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A Theory of Intrinsic Inflation Persistence
Takushi Kurozumi and Willem Van Zandweghe

We propose a novel theory of intrinsic inflation persistence by introducing trend inflation and variable elasticity of demand in a model with staggered price and wage setting. Under nonzero trend inflation, the variable elasticity generates intrinsic persistence in inflation through a measure of price dispersion stemming from staggered price setting. It also introduces intrinsic persistence in wage inflation under staggered wage setting, which affects price inflation. With the theory we show that inflation exhibits a persistent, hump-shaped response to a monetary policy shock. We also show that a credible disinflation leads to a gradual decline in inflation and a fall in output, and lower trend inflation reduces inflation persistence, as observed around the time of the Volcker disinflation.

Keywords: Trend inflation, variable elasticity of demand, price dispersion, intrinsic inflation persistence, credible disinflation.

JEL Classification: E31, E52.


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1 Introduction

The well-known persistent response of inflation to monetary policy shocks has been documented by a large empirical literature. Christiano et al. (2011), for instance, use a structural vector autoregression to show that inflation responds gradually to a shock to the monetary policy rate and that its peak response is delayed until some time after the shock. Understanding the source of inflation persistence has been of crucial importance not only for academic economists but also for monetary policymakers. Many previous studies have accounted for inflation persistence by embedding price indexation to past inflation (Christiano et al. (2005), Smets and Wouters (2007)) or backward-looking rule-of-thumb price-setters (Galí and Gertler (1999)) in dynamic stochastic general equilibrium (DSGE) models.¹ These assumptions based on backward-looking price-setting behavior give rise to intrinsic persistence in inflation, but remain controversial because they are ad hoc assumptions.² Moreover, the price indexation implies that all prices change in every period, which contradicts the micro evidence that many individual prices remain unchanged for several months, as argued by Woodford (2007). In addition, Benati (2008) questions such assumptions that “hardwire” inflation persistence in models, based on the result of his historical empirical analysis that the degree of inflation persistence varies across monetary policy regimes, which contrasts sharply with the implication of the assumptions that the degree of intrinsic persistence in inflation is policy invariant.³

Our paper proposes a novel theory of inflation persistence by introducing trend inflation and variable elasticity of demand in a DSGE model with Calvo (1983)-style staggered price and wage setting. Nonzero trend inflation affects inflation dynamics in the model because some prices remain unchanged in each period, consistent with micro evidence.⁴ The variable elasticity of demand then gives rise to intrinsic persistence in inflation through a measure

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¹Woodford (2007) reviews different theories of intrinsic persistence in inflation. Fuhrer (2011) discusses the distinction between “intrinsic” versus “inherited” persistence in inflation.

²Galí and Gertler (1999) suggest that “it is worth searching for explanations of inflation inertia beyond the traditional ones that rely heavily on arbitrary lags” (p. 219).

³Hofmann et al. (2012) present empirical evidence of changes in wage dynamics over time and similarly argue that hardwiring the degree of intrinsic persistence in wage inflation can be misleading.

⁴For micro evidence on price setting, see, e.g., Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), and Nakamura et al. (2018). Ascari and Sbordone (2014) survey the literature on the role of trend inflation in inflation dynamics.
of price dispersion stemming from staggered price setting. Likewise, variable elasticity of demand for labor introduces intrinsic persistence in wage inflation under staggered wage setting, which affects real wages and therefore price inflation. A plausibly calibrated version of the model shows that inflation exhibits a persistent, hump-shaped response to a monetary policy shock, as documented by the empirical literature.

Why does variable elasticity of demand generate intrinsic inflation persistence under nonzero trend inflation? To see this, we first note that in the presence of the variable elasticity, a measure of price dispersion becomes an important driver of inflation under nonzero trend inflation. Suppose an expansionary monetary policy shock hits the economy under positive trend inflation. Then, firms that can adjust their products’ prices raise them, while other firms keep their prices unchanged and thus have their relative prices eroded by inflation. Consequently, the dispersion of relative prices increases. A composite-good producer aggregates firms’ products using a technology that favors product variety, so that dispersion of demand for products decreases the efficiency in composite-good production. Although the increase in price dispersion would increase demand dispersion and thus reduce aggregate output, the composite-good producer can lessen the increase in demand dispersion by expanding the demand for each product. An outward shift in demand prevents a sharp decline in market share for products with higher relative prices, without generating a substantial increase in market share for products with lower relative prices. The asymmetric effect of a shift in demand on market shares is due to the variable elasticity of demand, because that elasticity assigns a higher price elasticity of demand to products with higher relative prices. The shift in demand also raises relative prices of individual products and thus the real marginal cost of composite-good production. This way, the price dispersion becomes a key driver of inflation. Then, because the price dispersion is determined by current and

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5Variable elasticity of demand, initially investigated by Kimball (1995), has been widely used as a source of strategic complementarity in DSGE models; see, e.g., Eichenbaum and Fisher (2007), Smets and Wouters (2007), and Levin et al. (2008). Bergin and Feenstra (2000, 2001) embed a translog demand structure in a closed or open economy model with staggered price setting to generate persistence in output or the real exchange rate, respectively. Dotsey and King (2005) employ a state-dependent pricing model to show that the persistence of output and inflation increases in the presence of variable elasticity of demand. Shirot (2015) and Kurozumi and Van Zandweghe (2016) incorporate not only variable elasticity but also trend inflation in staggered price-setting models to analyze their implications for the relationship between output and inflation and for determinacy of equilibrium (but not for inflation persistence).

6The relevant measure of price dispersion differs from the relative price distortion, which captures the loss in aggregate output due to demand dispersion. The variable elasticity of demand weakens the relationship
past inflation rates under staggered price setting, intrinsic persistence emerges in inflation. Therefore, our model provides a theoretical justification for intrinsic persistence in inflation without relying on arbitrary ad hoc backward-looking price-setting behavior.

This paper also contributes to the literature on disinflation. As Fuhrer (2011) points out, intrinsic persistence in inflation plays a key role in canonical New Keynesian (NK) models, where a credible permanent reduction in trend inflation induces a gradual adjustment of inflation to its new trend rate and a decline in output. These responses align closely with historical experiences; for instance, they are reminiscent of the U.S. economy’s evolution during the Volcker disinflation. Without the intrinsic persistence in NK models, inflation jumps to its new trend rate, while output never deviates from its steady-state value. By contrast, in our model, a credible disinflation leads to a gradual decline in inflation and a fall in output even though price-setting behavior is purely forward-looking. This is because our model has intrinsic persistence of inflation through the price dispersion, as noted above.

Our model provides a microfoundation of intrinsic persistence in price and wage inflation by relating the degrees of intrinsic persistence to structural parameters of the model. Consequently, the model is not subject to the criticism by Benati (2008) of models in which intrinsic inflation persistence is policy invariant. In particular, the degrees of intrinsic persistence in price and wage inflation are related to the rate of trend inflation in our model. We then show that lower trend inflation reduces inflation persistence. A number of empirical studies indicate that inflation persistence has decreased in the U.S. since the early 1980s, around the time of the Volcker disinflation. The leading explanation for the decrease emphasizes a more active monetary policy response to inflation. Our paper provides a new explanation: the fall in trend inflation caused the decrease in inflation persistence.

between the price dispersion and the relative price distortion in our model. As a consequence, the relative price distortion shows a relatively weak response to a monetary policy shock. This finding is consistent with that of Nakamura et al. (2018), who indicate little sensitivity of the relative price distortion—“inefficient price dispersion” in their terms—to changes in inflation, using the BLS microdata on consumer prices.

7See, e.g., Ball (1994), Fuhrer and Moore (1995), and Mankiw and Reis (2002).

8Empirical studies that point to a decrease in inflation persistence in the early to mid 1980s include Cogley and Sargent (2001), Stock and Watson (2007), Cogley et al. (2010), and Fuhrer (2011). Owing to differences in methodology and measures of inflation, not all studies indicate a change in inflation persistence in the post-World War II period (e.g., Pivetta and Reis (2007)).

9For studies on the source of the change in inflation persistence, see, e.g., Benati and Surico (2008), Carlstrom et al. (2009), Cogley et al. (2010), and Davig and Doh (2014).

10Bils et al. (2012) criticize the model of Smets and Wouters (2007), in particular its two key ingredi-
A few previous studies have also explained inflation persistence in DSGE models without backward-looking price-setting behavior. Mankiw and Reis (2002) develop a sticky information model to account for the persistent response of inflation to monetary policy shocks.\textsuperscript{11} Dupor et al. (2010) introduce sticky information in a model with staggered price setting and find that lagged inflation appears in the model-implied Phillips curve. A similar finding is obtained by Sheedy (2010), who instead incorporates an upward-sloping hazard function in the model so that prices are more likely to be changed as they have remained fixed for longer. Compared to these studies, our paper offers novel policy implications, including the effect of trend inflation on inflation persistence. Cogley and Sbordone (2008) embed not only price indexation to past inflation but also drifting trend inflation in a staggered price-setting model, and empirically show that intrinsic persistence of inflation is not needed for the model to explain U.S. inflation dynamics in the presence of drifting trend inflation. Phaneuf et al. (2018) employ a medium-scale DSGE model with a roundabout production structure and working capital to demonstrate that, even in the absence of intrinsic persistence, inflation can exhibit persistence inherited from real marginal cost.

The remainder of the paper proceeds as follows. Section 2 presents a model with staggered price and wage setting, trend inflation, and variable elasticity of demand. Section 3 shows that a plausibly calibrated version of the model can explain the well-known persistent response of inflation to monetary policy shocks. Using the calibrated model, Section 4 shows that a credible disinflation leads to a gradual decline in inflation and a fall in output. Section 5 demonstrates that lower trend inflation reduces inflation persistence. Section 6 concludes.

