Imperfect Information, Heterogeneous Demand Shocks, and Inflation Dynamics*

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Abstract

We use new survey data for the universe of Japanese firms to show that expectations about aggregate demand positively co-move with the expectations about sector-specific demand. We show that a simple model with imperfect information on the *current* aggregate and sector-specific components of demand explains the positive co-movement in the expectations. Our theoretical framework shows that under imperfect information on the *current* components of demand, an increase in the volatility of sector-specific demand reduces the sensitivity of inflation to changes in aggregate demand. We test and corroborate this theoretical prediction on Japanese data, showing that the movements in the volatility of sector-specific shocks explains the reduction in the sensitivity of inflation to economic activity in Japan over the past three decades.

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1 Introduction

A large class of macroeconomic models builds on the premise that firms set prices to fulfil demand, and several studies show that shocks to demand are heterogeneous and reflect aggregate and sector-specific disturbances.[1] Knowing the source for the change in demand has implications for the optimal price. The classic study by Ball and Mankiw (1995) shows that the optimal price adjusts if the change in demand originates from the aggregate shock, but it remains unchanged if the change originates from the sector-specific shock. In reality, however, information is imperfect and firms cannot observe the source of any changes in demand in real time. Instead, they form expectations on whether the source of changes stems from the aggregate or sector-specific component of demand based on their observations. Empirical evidence on the expectations on the aggregate and sector-specific components of demand is scarce. Moreover, despite the theory shows a tight link between the source of the shock and the firm's optimal pricing decision, there are no studies that connect imperfect information on the different components of demand to the sensitivity of inflation to economic activity.

Our analysis fills this gap. We use new survey data for the universe of Japanese firms to study the comovement in the firms' expectations about aggregate and sector-specific components of demand, and we develop a simple model of imperfect information that links the expectations on the distinct components of demand to the response of inflation to economic activity. We show that imperfect knowledge on the components of demand plays a critical role to explain the observed comovement in the expectations on the different components of demand, and account for the reduction in the sensitivity of inflation to changes in economic activity in Japan over the past three decades.

We establish four new results. First, we document novel evidence on the positive co-movements between expectations on aggregate and sector-specific components of demand using a sector-level survey for the universe of Japanese firms across 26 sectors. This evidence is important since it shows that expectations about the aggregate and sector-specific components of demand are not independent, as postulated by models based on perfect information.

Second, we demonstrate that imperfect information on the *current* shocks to demand is critical to generate the observed positive co-movement in the expectations. Motivated

[1] See di Giovanni et al. (2014) and references therein.

by our empirical results, we develop a simple model that embeds nominal price rigidities in the Lucas (1972) island framework, where firms cannot separately observe the different aggregate and sector-specific components that jointly move the observed demand. We prove analytically that imperfect information generates co-movements in the expectations about the different components of demand that are consistent with the positive co-movements in the survey data.

Third, we use our model to study the the sensitivity of inflation to changes in aggregate demand. Nominal price rigidities link inflation to the expectations of demand that in our model comprise the expectations on the different aggregate and sector-specific components. We show that the degree of sectoral heterogeneity in demand shocks – encapsulated by the ratio of volatility of sector-specific demand shocks compared to the volatility of aggregate demand shocks – is critical for the sensitivity of inflation to demand. Under perfect information, if the change in total sectoral demand originates from the aggregate component of demand, the price adjustment is large as a result of strategic complementarity in price-setting because the aggregate shock should be common to all firms. If instead the change in total sectoral demand originates from the sector-specific component of demand specific to each sector, the price adjustment in the sector is contained since firms would either lose customers (if the price rises) or forego earnings for a lower markup (if the price falls), given competitors in other sectors retain prices unchanged. The presence of imperfect information in our model prevents firms from perfectly disentangling the different contributions of aggregate and sector-specific components to total demand in the sector. Therefore, firms optimally attribute part of a change in total sectoral demand to movements in the sector-specific component of demand and thus underreact to shocks compared to the setting with perfect information. A testable prediction of our theoretical framework is that the response of prices to aggregate demand is inversely related to the ratio between the volatility of sector-specific and aggregate demand shocks.

Fourth, we use the predictions from the model on the inverse relation between the volatility of sector-specific shocks and the response of inflation to aggregate demand to test the relevance of imperfect information for the reduction in the sensitivity of inflation to demand on Japanese data. We estimate the volatility of the sector-specific component of demand relative to the volatility of the aggregate component of demand by using principal component analysis on sector-level data for Japanese firms across 29 sectors for the period 1975-2018.

In line with our theory, we show that the increase in the ratio of the volatility in sector-specific shocks compared to the volatility in aggregate shocks played a significant role in the reduction of the sensitivity of inflation to aggregate demand.

Our analysis is linked to four strands of literature. First, we relate to the literature on the formation of expectations under imperfect information. The study closest to us is Andrade et al. (2022) who examine the empirical plausibility of information frictions in the Lucas-island model by studying the relation between firms' expectations about aggregate variables and estimated industry-specific shocks. We relate to studies that develop imperfect information in models with flexible prices (Woodford, 2003; Hellwig and Venkateswaran, 2009; Crucini et al., 2015; Afrouzi, 2018; and Kato et al., 2021) and nominal price rigidities (Fukunaga, 2007; Nimark, 2008; Angeletos and La'O, 2009; Melosi, 2017; and L'Huillier, 2020). We also relate to studies that allow for coexistence of aggregate and idiosyncratic shocks in the presence of costly information acquisition (Veldkamp and Wolfers, 2007; and Acharya, 2017). Coibion et al. (2021) and Coibion et al. (2020) provide broad evidence on the relevance of firms' expectations to firms' decisions. Compared to the aforementioned studies, we provide novel evidence on firms' expectations about aggregate and disaggregate components of demand and assess the role of expectations for the sensitivity of inflation to aggregate demand.

Second, our analysis relates to the literature that investigates the effect of imperfect information on the Phillips curve. Mankiw and Reis (2002) and Dupor et al. (2010) develop sticky-information models to investigate the effect of informational frictions on the empirical performance of the Phillips curve. Coibion and Gorodnichenko (2015) establish that information frictions are critical in generating an empirically-consistent formation of expectations that explain the missing disinflation between 2009 and 2011. Coibion et al. (2018) show that information frictions are important to formulate an empirically congruous Phillips curve. Afrouzi (2020) and Afrouzi and Yang (2021) investigate the effect of rational inattention on the Phillips curve, showing that the endogenous attention allocation of firms to economic variables is critical for the sensitivity of inflation to the aggregate conditions.

Third, we are related to studies that investigate changes in the sensitivity of inflation to economic slack, as generated by the anchoring effect of inflation targets (Roberts, 2004, and L'Huillier and Zame, 2020), the increase in competition in the goods market (Sbordone, 2008, and Zanetti, 2009), downward wage rigidities (Akerlof et al., 1996), structural reforms

(Thomas and Zanetti, 2009, Zanetti, 2011, and Cacciatore and Fiori, 2016), and lower trend inflation (Ball and Mazumder, 2011).[2] Unlike these studies, however, our focus is on the relation between imperfect information and the sensitivity of inflation to changes in aggregate demand.

Finally, our analysis relates to studies that investigate the formation of expectations under imperfect information using firm-level survey data. Several studies focus on inflation expectations (Andrade et al., 2022 use a survey of French manufacturing firms, Coibion et al., 2020 and Bartiloro et al., 2017 use a survey of Italian firms, and Kumar et al., 2015 use a survey of firms in New Zealand). We are the first to use a survey on Japanese firms to study the formation of expectations about the aggregate and sector-specific components of demand.

The remainder of the paper is organized as follows. Section 2 provides evidence on the co-movement in expectations about aggregate and sector-specific demand from survey data. It develops a simple model with imperfect information that explains the positive co-movement in the expectations of the separate components of demand. Section 3 augments the model to incorporate general equilibrium and derive equilibrium pricing with and without nominal rigidities. Section 4 studies the sensitivity of inflation dynamics to demand, and it shows that the data corroborates the theoretical predictions. Section 5 concludes.

2 Evidence from Survey Data

In this section, we study the relation between the firms' expectations about the growth rate of aggregate and sector-specific components of total sectoral demand. We conduct the analysis using two approaches. First, we develop a VAR model that uses a Cholesky decomposition to identify sector-specific shocks that are orthogonal to the aggregate shocks by assuming that sector-specific shocks do not affect aggregate GDP in the same year. We use these identified disturbances to test whether they are insignificant to the firm's *expectations* about aggregate demand, proxied by the firms' expectations about the growth rate of aggregate GDP. We find a significant and systematic relation between sector-specific shocks and the expectations about aggregate demand that rules out the orthogonality of sector-specific shocks to the firms' expectations on aggregate demand. Second, we use regression analysis to test directly

[2] Several studies show a decline in the sensitivity of inflation to real activity. See survey by Mavroeidis et al. (2014) for a recent review of the literature on U.S. data. Kaihatsu et al. (2017) and Bundick and Smith (2020) provide evidence on the reduced sensitivity of inflation to real activity on Japanese data.

the correlation between firms' expectations about the growth rates of aggregate demand and sector-specific component of total sectoral demand using our survey data. We find that both approaches reach the same conclusion, showing positive and significant co-movement between firms' expectations about sector-specific and aggregate components of demand.

Survey data. We use the Annual Survey of Corporate Behavior (ASCB).[3] The survey is administered by the Cabinet Office of Japan across 26 sectors over the period 2003-2019.[4] Firms complete a mandatory questionnaire that records the separate expectations about the growth rate of total sectoral and aggregate demand, thus providing an account on the firm expectations about the different aggregate and sector-specific components of total demand.[5] Appendix F provides a description and summary statistics for the ASCB.

VAR approach. Our first approach in studying the comovements between the different components of demand consists in developing a VAR model with Cholesky identification that obtains shocks to the sector-specific demand that are orthogonal to aggregate shocks by assuming that sector-specific shocks do not affect aggregate GDP in the same year. We then test the co-movement between the identified sector-specific shocks and firms' expectations about aggregate demand, proxied by firms' expectations about aggregate GDP. If the firms' expectations about aggregate demand is independent from sector-specific fluctuations in demand, as it occurs when firms perfectly observe the separate components of demand, the correlation between them should be zero. We show that the data entails significant, positive correlation between sector-specific shocks and firms' expectations about aggregate demand.

The VAR model comprises two variables: (i) the yearly growth rate of the nominal aggregate GDP (GDP_t), and (ii) the yearly sale growth in each sector i (sectoral $\text{sales}_t(i)$).[6] We include 22 sectors to maintain correspondence between survey and sales data over the sample period 2004-2020. The data on nominal aggregate GDP is produced by the Cabinet Office of Japan, and the sectoral sales growth is from the Financial Statement Statistics of

[3] Appendix F provides details on the survey.

[4] Since the survey entails significant changes in the number of sectors before 2005, we start our sample period in 2005 when the size of sectors is stable between the different years.

[5] The question asked in the survey is: "Please enter a figure up to one decimal place in each of the boxes below as your rough forecast of Japan's nominal economic growth rates and the nominal growth rates of demand in your industry for FY20XX". The questionnaire of the survey is available at: https://www.esri.cao.go.jp/en/stat/ank/ank-e.html.

[6] We set the number of lags is equal to two based on Akaike's Information Criterion.

Corporations collected by the Ministry of Finance in Japan.[7] We identify the sector-specific shocks with a Cholesky decomposition that imposes orthogonality between sector-specific disturbances and aggregate shocks by assuming that sector-specific shocks do not affect aggregate GDP in the same year. This enables us to extract the sector specific shocks on sectoral-sale growth that are orthogonal to those of aggregate shocks. We separately estimate the structural VAR model for each sector to allow for different sensitivity of the sectoral sales growth to aggregate shocks across sectors. Our VAR model is:

$$\begin{aligned} A_0 \begin{bmatrix} \text{GDP}_t \\ \text{sectoral sales}_t(i) \end{bmatrix} &= C + A_1 \begin{bmatrix} \text{GDP}_{t-1} \\ \text{sectoral sales}_{t-1}(i) \end{bmatrix} \\ &+ A_2 \begin{bmatrix} \text{GDP}_{t-2} \\ \text{sectoral sales}_{t-2}(i) \end{bmatrix} + \begin{bmatrix} \epsilon_{\text{aggregate}_t} \\ \epsilon_{\text{sector-specific}_t}(i) \end{bmatrix}, \end{aligned} \tag{1}$$

where the matrix A_0 is lower triangular, the vector C is of constant terms, the matrices A_1 and A_2 are for the lag terms, and $\epsilon_{\text{aggregate}_t}$ and $\epsilon_{\text{sector-specific}_t}(i)$ are the exogenous aggregate and sector-specific shocks, respectively.

Using the VAR model in equation (1), we obtain estimates for sector-specific shocks $(\epsilon_{\text{sector-specific}}(i))$ that we use as instruments for sector-specific shocks on demand given the exogeneity from aggregate demand inherent in our Cholesky identification. We then estimate the correlation between the firms' expectations on aggregate demand, proxied by the growth rate of one-year-ahead aggregate output $(\mathbb{E}_{t-1}[GDP_t])$, and the expected sectoral demand at time $t-1$ for time t for each sector i $(\mathbb{E}_{t-1(i)}[SD_t(i)])$ that we proxy by the estimated sector-specific shocks $(\epsilon_{\text{sector-specific}}(i))$. We use Generalized Methods of Moments to estimate the correlation between $\mathbb{E}_{t-1}[GDP_t]$ and $\epsilon_{\text{sector-specific}}(i)$ by using the sector-specific shocks for the present, one-year-ago and two-years-ago shocks $(\epsilon_{\text{sector-specific}_t(i)}, \epsilon_{\text{sector-specific}_{t-1}(i)}, \epsilon_{\text{sector-specific}_{t-2}(i)})$ as our instrumental variables.

Table 1 shows in column (1) the estimated co-movement between sector-specific shocks on demand and firms' expectations about aggregate demand estimated using GMM. The regression coefficient (bold entry) that captures the co-movements between the sector-specific shocks and expectations about aggregate demand is positive and significant in all the specification of the model. Hence, our baseline estimates reject the hypothesis that expectations about aggregate and sector-specific components of demand are independent. To ensure the robustness of the estimation results, columns (2), (3), and (4) show estimates from the al-

[7] Appendix G provides a description of the data on yearly sales growth.

Table 1: Firms' expectations on aggregate and sectoral demands

	Firms' expectations on the growth rate of the aggregate demand (One year ahead expectations)			
	(1) Pooled GMM model	*(2) fixed effect model*	*(3) Period effect model*	*(4) Period-fixed effect model*
Firms' expectations on the growth rate of the sectoral demand (IV)	**0.64***** (0.14)	**0.65***** (0.13)	**0.19*** (0.12)	**0.22**** (0.11)
Fixed effect	No	Yes	No	Yes
Period effect	No	No	Yes	Yes
Observations	265	265	265	265
Adjusted-R^2	0.57	0.61	0.93	0.94

Note: Estimated by GMM. Instrument variables are the sector specific shocks in the current period, one year ago, and two years ago. The standard errors are HAC estimators.
**** Significant at the 1 percent level. ** Significant at the 5 percent level.* Significant at the 10 percent level.*

ternative specifications of the model that include sector-specific fixed effects, period effects, and period-fixed effects, respectively. The estimates are positive and significant across the different specification, showing robust correlation between the sector-specific demand shocks and firms' expectations about aggregate demand, implying that firms cannot disentangle aggregate shocks and sector-specific shocks, and firms' expectations about aggregate and sector-specific demand comove.

Regression approach. Our second approach consists in directly estimating the co-movement between the expectations of aggregate and sector-specific components of demand from survey data. If the expectations on the separate components of demand were independent of one another, as occurs when firms perfectly observe the separate components of demand, the co-movement in the different expectations is not significantly different from zero.

Table 2: Firms' expectations on aggregate and sector-specific demands

	Firms' expectations on the growth rate of the aggregate demand (One year ahead expectations)			
	(1) Pooled OLS model	*(2) fixed effect model*	*(3) Period effect model*	*(4) Period-fixed effect model*
Firms' expectations on the growth rate of the sector-specific demand	**0.30***** (0.08)	**0.28***** (0.08)	**0.65***** (0.25)	**0.51**** (0.26)
Fixed effect	No	Yes	No	Yes
Period effect	No	No	Yes	Yes
Observations	389	389	389	389
Adjusted-R^2	0.06	0.23	0.10	0.27

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators.
**** Significant at the 1 percent level. ** Significant at the 5 percent level.* Significant at the 10 percent level.*

Table 2 shows in column (1) the estimated co-movement obtained from regressing the firm's expectations of the growth rate of aggregate demand on the firm's expectations about the growth rate of sector-specific demand, which is calculated by subtracting the growth rate of aggregate demand from that of total sectoral demand.[8] The regression coefficient (bold

[8]Appendix B show consistency of the survey data with the Dixit-Stiglitz demand function that we use in

entry) that captures the co-movements between the expectations in aggregate and sector-specific demand is positive and significant, evincing a positive co-movement in the separate expectations. Thus, our baseline estimates reject the hypothesis that the expectations about different components of demand are independent. To ensure results are robust, columns (2), (3) and (4) show estimates from regressions that include sector-specific fixed effects, period effects, and period-fixed effects, respectively, and show that the positive correlation between aggregate and sector-specific components of demand remains significant and positive.

To summarize, our results based on VAR and regression analyses consistently show that the positive co-movement between firms' expectations about aggregate and sector-specific components of demand is a significant feature of the firms' expectations.

2.1 Expectations under Imperfect Information

We develop a parsimonious model of imperfect information on the different components of demand that explains the positive co-movement of the firms' expectations about the components of aggregate and sector-specific demand observed in survey data. We will extend the model to a general equilibrium framework to study the sensitivity of inflation to demand in Section 3.

We assume the economy is populated by a representative household and a continuum of monopolistic competitive firms that produce differentiated goods indexed by $j \in [0,1]$ in a continuum of sectors indexed by $i \in [0,1]$. Each firm j in sector i observes total sectoral demand ($x_t(i)$) that changes in response to aggregate demand and sector-specific demand, according to $x_t(i) = q_t + v_t(i)$, without observing the separate realizations for the aggregate (q_t) and sector-specific components ($v_t(i)$).[9] Aggregate demand follows the stochastic process:

$$q_t = q_{t-1} + u_t, \tag{2}$$

where u_t is an AR(1) process:

$$u_t = \rho_u u_{t-1} + e_t, \tag{3}$$

with $0 \leq \rho_u < 1$, and $e_t \sim \mathcal{N}(0, \sigma_t^2)$. The sector-specific demand follows the AR(1) process:

$$v_t(i) = \rho_v v_{t-1}(i) + \epsilon_t(i), \tag{4}$$

the model.

[9] We will derive and revisit this relation in a general equilibrium framework in Section 3. A recent study by Chahrour and Ulbricht (2019) shows that imperfect information on disaggregate shocks of the type we have in our simple model generate realistic business cycle statistics.

where $0 \leq \rho_v < 1$, and $\epsilon_t(i) \sim \mathcal{N}(0, \tau_t^2)$.

In each period t, firms set prices without observing the current aggregate and sector-specific components of total sectoral demand and therefore are unable to infer the current aggregate price.[10] Thus, each firm uses information from the common signal of total sectoral demand (i.e., $x_t(i) = q_t + v_t(i)$) and the past realizations of aggregate and sector-specific components of demand to make inference on the current components of aggregate (q_t) and sector-specific demand ($v_t(i)$), such that $q_t \sim \mathcal{N}\left(q_{t-1} + \rho_u u_{t-1}, \sigma_t^2\right)$ and $v_t(i) \sim \mathcal{N}\left(\rho_v v_{t-1}(i), \tau_t^2\right)$.[11] Hence, in each period t, the information set for the firms in sector i is:

$$\mathcal{H}_t(i) \equiv \left\{\{x_s(i)\}_{s=0}^{t}, \{q_s, u_s, v_s(i), e_s, \epsilon_s(i)\}_{s=0}^{t-1}\right\}, \tag{5}$$

and hereafter we denote the expectations under imperfect information as: $\mathbb{E}_t \equiv \mathbb{E}\left[\bullet|\mathcal{H}_t(i)\right]$.

