Labor Supply Shocks, Labor Force Entry, and Monetary Policy

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Abstract

Should monetary policy offset the effects of labor supply shocks on inflation and the output gap? Canonical New Keynesian models answer yes. Motivated by weak labor force participation during the pandemic, we reexamine the question by introducing labor force entry and exit in an otherwise canonical model with sticky prices and wages. The entry decision generates an employment channel of monetary policy, and a labor supply shock to the value of nonparticipation in the labor market induces a policy trade-off between stabilization of the employment gap and wage growth. For an adverse labor supply shock, optimal policy dampens the decline in employment to rein in wage growth, which entails a period of higher inflation and a positive output gap. A welfare analysis of policy rules shows that monetary policy should not lean against the employment gap.

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1 Introduction

The recovery of the US economy from the COVID-19 induced recession was distinguished by weak labor force participation and strong wage growth and inflation. Even though real GDP had already fully recovered and cyclical labor market indicators, including the vacancy-unemployment ratio, were at historically high levels, the labor force participation rate remained well below its pre-pandemic level and anecdotes about labor shortages were widespread.\footnote{The labor force participation rate fell sharply during the recession, from 63.2 percent in the first quarter of 2020 to 60.8 percent in the second quarter, and rebounded partially to 61.5 percent in the third quarter of 2020. Subsequently, the rate edged up to only 61.8 percent in the fourth quarter of 2021. Anecdotally, the Federal Reserve’s Beige Book for March 2022 contained 14 mentions of “labor shortages,” up from 3 a year earlier.}

Against this backdrop, inflation rose much more than anticipated and was accompanied by high wage growth. In a study of survey data, Faberman et al. (2022) analyze the willingness to work of individuals in or out of the labor force. They document a decline in the willingness to work during the COVID-19 pandemic, driven primarily by individuals out of the labor force, and conclude that the adverse labor supply effect of the pandemic at the end of 2021 was even worse than indicated by the labor force participation rate.\footnote{Based on another survey, Barrero et al. (2022) estimate that continuing social distancing reduced the labor force participation rate by about 2.5 percentage points in early 2022 and the drag from social distancing did not diminish from a year earlier.}

While COVID-19 may have reduced individuals’ willingness to work for a variety of reasons, such as illness, fear of illness, lack of childcare, or a change in priorities, these developments bear the marks of a labor supply shock from a macroeconomic viewpoint. Thus, a question is raised for monetary policymakers: How should monetary policy respond to the labor supply shock? New Keynesian (NK) models are the go-to framework for monetary policy analysis. The conventional wisdom based on canonical NK models answers that monetary policy should offset the effects of labor supply shocks on inflation and the output gap. However, the basic framework is ill-suited to the analysis of labor supply shocks. In canonical NK models with sticky prices and wages, labor supply and wage markup shocks are observationally equivalent; both adverse shocks lead to a contraction in output and increases in inflation and wage growth. Yet the two shocks have distinct implications for monetary policy, as emphasized by Chari et al. (2009). Adverse labor supply shocks reduce the natural rate of output that would prevail under flexible prices and wages. Monetary policy then
should let output decline to close the output gap and offset inflationary pressure arising from the shocks. In contrast, wage markup shocks should have no effect on the natural rate of output, thereby generating a policy trade-off: For adverse wage markup shocks, stabilizing output leads to higher inflation, while stabilizing inflation induces a negative output gap.

To make NK models more suitable for analyzing the implications of labor supply shocks, we introduce worker entry into and exit from the labor force in an otherwise canonical model with sticky prices and wages. As pointed out by Foroni et al. (2018), incorporating labor force participation can break the observational equivalence of labor supply and wage bargaining shocks.\(^3\) In our model, individuals enjoy an exogenous value of nonparticipation in the labor market, which they weigh against the value of market work. The labor force entry decision gives rise to an equilibrium condition that relates the benefit of market work (the wage markup) to its opportunity cost (the value of nonparticipation).\(^4\) This additional equilibrium condition enables us to identify labor supply and wage markup shocks separately, since they lead to opposite responses of employment.

In the model with labor force entry, we consider another type of labor supply shock, motivated by weak labor force participation during the pandemic. This type of shock is added to the value of nonparticipation in the labor market and can thus be classified as labor supply shocks to the extensive margin of labor, while the canonical labor supply shocks to hours worked are those to the intensive margin. We then show that, in contrast with the canonical shocks noted above, the shocks to the value of nonparticipation induce a monetary policy trade-off between the variability of the employment gap (i.e., the gap between actual employment and its natural rate) and the variabilities of wage growth and inflation.

The monetary policy trade-off can be ascribed to the NK Phillips curve (NKPC) for wage growth (or, equivalently, wage inflation). The canonical wage-NKPC relates wage growth to expected future wage growth and the average wage markup. Introducing labor force entry alters the wage-NKPC by introducing past, present, and expected future employment as a driver of wage growth in addition to the average wage markup. Through this additional driver, a decline in employment tends to raise wage growth, thus revealing an employment

\(^3\)Wage bargaining shocks in models with labor market search and matching frictions are the counterpart of wage markup shocks in models with sticky wages.

\(^4\)This paper assumes exogenous exits from the labor force.
channel of monetary policy. This new policy transmission channel arises in the model because a decline in employment reduces the variety of individual labor services available to firms, which lowers average labor productivity and thus raises the aggregate wage. Adverse labor supply shocks to the extensive margin reduce the natural rates of employment and output. To close the employment and output gaps, monetary policy must let actual employment and output decline, which in turn stabilizes the average wage markup and hence wage growth. However, the resulting decline in employment puts upward pressure on wage growth. Due to the employment channel of monetary policy, the labor supply shocks present policymakers with a trade-off: Letting employment decline to close the employment gap after the adverse shocks leads to higher wage growth. Moreover, our wage-NKPC implies that abstracting from labor force entry can cause the erroneous attribution of the additional endogenous drivers of wage growth to an exogenous labor market shock.\(^5\)

The policy trade-off generated by extensive-margin labor supply shocks overturns the conventional wisdom about monetary policy responses to labor supply shocks. We utilize the model to address the question as to how monetary policy should respond to extensive-margin labor supply shocks, motivated by the aforementioned recent US macroeconomic developments. To this end, we derive a welfare-maximizing (Ramsey) policy. Adverse labor supply shocks to the extensive margin reduce the natural rates of employment and output and raise wage growth and inflation in the model. The Ramsey policy prevents employment and output from declining as much as their natural rates, in order to rein in wage growth generated by the decline in employment. The economy then experiences a period of positive employment and output gaps and higher inflation. This suggests that monetary policy should respond less aggressively to extensive-margin labor supply shocks than to intensive-margin ones. This is because the decline in employment puts upward pressure on wage growth through the employment channel of monetary policy, in addition to the downward pressure from the conventional aggregate demand channel, which is the only policy transmission channel in the canonical NK counterpart of the model.\(^6\)

\(^5\)Smets and Wouters (2007) show that wage markup shocks—which are indistinguishable from intensive-margin labor supply shocks—account for large portions of the variances of output growth and inflation in an estimated dynamic stochastic general equilibrium model, which abstracts from labor force entry.

\(^6\)In the model with labor force entry, a tightening of monetary policy leads to higher wage growth through the negative effect of lower employment on average labor productivity. This feature is somewhat similar to
Impulse responses to extensive-margin labor supply shocks under the Ramsey policy are similar to those obtained under the Taylor (1993) rule, indicating that the policy rule provides a reasonable policy prescription if the economy is buffeted by such shocks. The Taylor rule generates a modest welfare loss for the labor supply shocks, compared to the Ramsey policy (a permanent consumption loss of 0.1 percent). A policy rule that responds to the employment gap instead of the output gap has a more sizeable welfare loss (0.5 percent). This indicates that the presence of employment in the wage-NKPC plays an important role in the optimal policy response to the labor supply shocks. Leaning against the employment gap obliges employment to change, which exacerbates the variability of wage growth.

Our paper contributes to two strands of the literature, on the role of labor market shocks in NK models and on the monetary policy implications of labor force fluctuations. Regarding the former literature, Galí et al. (2012) tackle the identification problem of (intensive-margin) labor supply and wage markup shocks by reformulating the labor market in the dynamic stochastic general equilibrium (DSGE) model of Smets and Wouters (2007). Their reformulation views a decline in employment due to workers’ market power as a rise in unemployment, thus connecting the average wage markup to the unemployment rate. Using the unemployment rate data, they identify the two shocks separately. Foroni et al. (2018) employ an NK model with labor market search and matching frictions and show that labor supply and wage bargaining shocks bring about opposite responses of unemployment, thus providing an alternative way of tackling the identification issue. Yet both papers study only the business cycle implications of labor market shocks. As for monetary policy implications, Galí (2011) proposes an NK model with sticky prices and wages, unemployment, and labor force participation, and uses the model to analyze an optimal policy conditional on technology shocks, but not on labor market shocks. Erceg and Levin (2014) construct an NK model with sticky prices and fluctuations in unemployment and the labor force. They show that the gradual adjustment of the labor force to changes in unemployment can justify a policy of letting the unemployment rate decline temporarily below its natural rate. Campolmi and Gnocchi (2016) build a DSGE model with labor market search frictions and labor force entry, and analyze the implications of technology shocks and labor supply shocks—market technology.
shocks and home technology shocks in their terms—for monetary policy. Since their model assumes flexible wages, an optimal policy calls for price stability.\(^7\)

A key distinction between our paper and the related literature is that our model abstracts from unemployment. Conceptually, the decision to participate in the labor force is influenced by the institutional structure of the labor market, including search frictions and wage rigidities. Previous studies incorporating unemployment into NK models then find that different specifications for the process of wage determination have distinct implications not only for business cycle fluctuations but also for monetary policy.\(^8\) Leaving unspecified the labor market frictions that give rise to unemployment allows us to investigate the implications of labor force entry in an otherwise canonical NK model.\(^9\)

Moreover, our paper is complementary to the literature on firm entry and exit on the product side.\(^10\) As pointed out by Bilbiie et al. (2008), since NK models involve monopolistically competitive product markets, which result in positive profits, assuming no firm entry or exit is theoretically unappealing. Our model applies their argument to the labor side in NK models with monopolistically competitive labor markets, where workers have market power.

