Monetary Policy & Anchored Expectations
An Endogenous Gain Learning Model

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Abstract

This paper analyzes monetary policy in a model with a potential unanchoring of inflation expectations. The degree of unanchoring is given by how sensitively the public's long-run inflation expectations respond to inflation surprises. I find that optimal policy moves the interest rate aggressively when expectations unanchor, allowing the central bank to accommodate inflation fluctuations when expectations are well-anchored. Furthermore, I estimate the model-implied relationship that determines the extent of unanchoring. The data suggest that the expectations process is nonlinear and asymmetric: expectations respond more sensitively to large or downside surprises than to smaller or upside ones.

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1 Introduction

> Inflation that runs below its desired level can lead to an unwelcome fall in *longer-term inflation expectations*, which, in turn, can pull actual inflation even lower, resulting in an adverse cycle of ever-lower inflation and inflation expectations. [...] *Well-anchored inflation expectations* are critical[.]
> Jerome Powell, Chairman of the Federal Reserve[1]
> *(Emphases added.)*

There is broad consensus among policymakers that anchoring inflation expectations is central to the modern conduct of monetary policy. As Powell emphasizes, policymakers think of expectations as anchored when long-run expectations do not fluctuate systematically with short-run inflation surprises. If expectations were to unanchor, policymakers fear this would result in an adverse cycle of self-enforcing movements in expectations. And yet, this is a topic that until recently had received little attention in the literature. While there existed an empirical literature proposing measures of central bank credibility often associated with the idea of expectations anchoring, there were few models with a formal concept of anchoring and unanchoring.

This paper studies how a concern to anchor inflation expectations affects the conduct of monetary policy. I augment a New Keynesian model of the type generally used to study monetary policy with inflation expectations that can unanchor to varying degrees, building on Carvalho et al. (2021)'s idea that the sensitivity of long-run inflation expectations to short-run inflation surprises can be thought of as a metric of unanchoring. I use survey data on inflation expectations to discipline how unanchoring happens in the model, and solve the Ramsey problem of how to optimally conduct monetary policy in this environment. The main contribution of the paper is to provide analytical and numerical policy prescriptions for monetary policy when the degree of expectations anchoring may vary.

In order to provide a convincing analysis of optimal policy in an environment with time-varying unanchoring, I need a qualitatively and quantitatively realistic model of expectation formation. To this end, I build on new work by Carvalho et al. (2021), who model the anchoring of expectations as a discrete sensitivity to surprises. In their model, agents choose whether they should revise their estimate of long-run inflation weakly or strongly in response to past expectations errors, yielding a well-anchored and an unanchored expectations regime. I extend this work along two dimensions. First, in order to be able to study optimal monetary policy analytically, I need to take derivatives of all equations of the model. I therefore consider a smooth, continuous sensitivity, which maintains the property in Carvalho et al. (2021) that agents choose what weight to put on past expectations errors based on recent inflation surprises. Second, I embed the anchoring model of expectation formation in a general equilibrium New Keynesian model, allowing me to formally consider the monetary policy problem.

As an empirical contribution, I employ a simulated method of moments strategy (Duffie and Singleton, 1990, Lee and Ingram, 1991, Smith, 1993) on Survey of Professional Forecasters (SPF)

[1] "New Economic Challenges and the Fed's Monetary Policy Review," August 27, 2020, Jackson Hole.

data to estimate how expectations unanchor in practice. I find that unanchoring in the data is nonlinear and asymmetric. On the one hand, larger inflation surprises upset the public's view on long-run inflation much more than smaller ones. On the other hand, in line with other studies such as Hebden et al. (2020), I also find that inflation surprises on the downside unanchor expectations more than same-sized surprises on the upside.

I then turn to the main contribution of the paper, the question of how to conduct policy when expectations can unanchor. I perform my analysis of policy in the model in three stages. First, I present an analytical characterization of the Ramsey problem of the monetary authority. This prescribes that the central bank should smooth out the effects of shocks over time, foreshadowing that the central bank's interest-rate policy will depend on the degree of unanchoring.

Second, I solve the optimality conditions of the Ramsey problem numerically. Because the anchoring function renders the model nonlinear, I rely on global methods to obtain the optimal interest-rate policy function. The key takeaway is that the optimal interest-rate setting is time-varying. The monetary authority acts aggressively when expectations unanchor, responding with large movements in the interest rate. When expectations are well-anchored, by contrast, the central bank accommodates fluctuations in inflation. This state-dependent behavior allows the central bank to anchor expectations in volatile times, but avoid inflicting volatility in stable times.

Third, I investigate how to deal with unanchoring when monetary policy simply sets the interest rate in response to fluctuations in inflation and the output gap. This "Taylor rule" specification of policy is interesting because academics and central bankers often think of monetary policy in practice as following a Taylor rule. I solve for the optimal response coefficient of the interest rate to inflation numerically in the case of both the anchoring and the rational expectations versions of my model. To avoid comparing apples with oranges when comparing the Taylor rule with the Ramsey policy, I abstract from the drifting interest-rate expectations specification of Eusepi et al. (2020) in the main analysis.

There are two key results. On the one hand, it is optimal to respond much more to inflation under varying levels of anchoring than under rational expectations because expectations in the anchoring model have a self-enforcing tendency. As has been shown in the literature, whenever this is the case, it is optimal to act aggressively to subdue fluctuations in expectations (Orphanides and Williams, 2004). On the other hand, the fact that expectations fluctuate smoothly between varying degrees of unanchoring introduces some trouble for the standard Taylor rule that always responds to inflation with the same aggressiveness. Very strong unanchoring calls for an aggressive response, while well-anchored expectations mean that a policymaker who intervenes only introduces excess volatility into the model (Eusepi et al., 2020). A fixed Taylor-rule coefficient cannot accommodate both cases, suggesting that in a model with varying levels of unanchoring, the size of the interest rate response should depend on the degree of unanchoring.

To understand the practical relevance of my findings, I also explore what the optimal Ramsey and Taylor-rule policies imply for the Great Inflation and the Great Moderation. I back out the structural shocks from this period using a simple structural VAR in the output gap, inflation and the federal funds rate and feed these into the model. Strikingly, both policies prevent the Great

Inflation from ever happening, thus eliminating the need to induce the Volcker recession to bring inflation back under control.

The difference between the two policies in this application is that the optimal Taylor rule renders the interest rate and the output gap much more volatile because it responds to inflation with the same aggressiveness no matter the extent of unanchoring. Just like the analysis of the Taylor rule suggested, an implication for policy in practice is that it is beneficial to allow the interest rate to respond more to inflation the more unanchored expectations are. A Taylor rule with aggressiveness that depends on the degree of unanchoring is both flexible and simple, combining the advantages of the optimal Ramsey and Taylor-rule policies of my model. It avoids inflicting volatility when expectations are well-anchored, but acts swiftly and strongly to reanchor expectations when long-run inflation expectations unanchor.

The paper is structured as follows. Section 2 introduces the model. Section 3 describes how anchoring works in the model. Section 4 estimates the anchoring process. Section 5 presents the results in three parts. First, Section 5.1 discusses an analytical characterization of the Ramsey policy. Second, Section 5.2 solves for the interest rate sequence that implements the optimal Ramsey allocation using global methods. Third, Section 5.3 investigates the optimal choice of the response coefficient on inflation if monetary policy is restricted to follow a Taylor rule. Section 6 examines the model's implications for the Great Inflation and the Great Moderation to draw practical policy conclusions. Section 7 concludes.

1.1 Related literature

The paper is related to two main strands of literature. The model I use to study the interaction between monetary policy and anchoring is a behavioral version of the standard New Keynesian (NK) model of the type widely used for monetary policy analysis. Monetary policy in the rational expectations (RE) version of this model has been studied extensively, for example in Clarida et al. (1999), Woodford (2003a) or Svensson (1999).

The behavioral part of the model is the departure from rational expectations on the part of the private sector. Instead, I allow the private sector's expectation of long-run inflation to fluctuate based on the history of observed inflation. This situates my paper in the adaptive learning literature advocated by Evans and Honkapohja (2001). This literature replaces the rational expectations assumption by postulating an ad-hoc forecasting rule that agents use to form expectations and that they update in every period using observed data. My contribution to this literature is to study optimal monetary policy in a model with expectations anchoring and unanchoring.

There are three main reasons for why adaptive learning is an attractive alternative to rational expectations in general, and a suitable framework to study anchoring in particular. First, many studies document the ability of adaptive learning models to match empirical properties of both expectations and macro aggregates. Adaptive learning models imply that forecast errors are correlated with forecast revisions, a feature of expectations documented by Coibion and Gorodnichenko (2015). The prediction of these models concerning the response of expectations to shocks exactly aligns with new evidence by Angeletos et al. (2021), suggesting that in response to shocks, expectations initially underreact, and then overshoot. As for the macro evidence, Milani (2007) demonstrates that estimated learning models match the persistence of inflation without recourse to backward-looking elements in the Phillips curve. Eusepi and Preston (2011) show that a calibrated adaptive learning version of the RBC model outperforms the rational expectations version. In particular, even with small deviations from RE, the learning model leads to persistent and hump-shaped responses to iid shocks, resolving the long-standing critique of real business cycle (RBC) models of Cogley and Nason (1993).

Secondly, there is very strong empirical backing for state-dependent expectations. Milani (2014) documents that state-dependence in expectations can generate endogenous time-varying volatility in models without any time-variation in the exogenous processes. Additionally, Carvalho et al. (2021) show that state-dependent sensitivity of long-run inflation expectations allows their model to fit both inflation and inflation expectations very well, both in- and out-of-sample. An entire empirical literature aimed at estimating inflation and trend inflation also arrives at the conclusion that modeling the private sector's expectations as reacting strongly to big surprises is key to fitting the inflation process (Leeper and Zha, 2003, Stock and Watson, 2016, Mertens, 2016 and Mertens and Nason, 2020).

Thirdly, an extensive experimental literature in the spirit of Anufriev and Hommes (2012) demonstrates that simple, state-dependent forecasting rules provide the best fit among competing models to how individuals form expectations in controlled lab settings. A key finding in this literature is that experiment participants rely on simple heuristics to form expectations, and they do so in a state-dependent way. In particular, changing the economic environment of the experiment induces the forecasters to change the forecasting rule they use.

Within the adaptive learning literature, the paper touches base with three sets of papers. First, Molnár and Santoro (2014) and Mele et al. (2019) show that adaptive learning in general introduces an intertemporal tradeoff to monetary policy that is absent under rational expectations. This general insight, which also implies that optimal policy under commitment coincides what that under discretion, also carries over to my model because it comes from the fact that agents learn from past data. Erceg and Levin (2003), Lu et al. (2016) and King and Lu (2021) also investigate monetary policy when the public learns from observed data and the policymaker seeks to influence the learning process. In these models, the public is endowed with the knowledge of the true model, while in my paper, the public has model-inconsistent expectations. This allows me to distinguish anchoring from the notion of reputation.

Second, Marcet and Nicolini (2003), Cho and Kasa (2015), Kostyshyna (2012) Milani (2014) and Carvalho et al. (2021) study time-varying and state-dependent sensitivities of expectations to surprises. My paper is intimately connected to Carvalho et al. (2021) in particular, who propose endogenous sensitivity as a model of anchoring. While their focus is validating the empirical performance of their anchoring model using inflation expectations surveys, I modify their framework to provide what I think is a novel contribution: the study of monetary policy for varying degrees of unanchoring.

Third, my results on the optimal Taylor rule speak to the literature on whether monetary policy should respond more or less to inflation when expectations are boundedly rational. On the one hand, Orphanides and Williams (2004) obtain that with non-rational expectations, monetary policy should respond more to inflation in order to subdue volatile expectations. In contrast, Eusepi et al. (2020) and Eusepi and Preston (2018) find that an aggressive response to inflation leads to excess volatility when expectations of the long-run interest rate have a drift. As a result, monetary policy cannot and should not respond strongly to inflation fluctuations. My paper instead emphasizes that when expectations unanchor to varying degrees, the optimal Taylor-rule coefficient on inflation should itself depend on the extent of unanchoring.

Finally, my work is also related to the literature attempting to explain features of expectations data using departures from the full information rational expectations (FIRE) paradigm. This literature consists of two main lines of attack. The first is relaxing the assumption of full information. In this body of work, information is either not fully or not symmetrically available, or information acquisition is costly (Mankiw and Reis, 2002, Sims, 2003, Maćkowiak and Wiederholt, 2009, Angeletos and Pavan, 2009). The second strand of this literature, to which this paper belongs, instead emphasizes that expectation formation departs from rational expectations. One example of this literature is Sargent (1999), who explores the role of adaptive learning on the part of the Federal Reserve for the Great Inflation. A more recent example is Bordalo et al. (2018). These authors complement the empirical results of Coibion and Gorodnichenko (2015) with new evidence to suggest that the pattern of over- and underreaction to news is consistent with diagnostic expectations instead of dispersed information. My model of anchored expectations is also consistent with these data facts.

2 The model

Apart from expectation formation, the model is a standard New Keynesian (NK) model with nominal frictions à la Calvo (1983). The advantage of having a standard NK backbone to the model is that one can neatly isolate the way anchoring alters the behavior of the model. Since the mechanics of the rational expectations version of this model are well understood, I only lay out the model briefly and pinpoint the places where the assumption of non-rational expectations matters.[2]

2.1 Households

The representative household is infinitely-lived and maximizes expected discounted lifetime utility from consumption net of the disutility of supplying labor hours:

$$\hat{\mathbb{E}}_t^i \sum_{T=t}^{\infty} \beta^{T-t} \Big[U(C_T^i) - \int_0^1 v(h_T^i(j))dj \Big]. \tag{1}$$

[2] For more details on the NK model, see Woodford (2003a).

In the above, the only non-standard element is the expectations operator, $\hat{\mathbb{E}}^i$. As detailed in Section 3, this operator captures non-rational expectations and is assumed to satisfy the law of iterated expectations, so that $\hat{\mathbb{E}}^i_t \hat{\mathbb{E}}^i_{t+1} = \hat{\mathbb{E}}^i_t$. The remaining elements are familiar from the rational expectations version of the NK model. $U(\cdot)$ and $v(\cdot)$ denote the utility of consumption and disutility of labor, respectively, and β is the discount factor of the household. I am using standard CRRA utility of the form $U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. $h^i_t(j)$ denotes the supply of labor hours of household i at time t in the production of good j, and the household participates in the production of all goods j. Similarly, household i's consumption bundle at time t, C^i_t, is a Dixit-Stiglitz composite of all goods in the economy:

$$C^i_t = \left[\int_0^1 c^i_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $\theta > 1$ is the elasticity of substitution between the varieties of consumption goods. Denoting by $p_t(j)$ the time-t price of good j, the aggregate price level in the economy can then be written as

$$P_t = \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{\theta-1}}. \tag{3}$$

The budget constraint of household i is given by

$$B^i_t \leq (1 + i_{t-1}) B^i_{t-1} + \int_0^1 w_t(j) h^i_t(j) + \Pi^i_t(j) dj - T_t - P_t C^i_t, \tag{4}$$

where $\Pi^i_t(j)$ denotes profits from firm j remitted to household i, T_t taxes, and B^i_t the riskless bond purchases at time t.[3]

The only difference to the standard New Keynesian model is the expectations operator, $\hat{\mathbb{E}}^i$. This is the subjective expectations operator that differs from its rational expectations counterpart, $\mathbb{E}$, in that it does not encompass knowledge of the model. In particular, households have no knowledge of the fact that they are identical. By extension, they also do not internalize that they hold identical beliefs about the evolution of the economy. This implies that while the modeler can suppress the index i, understanding that $\hat{\mathbb{E}}^i = \hat{\mathbb{E}}^j = \hat{\mathbb{E}}$, households cannot do so. As we will see in Section 2.3, this has implications for their forecasting behavior and will result in decision rules that differ from those of the rational expectations version of the model.