2 Model

To account for inflation persistence, this paper uses a DSGE model with Calvo (1983)-style staggered price and wage setting, trend inflation, and variable elasticity of demand. sticky prices and strategic complementarities, because the model has trouble matching low inflation persistence in the U.S. in recent decades. Our model with staggered price and wage setting, trend inflation, and variable elasticity of demand attributes the low persistence to the low trend inflation observed in recent decades.

\textsuperscript{11}Mankiw and Reis (2002) point out that “the key empirical fact that is hard to match, however, is not the high autocorrelations of inflation, but the delayed response of inflation to monetary policy shocks” (p. 1311).
for goods and labor. The model consists of a representative composite-good producer, individual-goods producing firms, a representative household, a representative labor packer, and a monetary authority. A key feature of the model is that, in each period, a fraction of individual goods’ prices remains unchanged in line with micro evidence, while the remaining fraction of prices is set by firms that face demand curves with variable elasticity. Likewise, a fraction of individual workers’ nominal wages remains unchanged, while the remaining fraction of wages is chosen for labor demand curves with variable elasticity. The behavior of each economic agent is described in what follows.

### 2.1 Composite-good producer

There are a representative composite-good producer and a continuum of firms $f \in [0, 1]$, each of which produces an individual differentiated good $Y_t(f)$. As in Kimball (1995), the composite good $Y_t$ is produced by aggregating individual goods $\{Y_t(f)\}$ with

$$\int_0^1 F_p\left(\frac{Y_t(f)}{Y_t}\right) df = 1. \quad \text{(1)}$$

Following Dotsey and King (2005) and Levin et al. (2008), the function $F_p(\cdot)$ is assumed to be of the form

$$F_p\left(\frac{Y_t(f)}{Y_t}\right) = \frac{\gamma_p}{(1 + \epsilon_p)(\gamma_p - 1)} \left[ (1 + \epsilon_p)\frac{Y_t(f)}{Y_t} - \epsilon_p \right]^{\gamma_p-1} + 1 - \frac{\gamma_p}{(1 + \epsilon_p)(\gamma_p - 1)},$$

where $\gamma_p \equiv \theta_p(1 + \epsilon_p)$. The parameter $\epsilon_p$ governs the curvature of the demand curve for each individual good, which is given by $-\epsilon_p \theta_p$. In the special case of $\epsilon_p = 0$, the aggregator (1) is reduced to the constant elasticity of substitution (CES) one $Y_t = \left[ \int_0^1 (Y_t(f))^{(\theta_p-1)/\theta_p} df \right]^{\theta_p/(\theta_p-1)}$, where $\theta_p > 1$ represents the elasticity of substitution between individual goods. In the case of $\epsilon_p < 0$, strategic complementarity arises in price setting, and this case is of particular interest in the paper.

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12 For a microfoundation of variable elasticity of demand, see Benabou (1988), Heidhues and Koszegi (2008), and Gourio and Rudanko (2014) among others. Benabou develops a model of customer search, where a search cost gives rise to a reservation price above which a customer continues to search for a seller. Heidhues and Koszegi consider customers’ loss aversion, which increases the price responsiveness of demand at higher relative to lower market prices. Gourio and Rudanko construct a model of customer capital, where firms have a long-term relationship with customers whose demand is unresponsive to a low price.
The composite-good producer maximizes profit \( P_t Y_t - \int_0^1 P_t(f)Y_t(f) \, df \) subject to the aggregator (1), given the composite good’s price \( P_t \) and individual goods’ prices \( \{P_t(f)\} \). Combining the first-order conditions for profit maximization and the aggregator (1) yields

\[
\frac{Y_t(f)}{Y_t} = \frac{1}{1 + \epsilon_p} \left[ \left( \frac{P_t(f)}{P_t} \right)^{-\gamma_p} + \epsilon_p \right], \tag{2}
\]

\[
d_{p,t} = \left[ \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{1-\gamma_p} \, df \right]^{\frac{1}{1-\gamma_p}}, \tag{3}
\]

\[
1 = \frac{1}{1 + \epsilon_p} d_{p,t} + \frac{\epsilon_p}{1 + \epsilon_p} e_{p,t}, \tag{4}
\]

where

\[
e_{p,t} \equiv \int_0^1 \frac{P_t(f)}{P_t} \, df. \tag{5}
\]

The variable \( d_{p,t} \) is the Lagrange multiplier on the aggregator (1), which represents the real marginal cost of producing the composite good, and it is a measure of price dispersion as shown in (3). Thus a larger value of the price dispersion leads to a higher value of the real marginal cost. In the special case of \( \epsilon_p = 0 \), where the aggregator (1) becomes the CES one as noted above, eqs. (2)–(4) can be reduced to

\[
Y_t(f) = Y_t(P_t(f)/P_t)^{-\theta_p}, \quad P_t = \left[ \int_0^1 (P_t(f))^{1-\theta_p} \, df \right]^{1/(1-\theta_p)}, \quad \text{and} \quad d_{p,t} = 1,
\]

respectively.

Eq. (2) is the demand curve for each individual good \( Y_t(f) \). The (price) elasticity of demand for the good is then given by \( \eta_{p,t} = \theta_p \left[ 1 + \epsilon_p - \epsilon_p (Y_t(f)/Y_t)^{-1} \right] \). Figure 1 illustrates the demand curve (2) using two values of the curvature parameter, \( \epsilon_p = 0 \) and \( \epsilon_p = -3 \). In the case of \( \epsilon_p = 0 \) (the dotted line), \( \eta_{p,t} = \theta_p \), that is, the demand curve has a constant elasticity of \( \theta_p \).

The case of \( \epsilon_p = -3 \) shows two features. First, the elasticity \( \eta_{p,t} \) varies inversely with relative demand \( Y_t(f)/Y_t \). That is, relative demand for an individual good becomes more price-elastic for an increase in the relative price of the good, whereas the demand becomes less price-elastic for a decrease in the price. As is well understood, this feature induces strategic complementarity in price setting, because in the face of the increasing elasticity, firms keep their products’ relative prices near those of other firms when they can adjust prices. Second, the price dispersion shifts the demand curve under positive trend inflation. The figure shows the demand curves with variable elasticity under a trend inflation rate \( \bar{\pi} \equiv 4 \log \pi \) of zero.
Figure 1: Demand curves with variable and constant elasticity.

Notes: The case of $\epsilon_p = 0$, that is, constant elasticity of demand, is displayed by the dotted line. The case of $\epsilon_p = -3$, that is, variable elasticity of demand, is illustrated by the dashed and the solid lines, which assume, respectively, a trend inflation rate $\bar{\pi}$ of zero and 2.5 percent annually. The values of other model parameters used here are reported in Table 1 below.

A rise in inflation shifts out the demand curve by increasing the price dispersion $d_{p,t}$ to a value exceeding one. The shift raises the relative price of differentiated goods that are inputs into the production of the composite good. Thus the figure illustrates how a larger value of the price dispersion leads to a higher value of the real marginal cost of producing the composite good. Note that the shift lessens the increase in the dispersion of demand associated with the higher price dispersion by preventing a sharp decline in market share for products with higher relative prices, without generating a substantial increase in market share for products with lower relative prices.\(^{13}\)

\(^{13}\)To see this, consider two products: one with a higher (log) relative price of 2 percent and the other with a lower (log) relative price of $-2$ percent. In Figure 1, the shift in the demand curve when trend inflation increases from zero to 2.5 percent annually increases market share by 9.4 ($= -18.8 - (-28.2)$) percent for the product with the higher relative price, versus only 2.8 ($= 18.1 - 15.3$) percent for the product with the lower relative price. Because the latter product has a larger market share than the former, the shift in demand lessens demand dispersion. In contrast, in the case of constant elasticity of demand, a shift in the demand curve would cause a proportional change in market shares that would leave demand dispersion unchanged,
2.2 Firms

Each firm $f$ produces one kind of differentiated good $Y_t(f)$ using the production technology

$$Y_t(f) = N_t(f),$$

(6)

where $N_t(f)$ is labor input of firm $f$, and minimizes cost $w_t N_t(f)$ subject to the technology (6), given the real wage $w_t$. The first-order condition for cost minimization shows that each firm’s real marginal cost is identical and equal to the real wage:

$$mc_t = w_t.$$  

(7)

In the face of the demand curve (2) and the marginal cost $mc_t$, firms set their products’ prices on a staggered basis as in Calvo (1983). In each period, a fraction $\alpha_p \in (0, 1)$ of firms keeps prices unchanged, while the remaining fraction $1 - \alpha_p$ of firms sets the price $P_t(f)$ so as to maximize relevant profit

$$E_t \sum_{j=0}^{\infty} \alpha_p^j q_{t,t+j} \left( \frac{P_t(f)}{P_{t+j}} - mc_{t+j} \right) \frac{Y_{t+j}}{1 + \epsilon_p} \left[ \left( \frac{P_t(f)}{P_{t+j} d_{p,t+j}} \right)^{-\gamma_p} + \epsilon_p \right],$$

where $E_t$ denotes the expectation operator conditional on information available in period $t$ and $q_{t,t+j}$ is the (real) stochastic discount factor between period $t$ and period $t+j$.

