Rational Sunspots

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1The views expressed are solely the responsibility of the authors and should not to be interpreted as reflecting the views of Sveriges Riksbank.
Introduction

In this paper:
We propose a generalization of the Rational Expectations framework to estimate temporary unstable paths

Premise
RE generally implies multiple solutions

- Explosive
- Stable

How can we get uniqueness? (Sargent and Wallace, 1973; Phelps and Taylor, 1977; Taylor, 1977; Blanchard, 1979)

Stability Criterion: Transversality conditions
In saddle paths dynamics only one solution is stable

This became the standard in Macroeconomics (Blanchard and Kahn, 1980)
Example: U.S. Great Inflation period

Is it appropriate to rule out unstable paths from the empirical analysis?

Is there any evidence that inflation is described (at least for a while) by unstable equilibria?

Figure: CPI inflation, quarterly data. Sample: 1960Q1 - 1997Q4
Generalization of the RE framework: Rational Sunspots

A novel way to introduce sunspots in a RE model to take into account the possibility of unstable paths.

Drifting parameters and stochastic volatility.

Develop an econometric strategy suited for our framework to verify if unstable paths are empirically relevant.

Application: Example of U.S. Great Inflation (LS model and data). U.S. inflation dynamics in the 70’s are better described by unstable rational equilibrium paths. Unstable paths can be empirically relevant, also within the context of RE.
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- A novel way to introduce sunspots in a RE model to take into account the possibility of unstable paths
1 Generalization of the RE framework: Rational Sunspots

- A novel way to introduce sunspots in a RE model to take into account the possibility of unstable paths
- Drifting parameters and stochastic volatility
Outline Paper / Talk

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   - A novel way to introduce sunspots in a RE model to take into account the possibility of unstable paths
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   - U.S. inflation dynamics in the 70’s are better described by unstable rational equilibrium paths
   - Unstable paths can be empirically relevant, also within the context of RE
A simple example: multiple RE solutions

Consider the following model inspired by Cochrane (2011), including the Fisher equation (1) and the Taylor rule (2):

\[ i_t = r + E_t \pi_{t+1} \]  
\[ i_t = r + \phi \pi_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \]  
\[ \pi_t = \frac{1}{\phi} E_t \pi_{t+1} + e_t, \quad e_t \sim i.i.d. N(0, \sigma^2_e) \]

Equation (3) has an infinite number of solutions:

\[ E_t \pi_{t+1} = \phi \pi_t - \phi e_t \Rightarrow \pi_{t+1} = \phi \pi_t - \phi e_t + \eta_{t+1} \]

where \( E_t \eta_{t+1} = 0. \)
Multiple RE solutions

Muth (1961) and Blanchard (1979): UCM $\Rightarrow$ All the solutions for $\pi_t$ are described by

$$
\pi_t = \sum_{j=1}^{\infty} \phi^j (b - 1) e_{t-j} + be_t + \sum_{j=1}^{\infty} \frac{b}{\phi^j} E_t e_{t+j} \\
\pi_t = \phi \pi_{t-1} - \phi e_{t-1} + be_t
$$

- Degree of freedom: the solution is parameterized by $b \in (-\infty, +\infty)$
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(4)

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- Easy to recognize two particular cases:
  - "pure" forward-looking ($b = 1$)
    $$
    \pi_t^F = \sum_{j=0}^{\infty} \left( \frac{1}{\phi} \right)^j E_t e_{t+j} = e_t
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- Degree of freedom: the solution is parameterized by $b \in (-\infty, +\infty)$
- Easy to recognize two particular cases:
  - "pure" forward-looking ($b = 1$)
    $$\pi^F_t = \sum_{j=0}^{\infty} \left( \frac{1}{\phi} \right)^j E_t e_{t+j} = e_t$$
  - "pure" backward-looking solution ($b = 0$)
    $$\pi^B_t = - \sum_{j=1}^{\infty} \phi^j e_{t-j} = \phi \pi^B_{t-1} - \phi e_{t-1}$$
The interpretation for $b$

- All the solutions can be written as a linear combination of the forward and the backward one (Blanchard, 1979):

$$\pi_t = (1 - b)\pi_t^B + b\pi_t^F$$
The interpretation for $b$

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- For $b \neq 0$, the expected value is an exponentially weighted average of the past observations (Muth, 1961)

\[ E_t\pi_{t+1} = (b - 1)\sum_{i=1}^{\infty} \left( \frac{\phi}{b} \right)^i \pi_{t+1-i} \]

*Natural interpretation for $b$: the way people form expectations*
The interpretation for $b$

- All the solutions can be written as a linear combination of the forward and the backward one (Blanchard, 1979):

$$\pi_t = (1 - b)\pi_t^B + b\pi_t^F$$

- For $b \neq 0$, the expected value $=$ an exponentially weighted average of the past observations (Muth, 1961)

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**Natural interpretation for $b$: the way people form expectations**

- $b$ defines the importance the agents give to the past data, both in *absolute* terms ($b$ vs 1), and in *relative* terms.

