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The Welfare Costs of Business Cycles Unveiled: Measuring the Extent of Stabilization Policies

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How can we measure the welfare benefit of ongoing stabilization? We develop a methodology to calculate the welfare cost of business cycles taking into account that observed consumption is partially smoothed. We propose a decomposition that disentangles consumption in a mix of laissez-faire (absent policies) and riskless components. With a novel identification strategy, we estimate the span of stabilization power. Our results show that the welfare cost of total fluctuations is 5.81 percent of lifetime consumption, in which 80 percent is smoothed by the status quo, yielding a residual 1.05 percent to be tackled by policy.

Keywords: business cycles, consumption, stabilization, macroeconomic history.

JEL Classifications: E32, E21, E63, N10.

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1 Introduction

In a tough challenge to conventional wisdom, Lucas (1987) asked how much Americans would be willing to pay, in terms of consumption, to live in an economy that is not subject to the macroeconomic volatility that the US witnessed during the post-war period. Finding that a representative consumer would sacrifice at most one-tenth of a percent of lifetime consumption, Lucas concluded that there would be little benefit in further attempting to stabilize the residual risk of business cycles.

Not surprisingly, Lucas's seminal result attracted a great deal of controversy and generated a wealth of literature that revisits his estimates. In this paper, we explore a critical point, which is subtly present in Lucas (1987), that calls for a new measurement effort when estimating the costs of business cycles: all observed consumption is already partially smoothed. That is, the data that we gather for consumption stem from a realized allocation that is subject to the status quo of economic stabilization policies.

In order to measure the contribution of ongoing policies as well as the residual to be smoothed, we then need to disentangle which part of the observed consumption pertains to each category. To accomplish such a task, we propose a tractable decomposition in which observed consumption is a weighted geometric mean of laissez-faire consumption, i.e., the counterfactual consumption series in the absence of any policy and a riskless consumption sequence.

Our decomposition allows us to map all stabilization policies to a single parameter θ, which we define as the the span of stabilization power. Within this structure, we are able to prove that the welfare cost of total economic fluctuations is equal to the sum of the benefit of ongoing policies plus the cost of residual fluctuations still to be smoothed. We apply our formulation to two classic shock structures for the consumption process: the one of Lucas (1987) with transitory shocks and the one by Obstfeld (1994) with permanent innovations.

We then proceed to estimate the parameters in our welfare decomposition but hit a measurement challenge: since the laissez-faire consumption is not observable, we need to identify θ. For this task, we resort to the literature of identification in macroeconomics and couple it with the relevant facts of US macroeconomic history. By using a log-normal consumption process, we are able to estimate our parameter exploring the heteroskedasticity of the consumption data, which show a significant decrease in volatility after WWII. We use an augmented version of the historical consumption series provided by Barro and Ursúa (2010) and divide the sample into pre- and post-war periods with distinct measured volatilities. Such a discontinuity-based strategy allows us to pin down the span of stabilization policies from 1946 until today, which we then take back as an input in our decomposition.

Our results show that stabilization policies smooth 40-70 percent of the consumption shocks. The cost of total economic fluctuations is 5.81 percent of consumption with permanent shocks in our preferred parameter space. Close to 20 percent of such costs could become the benefit of smoothing the residual from what is observed. Since this is our measure that is the easiest to compare with the literature, we are able to find a residual cost that is 45 percent higher when taking into account our measurement of the benefit of ongoing policies. Finally, a feature arises from the concave nature of the utility: the more risk averse consumers are, the more they value the benefit of ongoing stabilization policies.

2 Related Literature

Our paper is embedded in three major strands of the literature in macroeconomics: (i) the large body of work concerned with the calculus of the welfare costs of business cycles; (ii) the literature of identification in macroeconomics (Nakamura and Steinsson, 2018); and (iii) the classic literature that deals with macroeconomic history focusing on measurement and data.

Several papers build on Lucas's departing point and relax some of his assumptions. For example, Obstfeld (1994) switches the original transitory shocks for permanent ones, Reis (2009) further develops the time-series aspects, while Issler et al. (2008) and Guillén et al. (2014) combine both types of shocks. More recently, Barros et al. (2017) use a state-space decomposition and Hai et al. (2020) include memorable goods.[1] Another block in this body departs from the representative agent setting and estimates the costs under incomplete markets and heterogeneous agents such as in Imrohoruglu (1989), Krusell and Smith (1999), Storesletten et al. (2001), and De Santis (2007). Our contribution here is to model the observed consumption as a partially smoothed series and the proposal of a new and tractable decomposition that allows us to disentangle the reach of the ongoing policies.

We also view our work as building on the effort of calculating the costs of business cycles, with critical attention to measurement and identification that often appeared in what became the "disasters" approach in the literature. When developing our identification strategy, we start by drawing from Barro and Ursúa (2008)'s observation that for the OECD economies, there is a change in consumption volatility in the post-war period. We then resort to Nakamura et al. (2013)'s insight of using the variation in the volatility of the consumption series to better identify the shift in the role of stabilization policies. Moreover, we build on Nakamura et al. (2017) in our use of both transitory and permanent formulations for the shocks in conjunction with a time-varying volatility for the consumption series. More recently, at the intersection of the disasters and welfare costs literature, Jorda et al. (2020) find that substantial costs may arise from a novel estimate of frequent and small disasters.[2] In addition, by considering the asymmetric nature of economic fluctuations, Dupraz et al. (2019) develop a plucking model of business cycles and find welfare gains from eliminating economic fluctuations that are an order of magnitude larger than in standard models.

We conduct our data analysis grounding it in the literature on macroeconomic history. First, we build our sample directly from the historical data compiled by Barro and Ursúa (2010). We use references such as Blanchard et al. (2000) and Martin and Weaver (2005) to guide our choice of dates in our identification strategy and in the robustness checks.

[1] For an in-depth early discussion of this literature, see Barlevy (2005), who discusses other references such as Dolmas (1998) and Alvarez and Jermann (2004).

[2] Other examples in this literature are Barro and Jin (2011) and Gourio (2012).

Finally, we revisit the seminal work of Romer (1986) and Balke and Gordon (1989) that documents the challenges faced when measuring the volatility of macroeconomic aggregates and show how our methodology can reconcile improvements in both measurement and stabilization after WWII.

3 Model

3.1 Environment and Definitions

The economy is populated by a representative consumer, whose lifetime utility is given by $\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t)\right]$, where C_t is consumption in period t, $\beta \in (0,1)$ is an intertemporal discount factor, $u(\cdot)$ is the instantaneous utility function, and $\mathbb{E}_0[\cdot]$ is the expectation operator conditional on the information set $\mathcal{I}_0$.[3] We begin with a few definitions:

Definition 1. *Define $\bar{C}_t \equiv \mathbb{E}_0[C_t]$. Then $\{\bar{C}_t\}_{t=0}^{\infty}$ is the riskless consumption sequence.*

Definition 2. *Define $\tilde{C}_t$ as consumption in the absence of stabilization policies. Then $\{\tilde{C}_t\}_{t=0}^{\infty}$ is the laissez-faire consumption sequence.*

We can now define the welfare cost of the total economic fluctuations as the constant $\lambda^T > 0$ that solves the following condition:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left((1+\lambda^T)\tilde{C}_t\right)\right] = \sum_{t=0}^{\infty}\beta^t u\left(\bar{C}_t\right). \tag{1}$$

The parameter λ^T measures the constant compensation required by the consumer to be indifferent between the adjusted laissez-faire, $\{(1+\lambda^T)\tilde{C}_t\}_{t=0}^{\infty}$, and the riskless consumption sequences.

[3] We assume that the expectation is taken before the realization of any uncertainty in period 0, as in some calculations done by Obstfeld (1994) and Reis (2009). In that sense, consumption in that period is treated as a stochastic variable. Under this assumption we compare the expected utility in two worlds where the agent is still uncertain about all consumption flows, as in Lucas (1987).

Given that the observed time series on consumption is subject to the ongoing stabilization policies, we can view it as the combination of two extreme cases: (i) the (non-observed) consumption series in the absence of any stabilization policies, $\tilde{C}_t$, and (ii) the (non-observed) perfectly smoothed consumption, $\bar{C}_t$. We then model the (observed) partially smoothed consumption as a weighted geometric average:

$$C_t(\theta) \equiv \bar{C}_t^{\theta} \tilde{C}_t^{1-\theta}, \tag{2}$$

where the parameter $\theta \in [0, 1]$ measures the degree of consumption smoothing. Thus, θ can be interpreted as the span of the stabilization power of governmental policies.

We can now define the benefit of the ongoing stabilization policies as the constant $\lambda^B > 0$ that solves the following condition:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u \left((1 + \lambda^B) \tilde{C}_t \right) \right] = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t(\theta)) \right]. \tag{3}$$

The parameter λ^B is the compensation required by the consumer to be indifferent between the adjusted laissez-faire consumption sequence and the effective consumption sequence, $\{C_t(\theta)\}_{t=0}^{\infty}$.

