Multiperiod Loans, Occasionally Binding Constraints, and Monetary Policy: A Quantitative Evaluation

Kristina Bluwstein, Michał Brzoza-Brzezina, Paolo Gelain, and Marcin Kolasa
Working papers of the Federal Reserve Bank of Cleveland are preliminary materials circulated to stimulate discussion and critical comment on research in progress. They may not have been subject to the formal editorial review accorded official Federal Reserve Bank of Cleveland publications. The views stated herein are those of the authors and not necessarily those of the Bank of England, Narodowy Bank Polski, the Federal Reserve Bank of Cleveland, or the Board of Governors of the Federal Reserve System.

Working papers are available on the Cleveland Fed’s website:  
https://clevelandfed.org/wp
Multiperiod Loans, Occasionally Binding Constraints, and Monetary Policy: A Quantitative Evaluation
Kristina Bluwstein, Michał Brzoza-Brzezina, Paolo Gelain, and Marcin Kolasa

We study the implications of multiperiod mortgage loans for monetary policy, considering several realistic modifications—fixed interest rate contracts, a lower bound constraint on newly granted loans, and the possibility of the collateral constraint to become slack—to an otherwise standard DSGE model with housing and financial intermediaries. We estimate the model in its nonlinear form and argue that all these features are important to understand the evolution of mortgage debt during the recent US housing market boom and bust. We show how the nonlinearities associated with the two constraints make the transmission of monetary policy dependent on the housing cycle, with weaker effects observed when house prices are high or start falling sharply. We also find that higher average loan duration makes monetary policy less effective and may lead to asymmetric responses to positive and negative monetary shocks.

JEL Classification Codes: E44, E51, E52.
Keywords: mortgages, fixed-rate contracts, monetary policy.

1 Introduction

As highlighted by the recent financial crisis, mortgage markets can play an important role in driving business cycles. Moreover, they strongly interfere with macroeconomic policies, and monetary policy in particular. These observations led to an unprecedented boom in the creation (and publication) of macroeconomic models featuring financial intermediation, housing markets and mortgage loans, the early examples of which were Iacoviello (2005), Iacoviello and Neri (2010), and Gerali et al. (2010).

In spite of its apparent importance, structural modeling of mortgage and housing markets usually assumes – needless to say, counterfactually – that mortgage loans are granted for a single period, which in most models corresponds to one quarter. This makes it impossible to incorporate fixed-rate contracts, despite their prevalence in some countries, and most notably in the US. Second, most models that follow the seminal contribution of Iacoviello (2005) and build financial imperfections on the concept of collateral constraints ignore another important fact: that such constraints usually bind only occasionally. However, while a potential creditor can be prevented from taking a loan, she cannot be forced to take one. Last, but not least, another nonlinearity associated with mortgages seems important: a lower bound for new loans. Again, there is an asymmetry at play: a borrower can be prevented from taking a new loan, but usually cannot be forced to accelerate the repayment of old loans when the collateral value declines.

This brings three features of mortgage markets to our attention: the multiperiodicity of loan contracts, the occasionally binding nature of collateral constraints, and the existence of a lower bound for new loans. It should be highlighted that these are not only potentially important stand-alone features of the mortgage market, but they can also enter into powerful interactions. For instance, both the already mentioned fixed-rate contracts and the lower bound on new loans make sense only in a multi-period contract setting. While all three features have been dealt with in the literature before, we are not aware of any study that offers a thorough, quantitative assessment of the
role they played in shaping macroeconomic dynamics or that explains how they affect the transmission of monetary policy. We believe that these are highly relevant topics and we try to fill the existing gap in the literature.

Our paper offers three contributions to the literature reviewed in the next section. First, we confirm the key importance of multiperiod contracts and the two nonlinearities in explaining the US housing and financial cycle in the past 25 years, which linear models have a hard time matching. In particular, we document, period by period, when and how the nonlinearities were relevant. Second, we demonstrate and explain how the phase of the housing cycle affects the transmission of monetary policy. Last but not least, we are the first to provide a nonlinear Bayesian estimation of a DSGE model with housing and financial frictions, where mortgage contracts are multi-period, the collateral constraint faced by borrowers can be occasionally binding, and new loans are subject to a lower bound.

More specifically, we construct a DSGE model with real and nominal rigidities as well as housing and financial intermediaries. In contrast to much of the literature, we allow loans to be multi-period and carry a constant interest rate over the contract’s duration. We introduce the two nonlinearities mentioned above, which have also been recently highlighted by Guerrieri and Iacoviello (2017) and Justiniano, Primiceri, and Tambalotti (2015). As a result, and abstracting away for a moment from the fact that we assume mortgages to be of a fixed-rate type, a mortgage contract in our model can be thought of as a home equity loan as in Iacoviello (2005), but with the lower bound on new borrowing making it effectively a long-term loan. Whenever binding, this constraint prevents the borrower from being forced to prepay and thus undo some of the loans taken in the past. As a result, in some states of the world, past borrowing will matter for the current level of debt. This breaks the equivalence between one-period and multi-period debt, even when the mortgage cost is adjusted every period, as under variable rate contracts.\footnote{Our framework is thus different from that of Kydland, Rupert, and Šustek (2016), who essentially consider first mortgages rather than home equity loans. In their setup, the distinction between one-period and multi-period loans is always relevant, even in the absence of constraints on earlier repayment.}
We estimate the model in its nonlinear form using US data and the method developed by Guerrieri and Iacoviello (2017). We show that all of our extensions are important to understand the evolution of loans during the recent US housing market boom and bust. A standard DSGE model with housing, single-period loans, and a permanently binding collateral constraint generates a highly counterfactual series of mortgage loans. In contrast, we show that our framework is able to match the evolution of mortgage debt much better. In particular, and unlike in the standard model, our multi-period setting reproduces the fact that loans peaked almost two years after house prices did, and that their subsequent fall was moderate. Our estimation also allows us to precisely identify the periods when the two nonlinearities considered in our framework mattered. For instance, according to our estimation, the collateral constraint became slack in the second half of the housing market boom (i.e., between 2002 and 2007), while the lower bound constraint was most important during the credit crunch (between 2008 and 2013). Our interpretation of the latter is that during that episode, banks wanted to restrict lending by more than they were able to. We treat these results not only as telling an interesting and plausible story about the US housing market developments, but also as a proof of the significance of our extensions to the basic framework.

We next use our model to evaluate the impact of multiperiodicity and the two nonlinearities on the transmission of monetary policy. We show how the impact of monetary shocks depends on the momentum of the housing cycle as the latter affects the degree to which the financial accelerator mechanism amplifies the initial impulse. For instance, our model predicts that the initial impact of expansionary monetary policy on output would have been 40% stronger in 1995 (when the collateral constraint was binding) than in 2006 (when it was slack).

The occasionally binding nature of the constraints can also generate significant asymmetry in the economy’s responses to positive and negative monetary policy shocks. The degree of this asymmetry and time variation crucially depends on the average mortgage loan maturity, making this characteristic of the mortgage market an important determinant of the strength of the monetary policy transmission mechanism. For
instance, the impact of a monetary tightening on output and inflation sharply decreases in mortgage loan maturity. In contrast, expansionary policy is much less affected by this parameter.

The rest of the paper is structured as follows. In the remainder of this section we present a literature review. Sections 2 and 3 present the model and its estimation. Section 4 demonstrates the importance of our extensions using the recent housing boom and bust in the US as an example. Section 5 discusses the implications of our model for monetary policy transmission. Section 6 concludes.

