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Finding a Stable Phillips Curve Relationship: 
A Persistence-Dependent Regression Model 
Richard Ashley and Randal J. Verbrugge

We establish that the Phillips curve is persistence-dependent: inflation responds differently to persistent versus moderately persistent (or versus transient) fluctuations in the unemployment gap. Previous work fails to model this dependence, so it finds numerous “inflation puzzles”—such as missing inflation/disinflation—noted in the literature. Our model specification eliminates these puzzles; for example, the Phillips curve has not weakened, and inflation is not “stubbornly low” at present. The model’s coefficients are stable, and it provides accurate conditional recursive forecasts through the Great Recession. The persistence-dependent relationship we uncover is interpretable as being business-cycle-phase-dependent and is thus consistent with existing theory.

JEL Classification Codes: E00, E31, C22, C32, E5. 
Keywords: overheating; recession gap; persistence dependence; NAIRU.


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Executive Summary

Persistence-dependence in the relationship between inflation and the unemployment rate simply means that inflation responds differently to relatively persistent fluctuations in unemployment than it does to relatively transitory ones.

This persistence-dependent relationship sounds exotic, but it is actually just a systematic elaboration – only here applied to the unemployment rate coefficient in the Phillips curve – of the profession’s permanent income hypothesis insight in the 1960s, to the effect that the size of the “marginal propensity to consume” coefficient on a fluctuation in disposable income in a simple linear Keynesian consumption function depends on the persistence in this fluctuation. The only difference here is that the unemployment gap is partitioned into three components – with differing persistence levels – that add up to the original unemployment gap data.

Our paper is a detailed investigation into the full nature of the persistence-dependence in the Phillips curve relationship between inflation and unemployment. This investigation reveals surprisingly deep insights about the nature of the inflation process and its relationship to business cycle dynamics.

Using a one-sided filtering approach – crucial here because of likely feedback in the relationship – we decompose unemployment rate time-series data into three components: a persistent component, composed of all fluctuations that last longer than 48 months; a moderately persistent component, composed of all fluctuations that are completed between 12 and 48 months; and a transient component, which is the remainder. We allow for asymmetry (around zero) for each component’s coefficient,1 but permit the data to reject this hypothesis.

Consistent with existing theory, we find that all three of these persistence components have a statistically significant influence on inflation, and we reject symmetry in each of the three coefficients. The coefficient pattern is interesting: it is the negative fluctuations in the most persistent unemployment component that enter the model with a significant coefficient, whereas it is the positive fluctuations in the moderately persistent and transient unemployment components that enter with significant coefficients. These three statistically significant coefficients are stable across the sample; in our specification, there is no weakening in the Phillips curve relationship over the Great Recession. Of these coefficients, the one that is most statistically noteworthy is the strong negative coefficient on the positive fluctuations in the moderately persistent unemployment component.

Importantly, these results are economically interpretable and imply that the observed Phillips curve relationship varies across the phases of the business cycle. In particular, at the onset of a recession, when unemployment is rising rapidly and both the moderately persistent and transient components of the unemployment rate both swing positive, we find that there is a very strong downward force on inflation. Later on in the cycle, shortly after unemployment peaks and begins to fall again, the variation in these two unemployment components becomes negligible, and our estimated model then indicates that the relationship between labor market slack and inflation essentially vanishes. (As noted above, there is no apparent statistical relationship between inflation and positive fluctuations in the most persistent component.) Only when labor market slack becomes persistently negative – that is, when the most persistent component of the unemployment rate drops below the natural rate – does a Phillips curve relationship re-emerge in our model.

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1 We need to make use of an estimate of the natural rate of unemployment only to partition the persistent component into positive and negative parts. Our results are robust to different estimates of the natural rate.
Because our model is statistically rich enough to have stable coefficients, it is able to explain why the usual Phillips curve models—which are misspecified in that they ignore the persistence dependence in the relationship—yield estimated Phillips curve relationships that shift about over time and forecast poorly. In contrast, its good inflation forecasting performance validates our model: for example, using coefficients estimated on the pre-2007 data, our model forecasts inflation through the Great Recession quite well, and it provides generally good out-of-sample inflation forecasting throughout the sample period of 1985-2016.

Most importantly, however, our persistence-dependent Phillips curve model eliminates the “inflation puzzles” that have so troubled analysts (and monetary policy officials). Thus, for example, our Phillips curve relationship has not deteriorated in recent years, although—given the state of the business cycle—it does (in February 2020) predict a weaker link between inflation and the unemployment gap than was the case in the downturn, when the gap was rising rapidly. Therefore, our model is decidedly not now suffering from “missing inflation” nor is it finding that inflation at present is “stubbornly low.”

At the same time, our results are broadly consistent with much extant theoretical work, and the economic significance of the time pattern of a fluctuation in the unemployment gap can potentially point theorists in a productive direction.

Our work further has notable implications for monetary policy, inasmuch as it empirically clarifies the responsiveness of inflation to labor market slack as being business-cycle-phase-dependent. Evidently, it is much more difficult to increase inflation (which apparently can only be done when the economy is overheating) than to decrease inflation, which occurs rather abruptly when the economy slips into a recession. Later in the cycle, when the unemployment gap is positive but declining, we find that inflation only gradually adjusts toward its trend and is unresponsive to reductions in the unemployment rate.
1. Introduction

The Phillips curve relationship remains central to macroeconomics and plays an absolutely fundamental role in current monetary policy deliberations (see, for example, Brainard 2019), not least because this relationship lies at the core of the structural models that dominate current monetary policy discussions. But by most accounts, inflation dynamics over the Great Recession seem to have diverged markedly from their previous patterns, posing a number of puzzles to the existing understanding of the Phillips curve relationship. The most prominent puzzle is the missing disinflation: given the large amount of labor market slack that persisted for many years, standard pre-Great-Recession Phillips curve specifications predicted far lower inflation over the Great Recession than actually occurred (see, for example, Ball and Mazumder 2011; Coibion and Gorodnichenko 2015). It seems almost universally believed that the Phillips curve relationship has weakened. But the inflation puzzles extend beyond missing disinflation; for instance, the inflation puzzle at the time of this writing is the opposite: shouldn’t inflation be higher now, given how long the unemployment rate has been below conventional estimates of its natural rate? Inflation expectations are thought to be currently anchored at target, but this “missing inflation” is believed to threaten this anchoring.

These inflation puzzles – as well as a number of other findings and puzzles in the literature, such as the apparent time variation in the relationship, and the odd behavior of the reverse-engineered NAIRU in Coibion and Gorodnichenko (2015) – completely disappear with the persistence-dependent specification of the Phillips curve relationship proposed here. Using

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2 Bob Hall famously stated in his AEA presidential address (2011), “It is not news that NAIRU theory is a failure” and that standard New Keynesian Phillips curves “cannot explain the stabilization of inflation at positive rates in the presence of long-lasting slack.” Bullard (2017) stated, “The results shown here call into question the idea that unemployment outcomes are a major factor in driving inflation outcomes in the U.S. economy.” In July 2019 testimony, Federal Reserve Chair Jerome Powell stated, “The relationship between the slack in the economy or unemployment and inflation was a strong one 50 years ago … and has gone away” (Li, 2019). The Economist (2020) argues that strong jobs reports no longer cause markets to expect rate hikes, owing to the apparently vanished Phillips curve. For similar sentiments, see The Economist (2017), Summers (2017), and Blinder (2018).

3 See, for example, Clark and McCracken (2006) or Stock and Watson (2007). Similarly, Luengo-Prado, Rao, and Sherehiy (2018) document that the relationship appears to have vanished after 2009. (We agree!)

4 Persistence-dependence is defined below. We explain Great Recession inflation dynamics without reference to biased inflation expectations (cf. Coibion and Gorodnichenko 2015) or to the short-term unemployment rate (cf. Ball and Mazumder, 2019). Regarding the latter paper, we agree with those authors that so-called “core” inflation measures are deficient and mask the true Phillips curve relationship; see Appendix A.2.
coefficients estimated in 2006, recursive forecasts from our specification (conditioned only on
the time path of unemployment) well-predict inflation over the entire Great Recession: there is
no downward-speed puzzle, no missing disinflation, no missing inflation, and neither is inflation
now “stubbornly low” (FOMC minutes, July 30-31, 2019). Clearly, these facts are highly
relevant for policy deliberations today. We emphasize that, under our specification, even the
Great Recession did not alter the dynamics of inflation. Moreover, our findings have a natural
interpretation in terms of business-cycle-phase dependence and are consistent with extant theory.

Persistence-dependence (or frequency-dependence) in a relationship between two time-
series variables Y and X implies that Y responds differently to persistent fluctuations in X than it
does to transitory fluctuations in X. Inflation’s persistence-dependent relationship to
unemployment sounds exotic, but actually has deep roots in empirical macroeconomics:
Friedman (1968) and Phelps (1967, 1968) established that the most persistent unemployment rate
fluctuations, that is, natural rate fluctuations, are unrelated to inflation. Further, persistence-
dependence exists in many other macroeconomic relationships; here are four additional
eamples: First, the “permanent income hypothesis” implies that consumption should mainly
respond to persistent movements in income. Second, RBC modeling was built upon the
presumption that business cycle relationships are distinct from low-frequency relationships – and
this idea has recently regained traction, as we discuss below.5 Third, Friedman (1988) and
Cochrane (2018) both argued that (transient) measurement error would give rise to what we call
persistence-dependence. Finally, data are routinely de-seasonalized under the presumption that
relationships at seasonal frequencies differ from those at other frequencies. Until quite recently,
however, persistence-dependence was rarely examined, carefully and explicitly.