In what follows, we show that imperfect information on the *current* components of demand explains the observed positive correlation between firms' expectations on aggregate and sector-specific components of total demand.

Mapping the model to the data. The model characterizes the expectations on the *level* of total demand and its different components whereas the data refer to the expectations on the *changes* of total demand and its aggregate and sector-specific components. To link the model with the empirical measurements, we focus on the changes in total sectoral demand and its separate components by taking the first difference of $x_t(i)$: $\Delta x_t(i) = \Delta q_t + \Delta v_t(i)$. Thus, the model now provides a measure of changes in expectations in aggregate and sector-specific demands, Δq_t and $\Delta v_t(i)$, respectively, that is consistent with the measurement in the data.

To simplify notation, we label $\widetilde{x}_t(i) = \Delta x_t(i)$, $\widetilde{v}_t(i) = \Delta v_t(i)$, and by using equation (2), $u_t = \Delta q_t$. Combining equations (3)-(4), we write the change in total sectoral demand, $\widetilde{x}_t(i)$, as the sum of the change in aggregate demand, u_t, and the change in sector-specific demand, $\widetilde{v}_t(i)$:

$$\widetilde{x}_t(i) = u_t + \widetilde{v}_t(i). \tag{6}$$

Equation (6) shows that the change in total sectoral demand in the model comprises the changes in aggregate and sector-specific demand, as in the data. In the remaining part of this

[10] The assumption that q_t is unobservable in period t implies that the labor market clears after firms set prices. Therefore, firms base their profit-maximizing decisions on the expected nominal wage in period t, as in Angeletos and La'O (2009).

[11] See Guerron-Quintana et al. (2018) for an overview on solutions for filtering problems in economics.

section, we use equation (6) to study the effect of imperfect information for the co-movement between changes in expectations about aggregate and sector-specific demand.

The formation of expectations and co-movements in the components of total sectoral demand. Using equation (6), current expectations about total demand in k-period ahead are equal to:

$$\mathbb{E}_t\left[\sum_{h=1}^{k}\widetilde{x}_{t+h}(i)\right] = \mathbb{E}_t\left[\sum_{h=1}^{k}u_{t+h}\right] + \mathbb{E}_t\left[\sum_{h=1}^{k}\widetilde{v}_{t+h}(i)\right]. \tag{7}$$

Equation (7) shows that the current expectations of total demand k-period ahead comprises the expectations of the aggregate and sector-specific components of demand in k-period ahead. If firms are able to observe separately the components of aggregate and sector-specific demand, such that $\mathbb{E}_t\left[u_t\right] = u_t$ and $\mathbb{E}_t\left[\widetilde{v}_t\right] = \widetilde{v}_t$, the expectations of the different components of total sectoral demand are independent of each other and the co-movement between them is equal to zero. The next proposition shows that imperfect information renders the expectations on the separate components of demand dependent on the common change in total sectoral demand, therefore generating a co-movement in expectations.

Proposition 1 *Under imperfect information, the expectations at time t about the changes in aggregate and sector-specific demands are equal to:*

$$\mathbb{E}_t\left[u_t\right] = \rho_u u_{t-1} + \frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\left[e_t + \epsilon_t(i)\right] \tag{8}$$

and

$$\mathbb{E}_t\left[\tilde{v}_t(i)\right] = (\rho_v - 1)v_{t-1}(i) + \frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\left[e_t + \epsilon_t(i)\right], \tag{9}$$

respectively.

Proof: See Appendix E.1. □

Equations (8) and (9) show that the firm's expectations on the changes in aggregate and sector-specific demand depend on the changes in total sectoral demand, which comprises shocks to aggregate and sector-specific shocks ($e_t + \epsilon_t(i)$) that the firm cannot separately observe. The response of each expectation to movement in total sectoral demand depends on the ratio τ_t/σ_t, which represents the volatility of sector-specific shocks relative to aggregate shocks. If the volatility of the shock to sector-specific demand is larger than the volatility of the shock to aggregate demand (i.e., $\tau_t/\sigma_t > 1$) – reflecting the fact that changes in total sectoral demand are predominantly driven by the sector-specific component of demand – the response of firms' expectations on the sector-specific component of demand to the change in total sector demand increases while the response of firms' expectations on the aggregate component of demand to total sectoral demand decreases.

The next propositions characterize the sign of the co-movement between the current expectations of aggregate and sector-specific demand and the resulting co-movement in the *expectations* between the separate components of demand.

Proposition 2 *Under imperfect information, the co-movement in the current expectations about aggregate and sector-specific demand is equal to:*

$$\mathbb{C}(\mathbb{E}_t\left[u_t\right], \mathbb{E}_t\left[\tilde{v}_t(i)\right]) = \frac{\sigma_t^2 \tau_t^2}{\sigma_t^2 + \tau_t^2} > 0, \tag{10}$$

where $\mathbb{C}(\cdot)$ *is the unconditional covariance operator.*

Proof: See Appendix E.2. □

Proposition 2 shows that the presence of imperfect information generates a positive co-movement between the current expectations of aggregate and sector-specific components of total sectoral demand. This implies a positive co-movement between the expectations about the components of k-period ahead demand, as shown in the next proposition.

Proposition 3 *If total demand comprises unobservable aggregate and sector-specific components (i.e.,* $\widetilde{x}_t(i) = u_t + \widetilde{v}_t(i)$*), the positive co-moment in the current expectations generates the positive co-movement in the* k*-period ahead expectations:*

$$\mathbb{C}(\mathbb{E}_t\left[u_t\right], \mathbb{E}_t\left[\tilde{v}_t(i)\right]) > 0 \Rightarrow \mathbb{C}\left(\mathbb{E}_t\left[\sum_{h=1}^{k} u_{t+h}\right], \mathbb{E}_t\left[\sum_{h=1}^{k} \widetilde{v}_{t+h}(i)\right]\right) > 0.$$

Proof: See Appendix E.3. □

Proposition 3 provides the theoretical underpinning that explains the positive relation between the expectations on aggregate and sector-specific components of demand consistent with the data. If we use the model to estimate the regression coefficients in Table 2, it yields:

$$\mathbb{E}_t\left[\sum_{h=1}^{k} u_{t+h}\right] = \beta_0 + \beta_1 \mathbb{E}_t\left[\sum_{h=1}^{k} \widetilde{v}_{t+h}(i)\right], \tag{11}$$

where β_0 is the constant term in the regression and β_1 is the coefficient that captures the correlation between changes in aggregate and sector-specific demand. The value for β_1 is equal to:

$$\beta_1 = \frac{\mathbb{C}\left(\mathbb{E}_t\left[\sum_{h=1}^{k} u_{t+h}\right], \mathbb{E}_t\left[\sum_{h=1}^{k} \widetilde{v}_{t+h}(i)\right]\right)}{\mathbb{V}\left(\mathbb{E}_t\left[\sum_{h=1}^{k} \widetilde{v}_{t+h}(i)\right]\right)}. \tag{12}$$

Equation (12) shows that the value for the correlation coefficient β_1 depends on the covariance of expectations about future realizations of aggregate and sector-specific demand, which Proposition 3 shows is determined by the correlation between the *current* expectations on these components. To sum up, the analysis shows that imperfect information on the current components of total sectoral demand is critical to generate a positive co-movement in the expectations of aggregate and sector-specific components of demand, as observed in the data.

3 General Equilibrium

This section embeds the empirically-congruous expectations based on imperfect information in a general equilibrium framework to study the sensitivity of inflation to aggregate demand.

3.1 Model

The model is based on Woodford (2003) and Angeletos and La'O (2009). We maintain the information structure developed in the previous section. The economy is populated by a representative household and a continuum of monopolistic competitive firms that produce differentiated goods, indexed by $j \in [0, 1]$ in a continuum of sectors, indexed by $i \in [0, 1]$. The representative household consumes the whole income with no saving in equilibrium. Monopolistic competitive firms face a total sectoral demand that comprises aggregate and sector-specific shocks, as described in equations (2), (3), and (4). Firms observe current total sectoral demand and the past realizations of aggregate and sector-specific shocks to demand, but they are unable to separately observe the realizations of aggregate and sector-specific components of total sectoral demand in real time. Namely, firms form expectations at time t, using the information set $\mathcal{H}_t(i)$ in equation (5).

The rest of the section develops the problems of households and firms and derives the equilibrium.

Households. The following utility function describes the preferences of the representative household over consumption, C_t, and labor, N_t:

$$\sum_{t=0}^{\infty} \beta^t \left(\log C_t - N_t\right),$$

where $\beta \in (0,1)$ is the discount rate. The household's aggregate consumption, C_t, and consumption of goods in sector i, $C_t(i)$, are defined by the CES consumption aggregators:

$$C_t \equiv \left[\int_0^1 \left(C_t(i)\Theta_t(i)\right)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}, \text{and } C_t(i) \equiv \left[\int_0^1 \left(C_t(i,j)\right)^{\frac{\widetilde{\eta}-1}{\widetilde{\eta}}} dj\right]^{\frac{\widetilde{\eta}}{\widetilde{\eta}-1}},$$

where $\eta > 1$ is the elasticity of substitution across sectors, $\widetilde{\eta} > 1$ is the elasticity of substitution across goods within the same sector, $C_t(i,j)$ is consumption of good j in sector i, and $\Theta_t(i)$ is the sector-specific preference shocks (defined below).

Firms. Each firm j in sector i (referred as "firm (i,j)") faces the following demand:

$$C_t(i,j) = \Theta_t^{\eta-1}(i)\left(\frac{P_t(i,j)}{P_t(i)}\right)^{-\widetilde{\eta}}\left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t, \tag{13}$$

where $P_t(i) \equiv \left[\int_0^1 P_t^{1-\widetilde{\eta}}(i,j)dj\right]^{\frac{1}{1-\widetilde{\eta}}}$ is the price index for sector i, $P_t \equiv \left[\int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)di\right]^{\frac{1}{1-\eta}}$ is the aggregate price index, and the sector-specific preference shock, $\Theta_t(i)$, acts as an exogenous demand shifter for firm (i,j).[12]

Each firm (i,j) manufactures a single good $Y(i,j)$, according to the production technology:

$$Y_t(i,j) = AL_t^{\epsilon}(i,j), \tag{14}$$

where A is aggregate productivity and $\epsilon \in (0,1)$ determines the degree of diminishing marginal returns in production.

Market Clearing. In a symmetric equilibrium, market clearing implies $Y_t(i,j) = C_t(i,j)$ for each firm (i,j) and thus $Y_t = C_t$ in the economy. Aggregate nominal demand, Q_t, is given by the following cash-in-advance constraint:

$$Q_t = P_t C_t.$$

In the rest of the analysis, we use lower-case variables to indicate logarithms of the corresponding upper-case variables (i.e., $x_t \equiv \log X_t$).

[12]See Appendix A for the derivation of the demand function for each firm (i,j) and price indexes. Appendix B shows that total sectoral demand in equation (13) entails independent aggregate and sector-specific components consistent with the empirical analysis.

Optimal Price-Setting Rule and Total Sectoral Demand. In what follows, we derive the optimal price-setting rule as a function of total sectoral demand.

During each period t, the firm (i, j) sets the optimal price as a mark-up over the marginal cost:

$$p_t(i,j) = \mu + mc_t(i,j), \tag{15}$$

where $\mu \equiv \widetilde{\eta}/(\widetilde{\eta} - 1) > 0$ is the mark-up and $mc_t(i,j)$ is the nominal marginal cost faced by firm (i, j). The nominal marginal cost is the difference between the nominal wage, w_t, and the marginal product of labor:

$$mc_t(i,j) = w_t + (1-\epsilon)\, l_t(i,j) - a - log(\epsilon). \tag{16}$$

Using the production technology in equation (14), we express labor input as: $l_t(i,j) = [y_t(i,j) - a]/\epsilon$, and we use it in equation (16) to rewrite the nominal marginal cost as:

$$mc_t(i,j) = w_t + \frac{1-\epsilon}{\epsilon} y_t(i,j) - \frac{1}{\epsilon} a - log(\epsilon).$$

The optimal labor supply condition for the representative household is:

$$w_t - p_t = c_t, \tag{17}$$

and the linearized consumer demand in equation (13) is:

$$c_t(i,j) = -\widetilde{\eta}\,(p_t(i,j) - p_t(i)) - \eta\,(p_t(i) - p_t) + c_t + (\eta - 1)\,\theta_t(i), \tag{18}$$

where the sector-specific preference shock, $\theta_t(i)$, follows the AR(1) process:

$$\theta_t(i) = \rho_v \theta_{t-1}(i) + \widetilde{\epsilon}_t(i), \tag{19}$$

and $\widetilde{\epsilon}_t(i) \sim \mathcal{N}(0, (1-\epsilon)^{-2}\,(\eta-1)^{-2}\,\tau_t^2)$.[13]

We derive the optimal price-setting rule for firm (i, j) by using equations (17), (18), the equilibrium conditions, $y_t(i,j) = c_t(i,j)$, $y_t = c_t$, and the cash-in-advance constraint, $y_t = q_t - p_t$, which yields:[14]

$$p_t(i,j) = r_1 p_t(i) + r_2 p_t + (1 - r_1 - r_2)\, x_t(i) + \xi, \tag{20}$$

[13] Note that the information set is augmented with $p_s, \theta_s(i)$, and $\widetilde{\epsilon}_t(i)$. Namely, the following is the observed variables at time t: $\mathcal{H}_t(i) \equiv \left\{\{x_s(i)\}_{s=0}^{t}, \{p_s, q_s, u_s, v_s(i), \theta_s(i), e_s, \epsilon_s(i), \widetilde{\epsilon}_s(i)\}_{s=0}^{t-1}\right\}$. All propositions in the previous section continue to hold.

[14] Appendix D shows the derivation of the price setting rule.

where

$$\begin{aligned}
x_t(i) &= q_t + v_t(i), &(21)\\
v_t(i) &= (1-\epsilon)(\eta-1)\,\theta_t(i), &(22)\\
\xi &= \frac{\epsilon}{\epsilon+\widetilde{\eta}\,(1-\epsilon)}(\mu-\frac{1}{\epsilon}a-log(\epsilon)), &(23)\\
r_1 &= \frac{(\widetilde{\eta}-\eta)\,(1-\epsilon)}{\epsilon+\widetilde{\eta}\,(1-\epsilon)}, &(24)\\
r_2 &= \frac{(\eta-1)\,(1-\epsilon)}{\epsilon+\widetilde{\eta}\,(1-\epsilon)}, &(25)
\end{aligned}$$

and $p_t = \int_0^1 p_t(i)di$.[15] Equation (20) shows that the optimal pricing rule for firm (i,j) is a weighted average of the sectoral prices ($p_t(i)$), aggregate prices (p_t), and total sectoral demand ($x_t(i)$), which adds aggregate and sector-specific demand (i.e., $x_t(i) = q_t + v_t(i)$). The weights on each term of equation (20) are determined by the parameters r_1 and r_2, which reflect the degree of strategic complementarity among firms in the same sector and across sectors, respectively. Equation (21) shows that total demand ($x_t(i)$) additively combines the aggregate (q_t) and sector-specific components ($v_t(i)$). Equation (22) shows that the sector-specific demand depends on the sector-specific preference shock $\theta_t(i)$. The constant parameter ξ, defined by equation (23), is a linear transformation of the level of aggregate productivity, a. By normalizing aggregate productivity such that $\xi = 0$, the price level for firm (i,j) is uniquely determined by sector-specific and aggregate prices and total sectoral demand.[16]

Since firms in the same sector face the same marginal costs and have access to the same information, $p_t(i) = p_t(i,j) = p_t(i,j')$ for $j \neq j'$ in equilibrium, and equation (20) reduces to:

$$p_t(i) = rp_t + (1-r)\,x_t(i), \tag{26}$$

where

$$r \equiv \frac{r_2}{1-r_1} = \frac{(\eta-1)\,(1-\epsilon)}{\epsilon+\eta\,(1-\epsilon)}.$$

Equation (26) shows that the optimal pricing rule for firm (i,j) is a weighted average of aggregate prices (p_t) and total sectoral demand ($x_t(i)$). The weights for average prices and total sectoral demand are determined by the parameter r, which reflects the degree of strategic complementarity between firms in different sectors, consistent with equation (20).[17]

[15] Appendix C shows the derivation of the index of aggregate prices.

[16] Note that setting $\xi = 0$ is irrelevant for inflation since ξ affects the price level only.

[17] Equation (26) shows that if production technology converges to constant returns (i.e., $\epsilon \to 1$), average

3.2 Nominal Price Rigidities

To link expectations about total sectoral demand to the price-setting behavior of the firm, we enrich the model with nominal price rigidities that prevent firms from optimally adjusting prices in each period. In this environment, the optimal price depends on the expectations of future demand, which in our framework, reflects both the different aggregate and sector-specific components. Therefore, the co-movement between those expectations plays a critical role for the price-setting decision and ultimately inflation dynamics.

We embed nominal price rigidities, as in Calvo (1983), by assuming that a firm maintains the same price with exogenous probability $\theta \in (0,1)$ and otherwise changes the price optimally based on the expectations of demand. The optimal price for firms in sector i, denoted by $p_t^*(i)$, depends on expectations formed at time t on present and future prices, as described by the pricing rule:

$$\begin{aligned} p_t^*(i) &= (1-\beta\theta)\sum_{j=0}^{\infty}(\beta\theta)^j \mathbb{E}_t[p_{t+j}(i)] \\ &= (1-\beta\theta)\sum_{j=0}^{\infty}(\beta\theta)^j \left[r\mathbb{E}_t[p_{t+j}] \ + (1-r)\mathbb{E}_t[x_{t+j}(i)]\right], \end{aligned} \tag{27}$$

where the second equation is derived by substituting the optimal pricing rule in equation (26). Unlike standard full-information rational expectations models, the expectations in equation (27) are formed under imperfect information, and they are determined in accordance to Proposition 1. Equation (27) shows that each firm in sector i sets prices as a weighted average of the firm's expectations about current and *expected* future prices, and the expectations are formed based on the information available at time t. Since expectations about total sectoral demand ($\mathbb{E}_t[x_{t+j}(i)]$) depend on the different aggregate and sector-specific components of demand, as shown in equation (6), the co-movement of these components is critical to set the price.

The Equilibrium Average Price. Equation (27) provides the equilibrium average price once we derive the expectations for prices and total sectoral demand. The model is sufficiently

prices become less important in the determination of the price for firm i (i.e., $r \to 0$) since the marginal cost converges to the aggregate nominal wage across firms (i.e., $mc_t(i) \to w_t$) and heterogeneity in the firms' prices decreases. The magnitude of the sector-specific shock decreases (i.e., $v_t(i) \to 0$) as the production technology converges to constant returns (i.e., $\epsilon \to 1$). As a result, in the limiting case of a linear production technology (i.e., $\epsilon = 1$), the optimal pricing rule is $p_t(i) = q_t + \xi$.

simple to provide an analytical solution for the equilibrium average price, characterized in the next proposition.

Proposition 4 *The equilibrium average price and sectoral price are given by:*

$$p_t = [\theta + (1-\theta)a_1]\, p_{t-1} + (1-\theta)\, a_2 q_t + (1-\theta)\, a_3 q_{t-1} + (1-\theta)\, a_4 u_{t-1}, \tag{28}$$

$$p_t(i) = p_t + (1-\theta)\, a_2 v_t(i) + a_5 v_{t-1}(i) \tag{29}$$

where $(a_1, a_2, a_3, a_4, a_5)$ *are non-linear functions of the ratio in the volatility of sector-specific to aggregate shocks* (τ_t/σ_t)*.*

Proof: See Appendix E.4. □

Equations (28) and (29) show that the equilibrium aggregate and sectoral price depends on the equilibrium price in the period $t-1$ (p_{t-1}) and the sequence of present and past demands $(q_t, v_t(i), q_{t-1}, v_{t-1}(i))$. Important to our subsequent analysis, the proposition shows that the relative volatility of sector-specific shocks compared to aggregate shocks, encapsulated by the ratio τ_t/σ_t, plays a critical role for the sensitivity of the aggregate price to present and past aggregate demands, as we study in the next section.

