A Model of the Fed's View on Inflation

Thomas Hasenzagl[1], Filippo Pellegrino[2], Lucrezia Reichlin[3], and Giovanni Ricco[4]

[1]University of Minnesota
[2]London School of Economics
[3]London Business School, Now-Casting Economics, and CEPR
[4]University of Warwick and OFCE-SciencesPo, and CEPR

Inflation: Drivers and Dynamics 2020
Cleveland Fed & ECB, 21-22 May 2020

The Fed's View

> "Inflation is characterized by an underlying trend that has been essentially constant since the mid-1990s; .... Theory and evidence suggest that this trend is strongly influenced by inflation expectations that, in turn, depend on monetary policy. In particular, the remarkable stability of various measures of expected inflation in recent years presumably represents the fruits of the Federal Reserve's sustained effort since the early 1980s to bring down and stabilize inflation at a low level. The anchoring of inflation expectations ... does not, however, prevent actual inflation from fluctuating from year to year in response to the temporary influence of movements in energy prices and other disturbances. In addition, inflation will tend to run above or below its underlying trend to the extent that resource utilization–which may serve as an indicator of firms' marginal costs–is persistently high or low."
>
> — Janet Yellen, 60th Boston Fed Conference

The Fed's View

"Inflation is characterized by an underlying trend that has been essentially constant since the mid-1990s; .... Theory and evidence suggest that this **trend is strongly influenced by inflation expectations** that, in turn, depend on monetary policy. In particular, the remarkable stability of various measures of expected inflation in recent years presumably represents the fruits of the Federal Reserve's sustained effort since the early 1980s to bring down and stabilize inflation at a low level. The anchoring of inflation expectations ...does not, however, prevent actual inflation from fluctuating from year to year in response to the temporary influence of movements in energy prices and other disturbances. In addition, inflation will tend to run above or below its underlying trend to the extent that resource utilization–which may serve as an indicator of firms' marginal costs–is persistently high or low."

— Janet Yellen, 60th Boston Fed Conference

The Fed's View

"Inflation is characterized by an underlying trend that has been essentially constant since the mid-1990s; .... Theory and evidence suggest that this trend is strongly influenced by inflation expectations that, in turn, depend on monetary policy. In particular, the remarkable stability of various measures of expected inflation in recent years presumably represents the fruits of the Federal Reserve's sustained effort since the early 1980s to bring down and stabilize inflation at a low level. The anchoring of inflation expectations ...does not, however, prevent actual inflation from fluctuating from year to year in response to the temporary influence of movements in energy prices and other disturbances. In addition, inflation will tend to run above or below its underlying trend to the extent that **resource utilization–which may serve as an indicator of firms' marginal costs**–is persistently high or low."

— Janet Yellen, 60th Boston Fed Conference

The Fed's View

"Inflation is characterized by an underlying trend that has been essentially constant since the mid-1990s; .... Theory and evidence suggest that this trend is strongly influenced by inflation expectations that, in turn, depend on monetary policy. In particular, the remarkable stability of various measures of expected inflation in recent years presumably represents the fruits of the Federal Reserve's sustained effort since the early 1980s to bring down and stabilize inflation at a low level. The anchoring of inflation expectations ...does not, however, prevent actual inflation from fluctuating from year to year in response to the temporary influence of movements in **energy prices and other disturbances**. In addition, inflation will tend to run above or below its underlying trend to the extent that resource utilization–which may serve as an indicator of firms' marginal costs–is persistently high or low."

— Janet Yellen, 60th Boston Fed Conference

The Fed's View

> "Inflation is characterized by an underlying trend that has been essentially constant since the mid-1990s; .... Theory and evidence suggest that this **trend is strongly influenced by inflation expectations** that, in turn, depend on monetary policy. In particular, the remarkable stability of various measures of expected inflation in recent years presumably represents the fruits of the Federal Reserve's sustained effort since the early 1980s to bring down and stabilize inflation at a low level. The anchoring of inflation expectations ...does not, however, prevent actual inflation from fluctuating from year to year in response to the temporary influence of movements in **energy prices and other disturbances**. In addition, inflation will tend to run above or below its underlying trend to the extent that **resource utilization–which may serve as an indicator of firms' marginal costs**–is persistently high or low."
>
> — Janet Yellen, 60th Boston Fed Conference

The Fed’s View

Dynamic Components of Inflation:

1. A **trend in inflation** – reflecting long-term expectations

The Fed's View

Dynamic Components of Inflation:

1. A **trend in inflation** – reflecting long-term expectations
2. A reduced form **Phillips curve** – relating economic slack to prices

The Fed's View

Dynamic Components of Inflation:

1. A **trend in inflation** – reflecting long-term expectations
2. A reduced form **Phillips curve** – relating economic slack to prices
3. An **oil price component** unrelated to real variables

The Fed's View

Dynamic Components of Inflation:

1. A **trend in inflation** – reflecting long-term expectations
2. A reduced form **Phillips curve** – relating economic slack to prices
3. An **oil price component** unrelated to real variables

Challenges to this view:

- **Weak empirical evidence** on the PC
- Inflation can be **forecast** by statistical processes **unrelated to slack**
- Evidence of the **flattening** (or **disappearance** of the Phillips Curve)
- **Missing deflation**...
- Disanchoring of consumers' expectation due to **oil shocks**

