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The Welfare Costs of Business Cycles Unveiled: Measuring the Extent of Stabilization Policies*

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Abstract

How can we measure the welfare benefit of ongoing stabilization policies? We develop a methodology to calculate the welfare cost of business cycles taking into account that observed consumption is partially smoothed. We propose a decomposition that disentangles consumption in a mix of laissez-faire (absent policies) and riskless components. With a novel identification strategy, we estimate the span of stabilization power. Our results show that the welfare cost of total fluctuations is 11 percent of lifetime consumption, of which 82 percent is smoothed by the status quo policies, yielding a residual 1.8 percent of consumption to be tackled by policymakers.

Keywords: Business Cycles, Consumption, Stabilization, Macroeconomic History
JEL Classifications: E32, E21, E63, N10

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1 Introduction

In a tough challenge to conventional wisdom, Lucas (1987) asked how much Americans would be willing to pay, in terms of consumption, to live in an economy that is not subject to the macroeconomic volatility that the US witnessed during the post-war period. Finding that a representative consumer would sacrifice at most one-tenth of a percent of lifetime consumption, Lucas concluded that there would be little benefit in further attempting to stabilize the residual risk of business cycles.

Not surprisingly, Lucas's seminal result attracted a great deal of controversy and generated a wealth of literature that revisits his estimates. In this paper, we explore a critical point, which is subtly present in Lucas (1987), that calls for a new measurement effort when estimating the costs of business cycles: all observed consumption is already partially smoothed. That is, the data that we gather for consumption stem from a realized allocation that is subject to the status quo of economic stabilization policies.

In order to measure the contribution of ongoing policies as well as the relevance of the residual to be smoothed, we then need to disentangle which part of the observed consumption pertains to each category. To accomplish such a task, we propose a tractable decomposition in which observed consumption is a weighted geometric mean of laissez-faire consumption, i.e., the counterfactual consumption series in the absence of any policy and a riskless consumption sequence.

Our decomposition allows us to map all policies to a single parameter θ, which we define as the span of stabilization power. Within this structure, we are able to prove that the welfare cost of total economic fluctuations can be disentangled into the benefit of ongoing policies and the cost of residual fluctuations. We dialogue directly with the classic literature and use the flexibility of this approach to apply our formulation to three types of shock structures for the consumption process: the one of Lucas (1987) with transitory shocks, the one of Obstfeld (1994) with permanent innovations, and a third one that departs from the i.i.d. structure and uses an ARIMA process for the consumption series as proposed by Reis (2009), which we are able to incorporate in our framework with the use of the Beveridge-Nelson decomposition (Beveridge and Nelson, 1981; Issler et al., 2008; Guillén et al., 2014).

We then proceed to estimate the parameters in our welfare decomposition but hit a measurement challenge: since the laissez-faire consumption is not observable, we need to identify θ. For this task, we resort to the more novel literature of identification in macroeconomics and couple it with the relevant facts of US macroeconomic history. Our choice of data is an augmented version of the historical consumption series provided by Barro and Ursúa (2010), which shows a significant decrease in volatility after WWII. Such pattern is identified by the literature and the visual inspection of the data, as well as by a statistical test to find breaks in the variance of the series.

This evidence allow us to design our identification strategy and divide the sample into pre- and post-war periods with distinct measured volatilities, attributing them to the larger role and presence of stabilization policies in the second period (Blanchard, 2000). Such a discontinuity-based strategy enable us to pin down the span of stabilization policies from 1947 until today, which we then take back as an input in our decomposition. Assuming a log-normal form for consumption, we obtain the results for all three shock structures, but our preferred specification is the one stemming from the ARIMA process, which, among the three considered, best models and fits the consumption data.

We estimate that the span of stabilization policies smooths 61 to 73 percent of the laissez-faire consumption shocks. With that, the cost of total economic fluctuations is 11 percent of consumption.[1] Close to 82 percent of such costs are already covered by stabilization policies, yielding that more than 9 percent of smoothed lifetime consumption is left unveiled if one does not take into account the benefit of ongoing stabilization. Finally, we observe a theoretical feature that arises from the concave nature of the utility: the more risk averse consumers are, the more they value the relative benefit of ongoing stabilization policies.

[1]Our estimates are robust to different cuts in the data, including the removal of the inter-war period and the years surrounding a statistically identified structural break.

2 Related Literature

Our paper is embedded in three major strands of the literature in macroeconomics: (i) the large body of work concerned with the calculus of the welfare costs of business cycles; (ii) the literature that studies the measurement of historical macroeconomic data; and (iii) the literature of identification in macroeconomics.

Several papers build on Lucas's departing point and relax some of his assumptions. For example, Obstfeld (1994) switches the original transitory shocks for permanent ones and focuses on its interaction with recursive preferences, Reis (2009) further develops the time-series aspects, while Issler et al. (2008) and Guillén et al. (2014) combine both types of shocks.[2] Another block in this body departs from the representative agent setting and estimates the costs under incomplete markets and heterogeneous agents such as in İmrohoroğlu (1989), Krusell and Smith Jr. (1999), Storesletten et al. (2001), and De Santis (2007). More recently, Hai et al. (2020) include memorable goods and Constantinides (2021) focuses on the role of idiosyncratic shocks faced by households that are unrelated to the business cycle. Our contribution here is to model the observed consumption as a partially smoothed series and the proposal of a new and tractable decomposition that allows us to disentangle the reach of the ongoing policies.

We conduct our data analysis grounding it in the literature on macroeconomic history. Our sample is built directly from the historical data compiled by Barro and Ursúa (2010) and when developing our novel identification strategy, we base it on Barro and Ursúa (2008)'s observation that for the OECD economies, there is a change in consumption volatility in the post-war period. Our approach also dialogues with the seminal work of Romer (1986) and Balke and Gordon (1989) that documents the challenges faced when measuring the volatility of macroeconomic aggregates and show how our methodology can reconcile improvements in both measurement and stabilization after WWII.

We also view our work as building on the effort of calculating the costs of business cycles, with critical attention to measurement and identification that often appeared in

[2] For an in-depth early discussion of this literature, see Barlevy (2005), who discusses other seminal references such as Dolmas (1998) and Alvarez and Jermann (2004).

what became known as the "disasters" approach in the literature. We resort to Nakamura et al. (2013)'s insight of using the variation in the volatility of the consumption series to better identify the shift in the role of stabilization policies. Moreover, we build on Nakamura et al. (2017) in our use of both transitory and permanent formulations for the shocks in conjunction with a time-varying volatility for the consumption series. At the intersection of the disasters and welfare costs literature, Jorda et al. (2020) find that substantial costs may arise from a novel estimate of frequent and small disasters.[3] In addition, by considering the asymmetric nature of economic fluctuations, Dupraz et al. (2019) develop a plucking model of business cycles and find welfare gains from eliminating economic fluctuations that are an order of magnitude larger than in standard models.

3 Model

3.1 Environment and Definitions

The economy is populated by a representative consumer whose lifetime utility is given by $\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t)\right]$, where C_t is consumption in period t, $\beta \in (0,1)$ is an intertemporal discount factor, $u(\cdot)$ is the instantaneous utility function, and $\mathbb{E}_0[\cdot]$ is the expectation operator conditional on the information set $\mathcal{I}_0$.[4] We begin with a few definitions:

Definition 1. *Define $\bar{C}_t \equiv \mathbb{E}_0[C_t]$. Then $\{\bar{C}_t\}_{t=0}^{\infty}$ is the riskless consumption sequence.*

Definition 2. *Define $\tilde{C}_t$ as consumption in the absence of stabilization policies. Then $\{\tilde{C}_t\}_{t=0}^{\infty}$ is the laissez-faire consumption sequence.*

We can now define the welfare cost of the total economic fluctuations as the constant $\lambda^T > 0$ that solves the following condition:

[3] Other examples in this literature are Barro and Jin (2011) and Gourio (2012).

[4] We assume that the expectation is taken before the realization of any uncertainty in period 0, as in some calculations done by Obstfeld (1994) and Reis (2009). In that sense, consumption in that period is treated as a stochastic variable. Under this assumption we compare the expected utility in two worlds where the agent is still uncertain about all consumption flows, as in Lucas (1987).

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda^T)\tilde{C}_t\right)\right]=\sum_{t=0}^{\infty}\beta^t u\left(\bar{C}_t\right). \tag{1}$$

The parameter λ^T measures the constant compensation required by the consumer to be indifferent between the adjusted laissez-faire, $\{(1+\lambda^T)\tilde{C}_t\}_{t=0}^{\infty}$, and the riskless consumption sequences.

Given that the observed time series on consumption is subject to the ongoing stabilization policies, we can view it as the combination of two extreme cases: (i) the (non-observed) consumption series in the absence of any stabilization policies, $\tilde{C}_t$, and (ii) the (non-observed) perfectly smoothed consumption, $\bar{C}_t$. We then model the (observed) partially smoothed consumption as a weighted geometric average:

$$C_t\left(\theta\right)\equiv\bar{C}_t^{\theta}\tilde{C}_t^{1-\theta}, \tag{2}$$

where the parameter $\theta\in[0,1]$ measures the degree of consumption smoothing. Thus, θ can be interpreted as the span of the stabilization power of governmental policies.

We can now define the benefit of the ongoing stabilization policies as the constant $\lambda^B>0$ that solves the following condition:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda^B)\tilde{C}_t\right)\right]=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t(\theta))\right]. \tag{3}$$

The parameter λ^B is the compensation required by the consumer to be indifferent between the adjusted laissez-faire consumption sequence and the effective consumption sequence, $\{C_t(\theta)\}_{t=0}^{\infty}$.

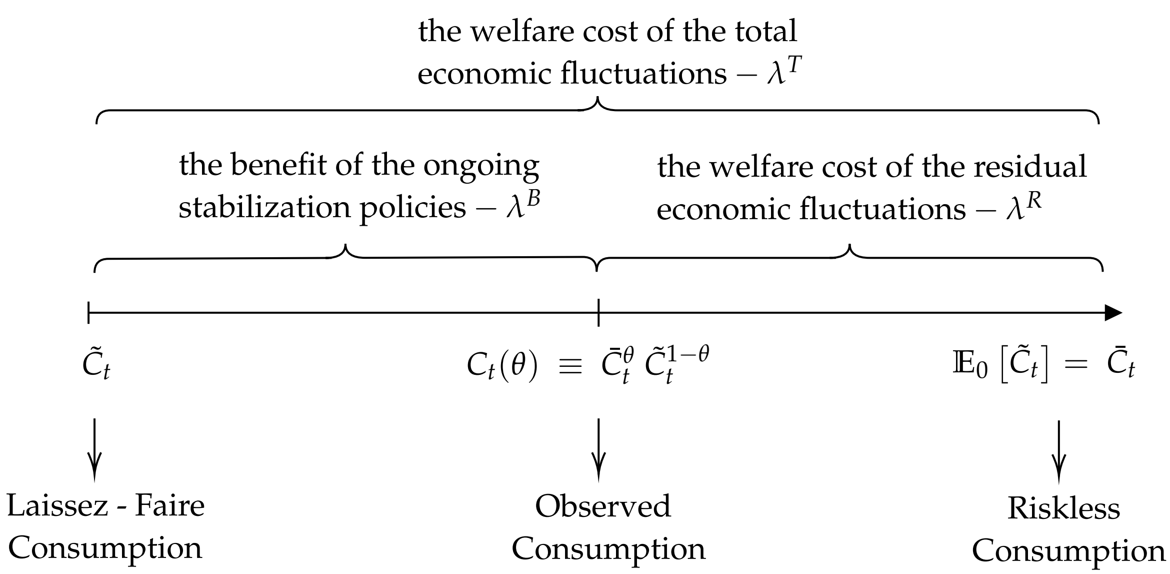
Finally, we can compute what is left to be stabilized by defining the welfare cost of the residual economic fluctuations as the constant $\lambda^R>0$ that solves the following condition:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda^R)C_t(\theta))\right)\right]=\sum_{t=0}^{\infty}\beta^t u\left(\bar{C}_t\right). \tag{4}$$

The parameter λ^R measures the constant compensation required by the consumer to be indifferent between the adjusted partially smoothed consumption sequence $\{(1 + \lambda^R)C_t(\theta)\}_{t=0}^{\infty}$ and the aforementioned riskless sequence.