Related literature

Our paper is related to two strands of the literature. First and foremost, we contribute to the relatively recent but growing stream of papers that introduce multi-period loans to study macroeconomic phenomena.

Beneš and Lees (2010) investigate the implications of the existence of multi-period fixed-rate loans for the behavior of a small open economy exposed to finance shocks and housing boom-bust cycles. Calza, Monacelli, and Stracca (2013) develop a DSGE model with two-period mortgage contracts and show that the strength of monetary transmission depends on the loan type (fixed vs. variable rate). Kydland, Rupert, and Šustek (2016) develop a multi-period loans model in which loans taken out in a given period are only used to finance new homes constructed in the same period, and study the business cycle implications of a longer time to build in housing construction. Garriga, Kydland, and Šustek (2017) use this setup to analyze how monetary policy functions in such a context. Gelain, Lansing, and Natvik (2018a) and Gelain, Lansing, and Natvik (2018b) use the same framework, the former to investigate whether a standard asset pricing model can account for the boom-bust patterns in U.S. data over the period 1993-2015 and the latter to address the leaning-against-the-wind argument. Alpanda and Zubairy (2017) compare the effectiveness of monetary policy, housing-related fiscal policy, and macroprudential regulations in reducing household indebtedness in an
estimated DSGE model featuring long-term fixed-rate borrowing. Andrés, Arce, and Thomas (2017) exploit the long-term debt framework to assess the effects of reforms in product and labor markets. Finally Kaplan, Mitman, and Violante (2019) develop a general equilibrium model with long-term debt and multiple aggregate shocks to study the housing boom and bust around the Great Recession. Our main contribution to this line of the literature is our focus on the nonlinearities associated with mortgage contracts and their implications for monetary policy transmission.

Our second point of reference is papers that estimate DSGE models with financial frictions. The estimation of linear frameworks has become highly popular over the past 10 years and has helped to cover numerous important topics such as (i) estimating the impact of housing market shocks in driving the business cycle (Iacoviello and Neri, 2010), (ii) assessing the impact of macroprudential tools on real and financial variables (Gerali et al., 2010), (iii) quantifying the relevance of financial frictions (Queijo von Heideken, 2009; Jermann and Quadrini, 2012) or (iv) optimally designing macroprudential policy (Bielecki et al., 2018).

However, estimation of nonlinear models using full information methods still poses a significant technical constraint. This has been somewhat relaxed with the introduction of the OccBin toolkit by Guerrieri and Iacoviello (2015). Nevertheless, to our knowledge, only very few models with occasionally binding constraints have been estimated so far. Guerrieri and Iacoviello (2017) estimate a model with occasionally binding collateral constraints and show that the contribution of rising house prices to business cycle fluctuations has become small during the US housing boom, once the constraint became slack. Bluwstein (2017) estimates a nonlinear DSGE model with a heterogeneous banking sector and an occasionally binding borrowing constraint, and shows that the asymmetry in macro-financial linkages arises from the borrowers’ balance sheet channel. Our paper is the first to address the two key nonlinearities associated with the mortgage market at the same time in a fully fledged econometric framework.
2 Model

We start from a standard medium-sized New Keynesian setup, extended to incorporate housing and credit frictions as in Iacoviello (2005), and modified to allow for multi-period loans. A key feature of our extension, particularly relevant in a multi-period contract environment, is that the collateral constraint is not assumed to hold with equality every period. Instead, borrowers’ total debt burden can occasionally exceed or fall below the value of collateralizable assets.

Following the common practice in the DSGE literature, we include several frictions that make the impulse responses to monetary shocks implied by our model consistent with the VAR literature. These are sticky prices, sticky wages, and investment adjustment costs. We also include a redistributive fiscal sector, which helps us account for the heterogeneity between borrowers and lenders observed in the microdata.

Our model economy is populated by two types of households, capital producers, goods producers, and the government authorities. Below we sketch the optimization problems facing each class of agents, focusing particularly on those that make up the key ingredients of our extension. The full list of equations making up the equilibrium in our model can be found in the Appendix.

2.1 Households

To introduce credit, we distinguish between two types of households that differ in their subjective discount rates. Those that are relatively patient are indexed by $P$ and make natural lenders, while the impatient group, denoted by $I$, are natural borrowers. The share of impatient households in the population is $\omega$. Within each group, a representative agent $i$ maximizes

$$
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \varepsilon_{u,t} \ln c_{i,t}(t) + A\varepsilon_{h,t} \ln h_{i,t}(t) - \frac{n_{i,t}(t)^{1+\sigma_n}}{1+\sigma_n} \right] \right\}
$$

(1)
for $i = \{I, P\}$, $\beta_I < \beta_P$ and $A > 0$. In the formula above, $c_t$ is consumption, $h_t$ denotes the housing stock, $n_t$ is labor supply, while $\varepsilon_{u,t}$ and $\varepsilon_{h,t}$ stand for consumption and housing preference AR(1) shocks with innovations $\epsilon_{u,t}$ and $\epsilon_{h,t}$.

Patient households’ maximization is subject to a standard budget constraint

$$P_t c_{P,t} + P_{h,t} (h_{P,t} - (1 - \delta_h) h_{P,t-1}) + P_{k,t} (k_t - (1 - \delta_k) k_{t-1}) + D_t =$$

$$= W_{P,t}(t)n_{P,t}(t) + R_{k,t} k_{t-1} + R_{t-1} D_{t-1} + \Pi_t + T_{P,t} + \Xi_{P,t}(t) \tag{2}$$

where $\delta_h \in (0, 1)$, $k_t$ denotes physical capital, $R_{k,t}$ is its rental rate, $\Pi_t$ stands for the financial result of firms and the banking sector, $T_{i,t}$ are lump-sum net transfers, $P_{h,t}$ and $P_{k,t}$ denote housing and physical capital prices, $W_{i,t}$ is the nominal wage, $D_t$ stands for one-period deposits paying a risk-free rate $R_t$ that is set by the central bank, and $\Xi_{i,t}$ is the payout from state-contingent securities traded between households of the same type and providing perfect insurance against household-specific labor income risk arising from wage stickiness.$^2$

Impatient households do not accumulate physical capital or hold any equity. They have access to multi-period mortgage loans, which are modeled as in Woodford (2001), i.e., as perpetuities with principal payments equal to a constant fraction $\frac{1}{m} \in (0, 1]$ of outstanding debt so that $m$ can be interpreted as average loan maturity, including possible prepayment that we do not consider among the set of choices made by households. As a result, outstanding mortgage debt $S_t$ follows the law of motion

$$S_t = L_t + (1 - \frac{1}{m}) S_{t-1} \tag{3}$$

where $L_t$ denotes newly originated mortgages.

In line with the prevalence of fixed-rate mortgages observed in the US in recent decades, we model them as contracts for which the interest payments are fixed at origination and apply for the whole loan duration. Hence, impatient households’ budget

$^2$The presence of these securities allows us to save on notation and drop indexing other households’ allocations with $i$. 
constraint can be written as (see, e.g., Greenwald, 2018)

\[ P_t c_{I,t} + P_{h,t} (h_{I,t} - (1 - \delta_h) h_{I,t-1}) + \frac{1}{m} S_{t-1} = W_{I,t}(t)n_{I,t}(t) + L_t + T_{I,t} + \Xi_{I,t}(t) \]  

(4)

where \( \Phi_t \) is the total promised payment on existing fixed-rate loans that evolves according to

\[ \Phi_t = (R_{h,t} - 1)L_t + (1 - \frac{1}{m})\Phi_{t-1} \]  

(5)

where \( R_{h,t} \) is the interest rate associated with loans originated at time \( t \). This rate will be determined in equilibrium by the optimal behavior of banks.