4 Demand Shocks and Inflation Dynamics

Using the definition of the average price in equation (28), we derive the analytical solution for the inflation rate, defined as the change in the average price from period $t-1$ to period t ($\pi_t \equiv p_t - p_{t-1}$), as characterized by the next proposition.

Proposition 5 *Under imperfect information on aggregate and sector-specific demand shocks, sectoral and average price inflation are equal to:*

$$\begin{aligned} \pi_t &= [\theta + (1-\theta)a_1]\, \pi_{t-1} + (1-\theta)\, a_2 u_t + (1-\theta)\,(a_3 + a_4)\, u_{t-1} - (1-\theta)\, a_4 u_{t-2} \\ &= \alpha_1 \pi_{t-1} + \alpha_2 u_t + \alpha_3 u_{t-1} + \alpha_4 u_{t-2}, \end{aligned} \tag{30}$$

$$\pi_t(i) = \pi_t + (1-\theta)\, a_2 \widetilde{v}_t(i) + a_5 \widetilde{v}_{t-1}(i) = \pi_t + \alpha_2 \widetilde{v}_t(i) + \alpha_5 \widetilde{v}_{t-1}(i), \tag{31}$$

where $\alpha_1 \equiv \theta + (1-\theta)a_1$*,* $\alpha_2 \equiv (1-\theta)a_2$*,* $\alpha_3 \equiv (1-\theta)(a_3 + a_4)$*,* $\alpha_4 \equiv -(1-\theta)a_4$*, and* $\alpha_5 \equiv (1-\theta)a_5$*.*

Proof: Taking the first difference of the equations (28) and (29) yields equations (30) and (31), respectively.□

Equations (30) and (31) provide the analytical solution for aggregate and sectoral inflation under imperfect information, respectively. Equation (30) shows that current inflation (π_t) depends on past inflation (π_{t-1}) and current and past changes in aggregate demand (u_t, u_{t-1}, and u_{t-2}, respectively), stemming from the assumption that demand in the past period $t-1$ is fully revealed in the current period t.[18] Similarly, equation (31) shows that current sectoral inflation ($\pi_t(i)$) depends on past average inflation (π_{t-1}) and current changes in total sectoral demand and past changes in aggregate and sector-specific demand ($\widetilde{x}_t(i)$, u_{t-1}, and $\widetilde{v}_{t-1}(i)$, respectively). The effect of τ_t/σ_t on the coefficients ($\alpha_2, \alpha_3, \alpha_4, \alpha_5$) is non-linear, and it interacts with the degree of nominal price rigidities θ.[19] Proposition 5 shows that if prices are flexible ($\theta = 0$), the parameter α_1 is equal to zero, showing that nominal price rigidities are the main driver of inflation persistence in this reduced form inflation dynamics. Since the effect of τ_t/σ_t on coefficients for equations (30) and (31), $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, and α_5, is highly non-linear and interplays with the degree of nominal price rigidities, we rely on numerical simulations to study the sensitivity of inflation to demand, developed in the next subsection.

We derive the Phillips curve under the simplified assumption $\rho_u = \rho_v = 0$.

Corollary 1 *Suppose $\rho_u = \rho_v = 0$. Phillips curve is given as follows:*

$$\pi_t = \frac{\alpha_2}{1-\alpha_2} y_t + \frac{\alpha_3}{1-\alpha_2} y_{t-1}, \tag{32}$$

Proof: See Appendix E.5.□

Corollary 1 shows the equation for aggregate inflation which has no term of lagged inflation (i.e. inflation persistence), as in the standard New Keynesian Phillips curve.[20] The lagged output gap emerges because firms face imperfect information about current economic variables and thus the expectations depend on past economic variables.

[18] The dynamics for inflation is related to Angeletos and La'O (2009), but it differs across two important dimensions. First, the coefficients ($\alpha_2, \alpha_3, \alpha_4, \alpha_5$) depend on the volatility of sector-specific shocks (τ^2), and second, inflation depends on the changes in demand two periods before u_{t-2} since aggregate shocks are persistent.

[19] See Appendix E.4 for the characterization of parameters a_1, a_2, a_3, a_4.and a_5.

[20] Inflation expectations vanish from the equation for inflation since they are determined by the linear combination of current and past economic variables in the information structure of the model.

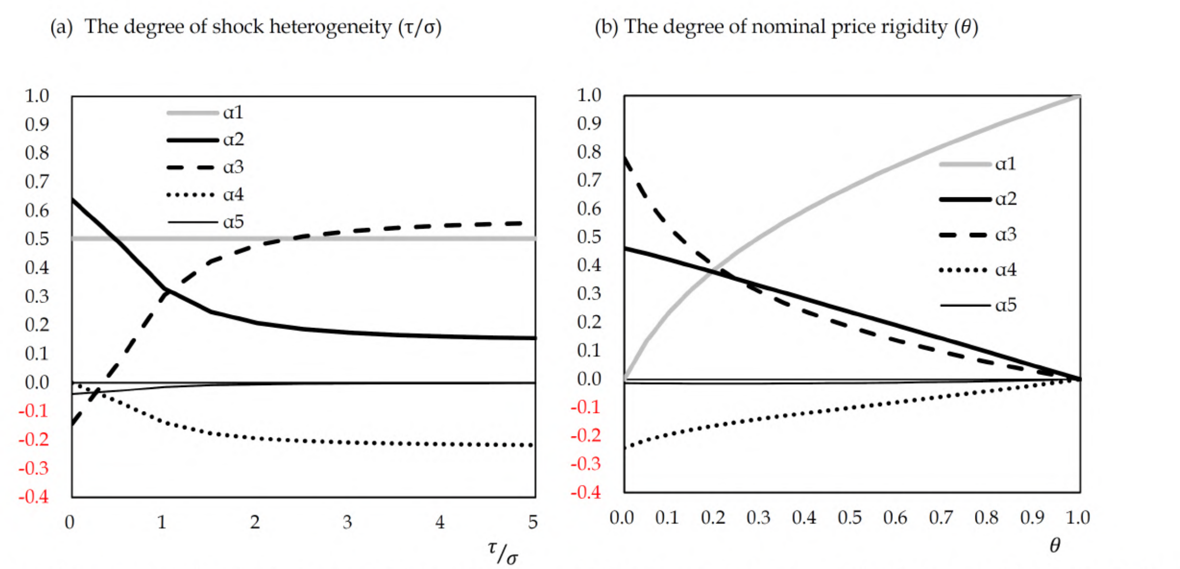
4.1 Numerical Simulations

The model shows that imperfect information makes the response of average and sectoral inflation to demands a non-linear function of the ratio of volatility of the sector-specific to aggregate shock (τ_t/σ_t) and the degree of nominal rigidities (θ), which jointly determine the response of inflation to demand, as encapsulated by the coefficients α_1, α_2, α_3, α_4, and α_5 in equations (30) and (31). In this section, we use numerical simulations to study the sensitivity of inflation to demand.

Sensitivity of Inflation to Changes in Demand. We simulate the model using a standard calibration. We set $\beta = 0.99$, $\eta = 8$, $\epsilon = 2/3$, and $r = [(\eta-1)(1-\epsilon)]/[\epsilon+\eta(1-\epsilon)] = 0.7$.

To investigate the role of shock heterogeneity, we allow the ratio $\tau_t/\sigma_t \in [0, 5]$ to cover a wide range of values. We will estimate this ratio in the next section. Similarly, we allow the degree of nominal price rigidity $\theta \in [0, 1]$ to cover the whole range of admissible values. We set the parameters for the persistence of aggregate and sector-specific shocks equal to $\rho_u = 0.45$ and $\rho_v = -0.08$ to replicate the estimates of first-order auto-correlation in Table 10 for both the aggregate and the median of sector-specific components of demand.

Figure 1: Sensitivity of coefficients



Notes: Parameters are $\tau/\sigma = 1$, $r = 0.7$, $\beta = 0.99$, $\rho_u = 0.45$, $\rho_v = -0.08$ *for (a), and* $\theta = 0.3$ $r = 0.7$, $\beta = 0.99, \rho_u = 0.45$, $\rho_v = -0.08$ *for (b).*

Figure 1 in panel (a) shows the coefficients α_1, α_2, α_3, α_4, and α_5 for different values of the relative volatility of sector-specific shocks (i.e., τ_t/σ_t). The coefficient α_1 on past inflation is insensitive to τ_t/σ_t, evincing that the relative volatility of sector-specific shocks plays no role in the relation between current inflation and past inflation, which instead is determined by the degree of nominal price rigidities, as we discuss below. The coefficient α_2 on current aggregate and sector-specific demand is instead highly sensitive to the relative volatility of sector-specific shocks, and inflation becomes less responsive to changes in current demand (i.e., α_2 decreases) when τ_t/σ_t increases. Strategic complementarity in the optimal price-setting, encapsulated by $r > 0$ in equation (26), induces the firm to hold the adjustment of prices if it attributes that the change in total sectoral demand is generated by the sector-specific component. Therefore, *ceteris paribus*, an increase in the volatility of the sector-specific component of demand decreases the response of prices to changes in total sectoral demand. The coefficient α_3 (past lag of aggregate demand) increases while the coefficient α_4 (past two lags of aggregate demand) and α_5 (past lag of sector-specific demand) decrease in response to the increase in τ_t/σ_t. The response of inflation is on average more sensitive to movements in past lags of demand. Overall, the numerical simulations show that the parameter α_2, which internalizes the effect of changes in τ_t/σ_t, plays a critical role in the sensitivity of inflation to demand.

Figure 1 in panel (b) shows the sensitivity of coefficients α_1, α_2, α_3, α_4, and α_5 to changes in the degree of nominal price rigidity (θ) in the inflation equation (30). The increase in nominal price rigidities generates a rise in the coefficient α_1 since a low frequency of price adjustment increases the importance of past inflation in the determination of current inflation. The increase in the degree of nominal price rigidity generates a decrease in the absolute value of the coefficients α_2, α_3, α_4 and α_5 since the sensitivity of individual prices to movements in current demand is lowered by the increase in nominal price rigidity (θ). [21]

4.2 Empirical Analysis on the Aggregate Inflation Dynamics

This section estimates the ratio of the volatility of the sector-specific component to the aggregate component of demand using principal component analysis on Japanese data. It then tests the empirical relevance of the increases in the relative volatility of sector-specific shocks for the reduced sensitivity of aggregate inflation to changes in aggregate demand.

Estimation of τ_t/σ_t. To estimate the ratio τ_t/σ_t, we derive the variances for the changes in the aggregate and sector-specific components of demand (σ_t^2 and τ_t^2, respectively). We proxy

[21] Appendix L.1 shows the impulse response function of the inflation to aggregate demand.

changes in aggregate demand by the principal component of the movements in sales growth across sectors, following the approach in Boivin et al. (2009). We use quarterly data on sector-level sales of Japanese firms from the Financial Statements Statistics of Corporations by Industry, compiled by the Ministry of Finance of Japan. The data cover the period 1975:Q3-2018:Q3 for 29 major sectors in the economy.[22]

We proxy the changes in the aggregate component of demand with sales, u_t, by the first principal component of $\widetilde{x}_t(i)$ across sectors, $i \in \{1, 2, ..., 29\}$, by calculating it as $u_t = (\sum_{i=1}^{29} \Lambda_i)_{i=1}^{-1} \sum_{i=1}^{29} \Lambda_i \widetilde{x}_t(i)$, where Λ_i is the loading factor of $\widetilde{x}_t(i)$ and the term $\left(\sum_{i=1}^{29} \Lambda_i\right)^{-1}$ normalizes $\sum_{i=1}^{29} \widetilde{x}_t(i)$.[23] We proxy sector-specific demand, $\widetilde{v}_t(i)$, by subtracting the estimated principal component from changes in total sectoral demand:[24] $\widetilde{x}_t(i) - u_t = \widetilde{x}_t(i) - (\sum_{i=1}^{29} \Lambda_i)^{-1} \sum_{i=1}^{29} \Lambda_i \widetilde{x}_t(i)$.[25]

We proxy the variance of aggregate fluctuations, σ_t^2, with the average of the square of residuals of equation (3) for alternative moving windows of size $2k + 1$:

$$\sigma_t{}^2 = \frac{1}{2k+1} \sum\nolimits_{s=-k}^{k} \widehat{e}_{t-s}^2. \tag{33}$$

Similarly, we proxy the variance of the sector-specific fluctuations, $\tau_t{}^2$, with the average of the square of the averages of the residuals of (4) across sectors for alternative moving windows of size $2k + 1$:

$$\tau_t{}^2 = \frac{1}{2k+1} \sum\nolimits_{s=-k}^{k} \left(\frac{1}{29} \sum\nolimits_{i=1}^{29} \frac{(\widehat{\epsilon}_{t-s}(i) - \widehat{\epsilon}_{t-s-1}(i))^2}{2} \right). \tag{34}$$

To ensure robustness of results across the different time windows, we compute the variance of each of the shocks in equations (33) and (34), using four alternative time windows: two

[22] Appendix H provides a description of the data.

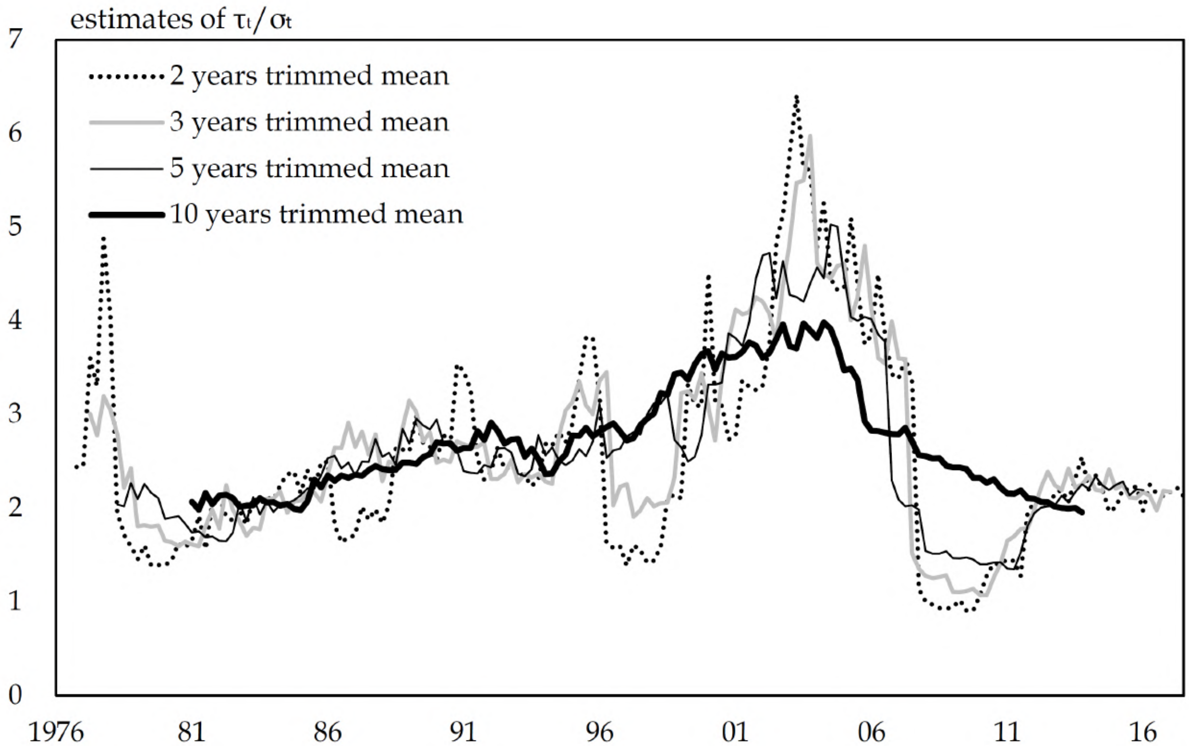
[23] The proportion of the variance of the first component is around 19%, which is considerably larger than the variance of the second component (7%), suggesting that the second principal component plays a limited role in aggregate shocks. Note that since the principal component is $\sum_{i=1}^{29} \Lambda_i \widetilde{x}_t(i)$ and changes in sectoral demand are $\widetilde{x}_t(i)$, the scale of the principal component $\sum_{i=1}^{29} \Lambda_i$ may differ from the scale of changes in sectoral demand. Estimation results reveal that $\sum_{i=1}^{29} \Lambda_i \approx 4.7$, which we use to normalize the principal component.

[24] To ensure results are robust to alternative normalization, we implement alternative specifications. First, we define $u_t = \sum_{i=1}^{29} \Lambda_i \widetilde{x}_t(i)$ and $\widetilde{x}_t(i) - u_t$, and second, we define $u_t = \left(\sum_{i=1}^{29} \Lambda_i\right)^{-1} \sum_{i=1}^{29} \Lambda_i \widetilde{x}_t(i)$ and $\widetilde{x}_t(i) - u_t$. Results remain unchanged across different normalization assumptions.

[25] Appendix I discusses the methodology we use to extract the sequence of shocks on aggregate and sector-specific components of total sectoral demand, and it provides summary statistics on the volatility of aggregate and sectoral-specific demand shocks. Appendix J shows that the changes in the series for aggregate demand extracted from the industry-level data are representative of aggregate movements in demand. Our series closely co-move with the average of industry-level data and with the measure of the output gap from the Bank of Japan that several studies use as a proxy for changes in aggregate demand.

years ($k = 4$), three years ($k = 6$), five years ($k = 10$), and ten years ($k = 20$), excluding the upper and lower 10% of the samples as outliers. Finally, we measure shock heterogeneity as the ratio of the square root of the estimate of the variance of sector-specific shocks (τ_t) to that of aggregate shocks (σ_t).

Figure 2: Estimates of shock heterogeneity (τ_t/σ_t)



Notes: Upper and lower 10% of the samples are excluded in estimation.
Source: Ministry of Finance "Financial statements statistics of corporations by industry".

Figure 2 shows the estimated series for the ratio of the variance of sector-specific shocks to the variance of aggregate shocks (τ_t/σ_t) for the alternative time windows. Entries show that the ratio τ_t/σ_t substantially varies throughout the sample period, rising steadily from a value of 2 in the mid-1980s to 4 in the mid-2000s and returning quickly to a value of approximately 2 after 2010 for the 10-year window. The shorter the time window, the larger the volatility, but the overall dynamics of the changes are similar across the alternative estimates. Overall, the analysis establishes substantial changes in the τ_t/σ_t ratio during the sample period.[26]

[26]Movements in τ_t/σ_t are primarily driven by changes in the volatility of sector-specific demand shocks (τ_t) while the value for volatility of aggregate demand shock (σ_t) remains broadly stable across the sample period, except during the period of the global financial crisis (2007:4Q to 2010:1Q).

Sensitivity of Inflation to Aggregate Demand. We use our proxy for the ratio τ_t/σ_t to study the empirical relevance of the increase (decrease) in the ratio for the reduced (increased) sensitivity of inflation to changes in aggregate demand.

We set up the empirical model using the insights from the price equation (30) that accounts for the effect of information frictions in the relation between inflation and aggregate demand. We regress current inflation (π_t) on past inflation (π_{t-1}),[27] changes in current aggregate demand (u_t), an interaction term between past inflation and the volatility ratio between sector-specific and aggregate shocks ($\pi_{t-1} \times \tau_t/\sigma_t$), and an interaction term between changes in current aggregate demand and the volatility ratio between sector-specific and aggregate shocks ($u_t \times \tau_t/\sigma_t$). The interaction terms $\pi_{t-1} \times \tau_t/\sigma_t$ and $u_t \times \tau_t/\sigma_t$ capture the differential effect of the ratio τ_t/σ_t for the effect of past inflation and aggregate demand on current inflation, respectively. In line with the theoretical model, we include aggregate demand with two lags and control for the degree of nominal price rigidities, motivated by the fact the comparative statics in the model described in section 4.1 show that the higher degree of nominal price rigidity increases the persistence of inflation and reduces the sensitivity of current inflation to changes in current aggregate demand. Specifically, we use an indicator variable equal to 1 for the period 2000-2018 ($\mathbf{1}_{\{2000-2018\}}$) when nominal price rigidities decrease (see evidence in Sudo et al., 2014 and Kurachi et al., 2016), and we enrich the estimation of the price equation with two additional interaction terms. The first term interacts the indicator variable for nominal price rigidities with past inflation ($\pi_{t-1} \times \mathbf{1}_{\{2000-2018\}}$) to capture the interplay between the degree of nominal price rigidity and the effect of past inflation on current inflation. The second term interacts the indicator variable for nominal price rigidities with current aggregate demand ($u_t \times \mathbf{1}_{\{2000-2018\}}$) to capture the interplay between nominal price rigidities and current aggregate demand. The empirical specification of the price inflation is summarized by the following equation:

$$\begin{aligned}\pi_t =& c_1 + \underbrace{\left(c_2 + c_3\mathbf{1}_{\{2000-2018\}} + c_4\left(\tau_t/\sigma_t\right)\right)}_{\alpha_1}\pi_{t-1} + \underbrace{\left(c_5 + c_6\mathbf{1}_{\{2000-2018\}} + c_7\left(\tau_t/\sigma_t\right)\right)}_{\alpha_2}u_t \\ &+ c_8 u_{t-1} + c_9 u_{t-2} + \varepsilon_t^c, \end{aligned} \tag{35}$$

where the coefficients $c_1, \ldots, c_9$ are regression coefficients, and ε_t^c is the error term.

