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Chengcheng Jia and Jing Cynthia Wu

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Average Inflation Targeting: Time Inconsistency and Intentional Ambiguity*

Chengcheng Jia
Federal Reserve Bank of Cleveland

Jing Cynthia Wu
University of Notre Dame and NBER

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Abstract

We study the implications of the Fed's new policy framework of average inflation targeting (AIT) and its ambiguous communication. The central bank has the incentive to deviate from its announced AIT and implement inflation targeting ex post to maximize social welfare. We show two motives for ambiguous communication about the horizon over which the central bank averages inflation as a result of time inconsistency. First, it is optimal for the central bank to announce different horizons depending on the state of the economy. Second, ambiguous communication helps the central bank gain credibility.

Keywords: inflation, monetary policy, central bank communication.

JEL Classification: E52, E31, E58.

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1 Introduction

At the 2020 Jackson Hole Economic Policy Symposium, Federal Reserve Chair Jerome Powell announced a revision to the Fed's long-run monetary policy framework, replacing inflation targeting (IT) with average inflation targeting (AIT) to achieve its dual mandate; see Powell (2020). However, a critical component that is missing from the Fed's communication is the horizon over which it targets average inflation at 2 percent; see Kozicki's (2019) discussion at the Fed Listens event in 2019 and Brunnermeier's (2021) panel discussion at the Jackson Hole Economic Policy Symposium in 2021. Our paper investigates the implications of such an ambiguous communication.

We focus on two key issues of AIT: time inconsistency and ambiguous communication. We show that AIT is not time consistent, which is the case for any path-dependent policy; see, for example, Eggertsson and Woodford (2003). By convincing the private sector about its intention to implement AIT, the central bank can improve the trade-off between inflation and real activity, which is captured by the Phillips curve. Ex post, the central bank has the incentive to deviate from its communication and implement IT instead to improve social welfare. This time-inconsistent strategy leads to two motives for ambiguous communication. First, the optimal horizon of the AIT announcement is time dependent, and ambiguous communication allows such flexibility. Second, the central bank's time-inconsistent strategy is welfare-improving, provided that the central bank is able to convince private agents with its communication. But can it? We show that although the central bank implements IT de facto, AIT believers can have a smaller nowcast error than IT believers. Moreover, announcing AIT without clearly specifying its horizon helps the central bank further gain credibility and improve welfare compared with the case with clear communication.

Our paper is the first paper that studies AIT that deviates from the full-information rational expectations (FIRE) framework. We introduce AIT into an otherwise textbook three-equation New Keynesian model by modifying the central bank's objective function. For a standard model, the central bank minimizes a quadratic loss function in inflation and the output gap, both of which are valued at the current period. AIT replaces current inflation in the objective function with average inflation over L periods between $t - L + 1$ and t.[1] In the FIRE benchmark, the central bank minimizes its loss subject to a forward-looking Phillips curve. Although the central bank is discretionary and solves its problem period by period, equilibrium is path dependent: inflation oscillates around its steady state after a cost-push shock.

The Phillips curve captures the trade-off between inflation and real activity the central bank faces. Although AIT does not change the forward-looking Phillips curve, it tilts its reduced form in a favorable way: compared with IT, AIT is associated with a Phillips curve that has a smaller intercept and slope. This works through the expectations channel of AIT: when inflation is above its target today, the expected inflation next period will be below the target. A lower expected inflation lowers inflation today through the expectation term in the forward-looking Phillips curve.

In the zero lower bound (ZLB) environment, AIT not only flattens the Phillips curve, but also alters the IS curve. The IS curve is vertical for IT but is downward sloping for AIT, and this difference arises from the expectation terms in the IS curve. Whether the central bank implements IT or AIT, the ZLB features below-target inflation and a negative output gap. However, AIT delivers a better equilibrium than IT with less negative inflation and a less negative output gap. This again works through the expectations channel of AIT: when current inflation is below the target, the private sector expects inflation next period to be above the target, which generates higher demand and inflation in the present. Consequently, AIT could be an effective tool to combat a liquidity trap.

Although AIT presents the central bank with a better inflation-output trade-off by tilting the Phillips curve in a favorable way, it does not necessarily yield higher welfare, because its objective function is different from social welfare. After the central bank convinces the private sector about its intention to implement AIT, it has an incentive to deviate from its

[1] We define AIT as flexible average inflation targeting, which also puts weight on the output gap. This definition is similar to flexible inflation targeting in the literature.

communicated objective and optimize social welfare instead. This strategy improves welfare but is time inconsistent.

The time-inconsistent nature of AIT motivates the central bank to make ambiguous announcements regarding the horizon over which it targets average inflation for two reasons. First, it is optimal for the central bank to announce different horizons for AIT depending on economic conditions. Specifically, the optimal strategy is to announce the largest feasible horizon when a cost-push shock hits and to announce two-period AIT thereafter. Instead of making self-conflicting announcements, the central bank can make communication ambiguous, which does not commit itself to a fixed horizon.

Second, the central bank's strategy to announce AIT but implement IT ex post could be welfare-enhancing, but is it sustainable? Or can agents learn the truth in the long run? We answer these questions through social learning, whereby agents meet and update their beliefs based on their performances. We allow two layers of heterogeneity. First, agents have different beliefs about the horizon L over which the central bank averages inflation. Second, agents with the same belief about L observe different private signals about economic fundamentals, and we use this as a device to capture uncertainty. By contrast, the central bank has perfect information.

We show ambiguous communication helps the central bank gain credibility and improve social welfare in the long run, which constitutes the second motive for ambiguous communication. This result does not depend on whether the central bank is credible initially, how agents weigh nowcast errors between inflation and the output gap, or if agents observe the underlying economic fundamentals perfectly. On the other hand, the optimal L is unclear. Two competing forces are at play: the largest L delivers the best Phillips curve but the fewest followers. However, the central bank does not need to face this trade-off if it communicates ambiguously, which delivers a larger following and better welfare than clear communication with any L.

The rest of the paper proceeds as follows: Section 1.1 draws a connection to the existing literature. Section 2 sets up the FIRE benchmark. Section 3 discusses time inconsistency and the flexibility motive for ambiguous communication, and the discussion assumes the central bank can convince the private sector with its communication. Via social learning, Section 4 assesses this assumption and focuses on the credibility motive of ambiguous communication. Finally, Section 5 concludes.

1.1 Literature

Only a few papers in the literature focus on the Federal Reserve's new policy framework of AIT. Coibion et al. (2020) use survey data to empirically assess how AIT changes expectations, whereas we use a theoretical framework. Also using a model, Amano et al. (2020) examine the optimal degree of history dependence under AIT and find it is relatively short. Their paper focuses on a simple instrument rule, whereas we introduce AIT into the central bank's optimization problem. In that sense, Nessén and Vestin (2005) is the paper closest to ours. However, their paper studies the case with FIRE, whereas our paper focuses on time inconsistency and ambiguous communication, which are more relevant to current policy discussions. In that sense, Budianto, Nakata, and Schmidt (2020) is the paper closest to ours. The difference is that they study the optimal horizon of AIT, whereas we focus on ambiguous communication where the horizon is unknown.

Several other papers discuss AIT as one of many alternative strategies when many advanced economies face a low-interest-rate environment. Mertens and Williams (2019a,b) show AIT can mitigate the effects of the ZLB by raising inflation expectations when inflation is low. Svensson (2020) compares average inflation targeting with annual inflation targeting, price-level targeting, temporary price-level targeting, and nominal GDP targeting, and discusses its advantages. Hebden et al. (2020) discuss the robustness of AIT to alternative assumptions about the slope of the Phillips curve and the uncertainty of economic slack. Andrade et al. (2021) show AIT can work as an alternative to a higher inflation target for the euro area, which faces a low r^*. Papell and Prodan (2021) show how the new policy framework is consistent with an alternative policy rule forward guidance.

Our paper is related to the literature on agent-based modeling. We focus on its applications in economics, which refers to it as social learning or social dynamics. Burnside, Eichenbaum, and Rebelo (2016) use social learning to study booms and busts in housing markets, and Bohren and Hauser (2021) study social learning in a theoretical framework. Our paper is related to Hachem and Wu (2017) in the sense that both papers study inflation. It is also related to Arifovic, Bullard, and Kostyshyna (2013) in the sense that both papers use a New Keynesian model. The difference is that we focus on the Fed's new framework of AIT.

Our paper speaks to the literature on the Phillips curve. Many recent papers, for example, Stock and Watson (2020), find empirical evidence suggesting the slope of the Phillips curve has flattened since the Great Recession. Our paper is especially related to Coibion and Gorodnichenko (2015) and Hazell et al. (2020) in the sense that we all emphasize the importance of inflation expectation formation to the slope of the Phillips curve. The difference is the above-referenced papers show empirically how the Phillips curve has changed recently, whereas our paper studies theoretically how AIT would change the Phillips curve.

Most of the discussion in the literature on the ZLB centers around conventional monetary policy (e.g., Eggertsson and Woodford (2003)), quantitative easing (e.g., Gertler and Karadi (2011), Gertler and Karadi (2013), Carlstrom, Fuerst, and Paustian (2017), Sims and Wu (2020), and Sims, Wu, and Zhang (forthcoming)), forward guidance (e.g., Levin et al. (2010)), or the negative interest rate policy (e.g., Wu and Xia (2020) and Ulate (2021)). A few papers study a mix of policy tools at the ZLB (e.g., Swanson (2018), Wu and Zhang (2019), Sims and Wu (2021), and Swanson (2021)). Different from these papers, we focus on AIT, which is the newest addition to central banks' toolkit.

Our paper is also broadly related to the literature on alternative expectations formation that deviates from the standard assumption of FIRE; for an extensive literature review, see Coibion, Gorodnichenko, and Kamdar (2018). One approach assumes agents are rational but have information frictions in the form of information rigidity (Mankiw and Reis (2002)), partial information (Angeletos and La'o (2020) and Jia (2019)), or rational inattention (Sims (2003)). Coibion and Gorodnichenko (2012) test these forms of information friction with a variety of survey forecasts. Another approach assumes that agents learn in an adaptive way (Evans and Honkapohja (1999) and Orphanides and Williams (2005)). The learning part of our paper broadly fits into this part of the literature, but we use social learning instead.

2 Full-Information Rational Expectations Benchmark

In this section, we lay out a benchmark model that characterizes the new policy framework of average inflation targeting under the FIRE assumption. After describing the model and its equilibrium in Section 2.1, we compare it with a standard model of inflation targeting in Section 2.2. We discuss AIT's implications for the Phillips curve in Section 2.3 and for the ZLB in Section 2.4. We deviate from the FIRE assumption and discuss the key issues of the paper, namely, time inconsistency and ambiguous communication, in Sections 3 and 4.

2.1 A Model of Average Inflation Targeting

We model average inflation targeting with the following objective function of the central bank: a weighted sum of squared average inflation over L periods and the squared output gap:

$$\mathbb{L}_t^{cb}(L) = \frac{1}{2}\left(\left(\frac{\pi_t + \pi_{t-1} + \ldots\pi_{t-L+1}}{L}\right)^2 + \lambda^{cb}(L)\hat{y}_t^2\right) + \beta\,\mathbb{E}_t\,\mathbb{L}_{t+1}^{cb}(L), \tag{2.1}$$

where $\hat{y}_t$ is the output gap, defined as the log deviation of real output from its natural level, and π_t is the rate of inflation. $\lambda^{cb}(L) > 0$ is the relative weight between the output gap and average inflation, and it could potentially depend on the horizon of AIT L.

In general, the central bank's objective function is different from social welfare, whose period loss is defined as

$$\mathcal{L}_t = \frac{1}{2}\left(\pi_t^2 + \lambda \hat{y}_t^2\right), \tag{2.2}$$

where λ is derived from the second-order approximation of the household's utility function (Rotemberg and Woodford, 1999). They only coincide when $L = 1$ and $\lambda^{cb}(1) = \lambda$.