A nascent body of research, however – building upon earlier work by Comin and Gertler
(2006) – is exploring the frequency domain for clues about business cycle drivers and dynamics.
Angeletos, Collard, and Dellas (2020) have recently argued that assessing the root causes of

5 For example, Beaudry, Galizia, and Portier (2020) state: “Therefore, in order to evaluate business cycle properties,
one needs to find a way to extract properties of the data that are unlikely to be contaminated by the lower-frequency
forces that are not of direct interest.” Below, we note how standard approaches, such as pre-filtering the data prior to
further analysis, will typically lead one astray, and offer a better alternative.
business cycles requires delving into the drivers of unemployment, output, consumption, and the like at different frequencies. Beaudry, Galizia, and Portier (2020) likewise highlight medium-frequency cycles and argue that it is crucial to examine the properties of the data at different frequencies in order to discriminate across classes of models. And numerous recent studies have located direct evidence for persistence-dependence in macroeconomic relationships. Cochrane (2018) and Ashley and Verbrugge (2015) find that the velocity of money has a persistence-dependent relationship to interest rates. Blundell, Low, and Preston (2013) find that at the micro level, consumption responds to persistent movements in after-tax income, and Arellano, Blundell, and Bonhomme (2018) find that persistent earned income shocks are harder to insure against, particularly for young families with low assets. Ashley and Li (2014) find that state-level consumption has a persistence-dependent relationship to movements in both housing wealth and stock wealth. Ciner (2015) finds that stock returns have a persistence-dependent relationship to inflation movements. And Ashley, Tsang, and Verbrugge (2020) find that historical FOMC policy responses to inflation and unemployment are persistence-dependent, with persistent movements in the unemployment rate, and transitory movements in inflation, being ignored by policymakers.6

The research question motivating the present research is the following: does the natural rate distinction fully capture persistence-dependence in the Phillips relationship? For instance, the work of Stock and Watson (2010) suggests that the Phillips curve relationship is concentrated at moderate persistence levels and that this relationship is asymmetric – that is, it holds only for increases in the unemployment rate. Similarly, transient fluctuations in the unemployment rate seem rather unlikely to influence inflation. And since persistence-dependence implies time-dependence in misspecified regression models, could persistence-dependence explain the recent apparent weakening of the Phillips curve that has occupied so much attention in recent years?

6 Other recent studies include Caraiani and Gupta (2018): the Bank of England responds only to persistent movements in the real exchange rate; Ciner (2014): stock returns have a persistence-dependent relationship to consumer sentiment; and Yanfeng (2013): Japanese industrial activity and inflation have a persistence-dependent relationship to oil prices. Earlier studies include Cochrane (1989): interest rates respond negatively to transitory movements in money growth; Reynard (2007): the money-inflation relationship depends upon persistence-dependence in velocity; and Benati (2009): inflation is mainly related to low-frequency movements in money.
With these research antecedents in mind, we here more fully explore the persistence-dependence in this relationship, using recently developed econometric tools that allow the data to speak very transparently as to the nature and form of this dependence, if it exists. Our approach is entirely empirical; while we review relevant extant theory in the Appendix – and we observe the consistency of our empirical findings with this body of theory – we leave the consequent further development of such theoretical modeling efforts for future work. Below we focus instead on the empirical specification of a statistically adequate reduced-form Phillips curve relationship between unemployment fluctuations and suitably-detrended trimmed-mean PCE inflation (whose superiority over “core” PCE inflation is discussed below).

The pattern of persistence-dependence we find has a natural interpretation. We find that there are three distinct empirical Phillips curve relationships, each apparently operative during a different phase of the business cycle. At the beginning of a recession, when unemployment is rising rapidly – so that the moderately persistent component of the unemployment rate becomes positive – we find a big downward impact on inflation. (This effect is reminiscent of the “recession gap” model of Stock and Watson 2010.) Once the economy has bottomed out and unemployment starts to recover, we find that the Phillips curve relationship essentially vanishes. Then – late in the expansion, when the unemployment rate persistently falls below the natural rate – we find that a Phillips curve relationship reappears, albeit in weaker form. One immediate implication of our findings is that DSGE models with conventional Phillips curves are likely to greatly underestimate the drop in inflation when a recession begins; we provide a brief review of theory that is consistent with these findings in the Appendix.

But does this new reduced-form Phillips curve specification perform well empirically? We find that it does. As noted above, using data only up through 2006 for coefficient estimation, we find that recursive forecasts from our specification (conditioned only on historical

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7 See Luengo-Prado, Rao, and Sheremirov (2018). Laxton, Meredith, and Rose (1995) and Debelle and Laxton (1997) also draw attention to the weak relationship between positive slack and inflation. This finding does not fit neatly into a typical empirical specification of a Phillips curve, but aligns nicely with standard industrial organization theory; see the Appendix.

8 We discuss the appropriate natural rate concept (and measurement) below.
unemployment data) well-predict inflation over the entire Great Recession. As a second test of our specification, we demonstrate that – even though our goal is to understand inflation, not to develop a forecasting model – out-of-sample forecasts generated by a simple variant of our model outperform some standard benchmarks.

2. A Persistence-Dependent Phillips Curve

2.1 Method

Our approach differs fundamentally from the existing Phillips curve literature in that we make use of the “persistence-dependent regression” econometric methodology developed in Ashley and Verbrugge (2009) and Ashley, Tsang, and Verbrugge (2020). These methods allow us to use the data to determine the relationship between inflation and movements in the unemployment rate at different persistence levels.

In particular, we take a standard reduced-form Phillips curve specification and then decompose the coefficient on the unemployment rate gap by persistence level (frequency) – allowing for asymmetry in each coefficient, but permitting the data to reject it. We select three persistence components (denoted “persistent,” “moderately persistent,” and “transient”) on largely a priori grounds: highly persistent fluctuations encompass all fluctuations whose reversion period exceeds 48 months; moderately persistent fluctuations encompass fluctuations with reversion periods greater than 12 months and less than 48 months; and transient fluctuations encompass all fluctuations with reversion periods of 12 months or less. Our empirical procedure allows the data to inform us as to whether or not this particular partitioning is useful.

The persistence-dependent regression method used here is both straightforward and, to an appropriate degree, flexible, leading to estimation/inference results that are easily interpreted. Essentially, we use one-sided filtering to partition the real-time unemployment rate into

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9 Asymmetry has been a hypothesized feature of the Phillips curve from its inception (Phillips 1958), and many previous studies have located evidence for asymmetry or convexity (for example, Debelle and Laxton 1997, Detmeister and Babb, 2017, or Murphy, 2017). Asymmetry is built into the Stock and Watson (2010) recession gap.

10 The 48-month cutoff was inspired by the Stock and Watson (2010) recession gap. We present a comparison to that gap in the Appendix.
components with differing levels of persistence, but which (by construction) sum up exactly to
the original unemployment rate series. In particular, we use a moving window to filter the real-
time $u_t$ data at each time $t$ in a one-sided (backward-looking) manner, partitioning the time $t$
observeration of the unemployment rate into the three persistence (or, equivalently, frequency)
components described above. (We subsequently decompose the persistent component into
positive and negative parts using a natural rate estimate, $u^*_t$.11) Since, by construction, these
components sum to the original time series – that is, to the real-time $u_t - u^*_t$ data – it is then
straightforward to use least squares fitting so as to assess whether inflation movements are
related differently to each of these three “persistence components” of the original $u_t - u^*_t$ data.

This partitioning, the one-sided filtering, and our restriction of the filtering solely to the
$u_t - u^*_t$ data are all essential here. Partitioning is necessary in order that – as noted above – these
three components of the unemployment rate gap add up to the original data, so that it is easy to
test the null hypothesis that the coefficients with which these three components enter a regression
model for the inflation rate are all equal. One-sided filtering is necessary for two reasons. First,
one-sided filtering – and only one-sided filtering – sensibly comports with the use of real-time
unemployment rate data. And second, two-sided filtering – such as ordinary HP-filtering or
ordinary spectral analysis not based on a moving window – inherently mixes up future and past
values of the unemployment rate gap in obtaining the persistence components, distorting the
causal meaning of inference in the resulting inflation model and limiting its use for practical
forecasting and/or policy analysis. These distortions from the use of two-sided filtering are
particularly severe when the dependent variable is also filtered and when the key relationship
likely (as here) involves feedback from the dependent variable (inflation) to the (filtered)
components of $u_t - u^*_t$ being used as explanatory variables. Fundamentally, this is because
filtering the dependent variable in a regression model implies that the model error term is
similarly filtered.12 For this reason – and because we want and need a model for inflation itself

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11 Henceforth in the manuscript, for brevity we refer to our procedure as “partitioning $u_t - u^*_t$.”
12 For more details, see Sargent (1987) or Ashley and Verbrugge (2009). For the same reason, and because such
calculations are incompatible with the use of real-time data, two-sided cross-spectral estimates or filtering with
wavelets are similarly ruled out for analyses of the present sort. Even absent feedback, transfer function gain and
(rather than for some filtered version of it) – we only decompose the unemployment rate gap into persistence components in our modeling below.

The length of the moving window of real-time data on $u_t - u_t^*$ specified for use below in partitioning this unemployment rate gap data into the three persistence components defined above was set here at 48 months. The “persistent” component of $u_t - u_t^*$ is thus aggregating all unemployment rate gap fluctuations with “reversion periods” greater than or equal to 48 months in length, including any trend in $u_t - u_t^*$. Other choices could be made in this regard – for example, a 60-month window – but the inflation modeling results reported below are not particularly sensitive to modest modifications in this window length.\(^\text{13}\)

13Stata and RATS software to implement the decomposition is available from the authors, and an extensive description of our moving-window-based persistence decomposition algorithm can be found in the Technical Appendix to Ashley, Tsang, and Verbrugge (2020). For replicability purposes we note that (per footnote 44 on page 48 of that paper) the standard Christiano-Fitzgerald bandpass filter was iteratively used here. Twelve projected values were used in padding out each window; extension of the data via forecasts to improve filter precision has a long history, starting with Dagum (1975). Here these projections were based on real-time quarterly $u_t$ forecasts from the Survey of Professional Forecasters, converted to monthly frequency (for this purpose) using linear interpolation. As with the window length, the inflation regression model results reported below are not very sensitive to these particular choices. In any case, since the three persistence components are constructed so as to add up to the original $u_t - u_t^*$ series, any nonoptimality in these choices will have only weakened the empirical results we report below.
2.2 Persistence Component Results

Figure 1 displays time plots of four time series. The top portion of this figure plots the real-time unemployment rate ($u_t$)\textsuperscript{14} and estimates of the natural rate of unemployment taken from Tasci (2018).\textsuperscript{15} Their difference is the unemployment rate gap ($u_t - u^*_t$), that is here decomposed, as

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\textsuperscript{14}Sources: Real-time unemployment rates: ALFRED, Federal Reserve Bank of Saint Louis; natural rate series: Tasci (2018).