The remainder of the paper proceeds as follows. Section 2 presents an NK model with sticky prices and wages, labor force entry, and labor market shocks. In the model, Section 3 analyzes impulse responses to the shocks under the Taylor rule, and shows a monetary policy trade-off induced by labor supply shocks to the extensive margin of labor. Section 4 examines optimal policy and conducts the welfare comparison of alternative policy strategies. Section 5 concludes.

\(^7\)Justiniano et al. (2013) use two measures of hourly labor income in estimating a DSGE model while allowing for their measurement errors, and evaluate a trade-off between the variabilities of inflation and the output gap in the US. Debortoli et al. (2019) assess the implications of the trade-off for central banks’ loss functions.

\(^8\)For examples, see Thomas (2008), Faia (2009), Blanchard and Galí (2010), and Sunakawa (2015). Galí (2011) argues that the main role of incorporating labor market frictions in models of monetary policy is to “make room” for wage rigidities.

\(^9\)Our approach follows the spirit of Benhabib et al. (1991), who introduce nonparticipation in the labor market in an otherwise canonical real business cycle model to facilitate comparison with its canonical counterpart.

\(^10\)See, e.g., Bilbiie et al. (2008), Lewis and Poilly (2012), Cavallari (2013), Bilbiie et al. (2014), and Bilbiie (2021).


2 New Keynesian Model with Labor Force Entry

In the model economy there are a representative household, firms, a representative composite-good producer and labor union, and a monetary authority. This section describes each economic agent’s behavior.

2.1 Composite-good producer and labor union

The representative composite-good producer combines the outputs of a continuum of firms $f \in [0, 1]$ using the CES aggregator $Y_t = \left[ \int_0^1 (Y_t(f))^{(\theta_p-1)/\theta_p} df \right]^{\theta_p/(\theta_p-1)}$, where $Y_t$ is the output of the composite good, $Y_t(f)$ is firm $f$’s output of an individual differentiated good, and $\theta_p > 1$ is the elasticity of substitution between individual goods. Given the composite good’s price $P_t$ and the prices of individual goods $\{P_t(f)\}$, the composite-good producer maximizes profit $P_t Y_t - \int_0^1 P_t(f) Y_t(f) df$ subject to the CES aggregator. The first-order condition for profit maximization yields the demand curve for each individual good

$$Y_t(f) = Y_t \left( \frac{P_t(f)}{P_t} \right)^{-\theta_p},$$

(1)

and thus the CES aggregator leads to the composite good’s price equation

$$P_t = \left[ \int_0^1 (P_t(f))^{1-\theta_p} df \right]^{1/\theta_p}.$$  

(2)

The representative labor union likewise combines the individual differentiated labor services of a continuum of workers $i \in [0, n_t]$ using the CES aggregator

$$l_t = \left[ \int_0^{n_t} (h_t(i))^{\theta_{w,t}-1} \frac{1}{\theta_{w,t}} \frac{\theta_{w,t}}{\theta_{w,t}-1} di \right]^{\theta_{w,t}}.$$  

(3)

where $n_t$ is the time-varying labor force, $l_t$ is aggregate labor, $h_t(i)$ is worker $i$’s hours worked for the differentiated labor service, and $\theta_{w,t} > 1$ is the time-varying elasticity of substitution between individual labor services.\(^{11}\) As in Erceg et al. (2000), the labor aggregator (3)

\(^{11}\)Because there is no unemployment in the model, the terms labor force and employment are used interchangeably.
combines each worker’s hours worked in the same proportion as firms would choose. The resulting aggregate labor exceeds total hours, reflecting that the variety in individual labor services makes the labor force more productive. Given the aggregate wage \( P_tW_t \) and individual wages \( \{P_tW_t(i)\} \), the labor union maximizes profit \( P_tW_t l_t - \int_0^{n_t} P_tW_t(i) h_t(i) di \) subject to the labor aggregator. The first-order condition for profit maximization yields the demand curve for each individual labor service

\[
h_t(i) = l_t \left( \frac{P_tW_t(i)}{P_tW_t} \right)^{-\theta_{w,t}},
\]

and thus the labor aggregator leads to the aggregate wage equation

\[
P_tW_t = \left[ \int_0^{n_t} (P_tW_t(i))^{1-\theta_{w,t}} di \right]^{-\frac{1}{\theta_{w,t}}}. \tag{5}
\]

A larger labor force, through the productivity-enhancing effect of a greater variety in individual labor services, reduces the aggregate wage, which will give rise to an employment channel of monetary policy as discussed later.

### 2.2 Households

The representative household consists of a large number of members who make joint decisions on consumption, savings, and labor force participation. Some members are not active in the labor market and receive a flow utility of nonparticipation \( F_t \), while the others supply differentiated labor services and choose their wages in a staggered fashion.

At the beginning of each period, a fraction \( 1 - \rho \) of workers exits the labor force, so \( \rho \in (0, 1] \) denotes workers’ survival probability. In each period a measure \( n_{e,t} \) of household members joins the labor force and forgoes the benefit of nonparticipation. Thus the law of motion of employment is

\[
n_t = \rho n_{t-1} + n_{e,t}. \tag{6}
\]

The representative household’s preferences over consumption, hours worked, and labor force participation are represented as the utility function
\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \int_0^{n_t} \frac{(h_t(i))^{1+\chi}}{1+\chi} \chi \exp z_{h,t} - n_{c,t} F \exp z_{n,t} \right],
\]

where \( E_t \) is the expectation operator conditional on information available in period \( t \), \( C_t \) is consumption of the composite good, \( \beta \in (0, 1) \) is the subjective discount factor, \( \chi \geq 0 \) is the inverse of the elasticity of labor supply, \( F_t \equiv F \exp z_{n,t} \) is the value of nonparticipation of a household member, and \( z_{h,t} \) and \( z_{n,t} \) are labor supply shocks to the intensive and extensive margins of labor, respectively. The household’s budget constraint is

\[
P_tC_t + B_t = \int_0^{n_t} P_t W_t(i) h_t(i) \chi + r_{t-1} B_{t-1} + D_t,
\]

where \( B_t \) is the stock of one-period bonds, \( r_t \) is the interest rate on the bonds, which is also the monetary policy rate, and \( D_t \) consists of lump-sum taxes and transfers as well as firms’ profits received. The household maximizes the utility function (7) subject to the budget constraint (8), the law of motion of employment (6), and the labor demand curves (4). Substituting the labor demand curves for individual labor services in the utility function introduces a relative wage distortion

\[
\Delta_{w,t} \equiv \frac{1}{n_t} \int_0^{n_t} \left( \frac{P_t W_t(i)}{P_t W_t} \right)^{-\theta_{w,t}(1+\chi)} d_i.
\]

The first-order conditions for consumption, bond holdings, and labor force participation are, respectively,

\[
\Lambda_t = \frac{1}{C_t},
\]

\[
1 = \beta E_t \left( \frac{\Lambda_{t+1}}{\Lambda_t} \frac{r_t}{\pi_{t+1}} \right),
\]

\[
n_t F \exp z_{n,t} = \Lambda_t W_t l_t - \frac{n_t^{1+\chi} \n_t \Delta_{w,t} \exp z_{h,t}}{1+\chi} + (\rho n_t) \beta E_t (F \exp z_{n,t+1}),
\]

where \( \Lambda_t \) is the marginal utility of consumption and \( \pi_t \equiv P_t/P_{t-1} \) is the inflation rate of the composite good’s price. The household increases the labor force until the opportunity cost of market work \( n_t F_t \equiv n_t F \exp z_{n,t} \) is equal to its benefit, which consists of the benefit of labor earnings net off its disutility and the benefit of reducing the expected future opportunity
cost as a portion $\rho n_t$ of workers will stay in the labor force.