Additionally, impatient households’ optimization is subject to two additional constraints. The first one is a standard collateral constraint, which is given by the following inequality

\[ R_t S_t \leq \vartheta (1 - \delta_h) \mathbb{E}_t \{ P_{h,t+1} h_{I,t} \} \]  

(6)

where \( \vartheta > 0 \) can be interpreted as a loan-to-value (LTV) ratio.

On top of this, we also introduce a lower bound constraint on the amount of new loans granted each period. Obviously, a loan cannot be negative, and this is the assumption implicitly introduced into the literature by Justiniano, Primiceri, and Tambalotti (2015). Given what can be observed from the data on new loan originations (described in detail in Section 3), we make this constraint even stronger by setting a (possibly strictly positive) lower bound on the amount of newly created loans

\[ L_t \geq \bar{l} P_t \]  

(7)

where \( \bar{l} \geq 0 \). This constraint can be rationalized by noting that, in reality, the housing market is heterogeneous and some households do have access to mortgages regardless of the economic situation.\(^3\) If \( \bar{l} = 0 \), the lower bound constraint can be interpreted as

\(^3\)Naturally, the way we introduce a lower bound on new loans can be considered ad hoc. However, developing a fully fledged heterogeneous household framework to derive this constraint from microfoundations goes much beyond the scope of this paper.
a nonnegativity constraint on new credit, which effectively means that banks cannot force borrowers to accelerate repayment of loans granted in the past.

The collateral constraint (6) is assumed to apply only if the lower bound constraint is slack. In other words, whenever equation (6) implies \( L_t < \bar{l}P_t \), new loans are equal to their lower bound \( L_t = \bar{l}P_t \). This means that, similarly to Justiniano, Primiceri, and Tambalotti (2015), our modeling setup allows for an increase in the observed LTV ratio above the level implied by bank policies during the episodes of plummeting house prices or sharp tightening of lending standards. It also provides an additional mechanism, on top of the possible slackness of the collateral constraint, that makes the effectiveness of policy interventions contingent on their scale and on the state of the economy.

Each household supplies differentiated labor in a monopolistically competitive fashion, with aggregation given by

\[
n_{i,t} = \left[ \int_0^1 n_{i,t}(\iota)^{\frac{1}{\mu_w}} d\iota \right]^{\mu_w}
\]  

for \( i = \{I, P\} \) and \( \mu_w > 1 \). Nominal wages are assumed to be sticky as in the Calvo scheme. More specifically, each period, only a randomly selected fraction \( 1 - \theta_w \) of households can reoptimize, while the remaining wages are automatically indexed to the steady state inflation.

### 2.2 Firms

There are several types of firms in our model. Perfectly competitive final goods producers aggregate intermediate goods indexed by \( \nu \) according to

\[
y_t = \left[ \int_0^1 y_t(\nu)^{\frac{1}{\mu}} d\nu \right]^\mu
\]  

where \( \mu > 1 \).

Intermediate goods producing firms operate in a monopolistically competitive envi-
ronment and use the following production function

\[ y_t(\nu) = \varepsilon_{z,t}k_{t-1}(\nu)^\alpha n_t(\nu)^{1-\alpha} \]  \hspace{1cm} (10)

where \( \varepsilon_{z,t} \) is an AR(1) productivity process with innovations \( \epsilon_{z,t} \), and homogeneous labor input is defined as

\[ n_t(\nu) = [\omega n_{I,t}(\nu)]^\gamma [(1 - \omega)n_{P,t}(\nu)]^{1-\gamma} \]  \hspace{1cm} (11)

Intermediate firms are subject to nominal rigidities so that, each period, only a random fraction \( 1 - \theta \) of them can reset their prices, while the remaining ones adjust their prices to the steady state inflation. Since these firms are owned by patient households, they use their marginal utility to discount future profits.

Finally, capital production is undertaken by perfectly competitive firms owned by patient households. They purchase undepreciated capital from the previous period and produce new stocks according to the following formula

\[ k_t = (1 - \delta_k)k_{t-1} + \left(1 - \Gamma \left( \frac{i_{k,t}}{i_{k,t-1}} \right) \right)i_{k,t} \]  \hspace{1cm} (12)

where \( i_{k,t} \) is final goods used for capital investment, while the adjustment costs function is parameterized such that \( \Gamma(1) = \Gamma'(1) = 0 \) and \( \Gamma''(1) = \kappa \geq 0. \)

### 2.3 Banks

Perfectly competitive banks collect deposits and use them to extend mortgage loans to impatient agents. Banks are owned by patient households that receive profits or cover losses generated in the sector. Their problem is to maximize

\[ \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_P^t \frac{\varepsilon_{n,t}}{c_{P,t}F_t} (\Phi_{t-1} + \frac{1}{m}S_{t-1} - L_t + D_t - R_{t-1}D_{t-1}) \right\} \]  \hspace{1cm} (13)
subject to the balance sheet constraint

\[ D_t = S_t \]  

(14)
as well as the law of motion for debt (3) and promised interest payments (5). The solution to banks’ optimization problem provides an equilibrium condition for the loan rate \( R_{h,t} \).

### 2.4 Government

The fiscal authority follows a passive policy, purchasing a constant volume \( g \) of final goods and financing their expenditures with lump-sum taxes levied on households so that the government budget is balanced every period

\[ P_t g = \omega T_{I,t} + (1 - \omega) T_{P,t} \]  

(15)

where \( P_t \) is the price of final goods. The tax policy is such that the share of impatient households in the total tax burden is fixed at \( \tau \).

The monetary authority sets the policy rate according to the standard Taylor-like rule

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\gamma_{\pi}} \left( \frac{y_t}{y} \right)^{\gamma_y} \right]^{1-\gamma_R} \epsilon_{R,t}
\]  

(16)

where variables without time subscripts denote their steady state values and \( \epsilon_{R,t} \) is an i.i.d. monetary policy shock.

### 2.5 Market clearing

The model is closed with a standard set of market clearing conditions. We assume that housing stock is fixed at the aggregate level at value \( h \) so that we have

\[ h = \omega h_{I,t} + (1 - \omega) h_{P,t} \]  

(17)
The aggregate resource constraint is

\[ y_t = \omega c_{I,t} + (1 - \omega) c_{P,t} + i_{k,t} + i_{h,t} + g \quad (18) \]

where \( g \) stands for government spending and

\[ i_{h,t} = \delta_h h \quad (19) \]

is housing investment compensating for housing depreciation.

3 Calibration and estimation

The model is partly calibrated and partly estimated. In what follows, we describe in detail the calibration and then move to the estimation part. Our country of reference is the US. The model and data frequency is quarterly.

The assumed parameter values for the calibration are reported in Table 1. Following the standard practice, a subset of parameters is taken from the literature or calibrated to match the long-run averages observed in the data. Households’ utility is parameterized such that it implies a moderate Frisch elasticity of labor supply. The discount factor of patient households is set to obtain an annualized average real interest rate of slightly below 3%. The relative impatience of borrowers is calibrated at around 1% quarterly. The steady state inflation rate is set to match the annual average of 2%. Physical capital is assumed to depreciate at 2% per quarter and its share in output is set to 0.3, both values being standard in the literature. The share of government purchases in output matches the long-run average of 16.5%. The LTV ratio, the share of housing in utility, and the housing depreciation rate are calibrated to jointly match the following three long-run proportions: the debt-to-GDP ratio of 0.46, the share of residential investment in output of 4.5%, and the housing-to-GDP ratio of 1.25.