Table 3 shows the estimates for equation (35), using the τ_t/σ_t ratio based on time-windows of two years (column 1), three years (columns 2), five years (column 3), and ten

[27] We use quarterly changes in consumer price index as a proxy for aggregate inflation. The CPI is from the Japanese Statistics Bureau and available here https://www.stat.go.jp/english/data/cpi/index.html

Table 3: Estimation of inflation dynamics

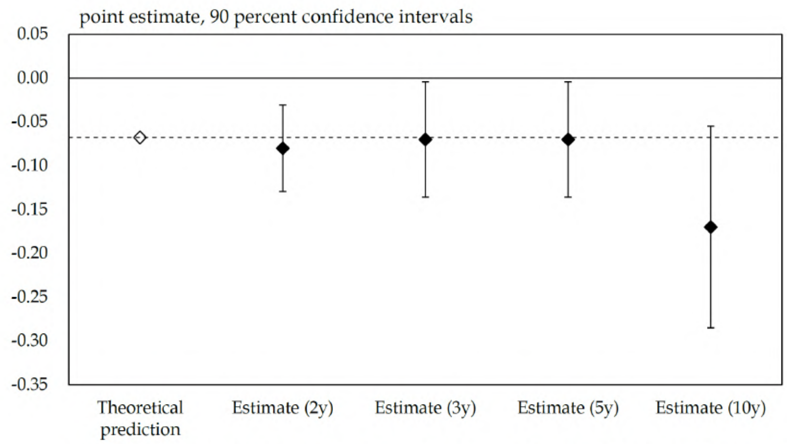
Dataset: Financial statement statistics of corporations by industry, consumer price index; 29 sectors; 1976/2Q-2018/3Q				
Dependent Variable: Inflation rate (π_t, core consumer price index, seasonally adjusted, QoQ, annualized)				
	2 years trimmed mean	*3 years trimmed mean*	*5 years trimmed mean*	*10 years trimmed mean*
Constant	0.09	0.08	0.06	0.05
	(0.09)	(0.09)	(0.08)	(0.07)
Lag of inflation (π_{t-1})	0.72 ***	0.62 ***	0.71 ***	0.64 ***
	(0.12)	(0.16)	(0.17)	(0.23)
Lag of inflation×time dummy (2000-2018)	-0.33 ***	-0.32 ***	-0.35 ***	-0.38 ***
($\pi_{t-1} \times 1_{\{2000-2018\}}$)	(0.11)	(0.11)	(0.11)	(0.11)
Lag of inflation	0.03	0.06	0.03	0.06
×shock heterogeneity ($\pi_{t-1} \times {}^{\tau_t}/_{\sigma_t}$)	(0.03)	(0.05)	(0.07)	(0.09)
Changes in aggregate demand (u_t)	0.37 ***	0.37 ***	0.38 **	0.63 ***
	(0.11)	(0.13)	(0.15)	(0.22)
Changes in aggregate demand ×time	-0.13	-0.13	-0.13	-0.08
dummy (2000-2018) ($u_t \times 1_{\{2000-2018\}}$)	(0.09)	(0.09)	(0.10)	(0.09)
Changes in aggregate demand	**-0.08 *****	**-0.07 ***	**-0.07 ***	**-0.17 ****
×shock heterogeneity ($\boldsymbol{u_t \times {}^{\tau_t}/_{\sigma_t}}$)	(0.03)	(0.04)	(0.04)	(0.07)
Observations	170	170	170	170
Adjusted-R^2	0.80	0.80	0.80	0.80

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. First and second lags of changes in aggregate demand are included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity. The series for the core consumer price index is "all items, less fresh food (impact of consumption taxes are adjusted)".
**** Significant at the 1 percent level. ** Significant at the 5 percent level.* Significant at the 10 percent level.*

years (column 4), respectively. All entries show that current inflation is positively correlated with past inflation and current demand, consistent with the theoretical prediction in the price equation (28). The estimation also shows that the coefficient for the interaction term of past inflation with the indicator variable ($\pi_{t-1} \times \mathbf{1}_{\{2000-2018\}}$) is negative and that for the interaction term of past inflation with shock heterogeneity is not significant, indicating that the positive correlation between current inflation and past inflation decreases with a decline in nominal price rigidities, again in line with the predictions of our model. The estimates for the interaction term of changes in demand with the indicator variable ($u_t \times \mathbf{1}_{\{2000-2018\}}$) are insignificant for all proxies of the τ_t/σ_t ratio. Important for our analysis, the interaction term between aggregate demand and the degree of shock heterogeneity ($u_t \times \tau_t/\sigma_t$) is negative and significant, implying that a rise in the τ_t/σ_t ratio reduces the positive correlation between inflation and aggregate demand, in accordance with the results of our analysis.

Figure 3 compares the estimates for the coefficient c_7 on the interaction term ($u_t \times \tau_t/\sigma_t$) for the alternative time windows of 2, 3, 5, and 10 years for the computation of the variance (dark diamond) against the the coefficient α_2 on the interaction term $u_t \times \tau_t/\sigma_t$ in equation (30), which represents the theoretical interaction between shock heterogeneity and aggregate demand (white diamond). The bands for the dark diamond show 90 percent confidence intervals of the empirical estimates. The figure shows that the estimates from the data

Figure 3: Shock heterogeneity and sensitivity of inflation to changes in aggregate demand



Notes: Parameters for theoretical prediction are $\theta = 0.3,\ r = 0.7,\ \beta = 0.99, \rho_u = 0.45,\ \rho_v = -0.08.$

are remarkably close to those generated by the theoretical model. Thus our theoretical framework is quantitatively consistent with the estimates in the data.[28]

Finally, to ensure that decline in nominal price rigidities is not driving the significance of the negative relation between τ_t/σ_t and inflation, Table 4 presents results for the benchmark regression that abstracts from the indicator variable $\mathbf{1}_{\{2000-2018\}}$ by omitting the interaction term between past inflation and the indicator variable (i.e., $\pi_{t-1} \times \mathbf{1}_{\{2000-2018\}}$) and the interaction term between changes in demand and the indicator variable ($u_t \times \mathbf{1}_{\{2000-2018\}}$) from equation (35). The regression coefficient on the term $u_t \times (\tau_t/\sigma_t)$ (bold entry) remains significant and negative, as in the benchmark regression

Our results suggest that the imperfect information on sectoral demand, together with the changes in shock heterogeneity, has contributed to the time-variation in the sensitivity of inflation to the aggregate demand shock in Japan.[29][30]

[28]For the robustness check, we run the regression in equation (35) enriched by an interaction term between aggregate demand and the sum of the variances of the aggregate and sector-specific components of sectoral demand ($u_t \times (\sigma_t + \tau_t)$). We also run the regression based on changes in inflation as specified by equation (35). In both cases, the interaction term between aggregate demand and the degree of shock heterogeneity ($u_t \times \tau_t/\sigma_t$) is negative and significant.

[29]Since shock heterogeneity has increased in the late 1990s, our result is relevant for the flattening of the Philips curve in Japan during the same period (see recent studies by Kaihatsu et al., 2017 and Bundick and Smith, 2020).

[30]Appendix L.2 shows the estimated impulse response of inflation to aggregate demand.

Table 4: Estimation of inflation dynamics

Dataset: Financial statement statistics of corporations by industry, consumer price index; 29 industries; 1976/2Q-2018/3Q				
Dependent Variable: Inflation rate (π_t, core consumer price index, seasonally adjusted, QoQ)				
	2 years trimmed mean	3 years trimmed mean	5 years trimmed mean	10 years trimmed mean
Constant	0.15 **	0.15 **	0.13 **	0.10
	(0.07)	(0.07)	(0.07)	(0.06)
Lag of inflation (π_{t-1})	0.69 ***	0.69 ***	0.57 ***	0.92 ***
	(0.12)	(0.12)	(0.16)	(0.26)
Lag of inflation	0.04	0.04	0.08	-0.08
×shock heterogeneity ($u_t \times {}^{\tau_t}/_{\sigma_t}$)	(0.04)	(0.04)	(0.06)	(0.11)
Changes in aggregate demand (u_t)	0.29 ***	0.29 ***	0.27 ***	0.61 ***
	(0.08)	(0.08)	(0.10)	(0.22)
Changes in aggregate demand	**-0.07** **	**-0.07** **	**-0.05**	**-0.18** **
×shock heterogeneity ($u_t \times {}^{\tau_t}/_{\sigma_t}$)	(0.03)	(0.03)	(0.04)	(0.08)
Observations	170	170	170	170
Adjusted-R^2	0.79	0.79	0.78	0.79

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. First and second lags of changes in aggregate demand are included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity. The series for the core consumer price index is "all items, less fresh food (impact of consumption taxes are adjusted)".
**** Significant at the 1 percent level. ** Significant at the 5 percent level.* Significant at the 10 percent level.*

4.3 Empirical Analysis on Sectoral Inflation Dynamics

This section estimates the ratio between the volatility of the sector-specific component and the aggregate component of demand for each sectors. It then tests the empirical relevance of the increases in the relative volatility of sector-specific shocks for the reduced sensitivity of sectoral inflation to changes in sector-specific demand.

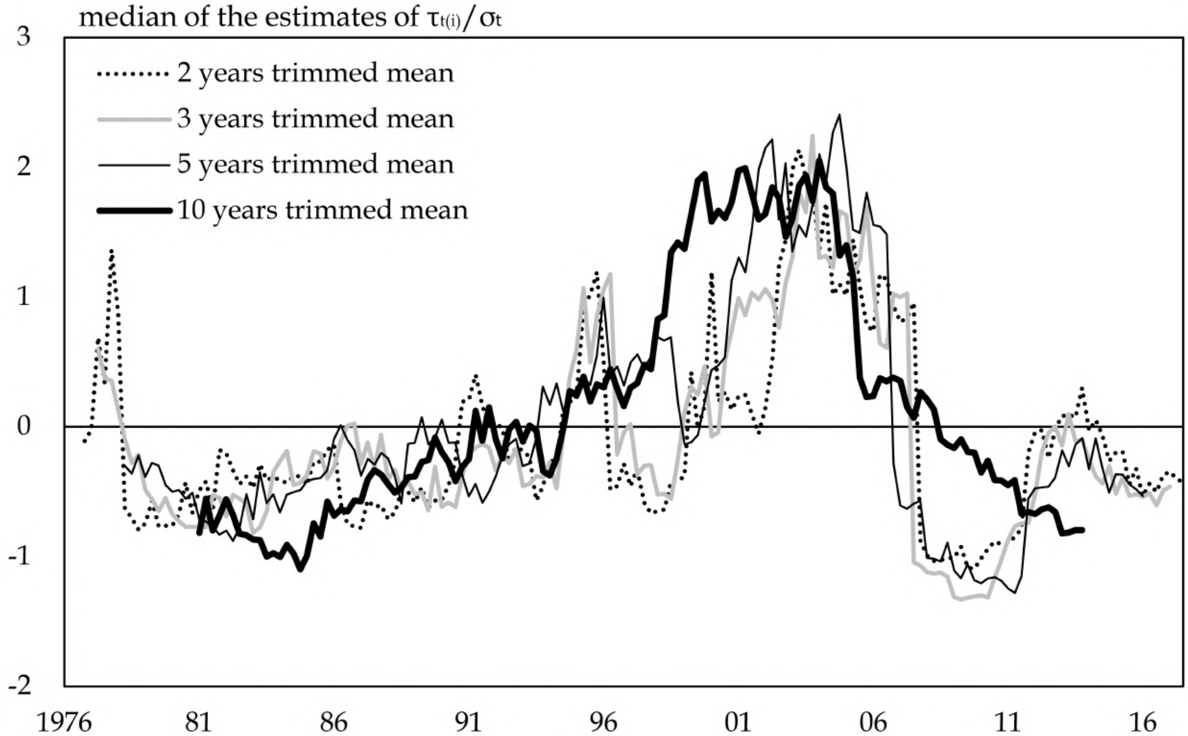
Estimation of $\tau_t(i)/\sigma_t$. To estimate the proxy for the shock heterogeneity in each sector, i.e. the ratio $\tau_t(i)/\sigma_t$, we follow the methodology in the previous section except that we do not take averages across sectors in equation (34) so that we can estimate heterogeneous $\tau_t(i)$. We also make the series in each sector standardized in that the average of the series is transformed to zero and the standard deviation is transformed to one. To match the data on shock heterogeneity with the sectoral inflation, we consider series for 13 manufacturing industries.[31]

Figure 4 shows the median of the 13 estimated series for the ratio of the variance of sector-specific shocks to the variance of aggregate shocks ($\tau_t(i)/\sigma_t$) for the alternative time windows: two years (k=4), three years (k=6), five years (k=10), and ten years (k=20). Similar to the developments in figure 2, entries show that the ratio $\tau_t(i)/\sigma_t$ substantially

[31]Specifically, the following sectors are included in our dataset: Foods, Textiles and Apparels, Wood Products, Pulp and Paper, Chemicals, Oil and Coal Products, Glass and Ceramics Products, Iron and Steel, Nonferrous Metals, Metal Products, Machinery, Electric Devices, and Transportation Equipment.

varies throughout the sample period.

Figure 4: Estimates of shock heterogeneity in each sector ($\tau_t(i)/\sigma_{tt}$)



Notes: Upper and lower 10% of the samples are excluded in estimation.
Source: Ministry of Finance "Financial statements statistics of corporations by industry".

Sensitivity of Sectoral Inflation to Sector-specific Demand. Equation (31) shows that the sensitivity of the sectoral inflation ($\pi_t(i)$) to changes in sector-specific demand ($\widetilde{v}_t(i)$) depends on α_2, which we know from our previous analysis in section 4.1 is negatively related to shock heterogeneity ($\tau_t(i)/\sigma_t$). In what follows, we investigates whether the model predictions are supported in the data.

To estimate the relation between the degree of shock heterogeneity and the sensitivity of the sectoral inflation to sector-specific demand, we follow the insights from the theoretical model, encapsulated by equation (31), and construct a panel dataset for the sectoral inflation rates ($\pi_t(i)$), sector-specific demand in each sector ($\widetilde{v}_t(i)$), and the measures for shock heterogeneity ($\tau_t(i)/\sigma_t$) that is heterogeneous across sectors. We use measures for aggregate inflation π_t, quarterly changes in consumer price index from Japanese Statistics Bureau, $\widetilde{v}_t(i)$ and $\tau_t(i)/\sigma_t$ from the Financial Statements Statistics of Corporations by Industry prepared by the Ministry of Finance, and we measure sectoral inflation $\pi_t(i)$ with the Producer Price index (PPI) in Japan, which is released by the Bank of Japan on a monthly basis.[32]

$$\pi_t(i) - \pi_t = d_1(i) + \underbrace{\left(d_2 + d_3 \mathbf{1}_{\{2000-2018\}} + d_4 \left(\tau_t(i)/\sigma_t\right)\right)}_{\alpha_2} \widetilde{v}_t(i) + d_5 \widetilde{v}_{t-1}(i) + \varepsilon_t^d, \tag{36}$$

where $d_1(i)$ is fixed effect indicator variable, parameters d_2-d_5 are regression coefficients, $\mathbf{1}_{\{2000-2018\}}$ is an indicator variable equal to 1 for the period 2000-2018 to control for the years with exogenous fall in price stickiness, as in our benchmark specification, and ε_t^d is the error term.

Table 5: Estimation of the sectoral inflation dynamics

Dataset: Financial statement statistics of corporations by industry, producer price index; 13 sectors; 1985/2Q-2018/3Q				
Dependent Variable: sector-specific component of inflation rate ($\pi_t(i) - \pi_t$, seasonally adjusted, QoQ, annualized)				
	2 years trimmed mean	*3 years trimmed mean*	*5 years trimmed mean*	*10 years trimmed mean*
Changes in sector-specific demand ($\widehat{v}_t(i)$)	0.47 ***	0.46 ***	0.41 ***	0.38 ***
	(0.09)	(0.09)	(0.09)	(0.09)
Changes in sector-specific demand ($\widehat{v}_t(i)$)	0.10	0.09	0.11	0.13
×time dummy (2000-2018)	(0.10)	(0.10)	(0.10)	(0.10)
Changes in sector-specific demand	**-0.11 *****	**-0.11 *****	**-0.08 ****	**-0.04**
($\widehat{v}_t(i)$) ×shock heterogeneity	(0.03)	(0.03)	(0.03)	(0.04)
Observations	1779	1779	1779	1779
Adjusted-R^2	0.11	0.10	0.10	0.10

Note: Estimated by ordinary-least-squares with fixed effect models. The standard errors are HAC estimators. First lag of sector-specific demand is included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity.
**** Significant at the 1 percent level. ** Significant at the 5 percent level.* Significant at the 10 percent level.*

Table 5 shows the estimates for equation (36) for alternative measures of shock heterogeneity based on time windows of two years (column 1), three years (columns 2), five years (column 3), and ten years (column 4), respectively. All entries show that sectoral inflation is positively correlated with current sector-specific demand ($\widetilde{v}_t(i)$). Important for our analysis, the interaction term between sector-specific demand and the degree of shock heterogeneity ($\widetilde{v}_t(i) \times \tau_t(i)/\sigma_t$) is negative in all entries and significant in most entries. Our results show that the data supports a decrease in the sensitivity of the sectoral inflation in response to a raise in shock heterogeneity, consistent with the prediction in our theoretical model.

To ensure that decline in nominal price rigidities is not driving the significance of the negative relation between $\tau_t(i)/\sigma_t$ and inflation, Table 6 presents results for the benchmark regression that abstracts from the indicator variable $\mathbf{1}_{\{2000-2018\}}$ by omitting the interaction

[32]For details, see https://www.boj.or.jp/en/statistics/pi/cgpi_release/index.htm/. For the summary statistics of the PPI data, see Appendix K.

term between past inflation and the indicator variable (i.e., $\pi_{t-1} \times \mathbf{1}_{\{2000-2018\}}$) and the interaction term between changes in demand and the indicator variable ($\widetilde{v}_t(i) \times \mathbf{1}_{\{2000-2018\}}$) from equation (36). The regression coefficient on the term $\widetilde{v}_t(i) \times (\tau_t(i)/\sigma_t)$ (bold entry) remains significant and negative, as in the benchmark regression.[33]

Table 6: Estimation of the sectoral inflation dynamics

Dataset: Financial statement statistics of corporations by industry, producer price index; 13 sectors; 1985/2Q-2018/3Q

Dependent Variable: sector-specific component of inflation rate ($\pi_t(i) - \pi_t$, seasonally adjusted, QoQ, annualized)

	2 years trimmed mean	*3 years trimmed mean*	*5 years trimmed mean*	*10 years trimmed mean*
Changes in sector-specific demand ($\hat{v}_t(i)$)	0.54 ***	0.52 ***	0.49 ***	0.47 ***
	(0.05)	(0.05)	(0.05)	(0.05)
Sector-specific demand ($\hat{v}_t(i)$)	**-0.11 *****	**-0.11 *****	**-0.08 ****	**-0.03**
×shock heterogeneity	(0.03)	(0.03)	(0.03)	(0.04)
Observations	1779	1779	1779	1779
Adjusted-R^2	0.11	0.10	0.10	0.10

Note: Estimated by ordinary-least-squares with fixed effect models. The standard errors are HAC estimators. First lag of sector-specific demand is included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity.
**** Significant at the 1 percent level. ** Significant at the 5 percent level.* Significant at the 10 percent level.*

5 Conclusion

Our study shows that imperfect information and shock heterogeneity play an important role on the expectations of firms. We use new sector-level survey data for the universe of Japanese firms to establish a positive co-movement in the expectations of aggregate and sector-specific components of demand. We show that imperfect information on the current components of demand is important to reproduce the observed positive co-movement in expectations. Our theoretical model shows that an increase in the volatility of sector-specific shocks relative to aggregate shocks reduces the sensitivity of inflation to aggregate demand. We test and corroborate this theoretical prediction using sector-level sales data for Japanese firms across 29 sectors.