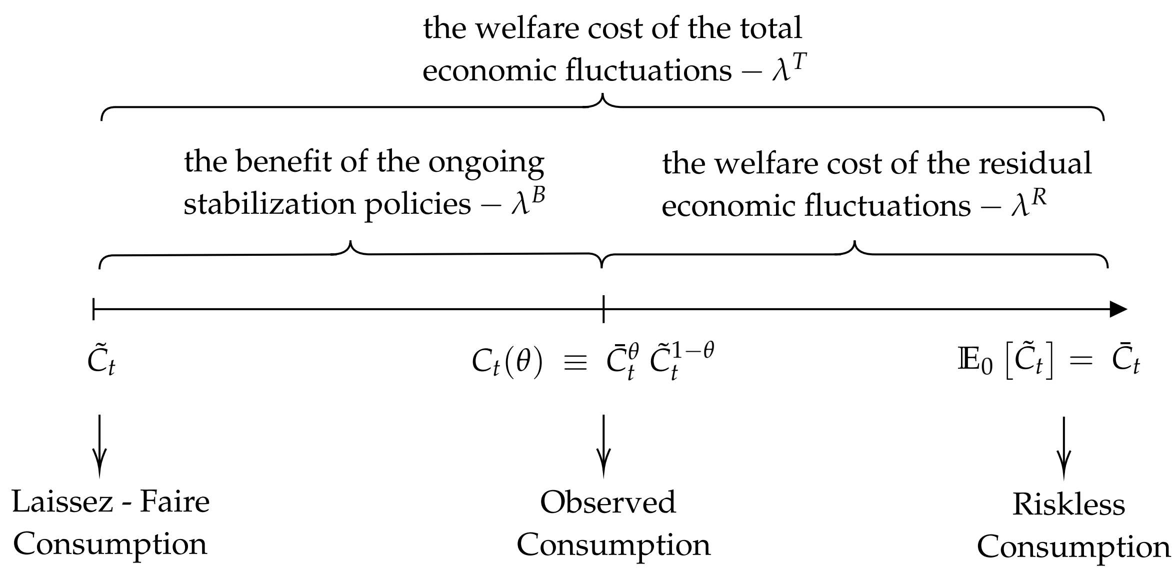
Finally, we can compute what is left to be stabilized by defining the welfare cost of the residual economic fluctuations as the constant $\lambda^R > 0$ that solves the following condition:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u \left((1 + \lambda^R) C_t(\theta) \right) \right] = \sum_{t=0}^{\infty} \beta^t u \left(\bar{C}_t \right). \tag{4}$$

The parameter λ^R measures the constant compensation required by the consumer to be indifferent between the adjusted partially smoothed consumption sequence $\{(1 + \lambda^R) C_t(\theta)\}_{t=0}^{\infty}$ and the aforementioned riskless sequence.

Figure 1 summarizes our modelling by showing where each parameter and measure defined is located in a spectrum of consumption that spans the highest to the lowest level of risk:

Figure 1: Decomposition of the welfare cost of the total economic fluctuations.



3.2 Assumptions

In order to calculate λ^T, λ^B, and λ^R and guarantee tractability, we assume a log-normal process for $\tilde{C}_t$, which implies that $C_t(\theta)$ is also a log-normal. We also need assumptions that guarantee that the sums in conditions (1), (3), and (4) are all finite. They are:

Assumption 1. *Log-Normal consumption process:* $\tilde{C}_t = \alpha_0(1+\alpha_1)^t X_t$, *where* $X_t = e^{x_t - 0.5\sigma_t^2}$, *with* $x_t|\mathcal{I}_0 \sim \mathcal{N}\left(0, \sigma_t^2\right)$.

Assumption 2. *The constant* $\Gamma \equiv \beta\left(1+\alpha_1\right)^{1-\gamma} \in (0,1)$.

Assumption 3. $\sum\limits_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\} < \infty$.[4]

Finally, following Lucas (1987), we assume a CRRA instantaneous utility with param-

[4] One should note that $\sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\left(1-\theta\right)\left(1-\gamma\right)\left(\gamma+\theta-\gamma\theta\right)\sigma_t^2\right\} < \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\}$ if $\gamma > 1$. This result ensures that the λ's are finite in some of our results.

eter γ:

$$u(C) = \begin{cases} \frac{C^{1-\gamma}}{1-\gamma}, \text{ if } \gamma > 0, \gamma \neq 1 \\ \ln(C), \text{ if } \gamma = 1 \end{cases} \tag{5}$$

Under Assumption 1, riskless consumption is given by $\bar{C}_t = \mathbb{E}_0[\tilde{C}_t] = \alpha_0(1+\alpha_1)^t$ and is deterministic. Furthermore, $\tilde{C}_t = \bar{C}_t X_t$, and the partially smoothed consumption can be rewritten as $C_t(\theta) = \bar{C}_t X_t^{1-\theta}$. From this formulation it is easy to see that the larger the parameter θ, the less important is the stochastic part of the partially smoothed consumption. As θ approaches one, consumption becomes fully smoothed.

3.3 Theoretical Results

We can now derive closed-form solutions for the parameters λ^B, λ^R and λ^T. We begin by proving three Lemmas that will be used in our main propositions. They proceed as follows: the right-hand side of the conditions (1) and (4) are the same, being calculated by Lemma 1. The left-hand side of conditions (1) and (3) are essentially the same, being calculated in Lemma 2. Finally, Lemma 3 allows us to calculate the right-hand side of condition (3) and the left-hand side of condition (4). We show in the text the proofs considering the simpler case of $\gamma = 1$. The general case with $\gamma > 0, \gamma \neq 1$ is straightforward using the reasoning shown in the proofs and can be found in Appendix A.

Lemma 1. *Under Assumption 1 and CRRA utility (5),*

$$\sum_{t=0}^{\infty} \beta^t u\left(\bar{C}_t\right) = \begin{cases} \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2} & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 \sum_{t=0}^{\infty} \Gamma^t & \text{if } \gamma > 0, \gamma \neq 1 \end{cases}, \tag{6}$$

where $\tilde{\alpha}_0 = (1-\gamma)^{-1}\alpha_0^{1-\gamma}$.

Proof. Consider a $\gamma = 1$. Then,

$$\sum_{t=0}^{\infty} \beta^t \ln\left(\bar{C}_t\right) = \sum_{t=0}^{\infty} \beta^t \left(\ln \alpha_0 + t \ln\left(1+\alpha_1\right)\right) \tag{7}$$

$$= \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln\left(1+\alpha_1\right)}{\left(1-\beta\right)^2}. \tag{8}$$

■

Lemma 2. *Under Assumption 1 and CRRA utility (5),*

$$E_0\left[\sum_{t=0}^{\infty} \beta^t u\left((1+k)\tilde{C}_t\right)\right] = \begin{cases} \frac{\ln(1+k)}{1-\beta} + \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1}{2}\sum_{t=0}^{\infty} \beta^t \sigma_t^2 & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 (1+k)^{1-\gamma} \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma\left(1-\gamma\right)\sigma_t^2\right\} & \text{if } \gamma > 0, \gamma \neq 1 \end{cases}. \tag{9}$$

Proof. For the case where $\gamma = 1$,

$$\begin{aligned} E_0\left[\sum_{t=0}^{\infty} \beta^t \ln\left[(1+k)\tilde{C}_t\right]\right] &= E_0\left[\sum_{t=0}^{\infty} \beta^t \left(\ln(1+k) + \ln \alpha_0 + t \ln\left(1+\alpha_1\right) + x_t - \frac{1}{2}\sigma_t^2\right)\right] \\ &= \sum_{t=0}^{\infty} \beta^t \left(\ln(1+k) + \ln \alpha_0 + t \ln\left(1+\alpha_1\right) + E_0\left[x_t\right] - \frac{1}{2}\sigma_t^2\right) \\ &= \frac{\ln(1+k)}{1-\beta} + \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln\left(1+\alpha_1\right)}{\left(1-\beta\right)^2} - \frac{1}{2}\sum_{t=0}^{\infty} \beta^t \sigma_t^2, \end{aligned}$$

using the fact that $E_0\left[x_t\right] = 0$.

■

Lemma 3. *Under Assumption 1 and CRRA utility (5),*

$$\sum_{t=0}^{\infty} \beta^t u\left((1+\ell)C_t(\theta)\right) =$$

$$\begin{cases} \frac{\ln(1+\ell)}{1-\beta} + \frac{\ln \alpha_0}{1-\beta} + \frac{\beta \ln(1+\alpha_1)}{(1-\beta)^2} - \frac{1}{2}\left(1-\theta\right)\sum_{t=0}^{\infty} \beta^t \sigma_t^2 & \text{if } \gamma = 1 \\ \tilde{\alpha}_0 (1+\ell)^{1-\gamma} \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\left(1-\gamma\right)\left(1-\theta\right)\left(\theta+\gamma-\gamma\theta\right)\sigma_t^2\right\} & \text{if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{10}$$

Proof. Again, when $\gamma = 1$,

$$\begin{aligned} E_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+\ell)C_t(\theta)\right]\right] &= E_0\left[\sum_{t=0}^{\infty}\beta^t \ln\left[(1+\ell)\alpha_0\left(1+\alpha_1\right)^t \exp\left\{(1-\theta)\left[x_t - 0.5\sigma_t^2\right]\right\}\right]\right] \\ &= \sum_{t=0}^{\infty}\beta^t\left(\ln(1+\ell) + \ln\alpha_0 + t\ln\left(1+\alpha_1\right) + (1-\theta)\left[E_0\left[x_t\right] - 0.5\sigma_t^2\right]\right) \\ &= \frac{\ln(1+\ell)}{1-\beta} + \frac{\ln\alpha_0}{1-\beta} + \frac{\beta\ln\left(1+\alpha_1\right)}{(1-\beta)^2} - \frac{1-\theta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2, \end{aligned}$$

given that $E_0\left[x_t\right] = 0$.

■

We now show Propositions 1, 2, and 3, which establish the welfare benefit of the ongoing stabilization policies, the welfare cost of the residual, and total macroeconomic uncertainty, respectively:

Proposition 1. *Under Assumptions 1 and 3 the benefit of the ongoing stabilization policies is given by*

$$\lambda^B = \begin{cases} \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}}{\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases}. \tag{11}$$

Proof. Replace k with λ^B in Lemma 2, use $\ell = 0$ in Lemma 3, and then solve equation (3) for λ^B. The assumptions guarantee that $\lambda^B < \infty$. ■

Proposition 2. *Under Assumptions 1, 2, and 3 the welfare cost of the residual macroeconomic fluctuations is given by*

$$\lambda^R = \begin{cases} \exp\left\{(1-\theta)\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases}. \tag{12}$$

Proof. We use $\ell = \lambda^R$ in Lemma 3 and the results in Lemma 1 for solving equation (4) to λ^R. The assumptions guarantee that $\lambda^R < \infty$. ∎

Proposition 3. *Under Assumptions 1, 2, and 3 the welfare cost of the total economic fluctuations is given by*

$$\lambda^T = \begin{cases} \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} - 1 & \text{if } \gamma = 1 \\ \left[\frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t \exp\left\{-0.5\gamma(1-\gamma)\sigma_t^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases}. \tag{13}$$

Proof. We use $k = \lambda^T$ in Lemma 2 and Lemma 1 in equation (1). Then, we solve it for λ^T. The assumptions guarantee that $\lambda^T < \infty$. ∎

We can now state our main result: the decomposition of the welfare cost of total economic fluctuations.