The central bank minimizes its loss in (2.1) by choosing inflation and the output gap subject to a standard forward-looking Phillips curve (see, e.g., Woodford (2003), Clarida, Galí, and Gertler (1999), and Galí (2015)):

$$\pi_t = \beta\, \mathbb{E}_t\, \pi_{t+1} + \kappa \hat{y}_t + u_t, \tag{2.3}$$

where β is the discount factor, and κ depends on nominal rigidity. We assume the cost-push shock, u_t, is the only shock in the economy, which introduces a trade-off between inflation and the output gap.

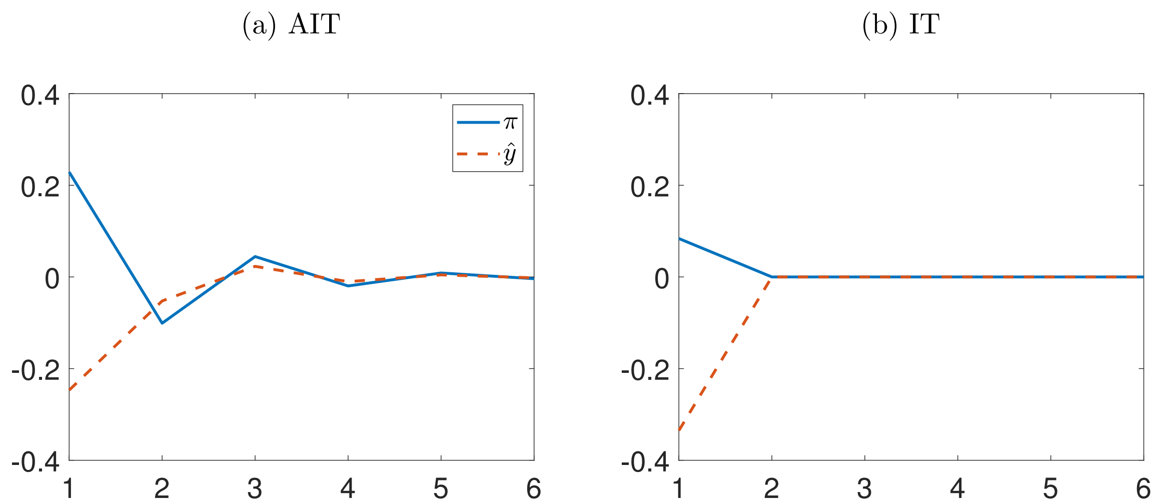
Our central bank is discretionary because it solves the optimization problem period by period. However, unlike in a standard model, the equilibrium is path dependent:

$$\pi_t = a_{\pi,1}^{(L)}\pi_{t-1} + ... + a_{\pi,L-1}^{(L)}\pi_{t-L+1} + b_\pi^{(L)} u_t \tag{2.4}$$

$$\hat{y}_t = a_{y,1}^{(L)}\pi_{t-1} + ... + a_{y,L-1}^{(L)}\pi_{t-L+1} + b_y^{(L)} u_t. \tag{2.5}$$

For analytic tractability and to gain some intuition, we start with two-period AIT, and we extend to multi-period in Sections 3.2 and 4. For two-period AIT ($L = 2$), the coefficients

Figure 1: Impulse responses to a cost-push shock



Notes: X-axis: horizon. Units on the y-axis: percentage points. Blue solid lines: inflation; red dashed lines: output gap. Left panel: AIT; right panel: IT. $\lambda^{cb}(2) = \lambda$, $u_t = 1$.

are given by the following set of fixed-point equations:

$$a_\pi \equiv a_{\pi,1}^{(2)} = -\vartheta \frac{\kappa}{4(1-\beta a_\pi)} \tag{2.6}$$

$$a_y \equiv a_{y,1}^{(2)} = \frac{1}{\kappa}\left(1-\beta a_\pi\right) a_\pi \tag{2.7}$$

$$b_\pi \equiv b_\pi^{(2)} = \vartheta \frac{\lambda^{cb}}{\kappa} \tag{2.8}$$

$$b_y \equiv b_y^{(2)} = \frac{1}{\kappa}\left(1-\beta a_\pi\right) b_\pi - \frac{1}{\kappa}, \tag{2.9}$$

where $\lambda^{cb} \equiv \lambda^{cb}(2)$. See Appendix A.1 for derivations and expression for ϑ.

We characterize the equilibrium with the following proposition:

Lemma 1 $a_\pi < 0$, $a_y < 0$.

Proof: See Appendix B.

Lemma 1 says higher inflation leads to lower inflation and a lower output gap in the next period. This lemma, especially a negative $a_\pi = \frac{\partial \pi_t}{\partial \pi_{t-1}}$, leads to an important result – the expectations channel of AIT, which we discuss in Section 2.3. We corroborate the result of a negative a_π with the left panel of Figure 1, which plots impulse responses of inflation and the output gap to a 1 percent cost-push shock.[2] The cost-push shock introduces a trade-off for the central bank by increasing inflation and decreasing the output gap. When the central bank implements AIT, inflation oscillates around zero. In particular, the central bank tightens monetary policy in the second period, which leads to a negative inflation rate and a negative output gap.

2.2 Comparison to Inflation Targeting

We define inflation targeting (IT) as the textbook version of optimal discretionary policy (see, e.g., Woodford (2003) and Galí (2015)). Its objective function is social welfare, which is the present discounted value of the period loss function in (2.2).

The equilibrium for IT is given by

$$\pi_t = b_\pi^{(1)} u_t = \left(\kappa + \frac{\lambda}{\kappa}\right)^{-1} \frac{\lambda}{\kappa} u_t \tag{2.10}$$

$$\hat{y}_t = b_y^{(1)} u_t = \left[\frac{1}{\kappa}\left(\kappa + \frac{\lambda}{\kappa}\right)^{-1} \frac{\lambda}{\kappa} - \frac{1}{\kappa}\right] u_t. \tag{2.11}$$

See Appendix A.2 for derivations.

Unlike the AIT equilibrium in (2.4) - (2.5), IT equilibrium in (2.10) - (2.11) does not depend on lagged inflation. On the right side of Figure 1, we plot the impulse responses under IT. After the first period, the economy is back to the steady state, which is in contrast to the oscillation in the left panel.

2.3 Phillips Curve

This section investigates how AIT affects the Phillips curve, which captures the central bank's available trade-off between inflation and the output gap. We begin with an important property of the model, which is a direct result of Lemma 1:

[2]Parameters are calibrated in line with the New Keynesian literature at an annual frequency. For details, see Appendix D.

Proposition 1 ***The Expectations Channel*** *implies lower (higher) positive (negative) inflation under AIT than that under IT for a given output gap.*

Proof: See Appendix B.

For AIT, the expectations channel works as follows: the private sector forms its expectations about future inflation conditional on current inflation. When inflation is positive, the expected inflation next period is negative per Lemma 1, which in turn lowers inflation today via the expectation term in the forward-looking Phillips curve (2.3). By contrast, IT does not have such an expectations channel.

As a result of the expectations channel, although IT and AIT share the same structural form of the Phillips curve as in (2.3), the reduced-form Phillips curves are different. For AIT, it is

$$\pi_t = \frac{\kappa}{1 - \beta a_\pi} \hat{y}_t + \frac{1}{1 - \beta a_\pi} u_t, \tag{2.12}$$

and the reduced-form Phillips curve for IT is

$$\pi_t = \kappa \hat{y}_t + u_t. \tag{2.13}$$

For (2.12), see derivations in Appendix A.1. The derivation of (2.13) is trivial because $\mathbb{E}_t \, \pi_{t+1} = 0$.

Comparing the Phillips curves under AIT in (2.12) and under IT in (2.13) leads to the following proposition:

Proposition 2 *AIT implies a smaller slope of the reduced-form Phillips curve than IT.*

Proof: See Appendix B.

Why does AIT flatten the reduced-form Phillips curve? We use the right panel of Figure 2 for illustration (a similar argument can be made using the left panel). Points A and B are both associated with more expansionary policy and have a 0.1 percent higher output gap than the origin, where the Phillips curves of IT and AIT intersect. The difference between them is that point A is on the IT Phillips curve, whereas point B is on the AIT Phillips curve. The direct effect on the forward-looking Phillips curve (2.3) is that both A and B have inflation that is κ percent higher than the origion. But they differ in terms of the indirect effect through the expectations channel described in Proposition 1. IT does not have such an indirect effect. By contrast, with an AIT policy, higher inflation today lowers expected inflation in the next period, which feeds back to lower inflation today. This expectations channel makes point B lower than point A. Therefore, AIT is associated with a smaller slope of the Phillips curve than IT.

With a flatter reduced-form Phillips curve, we next argue that AIT presents a better trade-off between inflation and the output gap than IT:

Proposition 3 *AIT yields a better available trade-off for the central bank between inflation and the output gap.*

Proof: See Appendix B.

This is a key result of the paper: AIT tilts the Phillips curve in a favorable way – closer to the origin in the relevant space (the second quadrant). We show the intuition of this result in two steps, and for each step, we focus on one state variable. First, we start with a cost-push shock.

Lemma 2 *The reduced-form Phillips curve under AIT has*

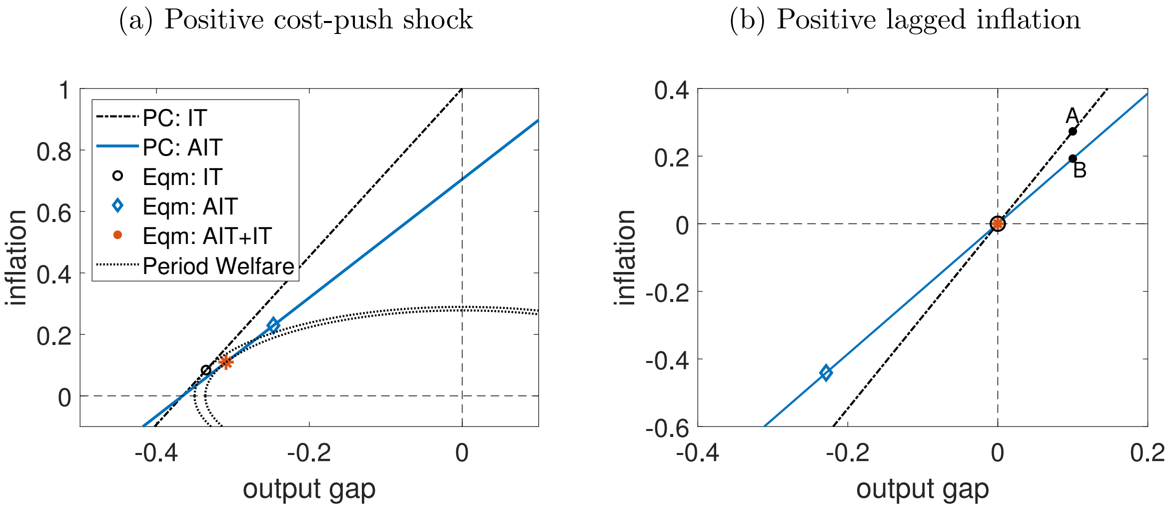
- *the same x-intercept as*
- *and a smaller absolute value of the y-intercept than*

that under IT after a cost-push shock.

Proof: See Appendix B.

The intercept (which we refer to as the y-intercept in the proposition) of the Phillips curve captures the equilibrium when the output gap is stabilized after a cost-push shock.

Figure 2: Reduced-form Phillips curves and equilibria



Notes: Left: $u_t = 1, \pi_{t-1} = 0$; right: $\pi_{t-1} = 1, u_t = 0$. Units: percentage points. Lines are Phillips curves. Black circle: IT equilibrium; blue diamond: AIT equilibrium with commitment; red star: the equilibrium where the private sector forms expectations based on the AIT announcement but the central bank deviates to implement IT ex post. The ellipses are period social welfare. $\lambda^{cb}(2) = \lambda$.

The two upward-sloping lines in the left panel of Figure 2 are Phillips curves after a positive cost-push shock, which leads to a positive inflation rate and hence a positive intercept. When the central bank implements AIT, the expectations channel that we discussed in Proposition 1 works as follows: positive inflation in the current period leads to negative expected inflation in the next period, which in turn lowers current inflation. Therefore, the intercept of the Phillips curve is smaller under AIT than under IT. The x-intercept of the Phillips curve represents the equilibrium when inflation is completely stabilized after a cost-push shock. Zero inflation this period implies zero expected inflation and a zero expected output gap next period for both policies, eliminating any feedback from the expectations to the current period. Therefore, the two policies share the same x-intercept.

