Expectations Formation, Sticky Prices, and the ZLB\textsuperscript{1}

Betsy Bersson, Patrick Hürtgen, Matthias Paustian

FRB, Bundesbank

May 15, 2019

\textsuperscript{1}These views are those of the authors, and not necessarily those of the Board of Governors of the Federal Reserve System, or of the Deutsche Bundesbank.
Introduction

- unusual macro phenomena occur at the ZLB with RE
  1. fiscal multipliers can be extremely large
  2. small delay in ELB liftoff can be very effective policy tool
  3. delaying liftoff for too many quarters can be a disaster
  4. to stabilize economy at ELB, may raise nominal rate

- expectations are crucial for all of these results
- we study these issues in a simple model of belief formation
Departures from RE: level-k thinking

- follow framework of Farhi and Werning (2018)
- framework is similar to Garcia-Schmitt and Woodford (2017)
- concept is called level-k thinking
- this is a plausible model for the process of belief revision
- mimics what might be going on in people’s mind
- expectations formed iteratively
- start from “partial” equilibrium effects of policy interventions
- add future general equilibrium effects imperfectly
The macro questions we address

- Use a model of bounded rationality to study
  1. optimal monetary policy at the ZLB
  2. effects of a delayed liftoff from the ZLB
  3. the fiscal multiplier under a transient peg
**Temporary equilibrium**

**Definition**

A temporary equilibrium in period $t$ is a collection of choices such that

1. given beliefs about future variables, household and firms optimize
2. goods, labor and asset markets clear
3. budget constraints for all agents are satisfied

A temporary equilibrium in period $t$ is the outcome of a mapping from beliefs $\{B_{t+j}\}_{j=1}^{\infty}$ about relevant variables $\{X_{t+j}\}_{j=1}^{\infty}$ into equilibrium values $X_t$ that satisfies the assumptions above.
**Temporary equilibrium, level-k equilibrium**

Temporary equilibrium

\[ X_t = \Phi \left( \{B_{t+j}\}_{j=1}^{\infty}, X_{t-1} \right) \]  

(1)

**Definition**

A level-\( k \) equilibrium is a temporary equilibrium where beliefs \( \{B_{t+j}\}_{j=1}^{\infty} \) are given by the level \( k-1 \) equilibrium sequences for \( \{X_{t+j}\}_{j=1}^{\infty} \). These are generated recursively given an initial belief

\[ X_t^k = \Phi \left( \{X_{t+j}^{k-1}\}_{j=1}^{\infty}, X_{t-1}^k \right). \]  

(2)
Further assumptions

- agents know the correct structure of the economy
- perfect foresight about exogenous government policies
- initial beliefs: RE equilibrium prior to policy intervention
- allows for fair comparison with RE (same baseline outlook)

What level of belief revision $k$ ?

- experimental evidence, $k \leq 3$ (Mauersberger et al. 2018)
- survey of firm managers $k < 5$ (Coibion et al. 2018)
Baseline New Keynesian model

For arbitrary expectations, the equilibrium conditions are

$$ y_t = E_t \sum_{s=0}^{\infty} \beta^s [(1 - \beta)y_{t+1+s} - \frac{1}{\sigma} (i_{t+s} - r_{t+s} - \pi_{t+1+s})] $$

(3)

$$ \pi_t = E_t \sum_{s=0}^{\infty} (\beta \phi)^s [\beta (1 - \varphi) \pi_{t+1+s} + \kappa y_{t+s}] $$

(4)

where $\pi_t$ is inflation, $y_t$ is output, $i_t$ is nominal interest rate

Under RE we use law of iterated expectations to obtain:

$$ \pi_t = \kappa y_t + \beta E_t \pi_{t+1} $$

(5)

$$ y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - r_t - E_t \pi_{t+1}) $$

(6)
Part I: Optimal Policy at the ZLB

- assume a natural rate shock \( r_t \) with persistence \( \rho = 0.9 \).
- benchmark Taylor rule: \( i_t = \max(r_t + 1.5\pi_t + 0.5y_t, ZLB) \)
- prices are fixed on average for about 7 quarters \( \varphi = 0.85 \)
- other parameters also standard

Compute optimal policy with commitment

- central bank loss function \( \mathcal{L} = \sum_{j=0}^{\infty} \beta^j \left( \pi_{t+j}^2 + \alpha y_{t+j}^2 \right) \)
- CASE 1: welfare based weight \( (\alpha \sim 0.0025) \)
- CASE 2: equal weight \( (\alpha = 1) \)
The benchmark outcome under a Taylor rule

**interest rate**

-1.5 -1 -0.5 0 0.5
5 10 15 20 25 30

**output gap**

-15 -10 -5 0
5 10 15 20 25 30

**inflation**

0 0.5
5 10 15 20 25 30

**real rate gap**

0 1 2 3
5 10 15 20 25 30
Optimal policy with rational expectations
Model under level-k thinking

