A Model of the Fed's View on Inflation

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Abstract

A view often expressed by central banks is that three components matter for inflation dynamics: a trend anchored by long-run expectations, a cycle connecting nominal and real variables, and energy prices. This paper proposes a novel semi-structural econometric model informed by this understanding of inflation, incorporating key economic relations such as the Phillips curve and Okun's Law, and supported by inflation expectation data. We identify a stable expectational trend, a steep and well identified Phillips curve and a sizeable energy price component. The latter often overpowers the Phillips curve and explains the inflation puzzles of the last ten years.

Keywords: Phillips curve, inflation dynamics, output gap, Okun's law, unobserved components, Bayesian estimation.

JEL Classification: C11, C32, C53, E31, E32, E52.

Inflation is characterized by an underlying trend that has been essentially constant since the mid-1990s; [...]. Theory and evidence suggest that this trend is strongly influenced by inflation expectations that, in turn, depend on monetary policy. In particular, the remarkable stability of various measures of expected inflation in recent years presumably represents the fruits of the Federal Reserve's sustained effort since the early 1980s to bring down and stabilize inflation at a low level. The anchoring of inflation expectations [...] does not, however, prevent actual inflation from fluctuating from year to year in response to the temporary influence of movements in energy prices and other disturbances. In addition, inflation will tend to run above or below its underlying trend to the extent that resource utilization – which may serve as an indicator of firms' marginal costs – is persistently high or low.

Yellen (2016), 'Macroeconomic Research After the Crisis'
Speech for the 60th Boston Fed Conference

The quote by Janet Yellen reflects a view, widely shared by policy makers and central bankers in particular, which maintains that three components matter for inflation dynamics: trend-expectations, oil prices, and the degree of resource utilisation in the economy. Similarly, most macroeconomic modelling is based on these three core ideas: some measure of slack affects short term fluctuations of inflation via a Phillips curve; monetary policy, via expectations, shapes its long run trend; and oil price and other idiosyncratic shocks explain the volatile component of headline inflation. While models that incorporate these ideas use a variety of different auxiliary assumptions, for example on the nature of expectations, the functional form of key equations, and the channels of propagation of the shocks, these three components remain the building blocks of a shared narrative. In this paper, we call this broadly and loosely defined understanding of inflation dynamics the 'Fed's view'.

Recent empirical evidence has challenged this view. Indeed, the literature presents a wide range of contrasting findings, including on the existence, stability, and steep-

ness of the slope of the Phillips curve, and regarding the degree of anchoring of inflation expectations.[1] First, many studies have found the Phillips curve to be unstable, hard to identify, and weak or disappearing in recent samples (see results and discussions in IMF, 2013, Ball and Mazumder, 2011, Blanchard et al., 2015 and McLeay and Tenreyro, 2018). Second, Phillips curve based forecasting models have been shown to perform poorly with respect to naive benchmarks, pointing to the irrelevance of slack measures for explaining inflation dynamics (see, Atkeson and Ohanian, 2001, Stock and Watson, 2007, 2009, and also Dotsey et al., 2011, Cecchetti et al., 2017, and Forbes et al., 2018 for recent evidence and relevant discussion). Third, a small but increasingly important literature has challenged the idea that expectations are fully anchored and forward looking. For example, papers have connected the 'missing disinflation puzzle' of the post-2008 crisis period to the partial dis-anchoring of consumers' inflation expectations that, in turn, can be accounted for by the evolution of oil prices (see Coibion and Gorodnichenko, 2015, and Coibion et al., 2017b).

This paper revisits some of this evidence by assessing the Fed's view of inflation dynamics through the lens of a stylised statistical model that is informed by economic theory and incorporates economic expectations while allowing for deviations from perfect information and full rationality. Our modelling strategy can be defined as 'semi-structural' since it incorporates minimal identifying assumptions from a general class of economic models, but lets the data speak on key aspects, such as expectation formation, the nature of the Phillips curve, and the role of oil prices. In this sense it occupies the middle ground between a fully specified Dynamic Stochastic General Equilibrium (DSGE) model and a Vector Auto Regressive (VAR) model. Our specification nests several potentially different forward and backward looking Phillips curve models, including the standard New-Keynesian Phillips curve

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[1]A survey of the extensive empirical literature on the PC is beyond the scope of this paper. For a recent survey of the New Keynesian Phillips curve focusing on univariate limited-information methods, see Mavroeidis et al. (2014). For a review of results using full-information methods to estimate dynamic stochastic general equilibrium (DSGE) models, see An and Schorfheide (2007). Nakamura and Steinsson (2013) review the use of microeconomic data to study price dynamics. Coibion et al. (2017b) discuss the incorporation of survey data on inflation expectations in models of inflation dynamics. Other surveys, providing complementary approaches, include Henry and Pagan (2004), Ólafsson (2006), Rudd and Whelan (2007), Nason and Smith (2008), Gordon (2011), and Tsoukis et al. (2011).

(NKPC), in which inflation is a purely forward-looking process, driven by expectations of future real economic activity. Moreover, the model incorporates professionals' and consumers' forecast survey data, as measures of agents' expectations, and allows them to depart from the full-information rational expectations benchmark without imposing any specific form of information friction. We do not require either of the two surveys to be an efficient and unbiased predictor of future inflation and allow for temporary and permanent deviations from a rational forecast, potentially capturing measurement and observational errors, as well as a time-dependent bias in inflation expectations. Finally, the model also captures the different channels through which energy prices can affect headline inflation. A first channel is through production marginal costs and the Phillips curve. We allow for this channel by extracting a business cycle component in oil prices.[2] However, energy prices can also impact inflation without affecting marginal costs and the real side of the economy. Indeed, in the model, oil disturbances can affect headline prices directly via energy services, which are part of the consumption basket, but also potentially via expectation formation, in line with the findings of Coibion and Gorodnichenko (2015). These two channels are captured by studying the differential impact of the energy cycles on headline and core inflation.

The empirical specification includes three price variables: CPI inflation, core CPI inflation, and oil prices; three real variables: GDP, employment, and the unemployment rate and two expectation variables: the median forecasts for 1-year-ahead CPI inflation from the University of Michigan consumers survey (UoM) and the Philadelphia Fed Survey of Professional Forecaster (SPF). The minimal restrictions we impose on the data allow us to identify three orthogonal components of inflation: (i) a unit root trend, common to inflation and inflation expectations – as represented in professionals' and consumers' surveys – which we interpret as a

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[2]A large and important literature has analysed the connection between demand and supply oil shocks and the business cycles (see, for example, Baumeister and Kilian, 2016, Hamilton, 2013, Kilian and Vigfusson, 2017). While this paper is not directly concerned with the identification of the channels of propagation of oil shocks, we incorporate some of the broad conclusions of this literature in our model by allowing for both a correlation of oil prices, inflation and slack in the economy, but also potentially for some other mechanisms through which oil shock can affect prices in the economy.

measure of monetary policy driven long term inflation expectations;[3] (ii) a stationary stochastic cycle, which, in the tradition of Burns and Mitchell (1946), captures multivariate and lagged commonalities in real, nominal (including energy prices), and labour market variables at business cycle frequencies and connects the output gap to prices and their expectations via a Phillips curve relationship, and to unemployment via Okun's law; (iii) a stationary stochastic cycle capturing the common dynamics between oil prices, inflation expectations, and CPI inflation but not affecting real variables. The model also identifies other key economic objects such as output potential, trend employment, and equilibrium unemployment, in the form of unit root trends.

Figure 1 portrays a synthetic view of our findings. In the upper chart we show the decomposition of the cycle of CPI inflation into component (ii) (the blue area), component (iii) (the red area) and an idiosyncratic residual (yellow area). In the bottom chart we show CPI inflation and its estimated trend.

The chart suggests (and the econometric analysis developed in the paper confirms it) that the Phillips curve – understood as a relationship connecting nominal variables with real variables and inflation expectations – is alive and well and has been fairly stable since the early 1980s.[4] Importantly, our cycle decomposition shows that the Phillips curve is not always the dominant component. Large oil price fluctuations can move prices away from the real-nominal relationship both by directly impacting energy services prices and by shifting consumers' expectations away from the rational forecast – 'disanchoring' them – and hence inducing expectation driven fluctuations in prices. This result confirms the intuition of Coibion and Gorodnichenko (2015) in a methodology which, in contrast to their approach, allows the Phillips curve to be recovered as orthogonal to oil-driven movements in the expectations and prices that are not transmitted to real variables. We provide

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[3]The choice of modelling inflation as non-stationary is supported by findings in the forecasting literature which suggests that models perform better when they allow for trend inflation (Faust and Wright, 2013).

[4]While we observe that a fixed parameter model is able to capture a stable Phillips curve from the 1980s, it is possible that time-variation in the parameters or stochastic volatility may be important over a longer sample (see Stock and Watson, 2007; Mertens and Nason, 2017). We do not explore this possibility in this paper. Indeed, estimation uncertainty is likely to obfuscate all gains coming from a more sophisticated model.
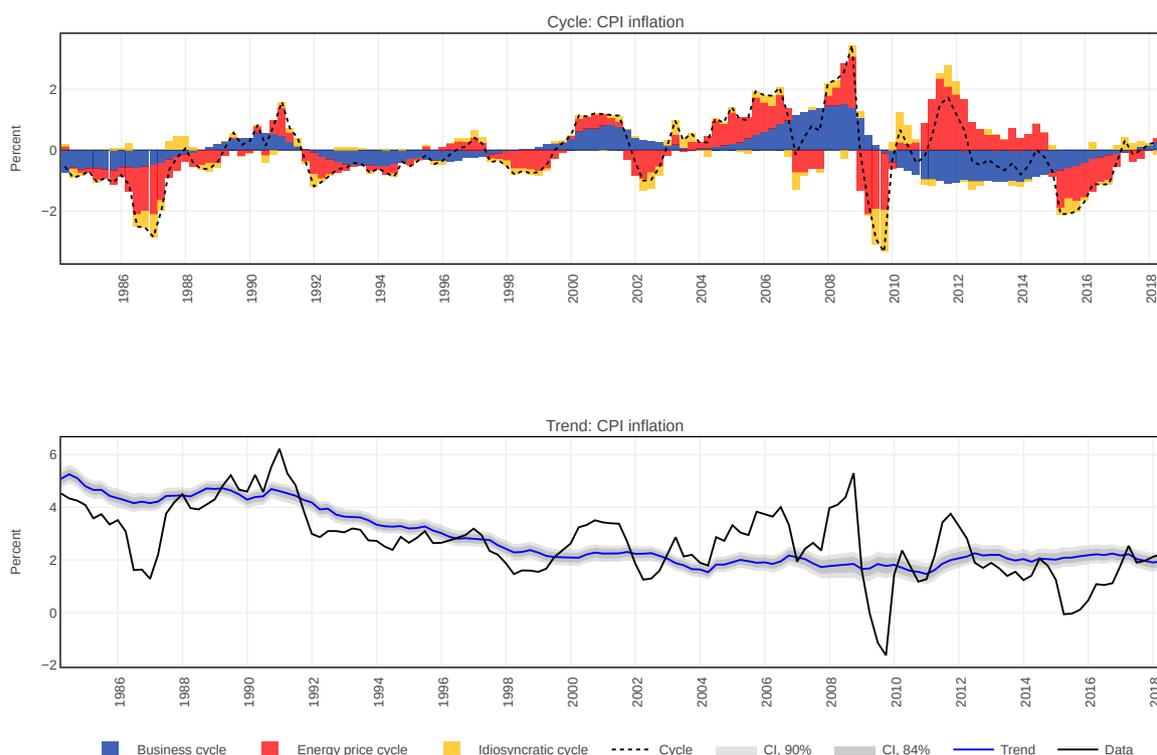
Figure 1: Top: Decomposition of the cycle of CPI inflation into common (in blue and red) and independent (in yellow) components, as estimated by the model. Bottom: Trend of CPI inflation (in blue), with relative coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model.

confirmation of the importance of using expectational data to identify both trend inflation and the Phillips curve, while dealing with disturbances to expectations that, albeit reflected in inflation, are unrelated to real variables and fundamentals. From a policy perspective, the stable inflation trend is an indication of the Fed's success in anchoring expectations. However, our results also point to the challenges that policymakers have to overcome in guiding expectations and stabilising the economy in the presence of large energy price disturbances.

