Short & Long Run impact of volatility on the effect monetary shocks

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Output’s impulse response $Y(t; \delta)$ to once-for all monetary shock $\delta$, cumulative IRF $\mathcal{M}(\delta)$.

Sticky price models w/ idiosyncratic shocks w/ variance $\sigma^2$ & w/o strategic complementarity:
- Menu costs: deterministic and random
- Single and Multi-product firms

Effect on $Y(t; \delta)$ and $\mathcal{M}(\delta)$) of permanent changes on idiosyncratic volatility $\sigma$: on impact and in the long run.

Long vs Short run, difference across models.

Help interpret state dependent or time varying VARs.
Price setting (ours) vs investment responses (traditional).
Once and for all monetary shock at $t = 0$. 

\[
\delta
\]

Percent deviation from steady state

time $t$

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Volatility & Monetary Shocks

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Aggregate shock hits x-section of desired adjustments

Price gap: $x$

Density function $p(x, 0)$
Impulse response of Price Level

\[ P(t) \]

\( \delta \)

percent deviation from steady state

0 0.2 0.4 0.6 0.8 1

0 5 10 15 20

time \( t \)
Impulse response of Prices and Output

\[ Y_t = \delta - P(t) \]

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Impulse response of Prices and Output

\[ Y_t = \delta - \mathcal{P}(t) \]
Price Setting Models

Lifetime Utility:
\[ \int_0^\infty e^{-rt} \left( \log c(t) - \alpha \ell(t) + \log \frac{M(t)}{P(t)} \right) dt \]

CES aggregate:
\[ c(t) = \left( \int_0^1 \sum_{i=1}^n \left( Z_{ki}(t) c_{ki}(t) \right)^{1-\frac{1}{n}} dk \right)^{\frac{n}{n-1}} \]

- Linear technology: \( c_{ki}(t) = \ell_{ki}(t) / Z_{ki}(t) \) and \( Z_{ki}(t) = \exp(\sigma W_{ki}(t)) \).
- \( W_{ki}(t) \) independent idiosyncratic Brownian motions.
- Equilibrium: constant nominal interest rate & wages \( W(t) \propto M(t) \).
- simultaneous adjustment of $n$ products, subject to random menu cost:

\[
\text{adjust } n \text{ prices paying } = \begin{cases} 
\psi & \text{with probability } 1 - \zeta \, dt \text{ or } \\
0 & \text{with probability } \zeta \, dt.
\end{cases}
\]

- Examples: Golosov-Lucas, Calvo, Calvo$^+$, Midrigan & extensions.

- Decision rule: change prices to ideal value when deviation of prices relative to (norm of) ideal price (vector) $x$ first hit $\bar{x}$ or when cost is free.

- Kurtosis $Kurt(Dp)$ & fraction free adjustment: $\frac{\zeta}{N(Dp)}$ depend on Calvo-ness $\phi = \frac{\zeta \bar{x}^2}{\sigma^2}$ and $n$

- “Sufficient statistic”: $\mathcal{M}(\delta) = \frac{\delta}{6} \frac{Kurt(Dp)}{N(Dp)}$ & shape of $Y(t; \delta)$ depend on Calvo-ness $\phi$ and $n$ for given $N(Dp)$.

- $Kurt(Dp) \& N(Dp)$ depends on structural parameters, among them $\sigma^2$. 
Set up for $\sigma$ and $\delta$ shocks (two MIT shocks)

- At $t = 0$ in steady state for $\sigma_0$
- Once and for all permanent change to $\sigma_1 > \sigma_0$.
- After $\tau > 0$ unexpected permanent monetary shocks $\delta$
  - **Shot run**: monetary shock occurs at $\tau = 0$, i.e. at the old steady state for $\sigma_0$, but new decisions rules corresponding to $\sigma_1$. Impact effect.
  - **Long Run**: monetary shock occurs at $\tau = \infty$, i.e. at new steady state for $\sigma_1$, and new decision rules corresponding to $\sigma_1$. Comparative static.
  - **Average speed of convergence**: from initial distribution with $\sigma_0$ to final distribution with $\sigma_1$.

- **Two forces**:
  - Effect of $\sigma$ on decision rules (barriers $\bar{x}$), and speed within barriers, which dominates.(same in SR and LR).
  - Effect of $\sigma$ on the initial distribution (different in SR and LR).
Long Run effect on IRF, Golosov-Lucas $n = 1$

Impulse response of output to monetary shock, $\sigma_0 < \sigma_1$

Long run
Short Run effect on IRF, Golosov-Lucas $n = 1$

Impulse response of output to monetary shock, $\sigma_0 < \sigma_1$

short run

$\text{Short run } (\sigma_0, \sigma_1)$
$\text{Long Run } (\sigma_0)$
Impulse response Golosov-Lucas $n = 1$

- **Long Run:**

$$Y(t; \sigma_1) = Y \left( t \frac{\sigma_1}{\sigma_0}; \sigma_0 \right) \quad \text{all } t \geq 0$$

$$M(\sigma_1) = \frac{\sigma_0}{\sigma_1} M(\sigma_0)$$

- **Short Run:**

$$Y(t; \sigma_0, \sigma_1) \approx Y \left( t \frac{\sigma_1}{\sigma_0}; \sigma_0 \right) \frac{\sigma_1}{\sigma_0} \quad \text{all } t \geq 0$$

$$M(\sigma_0, \sigma_1) \approx M(\sigma_0)$$

- **Expected time to converge:**

$$\frac{2}{\pi^2 N(Dp; \sigma_1)} \approx \frac{1}{5 N(Dp; \sigma_1)}$$
Higher $\sigma$ increases barriers $\bar{x}$, but less so that the increase in $\sigma$, so barriers are reached sooner. This decreases effect on output.

The previous point means that $N(Dp)$ increases with $\sigma$.

Higher barriers makes invariant distribution wider (in the long run).

In the short run, the initial distribution of gaps $x$ is narrower than new stationary distribution, so that it has fewer price increases. This increases the effect on output.

Short run, has two effect on opposite direction, twisting IRF $Y$, keeping same $M$.

Long run has only one effect, shifting inwards IRF $Y$, lowering $M$. 
Long run effect on cumulative IRF, different models

Elasticity of $M$ (=cumulative IRF to monetary shock) w.r.t. $\sigma$

Fraction of free adjustments $\ell = \zeta/N$
Long run effect on cumulative IRF, different models

Elasticity of $\mathcal{M}$ (=cumulative IRF to monetary shock) w.r.t. $\sigma$

$\mathcal{M}$ w.r.t. $\sigma$

Kurtosis of price changes $Kurt(Dp)$
Economies where, for given $N(Dp)$, monetary shocks have larger effects, have smaller sensitivity to shocks to volatility $\sigma$.

Kurtosis $Kurt(Dp)$ is increasing in “Calvo-ness” for each $n$, and increasing in $n$ for each level of “Calvo-ness”.

Main effect of increase in $\sigma$ is to increase $N(Dp)$. Price setting runs “faster”. Also it weakly decreases "Calvo-ness", but this effect is always dominated by the “speed".