\textsuperscript{15}We prefer this measure of the natural rate of unemployment because it is based on labor market flows, without reference to inflation data. See Occhino (2019) on why it is a poor idea to use a $u^*_t$ estimate based on a Phillips curve in a Phillips curve regression model. We use this measure only to partition the low-frequency component of the unemployment rate into positive and negative parts; however, our results are not sensitive to this choice. The two-sided Tasci (2018) estimate may be used because this natural rate estimate evolves independently of inflation.
noted above, into the three persistence components:

- the “persistent” gap component, including the sample variation in \( u_t - u^* \) with reversion periods greater than or equal to 48 months; this component includes any time trend in \( u_t - u^* \);
- the “moderately persistent” gap component, including the sample variation in \( u_t \) with reversion periods less than 48 months but greater than 12 months;
- and the “transient” gap component, comprising the remaining sample variation in \( u_t \), which mean-reverts within 12 or fewer months.

The “transient gap” component is, as one might expect, noisy enough so as to not admit of any clean economic interpretation. Consequently – while it is included in the regression models – this persistence component is, for clarity of exposition, only plotted in Appendix A5 and not plotted in the lower portion of Figure 1, which displays the time variation in the “persistent” and “moderately persistent” unemployment rate gap components.

Turning now to this lower portion of Figure 1, it is evident that the “persistent” gap component varies substantially across the sample and is recognizably capturing the smooth movements in \( u_t - u^* \). It, of course, traces out the three major recessions during the sample period.

The “moderately persistent” component is of chief interest here. This time-series is fairly smooth, because it excludes the relatively “noise-like” sample fluctuations in \( u_t \) with reversion periods less than 12 months in length: these sample variations were relegated to the “transient” gap component. As one should expect, the fluctuations in this “moderately persistent” component only last for three to four years. But what is most notable about this component is that its sample fluctuations are clearly not randomly located in time. Rather, this component has a marked tendency to fluctuate upward when the unemployment rate is rising sharply; it then dwindles to near zero shortly after \( u_t \) begins to fall. In other words, the “moderately persistent” unemployment rate gap component fluctuates upward at the onset of each recession and diminishes shortly after the recession begins to abate.

Thus, our persistence/frequency partitioning of \( u_t - u^* \) can be interpreted economically as decomposing the unemployment rate gap into its (highly persistent) overall trend, a moderately persistent “signal” (as it were) for the onset of each recession, and transient noise. The joint behavior of these components will play a role in explaining an otherwise puzzling finding in Coibion and Gorodnichenko (2015).
The empirical question then becomes, “Is this ‘moderately persistent’ component a recession-onset signal, significantly correlated with time variation in the price inflation rate?” There is good reason to think so, given the results of Stock and Watson (2010). And if this relationship differs notably from that of the more persistent component – again, quite likely given the aforementioned study – then it follows that the Phillips curve relationship is persistence-dependent to a degree that is economically and/or statistically significant. In that case, decomposing $u_t - u_t^*$ into these three persistence components can potentially yield a better-specified – and hence more stable and useful – estimated Phillips curve regression model. The estimation and testing of just such a model – also allowing for asymmetry in the way each component impacts the inflation rate – are the work of the remainder of this paper.

### 2.3 Persistence-Dependent Phillips Curve Regression Model

Our starting point is a relatively standard reduced-form Phillips curve, defined in terms of the unemployment gap. In particular, the baseline model specification is

$$\pi_{t+1}^{12} - \pi_t^* = \alpha + \beta_1 (\pi_t^{12} - \pi_{t-12}^*) + \beta_2 (\pi_t^{12} - \pi_{t-24}^*) + \lambda_1 (\text{gap}_t) + \epsilon_t$$

(1)

where $\pi_{t+12}^{12} = \ln \left( \frac{P_{t+12}}{P_t} \right)$ denotes the 12-month log-change in the price index, $\pi_t^*$ is an inflation trend measure relevant for this price index (the “PTR” measure from the Board of Governors, which adjusts and extends forecasts from the Survey of Professional Forecasts), and specification (1) includes a traditional unemployment gap term, $\text{gap}_t$, specified in terms of a natural rate: $\text{gap}_t = (u_t - u_t^*)$, where $u_t^*$ is taken from Tasci (2018). Our inclusion of 24 months of lagged inflation (via our use of the current 12-month inflation rate, and of the 12-month inflation rate from a year ago) is in line with typical practice. Most of our analysis focuses on trimmed-mean PCE inflation, the realized-inflation measure that arguably best removes noise from inflation (Mertens, 2016), but in the Appendix we consider other time series of inflation measures in robustness checks.

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16 Regression estimates using the CBO $u_t^*$ estimate are also investigated in the sequel.

17 As this working paper was undergoing final internal review, we learned of Ball and Mazumder (2019). Like us, these authors eschew the use of core PCE and estimate a Phillips curve using an alternative trend inflation indicator, in their case a weighted median PCE. (We present results for a weighted median PCE in the Appendix.) Similarly, both the Reserve Bank of New Zealand and the Bank of Canada now shun the use of “core” (exclusion-based) inflation measures as measures of trend inflation. See Carroll and Verbrugge (2019) and Verbrugge, Zaman, and Nunna (2018) for additional evidence regarding the superior forecasting ability of the trimmed mean PCE and
Prior studies, such as those of Clark and McCracken (2006), Faust and Wright (2013), Zaman (2013), and Clark and Doh (2014), have shown that inclusion of an accurate trend estimate improves the accuracy of inflation forecasts. In the context of Phillips curve estimation, modeling inflation in terms of deviation from the trend $\pi_t^*$ amounts to assuming that the Phillips curve relationship is silent about the long-run goals of monetary policy and, instead, pertains to fluctuations in inflation that are more closely related to business cycles. While Phillips curve forecasting models often include other variables such as the relative price of energy or imports, these variables are not found to be helpful for our 12-month projections. Here we estimate our models over the sample period 1985-2016, so as to focus on the recent period during which the Phillips curve is thought to have become weak and/or unstable.

Define $(un_{t,\text{persist}})$, $(un_{t,\text{mod\_persist}})$, and $(un_{t,\text{transient}})$ as the persistent part, the moderately persistent part, and the transient part of $un_t$, the (real-time) unemployment rate at $t$. Our specification then extends Equation (1) as follows:

$$
\pi_{t+12} - \pi_t^* = \alpha + \beta_1(\pi_{t+12} - \pi_{t-12}^*) + \beta_2(\pi_{t-12} - \pi_{t-24}^*) + \lambda_1^+ (un_{t,\text{persist}}) + \lambda_1^- (un_{t,\text{transient}}) + \lambda_2^+ (un_{t,\text{mod\_persist}}) + \lambda_2^- (un_{t,\text{transient}}) + \lambda_3^+ (un_{t,\text{transient}}) + \lambda_3^- (un_{t,\text{transient}}) + \epsilon_t
$$

where $(un_{t,\text{persist}})$ is the positive part of $(un_{t,\text{persist}} - un_{t,\text{transient}})$, $(un_{t,\text{mod\_persist}})$ is the positive part of $(un_{t,\text{mod\_Persist}})$, and $(un_{t,\text{transient}})$ is the positive part of $(un_{t,\text{transient}})$, and other terms are defined analogously.

We test whether asymmetry is warranted in any term; that is, we test whether $\lambda_1^+ = \lambda_1^-$, $\lambda_2^+ = \lambda_2^-$, or $\lambda_3^+ = \lambda_3^-$. We also perform a Chow test of coefficient stability to test whether the coefficient estimates change after 2006:12; after all, the coefficient estimate $\hat{\lambda}_i$ in Equation (1) drops from -0.23 to -0.09 as one extends the end of the sample from 2006:12 to 2016:12.
2.4 Discussion of Model Coefficient Estimates and In-Sample Inference Results

Table 1 below displays the OLS parameter estimates for the coefficients in Equation (1) and Equation (2), with estimated t-ratios quoted beneath each estimate.\(^{19}\)

Consider first the estimation results for the “standard” Phillips curve specification, Equation (1). Here notice that the evidence for the existence of any Phillips curve relationship between inflation and the unemployment rate gap \((u_t - u_t^*)\) is exceedingly weak for this model specification. In fact, with an estimated t-ratio of only -1.54, the estimate of \(\hat{\lambda}\), the coefficient on \((u_t - u_t^*)\) in Equation (1), has the right sign, but it is not statistically different from zero (on a two-tailed test) at even the 10 percent level of significance. And a standard Chow test rejects the null hypothesis that the parameters in Equation (1) are stable – when the sample data are partitioned into the period 1985:1 through 2006:12 versus 2007:1 through 2016:12 – with \(p = 0.048\).

This first set of results is hardly new or surprising: multiple studies in the literature have found ample evidence for a weak (and unstable) Phillips curve relationship over the past few decades.

Next consider the analogous estimation and inference results for Equation (2), our persistence-dependent re-formulation of the Phillips curve regression model specification. First, notice that the adjusted \(R^2\) is notably larger for this estimated model than for Equation (1) and that the BIC is notably smaller: evidently the improvement in the fit of Equation (2) to the sample data more than compensates for its greater complexity. Second, notice that there is ample evidence here for rejecting both the individual null hypotheses of coefficient symmetry on the two most persistent components – that is, \(H_0 : \hat{\lambda}_1^- = \hat{\lambda}_1^+\) and \(H_0 : \hat{\lambda}_2^- = \hat{\lambda}_2^+\) – and even stronger evidence to reject the joint null hypothesis of coefficient symmetry on all three persistence components. These results are consistent with evidence in Morris, Rich, and Tracy (2019) on asymmetry in the wage-based Phillips curve.