Given the labor demand curves (4), individual wages $\{P_t W_t(i)\}$ are chosen on a staggered basis as in Erceg et al. (2000). In each period, a fraction $\alpha_w \in (0, 1)$ of wages is indexed to the steady-state wage growth rate $\pi_{w,t}$, while the other fraction $1 - \alpha_w$ is chosen so as to maximize the relevant utility function

$$E_t \sum_{j=0}^{\infty} (\alpha_w \beta \rho)^j \left[ -\frac{(h_{t+j}|i(i))^1+\chi}{1+\chi} \exp z_{h,t+j} + \lambda_{t+j} \frac{P_t W_t(i) \pi_{w,t}^j}{P_t+j} h_{t+j|i(i)} \right]$$

subject to the labor demand curve

$$h_{t+j|i(i)} = l_{t+j} \left( \frac{P_t W_t(i) \pi_{w,t}^j}{P_t+j W_t+j} \right)^{-\theta_{w,t+j}}.$$

The first-order condition for utility maximization with respect to the wage is given by

$$0 = E_t \sum_{j=0}^{\infty} (\alpha_w \beta \rho)^j \frac{\lambda_{t+j}}{\lambda_t} l_{t+j} \left( (w_t^*)^{-\theta_{w,t+j}} \prod_{k=1}^{j} \left( \frac{\pi_{w,t+k}}{\pi_w} \right)^{\theta_{w,t+j}} \left\{ w_t^* \prod_{k=1}^{j} \left( \frac{\pi_{w,t+k}}{\pi_w} \right)^{-1} \right\} \right)$$

where $w_t^* \equiv W_t^*/W_t$ is the optimized relative wage and $\pi_{w,t}$ is the wage growth rate, i.e.,

$$\pi_{w,t} \equiv \frac{P_t W_t}{P_{t-1} W_{t-1}} = \pi_t \frac{W_t}{W_{t-1}}.$$  \hspace{1cm} (14)

2.3 Firms

Each firm $f$ produces one kind of differentiated good $Y_t(f)$ using the technology

$$Y_t(f) = A_t l_t(f),$$  \hspace{1cm} (15)

where $A_t$ represents the level of technology and $l_t(f)$ is firm $f$’s labor input. The technology level is assumed to be identical across firms and to follow the nonstationary stochastic process

$$\log A_t = \log a + \log A_{t-1} + \varepsilon_{a,t},$$  \hspace{1cm} (16)
where \( a \) is the steady-state rate of technological change \( A_t/A_{t-1} \) and \( \varepsilon_{a,t} \) is a technology shock.

Production cost minimization then implies that each firm faces the same real marginal cost

\[
m_{c_t} = \frac{W_t}{A_t}.
\]  

(17)

Taking into account the goods demand curves (1) and the real marginal cost (17), firms set their product prices on a staggered basis as in Calvo (1983). In each period, a fraction \( \alpha_p \in (0, 1) \) of firms indexes prices to the steady-state inflation rate \( \pi \), while the remaining fraction \( 1 - \alpha_p \) sets the price \( P_t(f) \) so as to maximize relevant profit

\[
E_t \sum_{j=0}^{\infty} \alpha_p^j Q_{t,t+j} \left( P_{t}(f) \pi^j - P_{t+j} m_{c_{t+j}} \right) Y_{t+j} \left( \frac{P_{t}(f) \pi^j}{P_{t+j}} \right)^{-\theta_p},
\]

where \( Q_{t,t+j} \) is the nominal stochastic discount factor between period \( t \) and period \( t + j \). Using the equilibrium condition \( Q_{t,t+j} = \beta^j (\Lambda_{t+j}/\Lambda_t)/(P_t/P_{t+j}) \), the first-order condition for profit maximization is given by

\[
0 = E_t \sum_{j=0}^{\infty} (\alpha_p^j \beta^j \Lambda_{t+j} Y_{t+j} \left( p^*_t \right)^{-\theta_p} \prod_{k=1}^{j} \left( \frac{\pi_{t+k}}{\pi} \right)^{\theta_p} \left[ p^*_t \prod_{k=1}^{j} \left( \frac{\pi_{t+k}}{\pi} \right) - \frac{\theta_p}{\theta_p - 1} m_{c_{t+j}} \right],
\]

(18)

where \( p^*_t \equiv P^*_t/P_t \) is the optimized relative price.

2.4 Monetary authority

The monetary authority conducts policy according to a Taylor-type rule. This rule adjusts the policy rate in response to the inflation rate and the output gap or employment gap:

\[
\log r_t = \log r + \phi_p (\log \pi_t - \log \pi) + \phi_y (\log Y_t - \log Y_t^n) + \phi_n (\log n_t - \log n_t^n),
\]

(19)

where \( Y_t^n \) and \( n_t^n \) are the natural rates of output and employment, which would prevail in the absence of nominal price and wage rigidities and time variation in the elasticity of substitution between individual labor services; and \( \phi_p, \phi_y, \) and \( \phi_n \) are the policy responses to the inflation rate, the output gap, and the employment gap.
2.5 Equilibrium

The composite good’s market clearing condition requires that its output be equal to the household’s consumption:

\[ Y_t = C_t. \]  \hfill (20)

The output is obtained by combining individual firms’ production functions (15) and the goods demand curves (1), which yields the aggregate production function

\[ Y_t \Delta_{p,t} = A_t l_t, \]  \hfill (21)

where the aggregate labor satisfies

\[ l_t = \int_0^1 l_t(f) df \]

and

\[ \Delta_{p,t} \equiv \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{-\theta_p} \]

is a relative price distortion that reflects inefficiency in production of the composite good due to dispersion in the relative prices of individual goods.

Staggered price-setting implies that the composite good’s price equation (23) and the relative price distortion equation (22) are reduced to, respectively,

\[ 1 = \alpha_p \left( \frac{\pi_t}{\pi} \right)^{\theta_p-1} + (1 - \alpha_p)(p_t^*)^{1-\theta_p}, \]  \hfill (23)

\[ \Delta_{p,t} = \alpha_p \left( \frac{\pi_t}{\pi} \right)^{\theta_p} \Delta_{p,t-1} + (1 - \alpha_p)(p_t^*)^{-\theta_p}. \]  \hfill (24)

For simplicity, the distribution of entrants’ wages is assumed to be the same as that of incumbent workers’ wages. Under this assumption, staggered wage-setting implies that the aggregate wage equation (5) and the relative wage distortion equation (9) are reduced to, respectively,

\[ 1 = \alpha_w \frac{n_t}{n_{t-1}} \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_{w,t}-1} + n_t \left( 1 - \alpha_w \right)(w_t^*)^{1-\theta_{w,t}}, \]  \hfill (25)

\[ \Delta_{w,t} = \alpha_w \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_{w,t}(1+\chi)} \Delta_{w,t-1} + (1 - \alpha_w)(w_t^*)^{-\theta_{w,t}(1+\chi)}. \]  \hfill (26)

The equilibrium conditions of the model consist of (6), (10)–(14), (16)–(21), and (23)–
along with the wage markup shock \( z_{w,t} \equiv -\log (\theta_{w,t}/\theta_w) / (\theta_w - 1) \) and the autoregressive processes for the shocks:

\[
z_{m,t} = \rho_m z_{m,t-1} + \varepsilon_{m,t},
\]

(27)

for \( m = w, h, n \). The disturbances \( \varepsilon_{m,t} \), \( m = a, w, h, n \) are drawn independently from the respective normal distributions with mean zero and variances \( \sigma_m^2 \). The natural rates of output and employment are obtained in the absence of nominal price and wage rigidities and the wage markup shock, i.e., \( \alpha_p = \alpha_w = 0 \) and \( z_{w,t} = 0 \).

Finally, it will be useful to define the average wage markup of the real wage over the marginal rate of substitution between consumption and leisure,

\[
\mu_{w,t} \equiv \int_0^{n_t} \omega_t(i) \mu_{w,t}(i) \, di = \frac{\Lambda_t W_t}{l_t^x \exp(z_{h,t}) n_t \Delta_{w,t}},
\]

where the weight \( \omega_t(i) \) is individual \( i \)'s share of the household’s disutility from hours worked given by \( \omega_t(i) = (h_t(i))^{1+\chi} / (l_t^{1+\chi} n_t \Delta_{w,t}) \) and the wage markup of individual \( i \) is given by \( \mu_{w,t}(i) = W_t(i) / [(h_t(i))^{\chi} \exp(z_{h,t}) / \Lambda_t] \).

3 Labor Market Shocks and Monetary Policy Trade-offs

In the model presented above, this section analyzes impulse responses to labor market shocks and shows that labor supply shocks to the extensive margin of labor induce a monetary policy trade-off.

3.1 Impulse responses to labor market shocks

We begin by deriving log-linearized equilibrium conditions of the model. Removing the nonstationary component of variables induced by the technology shock by defining \( y_t \equiv Y_t / A_t \), \( w_t \equiv W_t / A_t \), and \( \lambda_t \equiv \Lambda_t A_t \), log-linearizing the equilibrium conditions in terms of the detrended variables around the steady state, and assuming a unit measure of steady-state employment (i.e., \( n = 1 \)) lead to the following log-linearized equilibrium conditions:\(^{13}\)

\(^{12}\)A simple arithmetic average produces the same average wage markup to a first-order approximation.

\(^{13}\)The assumption of \( n = 1 \) facilitates comparison with the canonical NK counterpart of the model in the presence of wage markup shocks \( z_{w,t} \). For other values of \( n > 0 \), wage markup shocks are a driver of the
\[
\hat{y}_t = E_t \hat{y}_{t+1} - (\hat{r}_t - E_t \hat{n}_{t+1}) + E_t \epsilon_{a,t+1}, \tag{29}
\]
\[
\hat{r}_t = \phi_p \hat{n}_t + \phi_y (\hat{y}_t - \hat{y}_n) + \phi_n (\hat{n}_t - \hat{n}_n), \tag{30}
\]
\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_p \hat{w}_t, \tag{31}
\]
\[
\hat{\pi}_{w,t} = \beta E_t \hat{\pi}_{w,t+1} - \kappa_w [\hat{w}_t - (1 + \chi) \hat{y}_t - z_{h,t} - z_{w,t}]
\]
\[
\quad + \frac{1}{\theta_{w} - 1} \left[ \beta \rho (E_t \hat{n}_{t+1} - \alpha_w \hat{n}_t) - \frac{1}{\alpha_w} (\hat{n}_t - \alpha_w \hat{n}_{t-1}) \right], \tag{33}
\]
\[
\hat{w}_t - (1 + \chi) \hat{y}_t - z_{h,t} = \frac{1 + \theta_w \chi}{\theta_{w} - 1} \left( \frac{z_{n,t} - \beta \rho E_t \hat{z}_{n,t+1}}{1 - \beta \rho} - \hat{w}_t \right). \tag{34}
\]

In these conditions, hatted variables denote log-deviations from steady-state values, the relationship \(\hat{l}_t = \hat{y}_t = \hat{c}_t = -\hat{\lambda}_t\) is used, and the coefficients \(\kappa_p\) and \(\kappa_w\) are given by \(\kappa_p \equiv (1 - \alpha_p \beta)(1 - \alpha_p) / \alpha_p\) and \(\kappa_w \equiv (1 - \alpha_w \beta \rho)(1 - \alpha_w) / [(1 + \theta_w \chi) \alpha_w]\). Eqs. (29)–(32) are, respectively, the consumption Euler equation, the Taylor-type rule, the price-NKPC, and the definition of wage growth, which are all the same as in the case of a constant labor force, that is, the canonical NK counterpart of the model. Eq. (33) is the wage-NKPC, which relates wage growth to expected future wage growth and a number of driving variables. Eq. (34) is the log-linearization of the labor force entry condition (12).