Figure 1 summarizes our modelling by showing where each parameter and measure defined is located in a spectrum of consumption that spans the highest to the lowest level of risk.

Figure 1: Decomposition of the welfare cost of the total economic fluctuations



3.2 Assumptions

In order to calculate λ^T, λ^B, and λ^R and guarantee tractability, we assume a log-normal process for $\tilde{C}_t$, which implies that $C_t(\theta)$ is also log-normal. Following Lucas (1987), we assume a CRRA instantaneous utility with parameter γ:

$$u(C) = \begin{cases} \dfrac{C^{1-\gamma}}{1-\gamma}, \text{ if } \gamma > 1 \\ \ln(C), \text{ if } \gamma = 1 \end{cases} \tag{5}$$

We also need assumptions that guarantee that the sums in conditions (1), (3), and (4) are all finite. They are:

Assumption 1. *Log-normal consumption process:* $\tilde{C}_t = \alpha_0(1+\alpha_1)^t X_t$, *where* $X_t = e^{x_t - 0.5\sigma_t^2}$, *with* $x_t|\mathcal{I}_0 \sim \mathcal{N}\left(0, \sigma_t^2\right)$.

Assumption 2. *The constant* $\Gamma \equiv \beta\left(1+\alpha_1\right)^{1-\gamma} \in (0,1)$.

Assumption 3. $\sum\limits_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\} < \infty$.[5]

Under Assumption 1, riskless consumption is given by $\bar{C}_t = \mathbb{E}_0[\tilde{C}_t] = \alpha_0(1+\alpha_1)^t$ and is deterministic. Furthermore, $\tilde{C}_t = \bar{C}_t X_t$, and the partially smoothed consumption can be rewritten as $C_t\left(\theta\right) = \bar{C}_t X_t^{1-\theta}$. From this formulation it is easy to see that the larger the parameter θ, the less important is the stochastic part of the partially smoothed consumption.

3.3 Theoretical Results

We can now derive closed-form solutions for the parameters λ^B, λ^R and λ^T. Propositions 1, 2, and 3 establish, respectively, each of these parameters. The final step consists of using the propositions to obtain our main decomposition of the welfare cost of total economic fluctuations. All proofs are shown in Appendix A.

Proposition 1. *Under Assumptions 1 and 3 the benefit of the ongoing stabilization policies is given by*

$$\lambda^B = \begin{cases} \exp\left\{\theta \frac{1-\beta}{2} \sum_{t=0}^{\infty} \beta^t \sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}}{\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{6}$$

[5]Note that $\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\} < \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}$ if $\gamma > 1$. This result ensures that the λ's are finite in some of our results.

Proposition 2. *Under Assumptions 1, 2, and 3 the welfare cost of the residual macroeconomic fluctuations is given by*

$$\lambda^R = \begin{cases} \exp\left\{(1-\theta)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{7}$$

Proposition 3. *Under Assumptions 1, 2, and 3 the welfare cost of the total economic fluctuations is given by*

$$\lambda^T = \begin{cases} \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1, & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t \exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{8}$$

We can now state our main result in Theorem 1 below: the decomposition of the welfare cost of total economic fluctuations.

Theorem 1. *Under Assumptions 1 to 3 and CRRA utility (5), there is a decomposition of the welfare cost of total economic fluctuations in the form*

$$1 + \lambda^T = \left(1 + \lambda^B\right)\left(1 + \lambda^R\right). \tag{9}$$

4 Applications

In this section we characterize λ^T, λ^B, and λ^R using three different shock structures for the consumption process: the classic ones of Lucas (1987) with transitory shocks and of Obstfeld (1994) with permanent shocks, and one with an ARIMA process for consumption as proposed in Reis (2009) using the Beveridge-Nelson (BN) decomposition (Beveridge and Nelson, 1981; Issler et al., 2008; Guillén et al., 2014). The details of all calculations are shown in Appendix B.

Example 1 - Transitory Shocks (Lucas, 1987): Define $C_t = \alpha_0(1+\alpha_1)^t e^{-0.5\sigma_\varepsilon^2 + x_t^L}$, where $x_t^L|\mathcal{I}_0 \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. Hence,

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{\gamma}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{10}$$

$$\lambda^B = \begin{cases} \exp\left\{\frac{\theta}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{\gamma}{2}\sigma_\varepsilon^2 - \frac{1}{2}(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{11}$$

$$\lambda^R = \begin{cases} \exp\left\{\frac{1-\theta}{2}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{\frac{1}{2}(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma > 1 \end{cases} \tag{12}$$

For this process, the variance in Assumption 1 becomes $\sigma_t^2 = \sigma_\varepsilon^2$. Consequently, Assumption 3 is satisfied as long as Assumption 2 holds.

Example 2 - Permanent Shocks (Obstfeld, 1994): Define $C_t = \alpha_0(1+\alpha_1)^t e^{-0.5\sigma_\varepsilon^2 + x_t^O}$, where $x_t^O = \sum_{i=0}^t \varepsilon_i$, $\varepsilon_i | \mathcal{I}_0 \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.[6] Thus,

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\right\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{13}$$

$$\lambda^B = \begin{cases} \exp\left\{\theta\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \frac{\exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}}{\exp\left\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\right\}}{1-\Gamma\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{14}$$

$$\lambda^R = \begin{cases} \exp\left\{(1-\theta)\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1, & \text{if } \gamma = 1 \\ \frac{1}{\exp\left\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}\left[\frac{1-\Gamma\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{15}$$

In this case, $\sigma_t^2 = Var_0\left[\sum_{i=0}^t \varepsilon_i\right] = (t+1)\sigma_\varepsilon^2$, and the condition $\Gamma\exp\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\} <$

[6] In some calculations, Obstfeld (1994) treats C_0 as known. We consider the case where the expectation is taken before the realization of the shock ε_0.

1 is sufficient for Assumption 3 to be valid.

Example 3 - ARIMA-BN Process: Define

$$C_t = \alpha_0(1+\alpha_1)^t \exp\left\{-\frac{1}{2}\sigma^2_{x_t^{BN}}\right\} \exp\left\{x_t^{BN}\right\} \tag{16}$$

where, to obtain x_t^{BN}, we apply the Beveridge-Nelson decomposition: given a process, $C_t = f(t) + u_t$, where $f(t)$ is deterministic and $(1-L)u_t = \psi(L)\varepsilon_t$, where $\psi(L) = \sum_{j=0}^{\infty} \psi_j L^j$.

Define $\varphi_j = -\sum_{i=j+1}^{\infty} \psi_i$. Then, $x_t^{BN} = \psi(1)\sum_{j=0}^{t} \varepsilon_j + \sum_{j=0}^{t} \varphi_j \varepsilon_{t-j}$, with $\varepsilon_j | \mathcal{I}_0 \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. We rewrite $\sigma^2_{x_t^{BN}}$ as $\tilde{\sigma}^2_{x_t^{BN}} = \rho_0 + \rho_1 t$, where[7]

$$\rho_0 \equiv \psi(1)^2 \sigma_\varepsilon^2 + 2\psi(1)\sum_{j=0}^{\infty} \varphi_{t-j}\sigma_\varepsilon^2 + \sum_{j=0}^{\infty} \varphi_{t-j}^2 \sigma_\varepsilon^2 \quad \text{and} \quad \rho_1 \equiv \psi(1)^2 \sigma_\varepsilon^2 \tag{17}$$

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\{0.5\gamma\rho_0\}\left[\frac{1-\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}}, & \text{if } \gamma > 1 \end{cases} \tag{18}$$

$$\lambda^B = \begin{cases} \exp\left\{\frac{\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \frac{\exp\{0.5\gamma\rho_0\}}{\exp\{0.5(1-\theta)(\theta+\gamma-\gamma\theta)\rho_0\}} \times & \\ \quad \times \left[\frac{1-\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\}}{1-\Gamma\exp\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\rho_1\}}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{19}$$

$$\lambda^R = \begin{cases} \exp\left\{\frac{1-\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\{0.5(1-\theta)(\theta+\gamma-\gamma\theta)\rho_0\} \times & \\ \quad \times \left[\frac{1-\Gamma\exp\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\rho_1\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, & \text{if } \gamma > 1 \end{cases} \tag{20}$$

In this case, $\Gamma\exp\{-0.5\gamma(1-\gamma)\rho_1\} < 1$ is sufficient for Assumption 3 to be valid.

[7] Here we follow Issler et al. (2008). See calculations in Appendix B.3.

5 Empirical Approach

In this section, we first develop the regressions to be estimated and present the challenge in the identification of the span of θ. We then present the strategy we implement to overcome this difficulty, allowing us to pin down the values to be used in our results.

5.1 Estimation

5.1.1 Transitory Shocks

Assuming transitory shocks under Assumption 1 and applying the logarithm to both sides of equation (2), we have that:

$$\log\left(C_t(\theta)\right) = \log\left(\alpha_0\right) - (1-\theta)0.5\sigma_\varepsilon^2 + t\log\left(1+\alpha_1\right) + (1-\theta)\varepsilon_t. \tag{21}$$

We can reinterpret (21) as a time series regression of log per capita consumption c_t with coefficients π_0 and π_1, and error u_t:

$$\log(c_t) = \pi_0 + \pi_1 t + u_t, \tag{22}$$

Note that an identification problem arises when we try to estimate the parameters in equation (21) since $(\alpha_0, \theta, \sigma_\varepsilon^2)$ are all simultaneously mapped to π_0. Furthermore, σ_ε^2 is scaled by $(1-\theta)$, which lies in the background of u_t. Only parameter α_1 is well-identified and can be directly inverted from the estimates since $\alpha_1 = \exp\left(\pi_1\right) - 1$.

5.1.2 Permanent Shocks

Considering the case where permanent shocks hit consumption, we have that:

$$\log\left(C_t(\theta)\right) = \log\left(\alpha_0\right) - (1-\theta)0.5t\sigma_\varepsilon^2 + t\log\left(1+\alpha_1\right) + (1-\theta)\sum_{i=0}^{t}\varepsilon_i. \tag{23}$$

Taking first-differences,

$$\Delta \log \left(C_t(\theta)\right) = \log \left(1 + \alpha_1\right) - (1 - \theta) 0.5 \sigma_\varepsilon^2 + (1 - \theta) \varepsilon_t. \tag{24}$$

We can re-write equation (24) as:

$$\Delta \log(c_t) = \pi_0 + u_t. \tag{25}$$

The same identification issue arises: $(\alpha_1, \theta, \sigma_\varepsilon^2)$ are behind π_0 with σ_ε^2 scaled by $(1 - \theta)$.