More importantly, however, note that three of the six coefficients on the unemployment

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\(^{19}\) These estimated standard errors are (12-month) HAC standard error estimates, as the fitting errors (due to the cumulation in the dependent variable) appear to be serially correlated. Diagnostic checks with regard to possible heteroscedasticity in the model errors are therefore not quoted, as the HAC standard error estimates are consistent in this case anyway. The fitting errors for Equation (2) show no evidence of notable outliers.
rate gap persistence components are negative and quite statistically significant. In particular, notice that the coefficient on \( \lambda^+ \), quantifying the impact of positive fluctuations in the moderately persistent component of \(( u_t - u^*_t )\), is substantially negative (at -1.67), with an estimated t-ratio of -7.91; so \( H_0 : \lambda^+ = 0 \) can be rejected with \( p < 0.00005 \). Evidently, the positive fluctuations in the moderately persistent component of \(( u_t - u^*_t )\), observed (in Figure 1) at the onset of each recession in the sample, match up quite nicely with concomitant drops in the inflation rate at these times. This result is consistent with economic intuition and accords with the findings of Stock and Watson (2010).20

The remainder of this section discusses these Equation (2) estimation results in more depth. The regression results in Table 1, as noted above, indicate compelling evidence for asymmetry in the relationship between inflation and all three persistence components. The coefficient estimates differ notably for positive and negative gaps, and the formal hypothesis testing results very clearly reject the null hypotheses that these coefficients are equal, for either the most persistent unemployment component coefficients \(( \lambda^+_t \) and \( \lambda^-_t \)) and for the moderately persistent unemployment coefficients \(( \lambda^+_2 \) and \( \lambda^-_2 \)) – though we can only reject the null hypothesis \(( \lambda^+_3 = \lambda^-_3 \), corresponding to the transient unemployment components, at the 8 percent level of significance.21 We argue next that this asymmetry is eminently sensible and aligns well with economic theory.

These results have a natural interpretation in terms of business-cycle-phase dependence. There are three statistically significant coefficient estimates: that pertaining to the negative part of the persistent component \(( un^-_{\text{persist},t} \), that pertaining to the positive part of the moderately persistent component \(( un^+_{\text{mod.persist},t} \), and that pertaining to the positive part of the transient component \(( un^+_{\text{transient},t} \). The latter term is typically negligible except during periods when the moderately persistent part is also positive; see the Appendix. Conversely, the other

---

20We note that qualitatively similar results in this regard also obtain using different measures of inflation, or using the CBO estimates of \( u^*_t \) instead of the Tasci (2018) estimates used here; see Appendix A.1. We also obtain qualitatively similar results if we expand the range of the moderately persistent \( u_t - u^*_t \) component to include variation with reversion periods up to 60 months in length by using a larger window length, or if we re-specify the model using six-month changes in inflation. While this paper makes no attempt to be a multi-country study, we also note that preliminary analyses using data from Australia yield a similar pattern. These results are available on request.

21 The BIC also worsens notably when symmetry is imposed. We do not report these tests for brevity.
unemployment component coefficient estimates are not statistically distinguishable from zero. Why do we interpret this as business-cycle-phase dependence?

Table 1. Phillips Curve Regression Estimation Results

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Equation (1)</th>
<th>Equation (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_t - u_t^*$</td>
<td>$\lambda$ (t-stat)</td>
<td>-0.09 (-1.54)</td>
</tr>
<tr>
<td></td>
<td>Persistent component</td>
<td>0.03 (0.64)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_1^+$ (t-stat)</td>
<td>0.03 (0.64)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_1^-$ (t-stat)</td>
<td>-0.27 (-3.16)</td>
</tr>
<tr>
<td></td>
<td>Moderately persistent component</td>
<td>0.10 (0.14)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_2^+$ (t-stat)</td>
<td>-1.67 (-7.91)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_2^-$ (t-stat)</td>
<td>0.10 (0.14)</td>
</tr>
<tr>
<td></td>
<td>Transient component</td>
<td>-0.03 (-0.23)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_3^+$ (t-stat)</td>
<td>-0.51 (-2.36)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_3^-$ (t-stat)</td>
<td>-0.03 (-0.23)</td>
</tr>
<tr>
<td></td>
<td>Lagged inflation</td>
<td>$\beta_1$ (t-stat)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\beta_2$ (t-stat)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>constant (t-stat)</td>
</tr>
<tr>
<td></td>
<td>Adjusted R-squared</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>BIC</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>Hypothesis Test</td>
<td>$H_0$: $\lambda_1^+ = \lambda_1^-$</td>
</tr>
<tr>
<td></td>
<td>Rejection P-Values</td>
<td>$H_0$: $\lambda_2^+ = \lambda_2^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H_0$: $\lambda_3^+ = \lambda_3^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H_0$: $\lambda_4^+ = \lambda_4^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H_0$: $\lambda_5^+ = \lambda_5^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H_0$: (Chow test): {coefficients unchanged before and after 2006:12}</td>
</tr>
</tbody>
</table>

First focus on $\lambda_1^+$ and $\lambda_1^-$, coefficient estimates that pertain to the relationship of very persistent (low-frequency) movements in the unemployment rate gap to inflation, those with reversion periods of 48 or more months. A typical business cycle has a long recovery phase, beginning once the recession has bottomed out and unemployment has begun falling. During
much of this recovery, the unemployment rate is persistently above the natural rate; thus $u_{n_{\text{Persist},t}}$ is positive and $u_{n_{\text{Persist},t}}$ is zero. Our model thus predicts that the estimated impact of the unemployment gap on inflation is essentially zero during the early part of the recovery, despite the standard unemployment gap being large. (This follows because $\hat{\lambda}_t^+ \approx 0$ and because the persistent part of the unemployment gap is the only component that is nonnegligible during this period.) Putting this more starkly, a persistently high unemployment rate \textit{per se} has no influence on (more properly, no relationship to) inflation. However, once the unemployment rate has persistently dropped below the natural rate – which happens late in an expansion, during what is sometimes called the “overheating” phase – there is evidently a fairly strong upward influence of a negative (smooth and persistent) unemployment gap on inflation, as evidenced by the magnitude of $\hat{\lambda}_t^-$ (namely -0.27). The estimated size of this coefficient is directly comparable to, and much larger than, the estimate from conventional linear Phillips curves (given in the first column, for Equation (1), namely, -0.09). Most of the research investigating nonlinearity in the Phillips curve has focused upon a differential force from positive and negative unemployment gaps, but has not captured all the subtlety we find in the relationship, as we explain next.

Now focus on $\hat{\lambda}_2^+$ and $\hat{\lambda}_2^-$, coefficient estimates that pertain to moderately persistent fluctuations in the unemployment rate. Note that $\hat{\lambda}_2^-$ is essentially zero, whereas $\hat{\lambda}_2^+$ is highly significant, at -1.7. Inspection of Figure 1 reveals that, by and large, this moderately persistent component only departs from zero when the unemployment rate is rising fairly rapidly – namely, at the onset of a recession. We note also that $\hat{\lambda}_3^-$ is essentially zero, whereas $\hat{\lambda}_3^+$ (at -0.51) is statistically significant, but only at the 5 percent level. The transient component of the unemployment rate tends to positively co-move with the moderately persistent component (the correlation is 0.26; see Appendix A.5 for a plot). Thus, both the moderately persistent component and the transient component are associated with a strong downward force on inflation at the \textit{beginning} of a recession – this is the downward force noted by Stock and Watson (2010). The fact that this force vanishes once the recovery begins explains, for example, the findings in Luengo-Prado, Rao, and Sheremirov (2018): “...we find robust evidence of a structural break in

\footnote{We compare their specification to ours in the Appendix.}
the Phillips curve slope around 2009–2010. The co-movement of sectoral inflation rates and labor market slack has weakened, and it is now almost negligible” (p. 1).

Note that – in our persistence-dependent model – the coefficients in the Phillips curve relationship are stable over time. In particular, a Chow test of parameter stability for Equation (2) fails to reject the null hypothesis that the model coefficients change after 2006:12: the rejection p-value for this test is 0.26. Evidently, the Great Recession did not significantly weaken or alter the Phillips curve relationship. Section 3 below further examines the stability (and the consequential forecasting ability) of the persistence-dependent Phillips curve specification embodied in Equation (2), using out-of-sample validation methods.

In Appendix A.4, we discuss how our results compare to some other prominent findings in the literature. For example, we note there that we can use our results to reinterpret a finding in Coibion and Gorodnichenko (2015) regarding their reverse-engineered NAIRU. We also discuss there how our results naturally give rise to episodic forecast improvements and to time variation in Phillips curve coefficients for specifications that ignore persistence-dependence. Finally, we argue that our results explain numerous studies that find a convex-concave aspect to the Phillips curve relationship and studies that adduce evidence for regime switching. In Appendix A.8, we provide a synopsis of extant theory that is consistent with our findings.

3. Out-of-Sample Evidence

The statistical significance of the inference results discussed above strongly support our nonlinear (asymmetric) Phillips curve formulation, disaggregated according to persistence level per Equation (2). These statistical results fundamentally arise from the fact that this specification fits the historical sample data notably better than do the alternatives we considered, even – via consideration of the BIC measure – allowing for the number of additional coefficients estimated in Equation (2).

We find these results persuasive, but not necessarily definitive, in view of the usual concerns as to “data mining.” We also, in particular, wondered whether our specification can
explain inflation dynamics over the Great Recession. To address these issues, we conducted two additional – “out-of-sample” – exercises. First, we present an analysis of what can be called “partially recursive conditional” forecasts, to examine whether our model can resolve the various inflation puzzles apparently arising during the Great Recession; we also compare our conditional forecasts to those derived from the use of the conventional specification, Equation (1). Further, in a second set of calculations, we examine unconditional forecasts. In particular, we present supporting results based on out-of-sample (OOS) forecasting calculations using the Giacomini-Rossi and Diebold-Mariano testing frameworks. These exercises confirm our in-sample results.

3.1 Conditional Recursive Forecasts

For our forecasting exercises, we drop the Equation (2) regression terms with statistically insignificant estimated coefficients in Equation (2) – for example, $\lambda_1^+$, $\lambda_2^-$, and $\lambda_3^-$ – and use the pared-down forecasting model:

$$
\pi_{t+12}^{12} - \pi_t^* = \alpha + \beta_1 \left( \pi_t^{12} - \pi_{t-12}^* \right) + \beta_2 \left( \pi_{t-12}^{12} - \pi_{t-24}^* \right) + \lambda_1^- \left( un_{\text{Persist},t}^+ \right) + \lambda_2^+ \left( un_{\text{modPersist},t}^+ \right) + \lambda_3^+ \left( un_{\text{transient},t}^+ \right) + \epsilon_t
$$

(3)

In this section, we use Equation (3) to generate recursive conditional forecasts: these forecasts are conditioned on the historical unemployment values, but they are recursive in that each forecast draws its needed lagged inflation-deviation values from its own recent inflation forecasts. We compare these conditional forecasts to analogous ones obtained from Equation (1), the parallel model that instead conditions on the path of the CBO unemployment gap, and does not allow for the asymmetric business-cycle-phase dependence in Equation (3). In both cases, the model coefficients are fixed at the values estimated using the data prior to 2007:1.

