Heterogeneous Price Rigidities and Monetary Policy

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May 16, 2019
Introduction

What are the implications of heterogeneity for monetary policy (MP)?

- Normative perspective
- But also positive implications for MP transmission mechanism
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- Previous work:
  - Savers and debtors
  - Incidence of unemployment
  - Income composition
  - Cash holdings heterogeneity
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  - Normative perspective
  - But also positive implications for MP transmission mechanism

- Previous work:
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  - Incidence of unemployment
  - Income composition
  - Cash holdings heterogeneity

- Does heterogeneity in price rigidities across sectors matter?
  - Price stickiness is source of monetary non-neutrality in NK models
  - Price stickiness is known to be heterogeneous across sectors
  - What are the implications for distributional and/or aggregate effects of MP?
This paper

1. New stylized facts (BLS/CEX/ACS data): prices are more rigid in industries...
   - ... selling to richer/more educated households (“expenditure channel”)
   - ... employing richer/more education households (“earnings channel”)
   - Example: services and manufacturing
This paper

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   ▶ ... selling to richer/more educated households ("expenditure channel")
   ▶ ... employing richer/more education households ("earnings channel")
   ▶ Example: services and manufacturing

2. Heterogeneous Agent New Keynesian model with many sectors and household types
   ▶ Quantify the aggregate and distributional implications
   ▶ Consumption of college-educated households is 30% more sensitive
   ▶ Aggregate real effect of a 100bps MP tightening is dampened by 7%
Literature


Literature


**Our contribution:** We document and study a set of novel *earnings* and *expenditure* channels of monetary policy transmission


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**Our contribution:** We study an enriched HANK model with firm and household heterogeneity.
Outline

1. Conceptual framework

2. Data and stylized facts

3. Quantitative analysis
The simple model

- Two periods: $t = 1, 2$
- Two sectors: $s \in \{A, B\}$
- Finite household types $i$ with different sectoral exposures
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- Two sectors: \( s \in \{A, B\} \)
- Finite household types \( i \) with different sectoral exposures

Household \( i \) solves:

\[
\max \sum_{t=1}^{2} \beta^{t-1} U \left( (c_{i,t}^A)^{1-\alpha^i} (c_{i,t}^B)^{\alpha^i} \right)
\]
The simple model

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- Two sectors: \( s \in \{A, B\} \)
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**Household \( i \) solves:**

\[
\max \sum_{t=1}^{2} \beta^{t-1} U[(c_{i,t}^A)^{1-\alpha^i} (c_{i,t}^B)^{\alpha^i}]
\]

subject to

\[c_{i,1}^A + \frac{c_{i,2}^A}{R} + p_1 c_{i,1}^B + p_2 \frac{c_{i,2}^B}{R} = b_{i,1} \frac{\gamma^i(Y_1^A)}{\pi_1} + \gamma^i(Y_2^A) + p_1 \gamma^i(Y_1^B) + p_2 \frac{\gamma^i(Y_2^B)}{R}\]

where \( p_t = \frac{P_t^B}{P_t^A} \) is relative price, \( \alpha^i \) expenditure exposure and \( \gamma^i \) earnings exposure.
Simple perturbation: partial equilibrium

- Consider the general perturbation \( \{dR, dY^A_1, dY^B_1, dp, d\pi^A\} \)
Simple perturbation: partial equilibrium

- Consider the general perturbation \( \{dR, dY_1^A, dY_1^B, dp, d\pi^A\} \)
- Define:

\[
MPC_{i,1} \equiv \frac{\partial}{\partial y_i} p^{\alpha_i} c_{i,1}.
\]
Simple perturbation: partial equilibrium

- Consider the general perturbation \( \{dR, dY^A_1, dY^B_1, dp, d\pi^A\} \)

- Define:

\[
MPC_{i,1} \equiv \frac{\partial}{\partial y_i} p^{\alpha_i} c_{i,1}.
\]