This Paper

An **Econometric Formalisation** of the **Policymakers'/Median Economist's View**:

- A **semi-structural** unobserved components **Trend-Cycle model** á la Harvey (1985)
- ... employing **survey data on inflation**
- ... encompasses **full information rational expectations (FIRE)** but allows for **deviations**
- **Bayesian estimation** (Harvey et al., 2007, Del Negro et al. 2017, and Lenza and Jarociński, 2018)

This Paper

An **Econometric Formalisation** of the **Policymakers'/Median Economist's View**:

- A **semi-structural** unobserved components **Trend-Cycle model** á la Harvey (1985)
- ... employing **survey data on inflation**
- ... encompasses **full information rational expectations (FIRE)** but allows for **deviations**
- **Bayesian estimation** (Harvey et al., 2007, Del Negro et al. 2017, and Lenza and Jarociński, 2018)

Results:

1. **Stable** expectational trend
2. Sizeable and fairly steep reduced form **Phillips curve**
3. Some rationalisation of the **inflation puzzles**

An Empirical Trend-Cycle Model of Inflation Dynamics

A Stylised Rational Expectations Model

$$\begin{pmatrix} y_t \\ \pi_t \\ \mathbb{E}_t\left[\pi_{t+1}\right] \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta_\pi & 1 \\ \delta_{exp,1} + \delta_{exp,2}L & 1 \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \\ \mu_t^\pi \end{pmatrix} + \begin{pmatrix} \mu_t^y \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \psi_t^y \\ \psi_t^\pi \\ 0 \end{pmatrix}$$

- Can accommodate different specifications for the Phillips Curve

$$\boxed{\hat{\pi}_t = \sum_{i=1}^{2} \delta_i \hat{\pi}_{t-i} + \beta \mathbb{E}_t\left[\hat{\pi}_{t+1}\right] + \kappa \hat{y}_t + v_t}$$

- AR(2) $\widehat{\psi}_t$ with complex roots would be a solution to hybrid Phillips Curve
- AR(1) $\widehat{\psi}_t$ would be the solution to a **purely forward** looking NK Phillips Curve
- It also nests the **backwards looking** ‘Old-Keynesian’ Phillips curve connecting output gap and prices

A Richer Inflation Dynamics

① **Heterogenous dynamics** along the **business cycle**
$\Longrightarrow$ Lagged relations prices-slack

② Labour market dynamics along the **business cycle**
$\Longrightarrow$ **Okun's law** connecting slack-unemployment

③ Energy price can impact CPI

- as **markup shocks**
- directly as **consumption good**
- via expectations (**non-fundamental fluctuations**)

④ Deviations from full-information RE

An Empirical Trend-Cycle Model

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \end{pmatrix} = \begin{pmatrix} 1 \\ \delta_{e,1} + \delta_{e,2}L \\ \delta_{u,1} + \delta_{u,2}L \\ \delta_{oil,1} + \delta_{oil,2}L \\ \delta_{\pi,1} + \delta_{\pi,2}L \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \end{pmatrix}$$

- **Output gap** informs stationary **Business Cycle** fluctuations
- ... connects to labour market variables via **Okun's law**
- ... and to prices and expectations via the **Phillips curve**

An Empirical Trend-Cycle Model

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta_{e,1} + \delta_{e,2}L & 0 \\ \delta_{u,1} + \delta_{u,2}L & 0 \\ \delta_{oil,1} + \delta_{oil,2}L & 0 \\ \delta_{\pi,1} + \delta_{\pi,2}L & 1 \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L & 1 \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 & 1 \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 & 1 \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \\ \mu_t^\pi \end{pmatrix}$$

- **Output gap** informs stationary **Business Cycle** fluctuations
- ... connects to labour market variables via **Okun's law**
- ... and to prices and expectations via the **Phillips curve**
- Prices and expectations share common **trend inflation**

An Empirical Trend-Cycle Model

$$
\begin{pmatrix} y_t \\ e_t \\ u_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \delta_{e,1} + \delta_{e,2}L & 0 & 0 \\ \delta_{u,1} + \delta_{u,2}L & 0 & 0 \\ \delta_{oil,1} + \delta_{oil,2}L & 1 & 0 \\ \delta_{\pi,1} + \delta_{\pi,2}L & \gamma_{\pi,1} + \gamma_{\pi,2}L & 1 \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L & \gamma_{\pi^c,1} + \gamma_{\pi^c,2}L & 1 \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 & \gamma_{uom,1} + \gamma_{uom,2}L & 1 \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 & \gamma_{spf,1} + \gamma_{spf,2} & 1 \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \\ \psi_t^{EP} \\ \mu_t^{\pi} \end{pmatrix}
$$

- **Output gap** informs stationary **Business Cycle** fluctuations
- ... connects to labour market variables via **Okun's law**
- ... and to prices and expectations via the **Phillips curve**
- Prices and expectations share common **trend inflation**
- Oil price, prices and expectations are connected by an **Energy Cycle**