2.2 Firms

Firms are monopolistically competitive producers of the differentiated varieties $y_t(j)$. The production technology of firm j is $y_t(j) = A_t f(h_t(j))$, whose inverse, $f^{-1}(\cdot)$, signifies the amount of labor input. Noting that A_t is the level of technology and that $w_t(j)$ is the wage per labor hour, firm j profits at time t can be written as

$$\Pi^j_t = p_t(j) y_t(j) - w_t(j) f^{-1}(y_t(j)/A_t). \tag{5}$$

[3] For ease of exposition I have suppressed potential money assets here. This has no bearing on the model implications since it represents the cashless limit of an economy with explicit money balances.

Firm j's problem, then, is to set the price of the variety it produces, $p_t(j)$, to maximize the present discounted value of profit streams

$$\hat{\mathbb{E}}_t^j \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \Big[\Pi_t^j(p_t(j)) \Big], \tag{6}$$

subject to the downward-sloping demand curve

$$y_t(j) = Y_t \left(\frac{p_t(j)}{P_t} \right)^{-\theta}, \tag{7}$$

where

$$Q_{t,T} = \beta^{T-t} \frac{P_t U_c(C_T)}{P_T U_c(C_t)} \tag{8}$$

is the stochastic discount factor from households. Nominal frictions enter the model through the parameter α in Equation (6). This is the Calvo probability that firm j is not able to adjust its price in a given period.

Analogously to households, the setup of the production side of the economy is standard up to the expectation operator. Also here the model-consistent expectations operator $\mathbb{E}$ has been replaced by the subjective expectations operator $\hat{\mathbb{E}}^j$. This implies that firms, like households, do not know the model equations, nor do they internalize that they are identical. Thus, their decision rules, just like those of the households, will be distinct from their rational expectations counterparts.

2.3 Aggregate laws of motion

The model solution procedure entails deriving first-order conditions, taking a loglinear approximation around the nonstochastic steady state and imposing market clearing conditions to reduce the system to two equations, the New Keynesian Phillips curve and IS curve. The presence of subjective expectations, however, implies that firms and households are not aware of the fact that they are identical. Thus, as Preston (2005) points out, imposing market clearing conditions in the expectations of agents is inconsistent with the assumed information structure.[4]

Instead, I prevent firms and households from internalizing market clearing conditions. As Preston (2005) demonstrates, this leads to long-horizon forecasts showing up in firms' and households' first-order conditions. As a consequence, instead of the familiar expressions, the IS and

[4]Two ways have been proposed in the adaptive learning literature to deal with this issue. The first is the so-called Euler-equation approach, see for example by Bullard and Mitra (2002). This approach involves writing down the log-linearized first-order conditions of the model, and simply replacing the rational expectations operators with subjective ones. The other approach, advocated by Preston (2005) and adopted in this paper, instead involves rederiving the aggregate laws of motion from the individual agents' optimization problems that incorporate subjective expectations from the start. Lastly, a related but distinct alternative to Preston (2005)'s long-horizon approach is the shadow price learning framework advocated by Evans and McGough (2009).

Phillips curves take the following form:

$$x_t = \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_T - \pi_{T+1}) + \sigma r_T^n), \tag{9}$$

$$\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T). \tag{10}$$

Here x_t, π_t and i_t are the log-deviations of the output gap, inflation and the nominal interest rate from their steady state values, and σ is the intertemporal elasticity of substitution. The variables r_t^n and u_t are exogenous disturbances representing a natural-rate shock and a cost-push shock respectively.

The laws of motion (9) and (10) are obtained by deriving individual firms' and households' decision rules, which involve long-horizon expectations, and aggregating across the cross-section. Importantly, agents in the economy have no knowledge of these relations since they do not know that they are identical and thus are not able to impose market clearing conditions required to arrive at (9) and (10). Thus, although the evolution of the observables (π, x) is governed by the exogenous state variables (r^n, u) and long-horizon expectations via these two equations, agents in the economy are unaware of this. As I will spell out more formally in Section 3, it is indeed the equilibrium mapping between states and jump variables the agents are attempting to learn.

To simplify notation, I gather the exogenous state variables in the vector s_t and observables in the vector z_t as

$$s_t = \begin{bmatrix} r_t^n \\ \bar{i}_t \\ u_t \end{bmatrix} \qquad \text{and} \qquad z_t = \begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix}, \tag{11}$$

where $\bar{i}_t$ is a shock to the interest rate that only shows up in the model for particular specifications of monetary policy.[5] This allows me to denote long-horizon expectations by

$$f_{a,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} z_{T+1} \qquad \text{and} \qquad f_{b,t} \equiv \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\beta)^{T-t} z_{T+1}. \tag{12}$$

As detailed in Appendix A, one can use this notation to reformulate the laws of motion of jump variables (Equations (9) and (10)) and a given monetary policy rule compactly as

$$z_t = A_a f_{a,t} + A_b f_{b,t} + A_s s_t, \tag{13}$$

where the matrices $A_i,\ i = \{a, b, s\}$ gather coefficients and are given in Appendix A. Assuming that exogenous variables evolve according to independent AR(1) processes, I write the state transition matrix equation as

$$s_t = h s_{t-1} + \epsilon_t \qquad \text{with} \qquad \epsilon_t \sim \mathcal{N}(\mathbf{0}, \Sigma), \tag{14}$$

where h gathers the autoregressive coefficients ρ_j, ϵ_t the Gaussian innovations ε_t^j, and η the standard deviations σ_t^j, for $j = \{r, i, u\}$. $\Sigma = \eta\eta'$ is the variance-covariance matrix of disturbances.[6]

[5] For generality, I treat the exogenous state vector as three-dimensional throughout the paper, even when the monetary policy shock is absent and the second element of the state vector is zero.

[6] For the sake of conciseness, I have suppressed the expressions for these in the main text. See Appendix A.

3 The unanchoring of inflation expectations

The informational assumption of the model is that agents do not know the equilibrium mapping between states and jumps in the model. Without knowing the form of the observation equation, they are not able to form rational expectations forecasts. Instead, agents postulate an ad-hoc forecasting relationship between states and jumps and seek to refine it in light of incoming data. In other words, they act like an econometrician: they estimate a simple statistical model and constantly attempt to improve their model as new data arrive.

3.1 Perceived law of motion

I assume agents consider a forecasting model for the endogenous variables of the form

$$\hat{\mathbb{E}}_t z_{t+1} = a_{t-1} + b_{t-1} s_t, \tag{15}$$

where a and b are estimated coefficients of dimensions 3×1 and 3×3 respectively. This perceived law of motion (PLM) reflects the assumption that agents forecast jumps using a linear function of current states and a constant, with last period's estimated coefficients. Note that a can be interpreted as long-run expectations of the observables z. Since inflation is the first element of z, the first element of a corresponds to long-run inflation expectations. This object, which I will denote by $\bar{\pi}$, will be the main focus of the paper in what follows.

Summarizing the estimated coefficients as $\phi_{t-1} \equiv \begin{bmatrix} a_{t-1} & b_{t-1} \end{bmatrix}$, here 3×4, I can rewrite Equation (15) as

$$\hat{\mathbb{E}}_t z_{t+1} = \phi_{t-1} \begin{bmatrix} 1 \\ s_t \end{bmatrix}. \tag{16}$$

I also assume that

$$\hat{\mathbb{E}}_t \phi_{t+k} = \phi_t \quad \forall\, k \geq 0. \tag{17}$$

This assumption, known in the learning literature as anticipated utility (Kreps, 1998), means that agents do not internalize that they will update the forecasting rule in the future.[7] Since the states s_t are exogenous, I assume that agents know Equation (14), the equation governing the evolution of s_t.[8] Then, the PLM together with anticipated utility implies that k-period-ahead forecasts in the beginning of period t are constructed as

$$\hat{\mathbb{E}}_t z_{t+k} = a_{t-1} + b_{t-1} h^{k-1} s_t \quad \forall k \geq 1. \tag{18}$$

[7] Clearly, anticipated utility poses a higher level of irrationality than not knowing the model and using statistical techniques to learn it. Nevertheless, because Sargent (1999) demonstrates that it does not alter the dynamics of the linearized model, anticipated utility has become a standard simplifying assumption in the adaptive learning literature.

[8] This is another common simplifying assumption in the adaptive learning literature. In an extension, I relax it and find that when agents have to learn the evolution of state variables, they do so very quickly. Therefore, the dynamics in that case resemble the dynamics here, except that responses to shocks are muted as long as agents are learning about the state variables.

The timing assumptions of the model are as follows. In the beginning of period t, the current state s_t is realized. Agents then form expectations according to (15) using last period's estimate ϕ_{t-1} and the current state s_t. Given exogenous states and expectations, today's jump vector z_t is realized. This allows agents to evaluate the forecast error made at the end of last period

$$f_{t|t-1} \equiv z_t - \phi_{t-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \tag{19}$$

to update their forecasting rule.[9] The estimate is updated according to the following recursive least-squares algorithm:

$$\phi_t = \left(\phi'_{t-1} + k_t R_t^{-1} \begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} f'_{t|t-1} \right)', \tag{20}$$

$$R_t = R_{t-1} + k_t \left(\begin{bmatrix} 1 \\ s_{t-1} \end{bmatrix} \begin{bmatrix} 1 & s_{t-1} \end{bmatrix} - R_{t-1} \right). \tag{21}$$

Here R_t is the 4×4 variance-covariance matrix of the regressors and k_t is the learning gain, specifying to what extent the updated estimate loads on the forecast error. Clearly, a high gain implies high loadings and thus strong changes in the estimated coefficients ϕ_t. A low gain, by contrast, means that a given forecast error only has a small effect on ϕ_t. Thus, one can interpret the gain as the sensitivity of the expectations process to short-run surprises.

3.2 Endogenous gain as a metric of unanchoring

The vast majority of the learning literature specifies the gain either as a constant, $\bar{g}$, or decreasing with time, so that $k_t = t^{-1}$, where t indexes time. Instead, I follow Carvalho et al. (2021) in allowing the gain to fluctuate in a time-varying way in response to short-run forecast errors. I assume the gain evolves as

$$k_t = \mathbf{g}(f_{t|t-1}), \tag{22}$$

where $\mathbf{g}(\cdot)$ is a smooth, continuous function. I refer to $\mathbf{g}(\cdot)$ as the anchoring function since it specifies how the sensitivity of the expectations process is determined. As Carvalho et al. (2021) point out, one can think of the gain as a formal notion of the degree of unanchoring, because a high gain implies a high sensitivity to surprises, while a low gain a low one.

To understand the intuition, consider the learning vector a, the intercept in the private sector's forecasting rule in Equation (15). Recall that the first element of this vector, $\bar{\pi}$, corresponds to the expectation of long-run inflation. When the gain is high, agents update their expectations process strongly in light of surprises, thus also updating $\bar{\pi}$. What this means is that episodes of high gains correspond to episodes when the private sector changes its expectation of long-run inflation very strongly in response to current realized inflation. So if the central bank misses its inflation target, this surprise inflation will show up in long-run inflation expectations only if the gain is

[9] Alternatively, one could assume that agents use the forecast they made in the morning of the current period. This does not change the dynamics of the model in any way.

large enough. In other words, one can interpret the gain as a metric of unanchoring because it specifies how strongly current realized inflation will cause the public to revise its view of the inflation regime it is in.

Notice that the anchoring function is an extension to the conventional decreasing or constant gain specifications, as it nests both as special cases. A decreasing gain implies $\mathbf{g}(f_{t|t-1}) = t^{-1}$, while a constant gain $\mathbf{g}(f_{t|t-1}) = \bar{g}$. Furthermore, in both special cases, the derivative $\mathbf{g}_f = 0$.[10] The smooth gain function is also an extension to Carvalho et al. (2021)'s framework, as I explain below.

Apart from the smoothness and continuity assumptions, I also assume that

$$\mathbf{g}_{ff} \geq 0. \tag{23}$$

Equation (23) states that $\mathbf{g}(\cdot)$ is convex, meaning that the gain is increasing in the absolute value of forecast errors. To interpret this assumption, consider what time series of inflation surprises imply for the evolution of the gain. During the Great Inflation of the 1970s in the US, for example, when inflation kept exceeding the public's expectations, large positive forecast errors resulted in high gains through the convexity of $\mathbf{g}(\cdot)$. In other words, a volatile environment with lots of large inflation surprises drives up the gain, rendering long-run inflation expectations sensitive to short-run fluctuations, unanchoring them. The convexity makes sure that both upside and downside surprises raise the sensitivity of long-run expectations, so that what matters for the unanchoring of expectations is the absolute size of the surprise, not its sign.

The usefulness of specifying the evolution of the gain as endogenous is that it offers a state-dependent sensitivity of the expectations process to surprises. In both of the standard, exogenous gain schemes, the gain is divorced from the current environment. It either decreases deterministically ($k_t = t^{-1}$), or is a constant ($k_t = \bar{g}$). This means that the level of unanchoring is either deterministic or constant.

An endogenous gain, instead, generates periods of well-anchored expectations (low gain) as well as highly unanchored episodes (high gain), and everything in-between. The endogenous gain can thus be interpreted as a metric of the varying degrees of unanchoring. Furthermore, since monetary policy influences the volatility of the economic environment, an endogenous gain framework allows monetary policy to affect the anchoring and unanchoring of expectations directly.

State-dependent learning has significant empirical backing. The notion that forecasters ignore small surprises, yet revise their forecasting rules strongly following large surprises is a robust finding of the empirical literature estimating the inflation process (Leeper and Zha, 2003, Stock and Watson, 2016, Mertens, 2016 and Mertens and Nason, 2020). My model also implies a pattern of delayed reaction and overshooting of expectations in response to incoming information, in line with recent evidence by Angeletos et al. (2021). Furthermore, as demonstrated by Milani (2014), endogenous gain learning models can match time-varying volatility in the data, a feature that constant gain learning or rational expectations models cannot account for. Lastly, an expectation formation that allows individuals to regulate the extent to which they track incoming information

[10] To be precise, for a decreasing gain this statement only holds in the limit.

also aligns with micro-level evidence on state-dependent expectation formation in lab settings, such as in Anufriev and Hommes (2012).

To be clear, I am not the first to use an endogenous gain learning model in the macro context. Marcet and Nicolini (2003), Milani (2014) and Carvalho et al. (2021) propose models in which agents switch between a constant and a decreasing gain based on a switching criterion. In Kostyshyna (2012), agents use a Kushner and Yin (2003) algorithm to choose the size of the gain. Furthermore, Carvalho et al. (2021) propose an endogenous gain model as a metric of unanchoring.[11] Their focus is to show that an anchoring expectation formation can both match untargeted moments of long-run inflation expectations from surveys, as well as rationalize the historical dynamics of US inflation through a novel narrative of transitioning from an unanchored phase to an anchored one. To do so, they estimate a partial equilibrium model which incorporates their novel anchoring theory.

My focus here instead is to analyze the interaction between expectations unanchoring and monetary policy. I thus embed an endogenous gain model of anchoring in a general equilibrium context to understand how it affects the conduct of monetary policy. The modification of Carvalho et al. (2021)'s anchoring theory using a smooth gain function serves the purpose of rendering the optimal monetary policy problem tractable. With this modification, the derivatives of $\mathbf{g}(\cdot)$ exist, allowing me to obtain analytical expressions for the Ramsey problem of the monetary authority. Thus, the paper can shed light on what varying degrees of expectations unanchoring means for monetary policy.

3.3 Actual law of motion

To complete the model, I now use the specifics of the anchoring expectation formation to characterize the evolution of the jump variables under learning. Using the PLM from Equation (15), I write the long-horizon expectations in (12) as

$$f_{a,t} \equiv \frac{1}{1-\alpha\beta}a_{t-1} + b_{t-1}(I_3 - \alpha\beta h)^{-1}s_t \qquad \text{and} \qquad f_{b,t} \equiv \frac{1}{1-\beta}a_{t-1} + b_{t-1}(I_3 - \beta h)^{-1}s_t. \tag{24}$$

Substituting these into the law of motion of observables, Equation (13), yields the actual law of motion (ALM):

$$z_t = g^l_{t-1}\begin{bmatrix} 1 \\ s_t \end{bmatrix}, \tag{25}$$

where g^l is a 3×4 matrix given in Appendix B. Thus, instead of the state-space solution of the RE version of the model, Equations (14) and (B.1), the state-space solution for the learning model is characterized by the pair of equations (14) and (25), together with the PLM (18), the learning equations (20) and (21), as well as the anchoring function (22).