5.1.3 ARIMA-BN Process

Similarly, we have that:

$$\Delta \ln C_t(\theta) = \ln(1 + \alpha_1) - 0.5\rho_1 + (1 - \theta) \Delta x_t^{BN} \tag{26}$$

Hence,

$$\Delta \ln C_t(\theta) = \ln(1 + \alpha_1) - 0.5\psi(1)^2 \sigma_\varepsilon^2 + \psi(L) \tilde{\varepsilon}_t \tag{27}$$

where $\tilde{\varepsilon}_t \sim N\left(0, (1 - \theta)^2 \sigma_\varepsilon^2\right)$.

Here we use the fact that the per capita consumption series has a unit root and its first-difference is stationary.[8] Hence, we can switch to the ARMA(p, q) form:

$$\Phi(L) \Delta \ln C_t(\theta) = \Phi(1) \left[\ln(1 + \alpha_1) - 0.5\psi(1)^2 \sigma_\varepsilon^2\right] + \Theta(L) \tilde{\varepsilon}_t \tag{28}$$

At this step we estimate an ARMA(p, q) with an intercept for the first-difference of the

[8] The series is I(1) as identified by the ADF, PP, KPSS, and DF-GLS tests.

observed log consumption series. After that, we have $\hat{\Phi}(L)$ and $\hat{\Theta}(L)$ and invert the left-hand-side term to obtain:

$$\Delta \ln C_t (\theta) = \left[\ln (1 + \alpha_1) - 0.5\psi (1)^2 \sigma_\varepsilon^2\right] + \psi (L) \tilde{\varepsilon}_t \tag{29}$$

where $\hat{\psi}(L) = \hat{\Theta}(L)/\hat{\Phi}(L)$ was obtained in the estimation process.

5.2 Identification

From our previous characterization of the identification problem, we observed that the scaling of the structural parameters by θ means that the consumption series is partially smoothed due to the ongoing stabilization policies. This means that if we knew θ (or σ_ε^2) in advance, it would be possible to recover all parameters in our consumption model by running a simple regression like the ones shown previously. Since this is not possible, we need to design an identification strategy.

Our strategy consists of exploring an observed variation in the volatility of the historical consumption series in order to identify θ. We use a combination of three pieces of evidence: (i) the empirical fact documented in the literature that per capita consumption in the US became less volatile after WWII; (ii) a visual analysis in which we plot the series and observe a potentially unique break in the graph coinciding with the post-war period; and (iii) a statistical result in which we conduct a test to find any breaks in the variance series.

To apply this strategy in the data, we need to use a long series of consumption for the US. Our choice is to build on the data by Barro and Ursúa (2010). This database contains annual observations of US per capita consumption between 1834 and 2009. We complete the sequence of consumption between 2010 and 2019, maintaining their methodology and using the series available from the BEA's NIPA. Finally, we set the data in real terms to the year of 2012.[9]

[9]We use the series "Personal Consumption Expenditures" in Table 1.1.5, the price index series for the same category in Table 1.1.4, and the series"Population (midperiod thousands)" in Table 2.1 (US Bureau of

For the first factor, we follow Lucas (1987), Barro and Ursúa (2008), and Nakamura et al. (2017), who discuss and document the fact that the end of the Second World War marks a substantial decrease in the volatility of consumption over time, exhibiting a heteroskedastic pattern. The second piece of evidence with the visual analysis is depicted in Figure 2. For the third factor, we apply the iterated cumulative sums of squares (ICSS) algorithm developed by Inclan and Tiao (1994) to detect breaks in the variance of consumption growth. We use a 5 percent significance level to test for multiple breaks.[10] The ICSS algorithm identifies only one break in the variance of consumption growth indicating a sudden decrease in the volatility of consumption growth after 1947. We then profit from the approach of discontinuity-based identification as discussed in Nakamura and Steinsson (2018) and assume that no other factors, aside from the changes in stabilization policies, that affect the consumption series of the US change discontinuously at the end of WWII.

In formal terms, suppose that we have two periods of time, 1 and 2, and that $Var(\varepsilon_t) = \sigma_\varepsilon^2$ in both periods, but we observe a lower volatility in consumption in period 2. All else constant, we can attribute this difference in the measured volatility to a different span of stabilization power of policies in those periods. To see that, let θ_i and $\hat{\sigma}^2_{u,i}$ be, respectively, the stabilization power and the estimated variance of u_t in period $i \in \{1, 2\}$. Thus, we have that $\hat{\sigma}^2_{u,i} = (1-\theta_i)^2\sigma_\varepsilon^2$. If we knew θ_1 in advance, we could pin down θ_2 using the following identifying equation:

$$\hat{\theta}_2 = 1 - (1-\theta_1)\sqrt{\frac{\hat{\sigma}^2_{u,2}}{\hat{\sigma}^2_{u,1}}}. \tag{30}$$

The remaining parameter to delineate in the strategy is θ_1. For that, a natural candidate would be a period of incipient stabilization policies, i.e., one in which θ_1 is close to zero. In Figure 2, we show our identification strategy at work in the plot of the historical series

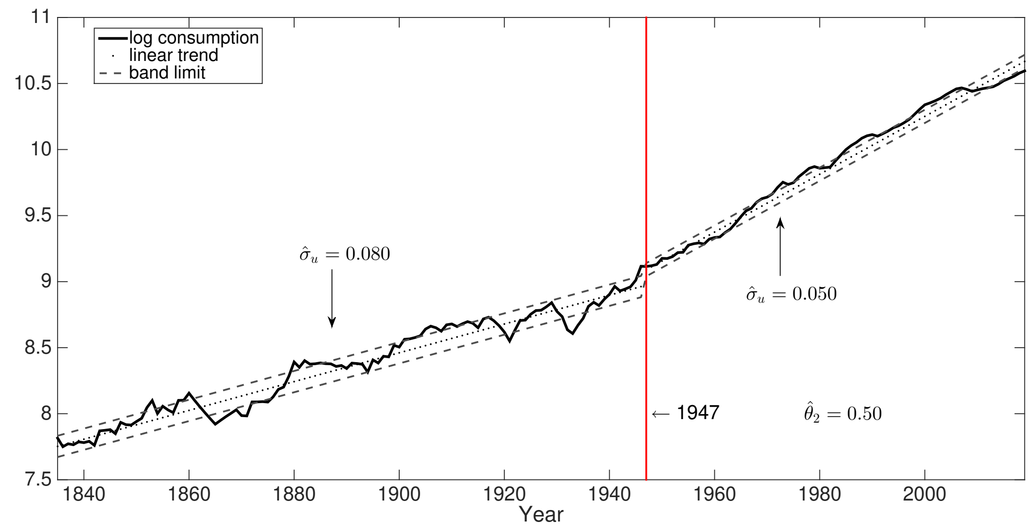
Economic Analysis, 2021a,b,c).

[10]We consider the critical value of 1.30 reported in Table 1 of Inclan and Tiao (1994) for a sample size of 200, which is the number closest to our sample. Considering the asymptotic value for the test (1.358) does not change our results.

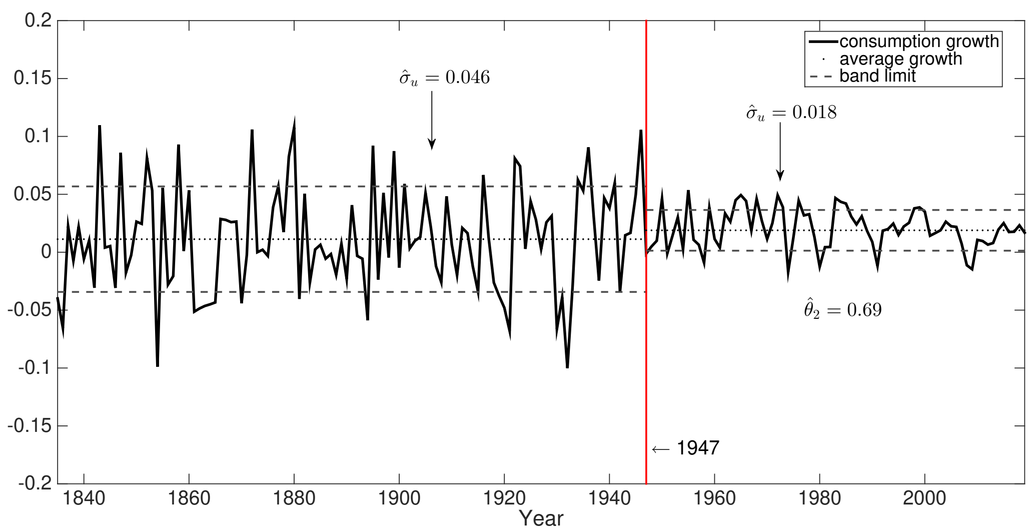
of consumption. The top panel shows the series in its log level for the identification with transitory shocks and the bottom panel shows the series' first-difference to accommodate the permanent shocks and ARIMA-BN process approaches as shown in equations (24) and (29).

If we divide the series into two periods, pre- and post-war, there is a substantial decrease in the measured standard error after 1947. Focusing on the series with first-differences in the bottom panel, for the period between 1835 and 1946, we have that $\hat{\sigma}_u = 0.046$, which then suffers a sharp decrease of more than 60 percent of its value, to $\hat{\sigma}_u = 0.018$, after WWII until today. With such a discontinuous decrease in the volatility of the series, we can plug these measures into equation (30) and, assuming $\theta_1 = 0.20$, for instance, we find that $\hat{\theta}_2 = 0.69$. This indicates a share of 69 percent of smoothed consumption in the observed series post-1947.

Figure 2: Time series of per capita consumption for the US between 1835 and 2019.



(a) Time series of log consumption.



(b) Time series of consumption growth.

Notes: The figure shows the time series for per capita consumption for the US between 1835 and 2019 with our augmented sample of the Barro and Ursúa (2010) data. There are two panels: the top one uses the series in log levels and the second in growth. The vertical line marks the year 1947, at the end of WWII. We report the standard errors for the two sub-periods generated by this line along with the average and band limits equivalent to $2\sigma_u$.

A critical point for our measurement of the decrease in consumption's standard error is the seminal argument by Romer (1986) about the spurious decrease in the unemployment rate's volatility after 1948, which was also emphasized for GNP in Balke and Gordon (1989) and revisited for GDP in the context of OECD economies by Barro and Ursúa (2008). The first differing factor in our approach is that we use the series for consumption collected by the BEA since 1929. On top of that, with our augmented sample of Barro and Ursúa (2010) data, we add an extra 90 years to the length of the original sample.

A second relevant consideration is the fact that our methodology allows us enough flexibility for a degree of discretion in the interpretation of the span during the incipient stabilization period. In equation (30), the greater θ_1, the smaller the impact of the volatility ratio in the identification of the second period's span. In that sense, the choice of θ_1 can be made larger to reflect both a historically motivated share of riskless consumption and to also take into account a certain degree of measurement error that undermined the mapping of such stabilization to the collected data.[11]

6 Empirical Results

6.1 Estimation

We run regressions (22) and the versions of (25) for permanent shocks and ARIMA-BN process – (24) and (29), respectively –, and obtain their estimated coefficients as well as the error volatility of the two distinct periods, $\hat{\sigma}^2_{u,i}$. We compute the span of stabilization power, $\hat{\theta}_2(\theta_1)$, for different levels in the grid $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, in order to allow different policy efficiencies in the initial period. With these two values at hand, we can then directly compute $\hat{\sigma}^2_{\varepsilon} = \hat{\sigma}^2_{u,1}/(1-\theta_1)^2$.