An Empirical Trend-Cycle Model

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \delta_{e,1} + \delta_{e,2}L & 0 & 0 \\ \delta_{u,1} + \delta_{u,2}L & 0 & 0 \\ \delta_{oil,1} + \delta_{oil,2}L & 1 & 0 \\ \delta_{\pi,1} + \delta_{\pi,2}L & \gamma_{\pi,1} + \gamma_{\pi,2}L & 1 \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L & \gamma_{\pi^c,1} + \gamma_{\pi^c,2}L & 1 \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 & \gamma_{uom,1} + \gamma_{uom,2}L & 1 \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 & \gamma_{spf,1} + \gamma_{spf,2} & 1 \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \\ \psi_t^{EP} \\ \mu_t^{\pi} \end{pmatrix} + \begin{pmatrix} \psi_t^y \\ \psi_t^e \\ \psi_t^u \\ \psi_t^{oil} \\ \psi_t^{\pi} \\ \psi_t^{\pi^c} \\ \psi_t^{uom} \\ \psi_t^{spf} \end{pmatrix}$$

- **Output gap** informs stationary **Business Cycle** fluctuations
- … connects to labour market variables via **Okun’s law**
- … and to prices and expectations via the **Phillips curve**
- Prices and expectations share common **trend inflation**
- Oil price, prices and expectations are connected by an **Energy Cycle**
- Stationary idiosyncratic disturbances

An Empirical Trend-Cycle Model

$$
\begin{pmatrix} y_t \\ e_t \\ u_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \delta_{e,1}+\delta_{e,2}L & 0 & 0 \\ \delta_{u,1}+\delta_{u,2}L & 0 & 0 \\ \delta_{oil,1}+\delta_{oil,2}L & 1 & 0 \\ \delta_{\pi,1}+\delta_{\pi,2}L & \gamma_{\pi,1}+\gamma_{\pi,2}L & 1 \\ \delta_{\pi^c,1}+\delta_{\pi^c,2}L & \gamma_{\pi^c,1}+\gamma_{\pi^c,2}L & 1 \\ \delta_{uom,1}+\delta_{uom,2}L+\delta_{uom,3}L^2 & \gamma_{uom,1}+\gamma_{uom,2}L & 1 \\ \delta_{spf,1}+\delta_{spf,2}L+\delta_{spf,3}L^2 & \gamma_{spf,1}+\gamma_{spf,2} & 1 \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \\ \psi_t^{EP} \\ \mu_t^{\pi} \end{pmatrix} + \begin{pmatrix} \psi_t^y \\ \psi_t^e \\ \psi_t^u \\ \psi_t^{oil} \\ \psi_t^{\pi} \\ \psi_t^{\pi^c} \\ \psi_t^{uom} \\ \psi_t^{spf} \end{pmatrix} + \begin{pmatrix} \mu_t^y \\ \mu_t^e \\ \mu_t^u \\ \mu_t^{oil} \\ \\ \\ \\ \\ \end{pmatrix}
$$

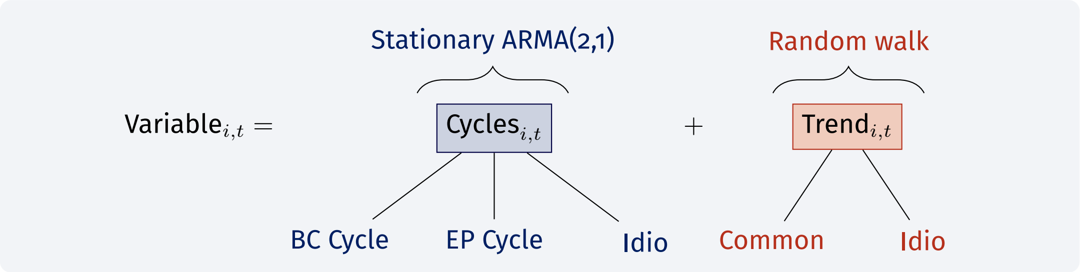
- **Output gap** informs stationary **Business Cycle** fluctuations
- ... connects to labour market variables via **Okun's law**
- ... and to prices and expectations via the **Phillips curve**
- Prices and expectations share common **trend inflation**
- Oil price, prices and expectations are connected by an **Energy Cycle**
- Stationary idiosyncratic disturbances
- Independent trend in output (**output potential**)
- Independent trend in employment/unemployment (**trend employment/equilibrium unemployment**)

An Empirical Trend-Cycle Model

$$\begin{pmatrix} y_t \\ e_t \\ u_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \delta_{e,1}+\delta_{e,2}L & 0 & 0 \\ \delta_{u,1}+\delta_{u,2}L & 0 & 0 \\ \delta_{oil,1}+\delta_{oil,2}L & 1 & 0 \\ \delta_{\pi,1}+\delta_{\pi,2}L & \gamma_{\pi,1}+\gamma_{\pi,2}L & 1 \\ \delta_{\pi^c,1}+\delta_{\pi^c,2}L & \gamma_{\pi^c,1}+\gamma_{\pi^c,2}L & 1 \\ \delta_{uom,1}+\delta_{uom,2}L+\delta_{uom,3}L^2 & \gamma_{uom,1}+\gamma_{uom,2}L & 1 \\ \delta_{spf,1}+\delta_{spf,2}L+\delta_{spf,3}L^2 & \gamma_{spf,1}+\gamma_{spf,2} & 1 \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \\ \psi_t^{EP} \\ \mu_t^{\pi} \end{pmatrix} + \begin{pmatrix} \psi_t^y \\ \psi_t^e \\ \psi_t^u \\ \psi_t^{oil} \\ \psi_t^{\pi} \\ \psi_t^{\pi^c} \\ \psi_t^{uom} \\ \psi_t^{spf} \end{pmatrix} + \begin{pmatrix} \mu_t^y \\ \mu_t^e \\ \mu_t^u \\ \mu_t^{oil} \\ 0 \\ 0 \\ \mu_t^{uom} \\ \mu_t^{spf} \end{pmatrix}$$