[11] See Appendix C for a detailed description of Carvalho et al. (2021)'s anchoring function.

3.4 Simplifying assumption

To simplify the analytical work in Section 5.1, I make one assumption that I maintain for the rest of the paper.

Assumption 1. $a_t = \begin{pmatrix} \bar{\pi}_t \\ 0 \\ 0 \end{pmatrix}$, *and* $b_t = g\ h, \quad \forall t.$

Assumption 1 amounts to restricting the intercepts in the forecasts of the output gap and the interest rate, as well as the slope coefficients of all forecasts to what they would be under rational expectations. This means that instead of learning the intercept and slope parameters for all three endogenous variables, the private sector only learns the intercept of the inflation process. Thus the single learning parameter is the long-run inflation expectation, $\bar{\pi}$.

The rationale behind this assumption is that it is the smallest possible deviation from rational expectations that has enough flexibility to study the unanchoring of inflation expectations. It renders the Ramsey problem very tractable and makes the comparison with the rational expectations benchmark more transparent. In particular, since the inflation intercept is learned using my anchoring model, the formulation is able to capture the time-varying sensitivity of long-run inflation expectations to short-run forecast errors in inflation.

Relaxing the assumption that slope coefficients are not learned does not change the implications of the model because it only makes impulse responses to shocks more bumpy after impact. As I explain in Section 5.3, the assumption on whether all variables are learned or not has stronger implications because interest-rate expectations are intricately linked with monetary policy, in particular with the question of whether strong policy responses are desirable or not. I maintain this assumption for the sake of tractability of the Ramsey problem, and for ease of comparison with rational expectations.

With Assumption 1, the updating equations (20) and (21) simplify to a single equation:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1}, \tag{26}$$

where, using the notation that b_1 is the first element of the vector b, the k-period-ahead inflation forecast in the beginning of period t is given by

$$\hat{\mathbb{E}}_t \pi_{t+k} = \bar{\pi}_{t-1} + b_1^k s_t, \quad \forall k \geq 1. \tag{27}$$

Lastly, the one-period-ahead forecast error in inflation from the end of last period simplifies to

$$f_{t|t-1} = \pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}). \tag{28}$$

An advantage of the simplifying assumption is that it offers insights into how the anchoring process works. Recursively substituting Equation (26) into Equation (28) yields the following expression

$$f_{t|t-1} = f_{t|t-1}^{RE} - \sum_{\tau=0}^{t-1} k_\tau f_{\tau|\tau-1}. \tag{29}$$

Here $f^{RE}_{t|t-1}$ stands for the rational expectations forecast error and captures unforecastable variation in inflation. Furthermore, I have initialized the long-run inflation expectation and the forecast error at the beginning of time with the rational expectations values of zero.

Equation (29) says that a forecast error today can arise from two sources. First, it can reflect a realization of shocks that was not foreseeable last period. This has the interpretation that a tail event like the Great Recession or the Covid-19 crisis can contribute to the unanchoring of expectations. The other source is the sum of the full history of gain-weighted forecast errors. This means that unanchoring is more likely to happen if the public has seen a long streak of similar surprises in the past, or if its expectations were highly or persistently unanchored in the past. In this way, even though the gain function only depends on the most recent forecast error, it still captures the dependence of anchoring on the full history of surprises.

4 Quantification of the anchoring channel

The numerical analysis of monetary policy requires a functional specification for the anchoring function $\mathbf{g}(\cdot)$ of Equation (22). For this reason, I back out the functional form of $\mathbf{g}(\cdot)$ from data. Apart from its usefulness for numerical analysis, the form of the anchoring function is interesting in its own right because it describes empirical properties of the anchoring of expectations. In particular, central bankers may want to know how much forecast errors of a particular sign and magnitude unanchor expectations.

I carry out the estimation in two steps. I first calibrate the parameters of the underlying New Keynesian model. Conditional on these parameter values, I estimate the anchoring function by simulated method of moments à la Lee and Ingram (1991), Duffie and Singleton (1990) and Smith (1993). I target the autocovariance structure of the Baxter-King filtered observables of the model and expectations. The observables are CPI inflation from the Bureau of Labor Statistics (BEA), the output gap and the federal funds rate from the Board of Governors of the Federal Reserve System.[12] For expectations, I rely on 12-month-ahead CPI inflation forecasts from the Survey of Professional Forecasters (SPF). The dataset is quarterly and ranges from 1981-Q3 to 2020-Q1. Appendix D contains a detailed description of the estimation methodology.

4.1 Calibration

For the calibration of the New Keynesian backbone of the model, I split the parameters into two subsets. The first is calibrated using values from the literature, while the second is calibrated to match the moments outlined above, for an initial set of expectations parameters. Tables 1 and 2 show the two subsets of calibrated parameters respectively. As Table 1 depicts, I adopt standard parameters from the literature where possible. In particular, for β, σ and other parameters underlying κ, the slope of the Phillips curve, I rely on the parameterization of Chari et al. (2000),

[12] The output gap measure is constructed as the difference between real GDP from the Bureau of Economic Analysis (BEA) and the Congressional Budget Office's (CBO) estimate of real potential output.

advocated in Woodford (2003a).

The composite parameter κ is given by $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\zeta$, where ζ is a measure of strategic complementarity in price setting. Assuming specific factor markets, constant desired markups with respect to output levels and no intermediate inputs, $\zeta = \frac{\omega+\sigma^{-1}}{1+\omega\theta}$. Here θ is the price elasticity of demand and ω is the elasticity of the marginal cost function with respect to output. Chari et al. (2000)'s calibration involves $\theta = 10, \sigma = 1, \omega = 1.25, \beta = 0.99$, so that together with my choice of α detailed below, κ is pinned down. Note that I lower β slightly (0.98 instead of Chari et al. (2000)'s 0.99). This allows the model to better match the autocovariance structure of the output gap because it lowers the pass-through of long-horizon expectations in the IS curve.

The probability of not adjusting prices, α, is set to match an average price duration of 2 quarters, which is slightly below the average number found in empirical studies of around 7 months.[13] I choose a slightly lower number in order to allow the learning mechanism, and not price stickiness, to drive the bulk of the model's dynamics.

To simplify the numerical analysis as well as the interpretation, I follow Molnár and Santoro (2014) in restricting the shocks to be iid. While this is a useful assumption to make sure that the optimal monetary policy problem has an analytical solution, it is actually not a restrictive assumption at all, as Molnár and Santoro (2014) show. The reason is that because learning models introduce endogenous persistence into inflation dynamics, autocorrelation in innovations becomes superfluous.

The volatilities of the disturbances and, where applicable, the output-coefficient of the Taylor rule, are set to match the above-mentioned moments for an initial set of expectations parameters. As shown in Table 2, this implies standard deviations of 0.01 for the natural rate and monetary policy shock and 0.5 for the cost-push shock. In sections of the paper where I assume a Taylor rule, the moment-matching exercise results in a 0.3 coefficient on the output gap. The last parameter of the New Keynesian core is the inflation coefficient of the Taylor rule, ψ_π. Unless otherwise specified, I set ψ_π to 1.5, the value recommended by Taylor (1993). Note that the asterisks in Tables 1 and 2 demarcate parameters that pertain to the Taylor rule and thus only to sections of the paper which assume that a Taylor rule is in effect.

I initialize the value of the gain at $\bar{g} = 0.145$. This is Carvalho et al. (2021)'s estimate for the case of unanchored expectations. In the case of a discrete anchoring function, as in Carvalho et al. (2021), this parameter has important implications for model dynamics because by construction, the gain takes on this value very frequently. However, since my specification for $\mathbf{g}(\cdot)$ is smooth, $\bar{g}$ does not have a strong bearing on model dynamics.

[13] On the lower end of the empirical values for α, Bils and Klenow (2004) find a mean duration of 4.3 months and Klenow and Malin (2010) 6.9 months. Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008) agree on between 7-9 months, while Eichenbaum et al. (2011)'s number is 10.6 months.

Table 1: Parameters calibrated from the literature

β	0.98	stochastic discount factor
σ	1	intertemporal elasticity of substitution
α	0.5	Calvo probability of not adjusting prices
κ	0.0842	slope of the Phillips curve
ψ_π	1.5	coefficient of inflation in Taylor rule*
$\bar{g}$	0.145	initial value of the gain

Table 2: Parameters set to match data moments

ψ_x	0.3	coefficient of the output gap in Taylor rule*
σ_r	0.01	standard deviation, natural rate shock
σ_i	0.01	standard deviation, monetary policy shock*
σ_u	0.5	standard deviation, cost-push shock

*Parameters with an asterisk refer to sections of the paper where a Taylor rule is in effect.

4.2 Estimation

In inferring the functional form of the anchoring function $\mathbf{g}(\cdot)$, I proceed as follows. First, since the analysis in Section 5 relies on Assumption 1, I similarly impose this on the estimation. In other words, I seek to back out the functional form of the relationship between the size of the gain and forecast errors in inflation, $k_t = \mathbf{g}(f_{t|t-1})$. In Appendix E, I investigate whether an alternative estimation strategy using long-run inflation expectations data is suitable to estimate a more general formulation for the anchoring function.

To be as close as possible to a non-parametric estimate of $\mathbf{g}(\cdot)$, while at the same time preserve the shape of the function, I employ a piecewise linear approximation of the form:

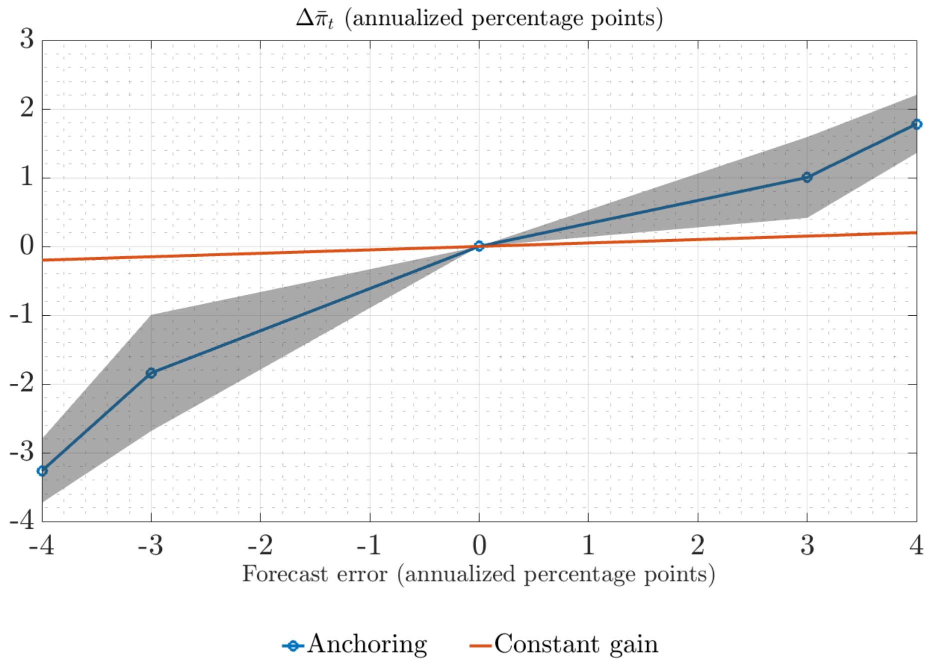
$$\mathbf{g}(f_{t|t-1}) = \sum_i \gamma_i b_i(f_{t|t-1}). \tag{30}$$

Here $b_i(\cdot)$ is a piecewise linear basis and γ is a vector of approximating coefficients. The index i refers to the breakpoints of the piecewise linear approximation. As explained in Appendix D, I estimate γ by simulated method of moments, targeting the autocovariance structure of the observables of the model and expectations.

The estimated coefficients are $\hat{\gamma}$ = (0.82; 0.61; 0; 0.33; 0.45). The elements of $\hat{\gamma}$ represent the value of the gain the private sector chooses when it observes a forecast error of a particular magnitude. For example, a forecast error of -4 pp in inflation is associated with a gain of 0.82.

To make these numbers easier to interpret, Figure 1 depicts what the elements of $\hat{\gamma}$ imply for the updating of long-run inflation expectations (blue dotted line). For comparison, the red line plots a constant gain model with the gain calibrated to the consensus value from the empirical literature (see below). The circles on the blue line indicate the nodes of the piecewise linear approximation.

Figure 1: Changes in long-run inflation expectations for various forecast errors given the estimated gain function



Estimates for 5 knots (indicated by circles), cross-section of size $N = 1000$. The shaded gray areas are 95% confidence intervals constructed using Wald's method from 100 bootstrap samples. The constant gain is calibrated to the consensus value of 0.05.

For instance, a gain of 0.82 that arises when the private sector observes a -4 pp forecast error implies a downward revision in long-run inflation expectations by about 3.2 pp. For the constant gain model, the -4 pp forecast error implies a 0.1 pp downward revision in long-run expectations.

Before interpreting the magnitudes of the estimated anchoring function, it is helpful to work out what amounts to a large gain. In particular, since the gain is the model's metric of the extent of unanchoring, from what size of the gain should one consider expectations to be significantly unanchored? The consensus in the literature on estimating learning gains is that if the true model is one with constant gain learning, then the gain lies between 0.01-0.05. On the higher end of the empirical estimates, Branch and Evans (2006) obtain a constant gain on inflation of 0.062. Milani (2007) finds 0.0183. Noting that a constant gain model corresponds to forecasting with a Kalman gain, where the Kalman gain is given by the signal-to-noise ratio of the underlying distribution, one can also calibrate the gain by measuring signal-to-noise ratios in the data. Doing so using SPF data, Erceg and Levin (2003) obtain a much higher gain of 0.13.

The estimates of the maximal value for endogenous gains lie closer to Erceg and Levin (2003)'s number, at 0.082 in Milani (2014) and at 0.145 in Carvalho et al. (2021). Estimating trend inflation using an unobserved components model with stochastic volatility, Mertens (2016) obtains time-varying Kalman gains for CPI inflation that fluctuate in the range between 0.1 and 0.8.

While Eusepi and Preston (2011) find that the value of 0.002 is sufficient to significantly alter the dynamic behavior of a standard RBC model, calibrated models tend to use values between 0.01-

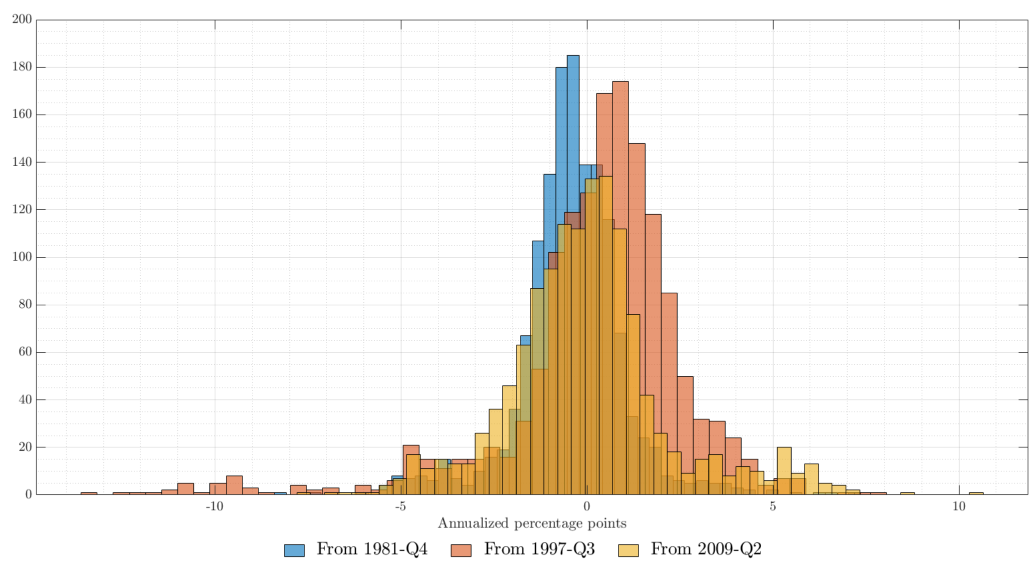
0.05. Since the estimation and calibration literature has settled on the number 0.05 in particular as the benchmark, I calibrate the constant gain model I use for comparison throughout the paper using this value. Accordingly, the constant gain model on Figure 1 also features a 0.05 gain.