From the statistical point of view, the model has a number of attractive features: it does not rely on arbitrary preliminary detrending of the data which may create distortions, it contains a rich lag structure allowing us to capture dynamic heterogeneity amongst variables, it allows us to perform conjunctural analysis and historical decompositions of variables in cyclical and trend components, and it is sufficiently efficient and parsimonious to be used as a forecasting tool. The unit root

trend common to inflation and inflation forecasts can be related to agents' long-term expectations, under the assumption that the 'law of iterated expectations' holds (see Beveridge and Nelson, 1981 and Mertens, 2016). In fact, the impact of all transitory components has to be zero in the long run.[5]

Our econometric representation is general in the sense described but has a structure that is motivated by the objective of parsimony. Indeed, our model can be understood as a restricted VAR model where, by adopting minimal economic restrictions to identify the potentially different dynamic components of inflation, we induce 'informed' parsimony thereby helping with signal extraction and forecasting. The proposed decomposition leads to a rather complex state space form. In order to deal with this complexity, we estimate the model using Bayesian methods. A Bayesian approach in the context of a similar but simpler model has been proposed by Planas et al. (2008) who implement a Bayesian version of the work of Kuttner (1994) and, more recently, by Lenza and Jarociński (2016). The latter paper is the closest to our work but focuses on estimating measures of the output gap in the Euro Area rather than on providing a decomposition that can be used for studying the drivers of inflation dynamics. Our paper also shares a similar approach and methodology with Del Negro et al. (2017), who employ a flexible VAR model that incorporates long-term survey expectations, to estimate common trends and study the natural rate of interest in the US.

Our work builds on the tradition of structural time series models (see Harvey, 1985), where observed time series are modelled as the sum of unobserved components: common and idiosyncratic trends and cycles. In doing this, and by focussing on inflation dynamics, this paper relates to the literature on the output gap, the Phillips curve, and trend inflation estimation with unobserved components models, started by Kuttner (1994). Similarly to Baştürk et al. (2014) and Lenza and Jarociński (2016), we do not pre-filter data to stationarity, model their low frequency behaviour by allowing for trends. As in Gordon (1982) and Basistha and Startz (2008), we use multiple real activity indicators to increase the reliability of

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[5]A discussion on the conditions under which survey data can be employed to study the PC is in Adam and Padula (2011).

the output gap estimates. Also, our work relates to a number of papers which have studied trend inflation in unobserved component models augmented with data on medium-/long-term inflation expectations, as for example, Clark and Doh (2014), and Mertens (2016).

There are several by-products of our analysis: we obtain an estimate of the output gap which is coherent with our modelling of real and nominal variables and exploits multivariate information, including both output and labor market variables; we also assess the stability of Okun's law and the quality of core inflation as an indicator of underlying inflation. Indeed, our approach generates an indicator of cyclical inflation which is clean not only from the direct effect of oil prices, as is the case for core inflation, but also from their indirect effects.

2 A Stylised Model for Inflation Dynamics

At the core of our empirical approach lies a stylised full information rational expectations model for inflation and output. In this section we discuss the intuition and basic building blocks. We assume that inflation and output can be decomposed into three components: (i) independent trends determining output potential μ_t^y and trend inflation μ_t^π; (ii) a common stationary cycle relating nominal and real variables (the output cycle is interpreted as the output gap) $\widehat{\psi}_t$; and (iii) some independent (white noise) disturbances to output and inflation, ψ_t^y and ψ_t^π, that can be thought of as classic measurement error or idiosyncratic shocks. We have:

$$y_t = \mu_t^y + \widehat{\psi}_t + \psi_t^y , \tag{1}$$

$$\pi_t = \mu_t^\pi + \delta_\pi \widehat{\psi}_t + \psi_t^\pi , \tag{2}$$

where the independent trends are assumed to be unit-root processes (with a drift in output)

$$\mu_t^y = \mu_0 + \mu_{t-1}^y + u_t^y , \tag{3}$$

$$\mu_t^\pi = \mu_{t-1}^\pi + u_t^\pi . \tag{4}$$

The economic interpretation of the different trend and cycle components is standard (see, for example the discussion in Yellen, 2015). The output trend – i.e. the output potential, capturing the long-term growth of the economy – is usually thought of as driven by technological innovation. Inflation fluctuates around a longer-term trend that, at least in recent times, has been essentially stable. Theory relates this trend inflation to inflation expectations that, in turn, are shaped by the conduct of monetary policy – for example, by policymakers' targets. Shocks of a different nature can impact marginal production costs and modify the intensity of resource utilisation in the economy, thus, temporarily pushing output away from its balanced growth path. The shortfall of actual GDP from potential output is the output gap $\widehat{\psi}_t$. The slack in the economy is reflected in the short-run cyclical fluctuations of inflation around its trend, in the presence of price rigidity. This relationship is generally described by an expectations-augmented Phillips curve in theoretical models. Finally, a nontrivial fraction of the quarter-to-quarter variability of inflation and output is attributable to independent and idiosyncratic shocks.

In line with the econometric literature on the output gap, we assume that $\widehat{\psi}_t$ is a stationary process with stochastic cyclical behaviour. The simplest process allowing for such a stochastic cycle is an AR(2) process with complex roots of the form

$$\widehat{\psi}_t = \alpha_1 \widehat{\psi}_{t-1} + \alpha_2 \widehat{\psi}_{t-2} + v_t \ . \tag{5}$$

Indeed, the AR(2) model can be written in a different and slightly more general form, displaying its pseudo-cyclical behaviour more clearly , i.e.

$$\widehat{\psi}_t = \rho \cos(\lambda)\widehat{\psi}_{t-1} + \rho \sin(\lambda)\widehat{\psi}_{t-1}^* + v_t \ , \tag{6}$$
$$\widehat{\psi}_t^* = -\rho \sin(\lambda)\widehat{\psi}_{t-1} + \rho \cos(\lambda)\widehat{\psi}_{t-1}^* + v_t^* \ ,$$

where the parameters $0 \leq \lambda \leq \pi$ and $0 \leq \rho \leq 1$ can be interpreted, respectively, as the frequency of the cycle and the damping factor on the amplitude while $\widehat{\psi}_t^*$ is a modelling auxiliary cycle and v_t and $\bar{v}_t$ are uncorrelated white noise disturb-

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ances (see Harvey, 1990).[6] The disturbances make the cycle stochastic rather than deterministic and, if $\rho < 1$, the process is stationary.

It is important to observe that, by assuming an output gap that is a stationary solution to an AR(2) process, the model in Eq. (1-2) admits a hybrid expectations-augmented New Keynesian Phillips Curve connecting the cyclical components of output, inflation, and inflation expectations, of the form

$$\widehat{\pi}_t = \sum_{i=1}^{2} \delta_i \widehat{\pi}_{t-i} + \beta \mathbb{E}_t \left[\widehat{\pi}_{t+1} \right] + \kappa \widehat{y}_t + \varepsilon_t \ , \tag{7}$$

where hats indicate deviations from trends.[7] In this model, rational expectations agents correctly form model-consistent expectations about inflation, that is

$$
\begin{aligned}
\mathbb{E}_t \left[\pi_{t+1} \right] = \mathbb{E}_t \left[\mu_{t+1}^{\pi} + \delta_{\pi} \widehat{\psi}_{t+1} + \psi_{t+1}^{\pi} \right] &= \mu_t^{\pi} + \delta_{\pi}(\alpha_1 \widehat{\psi}_t + \alpha_2 \widehat{\psi}_{t-1}) \\
&= \mu_t^{\pi} + \delta_{exp,1} \widehat{\psi}_t + \delta_{exp,2} \widehat{\psi}_{t-1} \ .
\end{aligned}
$$

The equations for output, inflation, and inflation expectations can be written in a compact reduced form representation in terms of a common cycle and a common trend

$$
\begin{pmatrix} y_t \\ \pi_t \\ \mathbb{E}_t \left[\widehat{\pi}_{t+1} \right] \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta_{\pi} & 1 \\ \delta_{exp,1} + \delta_{exp,2} L & 1 \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \\ \mu_t^{\pi} \end{pmatrix} + \begin{pmatrix} \mu_t^y \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \psi_t^y \\ \psi_t^{\pi} \\ 0 \end{pmatrix} \ . \tag{8}
$$

In principle this simple set of equations can also accommodate different specifications for the Phillips Curve, under suitable parameter restrictions. For example, an AR(1)

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[6]It is straightforward to show that the model can be rewritten as

$$(1 - 2\rho \cos(\lambda)L + \rho^2 L^2)\widehat{\psi}_t = (1 - \rho \cos(\lambda)L)v_t + (\rho \sin(\lambda)L)v_t^* \ .$$

Hence, under the restriction $\sigma_v^2 = 0$, the solution of the model is an AR(2), otherwise an ARMA(2,1). The intuition for the use of the auxiliary cycle is closely related to the standard multivariate AR(1) representation of univariate AR(p) processes.

[7]Empirical studies often feature hybrid Phillips curves to account for inflation persistence (a recent survey is in Tsoukis et al., 2011). Several different mechanisms have been proposed in the literature to introduce hybrid Phillips curves such as indexation assumptions (e.g. Gali and Gertler, 1999), state-contingent pricing (e.g. Dotsey et al., 1999), or deviations from rational expectations assumption (e.g. Erceg and Levin, 2003; Milani, 2007).

$\widehat{\psi}_t$ would be the solution to a purely forward looking New-Keynesian Phillips Curve. It also nests the backwards looking 'Old-Keynesian' Phillips curve connecting output gap and prices – as in the 'triangle model of inflation' (see Gordon, 1982, 1990).