We plot both of these forecasts (along with the actual inflation time series) below in Figure 2. The conditional forecasts generated by Equation (3) do a respectable job of tracking the broad contours of the evolution of inflation over the Great Recession and the recovery: the sharp dip in inflation, the partial bounceback, and the very slow movement toward long-run expected
inflation.\textsuperscript{23} Based on our new model specification, the Great Recession apparently did not substantially alter inflation dynamics; thus, in our framework, there are no “inflation puzzles” to worry about.

In sharp contrast, the similarly conditional forecasts generated by the linear model of Equation (1) are quite poor and do generate the well-known set of “puzzles”: inflation decelerated far more rapidly than this model predicts,\textsuperscript{24} yet overall inflation fell by less than this model predicts, as has often been noted. The divergence between the actual dynamics of inflation and this model’s predictions is striking. Indeed, tests of this divergence support the visual impressions from Figure 2: changes in the projections using the CBO gap are actually uncorrelated with changes in the inflation path (the estimated correlation is 0.22 +/- 0.19), whereas the correlation between changes in the Equation (3) projections and changes in the inflation path is substantial, at 0.48 +/- 0.10. These out-of-sample forecasting results reinforce a central message of this paper: a failure to properly specify the relationship between the unemployment rate and the inflation rate, allowing for both asymmetry and persistence dependence, yields unstable parameter estimates, strongly counterfactual conditional forecasts, and misleading conclusions about the nature of the inflation process.

\textsuperscript{23} Recall that we specify our models in terms of trimmed mean PCE inflation, detrended by long-run SPF forecasts. Faccini and Melosi (2020) provide a theory that generates low inflationary pressures over most of the recovery. \textsuperscript{24} As Clark (2014) has noted, once one properly accounts for trend inflation, a major disinflation puzzle pertains to why inflation fell so fast during the recession; our specification gracefully explains the rapid disinflation.
At the time this analysis was conducted (June 2019), the unemployment rate had arguably been below the natural rate for some time. Is there missing inflation? From the perspective of our model, the answer is again, perhaps surprisingly, “no”: from our Equation (3) model, the March 2018 prediction for 12-month trimmed mean PCE inflation in March 2019 was 2.06 percent, only about +0.1 percentage points above its realized value of 1.94 percent. The overheating force in the economy was moderate in early 2018, as implied by our model; one needs a much bigger unemployment rate gap, of the type seen in some previous recessions, to exert a strong upward force on the inflation rate.25

In Appendix A.6, we present an extension to Figure 2, which includes projections from both models when the coefficients in each are estimated a decade later, using data up through 2016:12. This extension figure underscores the stability of our Phillips curve – its recursive

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25 For instance, with an unemployment gap of -0.6, the upward force on inflation is (-0.24)(-0.6)=+.014 over 12 months. The June 2019 gap estimate is in the (-0.6,-0.8) range.
forecasts are nearly identical – while the recursive forecasts from the conventional – Equation (1) – model estimated in 2016 are much flatter, in keeping with the smaller unemployment gap coefficient estimate. This result further emphasizes the lesser fidelity of the conventional model; these circa-2016 conventional-model coefficient estimates heighten the downward speed puzzle while still under-predicting the inflation recovery.

3.2 GR and DM Out-of-Sample Forecast Tests

We conjecture, along the lines of Stock and Watson (2009, 2010), that the forecast improvement generated by our Phillips curve formulation over benchmark models is likely to be episodic, as the unemployment gap terms in our Equation (3) model are only substantially operative during two portions of the business cycle. To examine this issue, we use the Giacomini and Rossi (2010) fluctuation test, in addition to the Diebold-Mariano test of out-of-sample forecasting improvement.

The Giacomini-Rossi (GR) testing framework is well-suited for comparing the historical out-of-sample forecasting performance of competing models when the relative performance of these models may vary over time. However, as the authors point out, it has somewhat low power to detect overall forecasting quality differences; the Diebold-Mariano test is preferable in that context.

The GR “fluctuation: out-of-sample” ($F_{OOS}$) test statistic is given by

$$F_{t,m}^{OOS} = \sigma^{-1/2} m^{-1/2} \left( \sum_{j=t-m}^{t} \hat{\eta}^2_j - \sum_{j=t-m}^{t} \hat{\varepsilon}^2_j \right)$$

where $\sigma$ is a HAC estimate of the asymptotic variance of the difference; here we set $m$ equal to 48 months. The GR test is two-sided and is based on rolling-window estimates and forecasts. In the figures below, we plot the upper and lower 10 percent and 5 percent critical values and the GR $F_{OOS}$ test statistic for two forecast comparisons. When the $F_{OOS}$ statistic rises above the upper critical value, then the forecast performance of the “alternative” (persistence-dependent PC) model is significantly better (over the previous 48 months) compared to the baseline model.
Conversely, during time periods when the \( F^{OOS} \) term falls below the lower critical value, the reverse is true.

There is a large body of research on the performance of inflation forecasts based on economic activity gaps, relative to forecasts based on univariate benchmark models. A classic reference with regard to this point is Atkeson and Ohanian (2001), who famously found that a naïve univariate model generally outperformed the usual Phillips curve (PC) model, although some papers (such as Brayton, Roberts, and Williams (1999) and Stock and Watson (1999)) noted “deterioration” in PC forecasts prior to this study. Stock and Watson (2009) conclude that PC-based forecasts outperform univariate benchmarks sporadically, in particular, during episodes with a large unemployment gap, that is, exceeding positive or negative 1.5 percent. In contrast, previous research investigating the OOS performance of PC-based forecasting models vis-à-vis similar univariate benchmark models over the post-1985 period typically returns negative results, for example, Rossi and Sekhposyan (2010) and Dotsey, Fujita, and Stark (2017). Below we examine the conjecture that the asymmetric and persistence-dependent PC model proposed above – that is, Equation (3) – outperforms conventional models in which those features are omitted.\(^{26}\)

In particular, in the remainder of this section, we compare forecasts from our Equation (3) against Equation (1) – with the CBO gap – and against an Atkeson-Ohanian-type model. (Analogous comparisons against the Stock-Watson recession-gap model are provided in the Appendix.)

Figure 3 depicts the comparison against a standard (non-persistence-dependent) CBO gap model, Equation (1). Short windows are not appropriate here, since our model sharply differentiates between different portions of the business cycle. We consequently use a 20-year window and estimate models from 1985:1 onward; thus, our first forecast is for 2005:1, that is, for the 12-month movement in the (detrended) trimmed-mean PCE between 2005:1 and 2006:1. The \( F^{OOS} \) statistic looks back \( m=48 \) months, so the GR test itself thus runs from 2009:1 onward.

\(^{26}\) Unlike Dotsey, Fujita, and Stark (2017), we study trimmed-mean PCE inflation here, rather than headline inflation; other inflation estimators are examined in the Appendix.
In Figure 3, the \( F^{OOS} \) line is uniformly above zero, indicating that our Equation (3) specification outperforms the baseline CBO specification from 2009:1 to 2016:12. The forecasting improvement gain from the Equation (3) model is statistically significant at the 5 percent level until mid-2009 and from mid-2015 to the end of the sample.

The Diebold-Mariano rejection p-value is 0.01, indicating that taking the sample period as a whole, the forecast improvement of Equation (3) over the baseline model is convincing.

In Figure 4, we display analogous forecast comparison results comparing the OOS forecast performance of Equation (3) to that of an Atkeson-Ohanian-type model. The latter model “predicts that inflation over the next four quarters is expected to be equal to inflation over the previous four quarters” (Atkeson and Ohanian, 2001, p. 6). Thus, we compare forecasts from Equation (3) against those from the model

\[
\pi_{t+12}^{12} - \pi_t^{12} = (\pi_t^{12} - \pi_{t-1}^{12}) + \eta_t
\]  

(4)
Figure 4 displays convincing GR-test evidence for the episodic forecast improvement of our Equation (3) model over the Atkeson-Ohanian model. As in the comparison against the CBO gap model, our Equation (3) forecasts are better on average over the entire comparison period, and these gains are statistically significant at the 5 percent level from late 2010 to late 2012.

For this OOS forecast comparison, the Diebold-Mariano rejection p-value is 0.02, indicating that Equation (3) provides better forecasts than the Atkeson-Ohanian-type model (at the 2 percent level) over the forecasting period as a whole.

The test results discussed above show that our Equation (3) re-formulation of the Phillips curve yields statistically significant improvement in out-of-sample forecasting. We take this improved OOS forecasting performance for our re-formulation of the Phillips curve specification to indicate that the statistical inference results quoted in Section 2 reflect a new set of stable
statistical regularities in the historical data, rather than merely an improved in-sample fit of a more flexible model specification.\textsuperscript{27}

We note that the interpretation of all reduced-form Phillips curve forecasting models is inherently complicated by endogeneity due to extant monetary policy. In particular, as has been known since Lucas (1976), the empirical (reduced-form) Phillips curve will generally vary with monetary policy.\textsuperscript{28} Thus, to the extent that a central bank is successful in controlling inflation, one might expect that the reduced-form Phillips curve relationship will weaken. Further, consider a forecasting model with an unemployment gap term that is seriously misspecified. Least squares estimation of the coefficient on such a gap term might well be biased toward zero in that case, even though the estimated model will still provide unbiased forecasts on average. Consequently, the inflation forecasts generated by such a model will differ substantially from a better-specified model only episodically, for example, when the gap term is quite large, and we find this to be the case in the current exercise. Also, we note that accurate inflation forecasts at horizons longer than 12 months would require accurate forecasts of both $u_{mt}$ and $u_{mt}^*$. Lastly, we want to again emphasize that our goal in this paper is not to devise an improved forecasting model, but rather to provide insight into inflation dynamics; this will, in turn, be useful for both structural modeling and policy. Still, the conditional forecasting exercise undertaken above is useful in that it buttresses our claim that our persistence-dependent model – Equation (2) – is a better specification than the usual PC specification, as in Equation (1).

4. Conclusions
Being so central a topic to macroeconomics, the Phillips curve is the subject of a vast literature. We have argued above, however, that most of this literature suffers from fairly severe model misspecification in the posited Phillips curve regression equation. This widespread problem has

\textsuperscript{27} The examination of Equation (3) for routine use in forecasting inflation is a topic beyond the scope of the present paper, however.