Using the equilibrium condition $q_{t,t+j} = \beta^j C_t / C_{t+j}$ for the household’s log utility of consumption $C_t$ with its subjective discount factor $\beta \in (0, 1)$ and the composite-good market clearing condition

$$Y_t = C_t,$$

(8)

the first-order condition for profit maximization can be written as

$$E_t \sum_{j=0}^{\infty} (\alpha_p \beta)^j \left[ \left( \frac{p_t^*}{d_{p,t+j}} \prod_{k=1}^j \frac{1}{\pi_{t+k}} \right)^{-\gamma_p} - \left( \frac{p_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}}}{\gamma_p - 1 - mc_{t+j}} \right) - \frac{\epsilon_p}{\gamma_p - 1} p_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}} \right] = 0,$$

(9)

where $\pi_t = P_t / P_{t-1}$ is the gross inflation rate of the composite good’s price and $p_t^*$ is the

so that the demand curve does not shift.
relative price set by firms that can adjust prices in period $t$. Moreover, under the staggered price setting, eqs. (3) and (5) can be reduced to, respectively,

$$(d_{p,t})^{1-\gamma_p} = \alpha_p \left( \frac{d_{p,t-1}}{\pi_t} \right)^{1-\gamma_p} + (1 - \alpha_p)(p_t^*)^{1-\gamma_p}, \quad (10)$$

$$e_{p,t} = \alpha_p \left( \frac{e_{p,t-1}}{\pi_t} \right) + (1 - \alpha_p) p_t^*. \quad (11)$$

The labor market clearing condition is given by $N_t = \int_0^1 N_t(f) \, df$, where $N_t$ is labor input supplied by the labor packer. Combining this condition with the demand curve (2) and the production technology (6) yields

$$Y_t = \frac{N_t}{\Delta_t}, \quad (12)$$

where

$$\Delta_t \equiv \frac{s_t + \epsilon_p}{1 + \epsilon_p} \quad (13)$$

represents the relative price distortion and

$$s_t \equiv \int_0^1 \left( \frac{P_t(f)}{P_t d_{p,t}} \right)^{-\gamma_p} df, \quad (14)$$

which can be reduced, under the staggered price setting, to

$$(d_{p,t})^{-\gamma_p} s_t = \alpha_p \left( \frac{d_{p,t-1}}{\pi_t} \right)^{-\gamma_p} s_{t-1} + (1 - \alpha_p)(p_t^*)^{-\gamma_p}. \quad (15)$$

Combining (2), (13), and (14) shows that the relative price distortion coincides with a measure of demand dispersion:

$$\Delta_t = \int_0^1 \frac{Y_t(f)}{Y_t} df. \quad (16)$$

The relative price distortion $\Delta_t$ measures the inefficiency of aggregate production under staggered price setting. Because all firms share the same production technology (6), if all prices are flexible then all firms produce the same amount, and thus (16) demonstrates no relative price distortion (i.e., $\Delta_t = 1$) and the aggregate production equation (12) implies no inefficiency in producing aggregate output $Y_t$ using aggregate labor input $N_t$. On the other hand, staggered price setting generates price dispersion and hence demand dispersion, which increases the inefficiency of aggregate production, that is, the relative price distortion $\Delta_t$. 

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Whereas price dispersion is always proportional to demand dispersion in the case of constant elasticity of demand (i.e., $\epsilon_p = 0$), they have distinct dynamics and implications for inflation dynamics in the case of variable elasticity (i.e., $\epsilon_p < 0$), as shown later.

### 2.3 Household and labor packer

There is a representative household that consumes the composite good $C_t$, purchases one-period bonds $B_t$, and has a continuum of members $h \in [0,1]$, each of which supplies an individual differentiated labor service $N_t(h)$, so as to maximize the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log (C_t) - \int_0^1 \frac{(N_t(h))^{1+\sigma_n}}{1 + \sigma_n} \, dh \right]$$

subject to the budget constraint

$$P_tC_t + B_t = \int_0^1 W_t(h)N_t(h) \, dh + i_{t-1}B_{t-1} + T_t, \quad (17)$$

where $\sigma_n \geq 0$ is the inverse of the elasticity of labor supply, $W_t(h)$ is the nominal wage of the labor service $N_t(h)$, $i_t$ is the gross interest rate on the bonds and is assumed to coincide with the monetary policy rate, and $T_t$ consists of lump-sum taxes and transfers and firm profits received.

Assuming additive separability in preferences and complete contingent-claims markets for consumption implies that all members make a joint consumption–saving decision. Thus, combining the first-order conditions for utility maximization with respect to consumption and bond holdings yields the consumption Euler equation

$$1 = E_t \left( \frac{\beta C_t}{C_{t+1}} \frac{i_t}{\pi_{t+1}} \right). \quad (18)$$

There is a representative labor packer that supplies labor input $N_t$ to firms by aggregating individual labor services $\{N_t(h)\}$ with

$$\int_0^1 F_w \left( \frac{N_t(h)}{N_t} \right) \, dh = 1, \quad (19)$$
where the function $F_w(\cdot)$ takes the same form as $F_p(\cdot)$, but with parameters $\epsilon_w$, $\theta_w$, and $\gamma_w$ (instead of $\epsilon_p$, $\theta_p$, and $\gamma_p$). Note that $\theta_w > 1$ and $\gamma_w \equiv \theta_w(1 + \epsilon_w)$ and that the case of $\epsilon_w \leq 0$ is considered in the following sections. The labor packer maximizes profit $W_t N_t - \int_0^1 W_t(h) N_t(h) \, dh$ subject to the aggregator (19), given the labor input price $W_t$ and individual labor services’ nominal wages $\{W_t(h)\}$. Combining the first-order conditions for profit maximization and the aggregator (19) yields

$$\frac{N_t(h)}{N_t} = \frac{1}{1 + \epsilon_w} \left[ \left( \frac{W_t(h)}{W_t d_{w,t}} \right)^{-\gamma_w} + \epsilon_w \right],$$

$$d_{w,t} = \left[ \int_0^1 \left( \frac{W_t(h)}{W_t} \right)^{1-\gamma_w} \right]^{\frac{1}{1-\gamma_w}},$$

$$1 = \frac{1}{1 + \epsilon_w} d_{w,t} + \frac{\epsilon_w}{1 + \epsilon_w} e_{w,t},$$

where $d_{w,t}$ is the Lagrange multiplier on the aggregator (19) and coincides with a measure of wage dispersion as shown in (21), and

$$e_{w,t} \equiv \int_0^1 \frac{W_t(h)}{W_t} \, dh.$$

Given the demand curve (20), nominal wages are chosen on a staggered basis as in Calvo (1983). In each period, a fraction $\alpha_w \in (0, 1)$ of nominal wages remains unchanged, while the remaining fraction $1 - \alpha_w$ of wages is chosen so as to maximize the relevant utility function

$$E_t \sum_{j=0}^{\infty} (\alpha_w \beta)^j \left[ -\frac{(N_{t+j+1}(h))^{1+\sigma_n}}{1 + \sigma_n} + \Lambda_{t+j} \frac{W_t(h)}{P_{t+j}} N_{t+j+1}(h) \right]$$

subject to the demand curve

$$N_{t+j+1}(h) = \frac{N_{t+j}}{1 + \epsilon_w} \left[ \frac{W_t(h)}{W_{t+j} d_{w,t+j}} \right]^{-\gamma_w} + \epsilon_w,$$

where $\Lambda_t$ is the real value of the Lagrange multiplier on the household’s budget constraint (17) and meets the first-order condition $\Lambda_t = 1/C_t$ for the log utility of consumption. Using the composite-good market clearing condition (8), the first-order condition for utility maximization yields

$$\frac{N_t(h)}{N_t} = \frac{1}{1 + \epsilon_w} \left[ \left( \frac{W_t(h)}{W_t d_{w,t}} \right)^{-\gamma_w} + \epsilon_w \right],$$

$$d_{w,t} = \left[ \int_0^1 \left( \frac{W_t(h)}{W_t} \right)^{1-\gamma_w} \right]^{\frac{1}{1-\gamma_w}},$$

$$1 = \frac{1}{1 + \epsilon_w} d_{w,t} + \frac{\epsilon_w}{1 + \epsilon_w} e_{w,t},$$

where $e_{w,t}$ is the measure of wage dispersion as shown in (21), and

$$e_{w,t} \equiv \int_0^1 \frac{W_t(h)}{W_t} \, dh.$$

For the micro evidence on wages, see, e.g., Barattieri et al. (2014).