The case of a constant labor force or, equivalently, the canonical NK counterpart model can be retrieved by setting \(\rho = 1\). In this case, (33) and (34) are reduced to only the canonical wage-NKPC
\[
\hat{\pi}_{w,t} = \beta E_t \hat{\pi}_{w,t+1} - \kappa_w [\hat{w}_t - (1 + \chi) \hat{y}_t - z_{h,t} - z_{w,t}]. \tag{35}
\]

This wage-NKPC demonstrates the observational equivalence of intensive-margin labor supply shocks \(z_{h,t}\) and wage markup shocks \(z_{w,t}\), as emphasized by Chari et al. (2009).

However, in the model augmented with labor force entry, these two shocks can be identified separately. To see this, we analyze impulse responses to the shocks. This requires parameters of the model to be specified.

relative wage distortion \(\Delta_{w,t}\), which then has a first-order effect on the labor force entry condition. To target the value of \(n = 1\), this section assumes that the steady-state benefit of nonparticipation in the labor market satisfies \(F = [1/(1 - \beta \rho)](\theta_p - 1)/\theta_p][(1 + \theta_w \chi) / ((1 + \chi) \theta_w)].\)
Table 1 presents the quarterly calibration of parameters in the model. A key parameter is workers’ survival probability. Exits from the labor force have averaged about 2.5 percent of the population per month over the period 1991–2016 (for the evidence, see Figure 3 in Frazis, 2017), which implies that the quarterly exit rate is 7.5 percent of the population. Assuming that the labor force participation rate is 0.62, the quarterly exit rate is calculated as 0.075/0.62 = 12.1 percent of the labor force. In the model’s steady state, there are \( n_e \) exits in each quarter and the labor force is \( n \). Thus, the quarterly exit rate is \( n_e/n = 1 - \rho = 0.121 \). Consequently, the survival probability is set at \( \rho = 1 - 0.121 = 0.879 \) to target the ratio of \( n_e/n = 0.121 \).

Table 1: Calibration of parameters in the quarterly model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Workers’ survival probability</td>
<td>0.879</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Subjective discount factor</td>
<td>0.995</td>
</tr>
<tr>
<td>( a )</td>
<td>Gross steady-state rate of technological change</td>
<td>1.005</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Inverse of elasticity of labor supply</td>
<td>2</td>
</tr>
<tr>
<td>( \theta_p )</td>
<td>Elasticity of substitution between goods</td>
<td>4</td>
</tr>
<tr>
<td>( \theta_w )</td>
<td>Elasticity of substitution between labor services</td>
<td>4</td>
</tr>
<tr>
<td>( \alpha_p )</td>
<td>Degree of price rigidity</td>
<td>0.67</td>
</tr>
<tr>
<td>( \alpha_w )</td>
<td>Degree of wage rigidity</td>
<td>0.67</td>
</tr>
<tr>
<td>( \phi_p )</td>
<td>Policy response to inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>( \phi_y )</td>
<td>Policy response to output gap</td>
<td>0.125</td>
</tr>
<tr>
<td>( \phi_n )</td>
<td>Policy response to employment gap</td>
<td>0</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>Persistence of wage markup shock</td>
<td>0.9</td>
</tr>
<tr>
<td>( \rho_h )</td>
<td>Persistence of intensive-margin labor supply shock</td>
<td>0.9</td>
</tr>
<tr>
<td>( \rho_n )</td>
<td>Persistence of extensive-margin labor supply shock</td>
<td>0.9</td>
</tr>
<tr>
<td>( \sigma_w )</td>
<td>Standard deviation of wage markup shock</td>
<td>0.08</td>
</tr>
<tr>
<td>( \sigma_h )</td>
<td>Standard deviation of intensive-margin labor supply shock</td>
<td>0.08</td>
</tr>
<tr>
<td>( \sigma_n )</td>
<td>Standard deviation of extensive-margin labor supply shock</td>
<td>0.006</td>
</tr>
<tr>
<td>( \sigma_a )</td>
<td>Standard deviation of technology shock</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Values of the other structural parameters in the model are common to canonical NK models with sticky prices and wages, as presented in the upper panel of Table 1. The subjective discount factor is set at \( \beta = 0.995 \), the gross steady-state rate of technological change at \( a = 1.005 \) or 2 percent per year, and the inverse of the elasticity of labor supply at \( \chi = 2 \). The elasticities of substitution between individual goods and between individual
labor services are both chosen at $\theta_p = \theta_w = 4$. Nakamura and Steinsson (2010) observe that such a value of $\theta_p$ matches the estimates from the literature on industrial organization and international trade. The degrees of price and wage rigidities are equal to $\alpha_p = \alpha_w = 0.67$. The monetary policy responses to inflation, the output gap, and the employment gap are $\phi_p = 1.5$, $\phi_y = 0.125$, and $\phi_n = 0$, respectively.

The lower panel of Table 1 presents the calibration for the three labor market shocks. For wage markup shocks $z_{w,t}$ and intensive-margin labor supply shocks $z_{h,t}$, we set their persistence at $\rho_w = \rho_h = 0.9$ and the standard deviations of their innovations at $\sigma_w = \sigma_h = 0.08$, based on the estimates of Smets and Wouters (2007) for wage markup or labor supply shocks in their canonical wage-NKPC. Extensive-margin labor supply shocks are calibrated by adopting the same persistence of $\rho_n = 0.9$ and choosing the standard deviation of its innovation at $\sigma_n = 0.006$ consistent with the estimate of Chang et al. (2007) for labor supply shocks in their model without labor adjustment costs.

Figure 1 displays the impulse responses to one standard deviation adverse labor market shocks $z_{w,t}$ (first column), $z_{h,t}$ (second column), and $z_{n,t}$ (third column) in the model with labor force entry (solid blue lines) and its counterpart with a constant labor force or, equivalently, its canonical NK counterpart (dashed red lines). In the counterpart model, the wage markup shock $z_{w,t}$ and the intensive-margin labor supply shock $z_{h,t}$ produce practically indistinguishable impulse responses of the interest rate, output, inflation, the real wage, and wage growth, as noted above.\textsuperscript{16}

Introducing labor force entry enables us to identify the two shocks separately, because they lead to opposite responses of employment, as shown in the last row of Figure 1. The wage markup shock, by widening the average wage markup, stimulates an increase in employment. The intensive-margin labor supply shock reduces the average markup, which leads to a decline in employment.

\textsuperscript{15}The calibration for technology shocks presented in the last line of Table 1 is explained later.

\textsuperscript{16}The wage markup and intensive-margin labor supply shocks deliver identical impulse responses when the Taylor rule has no policy response to the output gap, i.e., $\phi_y = 0$. As for the average wage markup $\bar{\mu}_{w,t} = \bar{w}_t - (1 + \chi)\bar{y}_t - z_{h,t}$, the second to last row of Figure 1 shows that the adverse wage markup shock widens the markup by raising the real wage and reducing output, whereas the adverse labor supply shock to the intensive margin directly reduces the markup.
Figure 1: Impulse responses to three adverse labor market shocks.

**Notes:** The solid blue lines are impulse responses in the model with labor force entry, while the dashed red lines are those in its counterpart with a constant labor force. The panels in each column show the responses of the interest rate $r$, output $y$, the inflation rate $\pi$, the real wage $w$, the wage growth rate $\pi_w$, the average wage markup $\mu_w$, and employment $n$, respectively, to the wage markup shock $z_w$, the intensive-margin labor supply shock $z_h$, and the extensive-margin labor supply shock $z_n$. All responses are expressed as percentages; the responses of the interest, inflation, and wage growth rates are displayed at annualized rates. The values of model parameters used here are reported in Table 1.
Labor force entry also alters the impulse responses to the wage markup shock, as seen in the first column of Figure 1. The adverse shock raises the average wage markup and prompts an increase in employment as noted above, thus reducing wage growth in the wage-NKPC (40). Therefore, the employment increase offsets the effect of the shock on the real wage and hence on output and inflation. That is, labor force entry offsets the effects of sticky wages on the responses of output, the real wage, and inflation to the wage markup shock. This indicates that wage markup shocks may be of minor importance as a source of business cycle fluctuations, aside from fluctuations in employment.