**Proposition:** Household \( i' \)’s behavioral consumption response can be decomposed into

\[
dc_{i,1} = -\frac{1}{\gamma} MPS_{i,1} c_{i,1} \frac{dR}{R} + MPC_{i,1} \left\{ b_{i,2} \frac{dR}{R} - \frac{b_{i,1}}{\pi^A} \frac{dP^A}{P^A} \right\} + \gamma^A_i dY^A_1 + p\gamma^B_i dY^B_1 + \gamma^B_i p \left( Y^B_1 + \frac{1}{R} Y^B_2 \right) \frac{dp}{p} - \alpha_i p^{\alpha_i} \left( c_{i,1} + \frac{1}{R} c_{i,2} \right) \frac{dp}{p} \right\}.\]

Substitution effect  
Interest rate exposure  
Bond revaluation  
Heterogeneous earnings channel  
Relative price effect on real earnings  
Relative price effect on real expenditures
Simple perturbation: general equilibrium

**Proposition:** In response to our proposed aggregate perturbation, the change in aggregate demand can be decomposed as

\[
dY_1 = \left[ \text{Cov}_I \left( \mu \text{MPC}_{i,1}, b_{i,2} \right) - \frac{1}{\gamma} \mathbb{E}_I \left( \mu \text{MPS}_{i,1} c_{i,1} \right) \right] \frac{dR}{R} - \text{Cov}_I \left( \mu \text{MPC}_{i,1}, \frac{b_{i,1}}{\pi_A} \right) \frac{dP_A}{P_A} \\
+ \sum_s \frac{P^s_t}{P^A_t} \left( \mathbb{E}_I(\text{MPC}_{i,1}) + \text{Cov}_I(\mu \text{MPC}_{i,1}, \gamma^s_i) \right) dY_1^s
\]

Heterogeneous earnings effect

\[
+ \sum_t \frac{1}{R^{t-1}} p \left( \mathbb{E}_I(\text{MPC}_{i,1}) + \text{Cov}_I(\mu \text{MPC}_{i,1}, \gamma^B_i) \right) Y^B_t \frac{dp}{p}
\]

Relative price effect on earnings

\[
- \sum_t \frac{1}{R^{t-1}} \mathbb{E}_I \left( \mu \text{MPC}_{i,1} \alpha^i p^{\alpha^i} c_{i,t} \right) \frac{dp}{p}
\]

Relative price effect on expenditures
Outline

1. Conceptual framework

2. Data and stylized facts

3. Quantitative analysis
Data

- Build 3 linked datasets with price rigidities (consumer and producer prices), expenditures and payrolls
  - Covers full U.S. economy (except shelter in most cases)

- CPI-ACS sample:
  - merge price rigidity data from Nakamura and Steinsson (2008) (at the ELI level) to earnings data from the ACS (at the industry level)

- PPI-ACS sample:
  - match price rigidity data from Pasten et al. (2016) (at the 6-digit NAICS level) to ACS industries

- CPI-CEX sample:
  - merge price rigidity data from Nakamura and Steinsson (2008) (at the ELI level) to spending data from the CEX (at the UCC level).
New facts

Two empirical findings:

1. Prices more rigid in product categories selling to more educated/richer households (consistent with Cravino-Lan-Levchenko, 2019)

Examples:
- Services (frequency: 6.39%, share of sales to College: 37.9%)
- Taxi fares (frequency: 4.41%, share of sales to College: 62.3%)
- Fast food lunch (frequency: 7%, share of sales to College: 34.4%)
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Two empirical findings:

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   Examples:
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2. Prices more rigid in product categories employing more educated/richer households

   Examples:
   - Computer electronics (frequency: 28.95%, payroll share to College: 72.15%)
   - Poultry processing (frequency: 35.1%, payroll share to College: 14.43%)
Earnings channel: CPI-ACS

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### Earnings channel: CPI-ACS

<table>
<thead>
<tr>
<th>Frequency of Price Changes (%)</th>
<th>Share of Payroll to College Graduates (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>-0.9330***</td>
<td>-0.463**</td>
</tr>
<tr>
<td>(0.2649)</td>
<td>(0.2119)</td>
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</table>