Infinite solutions $=$ infinite way we can set that weights $\Rightarrow$ how to choose?
The stability criterium (e.g., Blanchard, 79)

\[ \pi_t = \phi \pi_{t-1} - \phi e_{t-1} + b e_t \]

Is the stability criterium sufficient to identify a unique path?

1. If \( \phi > 1 \quad YES \quad \) determinacy, by imposing \( b = 1 \) = f.l. solution
2. If \( \phi < 1 \quad NO \quad \) indeterminacy

=> "Sunspot equilibria can often be constructed by randomizing over multiple equilibria of a general equilibrium model, and models with indeterminacy are excellent candidates for the existence of sunspot equilibria since there are many equilibria over which to randomize."

Benhabib and Farmer (1999, p.390)
Introducing sunspot equilibria: any RE path

We have infinite equilibria because:

- there is an infinite number of ways of forming expectations
- all of them coherent with the Muth’s REH
- parametrized by $b$

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$$\pi_t = (1 - b)\pi_t^B + b\pi_t^F$$

hence we introduce sunspots randomizing over $b$:

$$b_t = b_t(\zeta_t)$$  \hspace{2cm} (5)$$

where $\zeta_t$ i.i.d., orthogonal to the fundamental shocks $e_s$ ($s = 1, 2, ...$), and $E_t\zeta_t = 0 \ \forall t$. 

Sources of multiplicity

Solution UCM:

\[ \pi_{t+1} = \phi \pi_t - \phi e_t + be_{t+1} \]

There are two sources of multiplicity \( \Rightarrow \) expectation error:

\[ \eta_{t+1}(e_{t+1}, \zeta_{t+1}) = be_{t+1} + \zeta_{t+1} \]

where \( \zeta_{t+1} = \) sunspot or non-fundamental error.

This paper considers the FIRST term: intrinsic multiplicity of RE solutions
Introducing sunspot equilibria: drifting parameters and unstable paths

If \( b_t = b_{t-1} + \zeta_t \), and \( \zeta_t \sim i.i.d. N(0, \sigma^2_\zeta) \), then

\[
\pi_t = \theta_t \pi_{t-1} - \theta_t e_{t-1} + b_t e_t
\]

with \( \theta_t = \phi \frac{(1 - b_t)}{(1 - b_{t-1})} \) (with \( b_{t-1} \neq 1 \) otherwise FL solution).

- Same form as \( \pi_t = \phi \pi_{t-1} - \phi e_{t-1} + b e_t \)
Introducing sunspot equilibria: drifting parameters and unstable paths

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- Drifting parameters and stochastic volatility within the rational expectations framework. Cogley and Sargent (2005), Primiceri (2005).
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- Drifting parameters and stochastic volatility within the rational expectations framework. Cogley and Sargent (2005), Primiceri (2005).
- Intuition: agents can modify in every period the expectation formation process
- Reconsidering unstable paths: \( |\phi| > 1 \) and \( b_t \) temporarily different from one \( \Rightarrow \) estimate the process for \( b_t \) and check
Example: U.S. Great Inflation period

Is it appropriate to rule out unstable paths from the empirical analysis?