Our study opens important avenues for future research. An interesting question left unanswered is the origins of the changes in the volatility of sector-specific shocks. Do the changes in the volatility of demand shocks reflect developments in the efficiency of production that reduce movements in relative prices, or are they a byproduct of the variations in the interlinkages among firms that are unaccounted in our standard model? Should monetary policy use strategically the attenuating effect of shock heterogeneity on the sensitivity of

[33] Appendix M examines the sensitivity of sectoral inflation to changes in total sectoral demand using the same dataset. The estimation results confirm the theoretical prediction.

inflation to achieve price stability? We plan to pursue some of these questions in future work.

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A Derivation of Demand Functions and Price Indexes

A.1 Demand Functions

The representative household first determines the allocation of consumption across sectors and then determines that to goods in each sector taking the expenditure level to each sector as given.

Define the expenditure level by $Z_t \equiv \int_0^1 P_t(i)C_t(i)di$, the Lagrangian is:

$$L = \left[\int_0^1 \left(C_t(i)\Theta_t(i)\right)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} - \lambda_t \left(\int_0^1 P_t(i)C_t(i)di - Z_t\right), \tag{37}$$

and the first-order conditions are:

$$C_t(i)^{-\frac{1}{\eta}} C_t^{\frac{1}{\eta}} \left(\Theta_t(i)\right)^{\frac{\eta-1}{\eta}} = \lambda_t P_t(i). \tag{38}$$

Thus, for any two sectors, the following equation holds:

$$C_t(i) = C_t(j) \left(\frac{P_t(i)}{P_t(j)}\right)^{-\eta} \left(\frac{\Theta_t(i)}{\Theta_t(j)}\right)^{\eta-1}. \tag{39}$$

By substituting equations (38) and (39) into the definition of consumption expenditures ($Z_t \equiv \int_0^1 P_t(i)C_t(i)di$), it yields:

$$\begin{aligned} &\int_0^1 P_t(i) \left[C_t(j) \left(\frac{P_t(i)}{P_t(j)}\right)^{-\eta} \left(\frac{\Theta_t(i)}{\Theta_t(j)}\right)^{\eta-1}\right] di = Z_t \\ \Leftrightarrow \quad & C_t(j) = P_t^{-\eta}(j)\Theta_t^{\eta-1}(j)Z_t \frac{1}{\int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)di}. \end{aligned} \tag{40}$$

By substituting the equation:

$$\int_0^1 P_t(i)C_t(i)di = Z_t = P_t C_t,$$

into equation (40), it yields:

$$C_t(i) = \Theta_t^{\eta-1}(i) \left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t \frac{P_t^{1-\eta}}{\int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)di}. \tag{41}$$

Using the definition of the price level, $P_t \equiv \left[\int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)di\right]^{\frac{1}{1-\eta}}$, we can re-write equation (41) as:

$$C_t(i) = \Theta_t^{\eta-1}(i) \left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t. \tag{42}$$

Applying the same calculation for $C_t(i) = \left[\int_0^1 (C_t(i,j))^{\frac{\widetilde{\eta}-1}{\widetilde{\eta}}} dj\right]^{\frac{\widetilde{\eta}}{\widetilde{\eta}-1}}$, it yields:

$$C_t(i,j) = \left(\frac{P_t(i,j)}{P_t(i)}\right)^{-\widetilde{\eta}} C_t(i). \tag{43}$$

By combining equations (42) and (43), we obtain the demand for good (i,j) as follows:

$$C_t(i,j) = \Theta_t^{\eta-1}(i)\left(\frac{P_t(i,j)}{P_t(i)}\right)^{-\widetilde{\eta}}\left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t.$$

A.2 Price Indexes

We show the derivation of aggregate price index $P_t \equiv \left[\int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)di\right]^{\frac{1}{1-\eta}}$, and we omit the derivation of sectoral price index $P_t(i) \equiv \left[\int_0^1 P_t^{1-\widetilde{\eta}}(i,j)dj\right]^{\frac{1}{1-\widetilde{\eta}}}$ since it can be similarly derived.

Recall that λ_t^{-1} indicates the shadow price of one unit of utility. The first-order condition in equation (38) can be re-written as:

$$\begin{aligned}
& C_t(i)^{-\frac{1}{\eta}} C_t^{\frac{1}{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}} = \lambda_t P_t(i) \\
\Leftrightarrow\ & C_t(i)^{\frac{\eta-1}{\eta}} C_t^{\frac{1}{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}} = \lambda_t C_t(i) P_t(i) \\
\Leftrightarrow\ & \int_0^1 \left(C_t(i)^{\frac{\eta-1}{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}}\right) di C_t^{\frac{1}{\eta}} = \lambda_t \int_0^1 C_t(i)P_t(i)di \\
\Leftrightarrow\ & C_t \lambda_t^{-1} = Z.
\end{aligned}$$

From the first-order condition (38) we derive the aggregate price index:

$$\begin{aligned}
& C_t(i)^{-\frac{1}{\eta}} C_t^{\frac{1}{\eta}} (\Theta_t(i))^{\frac{\eta-1}{\eta}} = \lambda_t P_t(i) \\
\Leftrightarrow\ & (C_t(i)\Theta_t(i))^{-\frac{1}{\eta}} C_t^{\frac{1}{\eta}} \Theta_t(i) = \lambda_t P_t(i) \\
\Leftrightarrow\ & (C_t(i)\Theta_t(i))^{\frac{1}{\eta}} = C_t^{\frac{1}{\eta}} \Theta_t(i) \lambda_t^{-1} P_t^{-1}(i) \\
\Leftrightarrow\ & (C_t(i)\Theta_t(i))^{\frac{\eta-1}{\eta}} = C_t^{\frac{\eta-1}{\eta}} \Theta_t^{\eta-1}(i) \lambda_t^{1-\eta} P_t^{1-\eta}(i) \\
\Leftrightarrow\ & \int_0^1 (C_t(i)\Theta_t(i))^{\frac{\eta-1}{\eta}} di = C_t^{\frac{\eta-1}{\eta}} \lambda_t^{1-\eta} \int_0^1 \left(P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)\right) di \\
\Leftrightarrow\ & 1 = \lambda_t^{1-\eta} \int_0^1 \left(P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)\right) di \\
\Leftrightarrow\ & \lambda_t^{-1} = \left[\int_0^1 \left(P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)\right) di\right]^{\frac{1}{1-\eta}}.
\end{aligned}$$

B Total Sectoral Demand and Aggregate and Sector-Specific Components

As shown in Appendix A, the demand for firm j in sector i in equation (13), can be expressed as:

$$C_t(i,j) = \left(\frac{P_t(i,j)}{P_t(i)}\right)^{-\widetilde{\eta}} C_t(i),$$

where the demand for sector i, $C_t(i)$, can be re-written as:

$$C_t(i) = \Theta_t^{\eta-1}(i) \left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t, \tag{44}$$

where C_t is the aggregate demand and $\left(\frac{P_t(i)}{P_t}\right)^{-\eta}$ is the cross-price elasticity term. $\Theta_t^{\eta-1}(i)$ is the sector-specific demand shifter driven by the preference shocks. We can express the demand in equation (44) in nominal terms as:

$$P_t C_t(i) = (P_t C_t)\, \Theta_t^{\eta-1}(i) \left(\frac{P_t(i)}{P_t}\right)^{-\eta}, \tag{45}$$

where we name $P_t C_t(i)$ is the total sectoral demand and the demand is composed of two components: the aggregate demand $P_t C_t$ and the sector-specific demand $\Theta_t^{\eta-1}(i) \left(\frac{P_t(i)}{P_t}\right)^{-\eta}$. By using $P_t \equiv \left[\int_0^1 P_t^{1-\eta}(i) \Theta_t^{\eta-1}(i) di\right]^{\frac{1}{1-\eta}}$ into equation (45), it yields the decomposition of the total sectoral demand $(P_t(i)C_t(i))$ into aggregate demand $(P_t C_t)$ and sector-specific demand $(\left(\frac{P_t(i)}{P_t}\right)^{-\eta} \Theta_t^{\eta-1}(i) / \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\eta} \Theta_t^{\eta-1}(i) di\right]^{\frac{-\eta}{1-\eta}})$, such that:

$$P_t C_t(i) = (P_t C_t) \left(\frac{\left(\frac{P_t(i)}{P_t}\right)^{-\eta} \Theta_t^{\eta-1}(i)}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\eta} \Theta_t^{\eta-1}(i) di\right]^{\frac{-\eta}{1-\eta}}} \right). \tag{46}$$

In equation (46) the relative sectoral price $(P_t(i)/P_t)$ depends on the exogenous sector-specific demand shifter, $\Theta_t(i)$, and aggregate demand and sector-specific demand are independent of each other.

To link the demand function in equation (45) to the empirical framework in Section 2, we show that the growth rates of total sectoral demand in our model can be decomposed into that of aggregate and sector-specific demand, as in the survey data. The growth rate of these term is given by

$$\frac{P_t C_t(i)}{P_{t-1} C_{t-1}(i)} = \frac{P_t C_t}{P_{t-1} C_{t-1}} \frac{\Theta_t^{\eta-1}(i)}{\Theta_{t-1}^{\eta-1}(i)} \left(\frac{\frac{P_t(i)}{P_{t-1}(i)}}{\frac{P_t}{P_{t-1}}} \right)^{-\eta}.$$

The log-linearization around the symmetric equilibrium yields:

$$[\Delta p_t + \Delta c_t(i)] = [\Delta p_t + \Delta c_t] + [(\eta - 1)\,\Delta\theta_t(i) - \eta\,(\Delta p_t(i) - \Delta p_t)]\,, \tag{47}$$

where lower-case variables to indicate logarithms of the corresponding upper-case variables (i.e., $x_t \equiv \log X_t$) and Δ indicates the difference the variables between two periods ($\Delta x_t \equiv x_t - x_{t-1}$). Equation (47) shows that the growth of the total sectoral demand ($\Delta p_t + \Delta c_t(i)$) is composed of that of aggregate demand ($\Delta p_t + \Delta c_t$) and that of sector-specific demand ($(\eta - 1)\,\Delta\theta_t(i) - \eta\,(\Delta p_t(i) - \Delta p_t)$), as in the survey data.

C Derivation of the Index of Aggregate Prices

Recall that: $P_t \equiv \left[\int_0^1 P_t^{1-\eta}(i)\Theta_t^{\eta-1}(i)di\right]^{\frac{1}{1-\eta}}$ can be expressed as, $P_t = \left[\int_0^1 \left(\frac{P_t(i)}{\Theta_t(i)}\right)^{1-\eta} di\right]^{\frac{1}{1-\eta}} = \left[\int_0^1 \left(\widetilde{P}_t(i)\right)^{1-\eta} di\right]^{\frac{1}{1-\eta}}$, where $\widetilde{P}_t(i) \equiv \frac{P_t(i)}{\Theta_t(i)}$. We then define $p_t \equiv \int_0^1 \widetilde{p}_t(i)di$, such that:

$$p_t \equiv \int_0^1 \widetilde{p}_t(i)di = \int_0^1 p_t(i)di - \int_0^1 \theta_t(i)di = \int_0^1 p_t(i)di,$$

since $\theta_t(i) \sim \mathcal{N}(0, (1-\epsilon)^{-2}\,(\eta - 1)^{-2}\,\tau_t^2)$ and $\int_0^1 \theta_t(i)di = 0$.

D Derivation of the Price Setting Rule

Using the following equations:

$$p_t(i,j) = \mu + mc_t(i,j),$$

$$c_t(i,j) = -\widetilde{\eta}\,(p_t(i,j) - p_t(i)) - \eta\,(p_t(i) - p_t) + c_t + (\eta - 1)\,\theta_t(i),$$

and

$$mc_t(i,j) = w_t + \frac{1-\epsilon}{\epsilon} y_t(i,j) - \frac{1}{\epsilon} a - log(\epsilon),$$

the price of firm j in sector i, $p_t(i,j)$, is equal to:

$$\begin{aligned}
p_t(i,j) &= \mu + mc_t(i,j) = \mu + y_t + p_t - \frac{1}{\epsilon}a - log(\epsilon) \\
&\quad + \frac{1-\epsilon}{\epsilon}\left[-\widetilde{\eta}\left(p_t(i,j) - p_t(i)\right) - \eta\left(p_t(i) - p_t\right) + c_t + (\eta-1)\,\theta_t(i)\right] \\
&= -\frac{1-\epsilon}{\epsilon}\widetilde{\eta}p_t(i,j) + \frac{1-\epsilon}{\epsilon}\left(\widetilde{\eta}-\eta\right)p_t(i) + \left(1 + \frac{1-\epsilon}{\epsilon}\eta\right)p_t \\
&\quad + (\mu - \frac{1}{\epsilon}a - log(\epsilon)) + \left(1 + \frac{1-\epsilon}{\epsilon}\right)y_t + \frac{1-\epsilon}{\epsilon}(\eta-1)\,\theta_t(i) \\
&= -\frac{1-\epsilon}{\epsilon}\widetilde{\eta}p_t(i,j) + \frac{1-\epsilon}{\epsilon}\left(\widetilde{\eta}-\eta\right)p_t(i) + (\mu - \frac{1}{\epsilon}a - log(\epsilon)) \\
&\quad + \left(1 + \frac{1-\epsilon}{\epsilon}\right)q_t + \frac{1-\epsilon}{\epsilon}(\eta-1)\,p_t + \frac{1-\epsilon}{\epsilon}(\eta-1)\,\theta_t(i) \\
&= \frac{\frac{1-\epsilon}{\epsilon}\left(\widetilde{\eta}-\eta\right)}{1+\frac{1-\epsilon}{\epsilon}\widetilde{\eta}}p_t(i) + \frac{1}{1+\frac{1-\epsilon}{\epsilon}\widetilde{\eta}}(\mu - \frac{1}{\epsilon}a - log(\epsilon)) \\
&\quad + \frac{1+\frac{1-\epsilon}{\epsilon}}{1+\frac{1-\epsilon}{\epsilon}\widetilde{\eta}}q_t + \frac{\frac{1-\epsilon}{\epsilon}(\eta-1)}{1+\frac{1-\epsilon}{\epsilon}\widetilde{\eta}}p_t + \frac{\frac{1-\epsilon}{\epsilon}(\eta-1)}{1+\frac{1-\epsilon}{\epsilon}\widetilde{\eta}}\theta_t(i) \\
&= \frac{\left(\widetilde{\eta}-\eta\right)(1-\epsilon)}{\epsilon+\widetilde{\eta}\,(1-\epsilon)}p_t(i) + \frac{\epsilon}{\epsilon+\widetilde{\eta}\,(1-\epsilon)}(\mu - \frac{1}{\epsilon}a - log(\epsilon)) \\
&\quad + \frac{1}{\epsilon+\widetilde{\eta}\,(1-\epsilon)}q_t + \frac{(1-\epsilon)(\eta-1)}{\epsilon+\widetilde{\eta}\,(1-\epsilon)}p_t + \frac{(1-\epsilon)(\eta-1)}{\epsilon+\widetilde{\eta}\,(1-\epsilon)}\theta_t(i).
\end{aligned}$$

E Proofs of Propositions

E.1 Proof of Proposition 1

The terms $\mathbb{E}_t[u_t]$ and $\mathbb{E}_t[v_t(i)]$ are equal to:

$$\begin{aligned}
\mathbb{E}_t[u_t] &= \frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\left(q_{t-1} + \rho_u u_{t-1}\right) + \frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\left[x_t(i) - \rho_v v_{t-1}(i)\right] - q_{t-1} \\
&= \rho_u u_{t-1} + \frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\left[x_t(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i)\right] \\
&= \rho_u u_{t-1} + \frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\left[e_t + \epsilon_t(i)\right] \\
\mathbb{E}_t[v_t(i)] &= \frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\rho_v v_{t-1}(i) + \frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\left[x_t(i) - q_{t-1} - \rho_u u_{t-1}\right] \\
&= \rho_v v_{t-1}(i) + \frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\left[x_t(i) - q_{t-1} - \rho_u u_{t-1} - \rho_v v_{t-1}(i)\right] \\
&= \rho_v v_{t-1}(i) + \frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\left[e_t + \epsilon_t(i)\right]
\end{aligned}$$

Thus, $\mathbb{E}_t\left[\tilde{v}_t\right]$ is given by,

$$\begin{aligned}\mathbb{E}_t\left[\tilde{v}_t(i)\right] &= \rho_v v_{t-1}(i)+\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\left[e_t+\epsilon_t(i)\right]-v_{t-1}(i)\\ &= (\rho_v-1)v_{t-1}(i)+\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\left[e_t+\epsilon_t(i)\right].\square\end{aligned}$$

E.2 Proof of Proposition 2

(i) is obvious. (ii) is obtained from

$$\mathbb{C}(\mathbb{E}_t\left[u_t\right],\mathbb{E}_t\left[\tilde{v}_t\right])=\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\mathbb{V}[e_t+\epsilon_t(i)]=\frac{\sigma_t^2\tau_t^2}{\sigma_t^2+\tau_t^2}>0.\square$$

E.3 Proof of Proposition 3

The terms $\mathbb{E}_t\left[\sum_{h=1}^k u_{t+h}\right]$ and $\mathbb{E}_t\left[\sum_{h=1}^k \widetilde{v}_{t+h}(i)\right]$ are equal to:

$$\mathbb{E}_t\left[\sum_{h=1}^k u_{t+h}\right]=\frac{1-\rho_u{}^{k+1}}{1-\rho_u}\mathbb{E}_t\left[u_t\right],$$

$$\mathbb{E}_t\left[\sum_{h=1}^k \widetilde{v}_{t+h}(i)\right]=\frac{1-\rho_v{}^{k+1}}{1-\rho_v}\mathbb{E}_t\left[\tilde{v}_t\right],$$

respectively. It follows that:

$$\mathbb{C}\left(\mathbb{E}_t\left[\sum_{h=1}^k u_{t+h}\right],\mathbb{E}_t\left[\sum_{h=1}^k \widetilde{v}_{t+h}(i)\right]\right)=\frac{1-\rho_u{}^{k+1}}{1-\rho_u}\frac{1-\rho_v{}^{k+1}}{1-\rho_v}\mathbb{C}\left(\mathbb{E}_t\left[u_t\right],\mathbb{E}_t\left[\tilde{v}_t\right]\right)>0.\square$$

E.4 Proof of Proposition 4

First, we guess that $p_t^*(i)$ takes the following form:

$$p_t^*(i)=a_1p_{t-1}+a_2x_t(i)+a_3q_{t-1}+a_4u_{t-1}+a_5v_{t-1}(i).$$

Given the guess, and since only a randomly selected fraction $1-\theta$ of firms adjusts prices in any given period, we infer that the sectoral and aggregate price level must satisfy:

$$\begin{aligned}p_t(i) &= \theta p_{t-1}(i)+(1-\theta)\int_0^1 p_t^*(i)di\\ &= \left[\theta+(1-\theta)a_1\right]p_{t-1}+(1-\theta)a_2x_t(i)+(1-\theta)a_3q_{t-1}+(1-\theta)a_4u_{t-1}+a_5v_{t-1}(i).\end{aligned}$$

$$\begin{aligned}p_t &= \int_0^1 p_t(i)di\\ &= \left[\theta+(1-\theta)a_1\right]p_{t-1}+(1-\theta)a_2q_t+(1-\theta)a_3q_{t-1}+(1-\theta)a_4u_{t-1}.\end{aligned}$$