Also, in line with the interpretation proposed, it is worth noting that trend inflation corresponds to the long-run forecast for inflation, which implies

$$\lim_{h \to \infty} \mathbb{E}_t[\pi_{t+h}] = \mu_t^\pi \ , \tag{9}$$

in the spirit of Beveridge and Nelson (1981), and that trend output informs expectations of growth in the long run:

$$\lim_{h \to \infty} \mathbb{E}_t[y_{t+h}] = \lim_{h \to \infty} \{\mu_0 h + \mu_t^y\} \ . \tag{10}$$

While such a stylised rational expectations model can provide the gist of the intuition for our econometric model, it is likely to be too simple as an empirical representation of business cycle dynamics.[8] First, it does not allow for dynamic heterogeneity, and hence nominal and real variables fluctuate only as contemporaneously connected by the slack in the economy, in contrast with the evidence that prices and labour market variables respond with lags to the slack in production. In fact, output is linked to unemployment via Okun's law and to inflation via the Phillips curve relationship which may involve lagging dynamics. These fundamental relationships connect potentially different measures of the slack in the economy, such as the output gap and the cyclical component of unemployment – i.e. the difference between the unemployment rate and its normal long-run level (equilibrium unemployment)[9] – and inform fluctuations at business cycle frequency in other real and nominal variables.

Second, in modelling price dynamics, forecasters and policymakers often distinguish between changes in food and energy prices – which enter into headline inflation

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[8]An estimated version of this model provides poor fit to the data. Results are available from the authors on request.

[9]For example, the measure of slack that is adopted in policy analysis by the Fed is obtained as the difference between the unemployment rate and the Congressional Budget Office's (CBO) historical series for the long-run natural rate (as in Yellen, 2015).

– and movements in the prices of other goods and services – that is, core inflation. This is because food and energy prices tend to be extremely volatile and influenced by factors that are disconnected from the slack in the economy and that are beyond the control of monetary policy. Examples are international political events – as is the case for oil price – as well as weather or diseases – as for food and beverages.[10] In fact, the price index for total consumer price (headline) inflation π_t is decomposed as

$$\pi_t = \pi_t^c + \upsilon_1 \pi_t^{en} + \upsilon_2 \pi_t^{food} \ , \tag{11}$$

where π_t^c is core CPI inflation, and π_t^{en} and π_t^{food} are, respectively, the growth rate for prices of consumer energy goods and services and prices of food, both expressed relative to core CPI prices; and υ_1, and υ_2 are the weights of energy and food in total consumption. This decomposition is important to study how slack in real output is transmitted to prices, by separating the direct impact of energy price shocks onto energy products, from their role as cost push shocks in production.

Finally, it has been argued in the literature that, once inflation expectations are admitted to a forward- or backward-looking Phillips curve equation, it is also possible that economic disturbances impact prices without any intermediating transmission through the output gap or other measures of slack in the economy (see, for example, Sims, 2008). In this spirit, Coibion and Gorodnichenko (2015) argue that the absence of disinflation during the Great Recession can be explained by the rise of consumers' inflation expectations between 2009 and 2011 due to the increase in oil prices in this period. Also, while macro-variables are likely to be affected by non-classical measurement error, agents' expectations, as captured by consumers' and professional forecasters' surveys, are likely to be only partially in line with national accounting definitions of aggregate prices and can introduce measurement errors and biases of a different nature.[11]

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[10]While the Federal Reserve's inflation objective is defined in terms of the overall change in consumer prices, core inflation is considered to provide a better indicator than total inflation for the developments in prices, in the medium term.

[11]For example, especially in consumer surveys the forecast horizon may be loosely defined while the relevant price index may be left unspecified. Also, projections are often reported at different frequencies and can have different forecasting points.

In the next section, we present an empirical model that expands on the core model to accommodate these possibly important aspects of business cycle and inflation dynamics.

3 An Empirical Trend-Cycle Model

Our benchmark empirical model expands on the core rational expectations model presented in the previous section to incorporate a rich information set including output, employment, and the unemployment rate – as measures of real activity and labor market developments – CPI inflation, core CPI inflation and consumers' and professionals' forecasts for one year ahead inflation – as proxies for economic agents' inflation expectations –, and oil prices to proxy for energy prices. To capture the complex dynamics relationships among the variables, we generalise the stylised model presented in the previous section by incorporating dynamic heterogeneity in the relationship linking real variables, labour market outcomes, and prices and by allowing for deviations from perfect rationality.

Importantly, our model provides an empirical specification of a number of key macroeconomic concepts. A unit root trend with drift provides a measure of output potential, while the trend in employment/unemployment captures the evolution of equilibrium unemployment. The cyclical component of unemployment connects to fluctuations in output at business cycle frequency via an Okun's law that involves the output gap and its lagged value. A unit root trend – common to headline and core CPI inflation, and inflation expectations – captures the inflation trend shaping long term expectations. The slack in the economy is reflected in the short-run cyclical fluctuations of inflation (and expectations) via a Phillips curve relationship involving the output gap and its lagged value. Also, oil prices are allowed to co-move along the business cycle with to the slack in the economy and possibly its lagged value, due to demand effects or mark-up shocks. The fact that the cyclical component of output informs economy-wide lead-lag fluctuations in both labour market and nominal variables supports the interpretation of the output gap as a measure of the business cycle.

Table 1: Data and transformations

Variable	Symbol	Mnemonic	Transformation
Real GDP	y_t	y	Levels
Employment	e_t	e	Levels
Unemployment rate	u_t	u	Levels
Oil price	oil_t	oil	Levels
CPI inflation	π_t	π	YoY
Core CPI inflation	π_t^c	π^c	YoY
UoM: Expected inflation	$F_t^{uom}\pi_{t+4}$	uom	Levels
SPF: Expected CPI	$F_t^{spf}\pi_{t+4}$	spf	Levels

Note: The table lists the macroeconomic variables used in the empirical model. 'UoM: Expected inflation' is the University of Michigan, 12-months ahead expected inflation rate. 'SPF: Expected CPI' is the Survey of Professional Forecasters, 4-quarters ahead expected CPI inflation rate. The oil price is the West Texas Intermediate Spot oil price.

We also design the model to be able to account for several potential deviations from the rational expectations benchmark. In particular, we allow for (i) oil price disturbances to affect prices either directly via energy prices in headline CPI, or via economic agents' forecasts by inducing a transitory disanchoring of expectations, with a stationary cycle connecting oil prices, expectations, and inflation but not the measure of slack in the economy; (ii) a time varying bias i.e. a permanent disanchoring of expectations in the form of unit root processes; (iii) non-classic measurement error in the variables and other sources of coloured noise.

We summarise these modelling choices in the following assumptions.

Assumption 1 CPI headline inflation, core CPI inflation. and agents' inflation expectations (consumers' and professional forecasters') share a common random walk trend (viz. **trend inflation**).

Assumption 2 Real output, employment, and unemployment have independent trends modelled with unit roots, with a drift for output and employment (i.e. **potential output** and **equilibrium employment/unemployment** respectively).

Assumption 3 Business cycle fluctuations in output are described by a stationary process with stochastic cycle in the form of an ARMA(2,1) process with complex roots (i.e. **output gap**).

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Assumption 4 Inflation, inflation expectations, and output are connected by a **Phillips curve** relationship defined as a moving average of the output gap and its first lag.

Assumption 5 Labour market variables are linked to output via the **Okun's Law** defined as a moving average of the output gap and its firs lag.

Assumption 6 Oil prices commove with the business cycle via a a moving average of the output gap and its firs lag (**business cycle component of oil prices**).

Assumption 7 Inflation expectations and inflation are connected, via a moving average of order one, to an ARMA(2,1) cycle in oil prices (**Energy cycle**).

Assumption 8 All variables can have an idiosyncratic ARMA(2,1) cycle component, possibly capturing **non-classic measurement error**, **differences in definitions** and other **sources of noise**.

Assumption 9 Agents' (consumers and professional forecasters) expectations have independent idiosyncratic unit roots without drift, capturing **time varying bias in the forecast**.

Assumption 10 All components are mutually orthogonal.

For the purpose of this analysis the University of Michigan (UoM) consumer survey and the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasts (SPF) one year ahead inflation forecast were chosen as proxies for consumers' and professionals' expectations. This was done because they both have relatively long histories and are available at quarterly frequency. Importantly, both of them target CPI inflation, either explicitly as is the case for the SPF or, implicitly, by surveying consumers, as is the case for UoM. For both surveys, we employ the median expected price change in the four quarters following the date of the survey, which is consistent with our use of year-on-year inflation. Data incorporated in the model are at quarterly frequency, with the sample starting in Q1 1984 and ending in Q2 2017. All variables enter the model in levels, except for price variables which are transformed to the year-on-year inflation rate (see Table 1 for details).

Our model in $x_t := \{y_t, e_t, u_t, oil_t, \pi_t, \pi_t^c, F_t^{uom}\pi_{t+4}, F_t^{spf}\pi_{t+4}\}$ can be written as

$$
\begin{pmatrix} y_t \\ e_t \\ u_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{uom}\pi_{t+4} \\ F_t^{spf}\pi_{t+4} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \delta_{e,1} + \delta_{e,2}L & 0 & 0 \\ \delta_{u,1} + \delta_{u,2}L & 0 & 0 \\ \delta_{oil,1} + \delta_{oil,2}L & 1 & 0 \\ \delta_{\pi,1} + \delta_{\pi,2}L & \gamma_{\pi,1} + \gamma_{\pi,2}L & \phi_\pi \\ \delta_{\pi^c,1} + \delta_{\pi^c,2}L & \gamma_{\pi^c,1} + \gamma_{\pi^c,2}L & \phi_{\pi^c} \\ \delta_{uom,1} + \delta_{uom,2}L + \delta_{uom,3}L^2 & \gamma_{uom,1} + \gamma_{uom,2}L & \phi_{uom} \\ \delta_{spf,1} + \delta_{spf,2}L + \delta_{spf,3}L^2 & \gamma_{spf,1} + \gamma_{spf,2}L & \phi_{spf} \end{pmatrix} \begin{pmatrix} \widehat{\psi}_t \\ \psi_t^{EP} \\ \mu_t^\pi \end{pmatrix} + \begin{pmatrix} \psi_t^y \\ \psi_t^e \\ \psi_t^u \\ \psi_t^{oil} \\ \psi_t^\pi \\ \psi_t^{\pi^c} \\ \psi_t^{uom} \\ \psi_t^{spf} \end{pmatrix} + \begin{pmatrix} \mu_t^y \\ \mu_t^e \\ \mu_t^u \\ \mu_t^{oil} \\ 0 \\ 0 \\ \mu_t^{uom} \\ \mu_t^{spf} \end{pmatrix} \quad (12)
$$

where ϕ_π, ϕ_{π^c}, ϕ_{uom}, and ϕ_{spf} are normalised to have unitary loading of inflation and inflation expectations on trend inflation.[12] It is worth noting that our empirical specification in Equation 12 would reduce to the stylised rational expectations model in Equation 8, under suitable parametric restrictions.