Figure: CPI inflation, quarterly data. Sample: 1960Q1 - 1997Q4
Example: Lubik and Schorfheide (2004) model

\[
\begin{align*}
\chi_t &= E_t(\chi_{t+1}) - \tau(R_t - E_t(\pi_{t+1})) + g_t \\
\pi_t &= \beta E_t(\pi_{t+1}) + \kappa(x_t - z_t) \\
R_t &= \rho_R R_{t-1} + (1 - \rho_R)(\psi_1 \pi_t + \psi_2(x_t - z_t)) + \epsilon_{R,t}
\end{align*}
\]

and

\[
\begin{align*}
g_t &= \rho_g g_{t-1} + \epsilon_{g,t}; \\
z_t &= \rho_z z_{t-1} + \epsilon_{z,t}
\end{align*}
\]

allow for non-zero correlation between the two shocks: \(\rho_{gz}\)

Compare two "models": \(M_S\) (stable solutions) and \(M_U\) (unstable solutions).
The estimation strategy

We use an econometric strategy to deal with the following issues:

i) the model has stochastic volatility, then the likelihood distribution is not Gaussian;

ii) we are interested in tracking the behavior of $b_t$, that can be considered as a stochastic latent process;

iii) we would like to study the fit of different models, and eventually compare them, during different periods.

Then, the econometric strategy is based on Bayesian methods, in particular on *Particle filtering*, and on *Sequential model monitoring*, based on Carvalho, Johannes, Lopes and Polson (2010)
Particle filter

Particle learning by Carvalho, Johannes, Lopes and Polson (2010)

1 Marginalization. \( \theta \): all latent states different from \( b \); \( y \): data

\[
p(\theta, b|y) = p(\theta|y, b) \cdot p(b|y)
\]

Kalman Filter Particle Filter

2 Parameter learning (Particle learning by CJLP 2010)
## Priors and Distributions

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Estimates Great Inflation sample

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90% credibility interval in brackets
Stable Model, Great Inflation

$M_S$: Comparison between the posterior distributions of the policy parameters and the probability intervals of LS

$\Rightarrow$ Very similar results to LS
Estimated path for $b_{1,t}$ - stable model $M_s$ - Great Inflation subsample ($1^{st}$ panel); sequential inference on the parameter $\psi_1$ ($2^{nd}$ panel).
### Unstable Model, Great Inflation

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Unstable Model, Great Inflation

The behavior of $b_{1,t}$

Figure: Estimated path of $b_{1,t}$ for the unstable model $M_U$ in the Great Inflation subsample
Comparing the relative fit of Ms/Mu

Sequential Bayes Factor West (1986): $2 \ln(W_t)$ and the inflation rate

The Bayes Factor strongly favours the unstable model
Estimates Great Moderation sample

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<td>[0.96 2.40]</td>
<td>[1.71 3.42]</td>
<td>[1.04 2.64]</td>
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<td>$\rho_g$</td>
<td>0.85</td>
<td>0.75</td>
<td>0.83</td>
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<td>[0.77 0.91]</td>
<td>[0.68 0.81]</td>
<td>[0.77 0.89]</td>
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<tr>
<td>$\rho_z$</td>
<td>0.77</td>
<td>0.74</td>
<td>0.85</td>
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<td>[0.63 0.88]</td>
<td>[0.66 0.80]</td>
<td>[0.77 0.93]</td>
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<tr>
<td>$\rho_{gz}$</td>
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<td>0.005</td>
<td>0.36</td>
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<td>[0.01 0.07]</td>
<td>[0.06 0.07]</td>
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<td>$\sigma_R$</td>
<td>0.17</td>
<td>0.12</td>
<td>0.18</td>
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<td>[0.14 0.21]</td>
<td>[0.10 0.14]</td>
<td>[0.14 0.21]</td>
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<tr>
<td>$\sigma_g$</td>
<td>0.14</td>
<td>0.14</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>[0.11 0.18]</td>
<td>[0.11 0.17]</td>
<td>[0.14 0.23]</td>
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<tr>
<td>$\sigma_z$</td>
<td>0.57</td>
<td>0.52</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>[0.49 0.69]</td>
<td>[0.46 0.71]</td>
<td>[0.52 0.76]</td>
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<tr>
<td>$\sigma_{\zeta}$</td>
<td>—</td>
<td>0.04</td>
<td>—</td>
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<tr>
<td></td>
<td>[0.03 0.06]</td>
<td>—</td>
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</tr>
</tbody>
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Under $M_S \Rightarrow$ determinacy $\Rightarrow b = 1$
$M_S$: Comparison between the posterior distributions of the policy parameters and the probability intervals of LS.
Unstable model: Great Moderation

The behavior of $b_{1,t}$

Figure: Estimated path of $b_{1,t}$ for the unstable model $M_U$ in the Post-82 subsample
Conclusions

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  - Asset prices and bubbles
Multiple RE solutions: Bubble literature