Therefore, $p_t^*(i)$ is obtained as:

$$\begin{aligned}
p_t^*(i) &= (1-\beta\theta)\left[(1-r)x_t(i) + r\mathbb{E}_t\left[p_t\right]\right] \;\; + \beta\theta\mathbb{E}_t[p_{t+1}^*(i)] \\
&= (1-\beta\theta)(1-r)x_t(i) + (1-\beta\theta)r\mathbb{E}_t\left[p_t\right] + \beta\theta\mathbb{E}_t[p_{t+1}^*(i)] \\
&= (1-\beta\theta)(1-r)x_t(i) + (1-\beta\theta)r\mathbb{E}_t\left[p_t\right] \\
&\quad +\beta\theta\mathbb{E}_t\left[a_1p_t + a_2x_{t+1}(i) + a_3q_t + a_4u_t + a_5v_t(i)\right] \\
&= (1-\beta\theta)(1-r)x_t(i) + \left[(1-\beta\theta)r + \beta\theta a_1\right]\mathbb{E}_t\left[p_t\right] \\
&\quad +\beta\theta a_2\mathbb{E}_t\left[x_{t+1}(i)\right] + \beta\theta a_3\mathbb{E}_t\left[q_t\right] + \beta\theta a_4\mathbb{E}_t\left[u_t\right] + \beta\theta a_5\mathbb{E}_t\left[v_t(i)\right] \\
&= (1-\beta\theta)(1-r)x_t(i) + \left[(1-\beta\theta)r + \beta\theta a_1\right]\mathbb{E}_t\left[p_t\right] \\
&\quad +\beta\theta a_2\mathbb{E}_t\left[q_t + u_{t+1} + v_{t+1}(i)\right] + \beta\theta a_3\mathbb{E}_t\left[q_t\right] + \beta\theta a_4\mathbb{E}_t\left[u_t\right] + \beta\theta a_5\mathbb{E}_t\left[v_t(i)\right] \\
&= (1-\beta\theta)(1-r)x_t(i) + \left[(1-\beta\theta)r + \beta\theta a_1\right]\mathbb{E}_t\left[p_t\right] \\
&\quad +\beta\theta\left(a_2+a_3\right)\mathbb{E}_t\left[q_t\right] + \beta\theta\left(a_2\rho_u + a_4\right)\mathbb{E}_t\left[u_t\right] + \beta\theta\left(a_2\rho_v + a_5\right)\mathbb{E}_t\left[v_t(i)\right].
\end{aligned}$$

The term $\mathbb{E}_t\left[p_t\right]$ is given by:

$$\mathbb{E}_t\left[p_t\right] = \left[\theta + (1-\theta)\,a_1\right]p_{t-1} + (1-\theta)\,a_2\mathbb{E}_t\left[q_t\right] + (1-\theta)\,a_3q_{t-1} + (1-\theta)\,a_4u_{t-1},$$

which yields:

$$\begin{aligned}
p_t^*(i) &= (1-\beta\theta)(1-r)x_t(i) + \left[(1-\beta\theta)r + \beta\theta a_1\right]\left[\theta + (1-\theta)\,a_1\right]p_{t-1} \\
&\quad + \left[\left[(1-\beta\theta)r + \beta\theta a_1\right](1-\theta)\,a_2 + \beta\theta\left(a_2+a_3\right)\right]\mathbb{E}_t\left[q_t\right] \\
&\quad + \left[(1-\beta\theta)r + \beta\theta a_1\right](1-\theta)\,a_3q_{t-1} \\
&\quad + \left[(1-\beta\theta)r + \beta\theta a_1\right](1-\theta)\,a_4u_{t-1} \\
&\quad +\beta\theta\left(a_2\rho_u + a_4\right)\mathbb{E}_t\left[u_t\right] + \beta\theta\left(a_2\rho_v + a_5\right)\mathbb{E}_t\left[v_t(i)\right] \\
&= (1-\beta\theta)(1-r)x_t(i) + \left[(1-\beta\theta)r + \beta\theta a_1\right]\left[\theta + (1-\theta)\,a_1\right]p_{t-1} \\
&\quad + \left[\left[\left[(1-\beta\theta)r + \beta\theta a_1\right](1-\theta)\,a_2 + \beta\theta\left(a_2+a_3\right)\right] + \left[(1-\beta\theta)r + \beta\theta a_1\right](1-\theta)\,a_3\right]q_{t-1} \\
&\quad + \left[\left[\left[(1-\beta\theta)r + \beta\theta a_1\right](1-\theta)\,a_2 + \beta\theta\left(a_2+a_3\right)\right] + \beta\theta\left(a_2\rho_u + a_4\right)\right]\mathbb{E}_t\left[u_t\right] \\
&\quad +\beta\theta\left(a_2\rho_v + a_5\right)\mathbb{E}_t\left[v_t(i)\right] \\
&\quad + \left[(1-\beta\theta)r + \beta\theta a_1\right](1-\theta)\,a_4u_{t-1} \\
&= (1-\beta\theta)(1-r)x_t(i) + \left[(1-\beta\theta)r + \beta\theta a_1\right]\left[\theta + (1-\theta)\,a_1\right]p_{t-1} \\
&\quad +b_1q_{t-1} + b_2\mathbb{E}_t\left[u_t\right] + b_3\mathbb{E}_t\left[v_t(i)\right] + b_4u_{t-1}.
\end{aligned}$$

where

$$\begin{aligned}
b_1 &= [[(1-\beta\theta)r+\beta\theta a_1](1-\theta)a_2+\beta\theta(a_2+a_3)]+[(1-\beta\theta)r+\beta\theta a_1](1-\theta)a_3,\\
b_2 &= [[(1-\beta\theta)r+\beta\theta a_1](1-\theta)a_2+\beta\theta(a_2+a_3)]+\beta\theta(a_2\rho_u+a_4),\\
b_3 &= \beta\theta(a_2\rho_v+a_5),\\
b_4 &= [(1-\beta\theta)r+\beta\theta a_1](1-\theta)a_4.
\end{aligned}$$

Since

$$\begin{aligned}
x_t(i) &= q_{t-1}+\rho_u u_{t-1}+e_t+\rho_v v_{t-1}(i)+\epsilon_t(i)\\
&\Leftrightarrow\ e_t = x_t(i)-q_{t-1}-\rho_u u_{t-1}-\rho_v v_{t-1}(i)-\epsilon_t(i),\\
&\Leftrightarrow\ \epsilon_t(i) = x_t(i)-q_{t-1}-\rho_u u_{t-1}-\rho_v v_{t-1}(i)-e_t,
\end{aligned}$$

the terms $\mathbb{E}_t[u_t]$ and $\mathbb{E}_t[v_t(i)]$ are equal to:

$$\begin{aligned}
\mathbb{E}_t[u_t] &= \rho_u u_{t-1}+\mathbb{E}_t[e_t]\\
&= \rho_u u_{t-1}+\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}[x_t(i)-q_{t-1}-\rho_u u_{t-1}-\rho_v v_{t-1}(i)]\\
\mathbb{E}_t[v_t(i)] &= \rho_v v_{t-1}(i)+\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}[x_t(i)-q_{t-1}-\rho_u u_{t-1}-\rho_v v_{t-1}(i)].
\end{aligned}$$

It follows that:

$$\begin{aligned}
p_t^*(i) &= (1-\beta\theta)(1-r)x_t(i)+[(1-\beta\theta)r+\beta\theta a_1][\theta+(1-\theta)a_1]p_{t-1}\\
&\quad +b_1q_{t-1}+b_2\mathbb{E}_t[u_t]+b_3\mathbb{E}_t[v_t(i)]+b_4u_{t-1}\\
&= (1-\beta\theta)(1-r)x_t(i)+[(1-\beta\theta)r+\beta\theta a_1][\theta+(1-\theta)a_1]p_{t-1}\\
&\quad +b_2\rho_u u_{t-1}+b_2\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}[x_t(i)-q_{t-1}-\rho_u u_{t-1}-\rho_v v_{t-1}(i)]\\
&\quad +b_3\rho_v v_{t-1}(i)+b_3\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}[x_t(i)-q_{t-1}-\rho_u u_{t-1}-\rho_v v_{t-1}(i)]\\
&\quad +b_4u_{t-1}+b_1q_{t-1}\\
&= [(1-\beta\theta)r+\beta\theta a_1][\theta+(1-\theta)a_1]p_{t-1}\\
&\quad +\left[(1-\beta\theta)(1-r)+b_2\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}+b_3\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\right]x_t(i)\\
&\quad +\left[b_1-b_2\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}-b_3\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\right]q_{t-1}\\
&\quad +\left[b_4+(b_2-b_3)\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\rho_u\right]u_{t-1}+[b_3-b_2]\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\rho_v v_{t-1}(i),
\end{aligned}$$

and thus the equilibrium conditions are:

$$
\begin{aligned}
a_1 &= \left[(1-\beta\theta)r+\beta\theta a_1\right]\left[\theta+(1-\theta)\,a_1\right],\\
a_2 &= (1-\beta\theta)(1-r)+b_2\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}+b_3\frac{\tau_t^2}{\sigma_t^2+\tau_t^2},\\
a_3 &= b_1-b_2\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}-b_3\frac{\tau_t^2}{\sigma_t^2+\tau_t^2},\\
a_4 &= b_4+(b_2-b_3)\,\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\rho_u,\\
a_5 &= \left[b_3-b_2\right]\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\rho_v,
\end{aligned}
$$

By simplifying the conditions, we obtain:

$$
a_1=\left[(1-\beta\theta)r+\beta\theta a_1\right]\left[\theta+(1-\theta)\,a_1\right],
$$

$$
\begin{aligned}
a_2 &= (1-\beta\theta)(1-r)\\
&\quad+\left[\left[(1-\beta\theta)r+\beta\theta a_1\right](1-\theta)\,a_2+\beta\theta\,(a_2+a_3)\right]\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\\
&\quad+\beta\theta\,(a_2\rho_u+a_4)\,\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}+\beta\theta\,(a_2\rho_v+a_5)\,\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\\
&= (1-\beta\theta)(1-r)\\
&\quad+\left[\left[\left[(1-\beta\theta)r+\beta\theta a_1\right](1-\theta)+\beta\theta\right]\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}+\beta\theta\left[\rho_u\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}+\rho_v\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\right]\right]a_2\\
&\quad+\beta\theta\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}a_3+\beta\theta\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}a_4+\beta\theta\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}a_5,
\end{aligned}
$$

$$
\begin{aligned}
a_3 &= \left[\left[(1-\beta\theta)r+\beta\theta a_1\right](1-\theta)\,a_2+\beta\theta\,(a_2+a_3)\right]\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\\
&\quad+\left[(1-\beta\theta)r+\beta\theta a_1\right](1-\theta)\,a_3-\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\beta\theta\,(a_2\rho_u+a_4)\\
&\quad-\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\beta\theta\,(a_2\rho_v+a_5)\\
&= \left[\left[\left[(1-\beta\theta)r+\beta\theta a_1\right](1-\theta)+\beta\theta\right]\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}-\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\beta\theta\rho_u-\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\beta\theta\rho_v\right]a_2\\
&\quad+\left[\beta\theta\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}+\left[(1-\beta\theta)r+\beta\theta a_1\right](1-\theta)\right]a_3\\
&\quad-\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}\beta\theta a_4-\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}\beta\theta a_5,
\end{aligned}
$$

$$
\begin{aligned}
a_4 &= [(1-\beta\theta)r + \beta\theta a_1](1-\theta)\, a_4 \\
&\quad + \left[\begin{array}{c} [[(1-\beta\theta)r + \beta\theta a_1](1-\theta)\, a_2 + \beta\theta\,(a_2 + a_3)] \\ +\beta\theta\,(a_2\rho_u + a_4) - \beta\theta\,(a_2\rho_v + a_5) \end{array} \right] \frac{\tau_t^2}{\sigma_t^2 + \tau_t^2}\rho_u \\
&= [[(1-\beta\theta)r + \beta\theta a_1](1-\theta) + \beta\theta + \beta\theta\rho_u - \beta\theta\rho_v] \frac{\tau_t^2}{\sigma_t^2 + \tau_t^2}\rho_u a_2 + \beta\theta \frac{\tau_t^2}{\sigma_t^2 + \tau_t^2}\rho_u a_3 \\
&\quad + \left[[(1-\beta\theta)r + \beta\theta a_1](1-\theta) + \beta\theta \frac{\tau_t^2}{\sigma_t^2 + \tau_t^2}\rho_u \right] a_4 - \beta\theta \frac{\tau_t^2}{\sigma_t^2 + \tau_t^2}\rho_u a_5,
\end{aligned}
$$

$$
\begin{aligned}
a_5 &= -\left[\begin{array}{c} [[(1-\beta\theta)r + \beta\theta a_1](1-\theta)\, a_2 + \beta\theta\,(a_2 + a_3)] \\ +\beta\theta\,(a_2\rho_u + a_4) - \beta\theta\,(a_2\rho_v + a_5) \end{array} \right] \frac{\sigma_t^2}{\sigma_t^2 + \tau_t^2}\rho_v \\
&= -[[(1-\beta\theta)r + \beta\theta a_1](1-\theta) + \beta\theta + \beta\theta\rho_u - \beta\theta\rho_v] \frac{\sigma_t^2}{\sigma_t^2 + \tau_t^2}\rho_v a_2 \\
&\quad -\beta\theta \frac{\sigma_t^2}{\sigma_t^2 + \tau_t^2}\rho_v a_3 - \beta\theta \frac{\sigma_t^2}{\sigma_t^2 + \tau_t^2}\rho_v a_4 + \beta\theta \frac{\sigma_t^2}{\sigma_t^2 + \tau_t^2}\rho_v a_5. \square
\end{aligned}
$$

E.5 Proof of Corollary 1

If $\rho_u = \rho_v = 0$ holds, then the conditions become $a_4 = a_5 = 0$,

$$
a_1 = [(1-\beta\theta)r + \beta\theta a_1][\theta + (1-\theta)\, a_1],
$$

$$
\begin{aligned}
a_2 = (1-\beta\theta)(1-r) \quad & + [[(1-\beta\theta)r + \beta\theta a_1](1-\theta) + \beta\theta] \frac{\sigma_t^2}{\sigma_t^2 + \tau_t^2} a_2 \\
& + \beta\theta \frac{\sigma_t^2}{\sigma_t^2 + \tau_t^2} a_3,
\end{aligned}
$$

and

$$
\begin{aligned}
a_3 &= [[(1-\beta\theta)r + \beta\theta a_1](1-\theta) + \beta\theta] \frac{\tau_t^2}{\sigma_t^2 + \tau_t^2} a_2 \\
&\quad + \left[\beta\theta \frac{\tau_t^2}{\sigma_t^2 + \tau_t^2} + [(1-\beta\theta)r + \beta\theta a_1](1-\theta) \right] a_3.
\end{aligned}
$$

Moreover, $a_1+a_2+a_3=1$ holds because if $a_1+a_2+a_3=1$ holds, in reality

$$\begin{aligned}
a_1+a_2+a_3 &= [(1-\beta\theta)r+\beta\theta a_1]\left[\theta+(1-\theta)\,a_1\right] \\
&\quad +(1-\beta\theta)(1-r)+\left[[(1-\beta\theta)r+\beta\theta a_1]\,(1-\theta)+\beta\theta\right]\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}a_2 \\
&\quad +\beta\theta\frac{\sigma_t^2}{\sigma_t^2+\tau_t^2}a_3+\left[[(1-\beta\theta)r+\beta\theta a_1]\,(1-\theta)+\beta\theta\right]\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}a_2 \\
&\quad +\left[\beta\theta\frac{\tau_t^2}{\sigma_t^2+\tau_t^2}+[(1-\beta\theta)r+\beta\theta a_1]\,(1-\theta)\right]a_3 \\
&= [(1-\beta\theta)r+\beta\theta a_1]\left[\theta+(1-\theta)\,a_1+(1-\theta)a_2+(1-\theta)\,a_3\right] \\
&\quad +(1-\beta\theta)(1-r)+\beta\theta a_2+\beta\theta a_3 \\
&= (1-\beta\theta)r+(1-\beta\theta)(1-r)+\beta\theta a_1+\beta\theta a_2+\beta\theta a_3 \\
&= 1-\beta\theta+\beta\theta\,(a_1+a_2+a_3)=1
\end{aligned}$$

holds. Next, given $a_1+a_2+a_3=1$ and cash-in-advance constraint $q_t=p_t+y_t$, the equation (28) is expressed as follows.

$$\begin{aligned}
p_t-p_{t-1} &= [\theta+(1-\theta)a_1]\,(p_{t-1}-p_{t-1})+(1-\theta)\,a_2\,(q_t-p_{t-1})+(1-\theta)\,a_3\,(q_{t-1}-p_{t-1})\,, \\
\Leftrightarrow &\quad \pi_t=(1-\theta)\,a_2\,(\pi_t+y_t)+(1-\theta)\,a_3\,(y_{t-1})\,, \\
\Leftrightarrow &\quad \pi_t=\frac{(1-\theta)\,a_2}{1-(1-\theta)\,a_2}y_t+\frac{(1-\theta)\,a_3}{1-(1-\theta)\,a_2}y_{t-1}.\square
\end{aligned}$$

F Survey Data

The Annual Survey of Corporate Behavior (ASCB) is administered by the Cabinet Office of Japan across 26 sectors in the economy over the period 2003-2019 fiscal year. The Economic and Social Research Institute in the Cabinet Office of Japan directly surveys approximately 1,000 public-listed Japanese firms on nominal and real growth rates of the Japanese economy as well as nominal and real growth rates of demand in their respective sectors.[34] The Cabinet Office of Japan releases the arithmetic averages of the individual firms' expectations within each sector while retaining the data on the expectations of the individual firms confidential.

The industries included in our sample are Foods, Textiles and Apparels, Pulp and Paper, Chemicals, Pharmaceutical, Rubber Products, Glass and Ceramics Products, Iron and Steel, Nonferrous Metals, Metal Products, Machinery, Electric Appliances, Transportation

[34] The survey is conducted each January.

Equipment, Precision Instruments, Other Products, Construction, Wholesale Trade, Retail Trade, Real Estate, Land Transportation, Warehousing and Harbor Transportation Services, Information and Communication, Electric Power and Gas, Services, Banks and Securities and Commodity Futures. We proxy expectations on aggregate demand with survey data on expectations on one-year-ahead GDP growth, and we proxy expectations on total sectoral demand with survey data on expectations on one-year-ahead growth rate in total sectoral demand (industry-level output).

The data provide aggregate responses from surveys for the universe of Japanese firms on expectations within the same enterprise about the one-year-ahead growth rate of total sectoral demand and aggregate demand. Since total sectoral demand compounds aggregate and sector-specific components of demand, we infer expectations on the sector-specific component of demand as the difference between the expectations of total sectoral demand and aggregate demand.