For the remaining parameters, in the case of transitory shocks we have that $\hat{\alpha}_1 = \exp(\hat{\pi}_1) - 1$. For the case of permanent shocks, $\hat{\alpha}_1 = \exp\left(\hat{\pi}_0 + (1-\hat{\theta}_2)0.5\sigma^2_{\varepsilon}\right) - 1$. Finally, for the case of the ARIMA-BN process we have that $\hat{\alpha}_1 = \exp\left(\hat{\pi}_0 + (1-\hat{\theta}_2)0.5\hat{\psi}(1)^2\sigma^2_{\varepsilon}\right) - 1$. Table 1 shows the results of our estimations.

[11] Here we also develop another subtle point mentioned in Lucas's original analysis. In Lucas (1987), footnote 4, there is a mention of Romer (1986) in which the author acknowledges that his calculations do not incorporate her findings and may rely on the 1930s experience.

Table 1: Estimated parameters.

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1946	7.7417	0.0109	0.0065	1835 - 1946	0.0113	-	0.0021	1835 - 1946	0.0113	-	0.0021
	(.0156)	(.0002)			(.0043)				(.0043)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3731	0.0222	0.0065	0.00	0.6284	0.0209	0.0021	0.00	0.6142	0.0277	0.0021
0.10	0.4358	0.0222	0.0080	0.10	0.6656	0.0209	0.0026	0.10	0.6528	0.0278	0.0026
0.20	0.4985	0.0222	0.0101	0.20	0.7027	0.0210	0.0032	0.20	0.6914	0.0278	0.0032
0.30	0.5612	0.0222	0.0132	0.30	0.7399	0.0210	0.0042	0.30	0.7300	0.0279	0.0042

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks and ARIMA-BN process. The time series used is our sample of the augmented Barro and Ursúa (2010) data with sub-periods divided at 1947. The series is I(1) as identified by the ADF, PP, KPSS, and DF-GLS tests. The first-difference of the series is identified, by both the AIC and BIC criteria, as an ARMA (0,0) for the pre-1947 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

Our preferred shock structure is the one with the ARIMA-BN process, since it is the one that most accurately models the data and allows for a more flexible structure without relying on the i.i.d. assumption of either the level or the first-difference of the series. We also focus the discussion on the results associated with our preferred choice of initial span, $\theta_1 = 0.20$, since it allows, as mentioned previously, for a combination of some degree of stabilization power and measurement error in the pre-1947 sample. The estimated span is 0.4985 with transitory shocks, 0.7027 with permanent shocks, and 0.6814 with the ARIMA-BN process.

These results show that the average post-war reach of stabilization policies is far from trivial and more than tripled after WWII. The results naturally vary according to the choice of θ_1, but with moderate sensitivity: had we considered a total absence of stabilization policies in the pre-war sample, i.e., $\theta_1 = 0$, we would have that the post-war smoothing factor would be 61 percent for the ARIMA-BN process. Moreover, for all shocks, as we increase the value of θ_1, the implied increase in $\hat{\theta}_2$ is incrementally smaller, further contributing to the robustness of the range estimated.

We relax the 1947 cutoff by conducting robustness checks with different windows of time. We find a structural break in the level of the series in 1931 and, given that, run another set of regressions, breaking the first half of the series in 1913 and in 1930 in order to allow for the break and to eliminate the highly unstable inter-war period. We find results similar to the estimates shown in Table 1. All details can be found in Appendix C.

6.2 Welfare Costs of Economic Fluctuations

With the estimated values for $\hat{\theta}_2$ and $\hat{\sigma}_\varepsilon^2$, we can now turn back to the calculation of our decomposition for λ^T, λ^B, and λ^R shown in Theorem 1. We show all our results in Table 2. The numbers are obtained by plugging in the estimates of Table 1 into equations (10) through (15). We also provide a measure that is more naturally comparable to the ones shown in the literature with the absence of the span θ, which is represented by λ^{lit} placed in the last column of the table. The derivation of this cost is straightforward and hence we leave it to Appendix B.4.

Table 2: Decomposition of the welfare cost of total economic fluctuations.

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	-
$\gamma = 1$	0.32	0.40	0.51	0.66	0.12	0.17	0.25	0.37	0.20	0.23	0.25	0.29	0.13
$\gamma = 2.5$	0.81	1.00	1.27	1.66	0.42	0.58	0.82	1.17	0.39	0.42	0.44	0.48	0.32
$\gamma = 5$	1.63	2.01	2.55	3.35	0.91	1.27	1.78	2.53	0.71	0.74	0.76	0.80	0.64
$\gamma = 7.5$	2.45	3.04	3.86	5.07	1.40	1.96	2.74	3.90	1.03	1.06	1.08	1.12	0.96
Permanent shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.63	3.92	4.99	6.65	1.64	3.02	4.11	5.74	0.97	0.88	0.84	0.86	0.36
$\gamma = 2.5$	3.25	4.89	6.33	8.91	2.15	3.92	5.40	7.97	1.08	0.94	0.88	0.88	0.52
$\gamma = 5$	4.14	6.28	8.34	13.01	2.89	5.21	7.34	11.99	1.21	1.02	0.93	0.90	0.63
$\gamma = 7.5$	5.44	8.39	11.60	15.47	4.00	7.19	10.52	14.39	1.39	1.12	0.98	0.95	0.69
ARIMA-BN													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	5.15	6.40	8.16	10.79	3.13	4.13	5.58	7.77	1.96	2.18	2.45	2.81	0.75
$\gamma = 2.5$	6.75	8.49	11.06	15.08	5.13	6.73	9.11	12.86	1.54	1.65	1.79	1.96	0.94
$\gamma = 5$	8.16	10.63	14.65	22.16	6.71	9.09	12.96	20.26	1.36	1.41	1.49	1.58	1.03
$\gamma = 7.5$	9.64	13.37	20.92	51.68	8.25	11.88	19.28	49.52	1.29	1.33	1.38	1.44	1.06

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations. The numbers are obtained using equations (10) through (20) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with a calibrated $\beta = 0.96$ for the permanent shocks and ARIMA-BN process.

We focus on the usual level of relative risk aversion used in the literature, $\gamma = 2.5$, and on the ARIMA-BN process structure. The total cost, λ^T, is 11 percent of lifetime consumption with $\lambda^B = 9.11$, or 82 percent of it represented as the benefit of ongoing policies. This leaves us with a residual of $\lambda^R = 1.79$ yet to be smoothed, almost double the value of λ^{lit}. More importantly, beyond finding high level costs for λ^T, the approach is able to unveil how much of the total welfare costs is left unaccounted for if one focuses only on the residual measures. Fixing γ at 2.5, even if we assume a zero effect of stabilization policies in pre-1947 period, there would still be 5.13 percent of lifetime consumption accruing to the benefit of ongoing policies. If we return to $\hat{\theta}_2 = 0.69$ and let $\gamma = 5$, we have that the total cost is 14.65 percent of lifetime consumption out of which 88 percent is already being stabilized.

The results also allow us to explore a theoretical aspect and understand how the concave utility interacts with our proposed decomposition and the parameter θ. If we fix a given level of the measured span of policies, the marginal benefit of smoothing the residual fluctuations in proportion to the total welfare cost, i.e., λ^R/λ^T, is decreasing in the relative degree of risk aversion. Risk-averse consumers tend to value relatively more the benefit generated by the ongoing stabilization policies, going up as much as 92 percent of the total welfare cost with the ARIMA-BN process when γ is at the highest level considered.

7 Conclusion

We revisited the long-standing issue of the welfare costs of business cycles, focusing on unveiling the extent to which ongoing stabilization policies are smoothing observed consumption. Our approach is rooted in the fact that all data we gather on consumption are subject to the policy status quo. We provided a decomposition for total macroeconomic fluctuations by disentangling them into the benefit of current policies and the residual to be flattened.

We also conducted an empirical analysis with the goal of identifying our key decomposition parameter, the span of stabilization power, using historical consumption data. Our estimate is approximately 69 percent and the welfare costs of total economic fluctuations are around 11 percent of permanent consumption, with 9 percent of it already being smoothed by ongoing policies and 1.8 percent left as a residual.

Our paper abstracts from key aspects relevant to our question, such as different types of consumption goods and agent heterogeneity. We also take a simplified view of the role of stabilization policies and technological changes in the post-war US economy. We understand that they are critical considerations and leave them for future research.

References

Alvarez, Fernando and Urban J. Jermann (2004). "Using asset prices to measure the cost of business cycles." *Journal of Political Economy*, 112(6), pp. 1223–1256. doi:10.1086/424738.

Bai, Jushan and Pierre Perron (1998). "Estimating and testing linear models with multiple structural changes." *Econometrica*, 66(1), pp. 47–78. doi:10.2307/2998540.

Bai, Jushan and Pierre Perron (2003). "Critical values for multiple structural change tests." *The Econometrics Journal*, 6(1), pp. 72–78. doi:10.1111/1368-423X.00102.

Balke, Nathan S. and Robert J. Gordon (1989). "The estimation of prewar gross national product." *Journal of Political Economy*, 97(1), pp. 38–92. doi:10.1086/261593.

Barlevy, Gadi (2005). "The cost of business cycles and the benefits of stabilization." In *Economic Perspectives*, pp. 32–49. Federal Reserve Bank of Chicago. URL https://www.chicagofed.org/publications/economic-perspectives/2005/1qtr2005-part3-barlevy.

Barro, Robert J. and Tao Jin (2011). "On the size distirbution of macroeconomic disasters." *Econometrica*, 79(5), pp. 1567–1589. doi:10.3982/ECTA8827.

Barro, Robert J. and José F. Ursúa (2008). "Macroeconomic crises since 1870." *Brookings Papers on Economic Activity, Spring*, 38(1), pp. 255–350. URL https://www.jstor.org/stable/27561619.

Barro, Robert J. and José F. Ursúa (2010). "Barro-Ursúa Macroeconomic Data." URL https://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data.

Beveridge, Stephen and Charles R. Nelson (1981). "A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'." *Journal of Monetary Economics*, 7(2), pp. 151–174. doi:10.1016/0304-3932(81)90040-4.

Blanchard, Olivier (2000). "What do we know about macroeconomics that Fisher and Wicksell did not?" *The Quarterly Journal of Economics*, 115(4), pp. 1375–1409. doi:10.1162/003355300554999.

Constantinides, George M. (2021). "Welfare costs of idiosyncratic and aggregate consumption shocks." Working Paper 29009, National Bureau of Economic Research. doi:10.3386/w29009.

De Santis, Massimiliano (2007). "Individual consumption risk and the welfare costs of business cycles." *American Economic Review*, 97(4), pp. 1408–1506. doi:10.1257/aer.97.4.1488.

Dolmas, Jim (1998). "Risk preferences and the welfare cost of business cycles." *Review of Economic Dynamics*, 1(3), pp. 646–676. doi:10.1006/redy.1998.0020.

Dupraz, Stéphane, Emi Nakamura, and Jón Steinsson (2019). "A plucking model of business cycles." Working Paper 26351, National Bureau of Economic Research. doi:10.3386/w26351.