Next, we turn to the case with non-zero lagged inflation. IT is not path dependent; therefore, lagged inflation does not introduce a trade-off between inflation and the output gap. Although lagged inflation is a state variable for AIT, it does not introduce an additional trade-off between inflation and the output gap for the two-period case (see the lines in the right panel of Figure 2).[3] We summarize this result in the following lemma:

Lemma 3 *The reduced-form Phillips curves under two-period AIT and IT both cross the origin after non-zero lagged inflation.*

Proof: See Appendix B.

What drives this result? The private sector forms its expectations next period conditional on current inflation only. Therefore, lagged inflation does not introduce an indirect effect via the expectations channel.

Although AIT allows a better available trade-off between inflation and the output gap, it does not guarantee an improvement in welfare, because the central bank's objective function is different from social welfare. In Section 3, we discuss a welfare-enhancing strategy in which the central bank implements a policy that is different from its announced AIT.

2.4 Implications of the Zero Lower Bound

The ZLB has plagued many advanced economies, including the US, Japan, the euro area, and many other countries, for over a decade. It is also a primary reason for the introduction of AIT; for example, see Powell (2020) and Diwan, Leduc, and Mertens (2020). This section takes a detour from the main theme of the paper, namely, time inconsistency and ambiguous communication, which we return to in Sections 3 and 4, and discusses the implications of the ZLB.

We model the ZLB with an interest rate peg and a negative shock to the natural rate of interest, for which we need to introduce the IS curve:

$$\hat{y}_t = \mathbb{E}_t \, \hat{y}_{t+1} - \frac{1}{\gamma} \left(\underline{i} - r_t^n - \mathbb{E}_t \, \pi_{t+1} \right), \tag{2.14}$$

where the nominal interest rate is $\underline{i} = 0$ and r_t^n is the natural rate of interest. γ is the inverse of the intertemporal elasticity of substitution. We model a one-period ZLB, which

[3] We later show that this is not the case for $L > 2$.

corresponds to one year in our calibration. The forward-looking Phillips curve is the same as equation (2.3).

We first solve the equilibrium during normal times when the ZLB does not bind. In this case, the natural rate shock does not change the equilibrium, because the central bank sets the interest rate optimally to track the natural rate and offset any effect the natural rate shock might have on the equilibrium. Therefore, the central bank can achieve dual stability when only the natural rate shock is present. See details in Appendix A.4. However, this is not the case when the ZLB binds, because the central bank cannot lower the nominal interest rate further to compensate a negative shock to the natural rate.

With our assumption on the ZLB, $\mathbb{E}_t\, \pi_{t+1} = \mathbb{E}_t\, \hat{y}_{t+1} = 0$ under IT. Therefore, the IS curve is

$$\hat{y}_t = \frac{1}{\gamma} r_t^n, \tag{2.15}$$

and the Phillips curve is the same as equation (2.13).

For two-period AIT,

$$\mathbb{E}_t\, \pi_{t+1} = a_\pi \pi_t \tag{2.16}$$

$$\mathbb{E}_t\, \hat{y}_{t+1} = a_y \pi_t, \tag{2.17}$$

where a_π and a_y satisfy equations (2.6) and (2.7). For derivation, see Appendix A.4.

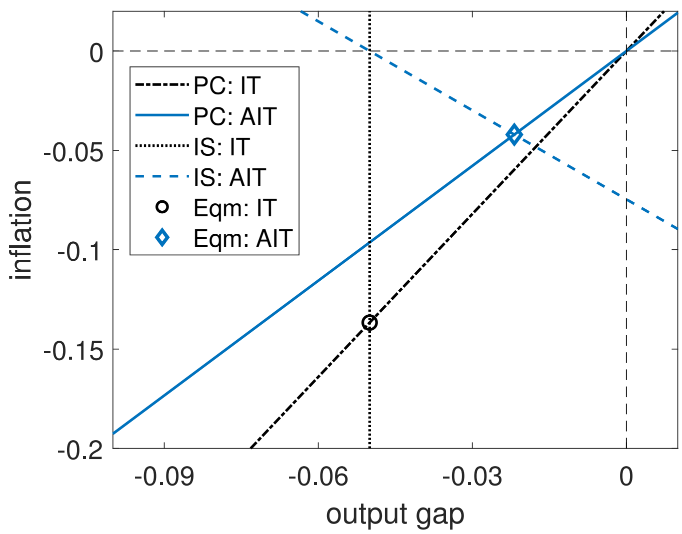
Therefore, the IS curve is

$$\hat{y}_t = \left(a_y + \frac{1}{\gamma} a_\pi \right) \pi_t + \frac{1}{\gamma} r_t^n, \tag{2.18}$$

and the Phillips curve is the same as equation (2.12).

Figure 3 plots the implications of the ZLB. The upward-sloping lines are the Phillips curves. The vertical dotted black and downward-sloping dashed blue lines are the IS curves,

Figure 3: Zero lower bound



Notes: $r_t^n = -1, u_t = 0$. Units: percentage points. The upward-sloping lines are the Phillips curves; the downward-sloping and vertical lines are the IS curves; the circle and diamond are the equilibria. Black represents IT and blue represents AIT. $\lambda^{cb}(2) = \lambda$.

and the circle and diamond are the equilibria. Black represents IT and blue is AIT. Although Figure 3 illustrates a case without the cost-push shock, our results go through as long as the cost-push shock is not too positive and the equilibria stay in the same quadrant.

Similar to our discussion pf Proposition 2, the AIT Phillips curve is flatter than the IT curve. The IS curve for IT is vertical per equation (2.15), whereas that for AIT is downward sloping per equation (2.18) and Lemma 1. Moreover, the two IS curves cross the horizontal axis at the same point because $\pi_t = 0$ implies $\mathbb{E}_t \pi_{t+1} = 0$ regardless of whether the central bank implements IT or AIT. In both cases, the equilibrium entails negative inflation and a negative output gap, which together are the hallmark of the ZLB.

However, AIT achieves a better equilibrium than IT with less negative inflation and a less negative output gap. This result again works through the expectations channel discussed in Proposition 1. Current inflation is below the target. With AIT, the private sector expects inflation next period to be above the target, which in turn raises both demand and inflation this period. Therefore, AIT could be a useful tool for an economy that is stuck in a liquidity trap.

3 Time Inconsistency and Flexibility Motive for Ambiguity

This section deviates from the FIRE framework in Section 2, demonstrates the time inconsistency of AIT, and argues that a strategy whereby the central bank announces AIT but implements IT ex post dominates both IT and AIT with commitment. To optimally implement this time-inconsistent strategy, the central bank wants to announce AIT with different horizons depending on the state of the economy. Ambiguous communication allows such flexibility.

In this section, we assume the central bank can successfully convince the private sector of its intention to implement AIT, although it deviates from its announcement ex post. In Section 4, we assess this assumption and investigate whether the central bank can fool the public consistently.

3.1 Time Inconsistency

Once the central bank convinces the private sector of its intended policy rule, the Phillips curve is fixed. The central bank can then improve welfare ex post by picking the point on the Philips curve that minimizes the period loss of social welfare in equation (2.2), and we formalize this idea in the following corollary:

Corollary 1 *The strategy whereby the central bank announces AIT but implements IT ex post dominates both IT and AIT.*

The equilibrium for this strategy is given by

$$\pi_t = \left(\frac{\kappa}{1-\beta a_\pi} + \lambda \frac{1-\beta a_\pi}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} u_t \tag{3.1}$$

$$\hat{y}_t = \left[\left(\frac{\kappa}{1-\beta a_\pi} + \lambda \frac{1-\beta a_\pi}{\kappa} \right)^{-1} \frac{\lambda}{\kappa} \frac{1-\beta a_\pi}{\kappa} - \frac{1}{\kappa} \right] u_t. \tag{3.2}$$

See Appendix A.3 for derivations.

We explain the intuition with Figure 2, where an equilibrium is the tangent point between an objective function, which could be the AIT objective or social welfare, and the constraint imposed by a Phillips curve. We discuss one state variable at a time.

First, after non-zero lagged inflation, equations (3.1) - (3.2) suggest the central bank deviates from AIT to the equilibrium of dual stabilization $\pi_t = x_t = 0$ (the red dot in the right panel of Figure 2). This new equilibrium has the same zero loss as the IT equilibrium (black circle), which improves welfare from the AIT equilibrium (blue diamond) that involves a positive loss.

Next, in response to a cost-push shock, in general, the central bank picks a different equilibrium than the equilibrium under AIT with commitment described in equations (2.4) - (2.5), or the red star could be different from the blue diamond in the left panel of Figure 2. The new equilibrium (the red star) is on the tangent point between the more favorable Phillips curve (the blue solid line) and the period loss of social welfare (the ellipses). Therefore, it dominates both IT, which is on the worse Phillips curve (the black dashed line), and AIT, where the central bank minimizes the AIT loss in equation (2.1) but not the period welfare.

3.2 Flexibility Motive of Ambiguous Communication

The time-inconsistent nature of AIT, which is discussed in Section 3.1, leads to a flexibility motive of ambiguous communication; that is, the central bank would like to announce AIT of different horizons depending on the state of the economy. As discussed, the central bank always maximizes the period welfare ex post regardless of its announcement. Therefore, comparing equilibria between announcing AIT with different horizons L is equivalent to comparing their Phillips curves, which are characterized by

$$\pi_t = \frac{\kappa}{1 - \beta a_{\pi,1}^{(L)}} \hat{y}_t + \sum_{l=1}^{L-2} \frac{\beta a_{\pi,l+1}^{(L)}}{1 - \beta a_{\pi,1}^{(L)}} \pi_{t-l} + \frac{1}{1 - \beta a_{\pi,1}^{(L)}} u_t, \tag{3.3}$$

where the coefficients are solved in Appendix C.1.

3.2.1 Optimal Multi-Period AIT

To make our comparison across different L meaningful, we need to first discipline $\lambda^{cb}(L)$, which determines how much weight the central bank puts on the output gap. Our results that compare IT with two-period AIT thus far hold for any $\lambda^{cb}(2)$. But $\lambda^{cb}(L)$ is no longer inconsequential when we compare AIT of different horizons.

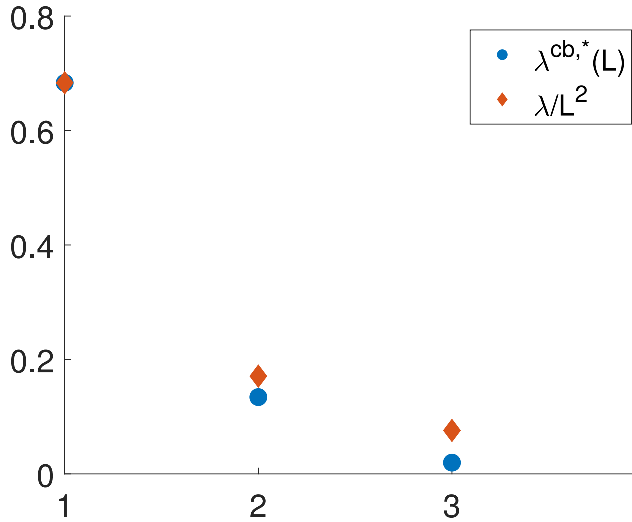
We choose the optimal $\lambda^{cb,*}(L)$ for each L such that it minimizes the unconditional welfare loss

$$\mathbb{E}_0 \mathcal{L}_t = \frac{1}{2} \left(var[\pi_t] + \lambda var[\hat{y}_t] \right), \tag{3.4}$$

where unconditional variances of inflation and the output gap are calculated using the AIT equilibrium conditions in equations (2.4) - (2.5); for details, see Appendix C.2. We use the optimal $\lambda^{cb,*}(L)$ hereafter.