14 For the micro evidence on wages, see, e.g., Barattieri et al. (2014).
maximization with respect to the nominal wage can be written as

\[
E_t \sum_{j=0}^{\infty} (\alpha_w \beta)^j \frac{N_{t+j}}{Y_{t+j}} \left[ \left( \frac{W_t^*/W_{t+j}}{\prod_{k=1}^{j} \pi_{w,t+k}} \right)^{\gamma_w} \times \left( \frac{W_t^*/W_{t}}{\prod_{k=1}^{j} \pi_{t+k}} \right)^{\gamma_w} - \gamma_w \frac{1}{\gamma_w - 1} \left( \frac{N_{t+j}}{1 + \varepsilon_w} \left( \frac{W_t^*/W_{t+j}}{\prod_{k=1}^{j} \pi_{w,t+k}} \right)^{\gamma_w} + \varepsilon_w \right) \right]^n \frac{\sigma_{\pi_{t+j}}}{\prod_{k=1}^{j} w_{t+k-1}}
\]

\begin{align*}
= 0,
\end{align*}

(24)

where \(W_t^*\) is the nominal wage that is chosen in period \(t\), and

\[
\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1}} \pi_t
\]

(25)

denotes wage inflation. Moreover, under the staggered wage setting, eqs. (21) and (23) can be reduced to, respectively,

\begin{align*}
(d_{w,t})^{1-\gamma_w} &= \alpha_w \left( \frac{d_{w,t-1}}{\pi_{w,t}} \right)^{1-\gamma_w} + (1 - \alpha_w) \left( \frac{W_t^*}{W_t} \right)^{1-\gamma_w}, \\
e_{w,t} &= \alpha_w \left( e_{w,t-1} \right)^{1-\gamma_w} + (1 - \alpha_w) \frac{W_t^*}{W_t}.
\end{align*}

(26)

(27)

2.4 Monetary authority

The monetary authority conducts policy according to a rule of the sort proposed by Taylor (1993). This rule adjusts the interest rate in response to deviations of the inflation rate from its trend rate and deviations of output from its trend level, and allows for policy inertia:

\[
\log i_t = \rho \log i_{t-1} + (1 - \rho) \left[ \log i + \phi_{\pi} (\log \pi_t - \log \pi) + \phi_Y (\log Y_t - \log Y) \right] + \varepsilon_{i,t},
\]

(28)

where \(i\) is the gross steady-state interest rate; \(\pi\) is the gross trend inflation rate; \(Y\) denotes steady-state output; \(\rho \in [0, 1)\), \(\phi_{\pi} \geq 0\), and \(\phi_Y \geq 0\) represent, respectively, the degrees of policy inertia, the policy response to inflation, and the one to output; and \(\varepsilon_{i,t}\) is an i.i.d. shock to monetary policy.
2.5 Log-linearized equilibrium conditions

The equilibrium conditions in the model consist of (4), (7)–(13), (15), (18), (22), and (24)–(28).

To demonstrate intrinsic persistence of inflation in the model, we derive a generalized NK Phillips curve (GNKPC) by log-linearizing the equilibrium conditions around the steady state with trend inflation $\pi$. For the steady state to be derived explicitly, we assume a unit elasticity of labor supply (i.e., $\sigma_n = 1$), which is a common value in the macroeconomic literature. We also assume, to ensure that the steady state is well defined, that the following conditions are satisfied:

$$\alpha_p \max(\pi^{\gamma_p}, \pi^{\gamma_p-1}, \pi^{-1}) < 1, \quad \alpha_w \max(\pi^{2\gamma_w}, \pi^{\gamma_w}, \pi^{\gamma_w-1}, \pi^{-1}) < 1. \tag{29}$$

These conditions are always met in the special case of zero trend inflation, i.e., $\pi = 1$.

The GNKPC is given by

$$\dot{\pi}_t = \beta \pi E_t \ddot{\pi}_{t+1} + \kappa_p \dot{m} c_t + \dot{d}_{p,t} + \ddot{d}_{p,t-1} + \beta \pi E_t \ddot{d}_{p,t+1} + \varphi_{p,t} + \psi_{p,t}, \tag{30}$$

where hatted variables denote log-deviations from steady-state values, and $\varphi_{p,t}$ and $\psi_{p,t}$ are auxiliary variables that are additional drivers of inflation under nonzero trend inflation and satisfy

$$\varphi_{p,t} = \alpha_p \beta \pi^{\gamma_p-1} E_t \varphi_{p,t+1} + \kappa_p \varphi \left( \gamma_p (1 - \alpha_p \beta \pi^{\gamma_p-1}) E_t \dot{d}_{p,t+1} + (\gamma_p - 1) E_t \ddot{\pi}_{t+1} \right), \tag{31}$$

$$\psi_{p,t} = \alpha_p \beta \pi^{\gamma_p-1} E_t \psi_{p,t+1} + \kappa_p \psi E_t \ddot{\pi}_{t+1}. \tag{32}$$

The law of motion of the price dispersion is given by

$$\dot{d}_{p,t} = \rho_p d_{p,t-1} + \kappa_{ped} \ddot{\pi}_t. \tag{33}$$

The composite coefficients $\kappa_p$, $\kappa_{pd}$, $\kappa_{pc}$, $\kappa_{ped}$, $\rho_p$, and $\kappa_{ped}$ in (30)–(33) consist of the model’s structural parameters, including the rate of trend inflation $\pi$, and are given in Appendix A.

The presence of the price dispersion is a novel feature of the GNKPC (30), and the price
dispersion is a source of intrinsic persistence in inflation. To see this, eq. (33) implies that the price dispersion is determined by current and past inflation rates: \( \hat{d}_{p,t} = \kappa_{pd} \sum_{j=0}^{\infty} \rho_{pd}^j \hat{\pi}_{t-j} \). Combining this and the GNKPC (30) leads to

\[
\hat{\pi}_t = b_{p1} \sum_{j=1}^{\infty} \rho_{pd}^{j-1} \hat{\pi}_{t-j} + b_{p2} E_t \hat{\pi}_{t+1} + b_{p3}(\kappa_p \hat{m}_c_t + \varphi_{p,t} + \psi_{p,t}),
\]  

(34)

where \( b_{p1} \equiv \kappa_{pd} b_{p3}[1 + \rho_{pd}(\kappa_{pd} + \beta \pi \rho_{pd})] \), \( b_{p2} \equiv \beta \pi b_{p3}(1 + \kappa_{pd}) \), and \( b_{p3} \equiv 1/[1 - \kappa_{pd}(\kappa_{pd} + \beta \pi \rho_{pd})] \). This shows that our model provides a theoretical justification for intrinsic persistence in inflation without relying on arbitrary ad hoc backward-looking price-setting behavior. The degree of intrinsic inflation persistence can be summarized as the sum of the coefficients on lagged inflation rates, \( \lambda_{p,\epsilon} \equiv b_{p1} \sum_{j=1}^{\infty} \rho_{pd}^{j-1} \rho_{pd} = b_{p1}/(1 - \rho_{pd}) \), and depends on the model’s structural parameters, including the rate of trend inflation \( \pi \).

In addition to intrinsic persistence in price inflation, the model has intrinsic persistence in wage inflation. The GNKPC for wage inflation (wage-GNKPC) is given by

\[
\hat{\pi}_{w,t} = \beta \pi_{\gamma w+1} E_t \hat{\pi}_{w,t+1} + \kappa_w \left(2 \hat{N}_t - \hat{\omega}_t\right) - (\kappa_{wd} - \kappa_{w}) \left(\hat{N}_t - \hat{\gamma}_t\right) + \kappa_{wd} \hat{d}_{w,t} + \hat{d}_{w,t-1} + \beta \pi_{\gamma w+1} E_t \hat{d}_{w,t+1} + \zeta_{w,t} + \varphi_{w,t} + \psi_{w,t},
\]

(35)

where \( \zeta_{w,t} \), \( \varphi_{w,t} \), and \( \psi_{w,t} \) are auxiliary variables that are additional drivers of wage inflation under nonzero trend inflation and satisfy

\[
\zeta_{w,t} = \alpha_{w,\beta \pi_{\gamma w}} E_t \zeta_{w,t+1} + \kappa_{w,\zeta} \left[ (1 - \alpha_{w,\beta \pi_{\gamma w}}) \left(2 \hat{E}_t \hat{N}_{t+1} - \hat{E}_t \hat{\omega}_{t+1} + \gamma_w E_t \hat{d}_{w,t+1}\right) + E_t \hat{w}_{t+1} - \hat{\omega}_t + \gamma_w E_t \hat{\pi}_{w,t+1} \right],
\]

(36)

\[
\varphi_{w,t} = \alpha_{w,\beta \pi_{\gamma w-1}} E_t \varphi_{w,t+1} + \kappa_{w,\varphi} \left[ (1 - \alpha_{w,\beta \pi_{\gamma w-1}}) \left(\hat{E}_t \hat{N}_{t+1} - \hat{E}_t \hat{\gamma}_{t+1} + \gamma_w E_t \hat{d}_{w,t+1}\right) + \gamma_w E_t \hat{\pi}_{w,t+1} - E_t \hat{\pi}_{t+1} \right],
\]

(37)

\[
\psi_{w,t} = \alpha_{w,\beta \pi_{-1}} E_t \psi_{w,t+1} + \kappa_{w,\psi} \left[ (1 - \alpha_{w,\beta \pi_{-1}}) \left(\hat{E}_t \hat{N}_{t+1} - \hat{E}_t \hat{\gamma}_{t+1}\right) - E_t \hat{\pi}_{t+1} \right].
\]

(38)
The law of motion of the wage dispersion is given by

\[ \hat{d}_{w,t} = \rho_{wd} \hat{d}_{w,t-1} + \kappa_{w\pi} \hat{\pi}_{w,t}. \]  

(39)

The composite coefficients \( \kappa_w, \tilde{\kappa}_{wd}, \kappa_{w\pi}, \kappa_{w\phi}, \kappa_{w\psi}, \rho_{wd}, \) and \( \kappa_{w\pi} \) in (35)–(39) are reported in Appendix A. Analogous to the price dispersion, eq. (39) implies that wage dispersion is determined by current and past rates of wage inflation:

\[ \hat{d}_{w,t} = \kappa_{w\pi} \sum_{j=0}^{\infty} \rho_{wd}^j \hat{\pi}_{w,t-j}. \]

Combining this and the wage-GNKPC (35) leads to

\[ \hat{\pi}_{w,t} = b_{w1} \sum_{j=1}^{\infty} \rho_{wd}^{j-1} \hat{\pi}_{w,t-j} + b_{w2} E_t \hat{\pi}_{w,t+1} + b_{w3} \left[ \kappa_w \left( 2 \hat{N}_t - \hat{w}_t \right) - \left( \kappa_{wd} - \kappa_{w\pi} \right) \left( \hat{N}_t - \hat{Y}_t \right) + \zeta_{w,t} + \varphi_{w,t} + \psi_{w,t} \right], \]

(40)

where \( b_{w1} \equiv \kappa_{w\pi} b_{w3} [1 + \rho_{wd} (\kappa_{wd} + \beta \pi^{\gamma w} + 1) \rho_{wd}] \), \( b_{w2} \equiv \beta \pi^{\gamma w} b_{w3} (1 + \kappa_{w\pi}) \), and \( b_{w3} \equiv 1 / [1 - \kappa_{wd} (\kappa_{wd} + \beta \pi^{\gamma w} + 1) \rho_{wd}] \). Therefore, the model also provides a theoretical justification for intrinsic persistence in wage inflation that does not rely on backward-looking wage-setting behavior. Moreover, the degree of intrinsic persistence in wage inflation can be summarized as the sum of the coefficients on past rates of wage inflation, \( \lambda_{w\pi} \equiv b_{w1} / (1 - \rho_{wd}) \), and depends on the model’s structural parameters, including the rate of trend inflation \( \pi \).