\textsuperscript{28} For recent studies focused on this point, see Fitzgerald and Nicolini (2014) or McLeay and Tenreyro (2018). Occhino (2019) also provides useful intuition. For a recent structural approach in an open economy New Keynesian model that focuses on inflation as a global phenomenon, see Kabukçuoğlu Dur and Martínez-Garcia (2019).
led to erroneous conclusions about the nature of the PC relationship and to the “inflation puzzles” prominently discussed in the recent literature.

We find that our re-specified reduced-form Phillips curve relationship produces stable coefficient estimates across the 1986-2016 sample period, but that this is not a simple linear relationship. Rather, it is what we term “persistence-dependent,” with the form of the relationship between inflation and unemployment fluctuations depending significantly – in both the statistical and the economic sense – on the persistence of these unemployment fluctuations.

Reviewing how the empirically stable specification that we obtain better explains inflation variation in the observed macroeconomic historical record, we find that our Phillips curve model is interpretable as a business-cycle-phase-dependent relationship, and one that is consistent with extant theory.29 In particular, we note that, at the beginning of a recession (precisely when the unemployment rate is rising rapidly), the moderately persistent and transient components of the unemployment rate become positive. Our coefficient estimates imply that this induces a large reduction in inflation – consistent with Stock and Watson (2010). After the unemployment rate peaks and begins to slowly descend, the aforementioned components effectively return to zero. The persistent component remains, of course, very large and positive during this descent, but our coefficient estimates imply that that imparts no force whatsoever on inflation. In other words, during most of the recovery – until the gap actually becomes persistently negative – the unemployment gap has little relationship to inflation. Finally, late in expansions, when the unemployment rate is persistently below the natural rate – that is, when the persistent component becomes negative – our coefficient estimates imply that this will induce a significant increase in inflation.

The in-sample fitting and out-of-sample forecasting results described in Sections 2 and 3 above show that our model specification well explains the time-evolution of inflation during the sample period – even over the Great Recession. In particular, under our model specification all of the “inflation puzzles” noted above disappear. Moreover, notably, the full Phillips curve relationship under our model specification has not materially changed over time – nor recently! – although our model does predict that the inflation-unemployment relationship will appear to be essentially nonexistent at certain times. In contrast, estimated versions of the standard Phillips curve specification effectively average the relationship across differing portions of the business cycle.

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29 See Appendix A.8 for details.
cycle; these portions feature differing “persistence profiles” in the unemployment rate gap, and hence differing inflation-unemployment relationships. Owing to the length of the recovery from the Great Recession, these misspecified formulations provide a single estimated Phillips curve model that is currently anomalously weak – so weak that its very existence is called into question. If (at this writing) the current pandemic induces a rapid rise in the unemployment rate, so that the moderately persistent component of the unemployment gap becomes positive – which, according to our model, will coincide with a rapid reduction in inflation – then these misspecified formulations will then apparently “strengthen” and faith in the conventional (Equation (1)) Phillips curve will be (erroneously) restored. When the economy begins to recover, our model predicts a partial rebound of inflation, followed by a very slow movement of inflation back to trend; and we also predict that standard PC formulations will again find “inflation puzzles.”

The reduced-form Phillips curve specification developed here is validated by its stable coefficients across the sample and by its historical out-of-sample forecasting effectiveness. It is not, however, presented here primarily as a contribution to the literature on inflation forecasting, although we hope that the present work can and will stimulate progress by others in that direction. Nor, as a reduced-form model, does the model specification formulated here directly contribute to the theoretical literature on inflation, although (as detailed in Appendix A.8) it is consistent with existing theories, both with regard to the asymmetry in its unemployment responsiveness and with regard to the manner in which it varies across the business cycle. And we hope that theorists will see our empirical finding of persistence-dependence in this relationship as a stimulus to investigate why and how this dependence arises. However, we see the main contribution of our work as identifying and documenting an important new statistical regularity – a new “stylized fact” – that any reasonably complete theoretical model for the US macroeconomy “ought to” imply.

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30 The current inflation predictions of our Equation (3) model differ sharply from those of standard Phillips curve models. On April 3 of this year, Goldman Sachs (Briggs and Mericle, 2020) predicted that end-2020 year-on-year core inflation would be 1.25 percent. Conditional on a currently-reasonable projection of unemployment over 2020 – peaking at 10.8 percent in June and slowly declining thereafter – a standard linear Phillips curve (Equation (1)) predicts that year-on-year trimmed mean inflation will reach +1.6 percent in April 2021 (and continue falling slowly, to bottom out near +1.2 percent in mid-2023), whilst our model (Equation (3)) predicts that it will bottom out in April 2021 below −1.0 percent, and then begin to recover.

31 Initial results with data on Australia indicate that our results are not unique to the US. Extending this work to a variety of other countries, including ones at differing levels of development, is beyond the scope of the present paper, however.
More broadly, we would like to emphasize the clear implications of the work presented here with regard to current and future monetary policy deliberations. As noted by John Cochrane (quoted in Steelman, Haltom, and Kenney, 2013), “The prevailing theory of inflation these days has nothing to do with money or transactions: the Fed sets interest rates, interest rates affect “demand,” and then demand affects inflation through the Phillips curve” (p. 36). The recent experience of year after year of zero nominal interest rates anchored inflation expectations, and low inflation suggests difficulty in fine-tuning inflation. Even with anchored inflation expectations, evidently the movement of inflation toward its long-run expected level is quite slow, and it takes an appreciable amount of overheating before there is a significant upward force on inflation. Conversely, we find that the Phillips curve mechanism is, in the other direction, very powerful: inflation can be slowed rapidly via a rapid upward movement of the unemployment rate – that is, a recession. The empirical re-formulation of the Phillips relation developed here harmonizes all of that experience in a relatively simple elaboration of the usual – but empirically unstable and unsuccessful – Phillips relation. This re-formulation explains the observed puzzles associated with the usual models, and its empirical implementation is sufficiently stable over the entire sample period as to provide reasonably accurate conditional forecasts of inflation over the Great Recession. These forecasts underscore the notion that inflation today is not “stubbornly low” but is – in the re-formulation of the Phillips curve described here – in fact exactly where its pre-2006 dynamics suggest it should be, given the state of the labor market.
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Appendix

A.1 Other Inflation Indicators and CBO Gap

In Table A1 we present the results using the CBO $u_t^*$ estimate rather than that of Tasci (2018) (in column 2), and several other inflation indicators. We provide the trimmed PCE results from Table 1 in column 1 for comparison. The final two rows in the table refer to GR and DM forecast comparisons against the baseline CBO model for the same dependent variable.

<table>
<thead>
<tr>
<th></th>
<th>Trimmerd PCE</th>
<th>Trimmerd PCE with CBO gap</th>
<th>Median PCE</th>
<th>Core PCE</th>
<th>Median CPI</th>
<th>Core CPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1^+$</td>
<td>0.03</td>
<td>0.02</td>
<td>0.05</td>
<td>0.03</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.67</td>
<td>0.41</td>
<td>2.58</td>
<td>0.62</td>
<td>1.30</td>
<td>0.16</td>
</tr>
<tr>
<td>$\lambda_1^-$</td>
<td>-0.27</td>
<td>-0.54</td>
<td>-0.35</td>
<td>-0.20</td>
<td>-0.35</td>
<td>-0.24</td>
</tr>
<tr>
<td>t-stat</td>
<td>-3.20</td>
<td>-2.72</td>
<td>-11.43</td>
<td>-2.05</td>
<td>-3.14</td>
<td>-2.41</td>
</tr>
<tr>
<td>$\lambda_2^+$</td>
<td>-1.67</td>
<td>-1.67</td>
<td>-1.91</td>
<td>-0.71</td>
<td>-2.13</td>
<td>-1.55</td>
</tr>
<tr>
<td>t-stat</td>
<td>-8.18</td>
<td>-6.98</td>
<td>-7.13</td>
<td>-2.59</td>
<td>-7.79</td>
<td>-5.59</td>
</tr>
<tr>
<td>$\lambda_2^-$</td>
<td>0.10</td>
<td>-0.08</td>
<td>0.17</td>
<td>-0.91</td>
<td>0.40</td>
<td>0.24</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.14</td>
<td>-0.11</td>
<td>0.27</td>
<td>-0.75</td>
<td>0.56</td>
<td>0.25</td>
</tr>
<tr>
<td>$\lambda_3^+$</td>
<td>-0.51</td>
<td>-0.54</td>
<td>-0.46</td>
<td>-0.48</td>
<td>-0.74</td>
<td>-0.24</td>
</tr>
<tr>
<td>t-stat</td>
<td>-2.44</td>
<td>-2.48</td>
<td>-2.10</td>
<td>-1.86</td>
<td>-2.76</td>
<td>-1.06</td>
</tr>
<tr>
<td>$\lambda_3^-$</td>
<td>-0.03</td>
<td>-0.02</td>
<td>-0.13</td>
<td>-0.05</td>
<td>0.08</td>
<td>-0.09</td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.32</td>
<td>-0.32</td>
<td>-0.90</td>
<td>-0.29</td>
<td>0.78</td>
<td>-0.63</td>
</tr>
</tbody>
</table>

Ho: $\lambda_1^+ = \lambda_1^-$
Ho: $\lambda_2^+ = \lambda_2^-$
Ho: $\lambda_3^+ = \lambda_3^-$
Ho: $\lambda_1^+ = \lambda_1^- = \lambda_2^+ = \lambda_2^- = \lambda_3^+ = \lambda_3^-$

Adjusted R-squared
GR Test p-value $<$0.05
DM Test p-value 0.01

Table A.1

In short, our results do not hinge on using the trimmed-mean PCE as the inflation indicator and, broadly speaking, are robust to using different inflation indicators. The partial exception is core PCE inflation, a topic we turn to next.
A.2 Deficiencies in Core PCE

We acknowledge that the core PCE tests do not reject symmetry in the moderately persistent component. Like Ball and Mazumder (2019), we suggest that this “puzzling” result stems from deficiencies of core PCE inflation as a measure of trend inflation. Theory predicts two major deficiencies of less-food-and-energy (“core”) inflation indexes, and both were exhibited in the post-1985 period. First, because the core PCE price index simply excludes items from the basket, core PCE inflation can be subject to bias over prolonged periods. And as Carroll and Verbrugge (2019) indicate, this bias has also been highly unstable over time. For example, between 1995 and 2007, core PCE inflation was downwardly biased by 0.25 percentage points, while it was upwardly biased by 0.3 percentage points between 1980 and 1985. This fact alone raises some doubts about its ability to truly match trend inflation. Second, despite their moniker, core inflation indexes are subject to large idiosyncratic transitory shocks that distort the estimate of trend inflation. (Indeed, the standard deviation of core inflation measures is so large that they are almost always examined in time-averaged form.) Large shocks are not confined to food and energy components. This sensitivity to transitory noise is significant in the present study: transitory shocks can occur at any time, but in the context of analyses that distinguish between phases of the business cycle, these shocks will be especially detrimental if they are correlated with the phase within the sample. One aspect of core PCE inflation is noteworthy: as discussed below, core PCE inflation is sensitive to the movements of prices that are not market-determined, and such movements may well be systematically related to the business cycle. In terms of its ability to reliably reflect trend inflation, as discussed above, when core PCE inflation departs from trimmed-mean PCE inflation, it is core PCE inflation that adjusts to eliminate the gap.