Recursions for updating beliefs:

\[
y^k_t = \sum_{s=0}^{\infty} \beta^s \left[ (1 - \beta)y^{k-1}_{t+1+s} - \frac{1}{\sigma} (i_{t+s} - r_{t+s} - \pi^{k-1}_{t+1+s}) \right]
\]

\[
\pi^k_t - \kappa y^k_t = \sum_{s=0}^{\infty} (\beta \varphi)^s \left[ \beta (1 - \varphi) \pi^{k-1}_{t+1+s} + \kappa \beta \varphi y^{k-1}_{t+1+s} \right]
\]

- initialize beliefs at RE under baseline Taylor rule
- this initialization makes for a fair comparison with RE
- policymaker knows how private sector forms belief and set policy optimally under commitment
Optimal policy with level 1 and 3 thinking
Discussion of CASE 1

- both settings give qualitatively similar results
- prescription is to delay liftoff relative to Taylor rule
- improvements in $\pi$ and $y$ smaller under level-k
- required stay at the ZLB is longer than under RE
- “central bankers need to work harder”
- consistent with Nakata (2018) work on ueber discounting
CASE 2: equal weights, same Taylor rule benchmark

- **interest rate**
  - 0 to -1 over 30 periods

- **output gap**
  - 0 to -15 over 30 periods

- **inflation**
  - 0 to 0.5 over 30 periods

- **real rate gap**
  - 3 to 0 over 30 periods
Optimal policy under RE with equal weights

- Interest rate
- Output gap
- Inflation
- Real rate gap

Optimal and baseline trajectories for the policy variables under RE with equal weights.
optimal policy has a neo-Fisherian flavor
liftoff from ELB occurs earlier than under Taylor rule
nominal rate is higher post liftoff than under simple rule
but real interest rate is lower
lower real rate stimulates demand and raises inflation
higher expected inflation raises nominal rate
prescription at odds with policy seen in practice
instantaneous move in inflation expectations crucial
Optimal policy under level 1-3 thinking (equal weights)
Discussion

- optimal policy stays at ZLB "lower for longer"
- delay in liftoff date not very sensitive to level $k$
- no neo-Fisherian feature
- macro outcomes worse than under RE
- worsening is substantial for low level of $k$
Convergence to rational expectations for high $k$?

- yes, level-$k$ converges to RE for high $k$
- that was surprising to me - why?
- RE interest rate path is above Taylor rule
- any uniformly higher rate path achieves worse outcomes
- but .... under RE, interest rate is below baseline in $T = 100$
- monetary easing in final period is tiny, but crucial
- Neo-Fisherian feature is entirely Keynesian
Part II: The reversal puzzle

- Adding indexation to past inflation to the model
- Under arbitrary expectations, Phillips curve given by

\[
p_t^* + \pi_t = (1 - \beta \varphi) E_t \sum_{s=0}^{\infty} (\beta \varphi)^s [\pi_{t+s} + (\omega + \sigma^{-1}) y_{t+s}]
\]

\[
p_t^* = \frac{\varphi}{1 - \varphi} (\pi_t - \pi_{t-1}),
\]

- \(p_t^*) is price of adjusting firms relative to aggregate price index
- with RE this amounts to well known Phillips curve

\[
\pi_t = \frac{1}{1 + \beta} \pi_{t-1} + \frac{\beta}{1 + \beta} E_t \pi_{t+1} + \frac{1}{1 + \beta} \kappa y_t
\]  

(7)
The policy experiment - time dependent forward guidance

- adverse demand shock drives the economy to ZLB
- stay at ZLB for $t^*$ periods under Taylor rule
- delay liftoff for additional $k$ periods
- return to Taylor rule from period $t^* + k + 1$ onwards
- central bank announces the following interest rate rule:

$$i_t = \begin{cases} 
ZLB & t = 1, 2, ..., t^* \cdot t^{*+1}, ..., t^{*+k} \\
\max(ZLB, r_t + \phi_\pi \pi_t + \phi_y y_t) & t \geq t^{*+k+1}
\end{cases}$$

- calibration: $\phi_\pi = 1.5$ and $\phi_y = 0.5$
- initial innovation is $r_1 = -0.015$
Reversal puzzle

- stay at ZLB for 5 quarters under Taylor rule (blue)
- delay liftoff by 1,2,3 quarters
Reversal puzzle

- common sense says that delay in liftoff should be stimulative
- this is true for short delay by 1 and 2 quarters
- delay by 3 quarters is contractionary!
- hence ZLB is binding for longer, have to delay by 5 quarters
- see in Carlstrom, Fuerst, Paustian JME (2015)
- would the same reversal occur with level-\(k\) thinking?
Level-2 thinking

delayed liftoff by 1, 2, and 3 quarters:

qualitatively similar results for higher levels
Discussion

- no reversal puzzle with level-k thinking
- level-$k$ converges to RE for small delay (conventional case)
- level-k does not converge to RE for large delay (perverse case)
- macro effect of delay grows without bound as $k$ increases
- bottom line: can discard reversal puzzle as implausible
- feature not shared by bounded rationality even in the limit
Part III: Fiscal multiplier with constant interest rates

- Fiscal multiplier can be large under an interest rate peg.
- Mechanism is well understood, but magnitude is surprising.
  - In Christiano et al. (2011) simple model fiscal multiplier is 3.7.
  - Same parametrization but with separable preferences it is 4.9.
- How sensitivity is the multiplier to bounded rationality (BR)?