An intuitive interpretation of the gain is that its inverse gives the number of past observations the private sector uses to form its current forecasts. Eusepi and Preston (2011)'s number, 0.002, thus implies that firms and households rely on the last 125 years of data. By contrast, the consensus number of 0.05 translates to using 5 years of data.

Seen from both of these perspectives, the consensus gain value of 0.05 corresponds to a sizable, but not excessive sensitivity to surprises. With this benchmark number in mind, the message of the estimation seems stark: the estimated coefficients are large. The highest gain value in my estimation, 0.82, can be interpreted as seeing forecast errors of 4 pp in absolute value prompting the private sector to discount any observations older than about 5 months. That is a very short time.

However, Figure 1 points out that there is considerable nonlinearity in the estimated gain function. One sees this by noticing how the approximation nodes show up as kinks on the figure. What this means is that large surprises unanchor expectations more than linearly compared to small ones. Intuitively, this captures the feature in the data that large surprises upset the public's view on inflation much more than small mistakes. One can thus interpret the nonlinearity of the gain function as an element of rationality on the part of the public sector, as it adjusts its expectation formation much more strongly when larger mistakes make revisions more urgent. By contrast, the constant gain model is by construction linear, failing to account for this feature of the data.

Figure 2: Forecast errors in the SPF data



1-quarter-ahead forecast errors of individual forecasters in the SPF, 1981-Q4 - 2021-Q4, annualized percentage points. The forecast errors are split into three non-overlapping bins with roughly the same number of observations.

Presumably, such large forecast errors are rare, however. To confirm this, Figure 2 plots individual-level forecast errors in the SPF, split into three distinct time periods so as to have roughly the same number of forecast errors in each subsample. As the figure shows, the bulk of surprises are approximately in the $[-3, 3]$ percentage point interval. What is also noticable is that forecast errors changed pattern over time. In the first time period, inflation tended to surprise forecasters on the downside, while in the second period (roughly between 1997 and 2010) it instead mostly surprised on the upside.[14] In the last decade, the tails of the distribution shrank, reflecting more stable inflation, less inflation surprises and thus a smaller need for forecasters to reconsider their view on the long-run mean of inflation.

A changing pattern in inflation surprises is thus a feature of the data. The nonlinearity of the anchoring framework embodies forecasters adjusting their forecasting behavior according to the volatility of their environment, updating a lot when there are large surprises, and barely updating if the environment is stable. As seen by comparing the red and blue lines on Figure 1, this is something agents would not be able to do if they used a constant gain forecasting model.

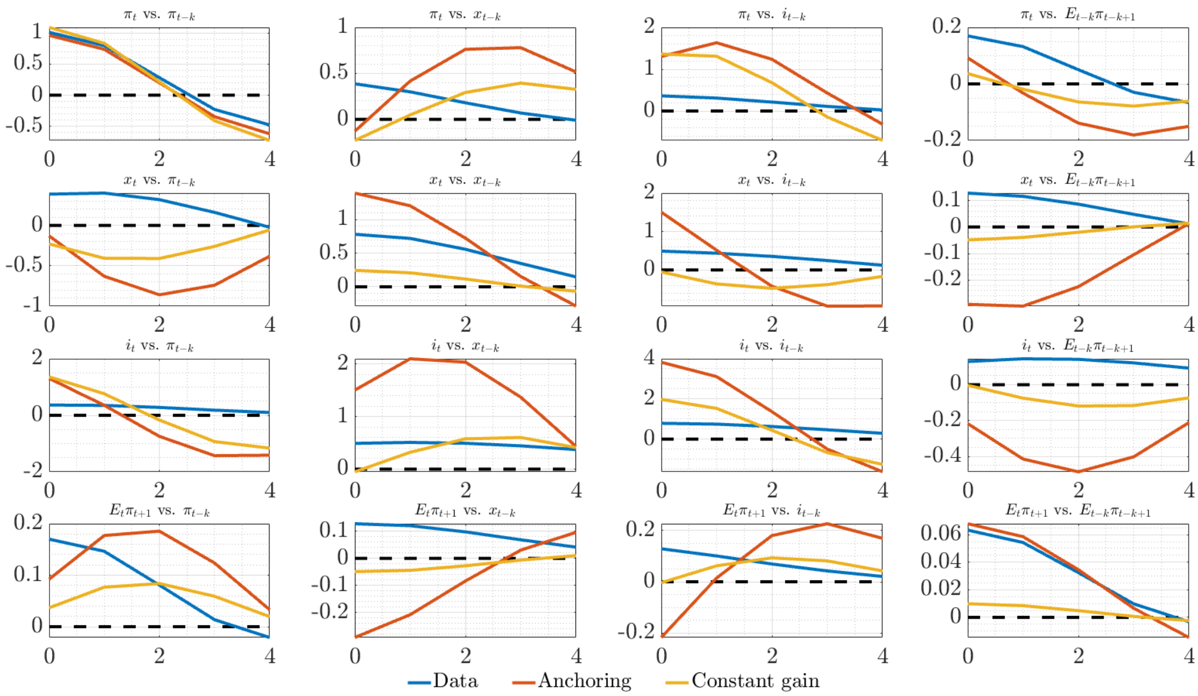
That time-varying anchoring is a feature consistent with data can be also seen on the autocovariogram, shown in Figure 3, which plots the moments of the model (red) against those in the data (blue). For comparison, I also plot moments generated by a constant gain learning model with the consensus value for the gain of 0.05 (yellow).

While a constant gain can match inflation dynamics (top left panel), its fit deteriorates compared to the anchoring model when it comes to the output gap (second diagonal panel) and, importantly, inflation expectations (bottom right panel). The reason the anchoring model fits the expectations data better than a constant gain alternative is because it does not constrain the private sector to always respond to surprises in the same way. In a volatile environment, such as the 1980s, high degrees of unanchoring allow the private sector to update its view very strongly on what kind of inflation regime it is in. With low and stable inflation instead, such as in the 2000s, occasional small inflation surprises do not incite the private sector to change its mind about average inflation. With a constant gain, instead, the private sector would respond as sensitively during the 2000s as during the 1980s. That is at odds with the data.

We also notice that unanchoring introduces another flexibility to expectation formation compared to constant gains: asymmetry. As seen on the estimates and on Figure 1, negative inflation surprises raise the gain about twice as much as positive ones do, and lead to almost twice as large updates in long-run inflation expectations. Consistent with the findings of Hebden et al. (2020), this indicates that long-run expectations are more sensitive to negative than to positive surprises. In other words, surprises on the downside are almost twice as likely to unanchor expectations as those on the upside. This has some interesting implications for the current time period, where rising inflation in the wake of the Covid-19 crisis has resulted in a series of very large inflation surprises. On the one hand, this is clearly putting upward pressure on the expectations anchor. On the other, the asymmetry of unanchoring implies that this situation would be more concerning if inflation was surprising on the downside rather than on the upside.

[14] In this period, there are some outliers with extreme downward surprises, which may be due to the Great Recession.

Figure 3: Autocovariogram



Estimates for a cross-section of $N = 1000$. Moments are computed for the best estimate for the sample length of 151 with 200 additional burn-in periods. The constant gain case shows the moments of the model for the consensus value of the gain of 0.05 from the literature.

5 Monetary policy and anchoring

This section sets up and solves the optimal monetary policy problem in the model with anchoring. In Section 5.1, I begin by analyzing the Ramsey problem of determining optimal paths for the endogenous variables that policy seeks to bring about. While anchoring introduces substantial nonlinearity into the model, it is possible to derive analytically an optimal target criterion for the policymaker to follow. As we shall see, the optimal rule prescribes for monetary policy to act conditionally on the stance of expectations, and will thus be time-varying and state-dependent. In particular, whether expectations are anchored or not matters for the extent to which there is a tradeoff between inflation and output gap stabilization, and also for the volatility cost of getting expectations anchored.

I then turn to the question of how to implement optimal policy. Section 5.2 uses global methods to solve for the interest rate sequence that implements the target criterion. I then discuss the properties of the optimal interest rate policy and why it is successful in stabilizing both inflation expectations and inflation.

The optimal interest rate policy will thus be a nonlinear function of all the states in the model - a complicated object to compute. In practice, however, monetary policy is most commonly modeled using time-invariant rules like the Taylor rule that are both simple to compute and to communicate to the public. In Section 5.3, I therefore restrict attention to Taylor-type feedback rules for the interest rate. I solve for the optimal Taylor-rule coefficient on inflation numerically and investigate how this choice affects the anchoring and unanchoring of expectations.

5.1 The Ramsey policy under anchoring

I assume the monetary authority seeks to maximize welfare of the representative household under commitment. As shown in Woodford (2003a), a second-order Taylor approximation of household utility delivers a central bank loss function of the form

$$L^{CB} = \mathbb{E}_t \sum_{T=t}^{\infty} \{(\pi_T - \pi^*)^2 + \lambda_x (x_T - x^*)^2\}, \tag{31}$$

where λ_x is the weight the central bank assigns to stabilizing the output gap. In the rational expectations New Keynesian model, λ_x is a function of deep parameters: $\lambda_x = \kappa/\theta$. Just as under rational expectations, one can show that it is optimal to set the central bank's targets, π^* and x^*, to zero. The central bank's problem, then, is to determine paths for inflation, the output gap and the interest rate that minimize the loss in Equation (31), subject to the model equations (9) and (10), as well as the PLM (18), the learning equations (20) and (21), and the anchoring function (22). The full statement of the Ramsey problem under Assumption 1 is as follows:

$$\begin{aligned}
&\min_{\{\pi_t, x_t, i_t, \bar{\pi}_{t-1}, k_t\}_{t=t_0}^{\infty}} \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \lambda_x x_t^2) \qquad \text{s.t.} \\
&x_t = \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} \beta^{T-t}((1-\beta)x_{T+1} - \sigma(\beta i_T - \pi_{T+1}) + \sigma r_T^n), \\
&\pi_t = \kappa x_t + \hat{\mathbb{E}}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}(\kappa\alpha\beta x_{T+1} + (1-\alpha)\beta\pi_{T+1} + u_T), \\
&\hat{\mathbb{E}}_t \pi_{t+k} = \bar{\pi}_{t-1} + b_1^k s_t \quad \forall k \geq 1, \\
&\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1}, \\
&f_{t|t-1} = \pi_t - \hat{\mathbb{E}}_{t-1}\pi_t, \\
&k_t = \mathbf{g}(f_{t|t-1}),
\end{aligned} \tag{32}$$

where $\mathbb{E}$ is the central bank's expectation. The use of the mathematical expectation $\mathbb{E}$ reflects the assumption that the central bank has model-consistent expectations and observes the private sector's expectations. This assumption, which Gaspar et al. (2010) refer to as "sophisticated central banking," is quite strong. In practice, it is likely that the central bank's measure of private sector expectations is noisy at best. Nevertheless, it is a useful benchmark case.

A last thing to note about the Ramsey problem in the anchoring model is that while it is set up under commitment, the optimal plans under commitment and discretion will coincide, and will thus not be subject to the time-inconsistency problem of Kydland and Prescott (1977). As shown by Mele et al. (2019) and Molnár and Santoro (2014), this is a general feature of adaptive learning models and comes from the fact that adaptive expectations cannot incorporate promises of the policymaker.

5.1.1 Optimal Ramsey policy as a target criterion

As foreshadowed above, the nonlinearity of the model due to the anchoring function prevents a full analytical solution to the Ramsey problem. Therefore I now characterize the first-order conditions of the problem analytically, and proceed in Section 5.2 to solve the full problem numerically. The details of the derivations are given in Appendix F, which also illustrates how the endogeneity of the gain introduces nonlinearity into the model.

The solution of the Ramsey problem is stated in the following proposition.

Proposition 1. *Target criterion in the anchoring model*
The targeting rule in the simplified learning model with anchoring is given by

$$\pi_t + \frac{\lambda_x}{\kappa} x_t = c\Big(k_t + f_{t|t-1}\mathbf{g}'_t\Big)\, \mathbb{E}_t \left((1-\beta)x_{t+1} + \beta \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} 1 - k_{t+j} - f_{t+j|t-1+j}\mathbf{g}'_{t-1+j}\right), \quad (33)$$

$$c \equiv \frac{\lambda_x}{\kappa} \frac{(1-\alpha)\beta^2}{1-\alpha\beta}. \quad (34)$$

Proved in Appendix F. For a target criterion for a more general specification of the gain function, see Appendix G.

Here $\mathbf{g}'_t$ denotes the derivative of $\mathbf{g}(\cdot)$ with respect to the forecast error $f_{t|t-1}$, evaluated at time t. Note that $\mathbf{g}' = \mathbf{g}_\pi = -\mathbf{g}_{\bar{\pi}}$.

The interpretation of Equation (33) is that the *intratemporal* tradeoff between inflation and the output gap due to cost-push shocks is complemented by two *intertemporal* tradeoffs. One is due to learning in general and corresponds to Molnár and Santoro (2014)'s result, and one is due to anchoring in particular, and is thus novel here. The first intertemporal effect comes from the current level of the gain, k_t, which captures how far learning is from converging to rational expectations. The second, novel intertemporal tradeoff is manifest in the derivative of the anchoring function today, $\mathbf{g}'_t$, as well as in all expected levels and changes in the gain in the future in the expression $(1 - k_{t+j} - f_{t+j|t-1+j}\mathbf{g}'_{t-1+j})$ in the second bracket on the right-hand side. These expressions say that the presence of anchoring qualifies the first intertemporal tradeoff because now the degree and direction in which the gain changes today and is expected to change in the future matter too. In other words, the central bank needs to consider whether its chosen interest rate sequence contributes to anchoring expectations in future periods, or whether it actually serves to unanchor them.

Let me investigate these channels in isolation. To see exactly what the role of anchoring is in the target criterion, consider first the special case of exogenous gain adaptive learning, for simplicity with a constant gain specification. In this case the anchoring function and the forecast error are irrelevant (since $\mathbf{g}'_t = 0,\ \forall t$) and (33) boils down to

$$\pi_t = -\frac{\lambda_x}{\kappa}x_t + \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta^2}{1-\alpha\beta}k\,\mathbb{E}_t\left((1-\beta)x_{t+1} + \sum_{i=0}^{\infty}x_{t+i+1}(1-k)^i\right), \tag{35}$$

which replicates Molnár and Santoro (2014)'s expression, with the exceptions that the parameters $\frac{(1-\alpha)\beta^2}{1-\alpha\beta}$ and an extra $(1-\beta)x_{t+1}$ appear.[15] These differences come from the fact that in my model the entire term structure of expectations enters the evolution equations of inflation and output, while in Molnár and Santoro (2014), only one-period-ahead expectations do.

As Molnár and Santoro (2014) emphasize, this result suggests that already the presence of learning by itself is responsible for the first intertemporal tradeoff between inflation and output gap stabilization. The fact that the central bank now has future output gaps as a margin of adjustment means that it does not have to face the full tradeoff in the current period. Learning allows the central bank to improve the current output gap without sacrificing inflation stability today; however, this results in a worsened tradeoff in the future. In other words, adaptive learning by itself allows the central bank to postpone the current tradeoff to later periods.

Intuitively, this happens because adaptive expectations are slow in converging to rational expectations. In the transition, the private sector's expectations do not adjust to fully internalize the intratemporal tradeoff. This gives the monetary authority room to transfer the tradeoff to the future.