The complete set of log-linearized equilibrium conditions consists of (30)–(33), (35)–(39), and

\[ \hat{Y}_t = E_t \hat{Y}_{t+1} - (\hat{t}_t - E_t \hat{\pi}_{t+1}), \]

(41)

\[ \hat{t}_t = \rho \hat{t}_{t-1} + (1 - \rho) \left( \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t \right) + \varepsilon_{i,t}, \]

(42)

\[ \hat{m}_c_t = \hat{w}_t, \]

(43)

\[ \hat{\pi}_{w,t} = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t, \]

(44)

\[ \hat{Y}_t = \hat{N}_t - \hat{\Delta}_t, \]

(45)

\[ \hat{\Delta}_t = \alpha_p \hat{\pi}^{\gamma p} \hat{\Delta}_{t-1} + \frac{s}{s + \epsilon_p} \frac{\gamma_p \alpha_p \hat{\pi}^{\gamma p-1} (\pi - 1)}{1 - \alpha_p \hat{\pi}^{\gamma p-1}} \left( \hat{\pi}_t + \hat{d}_{p,t} - \hat{d}_{p,t-1} \right), \]

(46)

where (41) is the spending Euler equation, (42) is the Taylor-type monetary policy rule, (43) is the marginal cost equation, (44) is the definition of wage inflation, (45) is the aggregate
production equation, and (46) is the law of motion of the relative price distortion $\hat{\Delta}_t$, where $s$ is the steady-state value of $s_t$ that is given by $s = (1 - \alpha_p)/(1 - \alpha_p \pi^\gamma_p)\left((1 - \alpha_p \pi^\gamma_p - 1)/(1 - \alpha_p)\right)\pi^\gamma_p/(\pi^\gamma_p - 1)$.

2.6 Canonical New Keynesian model

To show the implications of our model for inflation persistence, the model is compared with its canonical NK counterpart. The counterpart can be obtained by assuming that prices and nominal wages that are kept unchanged in the above setting are instead updated by indexing to a weighted average of trend inflation and recent past inflation: $P_t(f) = \pi_1^{1-\iota_p} \pi_{t-1}^{\iota_p} P_{t-1}(f)$ and $W_t(h) = \pi_1^{1-\iota_w} \pi_{t-1}^{\iota_w} W_{t-1}(h)$, where $0 \leq \iota_p, \iota_w \leq 1$. These assumptions give rise to the canonical NK Phillips curve (NKPC) and wage-NKPC with intrinsic persistence

$$\hat{\pi}_t = \frac{\iota_p}{1 + \beta \iota_p} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta \iota_p} E_t \hat{\pi}_{t+1} + \frac{(1 - \alpha_p)(1 - \alpha_p \beta)}{\alpha_p (1 + \beta \iota_p)(1 - \epsilon_p \theta_p/(\theta_p - 1))} \hat{m}_t, \tag{47}$$

$$\hat{\pi}_{w,t} = \frac{\iota_w}{1 + \beta \iota_w} \hat{\pi}_{w,t-1} + \frac{\beta}{1 + \beta \iota_w} E_t \hat{\pi}_{w,t+1} + \frac{(1 - \alpha_w)(1 - \alpha_w \beta)}{\alpha_w (1 + \beta \iota_w)(1 + \theta_w \sigma_n - \epsilon_w \theta_w/(\theta_w - 1))} \left(1 + \sigma_n\right) \hat{Y}_t - \hat{w}_t, \tag{48}$$

and imply that $\hat{\Delta}_t = \hat{d}_{p,t} = \varphi_{p,t} = \psi_{p,t} = \hat{d}_{w,t} = \zeta_{w,t} = \varphi_{w,t} = \psi_{w,t} = 0$. Thus, the canonical NK counterpart consists of (41)–(44) and (47)–(48). In the special case of full indexation to trend inflation (i.e., $\iota_p = \iota_w = 0$), this model coincides with our model at zero trend inflation, i.e., $\pi = 1$. Thus, we can demonstrate the effect of trend inflation on inflation persistence by comparing our model with nonzero trend inflation and the counterpart with full indexation to trend inflation. Moreover, we can compare our model that provides a microfoundation of intrinsic persistence in inflation, with the counterpart that assumes intrinsic persistence stemming from indexation to recent past inflation, i.e., $\iota_p, \iota_w > 0$.

3 Impulse Response Analysis

This section analyzes impulse responses to monetary policy shocks in the log-linearized model presented in the preceding section and shows that a plausibly calibrated version of the model can account for the well-known persistent response of inflation to monetary policy shocks.
3.1 Calibration of model parameters

The calibration of parameters in the quarterly model is summarized in Table 1. The elasticity of labor supply has already been fixed at $1/\sigma_n = 1$. As is common in the literature, we set the subjective discount factor at $\beta = 0.99$; the probability of no price change at $\alpha_p = 0.75$, which implies that the average frequency of price change is four quarters; and the parameter governing the elasticity of substitution between individual goods at $\theta_p = 10$, which implies a desired price markup of 11 percent. The corresponding parameters for wage setting and labor services are chosen at the same values $\alpha_w = 0.75$ and $\theta_w = 10$. For the parameters governing the curvature of demand curves, we set $\epsilon_p = -3$, which implies a curvature of the demand curves for goods of $-\epsilon_p \theta_p = 30$ that is within a wide range found in the literature surveyed by Dossche et al. (2010), and $\epsilon_w = -3$, which implies the curvature of the demand curves for labor services is $-\epsilon_w \theta_w = 30$. The trend inflation rate is chosen at 2.5 percent annually, which is the average inflation rate of the personal consumption expenditure (PCE) price index over the period 1985:Q1–2008:Q4.\(^{15}\) The degrees of policy inertia, the policy response to inflation, and the one to output are set at $\rho = 0.8$, $\phi_\pi = 1.5$, and $\phi_Y = 0.5/4$, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_n$</td>
<td>Inverse of the elasticity of labor supply</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>Probability of no price change</td>
<td>0.75</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>Probability of no wage change</td>
<td>0.75</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Parameter governing the elasticity of substitution between goods</td>
<td>10</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Parameter governing the elasticity of substitution between labor services</td>
<td>10</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Parameter governing the curvature of demand curves for goods</td>
<td>$-3$</td>
</tr>
<tr>
<td>$\epsilon_w$</td>
<td>Parameter governing the curvature of demand curves for labor services</td>
<td>$-3$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Gross trend inflation rate</td>
<td>$1.025^{1/4}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Degree of policy inertia</td>
<td>0.8</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Degree of policy response to inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_Y$</td>
<td>Degree of policy response to output</td>
<td>0.125</td>
</tr>
</tbody>
</table>

\(^{15}\)To meet the assumption (29) under the calibration, the trend inflation rate needs to be greater than −2.84 percent annually.
3.2 Impulse responses to monetary policy shocks

Empirical evidence indicates that the response of inflation to a monetary policy shock builds for some time before gradually diminishing. This subsection shows that our model can account for the evidence, using the calibration of parameters presented in Table 1.

Figure 2 illustrates the effects of an expansionary monetary policy shock on inflation in our model (solid lines) and in its canonical NK counterpart with full indexation to trend inflation (dashed lines) or with partial indexation to recent past inflation (dotted lines). For the counterpart with partial indexation to past inflation, the mean estimates of Smets and Wouters (2007) are used for values of the parameters $\iota_p = 0.24$ and $\iota_w = 0.58$. The policy shock leads to an immediate drop in the interest rate, which then returns gradually to its pre-shock level in the top right panel. The top left panel of the figure shows that inflation exhibits a persistent response to the policy shock in our model, with a hump shape and a gradual decline, consistent with the empirical evidence. Inflation rises for three quarters following the shock to a peak level and then declines gradually, similar to the response of inflation obtained in the canonical NK counterpart with partial indexation to past inflation, which rises for two quarters after the shock. The crucial role of trend inflation for inflation persistence in our model is evident by comparing the response of inflation in our model with that in the counterpart with full indexation to trend inflation. Because such a counterpart coincides with our model at zero trend inflation, the difference between the solid and the dashed lines shows the effect of trend inflation on the inflation response. Absent this effect, the response of inflation counterfactually peaks upon impact of the shock.

The difference between the cases of positive trend inflation (solid lines) and zero trend inflation (dashed lines) is caused mainly by the presence of the price dispersion $\hat{d}_{p,t}$ and the wage dispersion $\hat{d}_{w,t}$, as can be seen in the difference between the log-linearized equilibrium conditions (30)–(33) and (35)–(39) in the former case and (47)–(48) with $\iota_p = \iota_w = 0$ in the latter. The price dispersion exhibits a persistent, hump-shaped response to the shock, as displayed in the middle left panel of Figure 2, reflecting that it is determined by current and past inflation rates. The similar responses of inflation and the price dispersion are consistent with the empirical finding of Sheremirov (2019) that the correlation between inflation and dispersion of regular prices is positive. The price dispersion has a significant
Figure 2: Impulse responses to an expansionary monetary policy shock.