The optimal $\lambda^{cb,*}(L)$ is decreasing in L; see the blue dots in Figure 4. We can show the intuition for why $\lambda^{cb,*}(2) < \lambda^{cb,*}(L = 1) = \lambda$ with the left panel of Figure 2. The AIT equilibrium (blue diamond) is associated with higher inflation and a smaller output gap than the equilibrium where the central bank announces AIT but implements IT (red star). The reason is that the AIT central bank in equation (2.1) puts more weight on the output gap relative to current inflation than the social welfare objective function does when we assign $\lambda^{cb}(2) = \lambda$. To compensate for this effect, the optimal AIT needs to assign a weight that is

Figure 4: $\lambda^{cb}(L)$ for multi-period AIT



Notes: $\lambda^{cb}(L)$ (y-axis) as a function of L (x-axis). Blue dots: $\lambda^{cb}(L) = \lambda^{cb,*}(L)$; red diamonds: $\lambda^{cb}(L) = \lambda/L^2$.

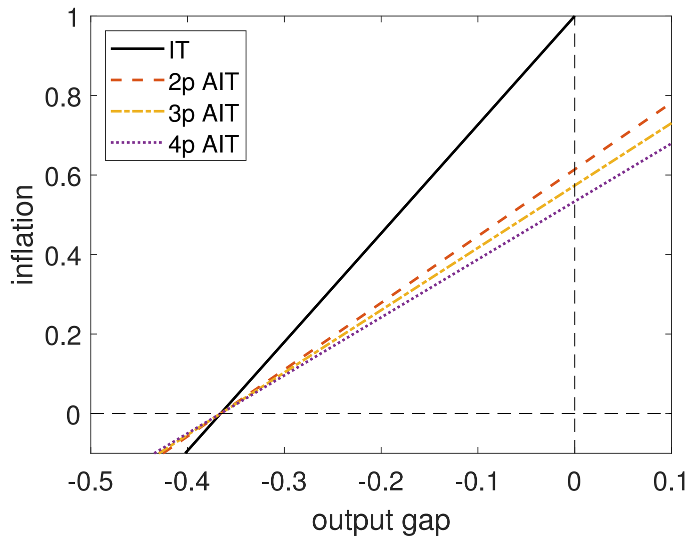
smaller than λ on the output gap.

For a general L, the weight that the central bank's period loss puts on current inflation is $1/L^2$, which decreases in L; see equation (2.1). To compensate for this, $\lambda^{cb,*}(L)$ needs to decrease in L as well. In Figure 4, we compare $\lambda^{cb,*}(L)$ with $\lambda^{cb}(L) = \lambda/L^2$. They are slightly different because π_t also enters $\mathbb{E}_t \mathbb{L}^{cb}_{t+1}(L)$, which makes $\lambda^{cb,*}(L)$ and λ/L^2 not identical. However, like the intuition, all our results go through if we set $\lambda^{cb}(L)$ to λ/L^2 instead of $\lambda^{cb,*}(L)$.

3.2.2 Ambiguous Communication and Optimal AIT

Suppose the economy only faces a cost-push shock at t. The central bank's optimal strategy is to announce the largest feasible L at time t and announce $L = 2$ subsequently. To show why, we begin with period t, for which the intercept of the Phillips curve (3.3), $\frac{1}{1-\beta a^{(L)}_{\pi,1}} u_t$, is a decreasing function of L for a positive shock; see Figure 5. In other words, the Phillips curve with the largest L ($L = 4$ years in this example) is associated with the strongest effect of the expectations channel and the best available trade-off between inflation and the output

Figure 5: Reduced-form Phillips curve for multi-period AIT



Notes: $u_t = 1, \pi_{t-1} = 0$. Units: percentage points. Black solid line: IT. Colored dashed lines are AIT with $L \in \{2, 3, 4\}$. $\lambda^{cb}(L) = \lambda^{cb,*}(L)$.

gap. Hence, the largest L forms the optimal horizon for the AIT announcement at time t.

At time $t + 1$, it is optimal for an AIT central bank to announce $L = 2$. In this case, the Phillips curve does not have an intercept; see equation (3.3). Consequently, we can achieve the equilibrium of zero inflation and a zero output gap. By contrast, if the central bank announces an $L > 2$, non-zero past inflation causes a non-zero intercept in the Phillips curve, which implies a non-zero loss in the period social welfare. At time $t + h$, the central bank can announce any $L \in \{2, ..., h + 1\}$ and implement IT to achieve dual stability.

However, switching between different L does not seem to be attainable under reasonable assumptions. We argue in Section 4 that agents might believe the central bank when its announcement is different from the actual implementation. But agents will not believe the central bank when it constantly changes its announcements. The fact that the central bank would like the flexibility to change its announcements about the length of AIT over time but cannot argues for ambiguous communication.

4 Social Learning and Credibility Motive of Ambiguous Communication

Section 3 demonstrates AIT's time-inconsistent nature and discusses the flexibility motive for ambiguous announcement about the horizon of AIT. In Section 3, we assume the central bank has full credibility. This section assesses this assumption and focuses on the following questions: Can agents eventually learn the truth about the central bank's behavior? Is the central bank's welfare-enhancing strategy sustainable and beneficial over time in light of learning? Section 4.1 sets up the environment of social learning. Section 4.2 argues ambiguous communication improves the central bank's credibility, and Section 4.3 explains when and why AIT can nowcast better than IT, which is the central bank's de facto strategy. Section 4.4 shows our results are robust regarding initial credibility and uncertainty about underlying economic conditions.

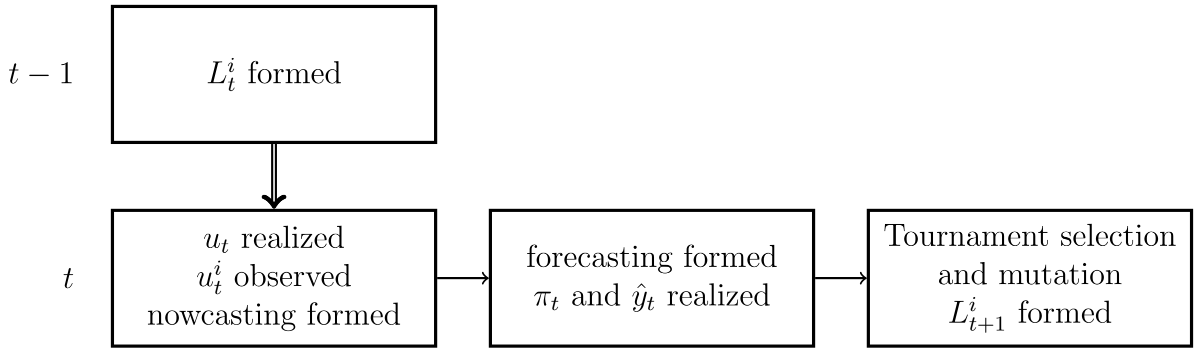
4.1 Social Learning

We model social learning similar to Arifovic, Bullard, and Kostyshyna (2013) and Hachem and Wu (2017). Agents are grouped by their beliefs about $L \in \{1, 2, ...L_{max}\}$, and beliefs can also be heterogeneous within each group. Agents update their beliefs via tournament selection and mutation. We assume each agent believes all other agents in the economy have the same information set that he/she does, whereas the central bank observes everything.

The sequence of events is drawn in Figure 6. Before entering period t, each agent has a belief about L, and we label it L_t^i. When agents enter period t, a shock u_t is realized, and agent i observes it with a private signal u_t^i:

$$u_t^i = u_t + \upsilon_t^i, \qquad \upsilon_t^i \sim N(0, \sigma_\upsilon^2). \tag{4.1}$$

Figure 6: The sequence of events



Within-group heterogeneity is present when $\sigma_v \neq 0$.[4] With the new information, agent i updates his/her expectations about aggregate variables at time t using equations (2.4) - (2.5):

$$\mathbb{E}(\pi_t|\mathcal{I}_{t-1}, L_t^i, u_t^i) = a_{\pi,1}^{(L_t^i)}\pi_{t-1} + ... + a_{\pi,L_t^i-1}^{(L_t^i)}\pi_{t-L_t^i+1} + b_\pi^{(L_t^i)}u_t^i \tag{4.2}$$

$$\mathbb{E}(\hat{y}_t|\mathcal{I}_{t-1}, L_t^i, u_t^i) = a_{y,1}^{(L_t^i)}\pi_{t-1} + ... + a_{y,L_t^i-1}^{(L_t^i)}\pi_{t-L_t^i+1} + b_y^{(L_t^i)}u_t^i, \tag{4.3}$$

where $\mathcal{I}_{t-1}$ is the realized equilibrium path up to time $t-1$. The nowcasting is used for social learning in the last subperiod in Figure 6.

Next, we describe the central bank's problem in the second subperiod, including how macro aggregates realize and agents form their forecasts. Agent i forms expectations about inflation in the next period:

$$\mathbb{E}_t^i\pi_{t+1} \equiv \mathbb{E}(\pi_{t+1}|\mathcal{I}_t, L_t^i) = a_{\pi,1}^{(L_t^i)}\pi_t + ... + a_{\pi,L_t^i-1}^{(L_t^i)}\pi_{t-L_t^i+2}. \tag{4.4}$$

The average expectation is

$$\bar{\mathbb{E}}_t\pi_{t+1} = \frac{1}{N}\sum_i \mathbb{E}_t^i\pi_{t+1}, \tag{4.5}$$

[4]We discuss the role of uncertainty, captured by σ_v, in Section 4.4.2.

where N is the total number of agents.

Therefore, the Phillips curve in equation (2.3) becomes

$$\pi_t = \beta \bar{\mathbb{E}}_t \pi_{t+1} + \kappa \hat{y}_t + u_t. \tag{4.6}$$

The central bank now picks π_t and $\hat{y}_t$ to minimize the period loss $\mathcal{L}_t$ defined in equation (2.2) subject to the Phillips curve in equation (4.6). The realized laws of motion for π_t and $\hat{y}_t$ are different from equations (2.4) - (2.5) and can be found in Appendix C.3.

Tournament selection works as follows: at each time period t, agents are randomly selected to meet in pairs. When two agents meet, they update their beliefs by comparing nowcast errors, which are defined as

$$\varepsilon_t^i = |\, \mathbb{E}(\pi_t | \mathcal{I}_{t-1}, L_t^i, u_t^i) - \pi_t| + w|\, \mathbb{E}(\hat{y}_t | \mathcal{I}_{t-1}, L_t^i, u_t^i) - \hat{y}_t|, \tag{4.7}$$

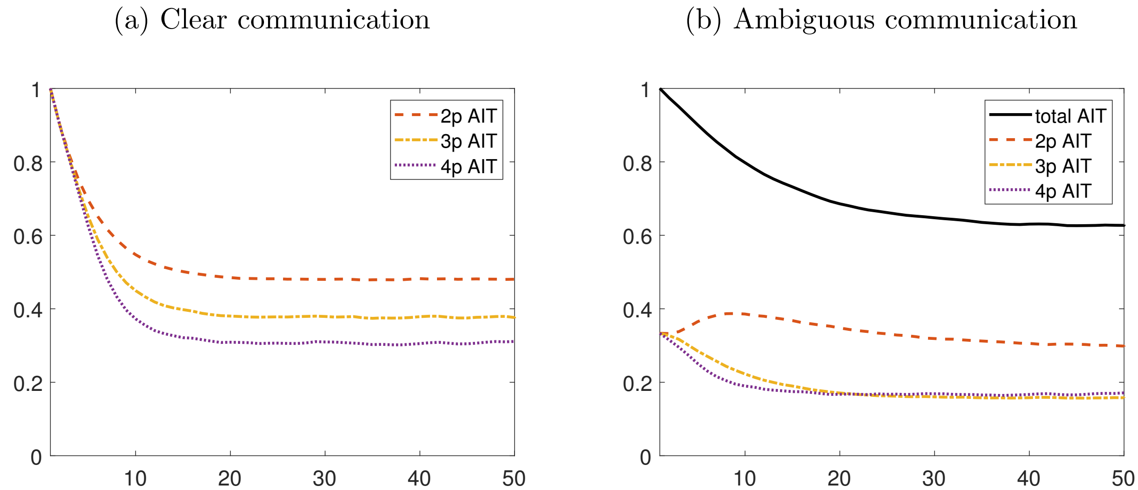
where we use nowcasting expectations in equations (4.2) - (4.3), and w is the weight agents put on the nowcast error on the output gap.[5] When two agents are from the same group, that is, $L_t^i = L_t^j$, they stay in this group. When two agents come from two different groups, suppose agent i has a smaller nowcast error $\varepsilon_t^i < \varepsilon_t^j$. Agent i stays in his/her current group, whereas agent j switches. At the end of period t, agents can mutate, that is, randomly switch to another rule. Mutation is a standard device in the literature to avoid the algorithm getting stuck in a corner solution prematurely. This step further updates the belief to L_{t+1}^i, which will be used at time $t+1$. Parameter values are in Appendix D.