Like the output gap in Equation 6, the energy cycle and the idiosyncratic ARMA(2,1) stationary cycles can be written in the following form:

$$
\begin{pmatrix} \psi_t^j \\ \psi_t^{*j} \end{pmatrix} = \rho^j \begin{pmatrix} \cos(\lambda^j) & \sin(\lambda^j) \\ -\sin(\lambda^j) & \cos(\lambda^j) \end{pmatrix} \begin{pmatrix} \psi_{t-1}^j \\ \psi_{t-1}^{*j} \end{pmatrix} + \begin{pmatrix} v_t^j \\ v_t^{*j} \end{pmatrix}, \quad \begin{pmatrix} v_t^j \\ v_t^{*j} \end{pmatrix} \sim \mathcal{N}(0, \varsigma_j^2 I_2)
$$

$$(13)$$

where $j \in \{EP, x_1, \ldots, x_n\}$ and ψ^{*j}, as discussed, is a term capturing an auxiliary cycle. For stationarity, we impose $0 < \lambda^j \le \pi$ and $0 < \rho^j < 1$ for all cycles, including the output gap. As discussed, the common and idiosyncratic trends are random walks (with/without drifts – μ_0^j) that can be written as

$$
\mu_t^j = \mu_0^j + \mu_{t-1}^j + u_t^j, \quad u_t^j \sim \mathcal{N}(0, \sigma_j^2) .
$$

All of the stochastic disturbances in the model are assumed to be mutually orthogonal and Gaussian. Finally, it is worth noting that the common and idiosyncratic trends in inflation and inflation expectations are identified up to a constant (see Bai and Wang, 2015, for a discussion on identification). For the sake of interpretation,

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[12]In the empirical model, the series are standardised so that the standard deviations of their first differences are equal to one. For this reason, we normalise ϕ_π, ϕ_{π^c}, ϕ_{uom}, and ϕ_{spf} to the reciprocal of the standard deviation of the first difference of the respective variable.

Table 2: Prior distributions

Name	Support	Density	Parameter 1	Parameter 2
δ, γ, ϕ and τ	$\mathbb{R}$	Normal	0	1000
σ^2 and ς^2	$(0, \infty)$	Inverse-Gamma	3	1
ρ	$[0.001, 0.970]$	Uniform	0.001	0.970
λ	$[0.001, \pi]$	Uniform	0.001	π

Note: Prior distribution for the model parameters adopted in estimating the model with US data. All of the priors are uniform over the range of the model parameters compatible with our modelling or weakly informative. Boundaries of the uniform priors ensure that the stochastic cycles are stationary and correctly specified according to the restrictions described in Harvey (1990).

we attribute the constant to the common trend so that it is on the same scale as the observed inflation variables.

4 Bringing the Model to the Data

Our estimation strategy builds on the approach recently suggested by Harvey et al. (2007), that adopts modern Bayesian techniques to support the estimation of 'structural' trend-cycle models á la Harvey (1985). In estimating the model, we elicit prior distributions that are either uniform over the range of the model parameters compatible with our modelling choices (i.e. $0 < \lambda^j \leq \pi$ and $0 < \rho^j < 1$), or weakly informative and in the form of very diffuse Normal and Inverse Gamma priors. Table 2 reports the parameters of our prior distributions.

We maximise and simulate the posterior distributions with a Metropolis-Within-Gibbs algorithm that is structured in two blocks. In the first block, we estimate the state space parameters by the Metropolis algorithm and, in the second block, we use the Gibbs algorithm to draw unobserved states conditional on model parameters. Relevant details and references are in the text and Appendix A.[13]

An important question concerns the role of the priors in identifying the model. Figure 2 and Figure 3 illustrate prior and posterior distributions for the variance of the error terms of the unobserved components, the frequency and persistence of the two common cycles, and the coefficients for the common cycles.[14] The charts

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[13]The lags for the survey variables in Equation 12 are implemented by including the auxiliary cycle ψ_t^{*j} from Equation 13.

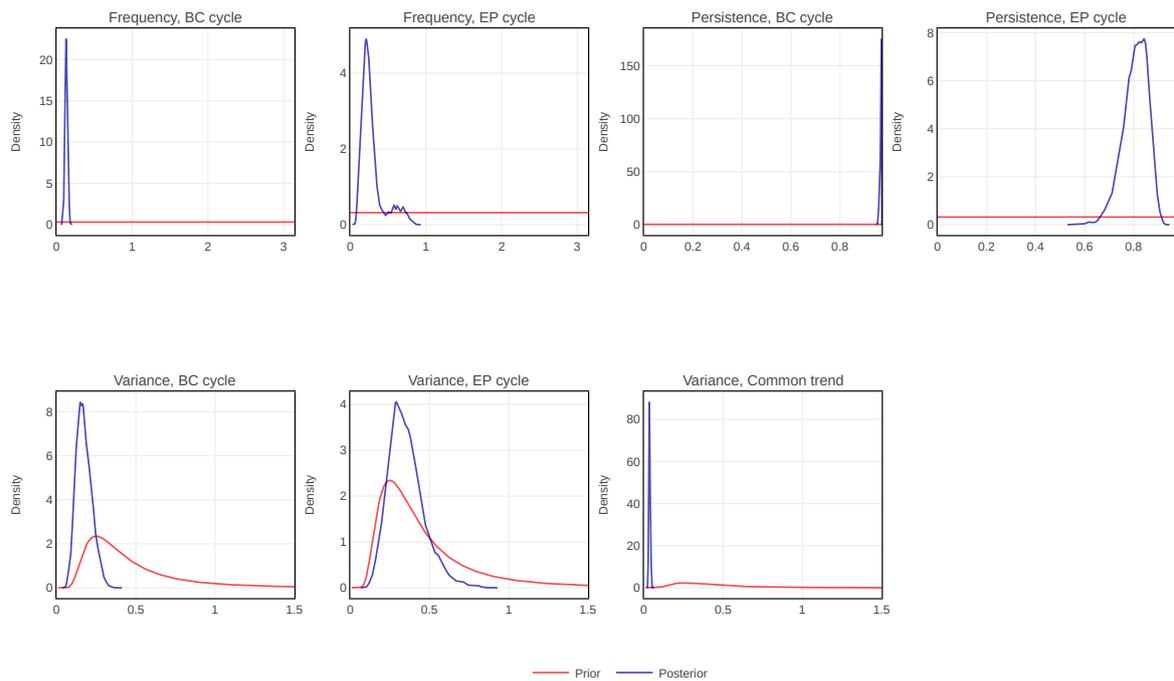[14]The posterior distributions of the full set of model parameters can be found in Appendix B.

Figure 2: Prior distributions (in red) and posterior distributions (in blue) of the frequency of the common cycles, persistence of the common cycles, and the variance of the shocks to the common cycles and common trend.

provide a good indication on whether data provide enough information to identify the model parameters. Indeed, the posterior distributions are well peaked and not shaped by the priors, and show that the data is very informative in estimating the many parameters of the model – in particular the variance of the shocks of the common components and the frequencies of the cycles. Importantly, the posterior distributions of the coefficients for the common cycles (Figure 3) indicate that coefficients equal to zero have negligible probability to be drawn in both cases. Moreover, our results are robust to changes in the parameters of the distributions of the more informative priors. See Appendix C.
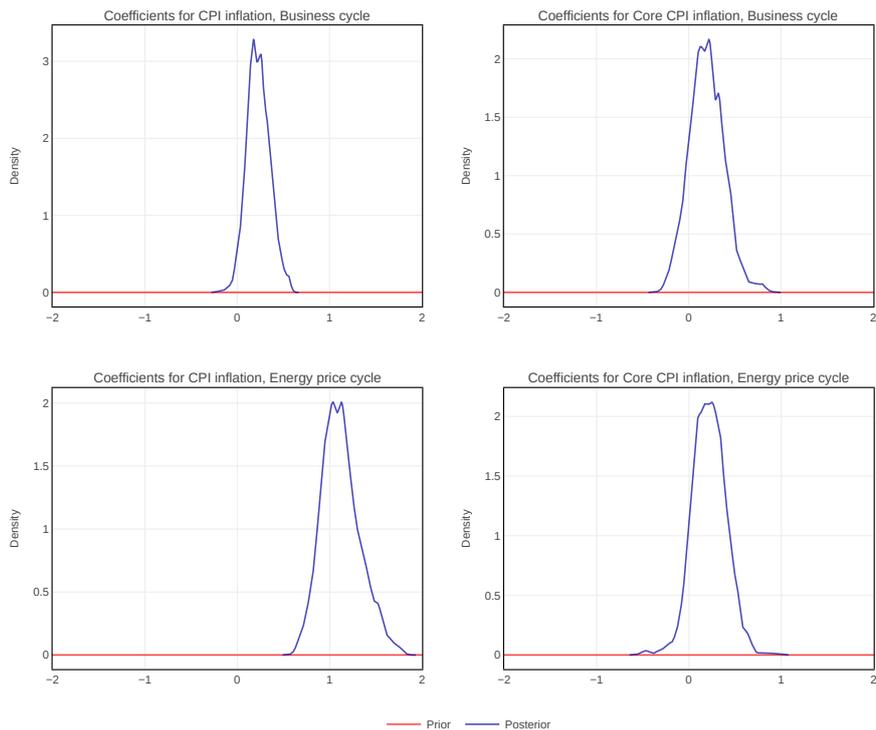
Figure 3: Prior distributions (in red) and posterior distributions (in blue) of the coefficients for the common cycles of CPI inflation and Core CPI inflation.

5 Trends and Cycles in the US Economy

The empirical model produces a coherent historical narrative of business cycle dynamics and an evaluation of how they impacted inflation dynamics, as well as a set of model-consistent measures for trend inflation, equilibrium unemployment, and output potential.

We start by analysing economic trends identified and estimated by the model in the next section and then move to economic cycles in the following one. We compare our assessment of trend-cycle dynamics with the estimates by the Congressional Budget Office (CBO) and the Board of Governors of the Federal Reserve.