Notes: The figure presents the impulse responses to a monetary policy shock of minus one percentage point in annualized terms under the calibration of model parameters reported in Table 1. The interest and inflation rates are displayed in annualized terms. The solid lines represent the model with trend inflation and variable elasticity of demand. The dashed and the dotted lines respectively show the canonical NK counterparts with full indexation to trend inflation ($\tau_p = \tau_w = 0$) and with partial indexation to past inflation ($\tau_p = 0.24$, $\tau_w = 0.58$), where the price dispersion exhibits no (first-order) response.
influence on inflation dynamics mainly through the GNKPC (30), where the past, present, and expected future values of the price dispersion drive inflation, thus making inflation depend on past inflation rates in the GNKPC (34). Regarding the wage dispersion, it can affect inflation dynamics indirectly through its effects on the real marginal cost (i.e., the real wage). However, the middle right panel indicates that the price and wage dispersion both have modest effects on the real marginal cost, as that cost displays a similar response to that in the counterpart with full indexation to trend inflation.

An economic intuition for why the price dispersion becomes a key driver of inflation is as follows. In response to an expansionary policy shock under positive trend inflation, firms that can adjust their products’ prices raise them, while other firms keep their prices unchanged and thus see inflation erode their relative prices. As a consequence, the dispersion of relative prices increases. This increase in price dispersion would increase demand dispersion, thereby reducing aggregate output; but the composite-good producer can lessen the increase in demand dispersion by expanding the demand for each product. An outward shift in demand prevents a sharp decline in market share for products with higher relative prices, without generating a substantial increase in market share for products with lower relative prices. The asymmetric effect of a shift in demand on market shares stems from the variable elasticity of demand, as that elasticity assigns a higher price elasticity of demand to products with higher relative prices. The shift in demand also raises relative prices of individual products and thus the real marginal cost of composite-good production. This way, the price dispersion becomes an important driver of inflation.

In contrast to the key role of the price dispersion, the relative price distortion $\Delta_t$ makes little contribution to the response of inflation to the policy shock in our model. Indeed, the bottom left panel of Figure 2 shows that the response of the relative price distortion to the expansionary policy shock is much weaker than that of the price dispersion. This is because,

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16The joint effect of the past, present, and expected future values of the price dispersion on inflation can give a sense of the direct effect of the dispersion on inflation (ignoring indirect effects through the real marginal cost, inflation expectations, and the auxiliary variables in the GNKPC). The term $x_t \equiv \beta \pi E_t \hat{d}_{p,t+1} + \kappa \hat{d}_{p,t} + \hat{d}_{p,t-1}$, which combines the dispersion terms in the GNKPC (30), declines on impact of the shock before gradually returning to the pre-shock level. The initial decline of $x_t$ mutes the response of inflation following the shock, thus generating a hump shape. Under the baseline calibration, $\kappa_{pd} = -2.026 \approx -2$ and $\beta \pi = 0.996 \approx 1$, so that $x_t$ is approximately equal to the second difference of $E_t \hat{d}_{p,t+1}$ (i.e., $x_t \approx (E_t \hat{d}_{p,t+1} - \hat{d}_{p,t}) - (\hat{d}_{p,t} - \hat{d}_{p,t-1})$). This suggests that a larger (more concave) response of $d_{p,t}$ would make the response of $x_t$ more negative and the response of inflation more hump-shaped.
as noted above, the relative price distortion captures demand dispersion, which is mitigated by the outward shift in the demand curve. The relative price distortion could affect real marginal cost and hence inflation dynamics through its effect on output, as the aggregate production equation (45) relates output to the relative price distortion. However, the bottom right panel of the figure shows that the response of output in our model is similar to that in the canonical NK counterparts with indexation to trend or past inflation, where output $\hat{Y}_t (= \hat{N}_t)$ is not affected, up to the first order, by the relative price distortion.\textsuperscript{17} Therefore, the relative price distortion plays little role for inflation dynamics in our model.

3.3 Roles of variable elasticity of demand and nominal rigidity

We have pointed out that the degree of intrinsic inflation persistence $\lambda_{pe}$ depends on structural parameters in our model. This subsection then shows that the variable elasticity of demand for goods ($\epsilon_p < 0$) and the rigidities of prices and nominal wages ($\alpha_p, \alpha_w$) play key roles for inflation persistence in our model.

Figure 3 presents impulse responses of inflation to an expansionary monetary policy shock for alternative values of four parameters: the parameters that govern the curvature of demand curves for goods and labor ($\epsilon_p, \epsilon_w$) and the Calvo probabilities for prices and wages ($\alpha_p, \alpha_w$). The top left panel of the figure shows the role of the variable elasticity of demand for goods by comparing the response of inflation under the baseline calibration presented in Table 1 with that in the case of constant elasticity of demand for goods. The latter case is obtained by setting $\epsilon_p = 0$ and implies that $\hat{d}_{p,t} = 0$ (and $\psi_{p,t} = 0$). With the constant elasticity, inflation peaks on impact of the shock, illustrating the importance of the variable elasticity for inflation persistence. The case of $\epsilon_p = -6$, which doubles the curvature of demand curves compared to the baseline, further accentuates the hump-shape of the inflation response, indicating that the curvature of demand curves for goods dampens the response of inflation early following the shock.

The top right panel of the figure illustrates a supporting role of the variable elasticity of

\textsuperscript{17}Empirical evidence points to a hump-shaped response of output to a monetary policy shock, but the response of output in our model is monotone. Adding habit formation in consumption preferences to the model would generate a hump-shaped response of output and would provide an additional source of inflation persistence. As this is well understood, our paper omits habit formation to clarify its contribution to the related literature.
Figure 3: Impulse response of inflation: Robustness.

Notes: The figure presents the impulse response of inflation to a monetary policy shock of minus one percentage point in annualized terms under the calibration of model parameters reported in Table 1, except as indicated in each panel. The solid lines represent the baseline case, while the dashed and the dotted lines represent the cases with alternative parameter values.

demand for labor. The panel considers two alternative values of the parameter $\epsilon_w = -0.5$ and $\epsilon_w = -6$, omitting the case of constant elasticity of demand for labor to avoid an issue of indeterminacy of equilibrium.\textsuperscript{18} In contrast with the effect of a variable elasticity of demand for goods, the response of inflation is quite robust to different degrees of curvature of the demand curves for labor. In addition, the panel shows that a higher curvature of demand curves for labor generates a larger response of inflation. That is because, under positive trend inflation, a higher curvature increases the slope of the wage-GNKPC, $\kappa_w$, which more than offsets the dampening effect of the wage dispersion in the wage-GNKPC. Nonetheless,

\textsuperscript{18}Trend inflation increases the likelihood of indeterminacy of equilibrium with constant elasticity of demand, as shown by Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011). Kurozumi and Van Zandwedge (2016) show that variable elasticity of demand largely prevents the indeterminacy caused by trend inflation.
the higher curvature still increases inflation persistence somewhat. A summary statistic of the persistence in impulse responses to a shock is the half-life, defined as the number of quarters until the size of the response falls to half of its size on impact of the shock. The half-life of the inflation response is 11 quarters under the baseline calibration, 10 quarters when $\epsilon_w = -0.5$, and 12 quarters when $\epsilon_w = -6$.

The bottom panels of Figure 3 show the importance of nominal rigidities for inflation persistence by comparing the response of inflation under the baseline calibration with those obtained under two alternative values of the probability of no price change (the bottom left panel) and the probability of no wage change (the bottom right panel). Higher price rigidity increases the persistence in the inflation response by increasing the price dispersion, whereas higher nominal wage rigidity increases inflation persistence via a more persistent response of the real marginal cost. Lower levels of the rigidities have the opposite effect on inflation persistence.\(^\text{19}\)

## 4 Credible Disinflation

Another approach for assessing inflation persistence is to examine the response of inflation to a credible disinflation. In this section, our model is used to analyze a transition from one steady state to another one with lower positive trend inflation.

During the Volcker disinflation in the early 1980s, the U.S. economy underwent a gradual decline in inflation and a recession. To account for this evolution, the existing literature has stressed that intrinsic persistence in inflation plays a key role in canonical NK models. As Fuhrer (2011) points out, when intrinsic persistence of inflation is absent in an NK model, a credible permanent reduction in trend inflation causes inflation to jump to its new trend rate and output to remain at its steady-state value. Once the intrinsic persistence is embedded in the model, the credible disinflation generates a gradual adjustment of inflation to its new

\(^{\text{19}}\)In a model with staggered price setting and constant elasticity of demand for goods, Damjanovic and Nolan (2010) show that a long average duration of price change of two years amplifies relative price distortion and makes it more persistent, thus generating a persistent response of inflation to a monetary policy shock. At the same time, however, their model generates a counterfactual decline in output due to the amplified relative price distortion after an expansionary policy shock, leading the authors to conclude that “further work is required to understand this and reconcile it with how one typically thinks the economy responds to such a shock” (p. 1096).
trend rate and a temporary decline in output.