- Excluding industries with price change frequency > p95:
  - Yes
  - Yes
  - No

- 2-digit Naics Code F.E.:
  - No
  - Yes
  - No

- Sample Size:
  - 86
  - 86
  - 94
Earnings channel: PPI-ACS

Notes: Includes All Prices Changes
### Earnings channel: PPI-ACS

<table>
<thead>
<tr>
<th>Frequency of Price Changes (%)</th>
<th>Share of Payroll to College Graduates (%)</th>
<th>Excluding industries with price change frequency $&gt; p95$</th>
<th>2-digit Naics Code F.E.</th>
<th>Sample Size</th>
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<td>Yes</td>
<td>No</td>
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<td>(2)</td>
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<td>-0.9823***</td>
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<td>(0.2149)</td>
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<td>-0.2027</td>
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<td>(0.1306)</td>
<td></td>
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<tr>
<td></td>
<td>-0.3771*</td>
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<tr>
<td></td>
<td>(0.1978)</td>
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</table>
Expenditure channel: CPI-CEX

Notes: Includes All Prices Changes
### Expenditure channel: CPI-CEX

<table>
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<tr>
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<th>Share of Sales to College Graduates (%)</th>
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<td>Frequency of Price Changes (%)</td>
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<td>(0.0824)</td>
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<tr>
<td>Excluding industries with price change frequency &gt; p95</td>
<td>Yes</td>
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<tr>
<td>Expenditure Category F.E.</td>
<td>No</td>
</tr>
<tr>
<td>Sample Size</td>
<td>242</td>
</tr>
</tbody>
</table>
Interaction between Earnings / Expenditure channels

Notes: OLS Coeff. 0.5416*** (s.e. 0.2264), N=88
New facts

- Implications for monetary policy tightening:
  - NK model prediction for sector with more rigid prices: less deflation, but bigger output gap
  - More educated households suffer more: preferred goods relatively more expensive, stronger labor demand contraction
  - Feedback loop on consumption of more educated households: demand for goods in more rigid sector falls even more (→ relative price, labor demand)
  - Monetary policy has relatively larger effect on richer, low-MPC households → dampened aggregate effect
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  - Monetary policy has relatively larger effect on richer, low-MPC households $\rightarrow$ dampened aggregate effect

- Robustness
  - Excluding sales
  - Different measures of income and education
  - Broad sector fixed effects (e.g. within goods)
Outline

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2. Data and stylized facts

3. Quantitative analysis
Model overview

- Start from one-asset heterogeneous-agent New Keynesian model
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- Four intermediate goods sectors $s$
  - Different price rigidity: $\delta^s$
  - Sectors employ two types of workers: $N_{C,t}^s$ and $N_{NC,t}^s$
  - Each sector has its own, fully segmented labor market (business cycle frequency)
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- Two household types $i \in \{C, NC\}$: college and non-college
  - Within type heterogeneity: uninsurable earnings risk (standard incomplete markets model)
  - Different sector-specific productivities: $Z^s_e$ (equivalent to $\gamma_i$ in simple model)
  - Different tastes: $\alpha^s_C$ and $\alpha^s_{NC}$

Policy experiment: contractionary 100bps monetary policy shock
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- Policy experiment: contractionary 100bps monetary policy shock
Model details

- CES consumption baskets

\[ c_{i,t} = \left[ \sum_{s}^{N} \left( \alpha_{i}^{s} \frac{1}{\eta} \left( c_{i,t}^{s} \right)^{\frac{n-1}{\eta}} \right) \right]^{\frac{n}{\eta-1}} \]

- Household budget constraint (assumptions on profit rebate important)

\[ \dot{a}_{i,t} = (i^{N} - \pi^{N})a_{i,t} + z_{i,t}n_{i,t}w_{i,t}p_{i,t} + \tau_{i,t}p_{i,t} - c_{i,t}p_{i,t}, \quad a_{i,t} \geq a \]

  - Interest income
  - Labor income
  - Transfer income
  - Consumption

- Intermediate goods producer production function

\[ Y^{s}_{t}(j) = \left[ \sum_{e \in C, NC} (Z^{s}_{e}) \frac{1}{\kappa} N^{s}_{e,t}(j)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}} \]

- Two Phillips Curves (under Rotemberg pricing)

\[ \dot{\pi}^{s}_{t} = \pi^{s}_{t} \left( i_{t} - \pi^{s}_{t} - \frac{Y^{s}_{t}}{Y^{s}_{t}} \right) - \frac{\epsilon - 1}{\delta^{s}} \left( \frac{\epsilon}{\epsilon - 1} MC^{s}_{t} - 1 \right) \]