Gourio, François (2012). "Disaster risk and business cycles." *American Economic Review*, 102(6), pp. 2734–2766. doi:10.1257/aer.102.6.2734.

Guillén, Osmani Teixeira de Carvalho, Joao Victor Issler, and Afonso Arinos de Mello Franco-Neto (2014). "On the welfare costs of business-cycle fluctuations and economic-growth variation in the 20th century and beyond." *Journal of Economic Dynamics and Control*, 39(C), pp. 62–78. doi:10.1016/j.jedc.2013.11.008.

Hai, Rong, Dirk Krueger, and Andrew Postlewaite (2020). "On the welfare costs of consumption fluctuations in the presence of memorable goods." *Quantitative Economics*, 11(4), pp. 1117–1214. doi:10.3982/QE1173.

İmrohoroğlu, Ayşe (1989). "Cost of business cycles with indivisibilities and liquidity constraints." *Journal of Political Economy*, 97(6), pp. 1364–83. doi:10.1086/261658.

Inclan, Carla and George C. Tiao (1994). "Use of cumulative sums of squares for retrospective detection of changes of variance." *Journal of the American Statistical Association*, 89(427), pp. 913–923. doi:10.1080/01621459.1994.10476824.

Issler, Joao Victor, Afonso Arinos de Mello Franco-Neto, and Osmani Teixeira de Carvalho Guillén (2008). "The welfare cost of macroeconomic uncertainty in the post-war period." *Economics Letters*, 98(2), pp. 167–175. doi:10.1016/j.econlet.2007.04.026.

Jorda, Oscar, Moritz Schularick, and Alan M. Taylor (2020). "Disasters everywhere: The costs of business cycles reconsidered." Working Paper 2020-11, Federal Reserve Bank of San Francisco. doi:10.24148/wp2020-11.

Krusell, Per and Anthony A. Smith Jr. (1999). "On the welfare effects of eliminating business cycles." *Review of Economic Dynamics*, 2(1), pp. 245–272. doi:10.1006/redy.1998.0043.

Lucas, Robert (1987). *Models of Business Cycles*. Yrjo Jahnsson Lectures. Basil Blackwell, Oxford and New York.

Nakamura, Emi, Dmitriy Sergeyev, and Jón Steinsson (2017). "Growth-rate and uncertainty shocks in consumption: Cross-country evidence." *American Economic Journal: Macroeconomics*, 9(1), pp. 1–39. doi:10.1257/mac.20150250.

Nakamura, Emi and Jón Steinsson (2018). "Identification in macroeconomics." *Journal of Economic Perspectives*, 32(3), pp. 59–86. doi:10.1257/jep.32.3.59.

Nakamura, Emi, Jón Steinsson, Robert Barro, and José F. Ursúa (2013). "Crises and recoveries in an empirical model of consumption disasters." *American Economic Journal: Macroeconomics*, 5(3), pp. 35–74. doi:10.1257/mac.5.3.35.

Obstfeld, Maurice (1994). "Evaluating risky consumption paths: The role of intertemporal substitutability." *European Economic Review*, 38(7), pp. 1471–1486. doi:10.1016/0014-2921(94)90020-5.

Reis, Ricardo (2009). "The time-series properties of aggregate consumption." *Journal of the European Economic Association*, 7(4), pp. 722–753. doi:10.1162/JEEA.2009.7.4.722.

Romer, Christina (1986). "Spurious volatility in historical unemployment data." *Journal of Political Economy*, 94(1), pp. 1–37. doi:10.1086/261361.

Storesletten, Kjetil, Chris Telmer, and Amir Yaron (2001). "The welfare cost of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk." *European Economic Review*, 45(7), pp. 1311–1339. doi:10.1016/S0014-2921(00)00101-X.

US Bureau of Economic Analysis (2021a). "Table 1.1.4. Price Indexes for Gross Domestic Product." URL https://apps.bea.gov/iTable/iTable.cfm?reqid=19&step=2#reqid=19&step=2&isuri=1&1921=survey, (accessed August 28, 2021).

US Bureau of Economic Analysis (2021b). "Table 1.1.5. Gross Domestic Product." URL https://apps.bea.gov/iTable/iTable.cfm?reqid=19&step=2#reqid=19&step=2&isuri=1&1921=survey, (accessed August 28, 2021).

US Bureau of Economic Analysis (2021c). "Table 2.1. Personal Income and Its Disposition." URL https://apps.bea.gov/iTable/iTable.cfm?reqid=19&step=2#reqid=19&step=2&isuri=1&1921=survey, (accessed August 28, 2021).

Appendix

A Proofs

Below we outline the proofs for Lemmas 1, 2, 3, Propositions 1, 2, 3 and for Theorem 1.

Lemma 1. *Under Assumption 1 and CRRA utility (5),*

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{C}_t\right) = \begin{cases} \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2}, & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 \sum_{t=0}^{\infty} \Gamma^t, & \text{if } \gamma > 1 \end{cases} \tag{31}$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 1. Consider a $\gamma = 1$. Then,

$$\sum_{t=0}^{\infty} \beta^t \ln\left(\bar{C}_t\right) = \sum_{t=0}^{\infty} \beta^t \left(\ln \alpha_0 + t \ln\left(1+\alpha_1\right)\right) = \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln\left(1+\alpha_1\right)}{(1-\beta)^2}. \tag{32}$$

When $\gamma > 1$,

$$\sum_{t=0}^{\infty} \beta^t (1-\gamma)^{-1} \left(\bar{C}_t\right)^{1-\gamma} = (1-\gamma)^{-1}\alpha_0^{1-\gamma} \sum_{t=0}^{\infty} \left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t = \tilde{\alpha}_0 \sum_{t=0}^{\infty} \Gamma^t. \tag{33}$$

■

Lemma 2. *Consider an arbitrary constant $k > 0$. Under Assumption 1 and CRRA utility (5),*

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t u\left((1+k)\tilde{C}_t\right)\right] = \begin{cases} \frac{\ln(1+k)}{1-\beta} + \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1}{2}\sum_{t=0}^{\infty} \beta^t \sigma_t^2, & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 (1+k)^{1-\gamma} \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}, & \text{if } \gamma > 1 \end{cases} \tag{34}$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 2. For the case where $\gamma = 1$,

$$
\begin{aligned}
\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+k)\tilde{C}_t\right]\right] &= E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\ln(1+k)+\ln\alpha_0+t\ln\left(1+\alpha_1\right)+x_t-\frac{1}{2}\sigma_t^2\right)\right] \\
&= \sum_{t=0}^{\infty}\beta^t\left(\ln(1+k)+\ln\alpha_0+t\ln\left(1+\alpha_1\right)+E_0\left[x_t\right]-\frac{1}{2}\sigma_t^2\right) \\
&= \frac{\ln(1+k)}{1-\beta}+\frac{\ln\alpha_0}{1-\beta}+\frac{\beta\ln\left(1+\alpha_1\right)}{(1-\beta)^2}-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2,
\end{aligned}
$$

using the fact that $E_0\left[x_t\right]=0$.

For the case where $\gamma>1$,

$$
\begin{aligned}
\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+k)\tilde{C}_t\right]^{1-\gamma}}{1-\gamma}\right] &= (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+k)\alpha_0\left(1+\alpha_1\right)^t\exp\left\{x_t-0.5\sigma_t^2\right\}\right]^{1-\gamma}\right] \\
&= \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t\times\ldots \\
&\quad \ldots\exp\left\{-0.5\left(1-\gamma\right)\sigma_t^2\right\}E_0\left[\exp\left\{\left(1-\gamma\right)x_t\right\}\right].
\end{aligned}
$$

Note that

$$
\mathbb{E}_0\left[\exp\left\{(1-\gamma)\,x_t\right\}\right]=\exp\left\{E_0\left[(1-\gamma)\,x_t\right]+0.5Var_0\left[(1-\gamma)\,x_t\right]\right\}=\exp\left\{0.5\left(1-\gamma\right)^2\sigma_t^2\right\}.
$$

Thus,

$$
\begin{aligned}
\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+k)\tilde{C}_t\right]^{1-\gamma}}{1-\gamma}\right] &= \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\left(1-\gamma\right)\sigma_t^2\right\}\exp\left\{0.5\left(1-\gamma\right)^2\sigma_t^2\right\} \\
&= \tilde{\alpha}_0(1+k)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}.
\end{aligned}
$$

■

Lemma 3. *Consider an arbitrary constant $\ell>0$. Under Assumption 1 and CRRA utility (5),*

$$
\sum_{t=0}^{\infty}\beta^t u\left((1+\ell)C_t(\theta)\right)=
$$

$$\begin{cases} \frac{\ln(1+\ell)}{1-\beta} + \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1}{2}(1-\theta)\sum_{t=0}^{\infty} \beta^t \sigma_t^2, & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 (1+\ell)^{1-\gamma} \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}, & \text{if } \gamma > 1 \end{cases} \tag{35}$$

where $\tilde{\alpha}_0 \equiv (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof of Lemma 3. Again, when $\gamma = 1$,

$$\begin{aligned} \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \ln\left[(1+\ell)C_t(\theta)\right]\right] &= \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \ln\left[(1+\ell)\alpha_0(1+\alpha_1)^t \exp\left\{(1-\theta)\left[x_t - 0.5\sigma_t^2\right]\right\}\right]\right] \\ &= \sum_{t=0}^{\infty} \beta^t \left(\ln(1+\ell) + \ln\alpha_0 + t\ln(1+\alpha_1) + (1-\theta)\left[E_0[x_t] - 0.5\sigma_t^2\right]\right) \\ &= \frac{\ln(1+\ell)}{1-\beta} + \frac{\ln\alpha_0}{1-\beta} + \frac{\beta\ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1-\theta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2, \end{aligned}$$

given that $E_0[x_t] = 0$. With $\gamma > 1$,

$$\begin{aligned} \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t \frac{\left[(1+\ell)C_t(\theta)\right]^{1-\gamma}}{1-\gamma}\right] &= (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+\ell)C_t(\theta)\right]^{1-\gamma}\right] \\ &= (1-\gamma)^{-1}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+\ell)\alpha_0(1+\alpha_1)^t + \ldots\right.\right. \\ &\ldots \quad \left.\left.\exp\left\{(1-\theta)\left[x_t - 0.5\sigma_t^2\right]\right\}\right]^{1-\gamma}\right] \\ &= \tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t \times \ldots \\ &\ldots \quad \exp\left\{-0.5(1-\theta)(1-\gamma)\sigma_t^2\right\} E_0\left[\exp\left\{(1-\theta)(1-\gamma)x_t\right\}\right]. \end{aligned}$$

Note that

$$\begin{aligned} \mathbb{E}_0\left[\exp\left\{(1-\theta)(1-\gamma)x_t\right\}\right] &= \exp\left\{\mathbb{E}_0\left[(1-\theta)(1-\gamma)x_t\right] + 0.5 Var_0\left[(1-\theta)(1-\gamma)x_t\right]\right\} \\ &= \exp\left\{0.5(1-\theta)^2(1-\gamma)^2\sigma_t^2\right\}. \end{aligned}$$

And,

$$\begin{aligned} & \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\sigma_t^2\right\}\mathbb{E}_0\left[\exp\left\{\left(1-\theta\right)\left(1-\gamma\right)x_t\right\}\right] \\ = \; & \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\sigma_t^2\right\}\exp\left\{0.5\left(1-\theta\right)^2\left(1-\gamma\right)^2\sigma_t^2\right\} \\ = \; & \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\}. \end{aligned}$$