4.2 Credibility Motive of Ambiguous Communication

This section demonstrates that ambiguous communication helps improve the central bank's credibility. In this section, we assess the full credibility assumption we made in Section 3

[5]We use equal weight $w = 1$ in the benchmark case and discuss the implications of different weighting schemes in Section 4.3.

Figure 7: Fraction of AIT believers



Notes: X-axis: horizon. Left panel: L is clearly communicated; right panel: AIT is announced without explicitly specifying L. Red dashed lines: fraction of two-period AIT believers, yellow dash-dotted lines: fraction of three-period AIT believers; purple dotted lines: fraction of four-period AIT believers. Black solid line in the right panel: fraction of total AIT believers. All the results are averaged over 1,000 simulations. $\lambda^{cb}(L) = \lambda^{cb,*}(L)$.

and ask if the central bank will eventually lose its following because agents figure out that the central bank deviates from its promise and implements IT ex post. For this purpose, we endow the central bank with full credibility. We further assess this assumption and show the robustness of our results in Section 4.4.1 when the central bank starts with no or partial credibility.

We define the fraction of agents in each group as $p^{(L)}$. The left panel of Figure 7 plots the fraction of AIT believers when the central bank announces AIT with a clear horizon $L \in (2, 3, 4)$ for two, three, or four years. In each case, there are only two groups of beliefs: IT and L-period AIT, or $p^{(1)} + p^{(L)} = 1$. The red dashed, the yellow dash-dotted, and the purple dotted lines represent a central bank that announces two-period, three-period, or four-period AIT, respectively. All our results are averaged over 1,000 simulations. At each point in time, the fraction of AIT believers is the highest when the central bank announces two-period AIT and lowest when the central bank announces four-period AIT.

The right panel of Figure 7 plots the evolution of beliefs under ambiguous communication, where the central bank only announces AIT as its tool to manage inflation but does not specify its length. At the beginning, an equal fraction of agents believe in AIT with different lengths: $p^{(2)} = p^{(3)} = p^{(4)}$. Over time, these fractions evolve through social learning with $p^{(1)} + p^{(2)} + p^{(3)} + p^{(4)} = 1$. The red dashed, yellow dash-dotted, and purple dotted lines capture the fraction of agents in each AIT group. The black solid line is the relevant metric, which is the sum of the three colored lines and captures the total fraction of AIT believers.

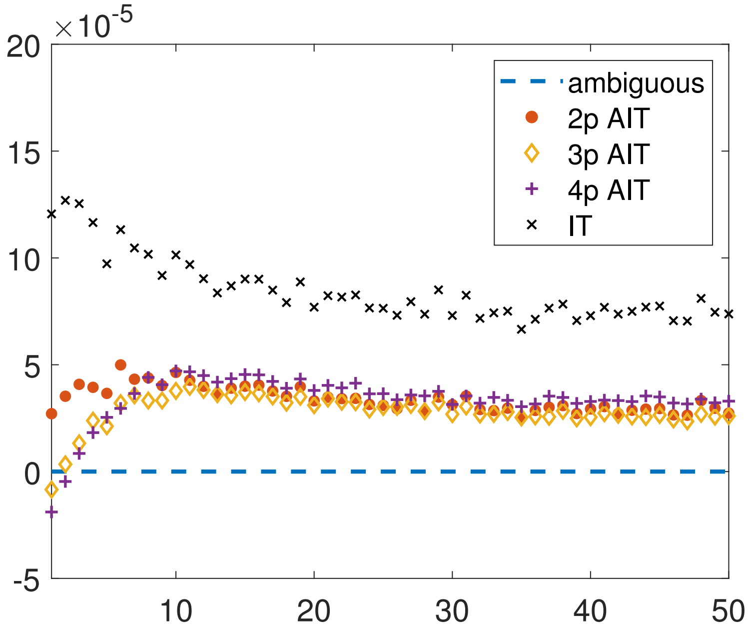
Comparing the black solid line in the right panel with each of the lines in the left panel, we find that ambiguous communication allows a larger fraction of agents to believe in the AIT strategy that the central bank announces, even though it implements IT ex post. Therefore, ambiguous communication helps the central bank gain more credibility in the long run.

Why does ambiguous communication help the central bank's credibility? The reason is that ambiguous communication gives the central bank's believers a bigger choice set. Depending on economic conditions, agents with different AIT beliefs might perform the best at different points in time. For example, 2-period AIT might outperform IT at time t_1. While it underperforms at time t_2 relative to IT, 3-period AIT outperforms both. If the central bank announces 2-period AIT, it gains believers at period t_1 but loses some at t_2. By contrast, if the central bank makes ambiguous communication, it gains credibility for both periods.

Comparing the three colored lines in the right panel, similar to the left panel, more agents believe in two-period AIT than in three- or four-period AIT. Notice that in the left panel, the three colors represent three different types of announcements. For each announcement, only the corresponding color is relevant, whereas in the right panel, there is only one type of announcement - ambiguous announcement - but agents can have different beliefs about L in this scenario.

We have established that ambiguous communication can help the central bank gain a larger following of its communicated AIT than clear communication. But does it improve welfare in the long run? In Figure 8, we plot the period loss of social welfare for five

Figure 8: Welfare loss



Notes: X-axis: horizon. y-axis: welfare loss. Blue dashed line: ambiguous communication, which is normalized to zero. Red dots: the central bank announces two-period AIT; yellow diamonds: the central bank announces three-period AIT; purple plus signs: the central bank announces four-period AIT; black crosses: IT. All the results are averaged over 1,000 simulations. $\lambda^{cb}(L) = \lambda^{cb,*}(L)$

different announcements. We take the ambiguous communication in the blue dashed line as the benchmark and plot loss in other cases relative to it. Therefore, the blue dashed line coincides with the horizontal axis by construction.

Initially, four-period AIT, which is represented by the purple "+"s, has the smallest loss. This result is consistent with what we find in Section 3.2: it is optimal for the central bank to announce the largest L on impact of the shock because it offers the best Phillips curve. But the advantage of announcing a large L disappears quickly, because four-period AIT has a smaller following over time; see Figure 7. At some point, we see a smaller welfare loss if the central bank announces $L = 3$ instead of $L = 4$.[6] This finding is consistent with the flexibility motive we discussed in Section 3.2. Two competing forces are at play: a

[6]Notice we interpret time in a relative sense instead of an absolute sense and focus our discussion on the long run. Although we calibrate structural parameters to an annual frequency, the time to converge depends on the specifics of social learning, which are not the focus of the paper.

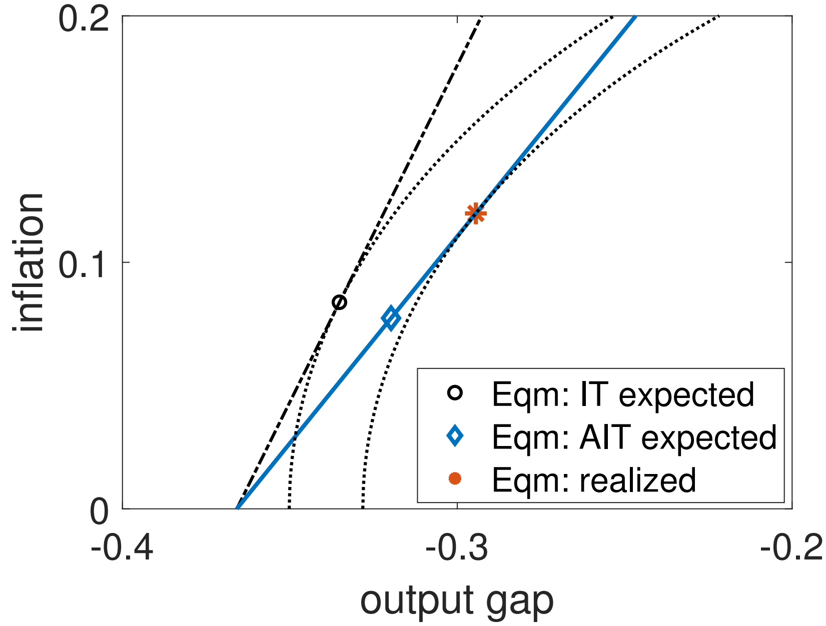


Figure 9: Expected and realized equilibria

Notes: $u_t = 1, \pi_{t-1} = 0$. Units: percentage points. Black circle: the equilibrium expected by the IT believers; blue diamond: the equilibrium expected by the AIT believers; red star: the realized equilibrium where the private sector forms expectations based on the two-period AIT announcement but the central bank deviates to implement IT ex post. Lines are Phillips curves. The ellipses are period social welfare. $\lambda^{cb}(2) = \lambda^{cb,*}(2)$.

larger L is associated with a more favorable Phillips curve but fewer followers. However, the central bank does not have to face this trade-off and can take advantage of ambiguous communication, which is associated with the smallest loss for all but the initial few periods; see the blue dashed line.

Comparing IT in the black "x"s with the rest of the colors, announcing IT and sticking to it, which is the operating framework before introducing AIT, is universally the worst in terms of welfare. This result is consistent with our argument in Corollary 1.

4.3 Why Can AIT Perform Better When the Truth Is IT

In Section 4.2, we showed that AIT believers' nowcast could potentially outperform IT believers'. The question is why, especially when the central bank always implements IT ex post.

We use Figure 9 to illustrate the intuition.[7] The red star marks the realized equilibrium after a positive cost-push shock in which all agents believe the central bank's announcement about two-period AIT but the central bank implements IT. The black circle and blue diamond mark the expected equilibria of the IT believers and the AIT believers. Neither AIT believers nor IT believers predict perfectly; or the red star does not overlap with either the blue diamond or the black circle. The IT believers have a smaller prediction error on inflation, whereas AIT believers have a smaller error on the output gap.