- HJB
- Taylor rule
- Kolmogorov forward equation
- Channel decomposition
Calibration strategy

- Heterogeneous expenditure shares: $\alpha_i^s$
- Heterogeneous sectoral skill intensities: $Z_e^s$
- Heterogeneous sectoral price stickiness: $\delta^s$
Summary of quantitative exercise

- Consider two cases: baseline (homogeneous price rigidities) and full model

- Cross-sectional effect: Compute distributional effects between C and NC as

\[ \Delta = \frac{\Delta C^C}{C^C_{SS}} - \frac{\Delta C^{NC}}{C^{NC}_{SS}}, \]

then difference full model from baseline, \( \Delta - \Delta^{baseline} \) and normalize by aggregate consumption response

- Aggregate effect: change in aggregate consumption response in full model relative to baseline

\[ \frac{\Delta C}{\Delta C^{baseline}} \]
Summary of quantitative exercise: cross-sectional
Summary of quantitative exercise: aggregate

![Graph showing the change in aggregate consumption response relative to baseline](image-url)
Conclusion

- This paper re-evaluates the implications of heterogeneous price stickiness for the transmission and the distributional effects of monetary policy.

- Establish new facts using micro data:
  1. Richer/more educated households purchase in more rigid sectors.
  2. Richer/more educated households work in more rigid sectors.

- Quantitative model to assess implications of these new facts:
  - Real effects of MP dampened in the presence of heterogeneous price stickiness.
  - Consumption of college households 30% more sensitive to MP shocks.
  - Aggregate effects of monetary policy muted by 5 - 10% due to novel earnings and expenditure channels.
Table 1: Parameters for Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>Curvature of (relative) labor supply curve</td>
<td>1.5</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$\theta_C$</td>
<td>$P(\text{Non-College}</td>
<td>\text{College})$</td>
<td>0.45/35</td>
</tr>
<tr>
<td>$\theta_{NC}$</td>
<td>$P(\text{College}</td>
<td>\text{Non-College})$</td>
<td>0.22/35</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Elasticity of substitution between intermediates</td>
<td>11</td>
<td>Basu and Fernald (1997)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>CRRA for upper-level utility function</td>
<td>1.5</td>
<td>N/A</td>
</tr>
<tr>
<td>$1 - \alpha^{NC}_A$</td>
<td>Non-college spending in A</td>
<td>41.5%</td>
<td>CEX</td>
</tr>
<tr>
<td>$1 - \alpha^{C}_A$</td>
<td>College spending in A</td>
<td>58.5%</td>
<td></td>
</tr>
<tr>
<td>$Z^{NC}_A$</td>
<td>Non-college prod in A</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>$Z^{C}_A$</td>
<td>College prod in A, normalized</td>
<td>1.14</td>
<td>QCEW</td>
</tr>
<tr>
<td>$Z^{NC}_B$</td>
<td>Non-college prod in B</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>$Z^{C}_B$</td>
<td>College prod in B</td>
<td>0.66</td>
<td></td>
</tr>
<tr>
<td>$\delta^A$</td>
<td>Price adj. cost in A</td>
<td>190</td>
<td>Nakamura and Steinsson (2008)</td>
</tr>
<tr>
<td>$\delta^B$</td>
<td>Price adj. cost in B</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
Baseline with 1 household type, 1 sector
Introducing sectoral price rigidity heterogeneity

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Comparison calibration: *add symmetric productivity differences*
Full calibration: asymmetric productivity differences and tastes
Differenced IRFs (Full – Comparison)

Household Consumption (% ss)

Labor Supply Sector A (% ss)

Labor Supply Sector B (% ss)

Sectoral PPI Inflation (\%)

Sectoral Production (% ss)

Sectoral Wages (College) (% ss)

Relative Price (% ss)

Nominal Interest Rate (% l)

Real Interest Rate (% l)

- Back.