Contrasting Equations (35) and (33) highlights the novel role of the anchoring channel. With anchoring, the extent to which policy can transfer the intratemporal tradeoff to future periods depends not only on the stance of the learning process, as in (35), but also on whether expectations are anchored, and in which direction they are moving. In fact, not only the current stance and change of anchoring matters, but also all expected future levels and changes.

Anchoring, however, complicates the possibility of transferring today's tradeoff to the future. One can see this on the fact that forecast errors and the derivatives of the anchoring function are able to flip the sign of the second term in (33). This means that anchoring can alleviate or worsen the intertemporal tradeoff. To see the intuition, consider the equation system of first-order conditions from solving the Ramsey problem. While the full system is presented in Appendix F, let us focus solely on Equation (F.2), the equation governing the dynamics of observables in the model:

$$2\pi_t = -2\frac{\lambda_x}{\kappa}x_t + \varphi_{5,t}k_t + \varphi_{6,t}\mathbf{g}'_t. \tag{36}$$

The Lagrange multipliers $\varphi_5 \geq 0$ and $\varphi_6 \geq 0$ are the multipliers of the updating equation (20) and the anchoring function respectively. This equation, upon substitution of the solutions for the two multipliers, yields the target criterion. It is therefore easy to read off the intuition at a glance. First, since $\varphi_{5,t}k_t \geq 0$, one immediately obtains the above-discussed conclusion that as long as the adaptive learning equation is a constraint to the policymaker ($\varphi_{5,t} > 0$), the central bank has

[15] Since Molnár and Santoro (2014) has no equation number at the relevant expression, I refer the reader to Equation (24) of Gaspar et al. (2010), who provide a parsimonious summary of Molnár and Santoro (2014).

more room to transfer the contemporaneous tradeoff between inflation and the output gap to the future.[16]

However, whether the anchoring equation alleviates or exacerbates the inflation-output gap tradeoff depends on the sign of g_t'. If the derivative is positive, the effect is the same as above, and the central bank has more leeway to postpone the tradeoff to the future. By contrast, if the derivative is negative, that is expectations are becoming anchored, the intratemporal tradeoff is worsened.

Why do unanchored expectations give the central bank the possibility to postpone its current inflation-output gap tradeoff? The reason is that when expectations become unanchored, the learning process is restarted. A not-yet converged learning process implies, as discussed above, that postponing the tradeoff is possible. Restarting the convergence process thus unlocks this possibility.

This seems to suggest that from a smoothing standpoint, the central bank should prefer to have unanchored expectations. As will be shown in Sections 5.2-5.3, volatility considerations will suggest otherwise. But in fact, even the smoothing viewpoint involves some ambiguity on whether expectations should be anchored from the perspective of the central bank. Clearly, the central bank prefers to face a learning process that on the one hand has not yet converged, and on the other is converging only slowly. A high gain under unanchored expectations implies both a sizable distance from convergence as well as faster learning and thus faster convergence. Therefore, ideally the central bank would like to have expectations anchored but the gain far from zero; a contradiction. Once the gain approaches zero, only unanchored expectations can raise it again to restart the learning process. But once the gain is large, the only way to slow down learning is to anchor expectations, that is, to lower the gain.

5.2 Implementing the Ramsey policy: the optimal interest rate sequence

Having a characterization of optimal policy in the anchoring model as a first-order condition, the next relevant question is how the central bank should set its interest rate tool in order to implement the target criterion in (33). In other words, we would like to know what time-path of interest rates implements the optimal sequence of inflation and output gaps. As emphasized in Section 5.1, the nonlinearity of the model does not admit an analytical answer to this question. I therefore solve for the optimal interest rate policy numerically using global methods. I rely on the calibration presented in Tables 1 and 2 and the estimated parameters of the expectations process in Section 4. Furthermore, I set the central bank's weight on output gap fluctuations, λ_x, to 0.05, the value estimated by Rotemberg and Woodford (1997).

Appendix H outlines my preferred solution procedure, the parameterized expectations approach, while Appendix I gives the details of the parametric value function iteration approach I

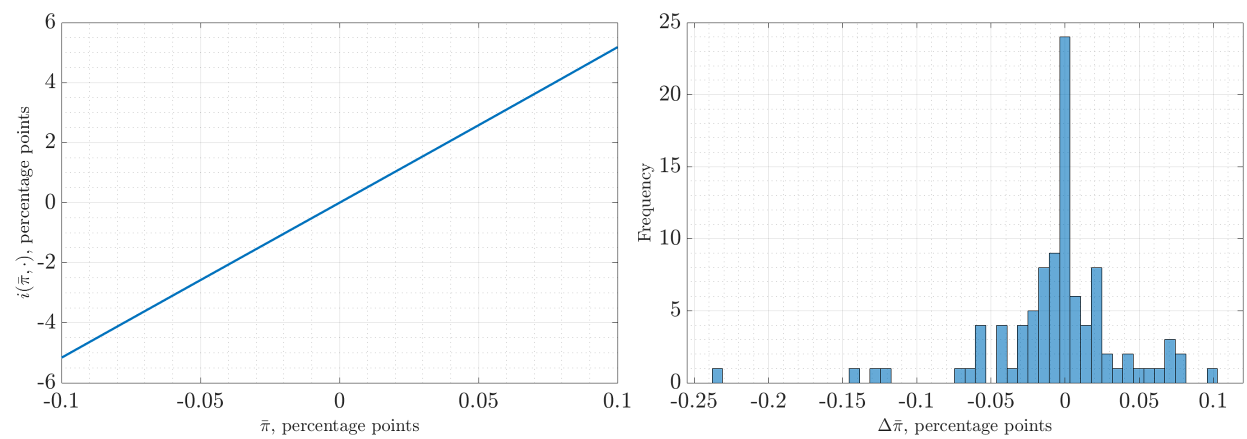
[16] φ_5 and φ_6 are zero if the learning process has converged. In absence of shocks, this eventually happens because decreasing forecast errors lead to decreasing gains. But with exogenous disturbances, forecast errors may increase throughout the lifetime of the economy, raising the gain and restarting the learning process. The model thus exhibits a weak form of weak expectational stability.

implement as a robustness check. The main output of this procedure is an approximation of the optimal interest rate policy as a function of the vector of state variables. The relevant state variables are expected mean inflation and the exogenous states at time t and $t-1$, rendering the state vector five-dimensional:

$$X_t = (\bar{\pi}_{t-1}, r_t^n, u_t, r_{t-1}^n, u_{t-1}). \tag{37}$$

As a first step, I plot how the approximated policy function depends on $\bar{\pi}_{t-1}$, while keeping all the other states at their mean. The result, depicted on Panel (a) of Figure 4, suggests that optimal interest-rate setting responds linearly and very sensitively to the stance of expectations, $\bar{\pi}_{t-1}$. If expected mean inflation increases by 5 basis points, the interest rate rises by about 250 basis points.[17]

Figure 4: Policy function and implied volatility in long-run expectations



(a) $i(\bar{\pi}$, all other states at their means)

(b) Histogram of changes in $\bar{\pi}$

This is a large response. Clearly, optimal policy involves subduing unanchored expectations by injecting massive negative feedback to the system. One may then wonder why optimal policy is so aggressive on unanchored expectations when the analysis of the target criterion in Section 5.1 suggested that learning can alleviate the stabilization tradeoff between output and inflation.

The reason is that the anchoring expectation formation introduces another intertemporal tradeoff to monetary policy: a volatility tradeoff. One can see this on Figure 5, portraying the dynamics of the system following a two-standard-deviation inflationary cost-push shock, conditional on a Taylor rule with baseline parameters. The figure contrasts the rational expectations version of the model with a well-anchored, weakly anchored and strongly unanchored scenario in the anchoring model, as well as with a constant gain model.

[17] This result is qualified if one relaxes Assumption 1. If the private sector learns about all observables, not just inflation, interest-rate expectations play a major role in determining forecasts of future inflation and thus add a stabilizing channel that is absent from the current specification. In the more general case, then, an order of magnitude smaller responses in the current interest rate are sufficient. For more on the role of interest-rate expectations, see Section 5.3.

The same shock that under rational expectations completely vanishes by the second period, triggers a large, persistent and oscillatory response in the anchoring model.[18] Clearly, the size and persistence of the shock, as well as the magnitude of oscillations increase the more unanchored expectations are. This comes from the fact that if expectations are anchored, stable expectations lower the pass-through between shocks and observables. Instead, if expectations are unanchored, they become volatile, passing through the shocks and amplifying them. Another way to say this is that anchored expectations flatten the reduced-form Phillips curve, echoing the argument of Hazell et al. (2022). Having unanchored expectations, then, comes at a volatility cost in the central bank's target variables.

This volatility cost is also present in the constant gain model, and its size depends on the size of the gain. But as the literature on estimating constant gain models shows (Branch and Evans, 2006, Milani, 2007), a too large constant gain is inconsistent with data, thus putting an upper bound on how much excess volatility can come from a constant gain model. Also, the constant gain model implies the same volatility cost to a particular shock in any state of the world. A state-dependent gain, instead, introduces the distinction between the effects of the same shock when expectations are unanchored as opposed to anchored. As seen on the difference between the blue and yellow lines, the same shock leads to much more volatility if expectations are unanchored when the shock hits than if they are strongly anchored.

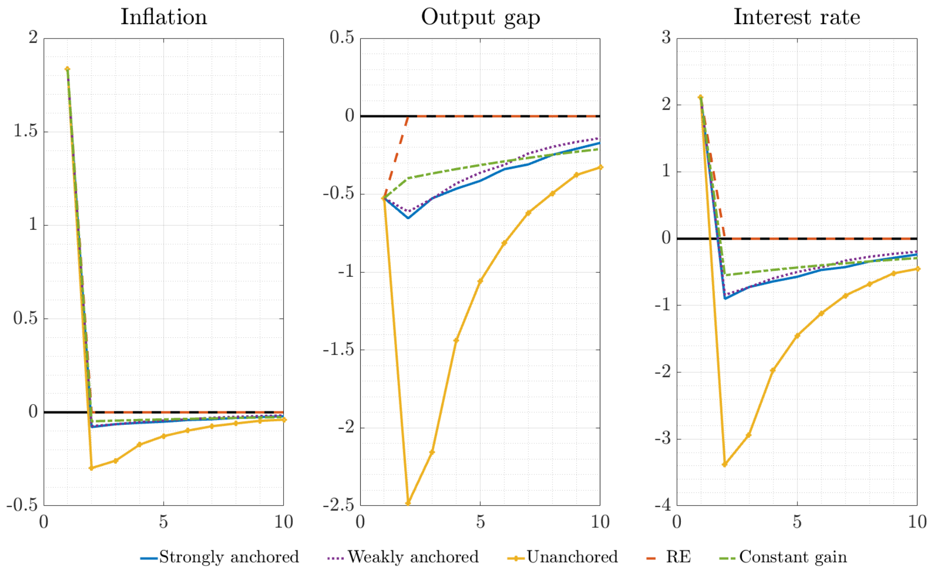
The fact that the volatility cost rises the more unanchored expectations are dictates that, in the long run, the central bank wishes to have expectations anchored. In the constant gain model, this would not be an option because the central bank would not be able to influence the size of the constant gain. In the anchoring model, instead, the central bank can affect the expectation formation process.

What complicates the central bank's life is that anchoring expectations itself comes at a convex volatility cost. Anchoring expectations requires an aggressive interest response because by these means the central bank can introduce negative feedback to the system. But changes in the interest rate surprise the private sector, raising forecast errors. The more unanchored expectations are, the more volatility the interest rate movement inflicts on the economy.

We can see from the policy function how optimal policy resolves this tradeoff: it reacts extremely aggressively to movements in long-run expectations. This way, the central bank hopes to avoid even larger interventions that would become necessary were expectations to unanchor further. To avoid having to pay so high a price, the central bank is extra aggressive in the short run to prevent massive unanchoring from ever materializing. Thus, the optimal response to the volatility tradeoff is to temporarily increase volatility in order to reduce it in the long-run. In this way, the central bank's aggressiveness in the model is driven by the desire to prevent upward (downward)

[18]As periodically noted in the literature, adaptive learning models tend to produce impulse responses that exhibit damped oscillations. Authors making explicit note of this phenomenon include Evans and Honkapohja (2001), Evans et al. (2013) and Anufriev and Hommes (2012). The reason is that under an adaptive learning framework, forecast errors following an impulse are oscillatory. In fact, the higher the learning gain, the higher the amplitude of forecast error oscillations. Appendix J presents a simple illustration for why this is the case.

Figure 5: Impulse responses after a cost-push shock



Shock imposed at $t = 25$ of a sample length of $T = 400$ (with 200 additional burn-in periods), cross-sectional average with a cross-section size of $N = 100$. The remark on whether expectations are anchored or not refers to whether the gain is below the 10^{th}, below the 50^{th}, or above the 90^{th} percentile of simulated gains at the time the shock hits. For the constant gain model, the gain is set to the consensus value of 0.05.

drifting long-run expectations from becoming a self-fulfilling inflationary (deflationary) spiral.

The large interest-rate responses resemble the idea advocated by Goodfriend (1993) that the central bank moves to offset "inflation scare" episodes. As Goodfriend shows, it was historically not uncommon to move the interest rate by hundreds of basis points to subdue inflation scares. For example, in March 1980, the Fed raised the interest rate by 230 bp to convince the public that it would not tolerate high inflation. The optimal policy function prescribes that this is exactly what the policymaker should do to fight unanchored expectations.

What Figure 4 also suggests is that acting aggressively in the short run indeed delivers the long-run benefits of stabilizing economic fluctuations. As Panel (b) depicts, employing the optimal policy implies that realized changes in long-run inflation expectations are very small. As seen on the histogram of $\Delta\bar{\pi}$, with the optimal policy in place, the model spends most of its time in the region of minuscule fluctuations in $\bar{\pi}$. The mode of the distribution is a change of 0.3 basis points in absolute value, implying that in normal times, the central bank only needs to raise or lower the interest rate by 15 basis points. In other words, the aggressive nature of optimal policy allows the central bank to keep expectations anchored or quickly reanchor them following shocks. In this way, the monetary authority eliminates as much volatility stemming from unanchored expectations as it possibly can.

5.3 Optimal Taylor rule under anchoring

Monetary policy is often formulated using a Taylor rule. Proponents of such a characterization, like Taylor (1993) himself, emphasize the benefits of having a simple, time-invariant and easily verifiable rule. Also in the anchoring model, a policymaker may thus be interested in using a Taylor-type approximation to optimal policy in order to combine the benefits of having a simple, yet near-optimal rule.[19] Therefore I now consider the restricted set of Taylor-type policy rules and ask what value of the time-invariant Taylor-rule coefficient on inflation is optimal in the case of the anchoring model.

In this section, I thus restrict attention to a standard Taylor rule:

$$i_t = \psi_\pi(\pi_t - \pi^*) + \psi_x(x_t - x^*) + \bar{i}_t, \tag{38}$$

where ψ_π and ψ_x represent the responsiveness of monetary policy to inflation and the output gap respectively. Lastly, $\bar{i}_t$ is a monetary policy shock. I also assume that when the Taylor rule is in effect, the central bank publicly announces this. In order to avoid comparing apples with oranges, I start by assuming that, exactly as in the Ramsey policy, interest-rate expectations are formed using the Taylor rule with rational expectations of inflation. This is a crucial assumption which has bearings on the debate on whether non-rational expectations necessitate a stronger policy response than rational expectations (Orphanides and Williams, 2004, Eusepi et al., 2020). I relax the assumption and discuss its implications at the end of the section.

I compute the optimal Taylor rule coefficient on inflation numerically by minimizing the central bank's expected loss in a cross-section of $N = 100$ simulations of the rational expectations, constant gain learning and anchoring models. I continue to use the calibration of Tables 1 and 2 and to parameterize the anchoring function using the estimated approximating coefficients of the gain function ($\hat{\gamma}$ in Section 4). For the constant gain model, I continue using the consensus gain value of 0.05.