Theorem 1. *Under Assumptions 1 to 3 and CRRA utility (5), there is a decomposition of the welfare cost of total economic fluctuations in the form*

$$1 + \lambda^T = \left(1 + \lambda^B\right)\left(1 + \lambda^R\right). \tag{14}$$

Proof. Straightforward from Propositions 1, 2, and 3. Details are left to Appendix A. ∎

4 Applications

In this section we outline below the closed-form solutions for the previously defined parameters λ^T, λ^B, and λ^R using two classical shock structures for the consumption process: Lucas (1987)'s transitory shocks and Obstfeld (1994)'s permanent shocks. In both of them, we apply the results obtained in Propositions 1, 2, and 3 to the different log-normal components X_t.

Example 1 (Lucas, 1987) : Define $C_t = \alpha_0(1+\alpha_1)^t e^{-0.5\sigma_\mu^2 + x_t^L}$, where $x_t^L \sim \mathcal{N}(0, \sigma_\mu^2)$. Hence,

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma = 1 \\ \exp\left\{\frac{\gamma}{2}\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{15}$$

$$\lambda^B = \begin{cases} \exp\left\{\frac{\theta}{2}\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma = 1 \\ \exp\left\{\frac{\gamma}{2}\sigma_\varepsilon^2 - \frac{1}{2}(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{16}$$

$$\lambda^R = \begin{cases} \exp\left\{\frac{1-\theta}{2}\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma = 1 \\ \exp\left\{\frac{1}{2}(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{17}$$

One should note that σ_t^2 becomes constant, being σ_ε^2. Consequently, Assumption 3 is satisfied as long as Assumption 2 holds. Therefore, we only need that $\Gamma \in (0, 1)$ in the case of transitory shocks to find out the closed-form solutions for λ^T, λ^B and λ^R.

Example 2 (Obstfeld, 1994) : Define $C_t = \alpha_0(1+\alpha_1)^t e^{-0.5\sigma_\mu^2 + x_t^O}$, where $x_t^O = \sum_{i=0}^t \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.[5] Thus,

$$\lambda^T = \begin{cases} \exp\left\{\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma = 1 \\ \exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\right\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{18}$$

$$\lambda^B = \begin{cases} \exp\left\{\theta\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma = 1 \\ \frac{\exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}}{\exp\left\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}\left[\frac{1-\Gamma\exp\left\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\right\}}{1-\Gamma\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\}}\right]^{\frac{1}{1-\gamma}} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{19}$$

$$\lambda^R = \begin{cases} \exp\left\{(1-\theta)\frac{1}{2}\frac{1}{1-\beta}\sigma_\varepsilon^2\right\} - 1 & \text{if } \gamma = 1 \\ \frac{1}{\exp\left\{0.5(1-\theta)[\gamma+\theta-\theta\gamma]\sigma_\varepsilon^2\right\}}\left[\frac{1-\Gamma\exp\left\{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_\varepsilon^2\right\}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1 & \text{if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{20}$$

[5] In some calculations, Obstfeld (1994) treats C_0 as known. We consider the case where the expectation is taken before the realization of the shock ε_0.

In this case, $\sigma_t^2 = Var_0\left[\sum_{i=0}^{t}\varepsilon_i\right] = (t+1)\sigma_\varepsilon^2$, and the condition $\Gamma\exp\{-0.5\gamma(1-\gamma)\sigma_\varepsilon^2\} < 1$ is sufficient for Assumption 3 to be valid. Therefore, in addition to $\Gamma \in (0,1)$, we need this condition when imposing permanent shocks in the consumption process.

5 Empirical Approach

In order to estimate the parameters of the underlying consumption process and compute λ^T, λ^B and λ^R, we have to be specific in our assumptions about the structure of the shocks. Following the examples in the last section, we consider the two aforementioned cases: transitory and permanent shocks. In this section, we first develop the regressions to be estimated in the data and characterize the challenge present in the identification of the span of stabilization power, θ, as well as the other parameters of our setting. We then present algebraically and visually the strategy we implement to overcome this difficulty, allowing us to pin down the values to be used in our results.

5.1 Estimation

5.1.1 Transitory Shocks

Assuming transitory shocks under Assumption 1 and applying the logarithm to both sides of equation (2), we have that:

$$\log\left(C_t(\theta)\right) = \log\left(\alpha_0\right) - (1-\theta)0.5\sigma_\varepsilon^2 + t\log\left(1+\alpha_1\right) + (1-\theta)\varepsilon_t. \tag{21}$$

It is straightforward to observe equation (21) above and reinterpret it as the following simple time series regression on log-consumption with coefficients π_0 and π_1, and error term u_t:

$$\log(c_t) = \pi_0 + \pi_1 t + u_t, \tag{22}$$

where c_t represents per capita consumption.

Note that an identification problem arises when we try to estimate the parameters in equation (21) since $(\alpha_0, \theta, \sigma_\varepsilon^2)$ are all scaled by $(1-\theta)$, with a convolution of both θ and σ_ε^2 lying in the background of u_t. Only parameter α_1 is well-identified and can be directly inverted from the estimates since $\alpha_1 = \exp(\pi_1) - 1$.

5.1.2 Permanent Shocks

Similarly, considering a case where permanent shocks hit consumption and taking the logarithm in equation (2), we have that:

$$\log(C_t(\theta)) = \log(\alpha_0) - (1-\theta)0.5t\sigma_\varepsilon^2 + t\log(1+\alpha_1) + (1-\theta)\sum_{i=0}^{t}\varepsilon_i \tag{23}$$

Taking first differences,

$$\Delta\log(C_t(\theta)) = \log(1+\alpha_1) - (1-\theta)0.5\sigma_\varepsilon^2 + (1-\theta)\varepsilon_t, \tag{24}$$

Once again, we can re-write equation (24) into a simple time-series regression:

$$\Delta\log(c_t) = \pi_0 + u_t, \tag{25}$$

The same identification problem arises when we try to estimate the parameters in equation (24): $(\alpha_1, \theta, \sigma_\varepsilon^2)$ are all scaled by $(1-\theta)$.

5.2 Identification

From our previous characterization of the identification problem, we are able to derive a critical fact that traces back to our main point about the nature of observed consumption: the scaling of the structural parameters by θ means that the consumption series is partially smoothed due to the ongoing stabilization policies. This means that if we knew θ (or σ_ε^2) in advance, it would be possible to recover all parameters in our consumption model by running a simple regression like the ones shown previously. Since this is not possible, we need to design an identification strategy.

Our strategy consists of exploring an observed variation in the volatility of the historical consumption series in order to identify θ. We build on the empirical fact that per capita consumption in the US has become less volatile over time, showing a heteroskedastic pattern. More specifically, we follow Lucas (1987), Barro and Ursúa (2008), and Nakamura et al. (2017), who show that the end of the Second World War marks a substantial decrease in the volatility of consumption. We profit from the approach of discontinuity-based identification (Nakamura and Steinsson, 2018) and assume that no other facts that affect the consumption series of the US change discontinuously at the end of WWII.

In order to understand how the identification works in formal terms, suppose that we have two periods of time, 1 and 2, and that $Var(\varepsilon_t) = \sigma_\varepsilon^2$ in both periods,[6] but we observe a lower volatility in consumption in period 2. All else constant, we can attribute this difference in the measured volatility to a different span of stabilization power of policies in those periods. To see that, let θ_i and $\hat{\sigma}_{u,i}^2$ be respectively the stabilization power and the estimated variance of u_t in period $i \in \{1, 2\}$. Thus, we have that $\hat{\sigma}_{u,i}^2 = (1-\theta_i)^2\sigma_\varepsilon^2$. If we knew θ_1 in advance, we could pin down θ_2 using the following identifying equation:

$$\hat{\theta}_2 = 1 - (1-\theta_1)\sqrt{\frac{\hat{\sigma}_{u,2}^2}{\hat{\sigma}_{u,1}^2}}. \tag{26}$$

The remaining parameter to delineate in the strategy is θ_1. For that, a natural candidate in the data would be a period of incipient stabilization policies, i.e., one in which θ_1 is close to zero. To apply this strategy in the data and estimate our parameters, we need to use a long series of consumption for the US. Our choice is to build on the data by Barro and Ursúa (2010). This database contains annual observations of US per capita

[6] In other words, the volatility of the laissez-faire consumption series, $\{\tilde{C}_t\}_{t=0}^{\infty}$, is time-independent.