5.1 Trend Inflation, Equilibrium Unemployment, GDP Potential

The model delivers very smooth and stable trends. Figure 4 plots real GDP, employment, unemployment, and oil prices against the median of the estimated independent
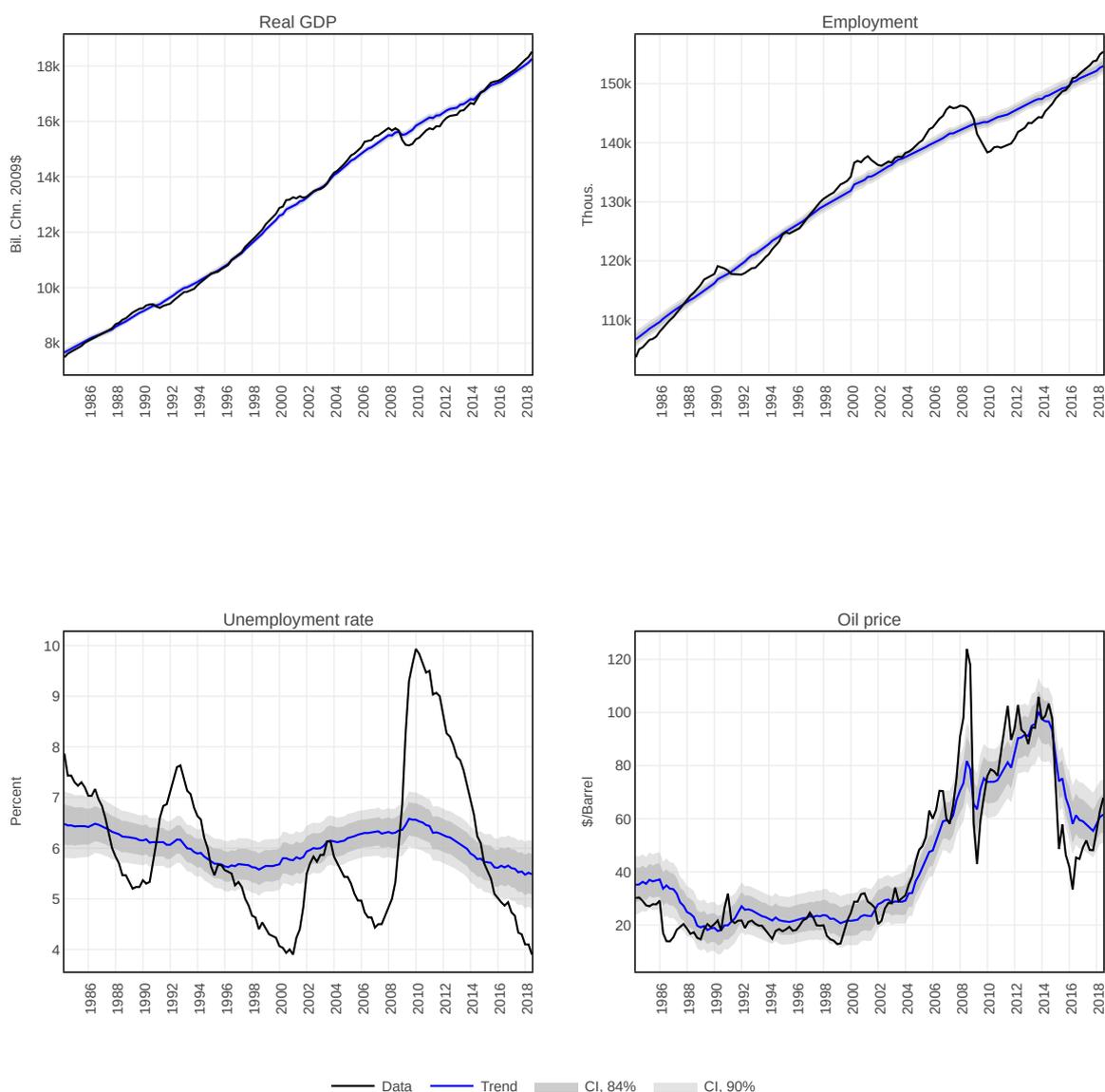
Figure 4: Independent trends of output, employment, unemployment, and oil prices (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The chart also reports the measures of potential outputs and NAIRU estimated by the CBO (in red).

trends, along with coverage bands (at 68% darker shade, and at 90% lighter shade coverage rate). Output trend, which can be thought of as a measure of potential output, is compared with the corresponding measure provided by the CBO.

While both trends are equally stable, they provide a different description of long term growth in the US. Since 2001, the model-implied trend lies below the CBO trend implying that, while the CBO's reading of the data is that the US economy

had only just reached its potential at the pre-crisis peak in 2008, our model signals an overheating of the economy from 2006 to 2008 and a marked slow-down of trend growth in the last part of the sample.

Figure 5 also compares the model-implied measure of equilibrium unemployment against the CBO's measure for the natural rate of unemployment (NAIRU). The two measures coincide in the first part of the sample while they diverge post-2000. While our model provides a very stable unemployment trend hoovering around 6% and with a temporary and small increase around the financial crisis in 2008, the CBO NAIRU shows a slow and persistent decline of the trend continuing through the crisis.

The trend in the oil price shows a hump-shaped increase in the second half of the sample that may be related to the global increase in oil demand post-2000. It is important to observe that, in our model, trends are jointly estimated with the cyclical components. Hence, the differences between our estimated trends and those of the Fed and the CBO have relevant implications for the reading of business cycle dynamics. This will be analysed in the remainder of the paper.

The inflation trend common to headline CPI, core CPI inflation, and consumers' and professional forecasters' inflation expectation variables is shown in Figure 5. Trend inflation appears to be roughly stable from 2000 to 2010 and, interestingly, is closely tracked by the SPF median forecast. The behaviour of UoM expectations, on the other hand, shows large and persistent deviations from the common trend (long-term inflation expectations) since 2004. We interpret this sizeable time-varying idiosyncratic trend as a bias in consumers' expectations.

The unit-root inflation trend can be connected to the long-term inflation expectations of rational agents under the assumption that the 'law of iterated expectations' holds (see Beveridge and Nelson, 1981 and Mertens, 2016). This interpretation is supported by Figure 6 where CPI inflation is plotted against the implied trend and the median 10-year ahead SPF inflation forecast. The chart provides a visual validation of our interpretation that the model trend estimate captures long-term expectations.
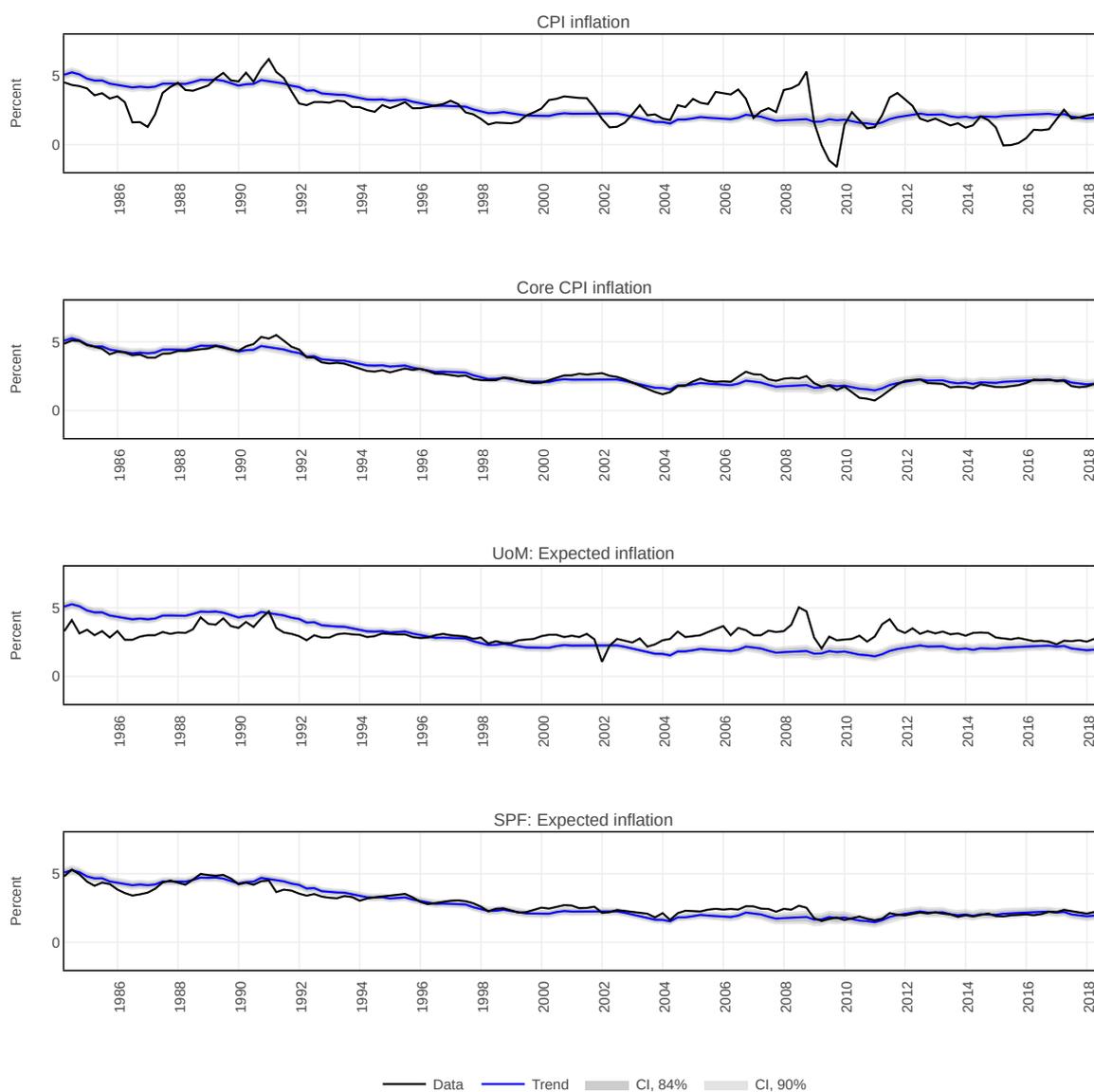
Figure 5: Trend common to CPI inflation, core CPI inflation, and inflation expectations (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model.

5.2 Business and Energy Price Cycles

Figure 7 shows the estimated common cycles in both the time and frequency domains. The first cycle provides a direct measure of the slack in the economy and captures fluctuations of output around its potential. It also connects real, labour market, and nominal variables and hence can be interpreted as a measure of the business cycle. For this reason, in what follows, we refer to it as 'business cycle'
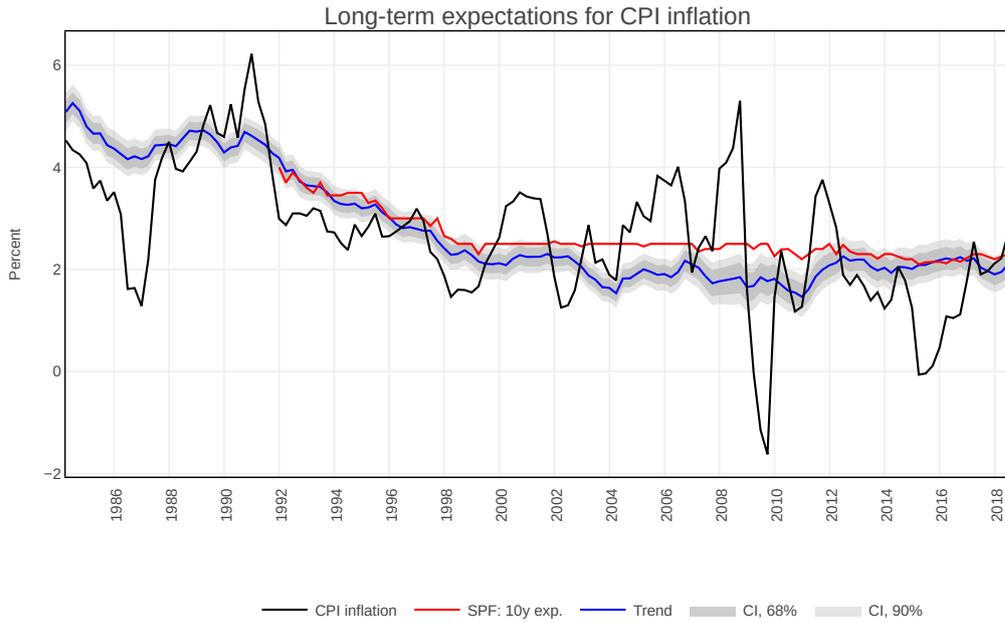
Figure 6: Common trend (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The chart also reports the measure of 10 year expectations for CPI inflation from the Survey of Professional Forecasters.

with a slight abuse of terminology. The upper charts in Figure 7 report the median of the posterior distribution of the business and energy price cycles with relative coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade). The lower charts show the associated spectral densities and coverage bands. The charts indicate that the 'business cycle' is quite regular and much less volatile than the energy price cycle. The spectral shape shows that the business cycle contributes to the inflation spectral shape with a relatively well defined peak and with a cycle between 7 and 8 years periodicity. Conversely, the energy price cycle occupies a broader range of frequencies with a less well defined peak and a periodicity about half as long as that of the business cycle.