While calibrating the parameters controlling the degree of heterogeneity between
patient and impatient households, we make sure that our choices are consistent with microdata evidence from the Survey of Consumer Finances (SCF) as extracted by Justiniano, Primiceri, and Tambalotti (2015). More specifically, the share of impatient households (borrowers) is set equal to the share of liquidity-constrained consumers (i.e., households with liquid assets worth less than two months of their income), which is 61% according to this data source. If one applies this classification, the SCF implies that, compared to other households, borrowers work on average 8% more hours and their average labor income is 36% lower. We use these two statistics to pin down the share of impatient households in production and the degree of redistribution via the tax system. This calibration also implies that the average total income of borrowers is about 40% of that of savers, which comes very close to the 46% reported in the SCF.

Another important parameter related to our extension is the lower bound on new loans. As already noted, Justiniano, Primiceri, and Tambalotti (2015) implicitly set it to zero. Our calibration is based on new mortgage originations in the US. Given the available data presented in Figure 1, a half of the average value of real new loans looks like a plausible floor for this variable so we chose $\bar{l}_t = 0.5l$, where $l$ denotes the steady state level of real loans in our model. This choice also plays very well in the exercise of replicating the US credit boom and crunch (see Section 4.2), which provides additional support for our calibration. Finally, it should be noted that setting the lower bound above rather than at zero does not alter our results in a qualitative sense. Regarding multi-period loans, we set their average duration at $m = 16$, which allows us to match the quarterly loan flow-to-stock ratio in the model’s steady state and in the data (6.7%).

The parameters controlling real and nominal rigidities, i.e., wage and price markups and stickiness, as well as capital investment adjustment costs are set to standard values assumed in the literature. Finally, the central bank rule is also parameterized in line with the original Taylor rule, except that we allow for some moderate interest rate smoothing.

Estimating models with nonlinearities is a particular challenge. While, in theory,
several techniques are available, in practice, they usually suffer from numerical problems and are time consuming. For this reason, in spite of the recent popularity of models with financial frictions, only a few have been estimated taking explicitly into account the nonlinearities arising, e.g., from collateral constraints binding only occasionally (see Guerrieri and Iacoviello, 2017). Our case is particularly challenging, since the model we consider features two nonlinear constraints. There is, however, a price to be paid: the range of parameters we are able to estimate is relatively narrow.

To be precise, we estimate the autocorrelations and standard deviations of structural shocks (technology, time preference, housing preference, and monetary): this gives us seven parameters (the monetary shock is i.i.d.). Fortunately, as shown above, the remaining structural parameters are either well established in the literature or their calibration can be done in line with the data but outside the model. We use a standard Bayesian random walk Metropolis-Hastings setup to estimate the posterior distribution but without applying a Kalman filter to evaluate the likelihood. Instead, to compute the likelihood, we use the method developed by Guerrieri and Iacoviello (2017) and applied in Bluwstein (2017), which builds upon the piecewise linear solution method and constructs the likelihood function by filtering the shock innovations, \( \epsilon_t = \{ \epsilon_u,t, \epsilon_h,t, \epsilon_z,t, \epsilon_R,t \} \), from the observed data \( Y_t \) recursively. The innovations are drawn from a multivariate Normal distribution, given the past unobserved components \( X_{t-1} \) and the current realization of \( Y_t \). The filtered innovations can then be used to evaluate the log-likelihood at each iteration of the Metropolis-Hastings algorithm

\[
\log(f(Y^T)) = -\frac{T}{2} \log(\det(\Sigma)) - \frac{1}{2} \sum_{t=1}^{T} \epsilon_t^\prime \Sigma^{-1} \epsilon_t - \sum_{t=1}^{T} \log(|\det \frac{\partial \epsilon_t}{\partial Y_t}|). \tag{20}
\]

where \( \Sigma \) is the covariance matrix of \( \epsilon_t \). As we are using the OccBin Toolkit, the Jacobian matrix \( \frac{\partial \epsilon_t}{\partial Y_t} \) is already computed as a part of the piecewise linear solution, which significantly speeds up our implementation. This method not only allows for parameter estimation, but it also shows us when and which constraint was binding over the time span of our sample.
To estimate the model, we use US quarterly data for real house prices, real GDP, core inflation, and the shadow interest rate for the period 1988q4-2016q2. We detrend the GDP series using an exponential trend and demean all data but the interest rate, for which we use the shadow rate by Wu and Xia (2016) to account for the zero lower bound period. Our prior assumptions are as follows. For all autocorrelations we assume a mean of 0.75, for the standard errors of technology, housing preference, and time preference shocks we assume a mean of 0.1, and for the monetary policy shock we assume a mean of 0.01. These are standard values in the literature.

Table 2 reports the results of the piecewise linear estimation procedure, in which both constraints were occasionally binding, and for the linear model, which assumes an always binding collateral constraint and never binding lower bound constraint. The three autoregressive shocks are relatively persistent, with autocorrelation coefficients between 0.72 and 0.98. The standard deviations of shocks seem in line with the values estimated in other papers, ranging from 0.37% for the monetary policy shock to 4.9% in the case of the time preference shock. The differences between the nonlinear and linear model are not huge, as is also the case, for instance, in Guerrieri and Iacoviello (2017). The differences in parameter estimates reflect the fact that some of the larger variations in the data can now be accounted for due to the inclusion of the two nonlinearities.

Three observations support our claim that the nonlinear estimation should be considered successful. First, the observable data and the series that the estimated model generates (by feeding the filtered errors that are used to compute the likelihood), match almost perfectly. While this would not be worth mentioning for linear filters, the practice of nonlinear estimation is such that filters are not always successful at replicating observable variables. Second, the posterior standard deviations for all parameters are much narrower than priors. This suggests that the estimation is supported by the data and not purely driven by our prior assumptions. Third, the marginal data density for the nonlinear model is 1589.3, while that for the linear model is 1580.0. This implies that the data support the nonlinear model by a posterior odds ratio of more than 11,000 to 1.
Having successfully estimated the model, we can move on to see what it tells us about the behavior of the US mortgage market and the effects of monetary policy under multi-period loans.

4 What does the model tell us about the data (and vice versa)?

As discussed in Section 2, our model features three crucial departures from the benchmark collateral constraint framework of Iacoviello (2005): multi-period loans, a lower bound constraint on new lending, and the possibility of the collateral constraint being slack. Before we discuss how these features work (and in particular how they affect monetary policy transmission), we document their importance in explaining the behavior of the US mortgage market. This section consists of two parts. First, we check what the model says about the periods when the constraints were (or were not) binding. Next we show how the three features help in replicating two key variables that were not treated as observable in estimation: the flow and stock of housing loans.

4.1 The role of nonlinear constraints

The estimation process allows us to check whether and when the two nonlinearities mattered. Figure 3 shows the Lagrange multipliers on the lower bound and collateral constraint in our estimation sample. According to our model, there were two periods when the lower bound constraint was binding: 1990-94 and 2008-13 (the multiplier is positive then). Both seem to be associated with credit-tightening periods. Figure 2 plots three selected measures of credit market tightness: two discontinued series from the Senior Loan Officer Survey on tightening standards for mortgage loans (total until 2007q1 and prime since 2007q2) and the Chicago Fed’s National Financial Conditions Credit Subindex. They show that 1990-92 and 2007-10 were the periods of extraordinarily tight credit supply conditions. Our interpretation is that banks sharply tightened
credit conditions and, as a consequence, hit the lower bound constraint (we show in the next subsection that this is also supported by the mortgage flow data).