- **Output gap** informs stationary **Business Cycle** fluctuations
- ... connects to labour market variables via **Okun's law**
- ... and to prices and expectations via the **Phillips curve**
- Prices and expectations share common **trend inflation**
- Oil price, prices and expectations are connected by an **Energy Cycle**
- Stationary idiosyncratic disturbances
- Independent trend in output (**output potential**)
- Independent trend in employment/unemployment (**trend employment/equilibrium unemployment**)
- ... and idiosyncratic trends in expectations

The Model in a Nutshell



- Stationary Cycles

$$\begin{pmatrix} \psi_t^j \\ \psi_t^{*j} \end{pmatrix} = \rho^j \begin{pmatrix} \cos(\lambda^j) & \sin(\lambda^j) \\ -\sin(\lambda^j) & \cos(\lambda^j) \end{pmatrix} \begin{pmatrix} \psi_{t-1}^j \\ \psi_{t-1}^{*j} \end{pmatrix} + \begin{pmatrix} v_t^j \\ v_t^{*j} \end{pmatrix}, \quad \begin{pmatrix} v_t^j \\ v_t^{*j} \end{pmatrix} \sim \mathcal{N}(0, \varsigma_j^2 \, I_2)$$

- Unit Root Trends (w/ or w/o drift)

$$\mu_t^j = \mu_0^j + \mu_{t-1}^j + u_t^j, \quad u_t^j \sim \mathcal{N}(0, \sigma_j^2) \; .$$

Bringing the Model to the Data

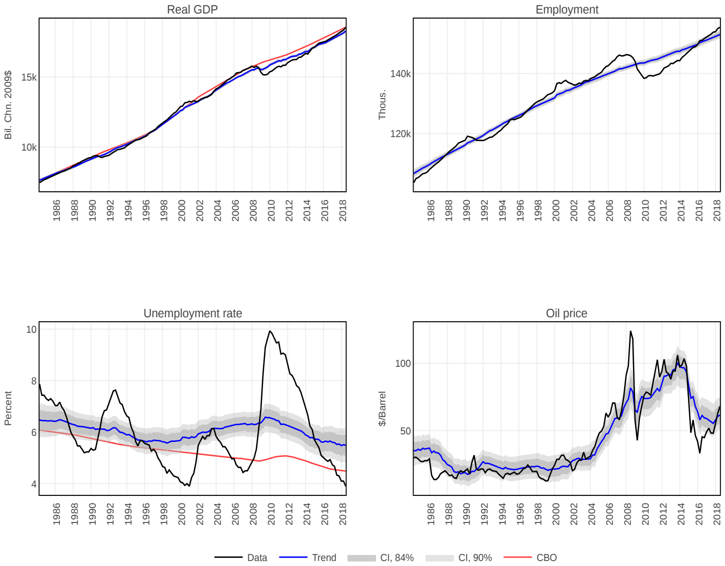
Variable	Transformation	Loads on		
		BC Cycle	EP Cycle	Common Trend
Gross Domestic Product	Levels	✓	✗	✗
Employment (or Empl/Pop)	Levels	✓	✗	✗
Unemployment Rate	Levels	✓	✗	✗
WTI Spot Oil Price	Levels	✓	✓	✗
CPI: All Items	YoY	✓	✓	✓
Core CPI	YoY	✓	✓	✓
UoM: Expected Inflation	Levels	✓	✓	✓
SPF: Expected Inflation	Levels	✓	✓	✓