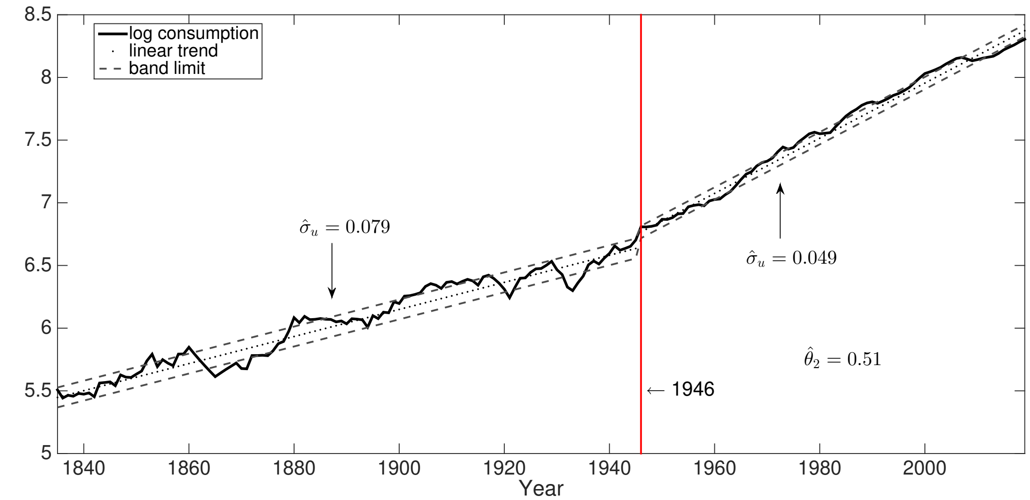
consumption between 1834 and 2009. We complete the sequence of consumption between 2010 and 2019, maintaining their methodology and using the series available from the BEA's NIPA.[7] Finally, we set the data in real terms to the year of 2012.

A large body of work points out differences in the way stabilization policies, both fiscal and monetary, were designed and conducted before 1930. For instance, Blanchard et al. (2000) argue that in the period prior to 1940, the study of macroeconomics was incipient. It is also not uncommon to find studies that describe how policy decisions, if existing, amplified the effects of economic shocks prior to World War II. A classical reference on this topic is Friedman and Schwartz (2008), describing how monetary policy exacerbated the 1929 crisis. On the other hand, during the 1930s many policies were established in order to stabilize the economy and provide insurance to the population. For example, the Social Security Act of 1935 created unemployment insurance, old-age insurance, and means-tested welfare programs, but it took years of expansion to reach (almost) universal coverage (Martin and Weaver, 2005).

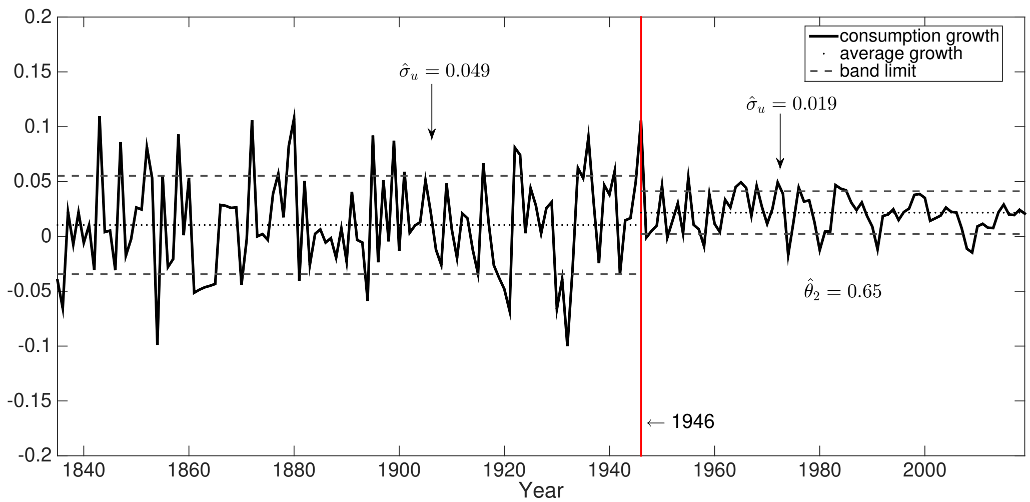
In Figure 2, we show our identification strategy at work in the plot of the historical series of consumption. The top panel shows the series in its log level for the identification with transitory shocks and the bottom panel shows the series first difference to accommodate the permanent shocks approach. One can see that, if we divide the series into two periods, pre- and post-war, there is a substantial decrease in the measured standard error after 1946. For instance, focusing on the series with first differences, for the period between 1834 and 1945, $\hat{\sigma}_u = 0.049$, which then suffers a decrease of more than 60 percent of its value, to $\hat{\sigma}_u = 0.019$, after WWII until today. With such a discontinuous decrease in the volatility of the series, we can apply our identification strategy to recover θ. We plug these two measures into equation (26) and, assuming $\theta_1 = 0.20$, we find that $\hat{\theta}_2 = 0.65$, indicating a share of 65 percent of smoothed consumption in the observed series.

[7] We use the"Real Personal Consumption Expenditures per Capita" series from the BEA available in FRED (link).

Figure 2: Time series of per capita consumption for the US between 1835 and 2019.



(a) Time series of log consumption.



(b) Time series of consumption growth.

Notes: The figure shows the time series for per capita consumption for the US between 1835 and 2019 with our augmented sample of the Barro and Ursúa (2010) data. There are two panels: the top one uses the series in log levels and the second in growth. The vertical line marks the year 1946, at the end of WWII. We report the standard errors for the two sub-periods generated by this line along with the average and band limits equivalent to $2\sigma_u$. We include an extra figure showing the data of panel (a) in an equivalent format to the data in panel (b) in Appendix B.

A critical point for our measurement of the decrease in consumption's standard error is the seminal argument by Romer (1986) about the spurious decrease in the unemployment rate's volatility after 1948. A similar point is emphasized for GNP in Balke and Gordon (1989) and revisited for GDP by Barro and Ursúa (2008). The first important consideration that differs in our approach is the fact that we use the series for consumption collected by the BEA since 1929, and together with the augmented Barro and Ursúa (2010) sample, we are able to add an extra 90 years to the original length that yielded the observation. A second and more important consideration is the fact that our methodology gives us enough flexibility for a degree of discretion in the interpretation of the span during the incipient stabilization period. From equation (26), one can see that, the greater θ_1, the smaller the impact of the volatility ratio in the identification of the second period's span. In that sense, the choice of θ_1 can be made larger to reflect both a historically motivated share of riskless consumption in the observed consumption mix and to also take into account a certain degree of measurement error that undermined the mapping of such stabilization to the collected data.[8]

6 Empirical Results

6.1 Estimation

We run regressions (22) and (25) and obtain their estimated coefficients as well as the error volatility of the two distinct periods, $\hat{\sigma}^2_{u,i}$. We obtain the span of stabilization power, $\hat{\theta}_2(\theta_1)$, for the grid $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, in order to allow different policies in the initial period. With those, we can also compute $\hat{\sigma}^2_\varepsilon = \hat{\sigma}^2_{u,1}/(1-\theta_1)^2$. For the remaining parameters, in the case of transitory shocks we have that $\hat{\alpha}_1 = \exp(\hat{\pi}_1) - 1$, while in the case of permanent shocks, $\hat{\alpha}_1 = \exp\left(\hat{\pi}_0 + (1-\hat{\theta}_2)0.5\sigma^2_\varepsilon\right) - 1$. Table 1 shows the results of our estimation.

[8] In effect, here we also develop another subtle point mentioned in Lucas's original analysis. In Lucas (1987), footnote 4, there is a mention of Romer (1986) in which the author acknowledges that his calculations do not incorporate her findings and may rely on the 1930s experience.

Table 1: Estimated parameters.

Transitory shocks				Permanent shocks			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$
1835 - 1945	5.4365	0.0108	0.0063	1835 - 1945	0.0104	-	0.0020
	(.0155)	(.0002)			(.0043)		
1946 - 2019	4.3028	0.0220	0.0024	1946 - 2019	0.0216	-	0.0004
	(.0402)	(.0003)			(.0023)		
Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3897	0.0222	0.0063	0.00	0.5650	0.0223	0.0020
0.10	0.4507	0.0222	0.0078	0.10	0.6085	0.0224	0.0025
0.20	0.5118	0.0222	0.0099	0.20	0.6520	0.0224	0.0031
0.30	0.5728	0.0222	0.0129	0.30	0.6955	0.0225	0.0041

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data with sub-periods divided at 1946. The implied parameters are obtained using the formulas described in the text and equation (26).

Our preferred choice of initial span θ_1 is 0.20, since it allows, as mentioned previously, for a combination of some degree of stabilization power and measurement error in the pre-1946 sample. With that level, the estimated span is 0.6520 with permanent shocks and 0.5118 with transitory shocks. That shows that the current reach of stabilization policies is far from trivial, since it is able to smooth approximately 70 percent of consumption volatility; plus it shows that the stabilization reach more than tripled after WWII.

Another feature of our strategy shown in Figure 2 is the strict division of the data in the year 1946. We relax this cut by conducting a series of robustness checks with different windows of time. We start by first searching for structural breaks in the series and identify one in 1931. Given that, we run another set of regressions, breaking the first half of the series in 1913 and in 1930 in order to allow for the break and to eliminate the highly unstable inter war period. We find results similar to the estimates shown in Table 1. We also conduct the same exercise with the original Barro and Ursúa (2010) sample and find no significant differences. All details, tests, and results can be found in Appendix C.

6.2 Welfare Costs of Business Cycles

With the estimated $\hat{\theta}_2$ and $\hat{\sigma}^2_\varepsilon$, we can now turn back to the calculation of our decomposition for λ^T, λ^B, and λ^R shown in Theorem 1. These numbers are obtained using equations (15) through (20). We also provide a measure that is comparable to the ones shown in the literature with the absence of the span θ, and is represented by λ without a superscript. The derivation of this application is shown in Appendix D. Table 2 shows the result:

Table 2: Decomposition of the welfare cost of total economic fluctuations

Transitory shocks													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.39	0.45	0.51	0.57	0.39	0.45	0.51	0.57	0.39	0.45	0.51	0.57	-
$\gamma = 1$	0.32	0.39	0.50	0.65	0.12	0.18	0.25	0.37	0.19	0.21	0.24	0.28	0.12
$\gamma = 2.5$	0.79	0.98	1.24	1.63	0.42	0.59	0.82	1.17	0.37	0.39	0.42	0.45	0.29
$\gamma = 5$	1.59	1.97	2.50	3.28	0.92	1.27	1.77	2.51	0.67	0.69	0.71	0.75	0.59
$\gamma = 10$	3.21	3.98	5.06	6.67	1.93	2.66	3.71	5.25	1.26	1.28	1.31	1.34	1.18
Permanent shocks													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.56	0.61	0.65	0.70	0.56	0.61	0.65	0.70	0.56	0.61	0.65	0.70	-
$\gamma = 1$	2.55	3.69	4.59	5.97	1.43	2.63	3.54	4.86	1.10	1.03	1.01	1.05	0.48
$\gamma = 2.5$	3.15	4.59	5.81	7.90	1.91	3.46	4.71	6.75	1.22	1.10	1.05	1.08	0.68
$\gamma = 5$	4.01	5.89	7.63	11.26	2.60	4.65	6.45	10.04	1.38	1.18	1.10	1.11	0.81
$\gamma = 10$	5.27	7.85	10.54	18.76	3.63	6.48	9.27	17.42	1.58	1.29	1.17	1.14	0.94

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations. The numbers are obtained using equations (15) through (20) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix D. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with a calibrated $\beta = 0.96$ for the permanent shock.