Table 7: Descriptive statistics about survey data

Dataset: Annual survey of corporate behavior; 26 sectors; 2005y-2019y

Sector	*Historical averages*		*Historical standard deviation*		*First-order auto correlation*		*Average sample size*
	(1) Growth of aggregate demand	*(2) Growth of sector-specific demand*	*(3) Growth of aggregate demand*	*(4) Growth of sector-specific demand*	*(5) Growth of aggregate demand*	*(6) Growth of sector-specific demand*	
Foods	1.09	-0.58	0.82	0.51	0.42	0.24	31
Textiles & Apparels	1.07	-0.73	0.84	0.32	0.27	-0.10	22
Pulp & Paper	1.03	-1.23	1.07	0.91	0.35	-0.06	9
Chemicals	1.14	-0.05	0.99	0.30	0.27	0.35	63
Pharmaceutical	1.20	0.28	0.72	0.80	0.53	-0.11	11
Rubber Products	0.98	-0.37	0.99	0.74	0.06	-0.05	5
Glass & Ceramics Products	0.91	-0.52	1.04	0.45	0.26	0.02	14
Iron & Steel	0.98	-0.71	1.05	1.70	0.34	-0.13	17
Nonferrous Metals	1.05	0.02	1.00	1.58	0.35	0.02	13
Metal Products	0.99	-0.66	0.88	0.86	0.49	0.14	25
Machinery	1.10	-0.02	0.98	1.46	0.44	-0.23	63
Electric Appliances	1.10	0.46	0.92	0.60	0.41	-0.18	76
Transportation Equipment	1.09	-0.17	0.99	1.39	0.29	0.12	30
Precision Instruments	1.22	0.16	0.99	0.90	0.63	0.42	11
Other Products	1.07	-0.54	0.87	0.37	0.51	0.24	28
Construction	1.14	-1.15	0.90	1.14	0.52	0.46	60
Wholesale Trade	1.07	-0.36	0.96	0.34	0.39	0.17	77
Retail Trade	0.96	-0.66	0.91	0.32	0.52	0.06	60
Real Estate	1.05	0.51	0.80	1.36	0.32	0.02	18
Land Transportation	1.11	-0.77	0.77	0.47	0.52	0.42	19
Warehousing & Harbor Transportation Services	1.24	-0.30	0.66	0.30	0.26	0.52	15
Information & Communication	1.01	0.55	0.94	0.71	0.16	-0.09	35
Electric Power & Gas	1.30	0.02	1.08	0.96	0.21	0.74	10
Services	1.01	0.30	0.86	0.62	0.46	0.03	46
Banks	1.20	-0.25	0.96	0.39	0.33	0.77	27
Securities & Commodity Futures	1.15	0.75	1.10	2.40	0.23	0.39	9

Table 7 provides summary statistics on salient stylized facts on the expectations in the aggregate and sector-specific components of demand. Columns (1) and (2) shows historical averages of the changes in the expectations of one-year-ahead growth rate of aggregate and sector-specific demand, respectively. Entries reveal large differences in the changes of expectations between the two distinct components of demand. Changes in the expectations of aggregate demand are broadly similar across sectors while changes in the expectations of sector-specific demand differ markedly across sectors. Columns (3) and (4) list historical standard errors of the sectoral average expectations and reveal that both components have sizeable similar volatility over the sample period. Historical standard deviations are computed as the time-series variation in the sector-level aggregate expectations about the growth of aggregate demand and that of sector-specific demand. Columns (5) and (6) show that the serial correlation of the aggregate component is twice as large as the serial correlation of the sector-specific component.

Figure 5: Firms' expectations about their total demand partitioned between aggregate and sector-specific demand

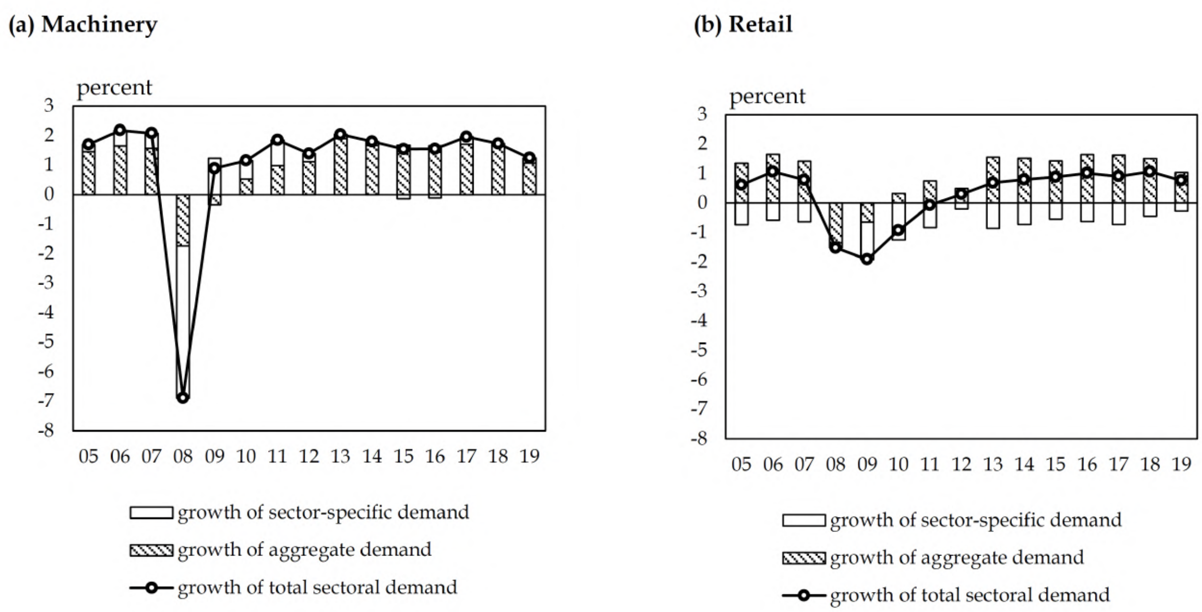


Figure 5 provides an illustrative example for the machinery (panel a) and the retail sectors (panel b), respectively, on the contribution of aggregate and sector-specific component of demand to total secotral demand. The panels show that expectations of the growth of total sectoral demand (black line) and sector-specific demand (while bar) are different across the two sectors. Importantly for us, while the contribution of the aggregate component of demand (gray bar) is similar in both sectors, the firms' expectations of the growth of aggregate demand exhibit positive correlation with those of sector-specific demand particularly around the global financial crisis. These facts are consistent with the broader evidence in Table 2.

G Sectoral Sales Data

We use yearly data on sector-level sales of Japanese firms from the Financial Statements Statistics of Corporations by Industry, compiled by the Ministry of Finance of Japan. The data cover the period 2005-2020 fiscal year for 22 major sectors in the economy. Specifically, the following sectors, which have unique correspondence with the survey data, are included in our dataset: Foods, Textiles, Pulp and Paper, Chemicals, Oil and Coal Products, Glass and Ceramics Products, Iron and Steel, Nonferrous Metals, Metal Products, Machinery, Electric Appliances, Transportation Equipment, Precision Instruments, Other Products, Construction, Wholesale Trade, Retail Trade, Real Estate, Hotel, Land Transportation, Warehousing and Harbor Transportation Services, Information and Communication, Electric Power and Gas, and Service.[35]

Table 8 reports summary statistics for the sector(industry)-level sales data. Column (1) lists the historical averages of the quarter-on-quarter sales growth in each sector, which are all positive. Columns (2) and (3) show the standard deviation and first-order auto correlation of the sales growth in each sector and confirm the high volatility and low persistent of the sectoral sales.

H Financial Statements Statistics of Corporations Data

We use quarterly data on sector-level sales of Japanese firms from the Financial Statements Statistics of Corporations by Industry, compiled by the Ministry of Finance of Japan. The data cover the period 1975:Q3-2018:Q3 for 29 major sectors in the economy. Specifically, the following sectors are included in our dataset: Foods, Textiles, Wood Products, Pulp and Paper, Printing, Chemicals, Oil and Coal Products, Glass and Ceramics Products, Iron and Steel, Nonferrous Metals, Metal Products, Machinery, Electric Devices, Cars and Related Products, Other Transportation Equipment, Other Products, Mining, Construction, Electric Power, Gas and Water Supply, Information and Communication, Land Transportation, Water Transportation, Wholesale, Retail, Real Estate, Hotel, Living-Related Service, Other

[35]The data is available at http://www.mof.go.jp/english/pri/reference/ssc/index.htm.

Table 8: Descriptive statistics about sectoral sales data

Dataset: Financial statement statistics of corporations by industry, 22sectors; 2005y-2020y			
Sector	*(1) Historical averages*	*(2) Historical standard deviation*	*(3) First-order auto correlation*
Foods	-0.16	7.33	-0.48
Textiles & Apparels	-2.90	11.12	-0.39
Pulp & Paper	3.06	28.38	-0.82
Chemicals	0.75	5.83	-0.20
Glass & Ceramics Products	-0.83	7.47	-0.02
Iron & Steel	-0.07	12.35	-0.13
Nonferrous Metals	2.25	13.99	-0.05
Metal Products	-0.58	9.31	-0.20
Machinery	-0.89	10.59	-0.03
Electric Appliances	-2.61	8.51	0.07
Transportation Equipment	1.47	7.38	0.17
Precision Instruments	-1.54	6.24	-0.10
Other Products	-0.51	8.26	-0.06
Construction	0.57	5.37	0.44
Wholesale Trade	-1.17	7.18	0.16
Retail Trade	1.08	5.71	-0.43
Real Estate	2.03	6.91	-0.21
Land Transportation	0.72	9.88	0.01
Warehousing & Harbor Transportation Services	-0.30	11.19	-0.29
Information & Communication	2.51	4.47	-0.12
Electric Power & Gas	-0.16	7.33	-0.48
Services	-2.90	11.12	-0.39

Note: The classification of the sectors in the Sectoral sales data for Japan are matched with those from the survey data in Table 7. We match the sectoral series in the sales data to (y) to those in the survey (x) as follows (x→y). (1) Foods → Foods, (2) Textiles → Textiles and Apparels, (3) Pulp and paper → Pulp and paper, (4) Chemicals → Chemicals, (5) Glass and ceramics products → Glass and ceramics products, (6) Iron and steel → Iron and steel, (7) Nonferrous metals → Nonferrous metals, (8) Metal products → Metal products, (9) Machinery → Machinery, (10) Electronic Device → Electronic appliances, (11) Transportation Equipment → Transportation Equipment, (12) Information and communication electronics equipment → Precision instruments, (13) Other products → Other products, (14) Construction → Construction, (15) Whole-sale → Wholesale trade, (16) Retail → Retail trade, (17) Real estate → Real estate, (18) Land transportation → Land transportation, (19) other transportation related services → Warehousing and harbor transportation services, (20) Information and communication → Information and communication, (21) Electric power → Electric power, and gas, and (22) Services → Services.

Service.[36]

Table 9: Descriptive statistics about sales data

Dataset: Financial statement statistics of corporations by industry, 29 sectors; 1975/3Q-2018/3Q			
Sector	*(1) Historical averages*	*(2) Historical standard deviation*	*(3) First-order auto correlation*
Foods	0.68	3.85	-0.15
Textiles	0.02	7.22	-0.14
Wood Products	0.18	10.73	-0.10
Pulp and Paper	0.42	6.15	0.04
Printing	0.49	7.15	-0.13
Chemicals	0.63	3.96	0.12
Oil and Coal Products	0.12	9.59	0.02
Glass and Ceramics Products	0.37	5.16	-0.08
Iron and Steel	0.32	5.43	0.24
Nonferrous Metals	0.67	6.53	0.34
Metal Product	0.79	6.53	-0.04
Machinery	0.92	4.58	0.13
Electric Device	1.01	4.59	0.26
Cars and Related Products	1.11	5.98	0.13
Other Transportation Equipment	0.25	9.47	-0.23
Other Products	0.87	7.53	-0.23
Mining	0.39	10.98	-0.16
Construction	0.94	3.50	0.01
Electric Power	1.03	3.81	0.11
Gas and Water Supply	1.01	3.83	0.22
Information and Communication	1.82	5.03	-0.02
Land Transportation	1.09	5.14	-0.04
Water Transportation	0.25	6.04	0.10
Whole-sale	0.53	4.26	-0.02
Retail	1.35	3.86	0.04
Real Estate	1.14	9.22	-0.14
Hotel	1.25	9.25	-0.03
Living-Related Service	1.65	10.86	-0.09
Other Service.	1.74	10.23	-0.29

Table 9 reports summary statistics for the sector(industry)-level sales data. Column (1) lists the historical averages of the quarter-on-quarter sales growth in each sector, which are all positive. Columns (2) and (3) show the standard deviation and first-order auto correlation of the sales growth in each sector and confirm the high volatility and low persistent of the sectoral sales.

[36]The data is available at http://www.mof.go.jp/english/pri/reference/ssc/index.htm.

I Extracting the Sequence of Shocks on Aggregate and Sector-Specific Components of Demand

To extract the sequence of shocks on aggregate and sector-specific components of demand $(e_t, \{\epsilon_t(i)\}_{i=1}^{29})$, we decompose fluctuations in aggregate and sector-specific components (i.e., $u_t, \{\widetilde{x}_t(i) - u_t\}_{i=1}^{29})$ into expected component and shocks for firms using the equations (3), (4) and (6). More concretely, we use equation (3 that characterizes the law of motion of aggregate demand as:

$$u_t = \rho_u u_{t-1} + e_t,$$

to decompose aggregate demand into the expected component ($\mathbb{E}_{t-1}[u_t] = \rho_u u_{t-1}$) and shock ($e_t$). We estimate the parameter ρ_u and the unobservable shock e_t using the equation:

$$u_t = c_u + \rho_u u_{t-1} + e_t,$$

where c_u is a constant term that normalizes e_t to have mean zero. We then proxy the shock to aggregate demand as:

$$\widehat{e}_t = u_t - \widehat{c}_u - \widehat{\rho}_u u_{t-1},$$

and the variance of the shock $\sigma_t^2 = \mathbb{V}(e_t) = \mathbb{E}\left[e_t^2\right]$ is approximated by $\frac{1}{2k+1}\sum_{s=-k}^{k} \widehat{e}_t^2$.

Similarly, we use equation (4) that characterizes the law of motion of sector-specific demand ($\{\widetilde{v}_t(i)\}_{i=1}^{29}$) as:

$$\widetilde{v}_t(i) = v_t(i) - v_{t-1}(i) = \rho_v \left(v_{t-1}(i) - v_{t-2}(i)\right) + \epsilon_t(i) - \epsilon_{t-1}(i) = \rho_v \widetilde{v}_{t-1}(i) + \epsilon_t(i) - \epsilon_{t-1}(i),$$

to decompose sector-specific demand into the expected component ($\mathbb{E}_{t-1}[\widetilde{v}_t(i)] = \rho_v \widetilde{v}_{t-1}(i) - \epsilon_{t-1}(i)$) and shock ($\epsilon_t(i)$). Since $(\rho_v, \epsilon_t(i), \epsilon_{t-1}(i))$ are unobservable for us, we estimate them from following empirical equation to obtain $(\rho_v, \epsilon_t(i) - \epsilon_{t-1}(i))$:

$$\left(\widetilde{x}_t(i) - u_t\right) = c_v(i) + \rho_v \left(\widetilde{x}_{t-1}(i) - u_{t-1}\right) + \left(\epsilon_t(i) - \epsilon_{t-1}(i)\right),$$

where $c_v(i)$ is a constant term to normalize $\epsilon_t(i) - \epsilon_{t-1}(i)$ as mean zero. We then obtain

$$\widehat{\epsilon}_t(i) - \widehat{\epsilon}_{t-1}(i) = \left(\widetilde{x}_t(i) - u_t\right) - \widehat{c}_v(i) - \rho_v \left(\widetilde{x}_{t-1}(i) - u_{t-1}\right)$$

as the proxy for shock on sector-specific demand $(\epsilon_t(i) - \epsilon_{t-1}(i))$. Using the cross-sectional variation of $\widehat{\epsilon}_t(i) - \widehat{\epsilon}_{t-1}(i)$, we approximate

$\tau_t^2 = \mathbb{V}(\epsilon_t(i)) = \mathbb{E}[\epsilon_t^2(i)]$ by $\frac{1}{2k+1}\sum_{s=-k}^{k}\left(\frac{1}{29}\sum_{i=1}^{29}\frac{(\widehat{\epsilon}_t(i)-\widehat{\epsilon}_{t-1}(i))^2}{2}\right)$.[37]

Table 10 reports summary statistics for estimates of the aggregate and sector-specific components of demand $\left(u_t, \{\widetilde{x}_t(i) - u_t\}_{i=1}^{29}\right)$ for the average (columns 1 and 2), standard deviation (columns 3 and 4), and first-order autocorrelation (columns 5 and 6) of the series. Columns (7) and (8) report standard deviation of $\widehat{e}_t$ and $\{\widehat{\epsilon}_t(i)\}_{i=1}^{29}$, respectively.

Table 10: Descriptive statistics about aggregate and sector-specific components of demand

Dataset: Financial statement statistics of corporations by industry, 29 sectors; 1975/4Q-2018/3Q

Sector	*Historical averages*		*Historical standard deviation*		*First-order auto correlation*		*Historical standard deviation*	
	(1) Growth of aggregate demand	*(2) Growth of sector-specific demand*	*(3) Growth of aggregate demand*	*(4) Growth of sector-specific demand*	*(5) Growth of aggregate demand*	*(6) Growth of sector-specific demand*	*(7) Aggregate shocks*	*(8) Sector-specific shocks*
Foods	0.72	-0.04	6.22	4.08	0.45	0.06	2.54	2.88
Textiles		-0.69		6.68		-0.16		4.67
Wood Products		-0.54		10.48		-0.14		7.35
Pulp and Paper		-0.30		5.91		0.00		4.19
Printing		-0.23		7.08		-0.07		5.00
Chemicals		-0.09		3.10		-0.09		2.19
Oil and Coal Products		-0.60		8.38		-0.15		5.88
Glass and Ceramics Products		-0.35		4.95		-0.16		3.47
Iron and Steel		-0.39		4.06		0.04		2.87
Nonferrous Metals		-0.04		4.93		0.09		3.48
Metal Product		0.07		6.14		-0.16		4.30
Machinery		0.21		3.73		-0.20		2.59
Electric Device		0.29		3.35		-0.03		2.37
Cars and Related Products		0.39		4.67		-0.08		3.30
Other Transportation Equipment		-0.47		9.66		-0.21		6.69
Other Products		0.16		6.62		-0.27		4.51
Mining		-0.32		9.92		-0.27		6.76
Construction		0.22		4.10		0.09		2.87
Electric Power		0.32		4.24		0.07		2.99
Gas and Water Supply		0.30		4.12		0.12		2.89
Information and Communication		1.11		5.22		-0.01		3.70
Land Transportation		0.37		5.51		-0.07		3.89
Water Transportation		-0.47		5.14		-0.07		3.63
Whole-sale		-0.18		3.11		-0.26		2.13
Retail		0.64		3.90		0.03		2.75
Real Estate		0.43		8.65		-0.12		6.08
Hotel		0.53		9.52		-0.02		6.75
Living-Related Service		0.94		11.30		-0.11		7.97
Other Service.		1.02		10.09		-0.29		6.85

J Aggregate Demand and the Output Gap

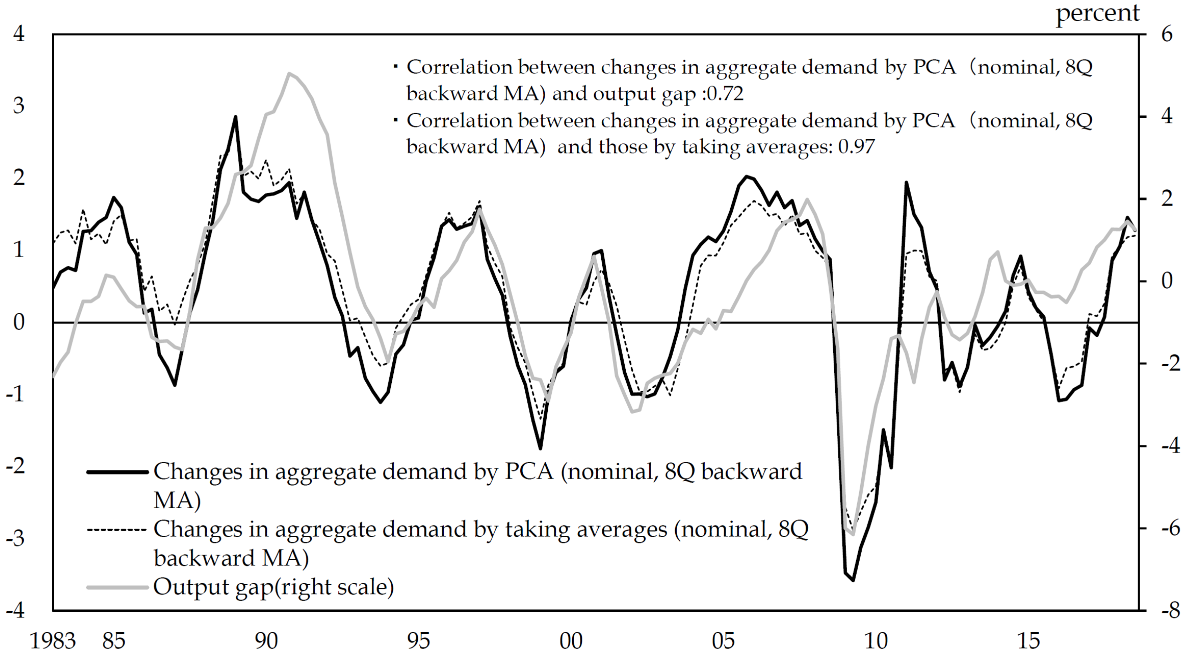
To evaluate whether the extracted (unnormalized) changes in aggregate demand ($u_t = \sum_{i=1}^{29}\Lambda_i\widetilde{x}_t(i)$) is a plausible measure of aggregate disturbances, and it is consistent with alternative measures, we compare the eight-quarters backward moving averages of the changes

[37]Note that the following equation holds,

$$\begin{aligned}\mathbb{V}(\widehat{\epsilon}_t(i) - \widehat{\epsilon}_{t-1}(i)) &= \mathbb{E}\left[(\widehat{\epsilon}_t(i) - \widehat{\epsilon}_{t-1}(i))^2\right] = 2\mathbb{V}(\epsilon_t(i)) \\ &\Leftrightarrow \mathbb{V}(\epsilon_t(i)) = \frac{1}{2}\mathbb{V}(\epsilon_t(i) - \epsilon_{t-1}(i)),\end{aligned}$$

and thus the variance of $\epsilon_t(i) - \epsilon_{t-1}(i)$ is monotonically increasing in τ_t^2.