Sample: Quarterly, Q1-1984 to Q2-2017

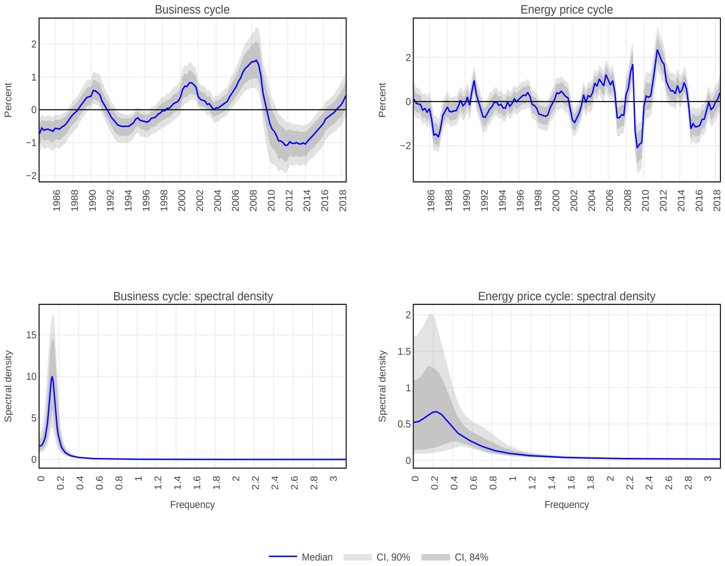
Bayesian Estimation

Trends & Cycles in US Inflation

Output Potential, Equilibrium Employment

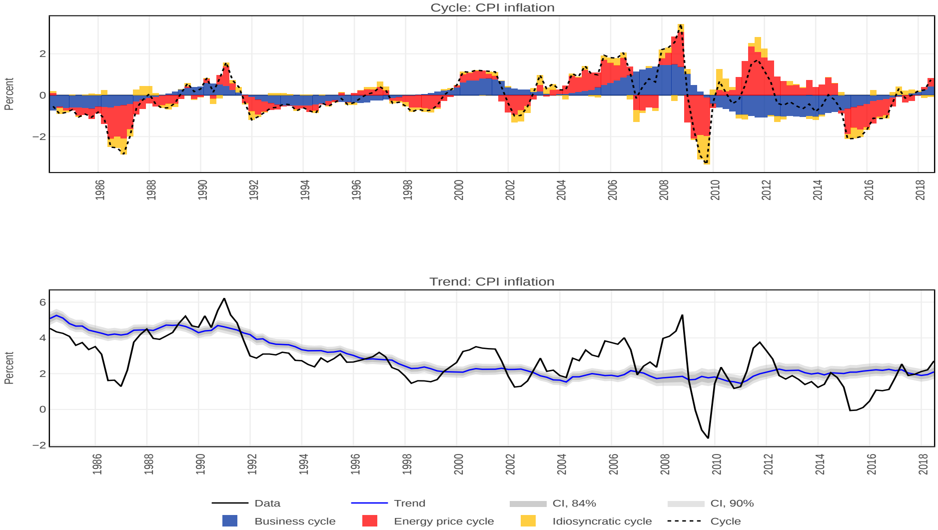


Common Cycles

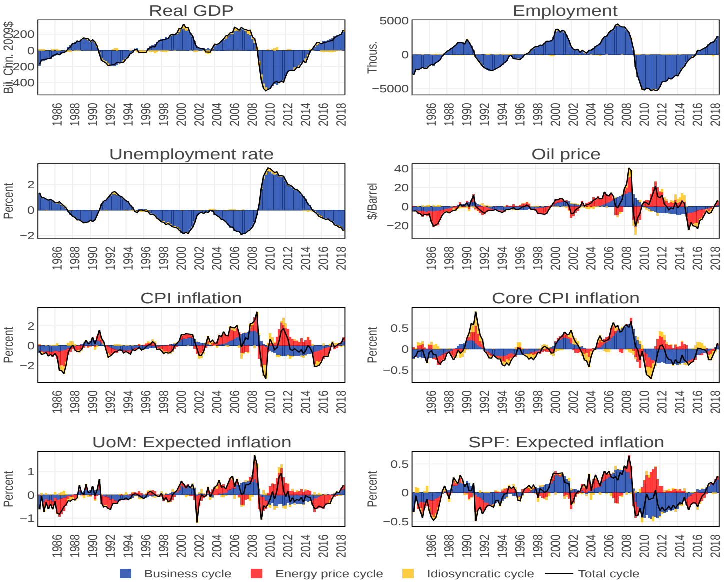


Historical Decomposition of the CPI

Headline CPI

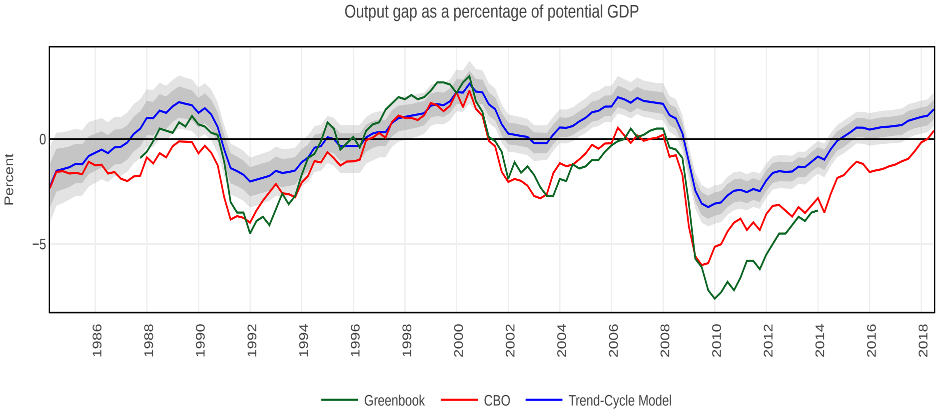


Common Cycles

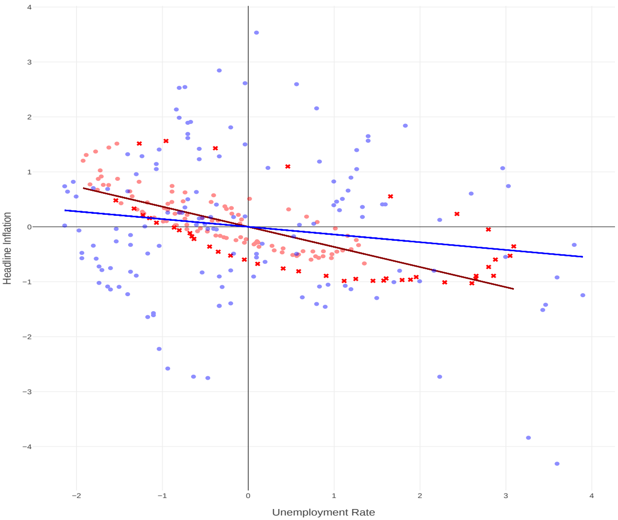


Output Gap

How Deep was Last Recession?