Before turning to the collateral constraint, we stress that our modeling assumption was that when the lower bound constraint binds, the collateral constraint does not bind by construction; otherwise, we could face two sharply binding and mutually inconsistent constraints at the same time. Hence, we do not evaluate the collateral constraint multiplier dynamics in the period 1990-94 and 2008-13, and accordingly, we do not plot it. The remaining periods give the following picture: the constraint was binding between 1994 and 2002. This shows that, in spite of rising house prices, households were still constrained in the first half of the housing market boom. In 2002 the constraint becomes slack and remains so until 2007, which suggests that once house prices increased sufficiently, households stopped using their whole borrowing potential to increase leverage. The collateral constraint becomes slack again in the last part of our sample, i.e., after 2013. This is a period when house prices started to rise again while – according to our results – households did not borrow up to their collateral constraint.

4.2 The credit boom and crunch

In this section we use different variants of our model to generate (using the Kalman smoother) two very interesting variables: the stock and flow of mortgage debt. These variables were not used as observables in the estimation process; hence, this makes our exercise a highly demanding test for the model. The goal is not only to show that our model is able to replicate these series to a surprisingly high degree, but also that our three key features matter for this success. The figures present the whole sample, but we focus our attention on the most interesting boom-bust period of 1998-2015.

The results for the stock of loans are presented in the upper panel of Figure 4. We start with the baseline one-period loan framework, i.e., with a permanently binding collateral constraint and no lower bound constraint, and then consecutively add our new features to document their impact. The solid gray line shows the actual data on
real home mortgages. The dashed gray line presents the implied path for this variable obtained from the baseline linear model with single-period loans. In this model, since the collateral constraint holds with equality, loans follow house prices very closely, which translates into a sharp overshooting of debt during the boom and its too dramatic collapse during the bust (as well as another overshooting from 2013 onward).

As a second step, we add multi-period loans and the lower bound constraint (dotted black line). As explained in Section 5.2, these features work only in interaction; each one introduced separately would produce zero or marginal change. When introduced together, they change the picture substantially as the slope of the bust phase now is less steep and resembles the actual data more closely. As already explained, the lower bound constraint played an important role in the years 2008-13, as banks could not force households to accelerate loan repayment despite the rapidly falling collateral value. However, during the boom phase, this model variant still overestimates the increase in mortgage debt.

We expect that this can be corrected by allowing for our second nonlinearity, as we have already shown that during the second half of the boom, the collateral constraint relaxed to an extent that households did not necessarily want to exploit it fully. The black dashed line shows the evolution of mortgage debt under this assumption (without the lower bound constraint). As expected, this model variant replicates the boom phase quite well (the collateral constraint is slack during this phase), but fails to explain the slow fall in debt during the bust. Moreover, allowing for the slackness allows us to solve the problem of a second boom forming since 2013; now loans decline as in the data.

Finally, the solid black line represents our complete model: with fixed-rate multi-period loans, an occasionally binding credit constraint, and the lower bound on new loans. Now the fit is much more in line with the real data. Moreover, the model-implied peak is very close to the actual one. This is in stark contrast to the linear model, for which mortgage debt just followed the movement of house prices, leading to a much higher (and narrower) peak and an overshooting at the end of the sample. The various models’ performance can also be evaluated by comparing the mean squared deviations
from the data: the nonlinear model’s error is only 21% of the linear model’s error, highlighting that even if our benchmark model is not perfect, it is much better than the baseline linear case.

Our second variable of interest is the flow of mortgages. While obviously related to the stock, this variable allows us to additionally demonstrate the importance of our modeling assumptions. First, for new loans the distinction between single- and multi-period loans is crucial. When loans are single-period, new loans equal the stock of debt. As a consequence, new loans show no similarity to the data, enough to mention that their steady state ratio to GDP is 45% (compared to 3.5% in the data). Only in the multi-period setting are the new loans generated from the model on a scale comparable to the data. The lower panel of Figure 4 presents two loan flow series generated by different model variants.\(^4\) On the top of this, we plot the actual data on new mortgage originations (gray solid line, source: Mortgage Bankers Association).

First, we demonstrate the multi-period linear case (i.e., with the collateral constraint binding permanently and the lower bound constraint not binding – gray dashed line). In some periods the simulation diverges sharply from the data. This is particularly the case in 2004-06, when the model predicts a strong expansion in lending, in 2007-09, when the linear model suggests a contraction much sharper than seen in the data, and in the following years, when the model predicts too strong a rebound. The solid black line presents the series generated from our complete nonlinear model. While it is not able to replicate the high-frequency volatility, it captures well the run-up to the boom and the post-crisis credit crunch. In contrast to the linear solution, we avoid the overshooting of 2004-06, the excessive contraction of 2007-09, and the subsequent over-reaction of the loan market. There are two short episodes when our nonlinear model fails to replicate the data that well: in 2003 we underpredict loan creation, and in 2013, we overpredict it. Nevertheless, the overall match is surprisingly good for a variable that was unobservable in estimation, and the nonlinear model fares much better than

\(^4\)Compared with the stock of loans, we limit the number of plotted lines since new loans are quite volatile and the picture would be hard to read with more series in one figure. The omitted series do not change the general conclusions and are available from the authors upon request.
the linear one. This can again be seen more formally by comparing the mean squared deviations from the data: their ratio is now even smaller than that for the stock and amounts to 16%.

Overall, the goal of this section was to show that our model can tell a consistent and plausible story about recent developments in the US housing and mortgage markets, and that all the departures from the baseline framework that we consider in the paper are important in this respect. Hence, we believe that our full model has the potential to deliver a more adequate description of the working of stabilization policies that affect these markets. This will be studied in the next section.

5 Implications for monetary policy

We are now ready to show how multi-period loans and the two associated nonlinearities work, and how their presence affects the transmission of monetary policy. Since, as we have shown in the previous section, the degree to which the collateral constraint and the lower bound on new loans bind varies over time, the propagation of shocks and policies will exhibit time dependence, and asymmetric responses to positive and negative shocks of the same size can be expected. Moreover, the presence of fixed-rate contracts and of the lower bound, which is more likely to bind when loan duration is high, creates interesting interactions between the transmission of shocks and the number of periods for which loans are effectively taken. Our goal is to demonstrate how these features work in our model and show that they can be quantitatively relevant. The time dependence, asymmetry, and interactions all essentially apply to any stochastic shocks or policy that one could consider in a model like ours. However, given our paper’s focus, we restrict our attention to the transmission of monetary policy innovations.
5.1 Time variation in the monetary transmission mechanism

To demonstrate how, according to our estimated model, the dynamic effects of monetary policy shocks have evolved over time due to the presence of nonlinearities associated with the mortgage market, we pick four dates in our sample, each representing a different degree of tightness of the collateral and lower bound constraints, using the evolution of the Lagrange multipliers presented in Figure 3 as a guideline. To calculate the dynamic responses to standard monetary policy shocks, we initialize the model from the values of the state variables identified on a given date during the estimation procedure by the Kalman filter, and simulate it forward, assuming either no further shocks or a one standard deviation innovation to the monetary policy feedback rule.