There were only three NBER recessions post-1985. This implies that the moderately persistent component experienced only three nonzero episodes after 1985: starting in 1991, starting in 2001, and starting in mid-2007. During two of these recoveries, core PCE inflation experienced dynamics that were at odds with limited-influence trend inflation indicators such as the trimmed-mean PCE or the median CPI, and even with the other prominent “less food and
energy” series, the core CPI. During the aftermath of the 2001 recession, year-over-year core PCE inflation displayed a prominent rebound from early 2002 to early 2003, including one month with 2.4 percent inflation, a reading not seen since the early 1990s. Other limited-influence trend inflation indicators displayed an essentially monotonic decline from 2001Q3 to 2003Q4. During the Great Recession, while other (year-over-year) trend inflation measures displayed an essentially monotonic decline from 2008Q4 through 2010, core PCE inflation again exhibited a strong rebound in the middle of this episode: starting from below 1 percent in September 2009, it rapidly rose to 1.7 percent during the first few months of 2010, then fell gradually back down to end below 1 percent in 2010Q4.

Conversely, during both of these episodes, inflation in the market-based core PCE displayed dynamics that were similar to other limited-influence trend inflation indicators; see Figure 4. This indicates that core PCE’s unusual dynamics during both of these episodes stemmed from the behavior of prices that were not market-determined. In short, core PCE inflation was evidently subject to countervailing idiosyncratic influences during the aftermath of both the 2001 recession and the Great Recession that all but masked trend inflation movements during critical periods. The anomalous behavior of core PCE inflation during these crucial episodes surely calls into serious question its usefulness as a trend inflation estimator.

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32 This may have been due to insurance payments related to 9/11 that caused m/m core PCE inflation to run negative in the fall, which showed up in y/y core PCE inflation a year later.
33 The PCE market-based price index is based primarily on observed market transactions for which there are corresponding price measures. It includes owners’ equivalent rent, but excludes most imputed expenditures, such as “financial services furnished without payment,” most insurance purchases, gambling, margins on used light motor vehicles, and expenditures by US residents working and traveling abroad.
34 See also Peach, Rich, and Linder (2013), who display a decomposition into goods and services. The anomalous movements during the Great Recession were almost entirely driven by imputed financial services price movements.
35 For more details, see Verbrugge (2020).
Figure 4: Four Inflation Measures

We plot four trend inflation indicators in Figure 4. Only one of these, “core” PCE, displays a notable inflation rebound in 2010. This came from unusual behavior of nonmarket goods, and in particular, imputed financial services. The susceptibility of core PCE inflation to such movements reduces its usefulness as a trend inflation measure.
A.3 Comparison to Stock-Watson Recession Gap

For the post-1985 period, Figure 5 plots the 12-month trimmed PCE inflation rate (leaded 12 months) along with the (monthly) recession gap term and the positive part of the moderately persistent unemployment rate fluctuations, which we here term the “bust gap.”

Figure 5: Stock-Watson Recession Gap and Scaled Bust Gap

In this figure, the latter series has been scaled by multiplying it by 5 so as to render its peak magnitude comparable to that of the recession gap during the middle two recession episodes. Regarding ocular econometrics, the bust gap has an edge in timing, in that the peak inflation deceleration is relatively close to the peak of the bust gap (but well prior to the peak of the recession gap) and ends roughly when the bust gap vanishes (while the recession gap stays significantly positive for much longer). However, this is merely suggestive. We now provide out-of-sample forecast evidence that our specification is superior: at least over the post-1985 period, the bust gap better captures the impact of recessions on inflation dynamics.

In Figure 6, we display the Giacomini-Rossi forecast comparison results from our Equation (3) model versus the Stock and Watson recession gap model. While our model outperforms the Stock-Watson analogue over the entire period, this is only statistically significant (at the 10 percent level) from September 2011 through June 2012. However, the
Diebold-Mariano test, with a p-value of 0.03, indicates that the gain from our model is statistically significant when considering the sample as a whole.

**Figure 6: Stock-Watson Gap Model Versus Equation (5) Model**

![Graph showing Stock-Watson Gap Model Versus Equation (5) Model](image)

**A.4 Relation to Some Other Findings in the Literature**

Our results reinterpret a finding in Coibion and Gorodnichenko (2015). These authors constructed the “NAIRU” implied by their estimated model that would be necessary to explain the “missing disinflation” during the Great Recession. While differing in details, the gap implied by the Coibion-Gorodnichenko NAIRU has broad similarities with our bust gap: it starts opening up shortly after the unemployment rate started rising rapidly in 2008 but was virtually back to zero by mid-2009. (We emphasize that inflation data are not used in constructing our gap measures.) These authors concluded that these dynamics were implausible for a NAIRU. However, our findings indicate that the implausibility of their estimate stemmed not from the
possibility that a NAIRU might have dynamics that were at such great odds with conventional estimates, but rather with the notion that a NAIRU is just another way to describe the natural rate of unemployment. As we have noted above, there is no reason that these concepts should coincide. Implicitly, both Stock and Watson (2010) and Coibion and Gorodnichenko (2015) provide evidence supportive of our findings.

A mismeasured gap will likely lead to the conclusion that forecasting performance is episodic (for example, Stock and Watson 2009) or that there is time variation in the inflation process, such as time variation in the coefficient on the activity variable (see evidence in Clark and McCracken 2006, Stock and Watson 2009, Vavra 2013 and Luengo-Prado, Rao, and Sheremirov, 2018). This may explain the forecasting performance of the time-varying unobserved components model of Stock and Watson (2007).

Our findings also reconcile evidence in, for example, Filardo (1998), Barnes and Olivei (2003), Huh and Jang (2007), Baghli, Cahn, and Fraisse (2007), Stock and Watson (2009), Fuhrer and Olivei (2010), Peach, Rich, and Cororaton (2011), and Peach, Rich, and Linder (2013) that the PC is “convex-concave” (see also Xu, Jiang, and Huang (2015)). These studies, among others already noted above, find a steepening of the Phillips curve as slack becomes negative. Similarly, our findings are also consistent with regime-switching studies, such as Huh, Lee, and Lee (2008) or Donayre and Panovska (2016), that find three regimes in the wage Phillips curve. Our viewpoint, though, is that previous studies somewhat mischaracterize the reduced-form Phillips relationships, first because none (aside from Stock and Watson (2010)) can well approximate the positive part of our moderately persistent component, and second because they typically estimate a fixed lower threshold for slack rather than allowing for a time-varying natural rate of unemployment. In sum, the form of nonlinearity we uncover is well-supported in the data and is consistent with economic theory (see Appendix A.8), yet is not cleanly captured by the standard sorts of nonlinearity that most models admit.

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36 See further discussion in Tasci and Verbrugge (2014).
37 See also Nalewaik (2016) for a rich regime-switching approach.
A.5 Relationship between Moderately Persistent and Transient Components

Figure 7: Moderately Persistent and Transient Components of Unemployment

We plot $u_{\text{modP}}$ and $u_{\text{transient}}$. These series are positively correlated, especially at the beginning of a recession.
A.6 Conditional Recursive Forecasts Using Both 2006:12 and 2016:12 (“Full Sample”) Coefficient Estimates

**Figure 8: More Conditional Recursive Forecasts**

Notice, comparing the gray and orange lines, that the conditional forecasts of our persistence-dependent model are essentially identical, regardless of whether we estimate coefficients in 2006 or 2016. Conversely, conditional forecasts from a standard linear model are notably different if one estimates the Phillips curve in 2016:12 rather than 2006:12. This reflects the purported weakening of the Phillips curve. With coefficients estimated in 2016:12, the fit to the evolution of inflation is still poor, with a far worse fit at the beginning of the Great Recession.

**A.7 Comparison to Wage PC of Morris, Rich, and Tracy**

In the table below, we compare the wage Phillips curve parameter estimates of Morris, Rich and Tracy (2019) (MRT) to the parameter estimates derived from a similarly specified price Phillips curve. The dependent variable in MRT is a four-quarter growth in average wages term,
constructed in MRT on the basis of CPS data, detrended using long-run SPF inflation expectations as in this paper. The specification in MRT is

$$GAW_{t+4}^{**} - \pi_t^* = \alpha_0 + \alpha_1 \left( prod_{t}^{trnd} \right) + \alpha_1^+ \left( un_t^Q - un_t^{*,CBO} \right) + \alpha_1^- \left( un_t^Q - un_t^{*,CBO} \right)$$

$$+ \alpha_2^+ \left[ \Delta \left( un_t^Q - un_t^{*,CBO} \right) \right]^+ + \alpha_2^- \left[ \Delta \left( un_t^Q - un_t^{*,CBO} \right) \right]^- + \varepsilon_t$$

(6)

where $GAW_{t+4}^{**}$ refers to growth in average wages (constructed from CPS data, as detailed in MRT), $un_t^Q$ is quarterly unemployment, $un_t^{*,CBO}$ is the CBO estimate of the natural rate of unemployment, and $prod_{t}^{trnd}$ refers to trend productivity growth at time $t$. For comparison to (6), we re-specify Equation (3) alternatively as

$$\pi_t^{12} - \pi_t^* = \alpha + \beta_1 \left( \pi_{t-12}^{12} - \pi_{t-12}^* \right) + \beta_2 \left( \pi_{t-24}^{12} - \pi_{t-24}^* \right) + \lambda_1^+ \left( un_t^3 - un_t^* \right)^+ + \lambda_1^- \left( un_t^3 - un_t^* \right)^-$$

$$+ \lambda_2^+ \left[ \Delta \left( un_t^3 - un_t^* \right) \right]^+ + \lambda_2^- \left[ \Delta \left( un_t^3 - un_t^* \right) \right]^- + \varepsilon_t$$

(7)

or in a slightly simplified form,

$$\pi_t^{12} - \pi_t^* = \alpha + \beta_1 \left( \pi_{t-12}^{12} - \pi_{t-12}^* \right) + \beta_2 \left( \pi_{t-24}^{12} - \pi_{t-24}^* \right) + \lambda_1^+ \left( un_t^3 - un_t^* \right)^+ + \lambda_1^- \left( un_t^3 - un_t^* \right)^-$$

$$+ \lambda_2^+ \left[ \Delta \left( un_t^3 \right) \right]^+ + \lambda_2^- \left[ \Delta \left( un_t^3 \right) \right]^- + \varepsilon_t$$

(8)

where $un_t^3$ is a three-month moving average of the real-time unemployment rate.