The Slope of the Phillips Curve

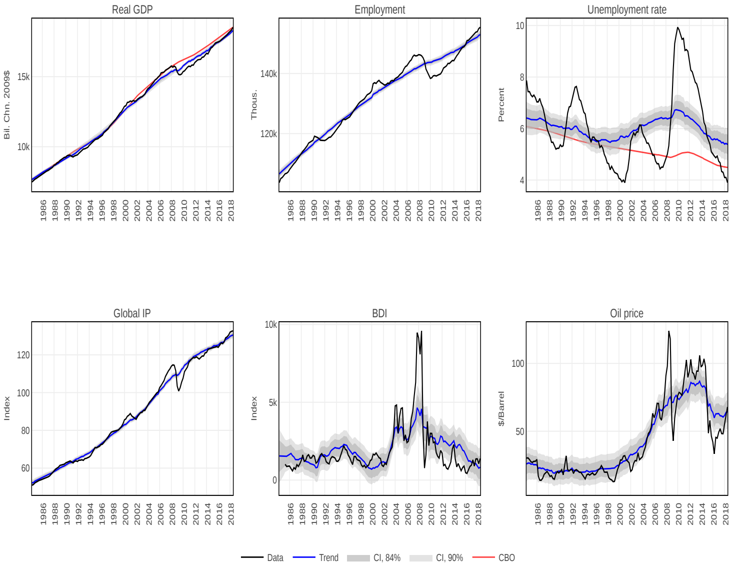


Phillips Curve slope

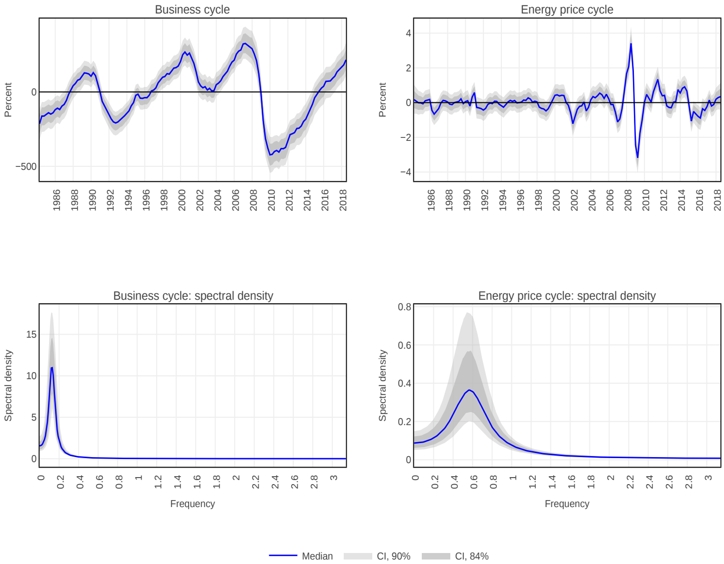
- Blue line is **-0.14**
- Red line is **-0.39**

Global Determinants of Inflation

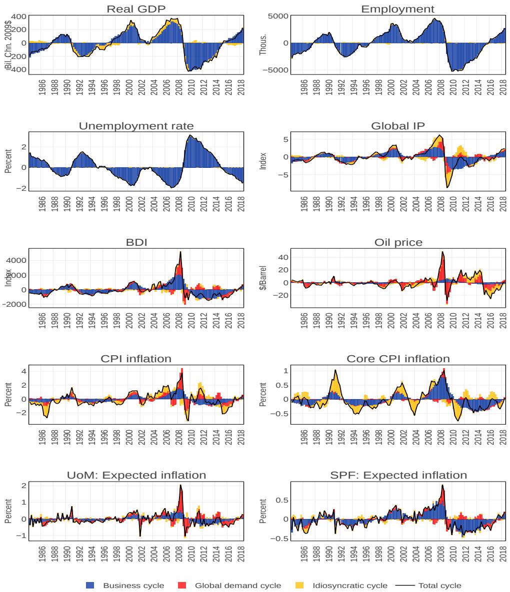
Global: Output Potential, Equilibrium Employment