Before analysing how these two cycles explain the historically observed inflation dynamics, it is useful to compare our measures of the business cycle and output gap with other commonly adopted measures such as those by the CBO and the Fed. This allows us to validate the business cycle 'dating' identified by the model and to assess the model based description of peak-to-trough fluctuations as compared to a data-informed judgmental description. Figure 8 shows the stationary deviations of
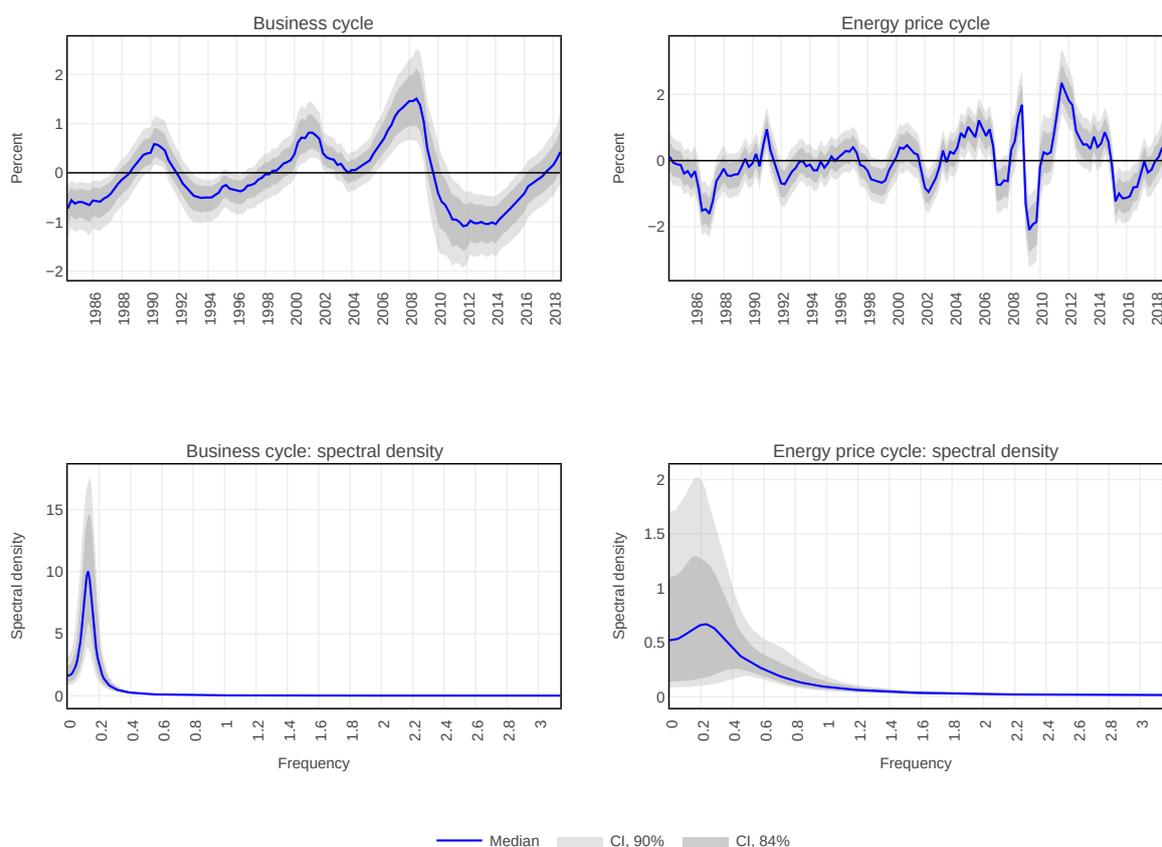
23

Figure 7: Top: Business cycle and Energy Price cycle, with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade). Bottom: Parametric spectrum of the Business cycle and Energy Price cycle.

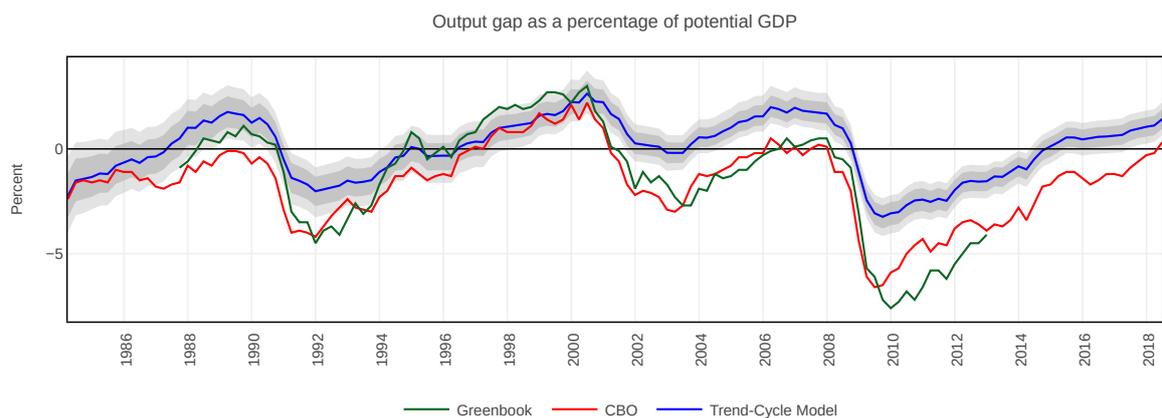**Figure 8:** Output gap (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The chart also reports the output gap from the CBO (in red) and the Fed's Greenbook (in green).

output from its trend, as estimated by the model which is obtained by rescaling the business cycle to match the GDP scale and by summing to it the output idiosyncratic

cycle component. Figure 8 also reports the output gap measure produced by the CBO and by the Fed in its Greenbook forecast (the latter is released with a 5 year delay).

Two features are worth observing. The model-based and the CBO/Fed business cycle dating of the turning points perfectly coincide as the peaks and troughs alignment shows. However, the model-consistent measure and the other two differ in their assessment of the the degree of slack in the economy since 2001. In fact, at the time of the slowdown of 2001-2002, our model indicates that the economy went from over-capacity to trend growth but, unlike the CBO's, does not identify a protracted period of slack. Moreover, we estimate a milder recession in 2008-2009 and find the economy to have been above full capacity since 2015. These differences can be better understood by contrasting both the model-consistent estimates of output potential and of the output gap against the CBO measures in Figure 4 and Figure 8. The CBO provides a more optimistic assessment of the trend growth and attributes the slowdown since the early millennium to a very deep contraction in the cyclical component of output. Its estimated output gap considers the US economy to have been below potential since 2001. Conversely, our model, which has a variable random trend and constant-parameter cyclical components, attributes the slowdown since the early millennium to a decline in trend growth rather than to a widening output gap in line with the research that has pointed to a slowdown in productivity growth since that date (see, for example, Hall et al. 2017). We should obviously recognise that the two different readings of the economic developments since 2001 are based on different and untestable assumptions about the long run behaviour of output and there is no obvious criterion to choose the 'correct' one. From a statistical perspective, the model identify a very persistent slowdown in output growth and interprets it as due to a permanent component – the unit-root output potential –, but cannot disentangle the source or the nature of this slowdown (see Coibion et al., 2017a for a discussion on the issue). However, one of the advantages of our modelling approach is that it allows for a transparent discussion on the assumptions about the long-run components of the economic variables. It also potentially allows for the direct incorporation of time-variation or structural breaks in the model parameters.
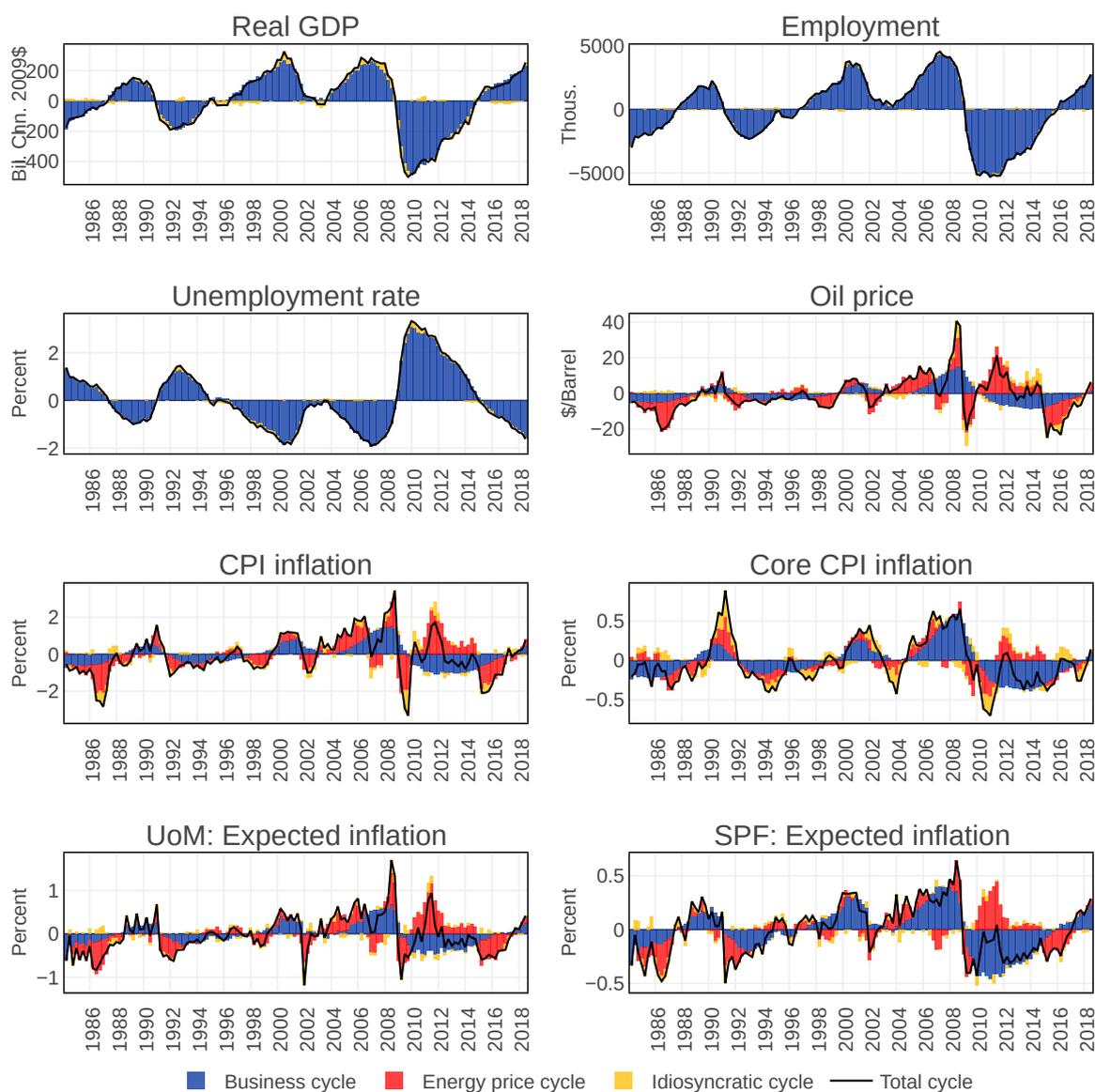
Figure 9: Historical decomposition of the cycles, as estimated by the model. The chart reports the Business cycle (in blue), Energy price cycle (in red), and idiosyncratic cycle (in yellow).