Table 3 presents the optimal Taylor rule coefficient ψ_π for the three models. The message is clear: in the anchoring model, the central bank needs to work much harder than in the other models and push strongly against inflation fluctuations. This is necessary because mistakes by the central bank in achieving the inflation target unanchor expectations, but unanchored expectations feed back into inflation, leading to further unanchoring. To get out of this unpleasant feedback loop, the central bank moves the interest rate exceedingly aggressively in response to movements in inflation. Corroborating the conventional wisdom in the adaptive learning literature, for example in Orphanides and Williams (2004), I thus find that monetary policy specified as a Taylor rule should be more aggressive on inflation than what would be optimal under rational expectations. As I explain below, however, one can overturn this result by maintaining alternative assumptions on interest rate expectations.

[19] Recall from Woodford (2003a) that even under rational expectations, a standard Taylor rule is not fully optimal because its purely forward-looking nature precludes the use of promises of future policy.

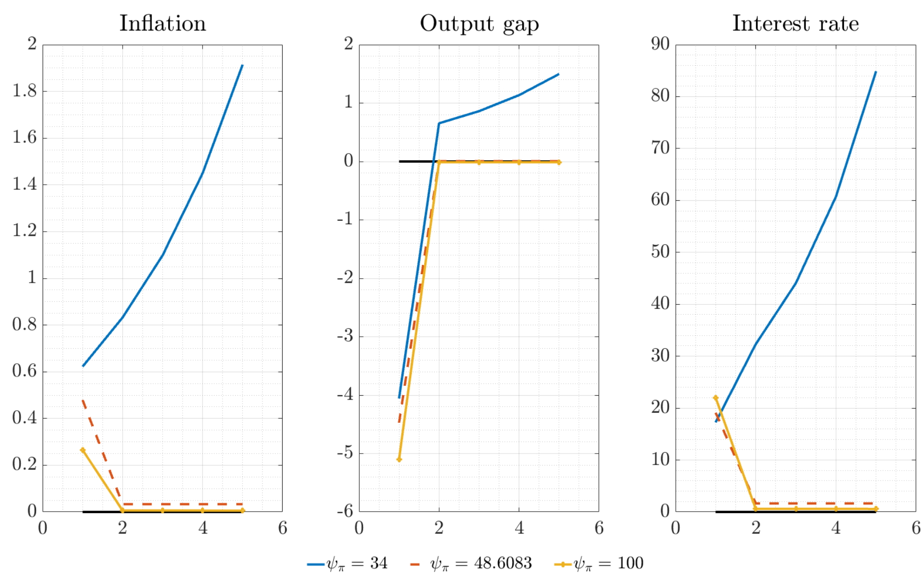
Table 3: Optimal coefficient on inflation, RE, anchoring and constant gain models

$\psi_\pi^{*,RE}$	$\psi_\pi^{*,Anchoring}$	$\psi_\pi^{*,Constant\ gain}$
2.2079	48.6083	45.6499

Sample length is $T = 400$ with a cross-section of $N = 100$ and 200 additional burn-in periods. For the constant gain model, the gain is set to the consensus value of 0.05.

The reason behind this high aggressiveness is that when the central bank's interest rate rule is not state-dependent, it moves the interest rate too little when expectations are unanchored, and too much when they are anchored. We can see this on Figure 6, which depicts impulse responses of the model for an inflationary cost-push shock for various levels of inflation aggressiveness ψ_π. A too small value of $\psi_\pi = 34$ relatively to the optimal value (blue line) imposes too little negative feedback to get inflation back to its pre-shock value. A too high value of ψ_π (yellow dotted line) instead is effective in stabilizing inflation, but also involves costs in terms of making interest rates and output very volatile. But as the figure suggests, it is relatively less costly to err on the upside, i.e. to choose a too high ψ_π, than on the downside, since a too low inflation coefficient in the Taylor rule is not sufficient to stabilize the economy at all.

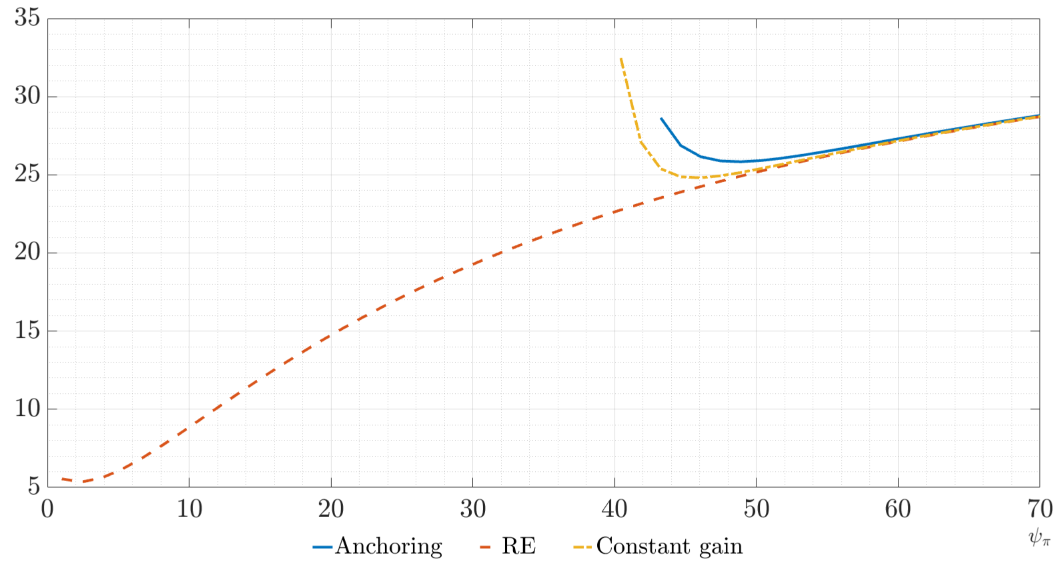
Figure 6: Impulse responses for various values of ψ_π



Cost-push shock imposed at $t = 25$ of a sample length of $T = 400$ (with 200 additional burn-in periods), cross-sectional average with a cross-section size of $N = 100$.

This intuition is reinforced in Figure 7, which depicts the central bank's loss as a function of the inflation coefficient ψ_π. One notices that for values of ψ_π below about 42, the loss in the anchoring model goes to infinity so fast that it exits the figure. Intuitively, this comes from the fact that since ψ_π is too low in this case to stabilize the model, the explosive paths for inflation, output gaps and interest rates lead to infinite losses for the central bank. A too high Taylor coefficient, instead,

Figure 7: Central bank loss as a function of ψ_π



Sample length is $T = 400$ with a cross-section of $N = 100$ and 200 additional burn-in periods. Note that for the anchoring and constant gain models, I only plot the loss function if it takes on sufficiently small values. For the constant gain model, the gain is set to the consensus value of 0.05.

while increasing the loss somewhat by virtue of making interest rates more volatile, is not nearly as costly. In fact, in this range of ψ_π-values (about > 50), the loss in the anchoring model approaches that in the rational expectations version of the model. This makes sense because since a sufficiently high inflation coefficient stabilizes inflation expectations, fluctuations in expectations cease being the main driver of losses. Thus the wedge between RE and anchoring disappears, and in both cases the central bank just incurs losses from volatility coming from volatile interest rates.

A similar intuition applies to the constant gain model as to the anchoring model: the central bank needs to lean against fluctuating expectations to stabilize the economy. But there is a crucial difference. In the constant gain world, the central bank does not need to work as hard as in the anchoring model, and its loss is also always weakly below that in the anchoring model. This is because expectations are always (un)anchored to the same degree in a constant gain model. The central bank can do nothing to influence anchoring, and thus there are no benefits of acting extra aggressively today to anchor expectations tomorrow. Thus a lower inflation coefficient is sufficient to do the job.

The anchoring and the constant gain models look similar here in the sense that the anchoring model involves a higher optimal aggressiveness to subdue higher losses, but the order of the magnitude of losses and optimal coefficients is similar. This is because the fact that the Taylor rule policy is not state-dependent constrains the policymaker to act *as if* the anchoring model was a constant gain model. There is simply not much the policymaker can do to take state-dependent anchoring into account apart from being a little more aggressive.

This highlights that the time-varying nature of Ramsey-optimal policy is its key characteristic. The ability to take the stance of anchoring into account is what enables the central bank to be exceedingly aggressive if and only if expectations are about to unanchor, and maintain a dovish stance otherwise. The way out for a central bank that conducts policy via a Taylor rule is to select state-dependent Taylor-rule coefficients. For example, if the Taylor-rule coefficients were a function of the gain, the model's metric of unanchoring, the central bank would choose low coefficients when expectations are anchored and high ones when expectations show signs of unanchoring. Such a policy rule would produce central bank behavior that appears to the econometrician as regime-switching, echoing the literature on estimating Taylor rules that frequently arrives at this conclusion (Lubik and Matthes, 2016).

Lastly, the optimal Taylor rule exercise entails an interesting lesson for optimal policy when the public is learning. I noted at the beginning of the section that interest-rate expectations are formed using a Taylor rule with rational inflation expectations. This assumption served to make the comparison between the Ramsey and the Taylor-rule policies fair. However, if one recomputes the inflation coefficient under the alternative assumption that the private sector forms interest-rate expectations with a Taylor rule with the anchoring inflation expectation, one obtains an optimal coefficient of 1.1, about half of the rational expectations value. In other words, in this case, optimal Taylor-rule policy should be *less* aggressive on inflation, not more.

In this case, then, one obtains Eusepi et al. (2020)'s result that when there is a drift in long-run interest-rate expectations, there is an upper limit to how aggressive the central bank can be. In such a case, interest-rate expectations inherit the volatility of inflation expectations induced by an aggressive interest rate response. This leads to a lot of additional volatility on top of the volatility coming from inflation expectations. This excess volatility outweighs the benefits of stabilization.

In fact, as I remark in the beginning of Section 5.2, a similar conclusion holds for the Ramsey policy as well. If one introduces a drift in interest-rate expectations, for example by allowing the private sector to form interest-rate expectations using a Taylor rule with the anchoring expectation of inflation in it, the aggressiveness of the optimal Ramsey policy is scaled down. This underscores Eusepi et al. (2020)'s point that understanding interest-rate expectations is really important.

But at the end of the day, in the anchoring model this boils down to a question of scale: how large should the policy response be quantitatively? My model is too simple to answer that question precisely. The key lesson for policy from the anchoring model, distinguishing it from constant gain models or from other models with private sector learning, is that the smoothly evolving degree of unanchoring introduces the need for policy to be state-dependent. This state-dependence is the main characteristic of the Ramsey policy, and, as I have argued, a central bank can overcome the deficiencies of a fixed-coefficient Taylor rule as well by conditioning the inflation-coefficient on the degree of unanchoring.

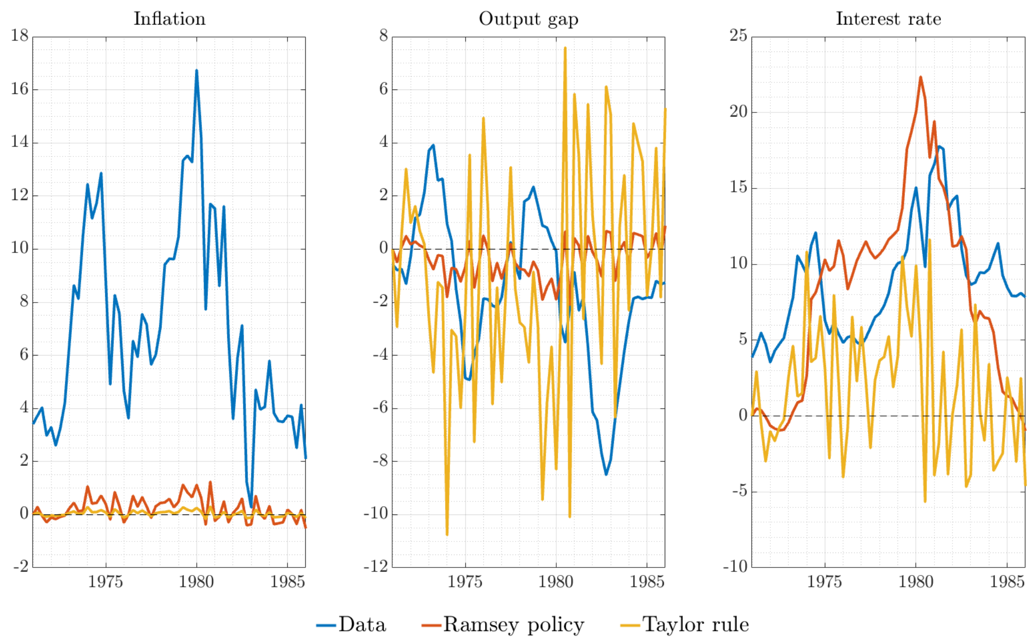
6 The Volcker disinflation

What are the practical implications of the optimal policies under anchoring for the conduct of monetary policy? To better understand this, I now consider what the model implies for the Great

Inflation and the Great Moderation. To do this, I first back out structural shocks in the US from a structural VAR for the period 1970-Q3 to 2021-Q3. I then feed the shocks into the model, both under the optimal Ramsey and Taylor rule policies, and compare the evolutions of the observables with the data.

I consider a two-lag VAR in the output gap, inflation and the federal funds rate (in this order). Following standard practice, I isolate a demand, supply and monetary policy shock by imposing a Cholesky decomposition on the variance-covariance matrix of the reduced-form residuals. This amounts to the assumption that the monetary policy shock only affects the federal funds rate on impact, while the demand shock is the only shock to affect all observables contemporaneously.

Figure 8: The Great Inflation and the Great Moderation through the lens of the anchoring model



Structural shocks are extracted from a trivariate VAR with a Cholesky identification, where the output gap is ordered first, inflation second and the federal funds rate last.

Figure 8 shows the historical evolutions of inflation, the output gap and the federal funds rate in blue against the counterfactual evolutions under the optimal Ramsey policy in red and the Taylor rule policy in yellow. I zoom in on the period between 1970 and 1985 to see in particular what the model implies for the Great Inflation and the Volcker period.

The first striking observation is that under the Ramsey policy, the Great Inflation never happens. Since the central bank that follows the Ramsey policy works hard to maintain the expectations anchor, it responds to inflationary pressures swiftly and strongly. This is most clearly seen when comparing the Ramsey policy interest rate with the true data between 1975 and 1980 (red against blue lines). Whereas the historical interest rate decreased around 1975, and only started rising slowly and with some backtracking toward 1980, the Ramsey policy involves a steadily rising interest rate throughout this period. In fact, under the Ramsey policy, the late 1970s would have seen even higher interest rates than the ones introduced by Paul Volcker in the early 1980s.

The aggressive response to inflationary pressures does not allow inflationary shocks to unanchor expectations, maintaining a strikingly stable inflation process. This also means that the normalization of the interest rate once inflationary pressures subside can happen much more quickly than in the data. Notice for example the difference between the historical and Ramsey policy interest rates in 1983-84. That there still are some inflationary pressures is reflected in that the Ramsey interest rate flattens momentarily. But then it continues decreasing steadily. Instead, historically, the Fed had to hike interest rates multiple times to convince markets that high inflation was not coming back.

A stable inflation process also helps to stabilize the output gap. As seen on the middle panel, the Ramsey policy essentially renders output acyclical. Since it pushes against inflationary pressures, it does not allow the booms in the early and late 1970s to materialize. At the same time, because high inflation does not have to be reigned in the 1980s, it also completely avoids the Volcker recession.

This goes to show how much policy can benefit from having anchored expectations: having paid the cost to get expectations anchored in the past pays off in terms of making life easy in the present. As discussed in Section 5.2, the central bank trades off the short-run cost of aggressive interest rate movements today with the long-run benefit of having anchored expectations. As Figure 8 suggests, the benefits outweigh the costs by far.