Global: Output Potential, Equilibrium Employment

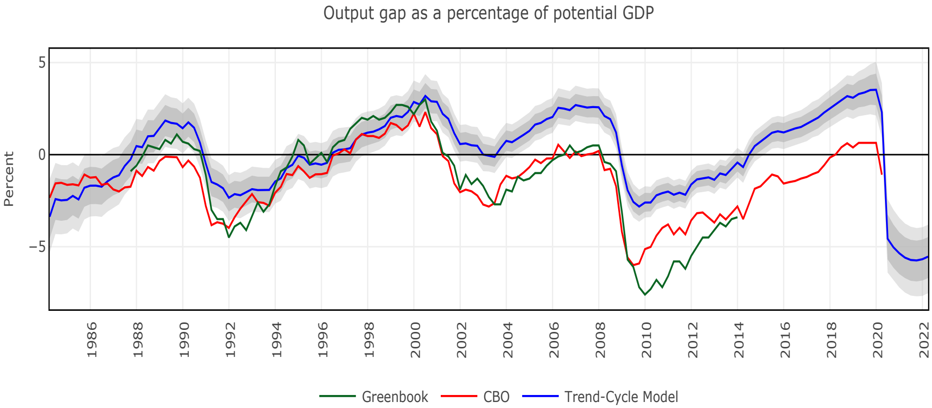


Global: Historical Decomposition

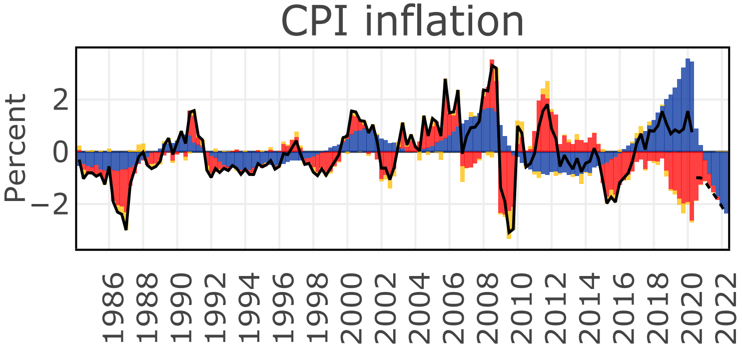


A Rough & Ready COVID Exercise

COVID Output Gap

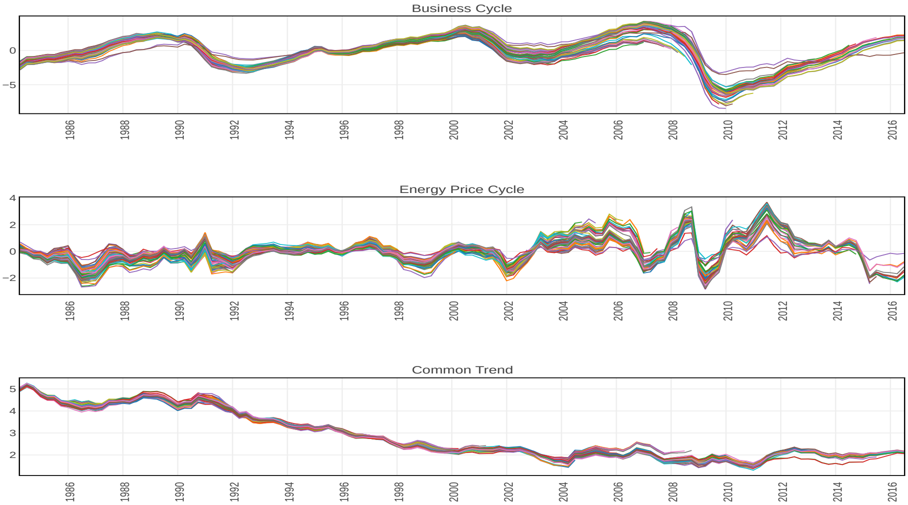


Deflation Ahead?



Out-of-Sample Performances

Out-of-Sample Cycle & Trend Revisions



Out-of-Sample Forecast Evaluation

Root Mean Squared Forecast Error relative to the Random Walk with drift

Horizon	Variable	TC Model	BVAR	UC-SV
h=1	Real GDP	1.04	**0.93**	x
	Employment	0.98	**0.76**	x
	Unemployment rate	0.85	**0.67**	x
	Oil price	**1.03**	1.08	x
	CPI Inflation	0.94	**0.91**	1.00
	Core CPI Inflation	**1.01**	1.04	**1.01**
	UOM: Expected inflation	**0.98**	1.04	x
	SPF: Expected CPI	**0.95**	1.06	x
h=2	Real GDP	1.06	**0.93**	x
	Employment	1.00	**0.76**	x
	Unemployment rate	0.85	**0.71**	x
	Oil price	**1.04**	1.18	x
	CPI Inflation	**0.90**	0.98	0.99
	Core CPI Inflation	**0.99**	1.15	**0.99**
	UOM: Expected inflation	**0.98**	1.09	x
	SPF: Expected CPI	**0.94**	1.18	x

Horizon	Variable	TC Model	BVAR	UC-SV
h=4	Real GDP	1.11	**1.01**	x
	Employment	1.06	**0.82**	x
	Unemployment rate	0.86	**0.83**	x
	Oil price	**1.03**	1.26	x
	CPI Inflation	**0.87**	1.13	0.97
	Core CPI Inflation	**0.95**	1.22	0.96
	UOM: Expected inflation	**0.96**	1.14	x
	SPF: Expected CPI	**0.92**	1.31	x
h=8	Real GDP	**1.17**	1.21	x
	Employment	1.13	**1.01**	x
	Unemployment rate	**0.85**	1.02	x
	Oil price	**0.99**	1.36	x
	CPI Inflation	**0.81**	1.09	0.95
	Core CPI Inflation	**0.84**	1.30	0.91
	UOM: Expected inflation	**0.92**	1.28	x
	SPF: Expected CPI	**0.88**	1.34	x

Conclusions

- Explicit modelling of **trends & cycles** and **parsimonious** characterisation of the **structural** relationships amongst macro variables
- Inflation dynamics since the 1980s shows a stable and fairly steep reduced form **Phillips Curve** with maximum power at around eight years periodicity.
- The Phillips Curve is **not** always the **dominant component** of cyclical inflation
- Long-term inflation expectations – common trend between inflation and expectations – roughly stable
- Consumer survey shows large and persistent deviations from the common trend (Coibion and Gorodnichenko, 2015)
- Important question on what happened to **trend growth/output gap** (Fernald et al. 2017, Coibion et al. 2018, Blanchard et al., 2015)

Appendix

Priors

Table: *Prior distributions*

Name	Support	Density	Parameter 1	Parameter 2
δ, γ, ϕ and τ	$\mathbb{R}$	Normal	0	1000
σ^2 and ς^2	$(0, \infty)$	Inverse-Gamma	3	1
ρ	$[0.001, 0.970]$	Uniform	0.001	0.970
λ	$[0.001, \pi]$	Uniform	0.001	π

Back...