The third column of Figure 1 presents the impulse responses to the extensive-margin labor supply shock $z_{n,t}$. The adverse shock reduces employment. Provided that the real wage is not too volatile, the shock increases the right-hand side of the entry condition (34). Consequently, the left-hand side of the condition, which consists of the component $\hat{w}_t - (1 + \chi)\hat{y}_t - z_{h,t}$ of the average wage markup, also rises. This raises the average wage markup on impact, in contrast to the response of the markup to the intensive-margin labor supply shock displayed in the second column. Yet the decline in employment induced by the shock subsequently reduces the average wage markup through its other component, $(1 + \theta_w \chi)/(\theta_w - 1)\hat{n}_t$. Despite the ambiguous response of the average wage markup, wage growth increases. That is because the wage markup is not the only driver of wage growth; the decline in employment directly boosts wage growth in the wage-NKPC. The resulting higher real wage raises inflation and reduces output.

### 3.2 Monetary policy trade-offs

To assess the implications of the three labor market shocks for monetary policy, it is useful to consider the gaps between the actual and natural rates of real variables.\(^{17}\) In the absence of nominal price and wage rigidities and wage markup shocks, the wage-setting condition implies that adverse labor supply shocks to the extensive and intensive margins of labor, $z_{n,t}$ and $z_{h,t}$, reduce the natural rate of output:\(^{18}\)

\(^{17}\)Wage markup shocks, by assumption, have no effect on the natural rates. The shocks drive a wedge between the efficient output level and the one that would prevail under flexible prices and wages. Therefore, the natural rate of output that excludes the effects of wage markup shocks tracks the efficient output level.

\(^{18}\)The autoregressive process for the extensive-margin labor supply shock $z_{n,t}$ ensures that the term in parentheses is increasing in the shock.
\[
\dot{y}_t^n = -\frac{1}{1+\chi} \left[ \frac{1+\theta_w \chi}{\theta_w - 1} \left( \frac{z_{n,t} - \beta \rho E_t z_{n,t+1}}{1 - \beta \rho} \right) + z_{h,t} \right].
\] (36)

The labor force entry condition implies that adverse labor supply shocks to the extensive margin also reduce the natural rate of employment:

\[
\dot{n}_t^n = -\frac{z_{n,t} - \beta \rho E_t z_{n,t+1}}{1 - \beta \rho}.
\] (37)

The log-linearization of the average wage markup (28) is

\[
\dot{\mu}_{w,t} = \frac{1+\theta_w \chi}{\theta_w - 1} \dot{n}_t + \dot{\nu}_t - (1+\chi)\dot{y}_t - z_{h,t}.
\] (38)

Intensive-margin labor supply shocks shift the average wage markup directly. Extensive-margin ones affect the markup indirectly through the labor force entry condition (34). Using (36)–(38), the entry condition can be rewritten as

\[
\dot{\mu}_{w,t} = \frac{1+\theta_w \chi}{\theta_w - 1} (\dot{n}_t - \dot{n}_t^n) - \frac{1+\theta_w \chi}{\theta_w} (\dot{y}_t - \dot{y}_t^n).
\] (39)

Adverse labor supply shocks to the extensive margin widen both the employment and the output gaps by reducing the natural rates of employment and output. In the entry condition (39), the increase in the employment gap tends to raise the average wage markup, whereas that in the output gap tends to reduce it, resulting in an ambiguous response of the average wage markup to the shocks. The average wage markup is stabilized if both the employment and output gaps are stabilized.

The wage-NKPC (33) can also be rewritten in terms of the average wage markup:

\[
\dot{\pi}_{w,t} = \beta \rho E_t \dot{\pi}_{w,t+1} + \kappa_w (\dot{\mu}_{w,t} - z_{w,t}) - \frac{1}{\theta_w - 1} [\dot{n}_t - \dot{n}_{t-1} - \beta \rho (E_t \dot{n}_{t+1} - \dot{n}_t)].
\] (40)

The wage-NKPC contains two drivers of wage growth, the wedge between the average wage markup and the wage markup shock, which remains even in the absence of labor force entry, and changes in employment adjusted for its expected future change. Consider again an adverse labor supply shock to the extensive margin that lowers the natural rates of employment and output. To close the employment and output gaps, monetary policy must let actual employment and output decline, which stabilizes the average wage markup and hence wage growth. However, the resulting decline in employment—the driver of wage
growth that reflects labor force entry—puts upward pressure on wage growth, which may be dampened by a further expected future decline or exacerbated by an expected future increase in employment, depending on the persistence of the shock. The negative effect of changes in employment on wage growth reflects that a smaller labor force implies less variety of individual labor services, which reduces average labor productivity and thereby raises the aggregate wage in the wage equation (25).19

The wage-NKPC (40) in the model with labor force entry implies that a decline in employment tends to raise wage growth, which constitutes an employment channel of monetary policy in addition to the conventional aggregate demand channel through which lower employment and output reduce wage growth. Due to the employment channel, extensive-margin labor supply shocks present policymakers with a trade-off between stabilization of the variabilities of wage growth and the employment gap. A decline in employment required to close the employment gap raises wage growth; to rein in wage growth, employment cannot decline as much as is required to close the employment gap.

As for intensive-margin labor supply shocks $z_{h,t}$ and wage markup shocks $z_{w,t}$, they do not produce the same policy trade-off as extensive-margin labor supply shocks $z_{n,t}$. Adverse labor supply shocks to the intensive margin reduce the natural rate of output, but do not affect the natural rate of employment. The lower natural rate of output in turn reduces the average wage markup, thus raising wage growth in (40). In response to the adverse shocks, monetary policy can reduce hours worked to lower output without changing employment, thereby closing both the employment and the output gaps and thus stabilizing the average wage markup, wage growth, and inflation. Thus, as in the canonical NK model, monetary policy is able to offset the effects of intensive-margin labor supply shocks in the model with labor force entry and exit.

Adverse wage markup shocks do not affect the natural rates, but boost wage growth in (40). This raises the real wage and thus causes inflation to rise and output to decline. Because the natural rate of output is unaffected by the shocks, the monetary authority faces

19 Defining total hours worked as $h_t = \int_0^{n_t} h_t(i) \, di$ and using the demand curves for individual labor services (4) to substitute for individual hours worked, we obtain the expression $\hat{h}_t = \hat{l}_t - (\theta_w - 1)^{-1} \hat{n}_t$. Because $\hat{l}_t = \hat{y}_t$, it follows that average labor productivity $\hat{y}_t - \hat{h}_t = (\theta_w - 1)^{-1} \hat{n}_t$ moves proportionally with employment. That lower employment increases the aggregate wage can be seen from the log-linearization of the wage equation (25), which is given by $\alpha_w \hat{w}_{w,t} = (1 - \alpha_w) \hat{w}_{l,t} - (\theta_w - 1)^{-1} (\hat{n}_t - \alpha_w \hat{n}_{t-1})$. 20
a trade-off between the variability of the output gap and the variabilities of inflation and wage growth. Thus, the shocks produce the same policy trade-off as in the case of a constant labor force, that is, the canonical NK counterpart of the model.

In the case of a constant labor force, the average wage markup equation (38) and the wage-NKPC (40) can be reduced respectively to

\[ \hat{\mu}_{w,t} = \hat{w}_t - (1 + \chi)\hat{y}_t - z_{h,t}, \]
\[ \hat{\pi}_{w,t} = \beta \rho E_t \hat{\pi}_{w,t+1} - \kappa_w (\hat{\mu}_{w,t} - z_{w,t}). \]

Comparing this canonical wage-NKPC with the one in (40) shows that labor force entry introduces current and expected future employment growth as a new driver of wage growth, which gives rise to the employment channel of monetary policy. This additional driver of wage growth can respond to each shock and the responses are shaped by the model’s propagation mechanisms, including monetary policy. Therefore, abstracting from labor force entry leads to the erroneous attribution of this endogenous component to wage markup shocks or intensive-margin labor supply shocks in the average wage markup.

3.3 Impulse responses to technology shocks

Before proceeding to the analysis of optimal policy, this section examines the impulse responses to technology shocks \( \varepsilon_{a,t} \) under the Taylor rule. The results of the impulse responses can facilitate comparison with the responses under optimal policy analyzed later. To this end, the standard deviation of innovations to technology shocks is set at \( \sigma_a = 0.007 \), as reported in the last line of Table 1.

Figure 2 displays the impulse responses to a one standard deviation positive technology shock \( \varepsilon_{a,t} \) in the model with labor force entry (solid blue lines) and its counterpart with a constant labor force or, equivalently, its canonical NK counterpart (dashed red lines). In the case of a constant labor force, the technology shock temporarily raises output and wage growth, and lowers the real marginal cost or, equivalently, the real wage and hence inflation. Higher consumption raises the marginal rate of substitution, which reduces the average wage markup along with the decline in the real wage.

The responses of the interest rate track those of inflation in each of the models.
Figure 2: Impulse responses to a positive technology shock.

Notes: The solid blue lines are impulse responses of the model with labor force entry, while the dashed red lines are those of its counterpart model with a constant labor force. The panels show the responses of output $y$, the inflation rate $\pi$, the real wage $w$, the wage growth rate $\pi_w$, the average wage markup $\mu_w$, and employment $n$ to the technology shock $\varepsilon_a$. All responses are expressed as percentages; the responses of the interest, inflation, and wage growth rates are displayed at annualized rates. The values of model parameters used here are reported in Table 1.
Introducing labor force entry alters the responses to the technology shock. The decline in the average wage markup caused by the shock induces a decline in labor force entry, which raises wage growth in the wage-NKPC (40) and boosts the real wage. The real wage rises enough to prevent the real wage from deviating from its pre-shock level. As a result, output and inflation remain unchanged as well. Indeed, the labor force entry condition (34) shows that the lower real wage and higher output after the shock reduce the left-hand side of the condition while increasing its right-hand side. The balance is only restored once the real wage and output return to their pre-shock levels. In this way, labor force entry offsets the effects of sticky wages on the responses of output, the real wage, and inflation to the technology shock. A decline in the labor force following a positive technology shock is consistent with the empirical evidence in Galí (2011) and Tüzemen and Van Zandweghe (2018).\textsuperscript{21}

4 Optimal Monetary Policy

We have established that extensive-margin labor supply shocks induce a monetary policy trade-off. This result overturns the conventional wisdom about monetary policy responses to labor supply shocks. Thus this section investigates how labor force entry affects optimal policy responses to extensive-margin labor supply shocks.