We can focus on the usual level of relative risk aversion used in the literature $\gamma = 2.5$. First, we look at transitory shocks. One can see that in this case, with a span of 0.51, the total cost λ^T is 1.24 percent of lifetime consumption, being divided into 0.82 stemming from the benefit of current stabilization policies and 0.42 of residual costs. Our preferred specification for the shocks, though, is the permanent one. With this structure, we have that $\lambda^T = 5.81$, with more than 80 percent of this cost stemming from λ^B and 1.05 percent of permanent consumption still left to be smoothed by policies. If we compare this result to the equivalent calculation obtained in the literature, λ, only our λ^R would keep the same reasoning in terms of a thought experiment. In that case, we can see that for our permanent shocks specification, our residual cost is almost 55 percent larger than what the literature would suggest.

The results also allow us to explore a theoretical aspect and understand how the concave utility interacts with our proposed decomposition and the parameter θ. If we fix a given level of the measured span of policies, the marginal benefit of smoothing the residual fluctuations in proportion to the total welfare cost, i.e. λ^R/λ^T, is decreasing in the relative degree of risk aversion γ. That means that more risk-averse consumers tend to value relatively more the benefit generated by the ongoing stabilization policies, going up as much as 88 percent of the total welfare cost with permanent shocks when γ is at the highest level shown.

7 Conclusion

In this paper we revisited the long-standing issue of the welfare costs of business cycles with a focus on unveiling the extent to which ongoing stabilization policies are smoothing observed consumption. We rooted our approach in the novel modelling that all data we gather on consumption are subject to the policy status quo and we provided a decomposition for macroeconomic fluctuations. We recovered the total welfare costs of business cycles by disentangling them into the benefit of current policies and the residual yet to be flattened.

We also conducted an empirical analysis with the goal of identifying our key decomposition parameter, the span of stabilization power, from the historical consumption data.

In doing so, we profited from the observation that there is a discontinuous decrease in the series' volatility after WWII, a fact widely documented by a vast literature in macroeconomics. With the proper strategy, we were able to recover estimates from the data and found that the span of stabilization power, in our preferred shock structure and parameter space, is approximately 70 percent and the welfare costs of total economic fluctuations are almost 6 percent of permanent consumption, with almost 5 percent of it already being smoothed by ongoing policies and 1 percent left as a residual for policy to smooth.

References

Alvarez, Fernando and Urban J. Jermann (2004). "Using Asset Prices to Measure the Cost of Business Cycles." *Journal of Political Economy*, 112(6), pp. 1223–1256. doi:10.1086/424738.

Bai, Jushan and Pierre Perron (1998). "Estimating and testing linear models with multiple structural changes." *Econometrica*, 66(1), pp. 47–78. doi:10.2307/2998540.

Bai, Jushan and Pierre Perron (2003). "Critical values for multiple structural change tests." *The Econometrics Journal*, 6(1), pp. 72–78. doi:10.1111/1368-423X.00102.

Balke, Nathan S. and Robert J. Gordon (1989). "The estimation of prewar gross national product." *Journal of Political Economy*, 97(1), pp. 38–92. doi:10.1086/261593.

Barlevy, Gadi (2005). "The cost of business cycles and the benefits of stabilization." In *Economic Perspectives*, pp. 32–49. Federal Reserve Bank of Chicago. URL https://www.chicagofed.org/publications/economic-perspectives/2005/1qtr2005-part3-barlevy.

Barro, Robert J and Tao Jin (2011). "On the Size Distirbution of Macroeconomic Disasters." *Econometrica*, 79(5), pp. 1567–1589. doi:10.3982/ECTA8827.

Barro, Robert J and José F Ursúa (2008). "Macroeconomic Crises since 1870." *Brookings Papers on Economic Activity, Spring*, 38(1), pp. 255–350. URL https://www.jstor.org/stable/27561620.

Barro, Robert J and José F Ursúa (2010). "Barro-Ursúa Macroeconomic Data." URL https://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data.

Barros, Fernando-Jr, Francisco Lima, and Diego M Silva (2017). "The Welfare Cost of Business Cycles for Heterogeneous Consumers: A State-Space Decomposition." *Economics Bulletin*, 33(3), pp. 1928–1941. URL http://www.accessecon.com/Pubs/EB/2017/Volume37/EB-17-V37-I3-P174.pdf.

Blanchard, Olivier et al. (2000). "What Do We Know about Macroeconomics that Fisher and Wicksell Did Not?" *The Quarterly Journal of Economics*, 115(4), pp. 1375–1409. doi:10.1162/003355300554999.

De Santis, Massimiliano (2007). "Individual consumption risk and the welfare costs of business cycles." *American Economic Review*, 97(4), pp. 1408–1506. doi:10.1257/aer.97.4.1488.

Dolmas, Jim (1998). "Risk preferences and the welfare cost of business cycles." *Review of Economic Dynamics*, 1(3), pp. 646–676. doi:10.1006/redy.1998.0020.

Dupraz, Stéphane, Emi Nakamura, and Jón Steinsson (2019). "A plucking model of business cycles." Working Paper 26351, National Bureau of Economic Research. doi:10.3386/w26351.

Friedman, Milton and Anna Jacobson Schwartz (2008). *A monetary history of the United States, 1867-1960*. Princeton University Press.

Gourio, François (2012). "Disaster Risk and Business Cycles." *American Economic Review*, 102(6), pp. 2734–2766. doi:10.1257/aer.102.6.2734.

Guillén, Osmani Teixeira de Carvalho, Joao Victor Issler, and Afonso Arinos de Mello Franco-Neto (2014). "On the welfare costs of business-cycle fluctuations and economic-growth variation in the 20th century and beyond." *Journal of Economic Dynamics and Control*, 39(C), pp. 62–78. doi:10.1016/j.jedc.2013.11.00.

Hai, Rong, Dirk Krueger, and Andrew Postlewaite (2020). "On the welfare costs of consumption fluctuations in the presence of memorable goods." *Quantitative Economics*, 11(4), pp. 1117–1214. doi:10.3982/QE1173.

Imrohoruglu, Ayse (1989). "Cost of Business Cycles with Indivisibilities and Liquidity Constraints." *Journal of Political Economy*, 97(6), pp. 1364–83. doi:10.1086/261658.

Issler, Joao Victor, Afonso Arinos de Mello Franco-Neto, and Osmani Teixeira de Carvalho Guillén (2008). "The welfare cost of macroeconomic uncertainty in the post-war period." *Economics Letters*, 98(2), pp. 167–175. doi:10.1016/j.econlet.2007.04.026.

Jorda, Oscar, Moritz Schularick, and Alan M. Taylor (2020). "Disasters everywhere: The costs of business cycles reconsidered." *Federal Reserve Bank of San Francisco Working Paper*, 11, pp. 1–34. doi:10.24148/wp2020-11.

Krusell, Per and Anthony A. Jr. Smith (1999). "On the Welfare Effects of Eliminating Business Cycles." *Review of Economic Dynamics*, 2(1), pp. 245–272. doi:10.1006/redy.1998.0043.

Lucas, Robert (1987). *Models of Business Cycles*. Yrjo Jahnsson Lectures. Basil Blackwell, Oxford and New York.

Martin, Patricia P. and David A Weaver (2005). "Social Security: A program and policy history." *Social Security Bulletin*, 66(1), pp. 1–15. URL https://www.ssa.gov/policy/docs/ssb/v66n1/v66n1p1.html.

Nakamura, Emi, Dmitriy Sergeyev, and Jon Steinsson (2013). "Crises and Recoveries in an Empirical Model of Consumption Disasters." *American Economic Journal: Macroeconomics*, 5(3), pp. 35–74. doi:10.1257/mac.5.3.35.

Nakamura, Emi, Dmitriy Sergeyev, and Jon Steinsson (2017). "Growth-rate and uncertainty shocks in consumption: Cross-country evidence." *American Economic Journal: Macroeconomics*, 9(1), pp. 1–39. doi:10.1257/mac.20150250.

Nakamura, Emi and Jon Steinsson (2018). "Identification in Macroeconomics." *Journal of Economic Perspectives*, 32(3), pp. 59–86. doi:10.1257/jep.32.3.59.

Obstfeld, Maurice (1994). "Evaluating risky consumption paths: The role of intertemporal substitutability." *European Economic Review*, 38(7), pp. 1471–1486. doi:10.1016/0014-2921(94)90020-5.

Reis, Ricardo (2009). "The time-series properties of aggregate consumption." *Journal of the European Economic Association*, 7(4), pp. 722–753. URL 10.1162/JEEA.2009.7.4.722.

Romer, Christina (1986). "Spurious volatility in historical unemployment data." *Journal of Political Economy*, 94(1), pp. 1–37. doi:10.1086/261361.