The optimal Taylor rule policy displays both similarities and differences to the Ramsey policy. Its main feature is an exceedingly volatile interest rate path, which comes from the fact that the Taylor rule is not state-dependent. On the one hand, this has the advantage that it leads to an inflation path which is even more stable than that in the Ramsey policy. But the volatility cost in terms of output fluctuations is very sizable - probably far too large for policymakers in practice. Again, the model suggests that adopting a Taylor rule with time-varying coefficients can approximate the benefits of the Ramsey policy while cutting down the volatility costs of a fixed-coefficient rule.

Of course, my model is too simple for the policy implications to be taken at face value. But what I think constitutes the practical lesson for policy is to learn from the key features of the optimal Ramsey and Taylor rule policies of the model. And those are the state-dependent nature of the Ramsey policy on the one hand, and the ability of a simple Taylor rule to anchor expectations on the other hand, especially if one conditions the inflation coefficient of the Taylor rule on the degree of unanchoring.

Combining these insights can yield practical guidance for policymakers. It suggests that a Taylor rule with state-dependent coefficients would do a good job in both guarding the expectations anchor as well as avoiding inflicting excess volatility. It could respond strongly to inflation fluctuations when expectations become unanchored, but avoid interfering too much when expectations are well-anchored. At the same time, such a policy rule would still be simple to design and to communicate to the public. What it would necessitate, however, would be an empirical measure of the extent of unanchoring like that in Mertens (2016) or Grishchenko et al. (2019) for the central bank and the public to rely on.

7 Conclusion

Central bankers frequently voice a concern to anchor expectations, that is, to render expectations of long-run inflation unresponsive to short-run economic conditions. The contribution of this paper is to investigate how this affects the conduct of monetary policy. I use a simple behavioral model in the spirit of Carvalho et al. (2021) to capture the notion of anchoring as the time-varying sensitivity of long-run expectations to short-run surprises. I quantify my anchoring channel by estimating the form of the function that determines the degree of unanchoring. I use the model to characterize monetary policy both analytically and numerically.

The simulated method of moments estimation establishes that anchoring in the data has two key properties: nonlinearity and asymmetry. On the one hand, expectations become more sensitive to forecast errors when the private sector made larger mistakes in predicting inflation in the past. On the other hand, like in Hebden et al. (2020), downside surprises unanchor expectations more than upside ones of the same magnitude do.

Using the thus quantified model, I provide three sets of results on monetary policy. I first consider the Ramsey policy of the central bank, deriving an analytical target criterion that prescribes how the monetary authority should respond to shocks. I show that the presence of my novel anchoring channel makes it desirable and feasible to smooth out shocks over time. However, the extent this is feasible varies over time in tandem with the current and expected future degrees of unanchoring.

Second, I use global methods to solve the nonlinear system of first-order conditions of the Ramsey problem numerically. I thus obtain an approximation to the optimal policy function, providing the fully optimal path of interest rates conditional on the sequence of exogenous disturbances. The main result is that optimal policy is state-dependent. Like in Goodfriend (1993)'s account of US monetary policy, the optimal policy involves responding aggressively when expectations unanchor in order to suppress the volatility that high degrees of unanchoring cause. By contrast, when facing well-anchored expectations, the central bank does not need to intervene and can accommodate inflation fluctuations.

Lastly, I explore the implications of the model for the most common specification of monetary policy, the Taylor rule. In the benchmark case with rational inflation expectations in the Taylor rule, the central bank should be much more aggressive on inflation in my model than under rational expectations, as in Orphanides and Williams (2004). This is because a strong response is required to anchor expectations. However, a time-invariant Taylor rule involves the same response to a given movement in inflation regardless of the degree of unanchoring, so that the central bank ends up inflicting excess volatility when expectations are well-anchored. As the application of the model to the Great Inflation and Volcker period shows, a Taylor rule that takes the degree of anchoring into account is a good candidate to combine the anchoring of expectations of the Ramsey policy with the simplicity of a simple Taylor rule.

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A Compact model notation

The A matrices are given by

$$A_a = \begin{pmatrix} g_{\pi a} \\ g_{xa} \\ \psi_\pi g_{\pi a} + \psi_x g_{xa} \end{pmatrix}, \quad A_b = \begin{pmatrix} g_{\pi b} \\ g_{xb} \\ \psi_\pi g_{\pi b} + \psi_x g_{xb} \end{pmatrix}, \quad A_s = \begin{pmatrix} g_{\pi s} \\ g_{xs} \\ \psi_\pi g_{\pi s} + \psi_x g_{xs} + \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \end{pmatrix}, \tag{A.1}$$

$$g_{\pi a} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \left[(1-\alpha)\beta, \kappa\alpha\beta, 0\right], \tag{A.2}$$

$$g_{xa} = \frac{-\sigma\psi_\pi}{w} \left[(1-\alpha)\beta, \kappa\alpha\beta, 0\right], \tag{A.3}$$

$$g_{\pi b} = \frac{\kappa}{w} \left[\sigma(1-\beta\psi_\pi), (1-\beta-\beta\sigma\psi_x, 0\right], \tag{A.4}$$

$$g_{xb} = \frac{1}{w} \left[\sigma(1-\beta\psi_\pi), (1-\beta-\beta\sigma\psi_x, 0\right], \tag{A.5}$$

$$g_{\pi s} = (1 - \frac{\kappa\sigma\psi_\pi}{w}) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta h)^{-1} - \frac{\kappa\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta h)^{-1}, \tag{A.6}$$

$$g_{xs} = \frac{-\sigma\psi_\pi}{w} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} (I_3 - \alpha\beta h)^{-1} - \frac{\sigma}{w} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} (I_3 - \beta h)^{-1}, \tag{A.7}$$

$$w = 1 + \sigma\psi_x + \kappa\sigma\psi_\pi. \tag{A.8}$$

The matrices of the state transition equation (14) are

$$h \equiv \begin{pmatrix} \rho_r & 0 & 0 \\ 0 & \rho_i & 0 \\ 0 & 0 & \rho_u \end{pmatrix}, \quad \epsilon_t \equiv \begin{pmatrix} \varepsilon_t^r \\ \varepsilon_t^i \\ \varepsilon_t^u \end{pmatrix}, \quad \text{and} \quad \eta \equiv \begin{pmatrix} \sigma_r & 0 & 0 \\ 0 & \sigma_i & 0 \\ 0 & 0 & \sigma_u \end{pmatrix}. \tag{A.9}$$

Note that this is the formulation for the case where a Taylor rule is in effect and is known by the private sector. It is straightforward to remove any of these two assumptions.

B The observation matrix for learning

Instead of the matrix g in the rational expectations observation equation

$$z_t = g s_t, \tag{B.1}$$

agents in the anchoring model use the estimated matrix g^l

$$g_{t-1}^l = \begin{bmatrix} F_{t-1} & G_{t-1} \end{bmatrix}, \tag{B.2}$$

with

$$F_{t-1} = \left(A_a \frac{1}{1-\alpha\beta} + A_b \frac{1}{1-\beta}\right) a_{t-1}, \tag{B.3}$$

$$G_{t-1} = A_a b_{t-1} \left(I_3 - \alpha\beta h\right)^{-1} + A_b b_{t-1} \left(I_3 - \beta h\right)^{-1} + A_s. \tag{B.4}$$

C An endogenous gain as a model of anchoring

Carvalho et al. (2021) define their anchoring function as follows. Let θ_t be a criterion to be defined. Then, for a threshold value $\tilde{\theta}$, the gain evolves according to

$$k_t = \begin{cases} (k_{t-1}+1)^{-1} & \text{if} \quad \theta_t < \tilde{\theta}, \\ \bar{g} & \text{otherwise.} \end{cases} \tag{C.1}$$

In other words, agents choose a decreasing gain when the criterion θ_t is lower than the threshold $\tilde{\theta}$; otherwise they choose a constant gain. The criterion employed by Carvalho et al. (2021) is computed as the absolute difference between subjective and model-consistent expectations, scaled by the variance of shocks:

$$\theta_t = \max|\Sigma^{-1}(\phi_{t-1} - \begin{bmatrix} F_{t-1} & G_{t-1} \end{bmatrix})|, \tag{C.2}$$

where Σ is the VC matrix of shocks, ϕ_{t-1} is the estimated matrix and $[F, G]$ is the ALM (see Appendix B).

As a robustness check, Carvalho et al. (2021) also compute an alternative criterion.[20] Let ω_t denote agents' time t estimate of the forecast error variance and θ_t be a statistic evaluated by agents in every period as

$$\omega_t = \omega_{t-1} + \tilde{\kappa} k_{t-1}(f_{t|t-1} f'_{t|t-1} - \omega_{t-1}), \tag{C.3}$$

$$\theta_t = \theta_{t-1} + \tilde{\kappa} k_{t-1}(f'_{t|t-1} \omega_t^{-1} f_{t|t-1} - \theta_{t-1}), \tag{C.4}$$

where $\tilde{\kappa}$ is a parameter that allows agents to scale the gain compared to the previous estimation and $f_{t|t-1}$ is the most recent forecast error, realized at time t. Indeed, this is a multivariate time series version of the squared CUSUM test.[21]

D Estimation procedure

The estimation of Section 4 is a simulated method of moments (SMM) exercise. As elaborated in the main text, I target the autocovariances of CPI inflation, the output gap, the federal funds rate and the 12-months ahead inflation forecasts coming from the Survey of Professional Forecasters. For the autocovariances, I consider lags $0, \ldots, 4$. The target moment vector, Ω, is the vectorized autocovariance matrices for the lags considered, 80×1.

Let Θ denote the set of parameters in the New Keynesian model that I calibrate. Then, for each proposed coefficient vector γ, the estimation procedure consists of simulating the model conditional on γ, Θ and N different sequences of disturbances, then computing model-implied moments for each simulation, and lastly choosing γ such that the squared distance between the data- and model-implied mean moments is minimized. Thus

[20] Note that for both criteria, I present the matrix generalizations of the scalar versions considered by Carvalho et al. (2021).

[21] See Brown et al. (1975) and Lütkepohl (2013) for details.

$$\hat{\gamma} = \arg\min \left(\Omega^{data} - \frac{1}{N} \sum_{n=1}^{N} \Omega^{model}(\gamma, \Theta, \{e_t^n\}_{t=1}^T) \right)' W^{-1} \left(\Omega^{data} - \frac{1}{N} \sum_{n=1}^{N} \Omega^{model}(\gamma, \Theta, \{e_t^n\}_{t=1}^T) \right), \tag{D.1}$$

where the observed data consist of $T = 151$ quarters. Here $\{e_t^n\}_{t=1}^T$ is a sequence of disturbances of the same length as the data; note that I use a cross-section of N such sequences and take average moments across the cross-section to wash out the effects of particular disturbances. Experimentation with the number N led me to choose $N = 1000$, as estimates no longer change upon selecting larger N.

Before computing moments, I filter both the observed and model-generated data using the Baxter and King (1999) filter, with thresholds at 6 and 32 quarters and truncation at 12 lags, the recommended values of the authors. I then compute the moments by fitting a reduced-form VAR to the filtered series and using the estimated coefficients to back out autocovariances. Because there are four observables to three structural shocks and occasionally low volatility in the expectation series, I estimate the VAR coefficients by ridge regression with a tuning parameter of 0.001. This is to ensure that the VAR coefficients are estimated with a lower standard error, so that estimated variances of the moments are more accurate. As the weighting matrix of the quadratic form in the moments, I use the inverse of the estimated variances of the target moments, W^{-1}, computed from 10000 bootstrapped samples.

To improve identification, I also impose restrictions on the estimates. First, I require that the γ-coefficients be convex, that is, that larger forecast errors in absolute value be associated with higher gains. Second, since forecast errors close to zero render the size of the gain irrelevant (cf. the learning equation (20)), I impose that the coefficient associated with a zero forecast error should be zero. Both restrictions are implemented with weights penalizing the loss function, and the weights are selected by experimentation.

Both additional assumptions reflect properties that the anchoring function should reasonably exhibit. The convexity assumption captures the very notion that larger forecast errors in absolute value suggest bigger changes to the forecasting procedure are necessary. This is thus a very natural requirement. As for the zero gain for zero forecast error assumption, the idea here is to supply the estimation with information where it is lacking. Since the updating of learning coefficients corresponds to gain times forecast error, as Equation (20) recalls, a zero forecast error supplies no information for the value of the gain. To impose a zero value here also seems natural, given that since forecast errors switch sign at zero, one would expect the zero forecast error to be an inflection point in the anchoring function (Gobbi et al., 2019). Furthermore, the objective function does not deteriorate upon imposing either assumption, suggesting that they are not at odds with the data. One does observe however that these additional moments do have bite in estimating the nodes at zero and at the edges, reflected in slightly tighter confidence intervals than at the other two nodes on Figure 1. Lastly, I supply 100 different initial points and select the estimate involving the lowest value for the loss (D.1).

E Alternative estimation strategy

One legitimate criticism of the estimation strategy in the main text is that since the autocovariances of the observables are likely to be correlated, the effective number of moments may be significantly lower than what is sufficient to properly identify the parameters of the gain function in Equation (22). If more moments were available, one may also hope to estimate more general forms of the gain function.

An alternative that I therefore now consider is to estimate the gain function directly using data on inflation and short- and long-run inflation forecasts (recall that in the main text, I did not use data on long-run inflation expectations). Next to the same CPI inflation series as in the main text, the idea is to rely on the 1-quarter-ahead and 10-year-ahead individual-level CPI inflation expectations from the Survey of Professional Forecasters. Then one can select the parameters of the gain function to make the following equation hold

$$\bar{\pi}_t^i - \bar{\pi}_{t-1}^i = \mathbf{g}(f_{t|t-1}^i) f_{t|t-1}^i \tag{E.1}$$

at each period t for each forecaster i. Here $f_{t|t-1}^i$ denotes the one-quarter-ahead forecast error of forecaster i. Since the individual-level SPF data at short and long horizons is available at the quarterly frequency between 1991-Q4 and 2021-Q4, and with about 30 forecasters in each wave, one obtains around 3600 moments. Of course, because of forecasters rotating in and out of the survey, the actual number of moments is significantly less, because in order to compute changing long-run expectations of a forecaster, one can in each period only use forecasters already present in the previous period.

The advantage of this approach is that a much larger number of moments allows many more parameters to be estimated. I also do not need to impose convexity as an additional restriction on the estimation. Furthermore, having more moments should make it possible to estimate more general specifications of the gain function. One obvious generalization of the gain function is to allow the gain to depend also on its own lag:

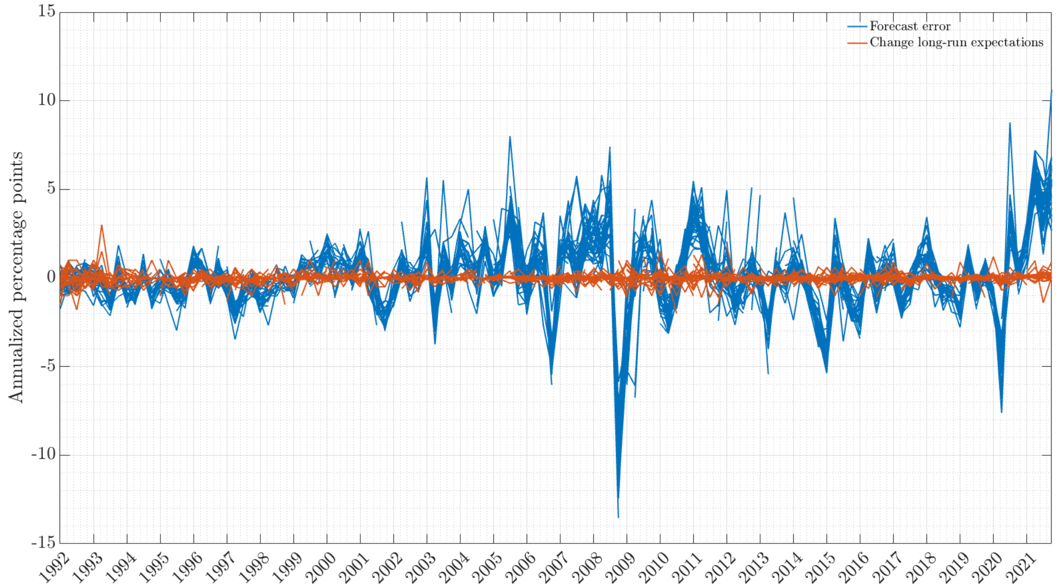
$$k_t = \mathbf{g}(k_{t-1}, f_{t|t-1}). \tag{E.2}$$

I therefore now hope to employ this alternative estimation strategy to estimate the piecewise linear approximating coefficients of the gain function, considering both the simple specification of the main text and the general specification of Equation (E.2).