5.3 Historical Decomposition

Let us now turn to the historical decomposition of the stationary components of the eight variables of interest into common and idiosyncratic cycles, as provided by the model. Figure 9 shows the results. Overall, the model provides a coherent description of inflation dynamics with a number of interesting features.

First, the business cycle (in blue) capture almost entirely the fluctuations around trend in real output, employment and unemployment. A negligible idiosyncratic
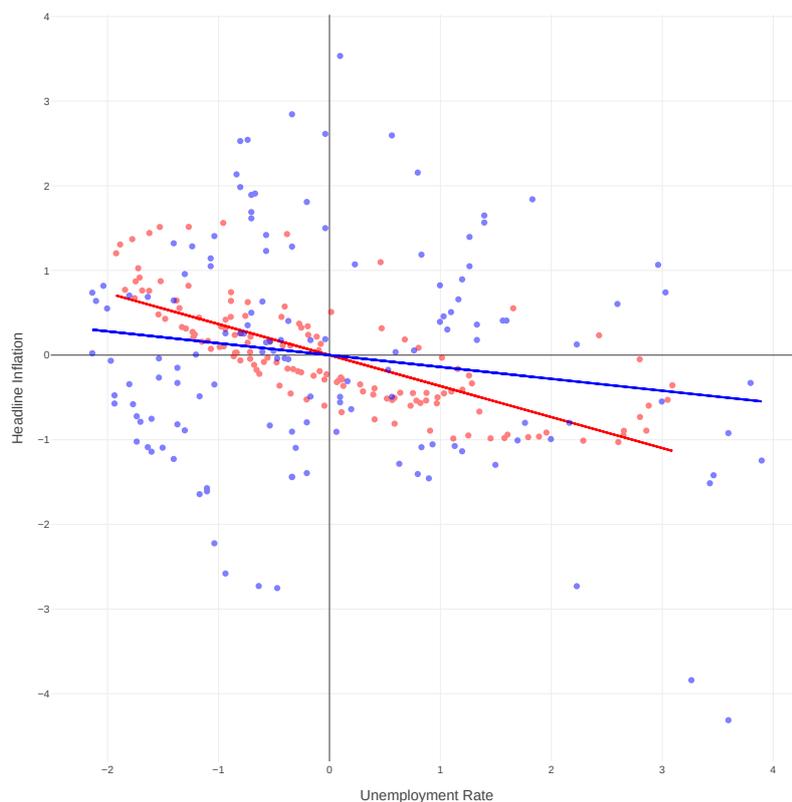
Figure 10: This chart plots the Business cycle component of CPI inflation against the Business cycle component of the unemployment rate (red dots) and the corresponding bivariate linear regression line (red line). The chart also plots demeaned CPI inflation against the demeaned unemployment rate (blue dots) and the corresponding bivariate linear regression line (blue line).

component (in yellow) is visible only in unemployment and almost non-existent in output and employment. This indicates that our measure of the output gap captures the slack in the economy well and is transmitted, via the lagged Okun's law relationship to the labour market. It should be stressed that lags are important in describing the delayed transmission from output dynamics to the labour market and may capture different types of labour market frictions.

Second, a small but non negligible share of oil price fluctuations is due to the comovement of this variable with the slack in the economy, along the business cycle. This may be due either to the demand effect of the US economy onto global oil prices, or the role of oil shocks as mark-up shocks in the aggregate production function.

Third, the slack in the economy is reflected in price dynamics via the Phillips curve which captures the lower frequency dynamics in the inflation cycle and accounts for a sizeable share of the variation in CPI inflation and most of the variation

in core CPI inflation. This 'real' component dominates SPF expectations while it provides a sizeable but not dominant share of variation in consumers' expectations. In our model the Phillips curve is a lagged relationship connecting prices, expectations and output (and hence labour market variables). Hence, a discussion about its 'steepness' may be slightly misleading since a reduced form relation between prices and unemployment would involve different lags of our business cycle. Nonetheless, in Figure 10, we compare a scatter plot showing how the business cycle components of CPI and unemployment would be related (red dots) with a scatter plot of (demeaned) CPI and unemployment variables (blue dots). The linear fit has a slope of -0.39 for the model based measures (red line), against a slope of -0.14 for a naïve estimate (blue line). This is a rough way to assess the strength of the Phillips curve identified by our model against that of a naïve estimate of its steepness.

Fourth, the stationary component of CPI inflation is dominated by the energy price cycle. This can be explained by the fact that energy prices are one of the components of the CPI basket and tend to be extremely volatile with a weak correlation with the slack in the national economy. Notice also that, while small, the energy price component is also visible and non-negligible in core CPI inflation where, by construction, energy prices are removed. This suggests that oil shocks impact core CPI inflation indirectly via expectations and not via the output gap or other measures of slack in the economy. In fact, as suggested by Coibion and Gorodnichenko (2015), household expectations are not fully anchored and respond strongly to oil price changes. Conversely, as observed above, the SPF median forecast tracks the unit-root trends while its cyclical component is dominated by the persistent business cycle component. In other words, the SPF forecasts are relatively unaffected by the volatile and less persistent energy price component. In this respect, the dynamics of the median SPF forecast seem to be consistent with a rational forecast.

Finally, overall, the cyclical part of inflation is well captured by the two common components and little is left to idiosyncratic forces. However, the two common cycles are not in any sense 'synchronised' . This sheds light on some of the puzzling behaviour of inflation since 2008. From 2011 to mid 2012 the inflation cycle is supported by oil prices while the Phillips curve exerts negative pressure. The opposite
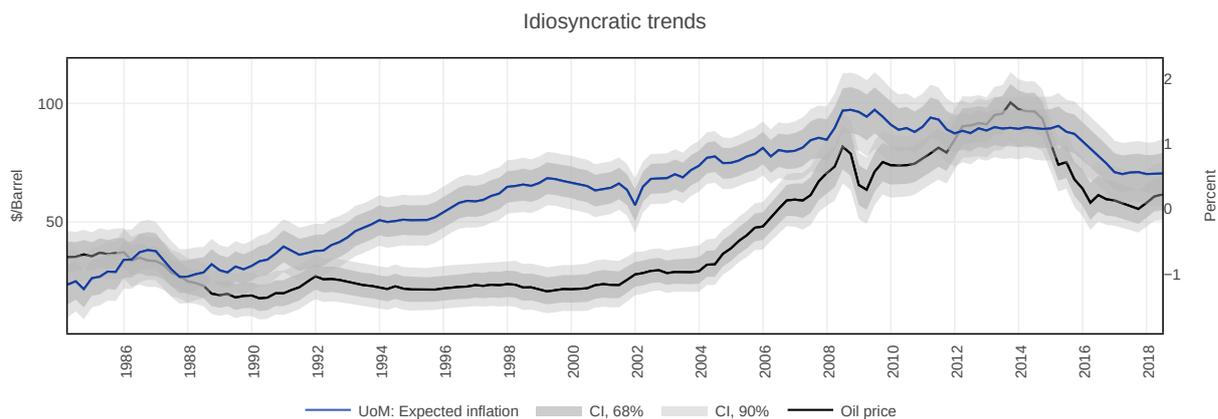
Idiosyncratic trends

Figure 11: Idiosyncratic trends of oil prices (left) and UOM Expected Inflation (right), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model.

is true from 2015 to the end of 2016 when oil prices drag inflation down while the Phillips curve exerts a small upward pressure.

5.4 The Role of Oil and Global Factors

As discussed, oil shocks can impact price dynamics via several different channels. First, as cost-push shocks in production, they impact prices via the Phillips curve. Also, oil prices can fluctuate due to US internal demand along the business cycle. These channels are directly captured by the common business cycle that connects the slack in the economy to oil prices and inflation. Secondly, they directly affect the prices of energy services which enter the consumption basket of headline CPI without affecting the output gap. This second channel is likely to explain most of the contribution of the energy price cycle to headline CPI inflation. Thirdly, they can generate 'non-fundamental' movement in consumers' inflation expectations and shift prices via this mechanism. This third channel is likely to explain the energy price cycle component in consumers' expectations and, importantly, in core CPI inflation which excludes energy prices. Overall, this channel is quantitatively non dominant in price dynamics albeit potentially very important since it is not under the control of standard monetary policy.

Much of the historical differences in inflation expectations between households and professional forecasters can be accounted for by the contribution of oil prices.

This was originally observed by Coibion and Gorodnichenko (2015) who also attribute to oil shocks a sizeable effect on consumer expectations. In our framework the effect can only be present through common stationary cycles and trends. However, our results show that there is a large idiosyncratic trend component in oil prices which, by construction, does not affect CPI inflation. Figure 11 plots it against the idiosyncratic consumers' expectation trend and provides suggestive evidence that consumer price expectations may actually have a persistent component related to oil prices. Our framework leaves it as unmodelled, and to future research.

A conjecture is that the oil price trend is connected to global demand and commodity price cycles as suggested, for example, by Delle Chiaie et al., 2018). Indeed, in recent years, the potential impact of globalisation on price dynamics has drawn attention from both policymakers and academics. The literature has suggested that the increase in international trade has negatively impacted the strength of the domestic Phillips curve relationship and increased the significance of 'global slack' and exchange rates in relation to CPI. It has for example maintained that the increasing impact of demand from emerging markets has affected volatility in commodity prices, that the increased price competition and the greater role of supply chains have reduced firms' pricing power, or that the reduced bargaining power of local workers has weakened the role for domestic slack (see Galí, 2010, for a theory-informed discussion of the literature on the topic).

A number of empirical works have identified a sizeable global common factor in inflation dynamics (e.g. Ciccarelli and Mojon, 2010, and Mumtaz et al., 2011), or proposed to add a measure of global slack (e.g. Borio and Filardo, 2007, Castelnuovo, 2010), supply chain intensity (e.g. Auer and Fischer, 2010; Auer et al., 2017) or exchange rates (e.g. Forbes et al., 2017) in the econometric specifications of price equations. We leave the investigation of these effects within our framework for future research. However, the stability of our results, obtained in a fixed parameter model, suggests that some of theese potential channels have had limited impact in US cyclical inflation dynamics.