4.1 Impulse responses under optimal monetary policy

We analyze the Ramsey policy to determine how monetary policy should respond to technology shocks and extensive-margin labor supply shocks. The monetary authority is assumed to maximize the household’s utility function (7) subject to the equilibrium conditions (6), (10)–(14), (16)–(18), and (20)–(27). The Lagrangian of the optimization problem of the monetary authority and the resulting equilibrium conditions are presented in Appendix A. After detrending variables and log-linearizing the equilibrium conditions under optimal policy, we obtain the impulse responses displayed in Figure 3.\textsuperscript{22} As pointed out by Erceg et al.\textsuperscript{21} The effects of labor force entry on the responses to the technology shock are similar to those on the responses to the wage markup shock displayed in Figure 1. Indeed, both shocks move output and the real wage in opposite directions, which necessitates a change in employment to satisfy the labor force entry condition (34) by returning the real wage and output to their pre-shock levels.

\textsuperscript{22}By abstracting from wage markup shocks $z_{w,t}$, the log-linearized model is described by the equilibrium conditions (29)–(34) for any value of the steady-state employment rate $n > 0$; so the steady-state value of nonparticipation does not affect equilibrium dynamics.
in general the monetary authority cannot simultaneously stabilize the output gap, inflation, and wage growth after a shock in the presence of sticky prices and wages.

The panels in the left column of Figure 3 plot the responses of the output gap $x_{y,t} \equiv \hat{y}_t - \hat{y}_n^t$, inflation, the real wage, wage growth, the average wage markup, and the employment gap $x_{n,t} \equiv \hat{n}_t - \hat{n}_n^t$ to a positive technology shock $\varepsilon_{a,t}$. The immediate effect of the shock is to raise wage growth (by the definition of wage growth (32)) and erode the real wage, which lowers inflation. To stabilize inflation, the monetary authority would have to accept stronger wage growth. In the case of a constant labor force, the monetary authority strikes a balance between inflation and wage growth that produces a relatively small output gap in the calibrated model (dashed red lines). Labor force entry worsens the policy trade-off (solid blue lines). The decline in the real wage following the shock reduces the average wage markup, which prompts a decline in labor force entry. The resulting decline in employment boosts wage growth in the wage-NKPC (40)—the endogenous effect of employment growth on wage growth—and dampens the declines in the real wage and the average wage markup. The optimal policy response in the calibrated model mitigates the decline in employment to rein in wage growth, but it entails accepting negative gaps of employment and output, as indicated in the bottom and top panels.23

The panels in the right column of Figure 3 illustrate how the monetary authority should respond to an adverse labor supply shock to the extensive margin $z_{n,t}$ in the model with labor force entry. The shock prompts a decline in employment, which boosts wage growth. Higher wage growth leads to a rise in the real wage, which raises inflation and reduces output. Although the shock reduces the natural rates of output (36) and employment (37), the policy response involves a trade-off between the variability of wage growth on the one hand and the variabilities of the employment gap, the output gap, and inflation on the other hand. Reducing output raises the average wage markup (38), thus dampening wage growth. Consequently, the real wage and inflation decline through the conventional aggregate demand channel of monetary policy. But the decline in employment prompted by the shock increases wage growth in the wage-NKPC (40) through the employment channel of monetary policy.

23Recall from Figure 2 that if the monetary authority follows the Taylor rule, which disregards wage growth, both inflation and output (and hence the output gap) are fully stabilized at the expense of a sharp decline in employment and a sharp increase in wage growth.
Figure 3: Impulse responses under optimal monetary policy.

Notes: The solid blue lines are impulse responses of the model with labor force entry, while the dashed red lines are those of its counterpart model with a constant labor force. The panels in each column show the responses of the output gap $\epsilon_a$, the inflation rate $\pi$, the real wage $w$, the wage growth rate $\pi_w$, the average wage markup $\mu_w$, and the employment gap $x_n$, respectively, to the technology shock $\epsilon_a$ and the labor supply shock $z_n$. All responses are expressed as percentages; the responses of the inflation and wage growth rates are displayed at annualized rates. The values of model parameters used here are reported in Table 1.
The optimal policy response in the calibrated model is to accept positive gaps of employment and output and positive inflation to rein in wage growth. After an initial burst of wage growth, the optimal policy response involves a period of wage growth below its long-run level.\textsuperscript{24}

To further illustrate the trade-off between the employment gap and wage growth, Figure 4 contrasts the responses to extensive-margin labor supply shocks under optimal policy with those obtained under alternative policy strategies that fully stabilize the output gap and the employment gap, respectively. The panels in the left column repeat the responses of inflation and wage growth under optimal policy from Figure 3 and display those of output and employment along with their natural rates. The panels in the middle column assume that monetary policy fully stabilizes the output gap \((x_{y,t} = 0)\). Such a policy offsets the effects of the shock on inflation and wage growth, but leads to a larger employment gap than under the optimal policy, as the decline in employment and the ensuing decline in average labor productivity both reduce output. The panels in the right column show that fully stabilizing the employment gap \((x_{n,t} = 0)\) leads to higher inflation and wage growth than under the optimal policy due to the employment channel of monetary policy.

It is worthwhile to note the similarity between the responses to extensive-margin labor supply shocks obtained when monetary policy follows the Taylor rule, shown in Figure 1, and those obtained under optimal policy, shown in Figure 3. Although the output and employment gaps increase under the optimal policy, output and employment still decline.

4.2 Welfare comparisons

We have seen that extensive-margin labor supply shocks entail a monetary policy trade-off and that labor force entry worsens the trade-off induced by technology shocks. This subsection broadens the analysis of optimal policy by considering the welfare loss from different monetary policy strategies. Two sets of strategies are considered. One consists of fully stabilizing inflation \((\hat{\pi}_t = 0)\) or wage growth \((\hat{\pi}_{w,t} = 0)\). The other consists of following simple rules: a strict inflation-targeting rule, the Taylor rule, or a rule that targets the employment gap instead of the output gap by setting \(\phi_y = 0\) and \(\phi_n = 0.125\) in the policy rule (19).

\textsuperscript{24}We confirmed that none of the variables displayed in Figure 3 respond to intensive-margin labor supply shocks \(z_{h,t}\) under the optimal policy, as the policy offsets the effects of such shocks.
Figure 4: Impulse responses under alternative policy strategies.

Notes: The panels in each column show the responses of output $y$, the inflation rate $\pi$, the wage growth rate $\pi_w$, and employment $n$ to the labor supply shock $z_n$, respectively, under each of the optimal policy and policy strategies that fully stabilize the output gap ($x_{y,t} = 0$) and the employment gap ($x_{n,t} = 0$). All responses are expressed as percentages; the responses of the inflation and wage growth rates are displayed at annualized rates. The values of model parameters used here are reported in Table 1.
The representative household’s welfare is represented by the utility function (7). Writing the welfare in recursive form

\[ W_t = \log c_t - \frac{l_e^{1+\chi} n_t \Delta w_t \exp z_{h,t}}{1 + \chi} - n e_t F \exp z_{n,t} + \beta E_t W_{t+1} + t.i.p, \]  

(41)

where t.i.p. stands for terms independent of policy, welfare can be included as an equilibrium condition in the solution of the model. The stochastic mean of welfare, denoted \( E(W) \), is obtained from a second-order solution of the model, following Schmitt-Grohé and Uribe (2004). Let \( E(W_a) \) and \( E(W_b) \) represent the mean of welfare under an arbitrary policy and the optimal policy benchmark, respectively, and denote by \( \delta \) the permanent consumption loss induced by the alternative strategy as a fraction of consumption under the optimal policy. This welfare loss depends on the two welfare levels as follows:

\[ \delta = 1 - \exp \left[ (1 - \beta) (E(W_a) - E(W_b)) \right]. \]

Table 2 presents the results of the welfare comparison. The top panel shows the welfare loss from each monetary policy strategy under the baseline calibration of parameters reported in Table 1.\(^{25}\) In the case of a constant labor force, fully stabilizing inflation at its target generates a large welfare loss (2.93 percent), whereas fully stabilizing wage growth performs as well as the optimal policy. This finding is due to the presence of sticky wages in the model and is in line with the result of Erceg et al. (2000). The simple rules perform well by leaning against inflation without fully stabilizing it.

In the model with labor force entry, two findings stand out. First, a strategy of fully stabilizing inflation performs much better than in the case of a constant labor force, conditional on technology shocks, and performs as well as the strategy of fully stabilizing wage growth, conditional on extensive-margin labor supply shocks. The large welfare loss in the case of a constant labor force is due to wage rigidity, and labor force entry offsets the effects of sticky wages on the responses to technology shocks, as shown in Figure 2. Second, following the rule with a policy response to the employment gap generates a sizeable welfare loss (0.53 percent), which is more than three times as large as the welfare loss associated with following

\(^{25}\)The steady-state benefit of nonparticipation \( F \) has to be specified for the welfare comparison, although the value of \( F \) does not affect the welfare comparison. The parameter value is set to target a unit measure of steady-state employment, i.e., \( n = 1 \).
Table 2: Welfare comparison of monetary policy strategies.