Thus,

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\frac{\left[(1+\ell)C_t\left(\theta\right)\right]^{1-\gamma}}{1-\gamma}\right]=\tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\}$$

■

Proof of Proposition 1. Replace k with λ^B in Lemma 2, use $\ell = 0$ in Lemma 3, and then solve equation (3) for λ^B. The assumptions guarantee that $\lambda^B < \infty$. ■

Proof of Proposition 2. We use $\ell = \lambda^R$ in Lemma 3 and the results in Lemma 1 for solving equation (4) for λ^R. The assumptions guarantee that $\lambda^R < \infty$. ■

Proof of Proposition 3. We use $k = \lambda^T$ in Lemma 2 and Lemma 1 in equation (1). Then, we solve it for λ^T. The assumptions guarantee that $\lambda^T < \infty$. ■

Proof of Theorem 1. For $\gamma = 1$, we have

$$\begin{aligned} \left(1+\lambda^B\right)\left(1+\lambda^R\right) &= \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}\exp\left\{\left(1-\theta\right)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} \\ &= \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} = 1+\lambda^T \end{aligned}$$

Now, for $\gamma > 1$, we have

$$\left(1+\lambda^B\right)^{1-\gamma}\left(1+\lambda^R\right)^{1-\gamma} = \frac{\sum_{t=0}^{\infty}\Gamma^t e^{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2}}{\sum_{t=0}^{\infty}\Gamma^t e^{-0.5\gamma(1-\gamma)\sigma_t^2}} \frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t e^{-0.5(1-\theta)(1-\gamma)(\gamma+\theta-\gamma\theta)\sigma_t^2}}$$

$$\Longleftrightarrow \left(1+\lambda^B\right)\left(1+\lambda^R\right) = \left[\frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t e^{-0.5\gamma(1-\gamma)\sigma_t^2}}\right]^{\frac{1}{1-\gamma}} = 1+\lambda^T$$

■

B Calculations for the Applications

B.1 Example 1 (Lucas, 1987):

For $\gamma = 1$:

$$\lambda^T = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{\frac{1}{2}\sigma_\varepsilon^2\right\} - 1 \tag{36}$$

$$\lambda^B = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{\theta\frac{1-\beta}{2}\sigma_\varepsilon^2\frac{1}{1-\beta}\right\} - 1 = \exp\left\{\frac{\theta}{2}\sigma_\varepsilon^2\right\} - 1 \tag{37}$$

$$\lambda^R = \exp\left\{(1-\theta)\frac{1-\beta}{2}\sigma_\varepsilon^2\frac{1}{1-\beta}\right\} - 1 = \exp\left\{\frac{1-\theta}{2}\sigma_\varepsilon^2\right\} - 1 \tag{38}$$

For $\gamma > 1$:

$$\begin{aligned}
\lambda^T &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t \exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \left[\frac{1}{\exp\left\{0.5\gamma\left(\gamma-1\right)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 = \exp\left\{\frac{1}{2}\gamma\sigma_\varepsilon^2\right\} - 1
\end{aligned} \tag{39}$$

$$\lambda^B = \left[\frac{\exp\left\{-0.5\,(1-\theta)\,(1-\gamma)\,[\gamma+\theta-\theta\gamma]\,\sigma^2\right\} \sum_{t=0}^{\infty}\left[\beta\,(1+\alpha_1)^{1-\gamma}\right]^t}{\exp\left\{0.5\gamma\,(\gamma-1)\,\sigma_\varepsilon^2\right\} \sum_{t=0}^{\infty}\left[\beta\,(1+\alpha_1)^{1-\gamma}\right]^t} \right]^{\frac{1}{1-\gamma}} - 1$$
$$= \left[\frac{\exp\left\{-0.5\,(1-\theta)\,(1-\gamma)\,[\gamma+\theta-\theta\gamma]\,\sigma_\varepsilon^2\right\}}{\exp\left\{-0.5\gamma\,(1-\gamma)\,\sigma_\varepsilon^2\right\}} \right]^{\frac{1}{1-\gamma}} - 1$$
$$= \exp\left\{\frac{1}{2}\left[\gamma-(1-\theta)\,(\gamma+\theta-\theta\gamma)\right]\sigma_\varepsilon^2\right\} \tag{40}$$

$$\lambda^R = \left[\frac{\sum_{t=0}^{\infty}\left[\beta\,(1+\alpha_1)^{1-\gamma}\right]^t}{\exp\left\{-0.5\,(1-\theta)\,(1-\gamma)\,[\gamma+\theta-\theta\gamma]\,\sigma_\varepsilon^2\right\} \sum_{t=0}^{\infty}\left[\beta\,(1+\alpha_1)^{1-\gamma}\right]^t} \right]^{\frac{1}{1-\gamma}} - 1$$
$$= \left[\frac{1}{\exp\left\{-0.5\,(1-\theta)\,(1-\gamma)\,[\gamma+\theta-\theta\gamma]\,\sigma_\varepsilon^2\right\}} \right]^{\frac{1}{1-\gamma}} - 1$$
$$= \exp\left\{\frac{1}{2}\,(1-\theta)\,(\gamma+\theta-\theta\gamma)\,\sigma_\varepsilon^2\right\} \tag{41}$$

B.2 Example 2 (Obstfeld, 1994)

For $\gamma = 1$:

$$\lambda^T = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\} - 1$$
$$= \exp\left\{\frac{1-\beta}{2}\left[\frac{\beta}{(1-\beta)^2}+\frac{1}{1-\beta}\right]\sigma_\varepsilon^2\right\} - 1 = \exp\left\{\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1 \tag{42}$$

$$\lambda^B = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\} - 1$$
$$= \exp\left\{\frac{\theta}{2}\left[\frac{\beta+1-\beta}{1-\beta}\right]\sigma_\varepsilon^2\right\} - 1 = \exp\left\{\frac{\theta}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1 \tag{43}$$

$$\lambda^R = \exp\left\{(1-\theta)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 = \exp\left\{(1-\theta)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\} - 1$$

$$= \exp\left\{\frac{1-\theta}{2}\left[\frac{\beta+1-\beta}{1-\beta}\right]\sigma_\varepsilon^2\right\} - 1 = \exp\left\{\frac{1-\theta}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1 \quad (44)$$

For $\gamma > 1$:

$$\lambda^T = \left[\frac{\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t\exp\{0.5\gamma(\gamma-1)(t\sigma_\varepsilon^2+\sigma_\varepsilon^2)\}}\right]^{\frac{1}{1-\gamma}} - 1$$

$$= \left[\frac{\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t}{\exp\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\}\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\exp\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\}\right]^t}\right]^{\frac{1}{1-\gamma}} - 1$$

$$= \left[\frac{1}{1-\beta(1+\alpha_1)^{1-\gamma}}\frac{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\right\}}{\exp\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\}}\right]^{\frac{1}{1-\gamma}} - 1$$

$$= \exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}\left[\frac{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\right\}}{1-\beta(1+\alpha_1)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}} - 1 \quad (45)$$

$$\lambda^B =$$

$$= \left[\frac{\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t\exp\left\{-0.5(1-\theta)(1-\gamma)[\gamma+\theta-\theta\gamma]\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}{\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t\exp\{0.5\gamma(\gamma-1)(t\sigma_\varepsilon^2+\sigma_\varepsilon^2)\}}\right]^{\frac{1}{1-\gamma}} - 1$$

$$= \left[\frac{\exp\left\{-0.5(1-\theta)(1-\gamma)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}{\exp\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\}}\frac{\frac{1}{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{-0.5(1-\theta)(1-\gamma)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}}{\frac{1}{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\right\}}}\right]^{\frac{1}{1-\gamma}} - 1$$

$$= \frac{\exp\left\{0.5\gamma\sigma^2\right\}}{\exp\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma^2\}}$$

$$\times\left[\frac{1-\beta(1+\alpha_1)^{1-\gamma}\exp\left\{0.5\gamma(\gamma-1)\sigma_\varepsilon^2\right\}}{1-\beta(1+\alpha_1)^{1-\gamma}\exp\{-0.5(1-\theta)(1-\gamma)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\}}\right]^{\frac{1}{1-\gamma}} - 1 \quad (46)$$

$$\begin{aligned}
\lambda^R &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\left(t\sigma_\varepsilon^2+\sigma_\varepsilon^2\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \left[\frac{1}{\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad\times\left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}\right]^t}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \frac{1}{\exp\left\{-0.5\left(1-\theta\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}} \\
&\quad\times\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left[\gamma+\theta-\theta\gamma\right]\sigma_\varepsilon^2\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}-1 \qquad (47)
\end{aligned}$$

B.3 Example 3 - ARIMA-BN Process (Reis, 2009):

From the Beveridge-Nelson decomposition,

$$\begin{aligned}
x_t^{BN} &= \psi\left(1\right)\sum_{j=0}^{t}\varepsilon_j+\sum_{j=0}^{t}\varphi_j\varepsilon_{t-j} \\
&= \left[\psi\left(1\right)+\varphi_t\right]\varepsilon_0+\left[\psi\left(1\right)+\varphi_{t-1}\right]\varepsilon_1+\cdots+\left[\psi\left(1\right)+\varphi_1\right]\varepsilon_{t-1}+\left[\psi\left(1\right)+\varphi_0\right]\varepsilon_t \\
&= \sum_{j=0}^{t}\left[\psi\left(1\right)+\varphi_{t-j}\right]\varepsilon_j \qquad (48)
\end{aligned}$$

Since ε_0 is revealed at the end of $t=0$, $\mathbb{E}_0\left[x_t^{BN}\right]=0$. Hence,

$$\begin{aligned}
\sigma^2_{x_t^{BN}} &\equiv \mathbb{E}\left[\left(x_t^{BN}-\mathbb{E}_0\left[x_t^{BN}\right]\right)^2\right]=\mathbb{E}\left[\left(x_t^{BN}\right)^2\right]=\mathbb{E}\left[\sum_{j=0}^{t}\left[\psi\left(1\right)+\varphi_{t-j}\right]^2\varepsilon_j^2\right] \\
&= \sum_{j=0}^{t}\left[\psi\left(1\right)^2+2\psi\left(1\right)\varphi_{t-j}+\varphi_{t-j}^2\right]\sigma_\varepsilon^2 \\
&= \left(t+1\right)\psi\left(1\right)^2\sigma_\varepsilon^2+2\psi\left(1\right)\sum_{j=0}^{t}\varphi_{t-j}\sigma_\varepsilon^2+\sum_{j=0}^{t}\varphi_{t-j}^2\sigma_\varepsilon^2 \qquad (49)
\end{aligned}$$

which can be rewritten into (17).

$$\lambda^B = \exp\left\{\theta \frac{1-\beta}{2} \sum_{t=0}^{\infty} \beta^t \left(\rho_0 + \rho_1 t\right)\right\} - 1 = \exp\left\{\frac{\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1 \tag{50}$$

$$\lambda^R = \exp\left\{(1-\theta)\frac{1-\beta}{2} \sum_{t=0}^{\infty} \beta^t \left(\rho_0 + \rho_1 t\right)\right\} - 1 = \exp\left\{\frac{1-\theta}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1 \tag{51}$$