Figure 6: Changes in aggregate demand and output gap



in aggregate demand, $\frac{1}{8}\sum_{s=0}^{7} u_{t-s}$,[38] with the averages of changes in total sectoral demand across sectors ($u_t = \frac{1}{29}\sum_{i=1}^{29} \widetilde{x}_t(i)$) and the output gap published by the Bank of Japan.[39]

Figure 6 examines the relation between the dynamics of our estimates for aggregate shocks and the output gaps. It shows that our measure of changes in aggregate demand highly co-moves with the averages of changes in sectoral demand across sectors, with a correlation coefficient equal to 0.97. It also shows that our measure of changes in aggregate demand and the output gap are highly correlated, with a correlation coefficient equal to 0.72, suggesting that our identified measure for the changes in aggregate demand is consistent with alternative measures of the changes in aggregate demand.

K Producer Price Index Data

We use monthly data on sector-level producer prices of Japanese firms from corporate Goods Price Index (CGPI), compiled by the Bank of Japan. The data cover the period 1981:M1-

[38]Our measure of the changes in aggregate demand is a flow rather than stock concept. By comparing moving averages of the changes in aggregate demand (i.e., the averages of flow data) with the output gap (i.e. stock data), we ensure that our measure is consistent with conventional measures.

[39]The series is available here. https://www.boj.or.jp/en/research/research_data/gap/index.htm/
The description of the methodology for the estimation is here https://www.boj.or.jp/en/research/brp/ron_2017/ron170531a.htm/.

2018:M9 for 13 major manufacturing sectors in the economy. The data is transformed to quarterly data by taking averages of samples in each quarter (i.e., three months). Specifically, the following sectors are included in our dataset: Beverages; foods; textile products; lumber and wood products; pulp, paper and related products; chemicals and related products; petroleum and coal products; ceramic, stone, and clay products; iron and steel; nonferrous metals; metal products; general purpose machinery; electronic components and devices; and electrical machinery and equipment.

Table 11: Descriptive statistics about PPI data

Dataset: producer price index (seasonally adjusted, QoQ, annualized), 13sectors; 1985/2Q-2018/3Q

Sector	*(1) Historical averages*	*(2) Historical standard deviation*	*(3) First-order auto correlation*
Foods	0.51	2.19	0.50
Textiles & Apparels	-0.21	3.29	0.48
Wood Products	0.81	8.31	0.30
Pulp and Paper	0.14	3.49	0.47
Chemicals	-0.36	6.35	0.38
Oil and Coal Products	1.60	26.83	0.30
Glass and Ceramics Products	0.13	2.09	0.54
Iron and Steel	0.68	9.35	0.67
Nonferrous Metals	0.53	17.19	0.36
Metal Product	0.54	2.58	0.51
Machinery	-0.50	1.53	0.24
Electric Device	-1.37	1.79	0.25
Transportation Equipment	-0.56	1.76	0.28

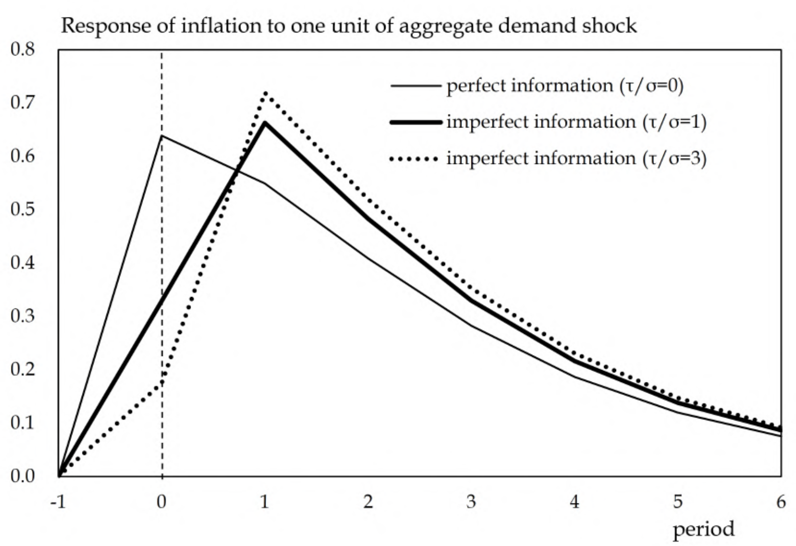
Note: The classification of the sectors in PPI data for Japan are matched with those from the sectoral sales data in Table 9. We match the sectoral series in the sales data to (y) to those in the survey (x) as follows (x→y). (1) Beverages and foods → Foods, (2) Textile products → Textiles and Apparels, (3) Lumber and wood products → Wood Products, (4) Pulp, paper and related products → Pulp and paper, (5) Chemicals and related products → Chemicals, (6) Petroleum and coal products → Oil and Coal Products, (7) Ceramic, stone and clay products → Glass and ceramics products, (8) Iron and steel → Iron and steel, (9) Nonferrous metals → Nonferrous metals, (10) Metal products → Metal products, (11) General purpose, production and business Oriented machinery → Machinery, (12) Electrical machinery and equipment → Electronic Device, (13) Transportation equipment → Transportation Equipment.

L IRF of Aggregate Inflation to Aggregate Demand Shocks

L.1 Numerical Assessment

How does the relative volatility of sector-specific demand shocks to aggregate shocks influence the sensitivity of inflation to changes in aggregate demand? To address this central question of our analysis, we simulate the model and determine the response of inflation to a one-period, positive aggregate demand shock for different values of τ_t/σ_t. Figure 7 shows that an increase in the ratio τ_t/σ_t reduces the response of inflation to changes in aggregate demand. Since the firm cannot disentangle changes in aggregate and sector-specific demand, it attributes changes in total sectoral demand partially to changes in sector-specific demand, which have no effect on the price-setting decisions of firms in other sectors in the economy. Attributing part of the movement in total sectoral demand to sector-specific demand induces the firm to decrease the response of prices to aggregate shocks. Therefore, inflation becomes less responsive to changes in total sectoral demand. If the ratio of τ_t/σ_t is large, the firm conjectures that a large fraction of the changes in total sectoral demand occurs because of sector-specific shock. Consequently, the firm expects that the average price in the period remains almost the same as that in the previous period and adjusts its prices less strongly to changes in aggregate demand. This makes the response more persistent.

Figure 7: Impulse response functions of aggregate inflation to aggregate shocks



Notes: Parameters are $\theta = 0.3,\ r = 0.7,\ \beta = 0.99, \rho_u = 0.45,\ \rho_v = -0.08.$

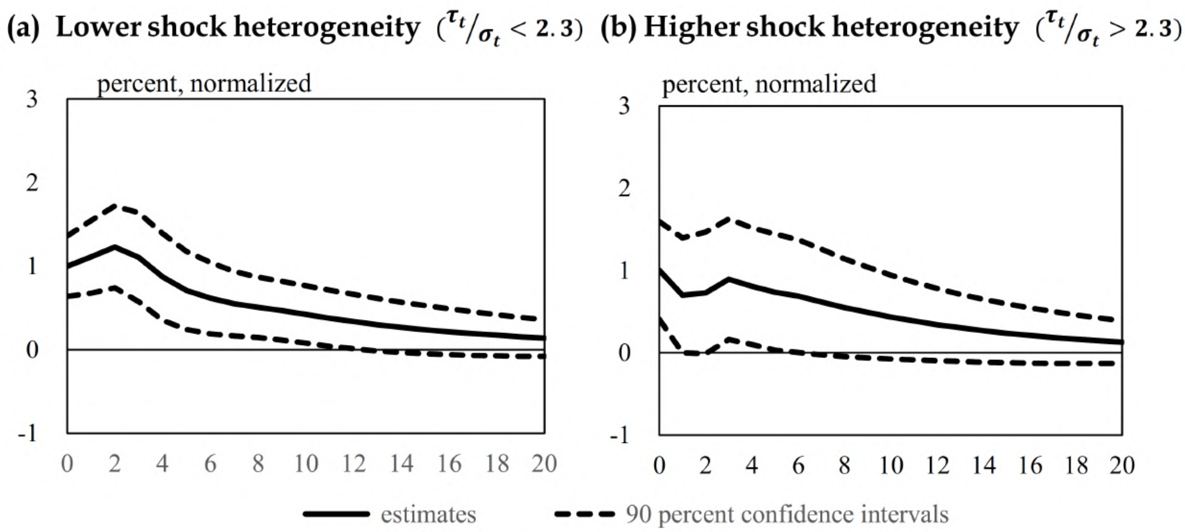
L.2 Empirical Assessment

As shown in Figure 7, in a reduced form the response of the aggregate inflation to aggregate demand shocks becomes more persistent as the shock heterogeneity τ_t/σ_t increases. In what follows, we investigate the difference in the dynamics responses of aggregte inflation to changes in aggregate demand. Specifically, we estimate the following Vector Auto-Regression model by dividing the samples to two groups, $\tau_t/\sigma_t > 2.3$ with 85 samples and $\tau_t/\sigma_t < 2.3$ with 84 samples. The number of lags is chosen based on Akaike's Information Criterion.

$$\begin{aligned} A_0 \begin{bmatrix} \Delta\text{demand}_t \\ \text{CPI}_t \end{bmatrix} &= C + A_1 \begin{bmatrix} \Delta\text{demand}_{t-1} \\ \text{CPI sales}_{t-1} \end{bmatrix} + A_2 \begin{bmatrix} \Delta\text{demand}_{t-2} \\ \text{CPI sales}_{t-2} \end{bmatrix} \\ &+ A_3 \begin{bmatrix} \Delta\text{demand}_{t-3} \\ \text{CPI sales}_{t-3} \end{bmatrix} + \begin{bmatrix} \epsilon_{\text{demand}_t} \\ \epsilon_{\text{CPI-specific}_t} \end{bmatrix} \end{aligned}$$

where the matrix A_0 is lower triangular, the vector C is of constant terms, the matrices A_1, A_2, and A_3 are for the lag terms, and $\epsilon_{\text{aggregate}_t}$ and $\epsilon_{\text{CPI-specific}_t}(i)$ are the exogenous aggregate and sector-specific shocks, respectively.

Figure 8: Responses of inflation to aggregate shocks



Note: Response to one standard deviation shock. Bold lines indicate the estimates and dotted lines indicate the 90 percent confidence intervals. The series for the core consumer price index is "all items, less fresh food (impact of consumption taxes are adjusted)". Sample period is 1975Q4-2018Q3 (the number of observations for panel (a) is 85 and that for panel (b) is 84). Shocks are identified by Cholesky decomposition with the assumption that aggregate shock is faster than inflation specific shocks. The number of lags is three chosen based on AIC.

Figure 8 shows the impulse responses of the inflation to aggregate shocks based on the estimated VAR model. The initial responses are normalized to unity. The comparison of panels (a) and (b) shows the relationship that the response under lower shock heterogeneity exhibits higher persistence than that under higher shock heterogeneity in two aspects. First, the response of panel (a) is positive and significant more than ten quarters while that of panel (b) is significant only four quarters. Second, only the response of panel (a) exhibits hump-shaped impulse response in the sense that the peak of the response is not in the initial period. It should be also noted that the first order auto-correlation in panel (a) (0.95) is higher thant that in panel (b) (0.83).

M Sensitivity of Sectoral Inflation to Total Sectoral Demand

This appendix assesses the empirical validity of our model from the perspective of total fluctuation of sectoral inflation dynamics. Equation (31) shows that the sensitivity of the sectoral inflation ($\pi_t(i)$) to changes in total sectoral demand ($x_t(i)$) depends on α_2, which we know from our previous analysis in section 4.1 is negatively related to shock heterogeneity ($\tau_t(i)/\sigma_t$). In what follows, we investigates whether the model predictions are supported in the data.

To estimate the relation between the degree of shock heterogeneity and the sensitivity of the sectoral inflation to total sectoral demand, we follow the insights from the theoretical model, encapsulated by equation (31), and construct a panel dataset for the sectoral inflation rates ($\pi_t(i)$), total demand in each sector ($x_t(i) \equiv u_t + \widetilde{v}_t(i)$), and the measures for shock heterogeneity ($\tau_t(i)/\sigma_t$) that is heterogeneous across sectors. We use measures for aggregate inflation π_t, quarterly changes in consumer price index from Japanese Statistics Bureau, u_t, $\widetilde{v}_t(i)$ and τ_t/σ_t from the Financial Statements Statistics of Corporations by Industry prepared by the Ministry of Finance, and we measure sectoral inflation $\pi_t(i)$ with the Producer Price index (PPI) in Japan, which is released by the Bank of Japan on a monthly basis.[40] Th empirical specification of sectoral inflation equation is:

$$\begin{aligned}\pi_t(i) =& d_1(i) + \underbrace{\left(d_2 + d_3\mathbf{1}_{\{2000-2018\}} + d_4\left(\tau_t(i)/\sigma_t\right)\right)}_{\alpha_1}\pi_{t-1} + \underbrace{\left(d_5 + d_6\mathbf{1}_{\{2000-2018\}} + d_7\left(\tau_t(i)/\sigma_t\right)\right)}_{\alpha_2}x_t(i) \\ &+ d_8u_{t-1} + d_9u_{t-2} + d_{10}\widetilde{v}_{t-1}(i) + \varepsilon_t^d, \end{aligned} \tag{48}$$

where $d_1(i)$ is fixed-effect indicator variable, the parameters d_2-d_{10} are regression coefficients,

[40]For details, see https://www.boj.or.jp/en/statistics/pi/cgpi_release/index.htm/. For the summary statistics of the PPI data, see Appendix K.

$\mathbf{1}_{\{2000-2018\}}$ is an indicator variable equal to 1 for the period 2000-2018 to control for the years with exogenous fall in price stickiness, as in our benchmark specification, and ε_t^d is the error term.

Table 12: Estimation of the sectoral inflation dynamics

Dataset: Financial statement statistics of corporations by industry, producer price index; 13 sectors; 1985/2Q-2018/3Q				
Dependent Variable: sectoral inflation rate ($\pi_t(i)$, seasonally adjusted, QoQ, annualized)				
	2 years trimmed mean	*3 years trimmed mean*	*5 years trimmed mean*	*10 years trimmed mean*
Lag of inflation (π_{t-1})	-0.21	-0.32	-0.32	-0.26
	(0.37)	(0.37)	(0.35)	(0.34)
Lag of inflation×time dummy (2000-2018)	-0.26	-0.33	-0.29	-0.15
($\pi_{t-1} \times 1_{\{2000-2018\}}$)	(0.56)	(0.56)	(0.54)	(0.48)
Lag of inflation	-0.28	-0.50	-0.43	-0.26
×shock heterogeneity ($\pi_{t-1} \times {}^{\tau_t}/_{\sigma_t}$)	(0.31)	(0.31)	(0.30)	(0.30)
Changes in total demand ($x_t(i)$)	0.48 ***	0.47 ***	0.43 ***	0.40 ***
	(0.09)	(0.09)	(0.08)	(0.08)
Changes in total demand ×time	0.13	0.12	0.15	0.17
dummy (2000-2018) ($x_t(i) \times 1_{\{2000-2018\}}$)	(0.10)	(0.10)	(0.10)	(0.10)
Changes in total demand	**-0.11** ***	**-0.10** ***	**-0.07** **	**-0.03**
×shock heterogeneity ($\boldsymbol{x_t(i) \times {}^{\tau_t}/_{\sigma_t}}$)	(0.03)	(0.03)	(0.03)	(0.04)
Observations	1778	1778	1778	1778
Adjusted-R^2	0.18	0.17	0.17	0.16

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. First and second lags of changes in aggregate demand and the first lag of changes in sector-specific demand are included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity.
**** Significant at the 1 percent level. ** Significant at the 5 percent level.* Significant at the 10 percent level.*

Table 12 shows the estimates for equation (48) for alternative measures of shock heterogeneity based on time windows of two years (column 1), three years (columns 2), five years (column 3), and ten years (column 4), respectively. All entries show that the sector-specific component of inflation is positively correlated with current total demand ($x_t(i)$). Important for our analysis, the interaction term between total sectoral demand and the degree of shock heterogeneity ($x_t(i) \times \tau_t(i)/\sigma_t$) is negative in all entries and significant in most entries. Our results show that the data supports a decrease in the sensitivity of the sectoral inflation in response to a raise in shock heterogeneity, consistent with the prediction in our theoretical model.

To ensure that decline in nominal price rigidities is not driving the significance of the negative relation between $\tau_t(i)/\sigma_t$ and inflation, Table 13 presents results for the benchmark regression that abstracts from the indicator variable $\mathbf{1}_{\{2000-2018\}}$ by omitting the interaction term between past inflation and the indicator variable (i.e., $\pi_{t-1} \times \mathbf{1}_{\{2000-2018\}}$) and the interaction term between changes in demand and the indicator variable ($x_t(i) \times \mathbf{1}_{\{2000-2018\}}$) from equation (48). The regression coefficient on the term $x_t(i) \times (\tau_t(i)/\sigma_t)$ (bold entry) remains significant and negative, as in the benchmark regression. The estimation results confirm the theoretical prediction.

Table 13: Estimation of the sectoral inflation dynamics

Dataset: Financial statement statistics of corporations by industry, producer price index; 13 sectors; 1985/2Q-2018/3Q				
Dependent Variable: sectoral inflation rate ($\pi_t(i)$, seasonally adjusted, QoQ, annualized)				
	2 years trimmed mean	*3 years trimmed mean*	*5 years trimmed mean*	*10 years trimmed mean*
Lag of inflation	-0.30	-0.44	-0.43	-0.34
×shock heterogeneity ($\pi_{t-1} \times {}^{\tau_t}/_{\sigma_r}$)	(0.28)	(0.29)	(0.27)	(0.26)
Lag of inflation	-0.25	-0.47	-0.42	-0.28
×shock heterogeneity ($\pi_{t-1} \times {}^{\tau_t}/_{\sigma_r}$)	(0.29)	(0.29)	(0.29)	(0.32)
Changes in total demand ($x_t(i)$)	0.58 ***	0.56 ***	0.53 ***	0.52 ***
	(0.05)	(0.05)	(0.05)	(0.05)
Changes in total demand	**-0.12 *****	**-0.10 *****	**-0.07 ****	**-0.02**
×shock heterogeneity ($\boldsymbol{x_t(i) \times {}^{\tau_t}/_{\sigma_r}}$)	(0.03)	(0.04)	(0.03)	(0.04)
Observations	1778	1778	1778	1778
Adjusted-R^2	0.17	0.17	0.17	0.16

Note: Estimated by ordinary-least-squares. The standard errors are HAC estimators. First and second lags of changes in aggregate demand and the first lag of changes in sector-specific demand are included in estimation as control variables. Data extrapolation using the values in the closest periods is applied for the missing values in the estimates of shock heterogeneity.

**** Significant at the 1 percent level. ** Significant at the 5 percent level.* Significant at the 10 percent level.*