Storesletten, Kjetil, Chris Telmer, and Amir Yaron (2001). "The welfare costs of business cycles revisited: Finite lives and cyclical variation in idiosyncratic risk." *European Economic Review*, 45(3), pp. 413–417. doi:10.1016/S0014-2921(00)00101-X.

Appendix

A Proofs

Below we outline the continuation of the proofs for Lemmas 1, 2, 3 and Theorem 1. All of them are the generalization of the proofs shown in the text but for the case with $\gamma \neq 1$.

For Lemma 1:

Proof.

$$\sum_{t=0}^{\infty} \beta^t (1-\gamma)^{-1} \left(\bar{C}_t\right)^{1-\gamma} = (1-\gamma)^{-1} \alpha_0^{1-\gamma} \sum_{t=0}^{\infty} \left[\beta \left(1+\alpha_1\right)^{1-\gamma}\right]^t \tag{27}$$

$$= \tilde{\alpha}_0 \sum_{t=0}^{\infty} \Gamma^t. \tag{28}$$

■

For Lemma 2:

Proof.

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \frac{\left[(1+k)\tilde{C}_t\right]^{1-\gamma}}{1-\gamma}\right] = E_0\left[\sum_{t=0}^{\infty} \beta^t \left[(1+k)\alpha_0 \left(1+\alpha_1\right)^t \exp\left\{x_t - 0.5\sigma_t^2\right\}\right]^{1-\gamma}\right]$$

$$= \tilde{\alpha}_0 (1+k)^{1-\gamma} \sum_{t=0}^{\infty} \left[\beta \left(1+\alpha_1\right)^{1-\gamma}\right]^t \times \dots$$

$$\dots \exp\left\{-0.5 \left(1-\gamma\right) \sigma_t^2\right\} E_0 \left[\exp\left\{\left(1-\gamma\right) x_t\right\}\right].$$

Note that

$$E_0 \left[\exp\left\{\left(1-\gamma\right) x_t\right\}\right] = \exp\left\{E_0 \left[\left(1-\gamma\right) x_t\right] + 0.5 Var_0 \left[\left(1-\gamma\right) x_t\right]\right\} = \exp\left\{0.5 \left(1-\gamma\right)^2 \sigma_t^2\right\}.$$

Thus,

$$E_0\left[\sum_{t=0}^{\infty} \beta^t \frac{\left[(1+k)\tilde{C}_t\right]^{1-\gamma}}{1-\gamma}\right] = \tilde{\alpha}_0 (1+k)^{1-\gamma} \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5 \left(1-\gamma\right) \sigma_t^2\right\} \exp\left\{0.5 \left(1-\gamma\right)^2 \sigma_t^2\right\}$$

$$= \tilde{\alpha}_0 (1+k)^{1-\gamma} \sum_{t=0}^{\infty} \Gamma^t \exp\left\{-0.5\gamma \left(1-\gamma\right) \sigma_t^2\right\}$$

■

For Lemma 3:

Proof.

$$
\begin{aligned}
E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{[(1+\ell)C_t(\theta)]^{1-\gamma}}{1-\gamma}\right] &= (1-\gamma)^{-1}E_0\left[\sum_{t=0}^{\infty}\beta^t[(1+\ell)C_t(\theta)]^{1-\gamma}\right] \\
&= (1-\gamma)^{-1}E_0\left[\sum_{t=0}^{\infty}\beta^t\left[(1+\ell)\alpha_0(1+\alpha_1)^t+\ldots\right.\right. \\
\ldots &\quad \left.\left.\exp\left\{(1-\theta)\left[x_t-0.5\sigma_t^2\right]\right\}\right]^{1-\gamma}\right] \\
&= \tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\gamma}\right]^t\times\ldots \\
\ldots &\quad \exp\left\{-0.5(1-\theta)(1-\gamma)\sigma_t^2\right\}E_0\left[\exp\left\{(1-\theta)(1-\gamma)x_t\right\}\right]
\end{aligned}
$$

Note that

$$
\begin{aligned}
E_0\left[\exp\left\{(1-\theta)(1-\gamma)x_t\right\}\right] &= \exp\left\{E_0\left[(1-\theta)(1-\gamma)x_t\right]+0.5Var_0\left[(1-\theta)(1-\gamma)x_t\right]\right\} \\
&= \exp\left\{0.5(1-\theta)^2(1-\gamma)^2\sigma_t^2\right\}
\end{aligned}
$$

And,

$$
\begin{aligned}
&\exp\left\{-0.5(1-\theta)(1-\gamma)\sigma_t^2\right\}E_0\left[\exp\left\{(1-\theta)(1-\gamma)x_t\right\}\right] \\
= &\exp\left\{-0.5(1-\theta)(1-\gamma)\sigma_t^2\right\}\exp\left\{0.5(1-\theta)^2(1-\gamma)^2\sigma_t^2\right\} \\
= &\exp\left\{-0.5(1-\theta)(1-\gamma)(\gamma+\theta-\gamma\theta)\sigma_t^2\right\}
\end{aligned}
$$

Thus,

$$
E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{[(1+\ell)C_t(\theta)]^{1-\gamma}}{1-\gamma}\right]=\tilde{\alpha}_0(1+\ell)^{1-\gamma}\sum_{t=0}^{\infty}\Gamma^t\exp\left\{-0.5(1-\theta)(1-\gamma)(\gamma+\theta-\gamma\theta)\sigma_t^2\right\},
$$

which concludes the proof. ■

For Theorem 1:

Proof. For $\gamma = 1$, we have

$$\begin{aligned}\left(1+\lambda^B\right)\left(1+\lambda^R\right) &= \exp\left\{\theta\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\}\exp\left\{(1-\theta)\,\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} \\ &= \exp\left\{\frac{1-\beta}{2}\sum_{t=0}^{\infty}\beta^t\sigma_t^2\right\} = 1+\lambda^T\end{aligned}$$

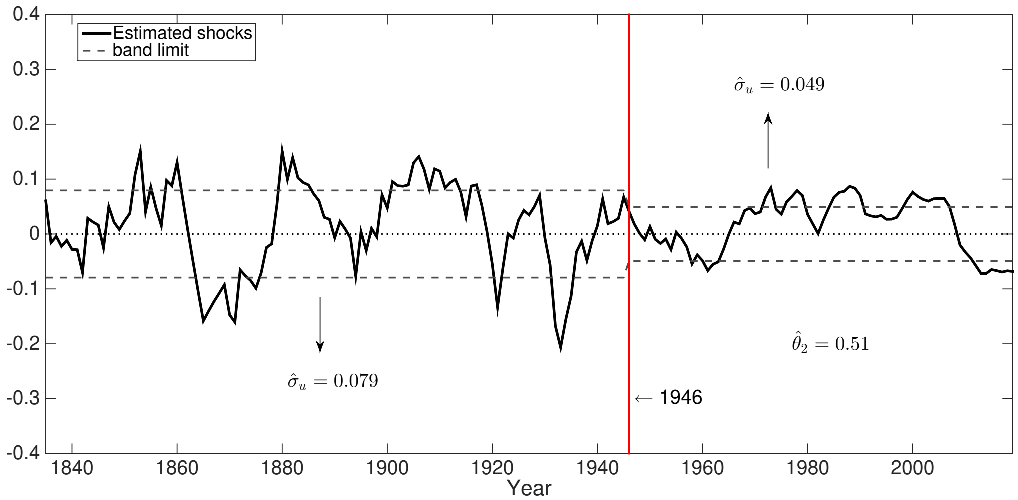
Now, for $\gamma \neq 1$, we have

$$\begin{aligned}\left(1+\lambda^B\right)^{1-\gamma}\left(1+\lambda^R\right)^{1-\gamma} &= \frac{\sum_{t=0}^{\infty}\Gamma^t e^{-0.5(1-\gamma)(1-\theta)(\theta+\gamma-\gamma\theta)\sigma_t^2}}{\sum_{t=0}^{\infty}\Gamma^t e^{-0.5\gamma(1-\gamma)\sigma_t^2}}\frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t e^{-0.5(1-\theta)(1-\gamma)(\gamma+\theta-\gamma\theta)\sigma_t^2}} \\ \Longleftrightarrow \left(1+\lambda^B\right)\left(1+\lambda^R\right) &= \left[\frac{\sum_{t=0}^{\infty}\Gamma^t}{\sum_{t=0}^{\infty}\Gamma^t e^{-0.5\gamma(1-\gamma)\sigma_t^2}}\right]^{\frac{1}{1-\gamma}} = 1+\lambda^T\end{aligned}$$

■

B Identification in Transitory Shocks

Figure 4: Time series of log-consumption between 1835 and 2019.



Notes: The figure shows the time series for per capita consumption for the US between 1835 and 2019 with our augmented sample of the Barro and Ursúa (2010) data. The vertical line marks the year 1946, at the end of WWII. We report the standard errors for the two sub-periods generated by this line along with the average and band limits equivalent to $2\sigma_u$.

C Robustness Exercises

C.1 Full Sample

In Table 3 we present our estimations of the welfare cost using the full sample as in the main text. We also compute the λ's for different values of β in the case of permanent shocks.