For $\gamma > 1$,

$$\begin{aligned}
\lambda^T &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}} - 1 \\
&= \left[\frac{1}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad \times \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^t}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}\right]^t}\right]^{\frac{1}{1-\gamma}} \\
&= \exp\left\{0.5\gamma\rho_0\right\}\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}
\end{aligned} \tag{52}$$

$$
\begin{aligned}
\lambda^{B} &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\left(\rho_0+\rho_1 t\right)\right\}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}\exp\left\{-0.5\gamma\left(1-\gamma\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \left[\frac{\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}}{\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad\times\left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}\right]^{t}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \frac{\exp\left\{0.5\gamma\rho_0\right\}}{\exp\left\{0.5\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}} \\
&\quad\times\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}}\right]^{\frac{1}{1-\gamma}}-1 \qquad (53)
\end{aligned}
$$

$$
\begin{aligned}
\lambda^{R} &= \left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\left(\rho_0+\rho_1 t\right)\right\}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \left[\frac{1}{\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\}}\right]^{\frac{1}{1-\gamma}} \\
&\quad\times\left[\frac{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\right]^{t}}{\sum_{t=0}^{\infty}\left[\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}\right]^{t}}\right]^{\frac{1}{1-\gamma}}-1 \\
&= \exp\left\{0.5\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_0\right\} \\
&\quad\times\left[\frac{1-\beta\left(1+\alpha_1\right)^{1-\gamma}\exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\rho_1\right\}}{1-\beta\left(1+\alpha_1\right)^{1-\gamma}}\right]^{\frac{1}{1-\gamma}}-1
\end{aligned}
$$

B.4 The Literature-based Cost λ^{lit}

Here we characterize in our three applications the welfare cost of business cycles in the absence of observed consumption as proposed in our decomposition. We simply substitute σ_{ε}^2 by σ_u^2 in our previous calculations and use the formula for λ^T for each type of shock. Recall that, in our methodology, $\sigma_u^2 = (1-\theta_2)^2\sigma_\varepsilon^2$.

Example 1 (Lucas, 1987) :

$$\lambda^{lit} = \begin{cases} \exp\left(\frac{\sigma_u^2}{2}\right) - 1, \text{ if } \gamma = 1 \\ \exp\left(\frac{\gamma\sigma_u^2}{2}\right) - 1, \text{ if } \gamma > 1 \end{cases} \tag{54}$$

Example 2 (Obstfeld, 1994) :

$$\lambda^{lit} = \begin{cases} \exp\left(\frac{\sigma_u^2}{2(1-\beta)}\right) - 1, \text{ if } \gamma = 1 \\ \exp\left\{0.5\gamma\sigma_u^2\right\}\left[\frac{1-\Gamma\exp -0.5\gamma(1-\gamma)\sigma_u^2}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, \text{ if } \gamma > 1 \end{cases} \tag{55}$$

Example 3 (Reis, 2009) :

In this case, the substitution is in equation (17).

$$\lambda^{lit} = \begin{cases} \exp\left\{\frac{1}{2}\left(\rho_0 + \frac{\beta}{1-\beta}\rho_1\right)\right\} - 1, & \text{if } \gamma = 1 \\ \exp\left\{0.5\gamma\rho_0\right\}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma\left(1-\gamma\right)\rho_1\right\}}{1-\Gamma}\right]\frac{1}{1-\gamma}, & \text{if } \gamma > 1 \end{cases} \tag{56}$$

C Robustness Exercises

C.1 Full Sample

In Table 3 we present our estimations of the welfare cost using the full sample as in the main text. We also compute the λ's for different values of β in the case of permanent shocks.

Table 3: Welfare cost - Full sample

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	0.37	0.44	0.50	0.56	-
$\gamma = 1$	0.32	0.40	0.51	0.66	0.12	0.17	0.25	0.37	0.20	0.23	0.25	0.29	0.13
$\gamma = 2.5$	0.81	1.00	1.27	1.66	0.42	0.58	0.82	1.17	0.39	0.42	0.44	0.48	0.32
$\gamma = 5$	1.63	2.01	2.55	3.35	0.91	1.27	1.78	2.53	0.71	0.74	0.76	0.80	0.64
$\gamma = 7.5$	2.45	3.04	3.86	5.07	1.40	1.96	2.74	3.90	1.03	1.06	1.08	1.12	0.96
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.10	2.60	3.30	4.33	1.31	1.72	2.31	3.18	0.77	0.86	0.97	1.11	0.29
$\gamma = 2.5$	3.42	4.26	5.46	7.27	2.63	3.41	4.53	6.23	0.77	0.82	0.89	0.98	0.46
$\gamma = 5$	4.58	5.79	7.60	10.51	3.77	4.94	6.69	9.51	0.78	0.81	0.86	0.91	0.58
$\gamma = 7.5$	5.44	7.03	9.56	14.12	4.61	6.16	8.63	13.11	0.80	0.82	0.86	0.90	0.65
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	2.63	3.92	4.99	6.65	1.64	3.02	4.11	5.74	0.97	0.88	0.84	0.86	0.36
$\gamma = 2.5$	3.25	4.89	6.33	8.91	2.15	3.92	5.40	7.97	1.08	0.94	0.88	0.88	0.52
$\gamma = 5$	4.14	6.28	8.34	13.01	2.89	5.21	7.34	11.99	1.21	1.02	0.93	0.90	0.63
$\gamma = 7.5$	5.44	8.39	11.60	15.47	4.00	7.19	10.52	14.39	1.39	1.12	0.98	0.95	0.69
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	0.63	0.67	0.70	0.74	-
$\gamma = 1$	3.52	4.36	5.55	7.31	2.20	2.88	3.87	5.36	1.29	1.44	1.62	1.85	0.48
$\gamma = 2.5$	4.60	5.74	7.40	9.92	3.54	4.60	6.14	8.51	1.02	1.09	1.18	1.30	0.61
$\gamma = 5$	5.48	6.97	9.23	12.96	4.52	5.96	8.14	11.76	0.92	0.96	1.01	1.07	0.69
$\gamma = 7.5$	6.23	8.13	11.21	17.13	5.29	7.14	10.16	15.97	0.89	0.92	0.96	1.00	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations expanding the one in the main text for different β's. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table 4: Welfare cost - Full sample - ARIMA-BN

$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	4.07	5.05	6.43	8.48	2.48	3.27	4.40	6.12	1.55	1.72	1.94	2.22	0.60
$\gamma = 2.5$	5.91	7.42	9.63	13.05	4.49	5.88	7.92	11.13	1.36	1.46	1.58	1.73	0.83
$\gamma = 5$	7.50	9.72	13.29	19.77	6.16	8.30	11.74	18.03	1.26	1.32	1.38	1.47	0.96
$\gamma = 7.5$	8.99	12.35	18.89	40.20	7.68	10.95	17.35	38.31	1.22	1.26	1.31	1.37	1.01
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	5.15	6.40	8.16	10.79	3.13	4.13	5.58	7.77	1.96	2.18	2.45	2.81	0.75
$\gamma = 2.5$	6.75	8.49	11.06	15.08	5.13	6.73	9.11	12.86	1.54	1.65	1.79	1.96	0.94
$\gamma = 5$	8.16	10.63	14.65	22.16	6.71	9.09	12.96	20.26	1.36	1.41	1.49	1.58	1.03
$\gamma = 7.5$	9.64	13.37	20.92	51.68	8.25	11.88	19.28	49.52	1.29	1.33	1.38	1.44	1.06
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	6.98	8.69	11.12	14.76	4.23	5.59	7.56	10.57	2.64	2.93	3.31	3.79	1.01
$\gamma = 2.5$	7.84	9.91	12.95	17.80	5.96	7.86	10.68	15.19	1.78	1.90	2.06	2.26	1.08
$\gamma = 5$	8.93	11.71	16.29	25.19	7.36	10.03	14.46	23.09	1.46	1.53	1.61	1.71	1.11
$\gamma = 7.5$	10.38	14.56	23.47	85.20	8.90	12.97	21.70	82.42	1.36	1.40	1.45	1.52	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations expanding the one in the main text for different β's. The numbers are obtained using equations (10) through (20) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.

C.2 Structural Break

One may be concerned with the presence of structural breaks in a long time series. We apply the methodology developed in Bai and Perron (1998, 2003) to test structural breaks in our sample. For the transitory shock version we test a structural break in the log-consumption and we find a break in 1931 (scaled F-statistic is 1221.88 with a critical value of 11.47). We also test a structural break in the first-difference of log-consumption for our example with permanent shocks. We obtain a scaled F-statistic of 9.31 (with critical value of 8.58) indicating a break in 1934.[12]

[12] In our tests, we allow for at most 5 breaks in the time series. The tests indicate only one break in the first-difference of log-consumption (1934) and indicate 3 breaks in the log-consumption (1879, 1931, and

We use the break to create sub-samples for our identification strategy. The first sub-sample considers the years from the beginning of the sample until the year of the structural break, and a second sub-sample, as in the main text, considers the years after WWII. To keep the sub-samples the same size for our estimations with transitory or permanent shocks, we set the first sub-sample for the years between 1835 and 1930. The results of the estimated parameters along with the implied θ are presented in Table 5. If we compare those results with the results in Table 1 in the main text, we note that the estimations imply only a marginal change in the implied parameters with all three shock structures. Table 6 presents the welfare cost using the implied parameters in Table 5.

1993).

Table 5: Structural break - Estimation

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1930	7.726	0.0114	0.0055	1835 - 1930	0.0096	-	0.0019	1835 - 1930	0.0096	-	0.0019
	(.0155)	(.0003)			(.0045)				(.0045)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3215	0.0222	0.0055	0.00	0.6158	0.0208	0.0019	0.00	0.6011	0.0277	0.0019
0.10	0.3893	0.0222	0.0068	0.10	0.6542	0.0209	0.0024	0.10	0.6410	0.0277	0.0024
0.20	0.4572	0.0222	0.0086	0.20	0.6926	0.0209	0.0030	0.20	0.6809	0.0278	0.0030
0.30	0.5250	0.0222	0.0113	0.30	0.7311	0.0210	0.0040	0.30	0.7208	0.0279	0.0040

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent and ARIMA-BN shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data with sub-periods 1835-1930 and 1947-2019. The series is I(1) as identified by the ADF, PP, KPSS, and DF-GLS tests. The first-difference of the series is identified, by both the AIC and BIC criteria, as an ARMA (0,0) for the pre-1947 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