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Heterogeneous Price Rigidities and Monetary Policy 6 / 13
Marginal propensities to consume (MPC)
Asset holdings and borrowing constraints
Disposable income and its decomposition

Disproportionate income (College) (% ss)

Disproportionate income (Non-College) (% ss)

Disproportionate (College) (d ss)

Disproportionate (Non-College) (d ss)

Back.
Households’ recursive optimization problem

- We collect households’ state variables in the vector $x_{i,t}$ with law of motion

$$
\begin{pmatrix}
(da_{i,t}) \\
(dz_{i,t})
\end{pmatrix} = \begin{pmatrix}
rt a_{i,t} + \sum_s z_{i,t}^s n_{i,t}^s w_{i,t}^s p_t^{\alpha^i} - p_t^{\alpha^i} c_{i,t} + \frac{T_{i,t}}{P_t^A} \\
\mu(z_{i,t}^s)
\end{pmatrix} dt + \begin{pmatrix}
0 \\
\sigma(z_{i,t}^s)
\end{pmatrix} dB_t.
$$

- This gives us the recursive, continuous-time Bellman equation

$$\rho v_{i,t}(x_{i,t}) = \partial_t v_{i,t}(x_{i,t}) + \max_{c_{i,t},n_{i,t}} u(c_{i,t}, n_{i,t}) + \theta_i \left( v_{-i,t}(x_{-i,t}) - v_{i,t}(x_{i,t}) \right)$$

$$+ \partial_a v_{i,t}(x_{i,t}) \left( r_t a_{i,t} + \sum_s z_{i,t}^s n_{i,t}^s w_{i,t}^s p_t^{\alpha^i} - p_t^{\alpha^i} c_{i,t} + \frac{T_{i,t}}{P_t^A} \right)$$

$$+ \mu(z_{i,t}^s) \partial_z v_{i,t}(x_{i,t}) + \frac{1}{2} \sigma(z_{i,t}^s)^2 \partial_{zz} v_{i,t}(x_{i,t})$$

- FOCs:

$$c_{i,t}^{-\gamma} = p_t^{\alpha^i} \partial_a v_{i,t}(x_{i,t})$$

$$c_{i,t}^{\gamma}(n_{i,t}^s)^{\phi} = z_{i,t}^s w_{i,t}^s.$$
The Taylor rule

- Assumptions on the Taylor rule are important
- For now, we assume equal weighting:

\[ i_t = i_t^* + \sum_s \left( \phi_s \pi_t^s + \phi_y (Y_t^s - Y) \right) + \xi_t, \]  

(1)
Aggregation in our model

- We write Kolmogorov forward (KF) equations separately for each household type.

- The KF equations characterizing the evolution of these density functions are given by

\[
\partial_t g_{i,t}(x_{i,t}) = - \partial_a \left( \left[ r_t a_{i,t} + \sum_s z_{i,t}^s n_{i,t} w_{i,t}^s p_{i,t}^{\alpha_i} - p_{i,t}^{\alpha_i} c_{i,t} + \frac{T_{i,t}}{P_{A,t}} \right] g_{i,t}(x_{i,t}) \right) \\
- \partial_z \left( \mu(z_{i,t}^s) g_{i,t}(x_{i,t}) \right) + \frac{1}{2} \partial_{zz} \left( \sigma(z_{i,t}^s)^2 g_{i,t}(x_{i,t}) \right) \\
- \theta_i g_{i,t}(x_{i,t}) + \theta_{-i} g_{-i,t}(x_{-i,t}).
\]
Channel decompositions

- Consider a perturbation $\{\xi_t\}$ that corresponds to a 100bps MP shock.
- We can decompose the effect on consumption as follows.

For College:

$$C_{C,0} (\{r_t, w_{C,t}, p_{C,t}, T_{C,t}\}_{t \in [0, \infty)}, g_0)$$

$$= \int_0^\infty \int_{\mathbb{Z}} c_C (a, z, \{r_t, w_{C,t}, p_{t}, T_{C,t}\}_{t \in [0, \infty)}) g_0 d(z, a)$$

$$dC_{C,0} = \int_0^\infty \frac{\partial C_{C,0}}{\partial r_t} dr_t + \frac{\partial C_{C,0}}{\partial w_{C,t}} dw_{C,t} + \frac{\partial C_{C,0}}{\partial p_{t}} dp_{t} + \frac{\partial C_{C,0}}{\partial T_{C,t}} dT_{C,t} dt$$