6 Model Forecasting Performance

In the previous sections we showed that a trend-cycle model, incorporating key economic relations and allowing for deviations of agents' forecasts from full information rational expectations, provides a coherent 'structural' interpretation of economic developments in the US from the 1980s onwards, based on fundamental and generally accepted economic relationships. While this is an important and desirable feature of the 'in-sample' behaviour of the model, an additional test of robustness and reliability of the model is provided by its out-of-sample behaviour.

In this section we provide an out-of-sample assessment of the model along two dimensions. First we look at trends and cycles extracted by the model in expanding samples, as it would happen in out-of-sample forecast, and check for their stability. This is important in assessing whether the historical decomposition provided by the model is reliable in a pseudo-real-time exercise. Second we test the out-of-sample forecasting performance of the model against two of the best performing models used for inflation forecasting. Forecasting inflation is notoriously difficult and good performance from such a complex model would provide indirect evidence of whether the model is able to capture important features of the data generating process.

Figure 12 shows the revisions of the two common cycles and of the inflation trend over time with an expanding data window. The model is re-estimated every quarter. The period from Q1 1984 to Q4 1998 is employed as the pre-sample, while the evaluation sample starts in Q1 1999 and ends in Q2 2017. Results show that trends and the common business cycle are fairly stable overall and provide an assessment of the development in the economy that is evenly consistent over the sample - including in the recessions. The energy price cycle provides a slightly less stable, albeit roughly coherent, reading of the contribution of energy fluctuations to prices.

The forecasting exercise is conducted in the same sample and again the period from Q1 1984 to Q4 1998 serves as the pre-sample. We use an expanding window and recursively forecast up to 8 quarters ahead. In every quarter we reestimate the model, including the unobserved components and the coefficients. Apart from
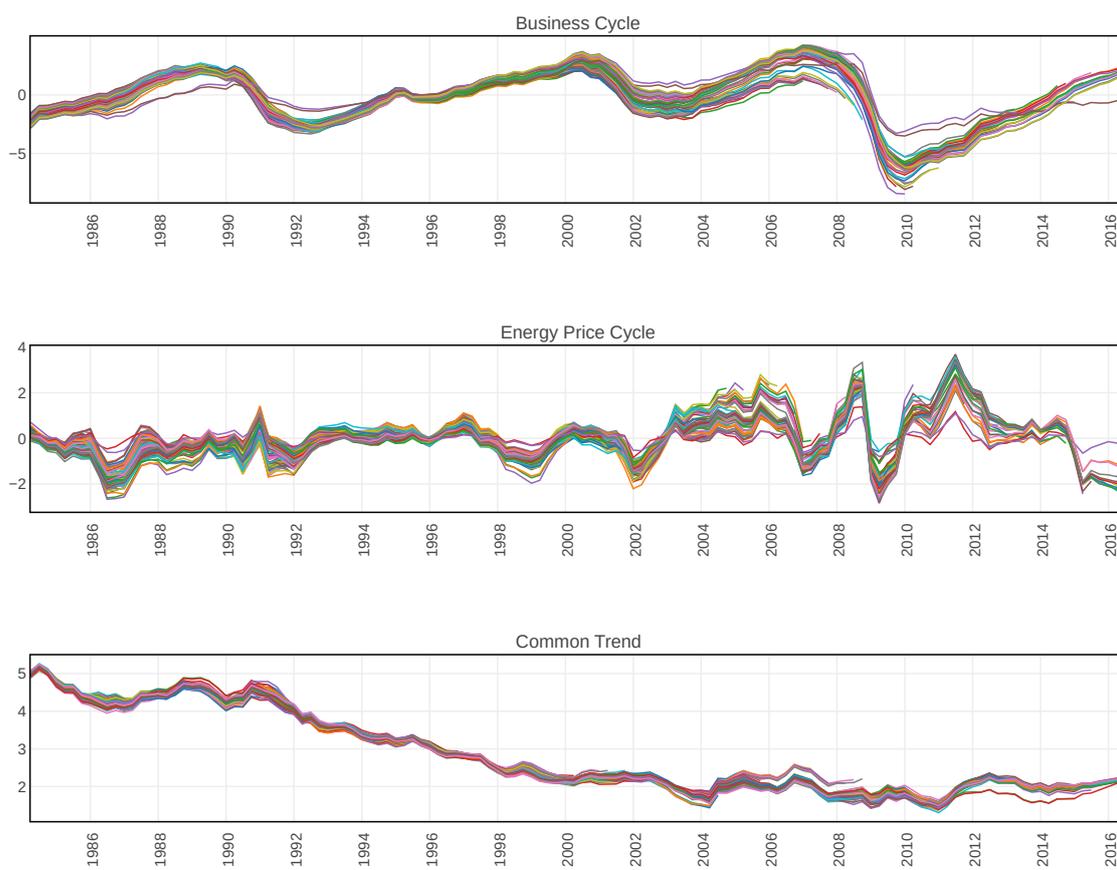
Figure 12: This chart shows the revisions of the business cycle (top), energy price cycle (middle), and common trend (bottom) as estimated during the OOS forecasting exercise.

our model (TC), we consider (i) a BVAR where priors are set as in Giannone et al. (2015) and (ii) an univariate unobserved components IMA(1,1) with stochastic volatility model as suggested by Stock and Watson (2007) to be tough benchmarks for inflation forecasts. For all models we report the root mean squared forecast errors relative to those of a random walk with drift for forecasting horizons of one, two, four, and eight quarters.

Results are reported in Table 3, and show that the trend-cycle model outperforms all others for CPI inflation and does particularly well at the two years horizon. Our conjecture is that our advantage with respect to the BVAR is driven by the random walk trend which captures the slow-moving, low frequency component while the advantage with respect to the UC-SV models is explained by the Phillips curve which captures cyclical co-movements. The trend-cycle model and the BVAR have

Table 3: Relative Root Mean Squared Errors

Horizon	Variable	TC Model	BVAR	UC-SV
h=1	Real GDP	1.00	**0.93**	x
	Employment	0.94	**0.76**	x
	Unemployment rate	0.82	**0.67**	x
	Oil price	**1.06**	1.08	x
	CPI Inflation	0.97	**0.91**	1.00
	Core CPI Inflation	**1.00**	1.04	1.01
	UOM: Expected inflation	**1.03**	1.04	x
	SPF: Expected CPI	**1.00**	1.06	x
h=2	Real GDP	1.02	**0.93**	x
	Employment	0.95	**0.76**	x
	Unemployment rate	0.80	**0.71**	x
	Oil price	**1.08**	1.18	x
	CPI Inflation	**0.95**	0.98	0.99
	Core CPI Inflation	**0.95**	1.15	0.99
	UOM: Expected inflation	**1.01**	1.09	x
	SPF: Expected CPI	**0.97**	1.18	x
h=4	Real GDP	1.04	**1.01**	x
	Employment	0.99	**0.82**	x
	Unemployment rate	0.81	**0.83**	x
	Oil price	**1.12**	1.26	x
	CPI Inflation	**0.95**	1.13	0.97
	Core CPI Inflation	**0.89**	1.22	0.96
	UOM: Expected inflation	**1.11**	1.14	x
	SPF: Expected CPI	**0.91**	1.31	x
h=8	Real GDP	**1.11**	1.21	x
	Employment	1.07	**1.01**	x
	Unemployment rate	**0.81**	1.02	x
	Oil price	**1.10**	1.36	x
	CPI Inflation	**0.85**	1.09	0.95
	Core CPI Inflation	**0.83**	1.30	0.91
	UOM: Expected inflation	**1.02**	1.28	x
	SPF: Expected CPI	**0.86**	1.34	x

Note: This table shows the RMSEs relative to the random walk with drift. The BVAR was estimated using Giannone et al. (2015). The UC-SV model was first proposed in Stock and Watson (2007).

similar performance in relation to the other variables with the exception of the unemployment rate one and two quarters ahead where the BVAR outperforms us.

Results seem to indicate that despite the large number of parameters and the imposition on the data of structural relationships dictated by economic theory, the model provides a stable historical decomposition in a pseudo real-time exercise and

very good performance in forecasting. We consider this as evidence providing support to the claim that the model is able to capture important features of the data generating process.

7   Concluding Comments

In this paper we propose a semi-structural approach for the empirical modelling of inflation which exploits minimal restrictions from economic theory while delivering a flexible empirical specification. The approach can be seen as a mid-way between the estimation of a fully specified DSGE model and a reduced form VAR model. We believe that this approach is promising beyond the particular application of this paper.

The results we have presented are informative for the debate on the Phillips curve. We point to a well identified and steep Phillips curve which captures a cyclical component of CPI inflation with maximum power at around eight years periodicity but also point to deviations from the standard rational expectations formulation since we identify a sizeable cycle in CPI inflation which is unrelated to real variables and captures the correlation between inflation expectations and oil prices. This cycle, which is of slightly shorter periodicity than the business cycle and is more volatile, points to a channel through which oil price developments temporarily affect consumer price expectations away from the nominal-real relationship captured by the Phillips curve. In the presence of large oil price shocks this component may dominate and cloud the signal on cyclical inflation.

Beyond that, our approach delivers empirical estimates of quantities which are the focus of policy analysis and does so within a unified framework. We provide a measure of the output gap, the Okun's relation, the natural rate of unemployment and potential output as well as long-run inflation expectations. We also produce a decomposition of CPI inflation and core inflation into a component related to the business cycle and one related to the expectationally driven energy price cycle. Interestingly, we identify an energy price cycle in both core and CPI inflation which suggests that core inflation provides a clouded signal of fundamental (trend and

cyclically driven) inflationary pressures. This result provides more motivation for our signal extraction approach to identify cyclical inflation. As for the real variables, the model's estimate of potential output identifies a slowdown around the beginning of the millennium, in line with recent research on productivity dynamics. This contrasts with the CBO's estimates and implies a difference between our estimate of the output gap and that of the CBO since the beginning of the productivity slow-down. While the CBO's view is the US economy was growing around potential before the 2008 crisis and below it since then, our model points to growth above potential between 2006 and 2008 and again since 2015. Although it is impossible to discriminate between these different views, a support for our story comes from the good inflation forecasting performance of our model which we show to out-perform traditional benchmarks.

From the policy perspective, our findings indicate that the central bank can exploit the Phillips curve trade-off but only in a limited way since the latter, although well identified, is a small component of inflation dynamics. Indeed, some of the so-called puzzles of inflation behaviour of the last decade can be explained by disentangling the Phillips curve from the energy price cycle. Moreover, while trend inflation appears to be roughly stable from 2000 to 2010, the behaviour of UoM expectations shows large and persistent deviations from the common trend (long-term inflation expectations) since 2004 which can be interpreted as sizeable time-varying idiosyncratic trend as a bias in consumers' expectations. Therefore, a problematic issue for the central bank is that, facing volatile and persistent oil price dynamics, consumer expectations can affect price dynamics producing large and persistent deviations from a stable trend. Our conclusions are therefore quite open-ended. The Fed's view that inflation is dominated by three components is supported by the data. However, the ability of the Central Bank to anchor expectations is limited especially because oil affects consumer expectations persistently and independently from the state of the real economy.

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