The differences between these two paths for the four selected starting points are plotted in Figure 5. Let us first concentrate on its left column, which documents the reactions to a monetary contraction. For a shock hitting in 2006q3, which is roughly in the middle of the period over which the collateral constraint was slack according to our model, and when house prices were at their peak, the reactions of output and inflation are in line with what is known from the literature on how the economy responds to a monetary tightening: both variables fall and then gradually return to the steady state. Since the higher cost of credit acts as a negative income shock for impatient households and a positive one for patient agents, both types smooth their consumption by increasing borrowing or saving, respectively. As a result, and also due to the working of the Fisherian debt deflation channel, total real debt in the economy goes up. This variable responds differently when the collateral constraint is binding, as in 1995q1, when houses were relatively cheap. Since a monetary policy tightening depresses house prices, and hence the value of collateral that can be used to secure loans, borrowers become even more financially constrained, so fewer new loans are taken and the stock of debt falls. This acts as a financial accelerator, amplifying the negative response of output. During a housing market bust, with steeply falling house prices, tightening credit conditions, and a binding lower bound on new loans (which is how our model
interprets the mortgage market’s stance in 2010q1), this variable does not move until the constraint becomes slack. As a consequence, real debt barely changes as its increase only reflects the debt deflation effect, and the contraction in output is only slightly deeper compared to the episodes during which the collateral constraint was not binding.

Focusing on asymmetries, we compare the left and right columns of Figure 5, where the latter shows reactions to a monetary expansion, presented with a reversed y axis to facilitate comparison. For the three starting points described above, the responses are symmetric as the considered monetary shocks are not large enough to trigger a regime switch. This is not the case for our fourth date, 2002q1, at which point only the collateral constraint is binding, but new loans are close to their lower bound. As a result, a monetary easing leads to an expansion in output, inflation, and debt that is very similar to that obtained for 1995q1. In contrast, as depicted in the left column, after an increase in the policy rate, new loans hit the lower bound, which effectively limits their adjustment for the first two periods after the shock. In turn, debt responds with a delay, and so does output, reaching its trough only after a year rather than on impact.

Overall, the simulations presented show how the effects of monetary policy may depend on the momentum of the housing cycle as the latter affects the degree to which the financial accelerator mechanism amplifies the initial impulse. The occasionally binding nature of the constraints associated with the mortgage market can also generate significant asymmetries, concerning both the magnitude and timing of responses to positive and negative shocks.

5.2 Interactions between mortgage market nonlinearities and loan maturity

As we already have stressed, the three features of mortgage markets that we consider, i.e., multiperiodicity of loans, the occasionally binding nature of collateral constraints, and the existence of a lower bound for new loans, are not only important stand-alone
modifications to the standard macroeconomic setup, but can also enter into powerful interactions. In particular, the distinction between fixed- and adjustable-rate loans does not make sense if contracts are single-period. Moreover, as explained below, the lower bound on new loans binds easier for longer loan maturities. In this section we take a closer look at these two particular interactions by demonstrating how the strength and asymmetries in the monetary policy transmission depend on debt duration that in our framework is represented by parameter $m$.\(^5\)

To this end, in Figure 6 we plot the peak and trough responses of output and inflation to negative and positive monetary policy shocks as a function of loan maturity. We consider shocks of one and two standard deviations. While calculating the responses, we assume that the economy is initially in the steady state equilibrium. Naturally, given the time dependence arising from the nonlinearities included in our model, the outcomes would be different if we considered alternative initial conditions. However, the steady state is a natural benchmark and sufficient to demonstrate our main point, which is the interaction of the strength of the responses with loan maturity.

The following observations can be made. First, the effect of monetary policy clearly decreases with loan maturity. This happens even if we ignore the occasionally binding nature of the constraints that result in discontinuities observed in the figure, and which we discuss later. The reason is that, under fixed-rate multi-period contracts, a change in the policy rate affects only the cost of newly granted loans. Therefore, the longer the loan duration, the lower the proportion of total debt to which new financial terms apply, and the less sensitive are borrowers to monetary easing or tightening.\(^6\)

The second observation that one can draw from Figure 6 is the presence of discontinuities for contractionary shocks. They are associated with the lower bound on new loans and, if loan duration or the size of the shocks is sufficiently large, result in asym-

\(^5\)In the discussion presented in this section, we abstract from the asymmetries associated with possible slackness in the collateral constraint as these have already been discussed, e.g., by Guerrieri and Iacoviello (2017), and they do not enter into interesting interactions with loan maturity.

\(^6\)Note that this experiment can also be interpreted as documenting the lower effectiveness of monetary policy under fixed- versus variable-rate loans, as the latter are equivalent to single-period loans if the lower bound does not bind.
metries in responses to positive and negative shocks. To see why loan duration matters here, note that the larger it is, the smaller the steady state share of new loans in total debt, see equation (3). Note that we calibrate the lower bound on new borrowing as half of its steady state value. As a result, the absolute magnitude by which new loans can fall before hitting the lower bound also decreases in $m$. Consequently, for a given adjustment in total debt implied by the collateral constraint, the lower bound is more likely to be hit if the contract maturity is higher. Since our simulations start from the steady state, in which the collateral constraint is binding and the lower bound is not, the latter nonlinearity may be activated only in experiments featuring a decrease in debt. Hence, the effects of a monetary tightening on total lending might be smaller compared to a monetary easing of the same scale. As our simulations show, this asymmetry is much more relevant for output than for inflation.

All this of discussion clearly indicates that the average mortgage loan maturity is an important parameter determining the strength and asymmetries in the monetary policy transmission mechanism.

6 Conclusions

In this paper we modify an otherwise standard DSGE model with housing and financial intermediaries in order to take into account some typical characteristics of residential mortgage markets – those that empirical studies have found to be relevant in many dimensions and that are largely ignored in the theoretical literature. The aim of considering these modifications is to evaluate to what extent they affect the transmission mechanism of monetary policy. The main nonstandard components we focus on are the introduction of multi-period loan contracts, as well as two features that make our model more realistic: a lower bound constraint on new loans and the possibility that the collateral constraint might become slack.

We first estimate the model using nonlinear Bayesian estimation techniques and demonstrate that all of these modifications are crucial in making our framework consis-
tent with housing market developments during the recent boom-bust cycle in the US. In particular, we document that the nonlinear setting is much better supported by the data. We also show that our estimated model with multiperiod loans and occasionally binding constraints does (in contrast to the standard linear model) a very good job in matching two key variables that we chose to be unobserved in the estimation process: the stock of debt and new mortgage originations.

As regards monetary policy transmission, we show that our modifications generate significant asymmetries and state-dependence of macroeconomic reactions. For instance, the transmission weakens in average debt maturity and can be up to 40% stronger when the collateral constraint is binding than when it is slack.

