction of small price changes
  - Average dispersion and skewness

- **Moment correlations:**
  - \( \text{Corr}(\text{Freq}, \pi) > 0 \)
  - \( \text{Corr}(\text{IQR}, \pi) < 0 \)
  - \( \text{Corr}(\text{Skew}, \pi) \geq 0 \)

- **Exploit variation in inflation over sample period**

Moment Values
Menu Cost, inflation = 0, skewness = 0

Menu Cost, inflation > 0, skewness < 0

Calvo, inflation = 0, skewness = 0

Calvo, inflation > 0, skewness = 0
Note: we estimate $p_0 = 0.069$, $a^+ = 1.224$, $b^+ = 0.208$, $c^+ = 0.244$, $a^- = -0.005$, $b^- = 1.665$, $c^- = -0.412$, $\sigma_\epsilon = 0.055$, $p_\epsilon = 0.485$ in the quadratic hazard function.
Key Features

Find that to match data, hazard function must have three important features:

1. Significant $p_0$: Calvo feature
2. Asymmetry: price increases more likely
3. Probability increases only slowly in $|x|$

Skewness correlation key to establish these features

Important implications for non-neutrality
Illustration with $\rho_0$
Illustration with $c^+$
Use of Skewness Correlation

\[ a^+ = a^- \]

Corr(Skew, Inflation)

\[ a^- = -0.5 \]

fix \( p_0 = .069 \)

\[ p_0 = 0.069, \ a^+ = a^- \]

Hazard function

\[ a^+ = 1.224, \ a^- = -0.5 \]

\[ a^- = -0.005, \ p_0 = 0.069 \]
Monetary Non-Neutrality

- Compute $\text{Var}(c_t)$ induced by aggregate shocks
- Use Hazard function to derive price level
- Similar results to Luo and Villar (2019)

<table>
<thead>
<tr>
<th>Hazard Function</th>
<th>$\text{Var}(c_t) \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calvo</td>
<td>0.537</td>
</tr>
<tr>
<td><strong>Non-Parametric</strong></td>
<td><strong>0.312</strong></td>
</tr>
<tr>
<td>Logistic</td>
<td>0.334</td>
</tr>
<tr>
<td><strong>Flexible Quadratic</strong></td>
<td><strong>0.329</strong></td>
</tr>
<tr>
<td>Midrigan/CalvoPlus</td>
<td>0.195</td>
</tr>
<tr>
<td>Caballero &amp; Engel</td>
<td>0.176</td>
</tr>
<tr>
<td>Golosov &amp; Lucas</td>
<td>0.064</td>
</tr>
</tbody>
</table>
Conclusion

- Hazard function allows us to directly evaluate question of selection effect, indirectly evaluate models
- Estimate hazard function using new data and moments
- Find hazard function has small extensive margin/selection effect
- Significant asymmetry between price increases and decreases
- Flexible framework to evaluate price flexibility
- Possible next steps/extensions:
  - Relevance of asymmetry for response to shocks
  - Use framework to evaluate imperfect information models
  - Better understand processes that determine price gaps
# Moment Values

## Table: Target Moments

<table>
<thead>
<tr>
<th>Unconditional Moments</th>
<th>Conditional Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Frequency</td>
<td>10.7%</td>
</tr>
<tr>
<td>Avg. Dispersion (IQR)</td>
<td>9.9%</td>
</tr>
<tr>
<td>Avg. Skewness</td>
<td>-0.14</td>
</tr>
<tr>
<td>Avg. absolute price change</td>
<td>7.5%</td>
</tr>
<tr>
<td>Fraction of Small Changes</td>
<td>13.2%</td>
</tr>
<tr>
<td>Avg. Frequency of Increases</td>
<td>7.64%</td>
</tr>
<tr>
<td>Avg. Frequency of Decreases</td>
<td>2.97%</td>
</tr>
<tr>
<td>Avg. Size of Increases</td>
<td>7.2%</td>
</tr>
<tr>
<td>Avg. Size of Decreases</td>
<td>7.9%</td>
</tr>
<tr>
<td>Dispersion of Price Increases</td>
<td>7.7%</td>
</tr>
<tr>
<td>Dispersion of Price Decreases</td>
<td>8.7%</td>
</tr>
</tbody>
</table>
## Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Quadratic</th>
<th>Logistic</th>
<th>Non-Parametric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Frequency</td>
<td>0.107</td>
<td>10.8</td>
<td>0.108</td>
<td>0.107</td>
</tr>
<tr>
<td>Frequency Increases</td>
<td>0.076</td>
<td>0.067</td>
<td>0.068</td>
<td>0.068</td>
</tr>
<tr>
<td>Frequency Decreases</td>
<td>0.030</td>
<td>0.041</td>
<td>0.040</td>
<td>0.039</td>
</tr>
<tr>
<td>Avg. Size</td>
<td>0.075</td>
<td>0.075</td>
<td>0.077</td>
<td>0.077</td>
</tr>
<tr>
<td>Avg. Size Increases</td>
<td>0.072</td>
<td>0.075</td>
<td>0.076</td>
<td>0.076</td>
</tr>
<tr>
<td>Avg. Size Decreases</td>
<td>0.079</td>
<td>0.076</td>
<td>0.080</td>
<td>0.078</td>
</tr>
<tr>
<td>Fraction Small</td>
<td>13.2%</td>
<td>12.4%</td>
<td>13.0%</td>
<td>13.5%</td>
</tr>
<tr>
<td>Corr(Frequency, $\pi$)</td>
<td>0.70</td>
<td>0.91</td>
<td>0.82</td>
<td>0.91</td>
</tr>
<tr>
<td>Corr(IQR, $\pi$)</td>
<td>-0.68</td>
<td>-0.91</td>
<td>0.88</td>
<td>-0.92</td>
</tr>
<tr>
<td>Corr(Skew, $\pi$)</td>
<td>0.36</td>
<td>0.22</td>
<td>31</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Two hazard functions estimated to match moments from one period:
Estimation for Sub-Periods

Two hazard functions estimated to match moments from one period:

(a) 1977-1984
   - Logistic
   - Quadratic
   - Non Parametric

(b) 1985 onwards
   - Logistic
   - Quadratic
   - Non Parametric