Bayesian Estimation

Metropolis-Within-Gibbs Algorithm

The algorithm is structured in two blocks

- The **first block** uses a Metropolis step for the **estimation of the state-space parameters**
- The **second block** uses a Gibbs step to draw the **unobserved states** conditional on the model parameters

Back...

Bayesian Estimation

- Metropolis algorithm draws the model parameters in the unbounded space in order to avoid a-priori rejections and to obtain a more efficient estimation routine
- The following transformations have been applied to parameters with Normal, Inverse-Gamma and Uniform priors

$$\theta_j^N = \Theta_j^N \qquad \theta_j^{IG} = \ln(\Theta_j^{IG} - a_j) \qquad \theta_j^U = \ln\left(\frac{\Theta_j^U - a_j}{b_j - \Theta_j^U}\right)$$

Where a_j and b_j are the lower and the upper bounds for the j-th parameter

- Jacobians of the transformations of the variables

$$\ln\left(\frac{d\Theta_j^N}{d\theta_j^N}\right) = 0 \qquad \ln\left(\frac{d\Theta_j^{IG}}{d\theta_j^{IG}}\right) = \theta_j^{IG}$$

$$\ln\left(\frac{d\Theta_j^U}{d\theta_j^U}\right) = \ln(b_j - a_j) + \theta_j^U - 2\ln(1 + \exp(\theta_j^U))$$

Back...

Bayesian Estimation

Algorithm: Metropolis-Within-Gibbs

Initialisation

For $s = 1, \ldots, n_s$ ($n_s = 40000$)

1. *Metropolis Algorithm*
   i. Draw a candidate vector for the unbounded parameters (θ_*), from a multivariate normal distribution with mean θ_{s-1} and variance $\omega\mathbb{I}$, where ω is a scaling constant used to get an acceptance rate between 25% and 35%
   ii. Set

$$\theta_s = \begin{cases} \theta_* & \text{with probability } \eta \\ \theta_{s-1} & \text{with probability } 1-\eta \end{cases} \tag{1}$$

   for

$$\eta = min\left(1, \frac{p(y \mid f(\theta_*)^{-1})\, p(f(\theta_*)^{-1})\, J(\theta_*)}{p(y \mid f(\theta_{s-1})^{-1})\, p(f(\theta_{s-1})^{-1})\, J(\theta_{s-1})}\right) \tag{2}$$

2. Discard the first $s = 1, \ldots, n_0$ ($n_0 = 20000$) draws of θ_s.

Back...

Bayesian Estimation

Algorithm: Metropolis-Within-Gibbs

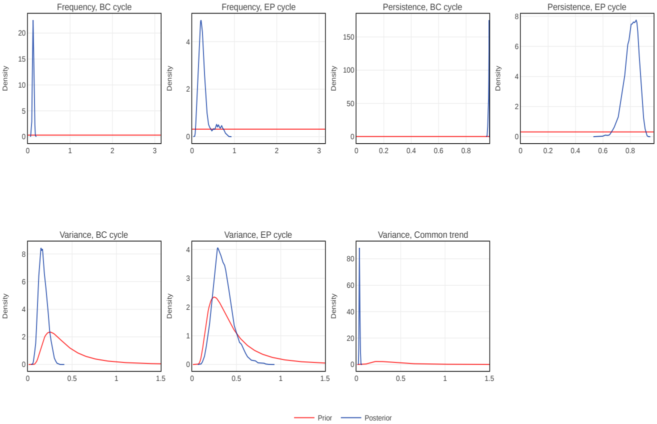
Recursion

1. *Metropolis Algorithm*
   Set Σ to the sample covariance of the chain of θ_s, $(s = \{n_0, \ldots, n_s\})$, from the Initialisation step.
   For $q = 1, \ldots, n_q$ $(n_q = 20000)$
   i. Draw a candidate vector for the parameters (θ_*), from a multivariate normal distribution with mean θ_{q-1} and variance $\omega\Sigma$, where ω is set to have an acceptance rate between 25% and 35%
   ii. Set
   $$\theta_q = \begin{cases} \theta_* & \text{with probability } \eta \\ \theta_{q-1} & \text{with probability } 1-\eta \end{cases} \tag{1}$$
   where η is defined as in the Initialisation step.
2. *Gibbs sampling*
   For $n_q > n_\emptyset$ for $n_\emptyset = 10000$ (burn-in period), apply the univariate approach for multivariate time series of of Koopman and Durbin (2000) to the simulation smoother proposed in Durbin and Koopman (2002) to sample the unobserved states, conditional on the parameters. In doing so, we follow the refinement proposed in Jarociński (2015).
3. Discard the first $q = 1, \ldots, n_\emptyset$ draws of θ_q.

Back...

Priors and Posteriors

Variance of Shocks to the Components



Back...