We believe that our results can be helpful in understanding the implications of the observed cross-country heterogeneity in the mortgage market design for the monetary policy transmission mechanism. Moreover, it is important to stress that the highlighted time dependence and possible asymmetries are not restricted to the effects of monetary shocks. In particular, the nonlinearities associated with the mortgage market may also limit the effectiveness of macroprudential policy. We leave this issue for possible future research.
References


## Tables and figures

### Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_P$</td>
<td>0.993</td>
<td>Discount factor, patient HHs</td>
</tr>
<tr>
<td>$\beta_I$</td>
<td>0.983</td>
<td>Discount factor, impatient HHs</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.009</td>
<td>Housing stock depreciation rate</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>0.02</td>
<td>Capital stock depreciation rate</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.61</td>
<td>Share of impatient HHs in population</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
<td>Share of impatient HHs in production</td>
</tr>
<tr>
<td>$A$</td>
<td>0.138</td>
<td>Steady state weight on housing in utility</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>1</td>
<td>Inverse of Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\mu_w$</td>
<td>1.2</td>
<td>Steady state wage markup</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>0.75</td>
<td>Calvo probability for wages</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.2</td>
<td>Steady state product markup</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>Calvo probability for prices</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.3</td>
<td>Output elasticity with respect to physical capital</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>5</td>
<td>Capital investment adjustment cost</td>
</tr>
<tr>
<td>$g/y$</td>
<td>0.165</td>
<td>Steady state share of government spending in output</td>
</tr>
<tr>
<td>$\tau$</td>
<td>-0.165</td>
<td>Share of taxes levied on impatient HHs</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>0.83</td>
<td>LTV ratio</td>
</tr>
<tr>
<td>$\pi$</td>
<td>1.005</td>
<td>Steady state inflation</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>0.9</td>
<td>Interest rate smoothing in monetary policy rule</td>
</tr>
<tr>
<td>$\gamma_\pi$</td>
<td>1.5</td>
<td>Response to inflation in monetary policy rule</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>0.5</td>
<td>Response to output in monetary policy rule</td>
</tr>
</tbody>
</table>
### Table 2: Estimated parameters

<table>
<thead>
<tr>
<th>Shock</th>
<th>Prior Mean</th>
<th>Prior St. dev.</th>
<th>Posterior (linear) Mean</th>
<th>Posterior (linear) St. dev.</th>
<th>Posterior (OccBin) Mean</th>
<th>Posterior (OccBin) St. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>AR coefficients</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing preference</td>
<td>0.75</td>
<td>0.1</td>
<td>0.9842</td>
<td>0.0040</td>
<td>0.9813</td>
<td>0.0009</td>
</tr>
<tr>
<td>Time preference</td>
<td>0.75</td>
<td>0.1</td>
<td>0.8774</td>
<td>0.0090</td>
<td>0.8840</td>
<td>0.0019</td>
</tr>
<tr>
<td>Technology</td>
<td>0.75</td>
<td>0.1</td>
<td>0.7303</td>
<td>0.0227</td>
<td>0.7242</td>
<td>0.0092</td>
</tr>
<tr>
<td><strong>Standard deviations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing preference</td>
<td>0.1</td>
<td>5</td>
<td>0.0447</td>
<td>0.0061</td>
<td>0.0419</td>
<td>0.0010</td>
</tr>
<tr>
<td>Monetary</td>
<td>0.01</td>
<td>0.5</td>
<td>0.0023</td>
<td>0.0002</td>
<td>0.0037</td>
<td>0.0004</td>
</tr>
<tr>
<td>Time preference</td>
<td>0.1</td>
<td>1</td>
<td>0.0475</td>
<td>0.0033</td>
<td>0.0491</td>
<td>0.0005</td>
</tr>
<tr>
<td>Technology</td>
<td>0.1</td>
<td>1</td>
<td>0.0124</td>
<td>0.0005</td>
<td>0.0129</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Note: The posterior statistics are based on 10,000 draws from the random walk Metropolis-Hastings algorithm. Starting values were chosen based on the estimated parameters of the model with a permanently binding borrowing constraint. The first 30% of draws are discarded as burn-in.
Figure 1: New mortgage originations

Note: This chart plots mortgage originations (new loans) in the US (source: Federal Housing Finance Agency data on mortgage originations for 1-4 family homes), deflated by CPI, divided by an exponential trend in real GDP over the period 1975-2016, and normalized so that period average = 1. The dotted line shows our choice of the lower bound on new loans.
Figure 2: Credit standards

Note: The two series from the Senior Loan Officer Survey (SLOS) show the net percentage of banks tightening standards for mortgage loans (total until 1Q2007 and prime since 2Q2007). Data are available only until 4Q2014. The third series is the Chicago Fed’s National Financial Conditions Credit Subindex.
Figure 3: Lagrange multipliers on borrowing and lower bound constraint

Note: The collateral constraint is nonbinding when its Lagrange multiplier is zero, and the lower bound is binding when its Lagrange multiplier is nonzero. By construction, the collateral constraint does not apply during the periods of a binding lower bound on new loans, and so its multiplier is not plotted.
Figure 4: Simulated and historical mortgage debt (stock and flow)

Note: The upper panel plots the evolution of the stock of real mortgage debt in the US (source: US Flow of funds (Z1); home mortgages to households and nonprofit organizations). The lower panel plots the evolution of the real new mortgage originations in the US (source: Mortgage Bankers Association). Both series are presented as demeaned deviations from a linear trend in real GDP, estimated over the period 1975-2016. Additionally, we plot the paths for these variables implied by different variants of our model after feeding it with the filtered errors obtained from the nonlinear estimation.
Figure 5: Reaction to a contractionary and expansionary monetary policy shock

Note: The figure plots the responses to a typical contractionary (left column) and expansionary (right column, reversed y axis) monetary policy shock at four dates in our sample. All responses are in percent deviations from the baseline; inflation is in annualized percentage points.
Figure 6: Loan maturity and asymmetric transmission of monetary policy shocks

Note: The figure plots the peak and trough responses of output and inflation to, respectively, expansionary and contractionary monetary shocks of different sizes (one standard deviation - solid lines, two standard deviations - dashed lines) as a function of average debt duration (m in the model). The simulations start at, and are presented in percent deviations from, the nonstochastic steady state. The responses for inflation are annualized percentage points.
Appendix

A  List of model equations

The following equations describe the equilibrium in our model. Small letters for variables defined in the main text with capital letters are used to indicate real quantities or prices, defined by dividing the nominal values by the aggregate price level $P_t$. The Lagrange multipliers associated with the collateral constraint and lower bound on new loans are denoted by $\lambda_{2,t}$ and $\lambda_{3,t}$, respectively.

Marginal utility of consumption (for $i = \{I, P\}$)

$$u_i' t = \frac{\varepsilon_{u,t}}{c_{i,t}} \quad \text{(A.1)}$$

Impatient households’ budget constraint

$$c_{I,t} + p_{h,t}(h_{I,t} - (1 - \delta_h)h_{I,t-1}) + \frac{\phi_{t-1}}{\pi_t} + \frac{s_{t-1}}{\pi_t} = w_t n_{I,t} + s_t - \frac{\tau}{\omega} g y_t \quad \text{(A.2)}$$

Debt accumulation

$$s_t = l_t + (1 - \frac{1}{m})\frac{s_{t-1}}{\pi_t} \quad \text{(A.3)}$$

Promised interest payments

$$\phi_t = (R_{h,t} - 1)l_t + (1 - \frac{1}{m})\frac{\phi_{t-1}}{\pi_t} \quad \text{(A.4)}$$

Demand for loans by impatient households

$$u'_{I,t} = \beta_I E_t \left\{ \frac{u'_{I,t+1}}{\pi_{t+1}} + \xi_{I,t} \right\} + \xi_{I,t} R_t + \psi_{I,t}(R_{h,t} - 1) - \beta_I (1 - \frac{1}{m}) E_t \left\{ \frac{\psi_{I,t+1}}{\pi_{t+1}} (R_{h,t+1} - 1) \right\} + \lambda_{3,t} \quad \text{(A.5)}$$

$$\psi_{I,t} = \beta_I E_t \left\{ \frac{u'_{I,t+1}}{\pi_{t+1}} \right\} + \beta_I (1 - \frac{1}{m}) E_t \left\{ \frac{\psi_{I,t+1}}{\pi_{t+1}} \right\} \quad \text{(A.6)}$$
Lower bound constraint on new loans

\[ l_t \geq \bar{l} \quad (A.7) \]