<table>
<thead>
<tr>
<th></th>
<th>Technology shock ($\varepsilon_{a,t}$)</th>
<th>Labor supply shock ($z_{n,t}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant LF</td>
<td>LF entry</td>
</tr>
<tr>
<td>$\alpha_p = 0.67$, $\alpha_w = 0.67$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\pi}_t = 0$</td>
<td>0.0293</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\hat{\pi}_{w,t} = 0$</td>
<td>0.0000</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\phi_p = 1.5$ ($\phi_y = \phi_n = 0$)</td>
<td>0.0001</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\phi_p = 1.5$, $\phi_y = 0.125$ ($\phi_n = 0$)</td>
<td>0.0001</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\phi_p = 1.5$, $\phi_n = 0.125$ ($\phi_y = 0$)</td>
<td>—</td>
<td>0.0053</td>
</tr>
<tr>
<td>$\alpha_p = 0.67$, $\alpha_w = 0.33$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\pi}_t = 0$</td>
<td>0.0009</td>
<td>0.0003</td>
</tr>
<tr>
<td>$\hat{\pi}_{w,t} = 0$</td>
<td>0.0000</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\phi_p = 1.5$ ($\phi_y = \phi_n = 0$)</td>
<td>0.0001</td>
<td>0.0003</td>
</tr>
<tr>
<td>$\phi_p = 1.5$, $\phi_y = 0.125$ ($\phi_n = 0$)</td>
<td>0.0000</td>
<td>0.0003</td>
</tr>
<tr>
<td>$\phi_p = 1.5$, $\phi_n = 0.125$ ($\phi_y = 0$)</td>
<td>—</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\alpha_p = 0.33$, $\alpha_w = 0.67$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\pi}_t = 0$</td>
<td>0.0294</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\hat{\pi}_{w,t} = 0$</td>
<td>0.0000</td>
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</tr>
<tr>
<td>$\phi_p = 1.5$ ($\phi_y = \phi_n = 0$)</td>
<td>0.0001</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\phi_p = 1.5$, $\phi_y = 0.125$ ($\phi_n = 0$)</td>
<td>0.0001</td>
<td>0.0018</td>
</tr>
<tr>
<td>$\phi_p = 1.5$, $\phi_n = 0.125$ ($\phi_y = 0$)</td>
<td>—</td>
<td>0.0090</td>
</tr>
</tbody>
</table>

**Notes:** The numbers in the table represent the permanent consumption loss induced by each monetary policy strategy as a fraction of consumption under the optimal policy. “LF” stands for labor force. The values of model parameters used here are reported in Table 1.

The larger welfare loss stems from the policy response to the employment gap, not from the lack of a policy response to the output gap, as indicated by the welfare loss under the strict inflation-targeting rule, which is almost the same as that under the Taylor rule. Intuitively, employment plays a significant role in the responses to shocks under optimal policy, by reining in wage growth. Leaning against the employment gap obliges employment to change, which exacerbates fluctuations in wage growth.²⁷

The bottom two panels of Table 2 repeat the welfare comparison under the assumption of a smaller degree of wage rigidity ($\alpha_w = 0.33$) and price rigidity ($\alpha_p = 0.33$), respectively.

²⁶The value of the elasticity of labor supply $1/\chi = 0.2$ can substantially increase these welfare losses. For example, the welfare loss induced by the policy rule with a response to the employment gap becomes 1.25–1.56 percent and that for the Taylor rule becomes 0.29–0.41 percent.

²⁷There is another argument against following a policy rule that responds to the employment gap: It shrinks the region of parameter values that ensure determinacy of equilibrium, as shown in Appendix B.
from fully stabilizing inflation in the case of a constant labor force and the welfare loss from following the rule with a policy response to the employment gap in the model with labor force entry. The bottom panel shows that a lower degree of price rigidity has the opposite effects. It exacerbates the welfare loss from fully stabilizing inflation in the case of a constant labor force.\textsuperscript{28} In the model with labor force entry, lower price rigidity exacerbates the welfare loss from following the rule with a policy response to the employment gap (0.55–0.90 percent).

5 Concluding Remarks

This paper has examined the implications of labor supply shocks to the extensive margin, motivated by US macroeconomic developments during the COVID-19 pandemic. Specifically, we introduce worker entry into and exit from the labor force in an otherwise canonical NK model with sticky prices and wages. The labor force entry decision gives rise to an employment channel of monetary policy, and a labor supply shock to the value of nonparticipation in the labor market induces a policy trade-off between stabilization of the variability of the employment gap and the variabilities of wage growth and inflation. Using the model, we have investigated the monetary policy implications of the labor supply shock and demonstrated that, in response to an adverse labor supply shock, optimal policy dampens the decline in employment to rein in wage growth, which results in a period of higher inflation and a positive output gap. Moreover, a welfare comparison of Taylor-type rules suggests that monetary policy should not lean against the employment gap.

The recovery of the US economy from the pandemic-induced recession was distinguished by weak labor force participation and strong wage growth and inflation. Our calibrated model suggests that, to the extent these developments reflected a labor supply shock, at least some fraction of the increase in US inflation observed in 2021 was desirable. Future research can quantify the distance between the observed US inflation and the counterfactual inflation under optimal policy, by embedding labor force entry and exit in a medium-scale DSGE model and estimating the model. Fiscal policy likely played a key role in US macroeconomic developments along with monetary policy. Accordingly, using the model with labor force

\textsuperscript{28}Galí (2008) reports a welfare comparison in the case of a constant labor force and demonstrates the sensitivity of welfare results to the degrees of price and wage rigidities. Our results are consistent with those reported in Table 6.1 of Galí (2008).
entry to analyze optimal fiscal and monetary policy in response to an adverse labor supply shock along the lines of Schmitt-Grohé and Uribe (2006) could be another fruitful agenda for future research.
Appendix

A Optimal Monetary Policy

This appendix derives the Ramsey policy. The first-order conditions for the optimized price and wage, (18) and (13), are rewritten recursively so that they can be included in the Lagrangian of the Ramsey policy problem. The Lagrangian is

\[ L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log y_t + \log A_t - \frac{1^{1+\chi} n_t \Delta w_{t,t} \exp z_{h,t}}{1 + \chi} - F \exp z_{n,t} n_{e,t} ight\} + M_{1,t} (p_t^* V_{p1,t} - V_{p2,t}) + M_{2,t} \left[ \lambda_t y_t + \alpha_p \beta E_t \left( \frac{\pi_{t+1}}{\pi} \right)^{\theta_p-1} V_{p1,t+1} - V_{p1,t} \right] 
+ M_{3,t} \left[ \frac{\theta_p}{\theta_p - 1} \lambda_t y_t w_t + \alpha_p \beta E_t \left( \frac{\pi_{t+1}}{\pi} \right)^{\theta_p} V_{p2,t+1} - V_{p2,t} \right] 
+ M_{4,t} \left[ (w_t^*)^{1+\theta_w \chi} V_{w1,t} - V_{w2,t} \right] 
+ M_{5,t} \left[ \lambda_t y_t + \alpha_w \rho E_t \frac{1}{\exp z_{a,t+1}} \left( \frac{\pi_{w,t+1}}{\pi_w} \right)^{\theta_w} \left( \frac{\pi_{t+1}}{\pi} \right)^{-1} V_{w1,t+1} - V_{w1,t} \right] 
+ M_{6,t} \left[ \frac{\theta_w}{\theta_w - 1} \frac{1^{1+\chi} \exp z_{h,t}}{w_t} + \alpha_w \rho E_t \left( \frac{\pi_{w,t+1}}{\pi_w} \right)^{\theta_w(1+\chi)} \frac{w_{t+1}}{w_t} V_{w2,t+1} - V_{w2,t} \right] 
+ M_{7,t} (l_t - y_t \Delta_p,t) + M_{8,t} \left( \frac{1}{y_t} - \lambda_t \right) + M_{9,t} \left[ \alpha_p \left( \frac{\pi_t}{\pi} \right)^{\theta_p-1} + (1 - \alpha_p) (p_t^*)^{1-\theta_p} - 1 \right] 
+ M_{10,t} \left[ \alpha_p \left( \frac{\pi_t}{\pi} \right)^{\theta_p} \Delta_{p,t-1} + (1 - \alpha_p) (p_t^*)^{\theta_p} - \Delta_{p,t} \right] 
+ M_{11,t} (pn_{t-1} + n_{e,t} - n_t) + M_{12,t} \left( \frac{\alpha \pi_t w_t}{w_{t-1}} \exp z_{a,t} - \pi_{w,t} \right) 
+ M_{13,t} \left( \lambda_t w_t l_t - \frac{1^{1+\chi} n_t \Delta w_{t,t} \exp z_{h,t}}{1 + \chi} + (pn_t) \beta F E_t \exp z_{n,t+1} - n_t F \exp z_{n,t} \right) 
+ M_{14,t} \left[ \alpha_w \left( \frac{n_t}{n_{t-1}} \right) \frac{\pi_{w,t}}{\pi_w} \theta_w^{-1} + (1 - \alpha_w) n_t (w_t^*)^{1-\theta_w} - 1 \right] 
+ M_{15,t} \left[ \alpha_w \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_w(1+\chi)} \Delta_{w,t-1} + (1 - \alpha_w) (w_t^*)^{-\theta_w(1+\chi)} - \Delta_{w,t} \right] \},

where \( M_{i,t}, \ i = 1, \ldots, 15 \) are Lagrange multipliers and the constraints associated with the multipliers \( M_{i,t}, \ i = 1, 2, 3 \) and \( M_{i,t}, \ i = 4, 5, 6 \) consist of the first-order conditions for the optimized price and wage, respectively.