Table 6: Structural break - Welfare cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.32	0.39	0.46	0.53	0.32	0.39	0.46	0.53	0.32	0.39	0.46	0.53	-
$\gamma = 1$	0.28	0.34	0.43	0.56	0.09	0.13	0.20	0.30	0.19	0.21	0.23	0.27	0.13
$\gamma = 2.5$	0.69	0.85	1.08	1.42	0.31	0.45	0.65	0.95	0.38	0.40	0.43	0.46	0.32
$\gamma = 5$	1.39	1.72	2.18	2.85	0.69	0.99	1.42	2.06	0.70	0.72	0.74	0.78	0.64
$\gamma = 7.5$	2.09	2.58	3.28	4.31	1.06	1.53	2.20	3.18	1.02	1.04	1.06	1.10	0.96
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	1.96	2.43	3.08	4.04	1.20	1.58	2.12	2.94	0.75	0.83	0.94	1.07	0.29
$\gamma = 2.5$	3.19	3.97	5.09	6.76	2.42	3.14	4.18	5.75	0.75	0.80	0.87	0.96	0.46
$\gamma = 5$	4.25	5.37	7.03	9.67	3.46	4.53	6.13	8.69	0.77	0.80	0.84	0.90	0.58
$\gamma = 7.5$	4.91	6.32	8.51	12.36	4.09	5.46	7.61	11.38	0.79	0.81	0.84	0.88	0.65
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	2.46	3.66	4.63	6.09	1.51	2.78	3.77	5.20	0.94	0.86	0.83	0.85	0.36
$\gamma = 2.5$	3.04	4.56	5.86	8.09	1.98	3.61	4.95	7.16	1.04	0.92	0.87	0.87	0.52
$\gamma = 5$	3.86	5.85	7.70	11.59	2.66	4.81	6.73	10.60	1.17	0.99	0.91	0.90	0.63
$\gamma = 7.5$	4.98	7.63	10.40	13.46	3.60	6.48	9.34	12.41	1.33	1.08	0.96	0.93	0.69
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	0.62	0.65	0.69	0.73	-
$\gamma = 1$	3.29	4.08	5.19	6.83	2.01	2.65	3.56	4.95	1.25	1.39	1.57	1.79	0.48
$\gamma = 2.5$	4.29	5.35	6.88	9.21	3.25	4.24	5.66	7.84	1.00	1.07	1.16	1.27	0.61
$\gamma = 5$	5.08	6.45	8.51	11.87	4.14	5.46	7.45	10.71	0.90	0.94	0.99	1.05	0.69
$\gamma = 7.5$	5.61	7.27	9.92	14.79	4.69	6.30	8.90	13.67	0.88	0.91	0.94	0.99	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that avoids the structural break in 1931. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 5. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table 7: Structural break - Welfare cost - ARIMA-BN

	$\beta = 0.95$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	3.80	4.71	6.00	7.91	2.27	3.00	4.05	5.64	1.50	1.67	1.88	2.15	0.60
$\gamma = 2.5$	5.50	6.90	8.94	12.08	4.12	5.40	7.28	10.21	1.33	1.43	1.54	1.69	0.83
$\gamma = 5$	6.92	8.94	12.13	17.79	5.60	7.54	10.62	16.11	1.25	1.30	1.36	1.45	0.96
$\gamma = 7.5$	8.19	11.12	16.56	31.28	6.90	9.75	15.07	29.52	1.21	1.25	1.30	1.35	1.01
	$\beta = 0.96$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	4.81	5.97	7.62	10.06	2.86	3.79	5.13	7.16	1.89	2.10	2.37	2.71	0.75
$\gamma = 2.5$	6.28	7.89	10.25	13.93	4.70	6.18	8.36	11.79	1.51	1.62	1.75	1.92	0.94
$\gamma = 5$	7.52	9.76	13.33	19.83	6.10	8.25	11.69	18.00	1.34	1.40	1.47	1.56	1.03
$\gamma = 7.5$	8.76	11.99	18.17	37.05	7.39	10.53	16.58	35.12	1.28	1.31	1.36	1.43	1.06
	$\beta = 0.97$												
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	6.51	8.10	10.36	13.75	3.87	5.12	6.95	9.73	2.55	2.84	3.20	3.66	1.01
$\gamma = 2.5$	7.29	9.19	11.99	16.41	5.46	7.20	9.78	13.90	1.74	1.86	2.01	2.21	1.08
$\gamma = 5$	8.22	10.73	14.78	22.39	6.68	9.08	12.99	20.37	1.44	1.51	1.58	1.68	1.11
$\gamma = 7.5$	9.41	13.00	20.15	46.47	7.96	11.45	18.44	44.30	1.35	1.39	1.44	1.50	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that avoids the structural break in 1931. The numbers are obtained using equations (10) through (20) with the estimates shown in Table 5. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.

C.3 Removing the Interwar Period

As the previous exercise used the disjoint periods (1835-1930 and 1947-2019), we run an additional experiment where we use the 1931 break in the time series as a reference point to design two new intervals. In the previous exercise, we have removed 15 periods - years 1931 to 1945 - from the full sample. Those periods were exclusively defined after the break. For this case, we remove a similar interval for the period before the 1931 break. We construct two sub-samples by excluding the interwar period from our data, which results in a first period with years 1835 to 1913 and a second period from 1947 to 2019, the last one as in our main analysis.[13]

Besides the structural break in the consumption series during the interwar period, many other relevant macroeconomic events happened during this window of time. For example, we have the 1929 crisis and the Great Depression that followed. In general, this period was marked by highly unstable macroeconomic outcomes, and hence, it is worth subtracting it from the sample to better measure pre-war volatility. Once again, the results are similar to our original analysis. Table 8 presents the estimated and implied parameters and Table 9 presents the computed λ's using the estimations in Table 8. Once again, there is no substantial change in the results.

[13]We also run an experiment by removing exactly 15 periods before and after the break, that is, using subssamples from 1835-1915 and 1947-2019. As expected, the results are so similar to the results in this subsection that we only report the exercise where we remove the interwar period.

Table 8: Removing the interwar period - Estimation

Transitory shocks				Permanent shocks				ARIMA-BN			
Estimated parameters				Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\phi}_1$	$\hat{\sigma}_u^2$
1835 - 1913	7.7099	0.0119	0.0057	1835 - 1913	0.0107	-	0.0019	1835 - 1913	0.0107	-	0.0019
	(.0175)	(.0004)			(.0049)				(.0049)		
1947 - 2019	6.6156	0.0219	0.0025	1947 - 2019	0.0203	-	0.0003	1947 - 2019	0.0189	0.2919	0.0003
	(.0427)	(.0003)			(.0020)				(.0030)	(.1321)	
Implied parameters				Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3309	0.0222	0.0057	0.00	0.6120	0.0208	0.0019	0.00	0.5972	0.0277	0.0019
0.10	0.3978	0.0222	0.0070	0.10	0.6508	0.0209	0.0023	0.10	0.6375	0.0277	0.0023
0.20	0.4647	0.0222	0.0089	0.20	0.6896	0.0209	0.0030	0.20	0.6777	0.0278	0.0030
0.30	0.5316	0.0222	0.0116	0.30	0.7284	0.0210	0.0039	0.30	0.7180	0.0279	0.0039

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent and ARIMA-BN shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data excluding the interwar period. The series is I(1) as identified by the ADF, PP, KPSS, and DF-GLS tests. The first-difference of the series is identified, by both the AIC and BIC criteria, as an ARMA (0,0) for the pre-1947 period and an ARMA(1,0) for the subsequent years. The implied parameters are obtained using the formulas described in the text and equation (30).

Table 9: Removing the interwar period - Welfare cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.33	0.40	0.46	0.53	0.33	0.40	0.46	0.53	0.33	0.40	0.46	0.53	-
$\gamma = 1$	0.28	0.35	0.44	0.58	0.09	0.14	0.21	0.31	0.19	0.21	0.24	0.27	0.13
$\gamma = 2.5$	0.71	0.88	1.11	1.46	0.33	0.47	0.68	0.99	0.38	0.40	0.43	0.46	0.32
$\gamma = 5$	1.43	1.76	2.24	2.93	0.72	1.04	1.48	2.14	0.70	0.72	0.75	0.78	0.64
$\gamma = 7.5$	2.15	2.66	3.38	4.43	1.12	1.60	2.29	3.30	1.02	1.04	1.07	1.10	0.96
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	1.92	2.38	3.02	3.96	1.17	1.54	2.07	2.87	0.74	0.82	0.93	1.06	0.29
$\gamma = 2.5$	3.13	3.89	4.98	6.62	2.36	3.07	4.08	5.62	0.75	0.80	0.86	0.95	0.46
$\gamma = 5$	4.16	5.25	6.87	9.44	3.37	4.42	5.98	8.47	0.77	0.80	0.84	0.89	0.58
$\gamma = 7.5$	4.91	6.32	8.51	12.36	4.09	5.46	7.61	11.38	0.79	0.81	0.84	0.88	0.65
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	2.41	3.58	4.53	5.94	1.47	2.71	3.67	5.05	0.93	0.85	0.83	0.85	0.36
$\gamma = 2.5$	2.98	4.47	5.73	7.87	1.93	3.52	4.83	6.94	1.03	0.91	0.86	0.87	0.52
$\gamma = 5$	3.79	5.73	7.52	11.22	2.60	4.69	6.55	10.24	1.16	0.99	0.91	0.89	0.63
$\gamma = 7.5$	4.98	7.63	10.40	13.46	3.60	6.48	9.34	12.41	1.33	1.08	0.96	0.93	0.69
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	0.61	0.65	0.69	0.73	-
$\gamma = 1$	3.22	3.99	5.08	6.69	1.96	2.58	3.48	4.83	1.24	1.38	1.55	1.77	0.48
$\gamma = 2.5$	4.20	5.24	6.74	9.01	3.17	4.13	5.52	7.65	0.99	1.06	1.15	1.26	0.61
$\gamma = 5$	4.97	6.31	8.31	11.58	4.04	5.32	7.25	10.42	0.90	0.94	0.99	1.05	0.69
$\gamma = 7.5$	5.61	7.27	9.92	14.79	4.69	6.30	8.90	13.67	0.88	0.91	0.94	0.99	0.73

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that excludes the interwar period. The numbers are obtained using equations (10) through (15) with the estimates shown in Table 8. All measures are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

Table 10: Removing the interwar period - Welfare cost - ARIMA-BN

$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	3.73	4.62	5.88	7.75	2.21	2.92	3.95	5.51	1.48	1.65	1.86	2.13	0.60
$\gamma = 2.5$	5.39	6.76	8.74	11.81	4.01	5.26	7.10	9.96	1.32	1.42	1.53	1.68	0.83
$\gamma = 5$	6.76	8.72	11.81	17.26	5.45	7.34	10.31	15.59	1.24	1.29	1.36	1.44	0.96
$\gamma = 7.5$	7.97	10.79	15.96	29.41	6.68	9.42	14.48	27.68	1.21	1.25	1.29	1.35	1.01
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	4.71	5.85	7.46	9.86	2.79	3.69	5.00	6.98	1.87	2.08	2.35	2.69	0.75
$\gamma = 2.5$	6.15	7.72	10.03	13.61	4.58	6.02	8.15	11.49	1.50	1.61	1.74	1.90	0.94
$\gamma = 5$	7.34	9.52	12.97	19.22	5.93	8.02	11.35	17.40	1.33	1.39	1.46	1.55	1.03
$\gamma = 7.5$	8.52	11.62	17.48	34.39	7.16	10.18	15.90	32.50	1.27	1.31	1.36	1.42	1.06
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ^{lit}
$\hat{\theta}_2$	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	0.60	0.64	0.68	0.72	-
$\gamma = 1$	6.38	7.94	10.15	13.46	3.76	4.99	6.77	9.49	2.52	2.81	3.16	3.63	1.01
$\gamma = 2.5$	7.14	8.99	11.73	16.03	5.32	7.02	9.54	13.54	1.73	1.85	2.00	2.19	1.08
$\gamma = 5$	8.03	10.46	14.37	21.65	6.50	8.82	12.60	19.65	1.44	1.50	1.58	1.67	1.11
$\gamma = 7.5$	9.15	12.58	19.32	42.06	7.70	11.05	17.64	39.96	1.34	1.38	1.43	1.50	1.12

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that excludes the interwar period. The numbers are obtained using equations (10) through (20) with the estimates shown in Table 8. All measures are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ^{lit}, described in Appendix B.4. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 7.5\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the ARIMA-BN process.