Table 3: Welfare Cost - Full Sample

	λ^T				λ^B				λ^R				λ
Transitory shocks													
$\hat{\theta}_2$	0.39	0.45	0.51	0.57	0.39	0.45	0.51	0.57	0.39	0.45	0.51	0.57	-
$\gamma = 1$	0.32	0.39	0.50	0.65	0.12	0.18	0.25	0.37	0.19	0.21	0.24	0.28	0.12
$\gamma = 2.5$	0.79	0.98	1.24	1.63	0.42	0.59	0.82	1.17	0.37	0.39	0.42	0.45	0.29
$\gamma = 5$	1.59	1.97	2.50	3.28	0.92	1.27	1.77	2.51	0.67	0.69	0.71	0.75	0.59
$\gamma = 10$	3.21	3.98	5.06	6.67	1.93	2.66	3.71	5.25	1.26	1.28	1.31	1.34	1.18
Permanent shocks													
$\beta = 0.95$													
$\hat{\theta}_2$	0.56	0.61	0.65	0.70	0.56	0.61	0.65	0.70	0.56	0.61	0.65	0.70	-
$\gamma = 1$	2.03	2.52	3.19	4.19	1.14	1.52	2.07	2.90	0.88	0.98	1.10	1.26	0.38
$\gamma = 2.5$	3.23	4.02	5.14	6.84	2.30	3.02	4.06	5.64	0.90	0.96	1.04	1.14	0.59
$\gamma = 5$	4.24	5.35	6.99	9.61	3.26	4.33	5.91	8.43	0.94	0.98	1.02	1.08	0.75
$\gamma = 10$	5.67	7.48	10.54	17.09	4.61	6.38	9.38	15.82	1.01	1.03	1.06	1.10	0.90
$\beta = 0.96$													
$\hat{\theta}_2$	0.56	0.61	0.65	0.70	0.56	0.61	0.65	0.70	0.56	0.61	0.65	0.70	-
$\gamma = 1$	2.55	3.69	4.59	5.97	1.43	2.63	3.54	4.86	1.10	1.03	1.01	1.05	0.48
$\gamma = 2.5$	3.15	4.59	5.81	7.90	1.91	3.46	4.71	6.75	1.22	1.10	1.05	1.08	0.68
$\gamma = 5$	4.01	5.89	7.63	11.26	2.60	4.65	6.45	10.04	1.38	1.18	1.10	1.11	0.81
$\gamma = 10$	5.27	7.85	10.54	18.76	3.63	6.48	9.27	17.42	1.58	1.29	1.17	1.14	0.94
$\beta = 0.97$													
$\hat{\theta}_2$	0.56	0.61	0.65	0.70	0.56	0.61	0.65	0.70	0.56	0.61	0.65	0.70	-
$\gamma = 1$	3.41	4.23	5.38	7.08	1.91	2.55	3.48	4.88	1.47	1.63	1.84	2.11	0.64
$\gamma = 2.5$	4.30	5.36	6.90	9.23	3.07	4.04	5.45	7.61	1.19	1.27	1.37	1.50	0.78
$\gamma = 5$	5.02	6.37	8.39	11.68	3.88	5.16	7.11	10.28	1.10	1.14	1.20	1.27	0.87
$\gamma = 10$	6.29	8.39	12.08	20.87	5.14	7.18	10.80	19.45	1.10	1.12	1.15	1.19	0.98

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations expanding the one in the main text for different β's. The numbers are obtained using equations (15) through (20) with the estimates shown in Table 1. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix D. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

C.2 Structural Break

One can be concerned with the presence of structural breaks in a long time series. We apply the methodology developed in Bai and Perron (1998, 2003) to test structural breaks in our sample. For the transitory shock version we test a structural break in the log-consumption and we find a break in 1931 (scaled F-statistic is 1221.88 with a critical value of 11.47). We also test a structural break in the first difference of log-consumption for our example with permanent shocks. We obtain a scaled F-statistic of 9.31 (with critical value of 8.58) indicating a break in 1934.[9]

We use the break in order to create sub-samples for our identification strategy. The first sub-sample considers the years from the beginning of the sample until the year of the structural break, and a second sub-sample as in the main text, considers the years after WWII. In order to keep the sub-samples same size for our estimations with transitory or permanent shocks, we set the first sub-sample for the years between 1835 and 1930.

The results of the estimated parameters along with the implied θ are presented in Table 4. If we compare those results with the results in Table 1 in the main text, we note that the estimations imply a marginal change in the implied parameters in the version with transitory shocks and a very small change in the case of permanent shocks.

[9] In our tests, we allow for at most 5 breaks in the time series. The tests indicate only one break in the first difference of log consumption (1934) and indicates 3 breaks in the log-consumption (1879, 1931 and, 1993).

Table 4: Structural Break - Estimation

Transitory shocks				Permanent shocks			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$
1835 - 1930	5.4181	0.0114	0.0055	1835 - 1930	0.0096	-	0.0019
	(.0156)	(.0003)			(.0045)		
1946 - 2019	4.3028	0.0220	0.0024	1946 - 2019	0.0216	-	0.0004
	(.0402)	(.0003)			(.0023)		
Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3463	0.0222	0.0055	0.00	0.5570	0.0223	0.0019
0.10	0.4117	0.0222	0.0068	0.10	0.6013	0.0224	0.0024
0.20	0.4771	0.0222	0.0086	0.20	0.6456	0.0224	0.0030
0.30	0.5424	0.0222	0.0113	0.30	0.6899	0.0225	0.0040

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data with sub-periods 1835-1930 and 1946-2019. The implied parameters are obtained using the formulas described in the text and equation (26).

Table 5 presents the welfare cost using the implied parameters in Table 4.

Table 5: Structural Break - Welfare cost

	λ^T				λ^B				λ^R				λ
Transitory shocks													
$\hat{\theta}_2$	0.35	0.41	0.48	0.54	0.35	0.41	0.48	0.54	0.35	0.41	0.48	0.54	-
$\gamma = 1$	0.28	0.34	0.43	0.56	0.10	0.14	0.21	0.31	0.18	0.20	0.23	0.26	0.12
$\gamma = 2.5$	0.69	0.85	1.08	1.42	0.33	0.47	0.68	0.98	0.36	0.38	0.40	0.44	0.29
$\gamma = 5$	1.39	1.72	2.18	2.85	0.73	1.04	1.47	2.11	0.65	0.67	0.70	0.73	0.59
$\gamma = 10$	2.79	3.46	4.40	5.79	1.53	2.17	3.07	4.40	1.25	1.27	1.29	1.33	1.18
Permanent shocks													
$\beta = 0.95$													
$\hat{\theta}_2$	0.56	0.60	0.65	0.69	0.56	0.60	0.65	0.69	0.56	0.60	0.65	0.69	-
$\gamma = 1$	1.96	2.43	3.08	4.04	1.09	1.45	1.98	2.77	0.86	0.96	1.08	1.24	0.38
$\gamma = 2.5$	3.11	3.87	4.95	6.58	2.20	2.89	3.88	5.40	0.89	0.95	1.03	1.12	0.59
$\gamma = 5$	4.07	5.14	6.71	9.20	3.11	4.12	5.63	8.03	0.93	0.97	1.02	1.08	0.75
$\gamma = 10$	5.42	7.12	9.97	15.85	4.37	6.03	8.82	14.60	1.01	1.03	1.06	1.09	0.90
$\beta = 0.96$													
$\hat{\theta}_2$	0.56	0.60	0.65	0.69	0.56	0.60	0.65	0.69	0.56	0.60	0.65	0.69	-
$\gamma = 1$	2.46	3.55	4.42	5.70	1.36	2.51	3.37	4.60	1.08	1.02	1.01	1.05	0.48
$\gamma = 2.5$	3.04	4.42	5.58	7.52	1.82	3.30	4.49	6.38	1.20	1.08	1.05	1.07	0.68
$\gamma = 5$	3.86	5.67	7.31	10.62	2.48	4.45	6.15	9.42	1.35	1.17	1.10	1.10	0.81
$\gamma = 10$	5.08	7.55	10.08	17.29	3.48	6.20	8.82	15.97	1.55	1.28	1.16	1.14	0.94
$\beta = 0.97$													
$\hat{\theta}_2$	0.56	0.60	0.65	0.69	0.56	0.60	0.65	0.69	0.56	0.60	0.65	0.69	-
$\gamma = 1$	3.29	4.08	5.19	6.83	1.82	2.43	3.32	4.66	1.44	1.61	1.81	2.07	0.64
$\gamma = 2.5$	4.14	5.16	6.63	8.87	2.92	3.86	5.21	7.28	1.18	1.26	1.35	1.48	0.78
$\gamma = 5$	4.82	6.11	8.03	11.15	3.69	4.92	6.77	9.77	1.09	1.13	1.19	1.26	0.87
$\gamma = 10$	6.01	7.97	11.37	19.08	4.86	6.77	10.11	17.68	1.09	1.12	1.15	1.19	0.98

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that avoids the structural break in 1931. The numbers are obtained using equations (15) through (20) with the estimates shown in Table 4. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix D. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

C.3 Removing the Interwar Period

As the previous exercise used the disjoint periods (1835-1930 and 1946-2019), we run an additional experiment where we use the 1931 break in the time series as a reference point to design two new intervals. In the previous exercise, we have removed 15 periods - years 1931 to 1945 - from the full sample. Those periods were exclusively defined after the break. For this case, we remove a similar interval for the period before the 1931 break. We construct two sub-samples by excluding the interwar period from our data, which results in a first period with years 1835 to 1913 and a second period from 1946 to 2019, the last one as in our main analysis. [10]

Besides the structural break in the consumption series during the interwar period, many other relevant macroeconomic events happened during this window of time. For example, we have the 1929 crisis and the Great Depression that followed. In general, this period was marked by highly unstable macroeconomic outcomes and hence it is worth subtracting it from the sample to better measure pre-war volatility. Once again, the results are similar to our original analysis.

Table 6 presents the estimated and implied parameters.

[10] We also run an experiment by removing exactly 15 periods before and after the break, that is, using subssamples from 1835-1915 and 1946-2019. As expected, the results are so similar to the results in this subsection that we only report the exercise where we remove the interwar period.

Table 6: Removing the Interwar Period - Estimation

Transitory shocks				Permanent shocks			
Estimated parameters				Estimated parameters			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$
1835 - 1913	5.402	0.0119	0.0057	1835 - 1913	0.0107	-	0.0019
	(.0175)	(.0004)			(.0049)		
1946 - 2019	4.3028	0.0220	0.0024	1946 - 2019	0.0216	-	0.0004
	(.0402)	(.0003)			(.0023)		
Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.3554	0.0222	0.0057	0.00	0.5527	0.0223	0.0019
0.10	0.4198	0.0222	0.0070	0.10	0.5974	0.0224	0.0023
0.20	0.4843	0.0222	0.0089	0.20	0.6421	0.0224	0.0030
0.30	0.5487	0.0222	0.0116	0.30	0.6869	0.0225	0.0039

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks. The time series used is our sample of the augmented Barro and Ursúa (2010) data excluding the interwar period. The implied parameters are obtained using the formulas described in the text and equation (26).