The equilibrium under optimal policy satisfies the constraints of the Lagrangian and the
following first-order conditions:

\[
\partial \pi_t : (\theta_p - 1) \alpha_p \left( \frac{\pi_t}{\pi} \right)^{\theta_p - 1} (V_{p1,t} M_{2,t-1} + M_{9,t}) + \theta_p \alpha_p \left( \frac{\pi_t}{\pi} \right)^{\theta_p} (V_{p2,t} M_{3,t-1} + M_{10,t} \Delta_{p,t-1}) \\
- \alpha_w \rho \frac{1}{\exp z_{a,t}} \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_w} \left( \frac{\pi_t}{\pi} \right)^{-1} V_{w1,t} M_{5,t-1} + M_{12,t} \pi_{w,t} = 0,
\]

\[
\partial \pi_{w,t} : \theta_w \alpha_p \rho \frac{1}{\exp z_{a,t}} \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_w} \left( \frac{\pi_t}{\pi} \right)^{-1} V_{w1,t} M_{5,t-1} + (\theta_w - 1) \alpha_w \frac{n_t}{n_{t-1}} \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_w - 1} M_{14,t} \\
+ \theta_w (1 + \chi) \alpha_w \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_w (1+\chi)} \left( \rho \frac{w_t}{w_{t-1}} V_{w2,t} M_{6,t-1} + M_{15,t} \Delta_{w,t-1} \right) = M_{12,t} \pi_{w,t},
\]

\[
\partial p^*_t : M_{1,t} V_{p1,t} (p^*_t)^{1+\theta_p} = (\theta_p - 1)(1 - \alpha_p) p^*_t M_{9,t} + \theta_p (1 - \alpha_p) M_{10,t},
\]

\[
\partial V_{p1,t} : M_{2,t} = M_{1,t} p^*_t + \alpha_p \left( \frac{\pi_t}{\pi} \right)^{\theta_p - 1} M_{2,t-1},
\]

\[
\partial V_{p2,t} : M_{3,t} = -M_{1,t} p^*_t + \alpha_p \left( \frac{\pi_t}{\pi} \right)^{\theta_p} M_{3,t-1},
\]

\[
\partial w^*_t : (1 + \theta_w \chi) (w^*_t)^{\theta_w (1+\chi)} V_{w1,t} M_{4,t} = (\theta_w - 1)(1 - \alpha_w) n_t M_{14,t} \\
+ \theta_w (1 + \chi) (1 - \alpha_w) (w^*_t)^{-\theta_w (1+\chi)} M_{15,t},
\]

\[
\partial V_{w1,t} : M_{5,t} = (w^*_t)^{1+\theta_w \chi} M_{4,t} + \alpha_w \rho \frac{1}{\exp z_{a,t}} \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_w} \left( \frac{\pi_t}{\pi} \right)^{-1} M_{5,t-1},
\]

\[
\partial V_{w2,t} : M_{6,t} = -M_{4,t} + \alpha_w \rho \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_w (1+\chi)} \frac{w_t}{w_{t-1}} M_{6,t-1},
\]

\[
\partial n_{c,t} : M_{11,t} = F \exp z_{n,t},
\]

\[
\partial \Delta_{w,t} : M_{15,t} - \alpha_w \beta E_t \left( \frac{\pi_{w,t-1}}{\pi_{w}} \right)^{\theta_w (1+\chi)} M_{15,t+1} = -\frac{l^t_{1+\chi} \exp z_{h,t}}{1 + \chi} (n_t + M_{13,t}),
\]

\[
\partial n_t : n_t (M_{11,t} - \beta p E_t M_{11,t+1}) = -\lambda_t w_t l_t \frac{M_{13,t}}{n_t} - \frac{l^t_{1+\chi} n_t \Delta_{w,t} \exp z_{h,t}}{1 + \chi} M_{14,t} + M_{14,t} \\
- \alpha_w \beta E_t \left( \frac{\pi_{w,t+1}}{\pi_w} \right)^{\theta_w - 1} \frac{n_{t+1}}{n_t} M_{14,t+1},
\]

\[
\partial y_t : \frac{1}{y_t} + \lambda_t \left( M_{2,t} + \frac{\theta_p}{\theta_p - 1} w_t M_{3,t} \right) - \Delta_{p,t} M_{7,t} - M_{8,t} \frac{y_t^2}{y_{t-1}^2} = 0,
\]

\[
\partial w_t : \frac{\theta_p}{\theta_p - 1} \lambda_t y_t w_t M_{3,t} + M_{12,t} \pi_{w,t} - \beta E_t M_{12,t+1} \pi_{w,t+1} + \lambda_t w_t l_t \frac{M_{13,t}}{n_t} \\
= \left[ M_{6,t} - \alpha_w \rho \left( \frac{\pi_{w,t}}{\pi_w} \right)^{\theta_w (1+\chi)} \frac{w_t}{w_{t-1}} M_{6,t-1} \right] V_{w2,t},
\]

\[
\partial \Delta_{p,t} : M_{10,t} = -y_t M_{7,t} + \alpha_p \beta E_t \left( \frac{\pi_{t+1}}{\pi} \right)^{\theta_p} M_{10,t+1},
\]

\[
\partial l_t : \lambda_t M_{5,t} + M_{7,t} + \left( \frac{\lambda_t w_t l_t}{n_t} - l^t \exp z_{h,t} \Delta_{w,t} \right) \frac{M_{13,t}}{n_t} = l^t \exp z_{h,t} \left( \frac{n_t \Delta_{w,t} - \theta_w (1 + \chi) M_{6,t}}{w_t} \right),
\]

\[
\partial \lambda_t : y_t \left( M_{2,t} + \frac{\theta_p}{\theta_p - 1} w_t M_{3,t} \right) + l_t M_{5,t} + (w_t l_t / n_t) M_{13,t} = M_{8,t}.
\]
B Equilibrium Determinacy

This appendix analyzes the implications of labor force entry for determinacy of equilibrium. We show that a policy response to the employment gap in the Taylor-type rule shrinks the region of parameter values that ensure determinacy of equilibrium. Following Galí (2008), the Taylor-type rule (30) is generalized, for this exercise, to allow for a policy response to wage growth:

\[
\hat{r}_t = \phi_p \hat{\pi}_t + \phi_w \hat{\pi}_{w,t} + \phi_y (\hat{y}_t - \hat{y}_n^n) + \phi_n (\hat{n}_t - \hat{n}_n^n),
\]

where \(\phi_w\) denotes the policy response to wage growth. By combining the long-run log-linearized equilibrium conditions (31)–(34), we obtain the long-run version of the Taylor principle, which holds that the policy rate should respond more than one-for-one with inflation in the long run. In the model with labor force entry and the policy rule (42), the principle requires that:

\[
\phi_p + \phi_w + \frac{1 - \beta}{\kappa_p} \frac{\theta_w}{\theta_w - 1} \phi_y - \frac{(\theta_w - 1)(1 - \beta \rho) - \kappa_w (1 + \theta_w \chi)(1 - \beta)}{(1/\alpha_w - \beta \rho)(1 - \alpha_w)} \phi_n > 1.
\]

Condition (43) shows that a policy response to the output gap (i.e., \(\phi_y > 0\)) reduces the minimum values of \(\phi_p\) and \(\phi_w\) that are required for equilibrium determinacy. In contrast, a policy response to the employment gap (i.e., \(\phi_n > 0\)) raises the minimum values of \(\phi_p\) and \(\phi_w\) that ensure equilibrium determinacy, if the coefficient on \(\phi_n\) in the condition is positive. It can be verified that the condition is positive if \(\alpha_p = \alpha_w\), as in the calibration of parameters reported in Table 1.

Figure A1 plots the determinacy and indeterminacy regions for three types of policy rules. First, the dotted black line is the boundary between the determinacy and indeterminacy regions of the parameter space obtained for policy rules that strictly target inflation and/or wage growth (i.e., \(\phi_y = \phi_n = 0\)). This line shows that any values of \(\phi_p\) and \(\phi_w\) such that \(\phi_p + \phi_w > 1\) ensures determinacy. Second, the dashed red line is the boundary obtained for policy rules that include a response to the output gap (i.e., \(\phi_y = 0.125, \phi_n = 0\)). The policy response to the output gap enlarges the determinacy region slightly in the calibrated model. Third, the solid blue line is the boundary obtained for policy rules that include a response to the employment gap (i.e., \(\phi_n = 0.125, \phi_y = 0\)). The policy response to the employment gap...
shrinks the determinacy region noticeably in the calibrated model by shifting the boundary in the northeast direction. Due to the policy response to the employment gap, only values of $\phi_p$ and $\phi_w$ such that $\phi_p + \phi_w > 1.23$ ensure determinacy. The boundaries plotted in the figure are obtained numerically, but coincide with the boundaries implied by the long-run version of the Taylor principle (43).

Figure A1: Equilibrium determinacy region of the parameter space.

Notes: Each line plots the boundary between the determinacy region in the northeast and the indeterminacy region in the southwest area of the figure. The dotted black line is the boundary obtained for policy rules that strictly target inflation and/or wage growth (i.e., $\phi_y = \phi_n = 0$). The dashed red line is the boundary obtained for policy rules that include a response to the output gap (i.e., $\phi_y = 0.125, \phi_n = 0$). The solid blue line is the boundary obtained for policy rules that include a response to the employment gap (i.e., $\phi_n = 0.125, \phi_y = 0$). The values of model parameters used here are reported in Table 1.
References


Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti (2013). “Is there a trade-