Answer:
- When RE multiplier is huge, BR multiplier very different.
- When RE multiplier is modest, BR multiplier is more similar.
The experiment

We follow Christiano et. al (2011), use three equation NK model

- model a joint monetary-fiscal expansion regime
- i.e. regime with fixed rates and high government spending
- \( p \) is the probability of being in an expansion regime
  - regime continues with probability \( p \)
  - exits with probability \( 1 - p \) into an absorbing state
  - after exit, we are back in steady state
  - expected duration the regime is \( T = \frac{1}{1-p} \)
- \( p < p^* \) : determinate case ("fundamental equilibrium")
- \( p > p^* \) : indeterminate case ("expectations driven trap")
The model

Under arbitrary expectations, model is given by

\[ c_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s p^{s+1} [(1 - \beta)c_{t+1+s} - \frac{1}{\sigma}(i_{t+s} - \pi_{t+1+s})] \]

\[ \pi_t = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \varphi)^s \left[ p^{s+1} \beta (1 - \varphi) \pi_{t+1+s} + p^s \kappa (\sigma c_{t+s} + \omega^{-1} y_{t+s}) \right] \]

\[ y_t = (1 - s) c_t + sg_t \]

Under RE we have:

\[ i_t - p\mathbb{E}_t \pi_{t+1} = -\sigma (c_t - p\mathbb{E}_t c_{t+1}) \]

\[ \pi_t = \beta p\mathbb{E}_t \pi_{t+1} + \kappa mc_t \]

\[ mc_t = \sigma c_t + \omega^{-1} y_t \]

\[ y_t = (1 - s) c_t + sg_t \]
**Rational expectations multiplier**

Under RE, fiscal multiplier during the peg is given by:

\[
\frac{dY}{dG} \equiv \left( \frac{1}{s} \right) \frac{dy_t}{dg_t} = \left[ \frac{\sigma [(1 - p) (1 - \beta p) - \kappa p]}{\Delta} \right]
\]

where

\[
\Delta \equiv \sigma (1 - p) (1 - \beta p) - \kappa [\sigma + \omega^{-1} (1 - s)] p.
\]

Unique stable equilibrium whenever \( \Delta > 0 \).

\( \beta = 0.99, \kappa = 0.028, \omega^{-1} = 0.5, \sigma = 2, s = 0.2 \) and \( p = \frac{5}{6} \).

Under this calibration, fiscal multiplier is 4.9.
Fiscal Multiplier under level-$k$ thinking: $p = \frac{5}{6}$

<table>
<thead>
<tr>
<th>level-k</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>multiplier</td>
<td>1</td>
<td>1.03</td>
<td>1.2</td>
<td>1.46</td>
<td>3.02</td>
<td>4.02</td>
<td>4.7</td>
<td>4.9</td>
</tr>
<tr>
<td>(% of RE)</td>
<td>(20)</td>
<td>(20)</td>
<td>(24)</td>
<td>(30)</td>
<td>(62)</td>
<td>(82)</td>
<td>(96)</td>
<td>(100)</td>
</tr>
</tbody>
</table>

- convergence to RE is extremely slow
- when RE multiplier is huge, BR multiplier is plausible
Fiscal Multiplier under level-\( k \) thinking: \( p = 0.8 \)

<table>
<thead>
<tr>
<th>level-k</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>multiplier</td>
<td>1</td>
<td>1.02</td>
<td>1.14</td>
<td>1.24</td>
<td>1.3</td>
</tr>
<tr>
<td>(% of RE)</td>
<td>(77)</td>
<td>(78)</td>
<td>(88)</td>
<td>(95)</td>
<td>(100)</td>
</tr>
</tbody>
</table>

- convergence to RE is much faster
- when RE multiplier is plausible, BR multiplier is similar
An expectations driven trap

- Mertens and Ravn (2014) focus on expectations driven trap
- use non-linear NK model, but nonlinearity is really minor
- when $p > p^*$, model is indeterminate
- sunspot equilibria (pessimism) can bring us to ZLB
- how large is fiscal multiplier in expectations driven trap?
- Mertens and Ravn (2014) focus on MSV solution under RE
- under level-k, equilibrium unique for given initial belief
Fiscal multiplier as a function of $p$

Figure: Fiscal multiplier as a function of the probability $p$
Conclusion

Expectations formation matter! Level-$k$ thinking does...

- ... support *lower for longer* strategies at ZLB
- ... not support neo-Fisherian strategies
- ... not support worries about reversals
- ... suggest fiscal multipliers at ZLB only mildly $>1$
- ... not suggest fiscal policy less effective in expectations trap