The U.S. economy’s evolution during the Volcker disinflation can be explained by our model even though price-setting behavior is purely forward-looking. To see this, the following experiment is carried out. In period 0, the economy is in the steady state with a trend inflation rate of 6.6 percent annually. At the start of period 1, trend inflation is reduced suddenly and credibly to 2 percent annually. The former value is the average inflation rate of the PCE price index over the period 1970:Q1–1979:Q4, while the latter is the Federal Reserve’s target for the PCE inflation rate since 2012. For the disinflation we assume that the policy has no inertia, i.e., $\rho = 0$. Denote the vector of endogenous state variables in the log-linearized models by $\hat{k}_t = \log k_t - \log k(\pi)$; for instance, $k_t = [w_t, \Delta_t, d_{p,t}, d_{w,t}]'$ in our model, $k_t = [w_t, \pi_t, \pi_{w,t}]'$ in the canonical NK counterpart with partial indexation to past inflation, and $k_t = w_t$ in the counterpart with full indexation to trend inflation. Here $k(\pi)$ denotes the vector of steady-state values of $k_t$, which stresses that some of these values are functions of trend inflation $\pi$. Because in period 0 all variables are in the steady state, in period 1 the lagged endogenous state variables under the new trend inflation rate are given by $\log k(\pi^0) - \log k(\pi^1)$, where $\pi^0 = 1.066^{1/4}$ and $\pi^1 = 1.02^{1/4}$. Then, the solution of the log-linearized model under the trend inflation rate $\pi^1$ is used to compute inflation and output in period $t = 1, 2, 3, \ldots$

Figure 4 displays the responses of inflation and output to the sudden and credible reduction in trend inflation from 6.6 percent to 2 percent annually, using the calibration of other model parameters reported in Table 1, except for $\rho = 0$. In this figure the dotted lines represent the responses in the canonical NK counterpart with partial indexation to past inflation ($\iota_p = 0.24$, $\iota_w = 0.58$). In this model, inflation declines gradually toward its new trend rate, while output falls temporarily and then rebounds gradually to the initial steady-state value, in line with the responses indicated by Fuhrer (2011).

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20 The disinflation is sudden in that agents did not anticipate the possibility of a change in trend inflation before period 1. The disinflation is credible in that agents believe that the new rate of trend inflation is permanent.

21 The results illustrated in Figure 4 are qualitatively unchanged in the absence of nominal wage rigidity (i.e., $\alpha_w = 0$) in our model and the canonical NK counterpart with partial indexation to past inflation, because the presence of $(w_t, d_{w,t})$ and $(w_t, \pi_{w,t})$, respectively, in the vectors of endogenous state variables is not needed for the results.

22 In the canonical NK counterpart with full indexation to trend inflation, the responses of inflation and output to the sudden and credible reduction in trend inflation are displayed by the dashed lines in Figure...
Similar responses are obtained in our model, as illustrated by the solid lines in Figure 4. This is because our model has intrinsic persistence of inflation through the price dispersion, as shown in the GNKPC (34). One difference between our model and the canonical NK counterpart with partial indexation to past inflation is that output in our model rebounds to its new steady-state value associated with the new rate of trend inflation, which is lower than the initial value of steady-state output.23

Figure 4: Credible disinflation.

Notes: The figure displays the responses of inflation and output to a sudden and credible reduction in trend inflation from 6.6 percent to 2 percent annually, using the calibration of other model parameters reported in Table 1, except for $\rho = 0$. The solid lines show the responses in our model, while the dotted and the dashed lines illustrate those in the canonical NK counterparts with partial indexation to past inflation ($\iota_p = 0$, $\iota_w = 0.58$) and with full indexation to trend inflation ($\iota_p = \iota_w = 0$), respectively.

5 Effect of Trend Inflation on Inflation Persistence

Our model provides a microfoundation of intrinsic persistence in price and wage inflation by relating the degrees of intrinsic persistence to structural parameters of the model. A

4. In this model, inflation drops instantly to the new rate of trend inflation, while output remains at its steady-state value.

23Kurozumi and Van Zandweghe (2016) show that variable elasticity of demand can cause steady-state output to become an increasing function of trend inflation, in contrast with the case of constant elasticity of demand. This is because the variable elasticity alters the effects of trend inflation on the two components of steady-state output: the steady-state average markup and the steady-state relative price distortion.
parameter of particular relevance for monetary policy is the rate of trend inflation, as it also represents the inflation target of the monetary authority in the model. This section examines the effect of a decline in the trend inflation rate on inflation persistence.

The degree of intrinsic persistence in inflation $\lambda_{pe}$ and its analogue for wage inflation $\lambda_{we}$ give a sense of the effect of trend inflation on inflation persistence in our model. Recall from Section 2 that $\lambda_{pe}$ and $\lambda_{we}$ are defined as the sum of the coefficients on lagged inflation rates in the GNKPC (34) and the wage-GNKPC (40), respectively. Figure 5 plots $\lambda_{pe}$ and $\lambda_{we}$ for values of the annualized trend inflation rate $\bar{\pi}$ ranging from zero to 10 percent, using the calibration of other model parameters reported in Table 1. For instance, at the baseline value for the trend inflation rate of 2.5 percent annually, $\lambda_{pe} = 0.12$ and $\lambda_{we} = 0.20$. To compare the degrees of intrinsic persistence in price and wage inflation in our model with the estimates of Smets and Wouters (2007), we can consider the values in the figure at the annualized trend inflation rate of 4.2 percent, which corresponds to the average inflation rate of the PCE price index over the sample period of Smets and Wouters (1966:Q1–2004:Q4). Those values are $\lambda_{pe} = 0.20$, which is close to the estimate of $\iota_p = 0.24$, and $\lambda_{we} = 0.32$, below the estimate of $\iota_w = 0.58$.

A number of empirical studies indicate that inflation persistence has decreased in the U.S. since the early 1980s. Cogley and Sargent (2001) employ spectral analysis to estimate inflation persistence and find that the persistence displays a similar pattern to the level of inflation: both the level and the persistence of inflation increased in the 1970s and decreased gradually from the early 1980s onward. Cogley et al. (2010) use predictability as a measure of persistence, as shocks that are more persistent make time series more predictable. They show that the persistence of the inflation gap (i.e., the gap between actual and trend inflation) rose in the 1970s and fell during and after the Volcker disinflation in the early 1980s. Stock and Watson (2007) characterize inflation as consisting of a transitory and a permanent component and show empirically that the variance of the permanent component increased in the 1970s before declining in the mid 1980s. Fuhrer (2011) examines the persistence in

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24 Benati (2008) conducts an empirical analysis of inflation persistence across countries and time periods and finds that the degree of inflation persistence varies depending on monetary policy regimes. He therefore argues against the assumption that intrinsic inflation persistence is policy invariant, as is embedded in the NKPC (47) and in many existing DSGE models. Based on an empirical analysis of wage dynamics, Hofmann et al. (2012) make a similar argument concerning intrinsic persistence in wage inflation.
Figure 5: Degree of intrinsic persistence in price and wage inflation.

Notes: The figure presents the degree of intrinsic persistence in price inflation, $\lambda_{pe}$, and the degree of intrinsic persistence in wage inflation, $\lambda_{we}$, for a range of values of the annualized trend inflation rate. The degree of intrinsic persistence in price inflation is defined as the sum of the coefficients on lagged inflation rates in the GNKPC (34), and the degree of intrinsic persistence in wage inflation is its analogue in the wage-GNKPC (40).

Consistently, estimated DSGE models indicate a decline in the degrees of intrinsic persistence in price and wage inflation from the period including the 1970s to the period since the mid 1980s; see, e.g., the subsample estimates of $(\iota_p, \iota_w)$ of Smets and Wouters (2007) and the corresponding estimated parameters of Hofmann et al. (2012).

This section uses our model to explain the measured decrease in inflation persistence from a high level in the 1970s to a lower level beginning in the 1980s, around the time of the Volcker disinflation. Most previous studies attribute the decrease in inflation persistence to a more active monetary policy response to inflation, sometimes in combination with changes

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25Because there are multiple ways of measuring persistence, and because various inflation measures have different properties, the evidence on changes in inflation persistence is not as clear-cut as the observation that trend inflation has declined. Notably, Pivetta and Reis (2007) find no evidence of a significant change in U.S. inflation persistence in the post-World War II period.
in the volatility of shocks to the U.S. economy (Benati and Surico (2008), Carlstrom et al. (2009), Davig and Doh (2014)).\footnote{Cogley et al. (2010) attribute the decrease in inflation-gap persistence primarily to a decline in the volatility of shocks to drifting trend inflation, with a secondary role for the monetary policy response to inflation. A shock to drifting trend inflation in their estimated model is reminiscent of the credible disinflation examined in Section 4, although in their model a decline in trend inflation leads inflation to undershoot the new trend rate initially.} In our calibrated model, a decline in the trend inflation rate from 6.6 to 2 percent annually reduces the degree of intrinsic persistence in price and wage inflation from \((\lambda_{pe}, \lambda_{we}) = (0.32, 0.45)\) to \((0.09, 0.16)\). Thus, the model suggests an alternative explanation: the decline in trend inflation caused the decrease in inflation persistence.

Figure 6: Impulse responses at high and low trend inflation rates.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{impulse_responses.png}
\caption{Impulse responses at high and low trend inflation rates.}
\end{figure}

Notes: The figure presents impulse responses to a monetary policy shock of minus one percentage point in annualized terms under the calibration of model parameters reported in Table 1. The inflation rate is displayed in annualized terms. The solid and the dashed lines assume respectively a trend inflation rate \(\bar{\pi}\) of 6.6 percent and 2 percent annually.