It is interesting to note that estimates of this wage Phillips curve yield qualitatively similar results, though they differ in details. For instance, there is a modest, but nonzero, downward force on wage growth when labor force slack is high. But the strongest pressures occur on wage growth from upward movements in the unemployment rate (though this force is not as strong as the downward force on prices during these periods), and from overheating (and this force is much stronger than the upward force on prices during these periods).
# Table A.2

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>( \text{GAW}^{t+4-\pi_e} )</th>
<th>( \pi^{t+12-\pi_e} )</th>
<th>( \pi^{t+112-\pi_e} )</th>
</tr>
</thead>
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<tr>
<td>MRT (2019)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Eq. (6)</td>
<td></td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>Eq. (7)</td>
<td></td>
<td></td>
<td>-0.3**</td>
</tr>
</tbody>
</table>

1. In Morris, Rich, and Tracy (2019) (MRT), this is the CBO gap; in the second and third columns, this is the gap as defined in this paper. The MRT estimation period is 1983:Q1-2018Q4. ** denotes statistical significance at the 1% level, using Newey-West (1987) standard errors. Time \( t \) subscripts are suppressed in the table.

We again see clear evidence of nonlinearity in the Phillips curve. However, while appealing in their greater simplicity, specifications (7) and (8) do not fit the data quite as well as (3), nor do they yield quite as accurate a conditional forecast of inflation over the post-2006 period. One might have guessed this, given the estimated differential between \( \lambda_2^+ \) and \( \lambda_3^+ \) in conjunction with Appendix A.5. It would be hard for a single term in \( \Delta un \) to capture what appears to be two different relationships.

## A.8 Theory

### A.8.1 Overheating and Inflation

From its inception in Phillips (1958), it was generally believed (see also Lipsey (1960)) that the general shape of the Phillips curve is convex, so that a negative unemployment gap (an overheating economy) has a bigger price impact than the same percentage positive unemployment gap (slack). Many theories naturally give rise to a convex wage Phillips curve. Layard, Nickell, and Jackman (1991) demonstrate that the shirking model of Shapiro and Stiglitz (1984) implies a nonlinear wage Phillips curve. The “bottlenecks” model of Evans (1985) and
the bargaining model of Blanchflower and Oswald (1990) also imply a nonlinear wage Phillips curve. We would expect such convexity to spill over into convexity in the price Phillips curve.

A convex shape to the price Phillips curve is suggested by models in which prices are downwardly rigid, such as Ball, Mankiw, and Romer (1988). In this model, which features menu costs of price adjustment in the presence of generally positive inflation, prices are more sticky downward because the relative price declines can “automatically” occur via inflation. Thus, even if a firm desires a relative price decline, it will optimally choose inaction and wait for inflation to deliver that decline in the near future.

In the standard New Keynesian model, the output gap maps directly into inflationary pressure. In the standard DMP model, the value of unemployment determines the worker’s outside option. Moscarini and Postel-Vinay (2017) draw attention to the fact that individual wage growth co-varies more strongly with the aggregate job-to-job transition rate than with the aggregate unemployment rate. Moscarini and Postel-Vinay (2019) provide a New Keynesian job-ladder model that is consistent with this fact and that explains how an overheating labor market can translate into price pressures. In this model, workers’ bargaining power derives from the ability to receive outside offers, not from the unemployment outside option. After a downturn, many employed workers are mismatched and easily poachable, and numerous unemployed workers are profitably hired. But late in the cycle, the stiff competition for employed productive workers leads to many outside wage offers being matched by current employers, and these wage increases effectively become cost-push shocks.

Another class of models that naturally deliver a Phillips curve relationship of this sort – that is, strong upward price pressure when the economy is overheating – is capacity-constraints models. Bils and Klenow (1998) find procyclical relative price and TFP movements in highly

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38 Downward nominal wage rigidity is a classic explanation for a convex wage Phillips curve (see Phillips (1958) and Daly and Hobijn (2014), and see Dupraz, Nakamura and Steinsson (2019) for a recent model delivering asymmetric unemployment fluctuations.

39 The class of models expounded in Clark and Laxton (1997) or Clark, Laxton and Rose (2001) also feature capacity constraints. Alan Greenspan seems to have believed in a convex Phillips curve arising from capacity constraints. For example, in his testimony to the Subcommittee on Economic Growth and Credit Formation (Greenspan 1994b, p.12), he stated: “If the economy were nearing capacity, we would expect to see certain patterns in the statistical and anecdotal information ... To attract additional workers, employers would presumably step up
procyclical consumption good sectors and argue that this suggests the existence of varying
capacity utilization with occasionally binding capacity constraints. Capacity constraints naturally
induce business cycle asymmetries (Hansen and Prescott, 2005). In the New Keynesian model of
Alvarez-Lois (2004), the Phillips curve becomes
\[ \pi_t = \beta E \pi_{t+1} + \mu (\theta_t + m_c t) \]
where \( \theta_t \) is the share of firms in the economy that are operating at full capacity. (See also
for alternative New Keynesian models with capacity constraints.) There is supportive evidence.
The paper by Lein and Köberl (2009) is a micro study of Swiss manufacturing firms. These
authors find evidence of a strong relationship between price increases and being capacity
constrained (either due to labor or due to technical capacity).40

A.8.2 Busts and Inflation

It has been thought puzzling that large labor market slack does not weigh on inflation, leading to
the famous inflation puzzle of the Great Recession. Not only is this suggested by a conventional
Phillips curve, it is ostensibly an implication of standard New Keynesian theory (see, for
example, King and Watson 2012). That paper demonstrates, though, that the low-frequency
movements in inflation should line up with low-frequency movements in real unit labor costs.
Most of the empirical work in the New Keynesian paradigm has used a variant of labor’s share
as the proxy for real marginal costs, but Bils (1987), Petrella and Santoro (2012), and Madeira
(2014) demonstrate that this can be a misleading proxy. Petrella and Santoro (2012) use the
income share of intermediate goods (and stress the importance of disaggregated data; see also

40 Using these same data, Köberl and Lein (2011) find that an aggregated capacity constraint measure is useful in a
Phillips curve. Similarly, at the micro level, Mikosch (2012) finds that the slope of the micro Phillips curve is
increasing as capacity constraints become tighter, although this effect disappears for firms facing intense
competition.
Bouakez, Cardia, and Ruge-Murcia 2014); Madeira (2014) constructs a proxy using overtime costs. Both alternatives improve the fit of New Keynesian Phillips curves.

Standard industrial organization theory predicts that, at the onset of a recession, we might see an *initial* drop in inflation, but not *continued* downward pressure – even though slack (as conventionally measured) remains high. In particular, the received wisdom in the industrial organization literature is that demand shortfalls tend to provoke price wars. But this behavior is forward-looking, and price declines are front-loaded. After a time, the price war effect ceases, and prices then start to drift slowly upward again. More generally, as is well known, countercyclical markups will mitigate aggregate price drops during recessions. Fernández et al. (2015) demonstrate that, in Spain, average markups rose in half of the sectors after 2008. Gilchrist et al. (2017) develop a New Keynesian model, extended in Gilchrist et al. (2018), that builds upon these insights, and provides supportive empirical evidence. These authors draw attention to the standard IO theory, but further note a nuance to this basic relationship. In customer markets, pricing decisions are investment decisions, and factors that influence investment will influence pricing. Thus, in the theory of Gottfries (1991) and Chevalier and Scharfstein (1996), under financial frictions, constrained firms in customer markets facing a fall in demand may find it optimal to maintain, or even increase, their prices to boost cash flow and avoid costly external financing. Financially unconstrained firms have the opportunity to reduce prices and invest in market share. In the model of Gilchrist et al. (2017), financial frictions imply that markups remain elevated after the initial adverse demand (or financial) shock. Evidence in both Gilchrist et al. (2017) and Gilchrist et al. (2018) is supportive; for instance, financially constrained firms in the US, on average, raised prices at the onset of the Great Recession, while

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41 In the price experimentation model of Bachmann and Moscarini (2012), a recession might trigger some firms to optimally increase prices, as the costly acquisition of information might allow them to “gamble for resurrection.”

42 This is not the same mechanism as in Christiano, Eichenbaum, and Trabandt (2015), in which a jump in credit spreads increases the cost of working capital, increasing marginal costs. Klemperer (1995) also draws attention to the notion of market share as an investment good, with the concomitant influence of the interest rate on prices. For a model featuring countercyclical markups driven by exit, in the absence of financial frictions, see Cheremukhin and Tutino (2016).
other firms dropped prices aggressively and increased their market share.\textsuperscript{43} Prices remained flat for about a year, then began to rise again. The resulting changes in market share were persistent. See also Hong (2019), who finds that markups are countercyclical (with cyclical variation systematically across firms) and who develops a customer-capital variant of a Hopenhayn (1992) model consistent with his findings. Finally, Alves (2019) demonstrates that a reduction in job-to-job flows during the recovery can worsen labor productivity, providing an upward force on inflation that is absent in standard models.

\textsuperscript{43} Asplund, Ericksson, and Strand (2005), Lundin et al. (2009), and Montero and Urtasun (2014) find similar evidence. Gilchrist et al. (2018) find a similar dichotomy between firms in financially weak versus financially strong countries in Europe. They further find that the deviations of price trajectories from the predictions of a standard Phillips curve can be related to financial constraints.