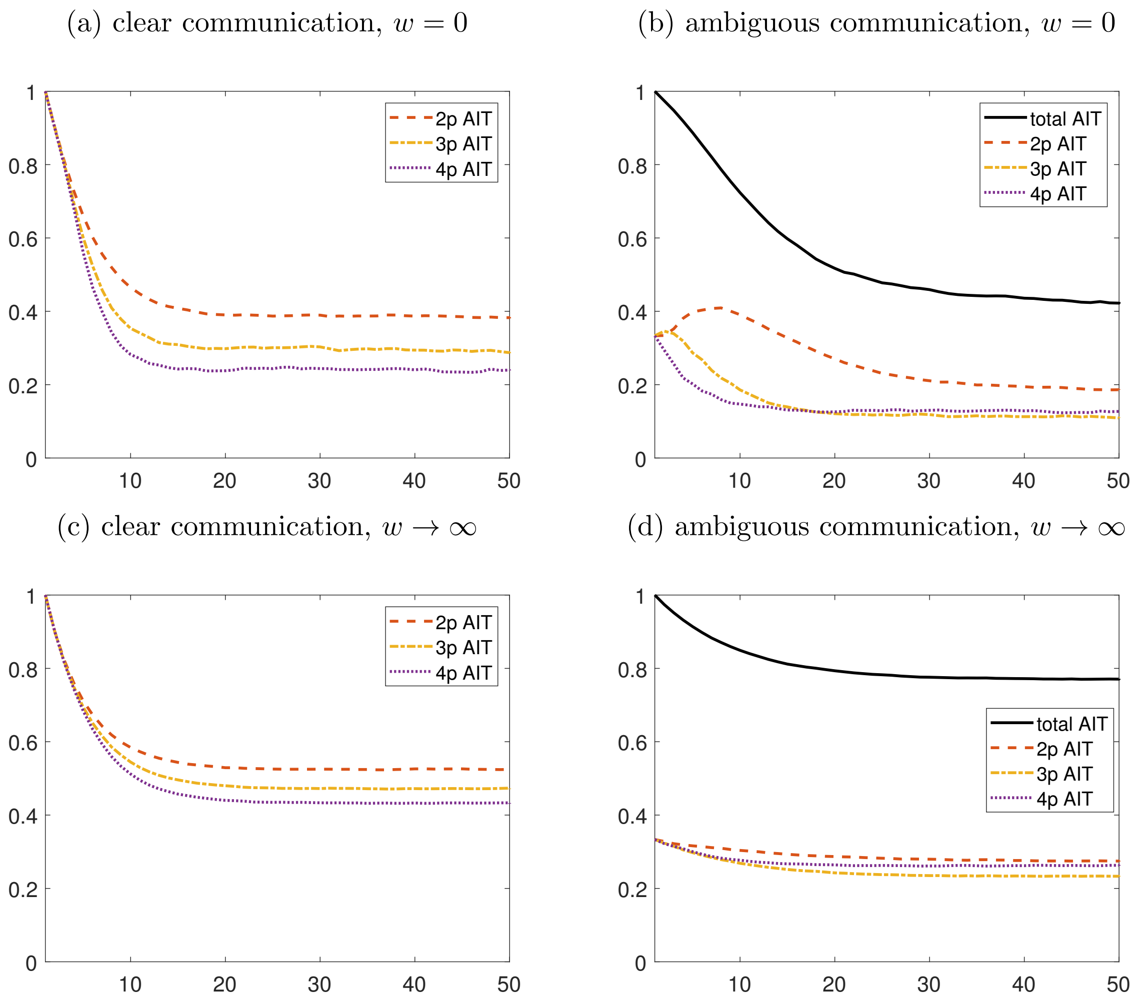
Although the central bank implements IT ex post, AIT believers could have a smaller nowcast error if agents put more weight on the output gap. We further elaborate on this point in Figure 10, where we extend the results in Figure 7 with two extreme cases: in the top panels, agents put all the weight on inflation, or $w = 0$; in the bottom panels, agents put all the weight on the output gap, or $w \to \infty$. Similar to Figure 7, the left panels are associated with a central bank that announces the horizon of AIT explicitly, whereas in the right panels, the central bank announces AIT without specifying the horizon.

Comparing the top panels with the bottom panels, we find that AIT has a larger following when agents put more weight on the output gap; that is, larger w. This result holds regardless of whether the central bank communicates the horizon of AIT clearly or ambiguously, which confirms the intuition provided by Figure 9.

Figure 10 also serves as a robustness check on the results in Figure 7. We compare the left panels with the right panels. In the long run, the central bank always arrives with more credibility when using ambiguous communication than when using clear communication; compare the black lines in the right panels with the colored lines in the left panels. This result further confirms the credibility motive of ambiguous communication and shows that the qualitative results in Figure 7 do not depend on how agents weigh nowcast errors between inflation and the output gap.

[7] Figure 9 and the left panel of Figure 2 only differ by $\lambda^{cb}(2)$ and scale.

Figure 10: Different weights in the nowcast error



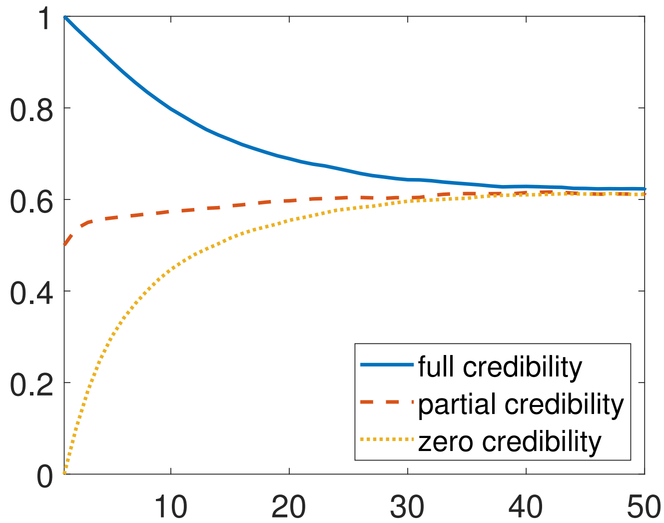
Notes: X-axis: horizon. Left panels: L is clearly communicated; right panels: AIT is announced without explicitly specifying L. Top panels: $w = 0$; bottom panels: $w \to \infty$. Red dashed lines: fraction of two-period believers, yellow dash-dotted lines: fraction of three-period believers; purple dotted lines: fraction of four-period AIT believers. Black solid line in the right panels: fraction of total AIT believers. All the results are averaged over 1,000 simulations. $\lambda^{cb}(L) = \lambda^{cb,*}(L)$

4.4 Robustness

4.4.1 Initial Credibility

Our analysis so far in this section starts with a central bank that is endowed with full credibility, which is a stark assumption. For example, Coibion et al. (2020) use survey data to show that the private sector does not understand the impact of AIT soon after its introduction. In this section, we assess what happens when the central bank is partially

Figure 11: Fraction of AIT believers with different initial credibility



Notes: X-axis: horizon. Blue solid line: the central bank starts with full credibility; red dashed line: the central bank starts with partial credibility; yellow dotted line: the central bank starts with zero credibility. All the results are averaged over 1,000 simulations. $\lambda^{cb}(L) = \lambda^{cb,*}(L)$

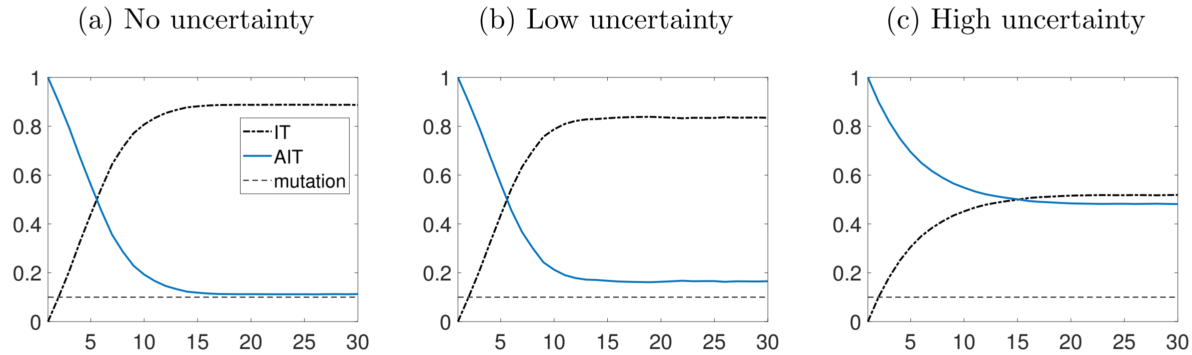
credible or not credible initially.

In Figure 11, we plot the evolution of the fraction of AIT believers with different levels of initial credibility. The blue solid line starts with 100 percent of agents believing AIT, which replicates the black line in the right panel of Figure 7. The red dashed line starts with partial credibility, where 50 percent of agents believe AIT, and the yellow dotted line starts with no credibility. In the long run, all three lines converge to the same point, where over 60 percent of agents believe AIT. In summary, the initial credibility does not matter for the long run.

4.4.2 Uncertainty

In this section, we show that uncertainty about underlying economic conditions does not change results qualitatively, although it does quantitatively. Figure 12 plots three scenarios: no uncertainty is present in the left panel, or $\sigma_v = 0$. The middle panel is associated with low uncertainty, and the right panel with high uncertainty. We compare IT with two-period AIT,

Figure 12: Fraction of IT/AIT believers



Notes: X-axis: horizon. Black dashed lines: fraction of IT believers $p^{(1)}$; blue solid lines: fraction of AIT believers $p^{(2)}$. Left: no uncertainty with $\sigma_\upsilon = 0$; middle: low uncertainty with $\sigma_\upsilon = 0.01$; right: high uncertainty with $\sigma_\upsilon = 0.1$. All the results are averaged over 1,000 simulations. $\lambda^{cb}(L) = \lambda^{cb,*}(L)$.

and the fractions of their believers are in black dashed lines for IT and blue solid lines for AIT. When uncertainty increases, more agents believe AIT in the long run, and convergence takes longer.

However, the following results are robust regardless of whether agents observe the underlying shocks with uncertainty. First, AIT has followers even with no uncertainty. In the left panel of Figure 12, a non-trivial fraction of agents believe the AIT announcement; that is, the fraction is higher than the mechanical mutation probability. Second, ambiguous communication allows the central bank to have more credibility than clear communication, regardless of the value of σ_υ.[8]

5 Conclusion

Our paper studies the implications of AIT. We focus on two key issues: time inconsistency and ambiguous communication. AIT can improve the available trade-off between inflation and real activity that the central bank faces, but a more favorable Phillips curve does not automatically improve social welfare. To improve social welfare, the central bank has the incentive to deviate from its communicated objective and implement IT ex post. The time-

[8] To save space, we do not show plots.

inconsistent nature calls for two motives for ambiguous communication. First, the optimal strategy for the central bank is to announce different horizons for AIT at different points in time, and ambiguous communication offers an implementable alternative to such self-conflicting announcements. Second, the time-inconsistent strategy is welfare improving, assuming the central bank can convince the private sector of its intention to implement AIT. We assess this assumption using social learning. We show that ambiguous communication helps the central bank gain credibility and improve welfare in the long run despite AIT being time inconsistent. These results do not rely on whether the central bank has credibility initially, how agents weigh nowcast errors between inflation and the output gap, or if agents observe economic shocks with uncertainty.

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A Derivations of Equilibria

A.1 Two-Period AIT Under FIRE

We solve the equilibrium with the method of undetermined coefficients. We guess the following linear functions for π_t and $\hat{y}_t$:

$$\pi_t = a_\pi \pi_{t-1} + b_\pi u_t \tag{A.1}$$
$$\hat{y}_t = a_y \pi_{t-1} + b_y u_t, \tag{A.2}$$

which yield

$$\mathbb{E}_t \, \pi_{t+j} = a_\pi^j \pi_t \tag{A.3}$$
$$\mathbb{E}_t \, \hat{y}_{t+j} = a_y a_\pi^{j-1} \pi_t. \tag{A.4}$$

Substitute the expression for $\mathbb{E}_t \, \pi_{t+1}$ from (A.3) into the Phillips curve in (2.3) and rearrange, and we get (2.12). Consequently, the slope of the Phillips curve is

$$\frac{\partial \pi_t}{\partial \hat{y}_t} = \frac{\kappa}{1 - \beta a_\pi}. \tag{A.5}$$

Rearranging (2.12), we get

$$\hat{y}_t = \frac{1}{\kappa} \left(1 - \beta a_\pi\right) \pi_t - \frac{1}{\kappa} u_t. \tag{A.6}$$

Rewrite the central bank's objective function in (2.1) by substituting out the expectation terms using (A.3) - (A.4):

$$\begin{aligned} \mathbb{L}_t^{cb}(2) &= \frac{1}{2}\left(\left(\frac{\pi_t + \pi_{t-1}}{2}\right)^2 + \lambda^{cb} \hat{y}_t^2\right) + \frac{1}{2} \sum_j \mathbb{E}_t \, \beta^j \left(\left(\frac{\pi_{t+j} + \pi_{t+j-1}}{2}\right)^2 + \lambda^{cb} \hat{y}_{t+j}^2\right) \\ &= \frac{1}{2}\left(\left(\frac{\pi_t + \pi_{t-1}}{2}\right)^2 + \lambda^{cb} \hat{y}_t^2\right) + \frac{1}{2} \pi_t^2 \sum_j \beta^j \left(\left(\frac{a_\pi + 1}{2}\right)^2 a_\pi^{2(j-1)} + \lambda^{cb} a_y^2 a_\pi^{2(j-1)}\right). \end{aligned} \tag{A.7}$$