Collateral constraint

\[ R_t s_t \leq \vartheta (1 - \delta_h) \mathbb{E}_t \{ p_{h,t+1} h_{I,t} \pi_{t+1} \} \quad \text{if} \quad \lambda_{3,t} = 0 \quad (A.8) \]

\[ \lambda_{2,t} = 0 \quad \text{if} \quad \lambda_{3,t} > 0 \quad (A.9) \]

Supply of deposits by patient households

\[ u'_{P,t} = \beta_P \mathbb{E}_t \left\{ R_t \frac{u'_{P,t+1}}{\pi_{t+1}} \right\} \quad (A.10) \]

Housing Euler equations

\[ \frac{A^e_{h,t}}{h_{I,t}} + \beta_I (1 - \delta_h) \mathbb{E}_t \left\{ u'_{I,t+1} p_{h,t+1} \right\} + \vartheta (1 - \delta_h) \lambda_{2,t} \mathbb{E}_t \{ p_{h,t+1} \pi_{t+1} \} = u'_{I,t} p_{h,t} \quad (A.11) \]

\[ \frac{A^e_{h,t}}{h_{P,t}} + \beta_P (1 - \delta_h) \mathbb{E}_t \left\{ u'_{P,t+1} p_{h,t+1} \right\} = u'_{P,t} p_{h,t} \quad (A.12) \]

Banks’ optimality conditions

\[ u'_{P,t} = \beta_P \mathbb{E}_t \left\{ \frac{u'_{P,t+1}}{\pi_{t+1}} \right\} + \psi_{P,t} (R_{h,t} - 1) - \beta_P (1 - \frac{1}{m}) \mathbb{E}_t \left\{ \frac{\psi_{P,t+1}}{\pi_{t+1}} (R_{h,t+1} - 1) \right\} \quad (A.13) \]

\[ \psi_{P,t} = \beta_P \mathbb{E}_t \left\{ \frac{u'_{P,t+1}}{\pi_{t+1}} \right\} + \beta_P (1 - \frac{1}{m}) \mathbb{E}_t \left\{ \frac{\psi_{P,t+1}}{\pi_{t+1}} \right\} \quad (A.14) \]

Capital accumulation

\[ k_t = (1 - \delta_k) k_{t-1} + \left( 1 - \Gamma \left( \frac{i_{k,t}}{i_{k,t-1}} \right) \right) i_{k,t} \quad (A.15) \]

Demand for capital

\[ u'_{P,t} p_{k,t} = \beta_P \mathbb{E}_t \left\{ u'_{P,t+1} \left[ (1 - \delta_k) p_{k,t+1} + r_{k,t+1} \right] \right\} \quad (A.16) \]
Investment demand

\[ 1 = p_{k,t} \left[ 1 - \frac{\dot{i}_{k,t}}{i_{k,t-1}} \right] - \Gamma \left( \frac{\dot{i}_{k,t}}{i_{k,t-1}} \right) \dot{i}_{k,t} + \beta_p \mathbb{E}_t \left\{ \frac{u'_{P,t+1}}{u'_{P,t}} p_{k,t+1} + \Gamma' \left( \frac{i_{k,t+1}}{i_{k,t}} \right) \frac{i_{k,t}^2}{i_{k,t}} \right\} \] (A.17)

Wages by household type (for \( i = \{I, P\} \))

\[ w_{i,t} = \left[ \theta_w \left( \frac{w_{i,t} - \pi}{\pi_t} \right)^{1 - \mu_w} + (1 - \theta_w) \ddot{w}_{i,t} \right]^{1 - \mu_w} \] (A.18)

Optimal reset wage (for \( i = \{I, P\} \))

\[ \frac{w_{i,t}^{1 + \sigma_n \mu_w - 1}}{w_{i,t}^{\mu_w - 1}} = \frac{\Omega_{i,t}^w}{\Upsilon_{i,t}^w} \] (A.19)

\[ \Omega_{i,t}^w = \mu_w w_{i,t}^{\mu_w - 1} (1 + \sigma_n) \nu_{i,t}^{1 + \sigma_n} + \beta_i \theta_{w,\mathbb{E}_t} \left\{ \left( \frac{\pi}{\pi_{t+1}} \right)^{1 - \mu_w} \Omega_{i,t+1}^w \right\} \] (A.20)

\[ \Upsilon_{i,t}^w = u'_{i,t} w_{i,t}^{\mu_w - 1} n_{i,t} + \beta_i \theta_{w,\mathbb{E}_t} \left\{ \left( \frac{\pi}{\pi_{t+1}} \right)^{1 - \mu_w} \Upsilon_{i,t+1}^w \right\} \] (A.21)

Average wage

\[ w_t = \left( \frac{w_{I,t}}{\gamma} \right)^{\gamma} \left( \frac{w_{P,t}}{1 - \gamma} \right)^{1 - \gamma} \] (A.22)

Labor demand

\[ n_{I,t} = \gamma \frac{w_t}{\omega} n_t \quad n_{P,t} = \frac{1 - \gamma}{1 - \omega} \frac{w_t}{w_{P,t}} n_t \] (A.23)

Aggregate inflation

\[ 1 = \theta \left( \frac{\pi}{\pi_t} \right)^{1 - \mu} + (1 - \theta) \tilde{p}_t \frac{1}{1 - \mu} \] (A.24)

Optimal reset price

\[ \tilde{p}_t = \mu \frac{\Omega_t}{\gamma_t} \] (A.25)

\[ \Omega_t = u'_{P,t} m_c y_t + \beta_p \theta_{\mathbb{E}_t} \left\{ \left( \frac{\pi}{\pi_{t+1}} \right)^{1 - \mu} \Omega_{t+1} \right\} \] (A.26)
\( \Upsilon_t = u_{P,t}^\prime y_t + \beta P \theta E_t \left\{ \left( \frac{\pi}{\pi_{t+1}} \right)^{\frac{1}{1-\mu}} \Upsilon_{t+1} \right\} \)  
(A.27)

Marginal cost

\[ mc_t = \frac{1}{\varepsilon_{z,t}} \left( \frac{r_{k,t}}{\alpha} \right)^{\alpha} \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha} \]  
(A.28)

Optimal factor proportions

\[ \frac{r_{k,t}}{w_t} = \frac{\alpha}{1-\alpha} \frac{n_t}{k_{t-1}} \]  
(A.29)

Monetary policy rule

\[ \frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\gamma_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\gamma_\pi} \left( \frac{y_t}{y} \right)^{\gamma_y} \right]^{1-\gamma_R} \epsilon_{R,t} \]  
(A.30)

Housing market clearing

\[ h = \omega h_{t,t} + (1-\omega)h_{P,t} \]  
(A.31)

Aggregate resource constraint

\[ y_t = \omega c_{I,t} + (1-\omega)c_{P,t} + i_{k,t} + \delta_h h + g \]  
(A.32)

Aggregate production function

\[ y_t \Delta_t = \varepsilon_{z,t} k_{t-1}^\alpha n_t^{1-\alpha} \]  
(A.33)

Price dispersion

\[ \Delta_t = (1-\theta)\bar{p}_t^{\frac{\mu}{1-\mu}} + \theta \left( \frac{\pi}{\pi_t} \right)^{\frac{\mu}{1-\mu}} \Delta_{t-1} \]  
(A.34)