Table 7 presents the computed λ using the estimations in Table 6.

Table 7: Removing the Interwar Period - Welfare Cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.36	0.42	0.48	0.55	0.36	0.42	0.48	0.55	0.36	0.42	0.48	0.55	-
$\gamma = 1$	0.28	0.35	0.44	0.58	0.10	0.15	0.21	0.32	0.18	0.20	0.23	0.26	0.12
$\gamma = 2.5$	0.71	0.88	1.11	1.46	0.35	0.50	0.70	1.01	0.36	0.38	0.41	0.44	0.29
$\gamma = 5$	1.43	1.76	2.24	2.93	0.77	1.08	1.53	2.18	0.66	0.68	0.70	0.73	0.59
$\gamma = 10$	2.87	3.56	4.53	5.95	1.60	2.26	3.19	4.56	1.25	1.27	1.30	1.33	1.18
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.55	0.60	0.64	0.69	0.55	0.60	0.64	0.69	0.55	0.60	0.64	0.69	-
$\gamma = 1$	1.92	2.38	3.02	3.96	1.06	1.41	1.93	2.70	0.86	0.95	1.07	1.22	0.38
$\gamma = 2.5$	3.05	3.79	4.85	6.44	2.14	2.82	3.79	5.27	0.89	0.95	1.02	1.12	0.59
$\gamma = 5$	3.99	5.02	6.56	8.98	3.03	4.02	5.49	7.82	0.93	0.97	1.01	1.07	0.75
$\gamma = 10$	5.29	6.93	9.68	15.24	4.24	5.85	8.53	14.00	1.01	1.03	1.06	1.09	0.90
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.55	0.60	0.64	0.69	0.55	0.60	0.64	0.69	0.55	0.60	0.64	0.69	-
$\gamma = 1$	2.41	3.48	4.32	5.56	1.32	2.44	3.29	4.46	1.07	1.01	1.00	1.05	0.48
$\gamma = 2.5$	2.98	4.33	5.46	7.32	1.77	3.22	4.37	6.18	1.19	1.08	1.04	1.07	0.68
$\gamma = 5$	3.79	5.55	7.14	10.30	2.42	4.34	5.99	9.10	1.34	1.16	1.09	1.10	0.81
$\gamma = 10$	4.98	7.39	9.84	16.58	3.39	6.05	8.58	15.27	1.53	1.27	1.15	1.13	0.94
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.55	0.60	0.64	0.69	0.55	0.60	0.64	0.69	0.55	0.60	0.64	0.69	-
$\gamma = 1$	3.22	3.99	5.08	6.69	1.77	2.37	3.23	4.55	1.43	1.59	1.79	2.05	0.64
$\gamma = 2.5$	4.05	5.06	6.49	8.68	2.85	3.76	5.08	7.10	1.17	1.25	1.35	1.47	0.78
$\gamma = 5$	4.72	5.97	7.85	10.88	3.59	4.79	6.59	9.51	1.09	1.13	1.18	1.25	0.87
$\gamma = 10$	5.86	7.75	11.01	18.22	4.71	6.56	9.76	16.84	1.09	1.12	1.14	1.18	0.98

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample that excludes the interwar period. The numbers are obtained using equations (15) through (20) with the estimates shown in Table 6. All measures are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix D. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5, 10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

C.4 Barro and Ursúa (2010) Data

We now show our results using Barro and Ursúa (2010) data only. The main difference occurs in the case of transitory shocks, where we observe a lower volatility in the post-war period. This leads to an increase in the span of stabilization power. Without the inclusion of our sample, we observe that this parameter is closer in value with both types of shocks.

Table 8: Barro and Ursúa (2010) Data - Estimation

Transitory shocks				Permanent shocks			
	$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$		$\hat{\pi}_0$	$\hat{\pi}_1$	$\hat{\sigma}_u^2$
1835 - 1945	5.4365	0.0108	0.0063	1835 - 1945	0.0104	-	0.0020
	(.0155)	(.0002)			(.0043)		
1946 - 2009	4.1278	0.0232	0.0011	1946 - 2009	0.0223	-	0.0004
	(.0326)	(.0002)			(.0026)		
Implied parameters				Implied parameters			
θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$	θ_1	$\hat{\theta}_2$	$\hat{\alpha}_1$	$\hat{\sigma}_\varepsilon^2$
0.00	0.5873	0.0235	0.0063	0.00	0.5380	0.0231	0.0020
0.10	0.6285	0.0235	0.0078	0.10	0.5842	0.0231	0.0025
0.20	0.6698	0.0235	0.0099	0.20	0.6304	0.0232	0.0031
0.30	0.7111	0.0235	0.0129	0.30	0.6766	0.0233	0.0041

Notes: The table displays the estimated parameters obtained by running regression (22) for transitory shocks and regression (25) for permanent shocks. The time series used is the Barro and Ursúa (2010) data. The implied parameters are obtained using the formulas described in the text and equation (26).

Table 9 presents the computed λ using the estimations in Table 8.

Table 9: Barro and Ursúa (2008) Data - Welfare Cost

Transitory shocks													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.59	0.63	0.67	0.71	0.59	0.63	0.67	0.71	0.59	0.63	0.67	0.71	-
$\gamma = 1$	0.32	0.39	0.50	0.65	0.19	0.25	0.33	0.46	0.13	0.15	0.16	0.19	0.05
$\gamma = 2.5$	0.79	0.98	1.24	1.63	0.58	0.75	1.00	1.36	0.21	0.23	0.24	0.27	0.13
$\gamma = 5$	1.59	1.97	2.50	3.28	1.24	1.60	2.11	2.87	0.35	0.36	0.38	0.40	0.27
$\gamma = 10$	3.21	3.98	5.06	6.67	2.58	3.33	4.39	5.95	0.62	0.63	0.65	0.67	0.54
Permanent shocks													
$\beta = 0.95$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	2.03	2.52	3.19	4.19	1.09	1.46	2.00	2.82	0.93	1.04	1.17	1.34	0.43
$\gamma = 2.5$	3.19	3.97	5.08	6.75	2.19	2.90	3.92	5.46	0.97	1.03	1.11	1.22	0.66
$\gamma = 5$	4.15	5.23	6.83	9.38	3.10	4.13	5.66	8.11	1.02	1.06	1.11	1.17	0.83
$\gamma = 10$	5.50	7.23	10.15	16.20	4.35	6.04	8.88	14.83	1.11	1.13	1.16	1.20	0.99
$\beta = 0.96$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	2.55	3.63	4.49	5.78	1.36	2.50	3.35	4.58	1.17	1.10	1.10	1.15	0.54
$\gamma = 2.5$	3.15	4.52	5.67	7.63	1.83	3.31	4.48	6.38	1.30	1.18	1.14	1.17	0.75
$\gamma = 5$	4.01	5.80	7.44	10.80	2.51	4.48	6.17	9.48	1.46	1.27	1.19	1.20	0.89
$\gamma = 10$	5.27	7.73	10.26	17.66	3.53	6.26	8.89	16.21	1.67	1.38	1.26	1.24	1.03
$\beta = 0.97$													
	λ^T				λ^B				λ^R				λ
$\hat{\theta}_2$	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	0.54	0.58	0.63	0.68	-
$\gamma = 1$	3.41	4.23	5.38	7.08	1.82	2.45	3.36	4.74	1.56	1.74	1.96	2.24	0.72
$\gamma = 2.5$	4.22	5.27	6.77	9.06	2.91	3.86	5.23	7.34	1.28	1.36	1.46	1.60	0.87
$\gamma = 5$	4.89	6.20	8.16	11.33	3.66	4.91	6.78	9.83	1.19	1.23	1.29	1.36	0.97
$\gamma = 10$	6.08	8.07	11.54	19.45	4.83	6.77	10.16	17.93	1.20	1.22	1.25	1.29	1.08

Notes: The table displays the computed parameters for the decomposition of the welfare cost of total economic fluctuations using a robustness sample with only the original Barro and Ursúa (2010) data. The numbers are obtained using equations (15) through (20) with the estimates shown in Table 8. All of the entries are in percentages of lifetime consumption. We also report an extra welfare cost measure, λ, described in Appendix D. We report numbers for the relative degree of risk aversion $\gamma \in \{1, 2.5, 5.10\}$, for the implied $\hat{\theta}_2$ along the grid for $\theta_1 \in \{0, 0.1, 0.2, 0.3\}$, and with $\beta \in \{0.95, 0.96, 0.97\}$ for the permanent shocks.

D Literature's λ

Here we characterize in our two applications the welfare cost of business cycles in the absence of observed consumption as proposed in our decomposition. Recall that $\sigma_u^2 = (1-\theta_2)^2\sigma_\varepsilon^2$.

Example 1 (Lucas, 1987) :

$$\lambda = \begin{cases} \exp\left(\frac{\sigma_u^2}{2}\right) - 1, \text{ if } \gamma = 1 \\ \exp\left(\frac{\gamma\sigma_u^2}{2}\right) - 1, \text{ if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{29}$$

Example (Obstfeld, 1994) :

$$\lambda = \begin{cases} \exp\left(\frac{\sigma_u^2}{2(1-\beta)}\right) - 1, \text{ if } \gamma = 1 \\ \exp\left\{0.5\gamma\sigma_\varepsilon^2\right\}\left[\frac{1-\Gamma e^{-0.5\gamma(1-\gamma)\sigma_u^2}}{1-\Gamma}\right]^{\frac{1}{1-\gamma}} - 1, \text{ if } \gamma > 0, \gamma \neq 1 \end{cases} \tag{30}$$