Minimize the central bank's objective function with respect to $\hat{y}_t$, which yields the following first-order condition (using (A.5) - (A.6)):

$$\begin{aligned} 0 &= \frac{1}{4}\left(\pi_t + \pi_{t-1}\right) \frac{\kappa}{1 - \beta a_\pi} + \lambda^{cb} \left(\frac{1}{\kappa}\left(1 - \beta a_\pi\right) \pi_t - \frac{1}{\kappa} u_t\right) \\ &\quad + \pi_t \frac{\kappa}{1 - \beta a_\pi} \sum_j \beta^j \left(\left(\frac{a_\pi + 1}{2}\right)^2 a_\pi^{2(j-1)} + \lambda^{cb} a_y^2 a_\pi^{2(j-1)}\right). \end{aligned} \tag{A.8}$$

Rearrange

$$0 = \left[\frac{\kappa}{4(1-\beta a_\pi)} + \lambda^{cb}\frac{1}{\kappa}(1-\beta a_\pi) + \frac{\kappa}{1-\beta a_\pi}\sum_j \beta^j\left(\left(\frac{a_\pi+1}{2}\right)^2 a_\pi^{2(j-1)} + \lambda^{cb}a_y^2 a_\pi^{2(j-1)}\right)\right]\pi_t \quad (\text{A.9})$$
$$+\frac{\kappa}{4(1-\beta a_\pi)}\pi_{t-1} - \lambda^{cb}\frac{1}{\kappa}u_t.$$

Define

$$\vartheta \equiv \left[\frac{\kappa}{4(1-\beta a_\pi)} + \lambda^{cb}\frac{1}{\kappa}(1-\beta a_\pi) + \frac{\kappa}{1-\beta a_\pi}\sum_j \beta^j\left(\left(\frac{a_\pi+1}{2}\right)^2 a_\pi^{2(j-1)} + \lambda^{cb}a_y^2 a_\pi^{2(j-1)}\right)\right]^{-1} \quad (\text{A.10})$$

then,

$$\pi_t = -\frac{\kappa}{4(1-\beta a_\pi)}\vartheta\pi_{t-1} + \lambda^{cb}\frac{1}{\kappa}\vartheta u_t. \quad (\text{A.11})$$

Comparing it with (A.1), we get (2.6) and (2.8).

Next, substituting (A.1) in (A.6) and rearranging,

$$\hat{y}_t = \frac{1}{\kappa}(1-\beta a_\pi)a_\pi\pi_{t-1} + \left(\frac{1}{\kappa}(1-\beta a_\pi)b_\pi - \frac{1}{\kappa}\right)u_t. \quad (\text{A.12})$$

Comparing it with (A.2), we obtain (2.7) and (2.9).

A.2 IT

The central bank minimizes the period loss of welfare in (2.2) with respect to $\hat{y}_t$, which results in the following first-order condition:

$$\pi_t\frac{\partial\pi_t}{\partial\hat{y}_t} + \lambda\hat{y}_t = 0. \quad (\text{A.13})$$

The constraint is the IT Phillips curve in (2.13). Using the Phillips curve to substitute out $\frac{\partial\pi_t}{\partial\hat{y}_t}$ and $\hat{y}_t$ in (A.13), we obtain

$$\kappa\pi_t + \frac{\lambda}{\kappa}(\pi_t - u_t) = 0, \quad (\text{A.14})$$

which is equivalent to (2.10). Combining it with (2.13) to cancel out π_t, we get (2.11).

A.3 Two-Period AIT with Time Inconsistency

The central bank minimizes the period loss of welfare in (2.2) with respect to $\hat{y}_t$, which results in the first-order condition in (A.13). The private sector believes AIT announcement.

Therefore, the Phillips curve is given by (2.12). Substituting (A.5) - (A.6) in, we obtain

$$\frac{\kappa}{1-\beta a_\pi}\pi_t + \frac{\lambda}{\kappa}\left(1-\beta a_\pi\right)\pi_t - \frac{\lambda}{\kappa}u_t = 0, \tag{A.15}$$

which is equivalent to (3.1). Substituting (3.1) into (A.6),

$$\hat{y}_t = \frac{1}{\kappa}\left(1-\beta a_\pi\right)\left(\frac{\kappa}{1-\beta a_\pi} + \lambda\frac{1-\beta a_\pi}{\kappa}\right)^{-1}\frac{\lambda}{\kappa}u_t - \frac{1}{\kappa}u_t, \tag{A.16}$$

which is equivalent to (3.2).

A.4 Two-Period AIT under FIRE with Natural Rate Shock

This appendix outlines an economy that faces both natural rate and cost-push shocks. The IS curve and the Phillips curve that characterize the economy are

$$\hat{y}_t = \mathbb{E}_t\,\hat{y}_{t+1} - \frac{1}{\gamma}\left(i_t - r_t^n - \mathbb{E}_t\,\pi_{t+1}\right) \tag{A.17}$$

$$\pi_t = \beta\,\mathbb{E}\,\pi_{t+1} + \kappa\hat{y}_t + u_t. \tag{A.18}$$

We conjecture the following equilibrium:

$$\pi_t = a_\pi\pi_{t-1} + b_\pi u_t + b_\pi^r r_t^n \tag{A.19}$$

$$\hat{y}_t = a_y\pi_{t-1} + b_y u_t + b_y^r r_t^n, \tag{A.20}$$

which leads to

$$\mathbb{E}_t\,\pi_{t+j} = a_\pi^j\pi_t \tag{A.21}$$

$$\mathbb{E}_t\,\hat{y}_{t+j} = a_y a_\pi^{j-1}\pi_t. \tag{A.22}$$

Because the expectations in (A.21) - (A.22) and the Phillips curve in (A.18) are the same as those in Appendix A.1, the remaining derivations in Appendix A.1 follow through and we find expressions for a_π, a_y, b_π, b_y as in (2.6) - (2.9).

Moreover, we obtain $b_\pi^r = 0$ and $b_y^r = 0$. Suppose the natural rate shock is the only shock in the economy, $\pi_{t+h} = \hat{y}_{t+h} = 0$ for any h. This equilibrium path can be achieved by setting the policy rate to track the natural rate:

$$i_t = r_t^n. \tag{A.23}$$

B Proofs

B.1 Proof of Lemma 1

We first prove $a_\pi < 0$ by contradiction. Suppose $a_\pi > 0$. For the system to have a stationary solution, $0 < a_\pi < 1$. Together with $0 < \beta < 1$, this implies $1 - \beta a_\pi > 0$. Together with

$\kappa > 0$ and $\lambda^{cb} > 0$, this implies $\vartheta > 0$ in (A.10). Consequently, $a_\pi = -\vartheta \frac{\kappa}{4(1-\beta a_\pi)} < 0$, and is thus a contradiction. We have proved $a_\pi < 0$, which implies $1 - \beta a_\pi > 0$, and it follows that $a_y = \frac{1}{\kappa}(1 - \beta a_\pi) a_\pi < 0$.■

B.2 Proof of Proposition 1

The expectation term in the forward-looking Phillips curve (2.3) is $\mathbb{E}_t \pi_{t+1} = a_\pi \pi_t$ for AIT per (2.4), and $\mathbb{E}_t \pi_{t+1} = 0$ for IT per (2.10). Lemma 1 shows $a_\pi < 0$. Therefore, $a_\pi \pi_t < 0$ when $\pi_t > 0$ and $a_\pi \pi_t > 0$ when $\pi_t < 0$.■

B.3 Proof of Proposition 2

The slope of the Phillips curve of AIT in (2.12) is $\frac{\kappa}{1-\beta a_\pi}$, and that of the IT Phillips curve in (2.13) is κ. We have shown $a_\pi < 0$ in Lemma 1. Therefore, $1 - \beta a_\pi > 1$, and hence, $\frac{\kappa}{1-\beta a_\pi} < \kappa$.■

B.4 Proof of Proposition 3

The AIT Phillips curve in (2.12) can be written as

$$(1 - \beta a_\pi)\pi_t^{AIT} = \kappa \hat{y}_t + u_t,$$

where we label inflation under AIT π_t^{AIT}. Compare it with the IT Phillips curve in (2.13):

$$\pi_t^{IT} = \kappa \hat{y}_t + u_t.$$

Given $\hat{y}_t$ and u_t,

$$(1 - \beta a_\pi)\pi_t^{AIT} = \pi_t^{IT}.$$

We have shown $a_\pi < 0$ in Lemma 1. Therefore, $1 - \beta a_\pi > 1$, and

$$|\pi_t^{IT}| > |\pi_t^{AIT}|.$$

■

B.5 Proof of Lemma 2

The y-intercept of the Phillips curve of AIT in (2.12) is $\frac{1}{1-\beta a_\pi} u_t$, and that of the IT Phillips curve in (2.13) is u_t. We have shown $a_\pi < 0$ in Lemma 1. Therefore, $1 - \beta a_\pi > 1$, and hence, $\frac{1}{1-\beta a_\pi}|u_t| < |u_t|$ for $u_t \neq 0$.

Next, the x-intercept of the Phillips curve of AIT in (2.12) is characterized by $0 = \frac{\kappa}{1-\beta a_\pi}\hat{y}_t + \frac{1}{1-\beta a_\pi} u_t$, or $0 = \kappa \hat{y}_t + u_t$. For the IT Phillips curve in (2.13), it is also $0 = \kappa \hat{y}_t + u_t$. Hence, they have the same x-intercept.■

B.6 Proof of Lemma 3

The AIT Phillips curve in (2.12) is

$$\pi_t = \frac{\kappa}{1-\beta a_\pi}\hat{y}_t,$$

when $u_t = 0$. When $\hat{y}_t = 0, \pi_t = 0$.

Similarly, the IT Phillips curve in (2.12) is

$$\pi_t = \kappa \hat{y}_t,$$

when $u_t = 0$. When $\hat{y}_t = 0, \pi_t = 0$.■

C Multi-Period AIT

C.1 Solving Equilibrium under FIRE

We solve multi-period AIT following the standard solution method of rational-expectation models with linear-quadratic optimization, which is developed in Oudiz and Sachs (1985) and further discussed in Svensson (2007).

The equilibrium in the private sector, which serves as the constraint of the central bank's optimization problem, is given by

$$\begin{bmatrix} X_{t+1} \\ \mathbb{E}_t\, x_{t+1} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + C\epsilon_{t+1}, \tag{C.1}$$

where X_t are the predetermined state variables, x_t are the forward-looking variables, and i_t are instruments.

Take conditional expectation of equation (C.1) at t:

$$\begin{bmatrix} \mathbb{E}_t\, X_{t+1} \\ \mathbb{E}_t\, x_{t+1} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t. \tag{C.2}$$

Then, we conjecture that

$$i_{t+1} = F_{t+1} X_{t+1} \tag{C.3}$$

$$x_{t+1} = G_{t+1} X_{t+1}, \tag{C.4}$$

where F_{t+1} and G_{t+1} are determined by the problem in period $t+1$ but are known in period t.