This estimation approach has one major drawback, however, which turns out to be prohibitive. The issue is that long-run inflation expectations data (5 or 10-year ahead) are only available starting in 1991-Q4 (as opposed to 1981-Q3 in the main text). But as Carvalho et al. (2021) show, this is the period in US monetary history that involved well-anchored expectations. This means that long-run inflation expectations do not really fluctuate in this period. That this is indeed the case can be seen from Figure 9, which plots changes in 10-year-ahead inflation expectations against 1-quarter-ahead individual forecast errors.

This means that restricting the sample to post-1990 leads to missing the time period where there is action in long-run expectations. This is a significant problem because since there is not enough

Figure 9: Time series of changes in individual long-run inflation expectations and forecast errors



variation in the long-run inflation expectations data, one cannot reliably estimate the parameters of the anchoring function. One still obtains the same convexity and asymmetry as in the main section, but the estimates are an order of magnitude lower. This is why I maintain the simulated method of moments strategy of the main text as my main estimation strategy.

F The policy problem in the simplified baseline model

Denote by $\mathbf{g}'_t = \mathbf{g}_{\pi,t} = -\mathbf{g}_{\bar{\pi},t}$ the derivative of the anchoring function $\mathbf{g}(\cdot)$ with respect to the forecast error, evaluated at time t. In this simplified setting, $\bar{\pi}_t = e_1 a_t$, the estimated constant for the inflation process. As in the main text, e_i is a selector vector, selecting row i of the subsequent matrix. I also use the notation $b_i \equiv e_i b$. The planner chooses $\{\pi_t, x_t, f_{a,t}, f_{b,t}, \bar{\pi}_t, k_t\}_{t=t_0}^{\infty}$ to minimize

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Bigg\{ & (\pi_t^2 + \lambda_x x_t^2) \\
& + \varphi_{1,t}\Big(\pi_t - \kappa x_t - (1-\alpha)\beta f_{a,t} - \kappa\alpha\beta b_2(I_3 - \alpha\beta h)^{-1}s_t - e_3(I_3 - \alpha\beta h)^{-1}s_t\Big) \\
& + \varphi_{2,t}\Big(x_t + \sigma i_t - \sigma f_{b,t} - (1-\beta)b_2(I_3 - \beta h)^{-1}s_t + \sigma\beta b_3(I_3 - \beta h)^{-1}s_t - \sigma e_1(I_3 - \beta h)^{-1}s_t)\Big) \\
& + \varphi_{3,t}\Big(f_{a,t} - \frac{1}{1-\alpha\beta}\bar{\pi}_{t-1} - b_1(I_3 - \alpha\beta h)^{-1}s_t\Big) \\
& + \varphi_{4,t}\Big(f_{b,t} - \frac{1}{1-\beta}\bar{\pi}_{t-1} - b_1(I_3 - \beta h)^{-1}s_t\Big) \\
& + \varphi_{5,t}\Big(\bar{\pi}_t - \bar{\pi}_{t-1} - k_t(\pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}))\Big) \\
& + \varphi_{6,t}\Big(k_t - \mathbf{g}(\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})\Big)\Bigg\},
\end{aligned}
\tag{F.1}
$$

where ρ_i are Lagrange-multipliers on the constraints. After a little bit of simplifying, the first-order conditions boil down to the following three equations:

$$
2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \varphi_{5,t}k_t - \varphi_{6,t}\mathbf{g}_t' = 0, \tag{F.2}
$$

$$
-\frac{2(1-\alpha)\beta^2}{1-\alpha\beta}\frac{\lambda_x}{\kappa}\,\mathbb{E}_t\, x_{t+1} + \varphi_{5,t} - \beta\,\mathbb{E}_t(1-k_{t+1})\varphi_{5,t+1} + \beta\mathbf{g}_t'\,\mathbb{E}_t\,\varphi_{6,t+1} = 0, \tag{F.3}
$$

$$
\varphi_{6,t} = (\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})\varphi_{5,t}. \tag{F.4}
$$

Note that Equation (F.2) is the analogue of Gaspar et al. (2010)'s Equation (22) (or, equivalently, of Molnár and Santoro (2014)'s (16)), except that there is an additional multiplier, φ_6. This multiplier reflects the fact that in addition to the constraint coming from the expectation process itself, with shadow value φ_5, learning involves the gain equation as a constraint as well. One can also clearly read off that when the learning process has converged, so that neither expectations nor the gain process are constraints ($\varphi_5 = \varphi_6 = 0$), the discretionary inflation-output gap tradeoff familiar from Clarida et al. (1999) obtains. Combining the above three equations and solving for $\varphi_{5,t}$, using the notation that $\prod_0^0 \equiv 1$, one obtains the target criterion (33).

The system of first-order conditions and model equations for this simplified system also reveals how the endogenous gain introduces nonlinearity to the equation system. In particular, notice how the gain k_t shows up multiplicatively with the Lagrange multiplier, φ_5. In fact, the origin of the problem is the learning equation for the long-run inflation expectation $\bar{\pi}_t$

$$
\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1} \tag{F.5}
$$

where the first interaction term between the gain and other endogenous variables show up. This results in an equation system of nonlinear difference equations that does not admit an analytical solution.

Considering equation (F.5) is instructive to see how it is indeed the endogeneity of the gain that causes these troubles. Were we to specify a constant gain setup, k_t would merely equal the constant $\bar{g}$. In such a case, all interaction terms would reduce to multiplication between endogenous variables and parameters; linearity would be restored and an analytical solution for the optimal time paths of endogenous variables would be obtainable. Similarly, a decreasing gain specification would also be manageable since for all t, the gain would simply be given by t^{-1}, and the anchoring function would also be deterministic and exogenous.

G Target criterion for a general gain function

Consider a more general anchoring mechanism than Equation (22) in which the current gain depends not only on the most recent forecast error, but also on the previous gain, as in Equation (E.2):

$$k_t = \mathbf{g}(k_{t-1}, f_{t|t-1}). \tag{G.1}$$

Using the notation $\mathbf{g}_{k,t}$ and $\mathbf{g}_{f,t}$ for the derivates of the anchoring function with respect to k and f evaluated at time t, the FOCs of the Ramsey problem are

$$2\pi_t + 2\frac{\lambda_x}{\kappa}x_t - \varphi_{5,t}k_t - \varphi_{6,t}\mathbf{g}_{f,t} = 0, \tag{G.2}$$

$$-\frac{2(1-\alpha)\beta^2}{1-\alpha\beta}\frac{\lambda_x}{\kappa}\mathbb{E}_t\, x_{t+1} + \varphi_{5,t} - \beta\,\mathbb{E}_t(1-k_{t+1})\varphi_{5,t+1} + \beta\mathbf{g}_{f,t}\,\mathbb{E}_t\,\varphi_{6,t+1} = 0, \tag{G.3}$$

$$\varphi_{6,t} - \mathbb{E}_t\,\mathbf{g}_{k,t+1}\varphi_{6,t+1} = (\pi_t - \bar{\pi}_{t-1} - b_1 s_{t-1})\varphi_{5,t}, \tag{G.4}$$

where the red term is the new element vis-à-vis the case where the anchoring function is specified in levels ($k_t = \mathbf{g}(f_{t|t-1})$). Thus a central bank that follows the target criterion in this general case has to compute the solution to this three-dimensional difference equation system, where the more involved algebra contains the same economic intuition.

H Parameterized expectations algorithm (PEA)

The objective of the parameterized expectations algorithm is to solve for the sequence of interest rates that solves the model equations including the target criterion, representing the first-order condition of the Ramsey problem. For convenience, I list the model equations:

$$x_t = -\sigma i_t + \begin{bmatrix}\sigma & 1-\beta & -\sigma\beta\end{bmatrix} f_{b,t} + \sigma \begin{bmatrix}1 & 0 & 0\end{bmatrix} (I_3 - \beta h)^{-1} s_t, \tag{H.1}$$

$$\pi_t = \kappa x_t + \begin{bmatrix}(1-\alpha)\beta & \kappa\alpha\beta & 0\end{bmatrix} f_{a,t} + \begin{bmatrix}0 & 0 & 1\end{bmatrix} (I_3 - \alpha\beta h)^{-1} s_t, \tag{H.2}$$

$$f_{a,t} = \frac{1}{1-\alpha\beta}\bar{\pi}_{t-1} + b(I_3 - \alpha\beta h)^{-1} s_t, \tag{H.3}$$

$$f_{b,t} = \frac{1}{1-\beta}\bar{\pi}_{t-1} + b(I_3 - \beta h)^{-1} s_t, \tag{H.4}$$

$$f_{t|t-1} = \pi_t - (\bar{\pi}_{t-1} + b_1 s_{t-1}), \tag{H.5}$$

$$k_t = \sum_i \gamma_i b_i(f_{t|t-1}), \tag{H.6}$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + k_t f_{t|t-1}, \tag{H.7}$$

$$\pi_t + \frac{\lambda_x}{\kappa} x_t = c\Big(k_t + f_{t|t-1}\mathbf{g}_t'\Big)\Big((1-\beta)\,\mathbb{E}_t\, x_{t+1} + \beta\,\mathbb{E}_t \sum_{i=1}^{\infty} x_{t+i} \prod_{j=0}^{i-1} 1 - k_{t+j} - f_{t+j|t-1+j}\mathbf{g}_{t-1+j}'\Big), \tag{H.8}$$

with $c \equiv \frac{\lambda_x}{\kappa}\frac{(1-\alpha)\beta^2}{1-\alpha\beta}$ as in the main text. Denote the expectation on the right hand side of (H.8) as E_t. The idea of the PEA is to approximate this expectation and to solve model equations given the approximation $\hat{E}_t$. The algorithm is as follows:[22]

Objective: Obtain the sequence $\{i_t\}_{t=1}^T$ that solves Equations (H.1) - (H.8) for a history of exogenous shocks $\{s_t\}_{t=1}^T$ of length T.

1. Conjecture an initial expectation $\hat{E}_t = \beta^0 s(X_t)$.
   The expectation is approximated as a projection on a basis, $s(X_t)$, where β^0 are initial projection coefficients, and $X_t = (k_t, \bar{\pi}_{t-1}, r_t^n, u_t)$ is the state vector. I use a monomial basis consisting of the first, second and third powers of X_t.

2. Solve model equations given conjectured $\hat{E}_t$ for a given sequence of shocks $\{s_t\}_{t=1}^T$.
   Compute residuals to the model equations (H.1) - (H.8) given $\{s_t\}_{t=1}^T$ and $\{\hat{E}_t\}_{t=1}^T$. Obtain a sequence $\{i_t\}_{t=1}^T$ that sets the residuals to zero. The output of this step is $\{v_t\}_{t=1}^T$, the simulated history of endogenous variables (Christiano and Fisher (2000) refer to this as a "synthetic time series").

3. Compute realized analogues of $\{E_t\}_{t=1}^T$ given $\{v_t\}_{t=1}^T$.

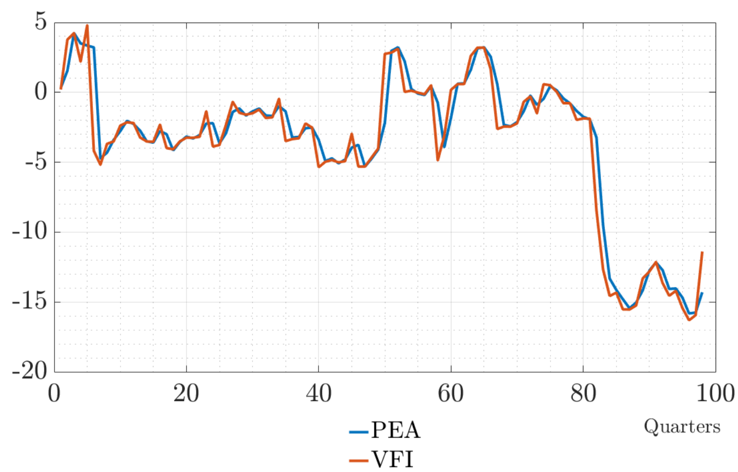
4. Update β regressing the synthetic E_t on $s(X_t)$.
   The coefficient update is $\beta^{i+1} = (s(X_t)'s(X_t))^{-1}s(X_t)'E_t$. Then iterate until convergence by evaluating at every step $||\beta^i - \beta^{i+1}||$.

[22] For a thorough treatment of the PEA, see Christiano and Fisher (2000).

I Parametric value function iteration

This is an alternative approach I implement as a robustness check to the PEA. The objective is thus the same: to obtain the interest rate sequence that solves the model equations. The general value function iteration (VFI) approach is fairly standard, for which reason I refer to the Judd (1998) textbook for details. Specific to my application is that the state vector is five-dimensional, $X_t = (\bar{\pi}_{t-1}, r_t^n, u_t, r_{t-1}^n, u_{t-1})$, and that I approximate the value function using a cubic spline. Thus the output of the algorithm is a cubic spline approximation of the value function and a policy function for each node on the grid of states. Next, I interpolate the policy function using a cubic spline as well. As a last step I pass the state vector from the PEA simulation, obtaining an interest rate sequence conditional on the history of states. Figure 10 shows the resulting interest rate sequence, obtained through the two approaches, conditional on a simulated sequence for the exogenous states.

Figure 10: Policy function for a particular history of states, PEA against VFI



J Oscillatory dynamics in adaptive learning models

Here I present an illustration for why adaptive learning models produce oscillatory impulse responses if the gain is high enough. Consider a stylized adaptive learning model in two equations:

$$\pi_t = \beta e_t + u_t, \tag{J.1}$$

$$e_t = e_{t-1} + k(\pi_t - e_{t-1}). \tag{J.2}$$

The reader can recognize in (J.1) a simplified Phillips curve in which I am abstracting from output gaps to keep the presentation as clear as possible. Like in the simple model of Section 5.1 in the main text, e_t represents the one-period inflation expectation $\hat{\mathbb{E}}_t \pi_{t+1}$. (J.2), then, represents the simplest possible recursive updating of the expectations e_t. My notation of the gain as k indicates a constant gain specification, but the intuition remains unchanged for decreasing (or endogenous) gains.

Combining the two equations allows one to solve for the time series of expectations

$$e_t = \frac{1-k}{1-k\beta} e_{t-1} + \frac{k}{1-k\beta} u_t, \tag{J.3}$$

which, for β close but smaller than 1, is a near-unit-root process. Defining the forecast error as $f_{t|t-1} \equiv \pi_t - e_{t-1}$, one obtains

$$f_{t|t-1} = -\frac{1-\beta}{1-k\beta} e_{t-1} + \frac{1}{1-k\beta} u_t. \tag{J.4}$$

Equation (J.4) shows that in this simple model, the forecast error loads on a near-unit-root process with a coefficient that is negative and less than one in absolute value. Damped oscillations are the result.

Note that even if the gain would converge to zero, the coefficient on e_{t-1} would be negative and less than one in absolute value. Thus even for decreasing gain learning, one obtains oscillations, but the lower the gain, the more damped the oscillations become. This corroborates my findings in the impulse responses of Figure 5. But importantly, the opposite extreme, when $k \to 1$, results in a coefficient of exactly -1, giving perpetual oscillations. This clearly illustrates how the oscillatory behavior of impulse responses comes from the oscillations in the forecast error that obtain when the gain is sufficiently large.