The solution can be rewritten as

\[
\pi_t = \sum_{j=1}^{\infty} \phi^j (b - 1) e_{t-j} + be_t + \sum_{j=1}^{\infty} b \phi^j E_t e_{t+j} = \\
= \sum_{j=0}^{\infty} \frac{1}{\phi^j} E_t e_{t+j} + (b - 1) \left[ \sum_{j=1}^{\infty} \phi^j e_{t-j} + e_t + \sum_{j=1}^{\infty} \frac{1}{\phi^j} E_t e_{t+j} \right] \\
\]

see Burmeister, Flood and Gaber (1983)
\( \lambda_2 \) \begin{cases} b_2 = 1 & \text{stable} \\ b_2 \neq 1 & \text{unstable} \end{cases}
\[ \begin{aligned}
\lambda_2 &= \begin{cases} 
  b_2 = 1 & \text{stable} \\
  b_2 \neq 1 & \text{unstable}
\end{cases}
\end{aligned} \]
\[ \begin{aligned}
\lambda_2 &= \begin{cases} 
b_2 = 1 & \text{stable} \\
b_2 \neq 1 & \text{unstable} 
\end{cases} \\
b_1 & \text{whatever}
\end{aligned} \]
\[ \lambda_1 \begin{cases} b_2 = 1 & \text{stable} \\ b_2 \neq 1 & \text{unstable} \end{cases} \]

\[ \lambda_2 \begin{cases} b_1 = 1 & \text{stable} \\ b_1 \neq 1 & \text{unstable} \end{cases} \]

\( b_1 \) whatever
Compare two "models": $M_S$ (stable solutions) and $M_U$ (unstable solutions).
Parameter learning

Particle learning by *Carvalho, Johannes, Lopes and Polson (2010)*

Assume that the posterior for some parameters $\psi$ is function of a set of sufficient statistics $s_t$ recursively updated

$$p(\psi | \theta_{0:t}, y_{0:t}) = p(\psi | s_t)$$

$$s_t = S(s_{t-1}, \theta_{0:t}, y_{0:t})$$

and consider $s_t$ as a latent state with deterministic evolution.

- When it is not possible use Liu and West approach. Approximating the posterior distribution with mixtures of Normals.
Bayesian model monitoring (West 1986)

Compare two models: $M_S$ and $M_U$.

For $t = 1 \ldots T$

- Compute the predictive likelihood: $p(y_t | y_{0:t-1}, M_i) \quad i = S, U$
- Compute the likelihood ratio

$$H_t = \frac{p(y_t | y_{0:t-1}, M_S)}{p(y_t | y_{0:t-1}, M_U)}$$

- Compute $W_t(k) = H_t H_{t-1} \ldots H_{t-k+1}$ (Kass and Raftery, 1995)

$W_t(k)$ is called the sequential Bayes factor and it assesses the fit of the most recent $k$ observations.
Transmission mechanism of structural shocks: GIRF in the $M_S$ model.
Stable Model, Great Inflation

Transmission mechanism of sunspot shock: GIRF in the $M_S$ model:
solid line: $b_1 = 1.3$, dashed line: $b_1 = 1.5$. 
Transmission mechanism of structural shocks: GIRF in the $M_U$ model
Transmission mechanism of sunspot shock: GIRF in the $M_U$ model: solid line: $b_1 = 1.3$, dashed line: $b_1 = 1.5$. 
Asymptotically equal stationary path (AES)

Process for $b$

$$b_t = \begin{cases} 
\theta b_{t-1} + 1 - \theta + u_t & \text{with probability } \frac{1}{\theta} \\
1 & \text{with probability } 1 - \frac{1}{\theta}
\end{cases}$$

where $u_t$ is white noise, so that $E(u_t) = 0$, $\forall t. \Rightarrow E(b_t) = b_{t-1}$

This process converge to 1 with probability 1, but it is perturbed by $u_t$. 

AES Model, Great Inflation

The behavior of $b_{1,t}$

Figure: Estimated path of AES $b_{1,t}$ for the unstable model $M_U$ in the Great Inflation subsample
Comparing the relative fit of Ms/AESMu

Sequential Bayes Factor West (1986): $2 \ln(W_t)$ and the inflation rate

The Bayes Factor strongly favours the AES model
Comparing the relative fit of Ms/Mu: Great Moderation

Figure: Sequential Bayes Factor West (1986): $2 \ln(W_t)$ and the inflation rate