Then, (C.4) and the upper block of (C.2) imply

$$\mathbb{E}_t\, x_{t+1} = G_{t+1}\, \mathbb{E}_t\, X_{t+1} = G_{t+1}(A_{11}X_t + A_{12}x_t + B_1 i_t) \tag{C.5}$$

where

$$A \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B \equiv \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad C \equiv \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}. \tag{C.6}$$

The lower block is

$$\mathbb{E}_t \, x_{t+1} = A_{21} X_t + A_{22} x_t + B_2 i_t. \tag{C.7}$$

Suppose

$$x_t = \bar{A}_t X_t + \bar{B}_t i_t. \tag{C.8}$$

Equating (C.5) and (C.7) and using (C.8), we get

$$\bar{A}_t = (A_{22} - G_{t+1} A_{12})^{-1} (G_{t+1} A_{11} - A_{21}) \tag{C.9}$$

$$\bar{B}_t = (A_{22} - G_{t+1} A_{12})^{-1} (G_{t+1} B_1 - B_2). \tag{C.10}$$

The upper block of (C.1) and (C.8) imply

$$X_{t+1} = \tilde{A}_t X_t + \tilde{B}_t i_t + C_1 \epsilon_{t+1}, \tag{C.11}$$

where

$$\tilde{A}_t = A_{11} + A_{12} \bar{A}_t \tag{C.12}$$

$$\tilde{B}_t = B_1 + A_{12} \bar{B}_t. \tag{C.13}$$

The period loss in the central bank's objective function is given by

$$L_t = \frac{1}{2} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}' W \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}. \tag{C.14}$$

Use (C.8), and we obtain

$$\mathcal{L}_t = \frac{1}{2} \begin{bmatrix} X_t \\ i_t \end{bmatrix}' \begin{bmatrix} Q_t & N_t \\ N_t' & R_t \end{bmatrix} \begin{bmatrix} X_t \\ i_t \end{bmatrix}, \tag{C.15}$$

where

$$Q_t = W_{XX} + W_{Xx} \bar{A}_t + \bar{A}_t' W_{xX} + \bar{A}_t' W_{xx} \bar{A}_t \tag{C.16}$$

$$N_t = W_{Xx} \bar{B}_t + \bar{A}_t' W_{xx} \bar{B}_t + W_{Xi} + \bar{A}_t' W_{xi} \tag{C.17}$$

$$R_t = W_{ii} + \bar{B}_t' W_{xx} \bar{B}_t + \bar{B}_t' W_{xi} + W_{ix} \bar{B}_t, \tag{C.18}$$

and

$$W \equiv \begin{bmatrix} W_{XX} & W_{Xx} & W_{Xi} \\ W_{xX} & W_{xx} & W_{xi} \\ W_{iX} & W_{ix} & W_{ii} \end{bmatrix}. \tag{C.19}$$

Since the period loss function is quadratic and the constraint is linear, the value function is quadratic and satisfies the following Bellman equation:

$$\frac{1}{2}\left[(1-\beta)\,X_t' V_t X_t + \beta w_t\right] = \min_{i_t}\left\{(1-\beta)L_t + \beta\,\mathbb{E}_t\,\frac{1}{2}\left[(1-\beta)X_{t+1}' V_{t+1} X_{t+1} + \beta w_{t+1}\right]\right\} \tag{C.20}$$

where V_{t+1} is a positive semidefinite matrix and w_{t+1} is a scalar, and both are known at time t.

Solve the first-order condition subject to (C.11) and (C.14):

$$0 = X_t' N_t + i_t' R_t + \beta\left(X_t' \tilde{A}_t' V_{t+1} \tilde{B}_t + i_t' \tilde{B}_t' V_{t+1} \tilde{B}_t\right). \tag{C.21}$$

Therefore,

$$i_t = F_t X_t, \tag{C.22}$$

which is consistent with the conjecture in (C.3) and

$$F_t = -\left(R_t + \beta \tilde{B}_t' V_{t+1} \tilde{B}_t\right)^{-1}\left(N_t' + \beta \tilde{B}_t' V_{t+1} \tilde{A}_t\right). \tag{C.23}$$

Substitute i_t in (C.8) with (C.22) and get

$$x_t = G_t X_t, \tag{C.24}$$

which is consistent with the conjecture in (C.4) and

$$G_t = \bar{A}_t + \bar{B}_t F_t. \tag{C.25}$$

Substituting (C.11), (C.14), and (C.22) into (C.20), we get

$$V_t = Q_t + N_t F_t + F_t' N_t' + F_t' R_t F_t + \beta\left(\tilde{A}_t + \tilde{B}_t F_t\right)' V_{t+1}\left(\tilde{A}_t + \tilde{B}_t F_t\right). \tag{C.26}$$

(C.9), (C.10), (C.12), (C.13), (C.16) - (C.18), (C.23), (C.25), and (C.26) define a mapping from (G_{t+1}, V_{t+1}) to (G_t, V_t), which also determine F_t.

Iterate the coefficients backward from (G_{t+1}, V_{t+1}) to (G_t, V_t) until they converge to a fixed point:

$$G = G_{t+1} = G_t \tag{C.27}$$

$$V = V_{t+1} = V_t. \tag{C.28}$$

Finally, the equilibrium conditions are

$$x_t = G X_t. \tag{C.29}$$

C.1.1 Example: $L = 3$

The state variables and forward-looking variables are

$$X_t = \begin{bmatrix} u_t \\ \pi_{t-2} \\ \pi_{t-1} \end{bmatrix}, \quad x_t = \begin{bmatrix} \pi_t \\ \hat{y}_t \end{bmatrix}, \tag{C.30}$$

and there is only one policy instrument in i_t, and $\epsilon_t = u_t$.

Write out linear system

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} & 1 \end{bmatrix} \begin{bmatrix} u_{t+1} \\ \pi_{t-1} \\ \pi_t \\ \mathbb{E}_t\, \pi_{t+1} \\ \mathbb{E}_t\, \hat{y}_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & -\kappa \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_t \\ \pi_{t-2} \\ \pi_{t-1} \\ \pi_t \\ \hat{y}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\gamma} \end{bmatrix} i_t + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_{t+1} \tag{C.31}$$

Map into (C.1),

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 & -\kappa \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\gamma} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \frac{1}{\gamma} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Next, the objective function maps into (C.14) with

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda^{cb}(L=3) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{C.32}$$

C.2 Unconditional Variances

This appendix solves the unconditional variances in (3.4). First, we write the equilibrium in (2.4) - (2.5) into the companion form:

$$X_t = \Phi X_{t-1} + \Sigma u_t \tag{C.33}$$

where

$$X_t = \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \vdots \\ \pi_{t-L+2} \\ \hat{y}_t \end{bmatrix}, \Phi = \begin{bmatrix} a^{(L)}_{\pi,1} & a^{(L)}_{\pi,2} & \dots & a^{(L)}_{\pi,L-2} & a^{(L)}_{\pi,L-1} & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 \\ a^{(L)}_{y,1} & a^{(L)}_{y,2} & \vdots & a^{(L)}_{y,L-2} & a^{(L)}_{y,L-1} & 0 \end{bmatrix}, \Sigma = \begin{bmatrix} b^{(L)}_{\pi} \\ 0 \\ \vdots \\ 0 \\ b^{(L)}_{y} \end{bmatrix} \tag{C.34}$$

and the coefficients are solved in Appendix C.1.

Take unconditional variances,

$$var(X_t) = \Phi var(X_{t-1})\Phi' + \Sigma var(u_t)\Sigma'. \tag{C.35}$$

The unconditional variance $var(X_t) = var(X_{t-1})$, and consequently,

$$vec\,(var(X_t)) = (I - \Phi \otimes \Phi)^{-1}\, vec\,(\Sigma var(u_t)\Sigma')\,, \tag{C.36}$$

where vec is the vectorization of a matrix, and I is an identity matrix.

C.3 Learning

Instead of writing out the general form, we write out how we implement the model where $\max L^i = 4$. Regardless of L^i_t, we can always write out agent i's inflation forecast as follows:

$$\mathbb{E}^i_t\, \pi_{t+1} = a^{(L^i_t)}_{\pi,1}\pi_t + a^{(L^i_t)}_{\pi,2}\pi_{t-1} + a^{(L^i_t)}_{\pi,3}\pi_{t-2}. \tag{C.37}$$

Then, the average expectation in the economy is

$$\bar{\mathbb{E}}_t\pi_{t+1} = \bar{a}_{\pi,1}\pi_t + \bar{a}_{\pi,2}\pi_{t-1} + \bar{a}_{\pi,3}\pi_{t-2}. \tag{C.38}$$

Substitute (C.38) into the Phillips curve in (2.3) and get

$$\pi_t = (1-\beta\bar{a}_{\pi,1})^{-1}\beta\bar{a}_{\pi,2}\pi_{t-1} + (1-\beta\bar{a}_{\pi,1})^{-1}\beta\bar{a}_{\pi,3}\pi_{t-2} + (1-\beta\bar{a}_{\pi,1})^{-1}\kappa\hat{y}_t + (1-\beta\bar{a}_{\pi,1})^{-1}u_t. \tag{C.39}$$

The central bank minimizes the current period welfare loss (2.2) subject to the Phillips curve (C.39). The first-order condition is

$$\pi_t\frac{\kappa}{1-\beta\bar{a}_{\pi,1}} + \lambda\hat{y}_t = 0. \tag{C.40}$$

Rewrite the Phillips curve in (C.39),

$$\hat{y}_t = \frac{1-\beta\bar{a}_{\pi,1}}{\kappa}\pi_t - \frac{\beta\bar{a}_{\pi,2}}{\kappa}\pi_{t-1} - \frac{\beta\bar{a}_{\pi,3}}{\kappa}\pi_{t-2} - \frac{1}{\kappa}u_t. \tag{C.41}$$

Substitute the expression of $\hat{y}_t$ into the first-order condition (C.40) and get the realized equilibrium:

$$\begin{aligned}
\pi_t &= \Delta\beta\bar{a}_{\pi,2}\pi_{t-1} + \Delta\beta\bar{a}_{\pi,3}\pi_{t-2} + \Delta u_t && \text{(C.42)} \\
\hat{y}_t &= \left(\frac{1-\beta\bar{a}_{\pi,1}}{\kappa}\Delta\beta\bar{a}_{\pi,2} - \frac{\beta\bar{a}_{\pi,2}}{\kappa}\right)\pi_{t-1} + \left(\frac{1-\beta\bar{a}_{\pi,1}}{\kappa}\Delta\beta\bar{a}_{\pi,3} - \frac{\beta\bar{a}_{\pi,3}}{\kappa}\right)\pi_{t-2} \\
&\quad + \left(\frac{1-\beta\bar{a}_{\pi,1}}{\kappa}\Delta - \frac{1}{\kappa}\right)u_t, && \text{(C.43)}
\end{aligned}$$

where $\Delta = \left(\frac{\kappa}{1-\beta\bar{a}_{\pi,1}} + \lambda\frac{1-\beta\bar{a}_{\pi,1}}{\kappa}\right)^{-1}\frac{\lambda}{\kappa}$.

D Calibration

D.1 Structural Parameters

We calibrate the model at an annual frequency, and our parameter values are in line with the standard New Keynesian literature; see parameters in Table D.1. The top panel displays the structural parameters we calibrate, some of which do not show up in the linear equations in our model, and the bottom panel displays implied parameter values. We set $\beta = 0.95$ to match the annual return on safe assets in the US. We assume a log utility for consumption, $\gamma = 1$, and a unitary Frisch elasticity of labor supply, $\varphi = 1$. We choose η, the price elasticity of demand, to be 4. We set the Calvo parameter, which governs price stickiness, to be $\frac{1}{3}$, implying the average duration between price changes is 1.5 years. We assume the cost-push shock is *i.i.d.* with a standard deviation of $\sigma_\epsilon = 0.1$.

With these underlying parameters, the slope of the Phillips curve, κ, is 2.73, and the weight on output gap stabilization in the welfare function, λ, is 0.6833.

Table D.1: Structural parameters

parameter	description	value
β	discount factor	0.95
γ	elasticity of intertemporal substitution	1
φ	elasticity of labor supply	1
η	the price elasticity of demand	10
θ	price rigidity	$\frac{1}{3}$
σ_ϵ	std. dev. of cost-push shock	0.1
κ	slope of the Phillips curve	2.73
λ	weight in welfare	0.6833

D.2 Learning Parameters

We summarize the learning parameters in Table D.2. We set the number of agents in the private sector to be 1,000. We further assume the number of random meetings per period is

1,000. Our results are reported as the average from 1,000 simulations. We follow Arifovic, Bullard, and Kostyshyna (2013) by setting the mutation probability to 10 percent. For the parameter that governs the level of uncertainty, we use $\sigma_v = 0.1$ for high uncertainty, which is our benchmark case, 0.01 for low uncertainty, and 0 for no uncertainty.

Table D.2: Learning parameters

parameter	description	value
N	number of agents	1,000
	number of simulations	1,000
	number of meetings every period	1,000
	probability of mutation	0.1
σ_v	std. dev. in private signals	0.1 (benchmark) / 0.01 (low)/ 0 (no)