The calibrated model shows that lower trend inflation reduces inflation persistence. Figure 6 illustrates impulse responses to a monetary policy shock at a trend inflation rate \(\bar{\pi}\)
of 6.6 percent annually (solid lines) and 2 percent annually (dashed lines). Using the calibration of Table 1 (except for the rate of trend inflation), the top left panel of the figure shows that the response of inflation to the policy shock is more persistent at the higher trend inflation rate of 6.6 percent annually than at the lower rate of 2 percent annually. Indeed, the half-life of the inflation response is 15 quarters at the higher trend inflation rate, and it declines to 10 quarters at the lower rate. At the same time, the delay between the shock and the maximum response of inflation declines from 5 quarters at the higher trend inflation rate to 2 quarters at the lower trend inflation rate. Under positive trend inflation, the variable elasticity of demand for goods and labor causes the price dispersion \( \hat{d}_{p,t} \) to generate intrinsic persistence in price inflation and the wage dispersion \( \hat{d}_{w,t} \) to generate intrinsic persistence in wage inflation, which affects price inflation through the real marginal cost. The price and wage dispersion increase with the level of trend inflation, as displayed in the figure, and therefore the lower trend inflation rate leads to lower persistence of inflation. Thus, our model provides a new explanation for the evidence that inflation persistence decreased around the time of the Volcker disinflation. According to this explanation, the decreases in trend inflation and inflation persistence are no coincidence; the decline in trend inflation reduced inflation persistence.

6 Conclusion

This paper has proposed a novel theory of intrinsic inflation persistence by introducing trend inflation and variable elasticity of demand in a model with Calvo-style staggered price and wage setting. Under nonzero trend inflation, the variable elasticity generates intrinsic persistence in inflation through a measure of price dispersion stemming from staggered price setting. It also introduces intrinsic persistence in wage inflation under staggered wage setting, which affects price inflation. The model provides a microfoundation of intrinsic inflation persistence without relying on arbitrary ad hoc backward-looking price-setting behavior. In a plausibly calibrated version of the model, inflation exhibits a persistent response to an expansionary monetary policy shock, with a hump shape and a gradual decline. With the calibrated model the paper has also demonstrated that a credible permanent reduction in trend inflation leads to a gradual decline in inflation and a fall in output as observed during
the Volcker disinflation. Moreover, the paper has shown that lower trend inflation reduces inflation persistence, providing a new explanation for the measured decrease in inflation persistence around the time of the Volcker disinflation.

Our results raise several questions for further research. Previous studies with DSGE models have suggested other sources of intrinsic inflation persistence, such as sticky information and an upward-sloping hazard function. This poses the question: what is the most empirically relevant among the competing sources? An empirical investigation of this question by estimating DSGE models with each of the sources is a fruitful avenue for future research.\(^{27}\)

Moreover, our model's implication that a decline in trend inflation leads to lower persistence of inflation provides an alternative view to the leading explanation, which holds that lower inflation persistence resulted from a more active monetary policy response to inflation. By estimating our model, future research could examine the relative importance of the two views. Conversely, the model implies that a rise in trend inflation increases inflation persistence and thus leads to longer-lasting deviations of inflation from a central bank's target rate. Therefore, the effect of trend inflation on inflation persistence is an additional factor that could be considered in research about the optimal inflation rate and in the debate on whether central banks should adopt a higher inflation target.

\(^{27}\text{Coibion (2010) and Dupor et al. (2010) show that an NK model with intrinsic persistence of inflation empirically outperforms a sticky information model. The latter authors also demonstrate that introducing sticky information in an NK model exhibits a similar empirical performance to incorporating intrinsic persistence of inflation in the model.}\)
Appendix A  Composite Coefficients in Log-Linearized Equilibrium Conditions

The composite coefficients in log-linearized equilibrium conditions (30)–(33) are given by

\[
\begin{align*}
\kappa_p &\equiv \frac{(1 - \alpha_p \pi^{\gamma_p - 1})(1 - \alpha_p \beta \pi^{\gamma_p})}{\alpha_p \pi^{\gamma_p - 1}[1 - \epsilon_p 2\beta/(\gamma_p - 1 - \epsilon_p 2)]}, \\
\kappa_{pd} &\equiv \gamma_p (\kappa_p - \tilde{\kappa}_{pd}) - \alpha_p \beta \pi^{\gamma_p} \frac{1}{\alpha_p \pi^{\gamma_p - 1}}, \\
\kappa_{pp} &\equiv \frac{\beta(\pi - 1)(1 - \alpha_p \pi^{\gamma_p - 1})}{1 - \epsilon_p 2(1 + \gamma_p)/(\gamma_p - 1)}, \\
\kappa_{pe} &\equiv \frac{\epsilon_p 2\beta(\pi^{\gamma_p - 1} - 1)(1 - \alpha_p \pi^{\gamma_p})}{\pi^{\gamma_p} [\gamma_p - 1 - \epsilon_p 2(1 + \gamma_p)]}, \\
\rho_{pd} &\equiv \frac{\alpha_p \pi^{-1}(1 + \epsilon_p 1 \pi^{\gamma_p})}{1 + \epsilon_p 1}, \\
\kappa_{ped} &\equiv -\frac{\epsilon_p 1 \alpha_p \pi^{-1}(\pi^{\gamma_p} - 1)}{(1 + \epsilon_p 1)(1 - \alpha_p \pi^{-1})}, \\
\end{align*}
\]

where

\[
\epsilon_p 1 \equiv \epsilon_p \frac{1 - \alpha_p}{1 - \alpha_p \pi^{\gamma_p - 1}}, \quad \epsilon_p 2 \equiv \epsilon_p 1 \frac{1 - \alpha_p \beta \pi^{\gamma_p - 1}}{1 - \alpha_p \pi^{\gamma_p} - 1}, \quad \tilde{\kappa}_{pd} \equiv \frac{(1 - \alpha_p \pi^{\gamma_p - 1})(1 - \alpha_p \beta \pi^{\gamma_p - 1})}{\alpha_p \pi^{\gamma_p - 1}[1 - \epsilon_p 2(1 + \gamma_p)/(\gamma_p - 1)]}.
\]

The composite coefficients in log-linearized equilibrium conditions (35)–(39) are given by

\[
\begin{align*}
\kappa_w &\equiv \frac{(1 + \epsilon_w 1)(1 - \alpha_w \pi^{\gamma_w - 1})(1 - \alpha_w \beta \pi^{2\gamma_w})}{\alpha_w \pi^{\gamma_w - 1}[1 - \epsilon_w 3][1 - \epsilon_w 2 \gamma_w/(\gamma_w - 1 - \epsilon_w 2)] + \gamma_w}, \\
\tilde{\kappa}_{wd} &\equiv \frac{(1 + \epsilon_w 3)[1 - \epsilon_w 2 \gamma_w/(\gamma_w - 1 - \epsilon_w 2)] + \gamma_w}{\alpha_w \pi^{\gamma_w - 1}[1 - \epsilon_w 3][1 - \epsilon_w 2 \gamma_w/(\gamma_w - 1 - \epsilon_w 2)] + \gamma_w}, \\
\kappa_{we} &\equiv \frac{\epsilon_w 2(1 + \epsilon_w 3)[1 - \epsilon_w 2(1 + \gamma_w)/(\gamma_w - 1)] + \gamma_w[1 - \epsilon_w 2/(\gamma_w - 1)]}{\alpha_w \pi^{\gamma_w - 1}[1 - \epsilon_w 3][1 - \epsilon_w 2 \gamma_w/(\gamma_w - 1 - \epsilon_w 2)] + \gamma_w}, \\
\kappa_{wd} &\equiv \gamma_w \left[\kappa_w \left(1 + \frac{1}{1 + \epsilon_w 1}\right) - \tilde{\kappa}_{wd}\right] - \alpha_w \beta \pi^{2\gamma_w} - \frac{1}{\alpha_w \pi^{\gamma_w - 1}}, \\
\kappa_{we} &\equiv -\frac{\epsilon_w 3 \pi^{\gamma_w - 1}(1 - \alpha_w \pi^{\gamma_w - 1})}{(1 + \epsilon_w 3)[1 - \epsilon_w 2 \gamma_w/(\gamma_w - 1 - \epsilon_w 2)] + \gamma_w}, \\
\kappa_{w2} &\equiv \frac{(1 + \epsilon_w 3)[1 - \epsilon_w 2(1 + \gamma_w)/(\gamma_w - 1)] + \gamma_w[1 - \epsilon_w 2/(\gamma_w - 1)]}{(1 + \epsilon_w 3)(1 - \alpha_w \pi^{\gamma_w - 1})}, \\
\kappa_{we} &\equiv \frac{\epsilon_w 2 \beta \pi^{\gamma_w - 1}}{\pi^{\gamma_w} [1 - \epsilon_w 2 \gamma_w/(\gamma_w - 1)] + \gamma_w[1 - \epsilon_w 2/(\gamma_w - 1)]}, \\
\rho_{wd} &\equiv \frac{\alpha_w \pi^{-1}(1 + \epsilon_w 1 \pi^{\gamma_w})}{1 + \epsilon_w 1}, \\
\kappa_{wed} &\equiv -\frac{\epsilon_w 1 \alpha_w \pi^{-1}(\pi^{\gamma_w} - 1)}{(1 + \epsilon_w 1)(1 - \alpha_w \pi^{-1})},
\end{align*}
\]

where

\[
\epsilon_w 1 \equiv \epsilon_w \frac{1 - \alpha_w}{1 - \alpha_w \pi^{\gamma_w - 1}}, \quad \epsilon_w 2 \equiv \epsilon_w 1 \frac{1 - \alpha_w \beta \pi^{\gamma_w - 1}}{1 - \alpha_w \beta \pi^{\gamma_w}}, \quad \epsilon_w 3 \equiv \epsilon_w 1 \frac{1 - \alpha_w \beta \pi^{2\gamma_w}}{1 - \alpha_w \beta \pi^{\gamma_w}}.
\]
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