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**The Informational Effect of
Monetary Policy and
the Case for Policy Commitment**

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**The Informational Effect of Monetary Policy
and the Case for Policy Commitment**

Chengcheng Jia

I explore how asymmetric information between the central bank and the private sector changes the optimal conduct of monetary policy. I build a New Keynesian model in which private agents have imperfect information about underlying shocks, while the central bank has perfect information. In this environment, private agents extract information about the underlying shocks from the central bank's interest-rate decisions. This informational effect weakens the direct effect of monetary policy: When the central bank adjusts the interest rate to offset the effects of underlying shocks, the interest rate also reveals information about the realization of underlying shocks. Because private agents have more precise information about the shocks and consequently react more aggressively to it, the economy becomes harder to stabilize with monetary policy. I show that committing to the optimal state-contingent policy rule alleviates this problem by controlling the information revealed through the interest rate.

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"Perhaps most importantly, we need to know more about the manner in which inflation expectations are formed and how monetary policy influences them. Ultimately, both actual and expected inflation are tied to the central bank's inflation target, whether that target is explicit or implicit. But how does this anchoring process occur?"

Yellen (2016)

1 Introduction

There is no doubt that central banks understand the importance of expectations and aim to anchor inflation expectations. However, an open question is how expectations are formed, and more importantly, what the role of monetary policy is in shaping expectations.

There has been much progress in the economics literature in modeling expectations that are different from the full-information rational expectations and in studying the implications for the optimal conduct of monetary policy. The majority of papers in this literature assume that private agents observe noisy signals that are exogenously distributed around the true state.¹ This assumption, although useful in many settings, excludes the possibility that private agents may learn the state of the economy from decisions made by other players, and one important example of "other players" is the central bank.

This paper studies how monetary policy should be conducted when expectations about the state of the economy are endogenous to monetary policy decisions. I assume that information about the true state of the economy is asymmetric between the central bank and the private sector. The private sector has partial information about the realization of underlying shocks, whereas the central bank has perfect information. In this environment, monetary policy has dual effects: The first is the traditionally studied direct effect on the borrowing cost of households, and the second is the informational effect on expectations in the private sector. Suppose that the economy is hit by a positive cost-push shock and it is partially observed in the private sector. The central bank now faces a trade-off when making interest rate decisions. If the central bank increases the interest rate, the direct effect of the tightening monetary policy is to decrease inflation by lowering demand, which offsets the effect of the positive cost-push shock on inflation. At the same time, however, firms also learn more about the shock from this interest rate response. This informational effect induces firms to increase prices, partially offsetting the direct effect of the tightening monetary policy.

To model asymmetric information between the central bank and the private sector, I introduce informational frictions to an otherwise canonical New Keynesian model with Calvo price rigidity.

¹Another way to model imperfect information is to assume lagged information, rather than partial information. For a comprehensive review of papers on imperfect information, see [Mankiw and Reis \(2010\)](#).

There are two types of shocks in the private sector: technology shocks and wage markup shocks. Their aggregate components map to natural-rate shocks in the output gap and cost-push shocks in inflation. Private agents, both the household and all firms, know the distributions of the shocks, but have partial information on the realization of the shocks. Under rational expectations, the private agents understand the way the interest rate reacts to the two shocks in equilibrium, and therefore, they extract information about the realization of the two shocks from interest rate responses.

I start the analysis from the baseline situation, in which case shocks do not have serial correlation and the current interest rate is the central bank's only policy instrument. I start by solving the equilibrium interest rate for a discretionary central bank that optimizes after the realization of shocks and takes as given how private agents form expectations from its interest rate decisions. In equilibrium, the interest rate is one signal that jointly provides information about two shocks. When the private agents form expectations about one shock, the prior distribution of the other shock becomes the source of noise in the signal.

The information on the realization of natural-rate shocks is beneficial, whereas the information on the realization of cost-push shocks is detrimental. This is because after a natural-rate shock, the central bank is able to completely stabilize inflation under perfect information. Therefore, when private agents regard a positive innovation in the interest rate as a response to a natural-rate shock, they expect inflation to be zero. The direct effect and the informational effect of monetary policy are aligned after natural-rate shocks. In contrast, the central bank only partially offsets the effect of a cost-push shock under perfect information, because cost-push shocks induce a trade-off between inflation and the output gap. Therefore, when private agents regard a positive innovation in the interest rate as a response to a cost-push shock, they expect inflation to be positive. The informational effect of tightening monetary policy is to increase inflation through expected inflation, partially offsetting the direct effect.

I then study the optimal state-contingent policy rule and compare it with the equilibrium interest rate under discretion. To isolate the gains from the informational effect, I focus on a simple rule in which the interest rate only responds to current shocks. This removes the traditionally studied gains from commitment to a delayed response, which changes the expectations on the future equilibrium. By committing to a policy rule, the central bank effectively chooses a direct mapping from the actual shocks to the expected shocks. I show that the optimal policy rule reduces the sensitivity of expected shocks to the interest rate, which consequently alleviates the degree to which the informational effect dampens the direct effect of the interest rate.

The informational gains from commitment lead to a novel time-inconsistency problem. Different from the traditional time-inconsistency problem, in which the incentive to deviate applies across time periods,² the time-inconsistency problem in my model applies across states. Once the

²Many papers have shown the time-inconsistency problem when the central bank optimally commits to a future

central bank has committed to a policy rule, it has fixed the informational effect of the interest rate. Ex-post, the central bank has an incentive to deviate from its committed rule, assuming that such a change will not change how expectations are formed in the private sector.

In practice, central banks commonly treat communication about the state of the economy as an important tool. The baseline model captures the situation in which information is conveyed through policy decisions, but what if the central bank is able to directly tell the private sector the information about the shocks? To address this question, I model central bank direct communication by adding external signals independent of the interest rate. This raises the question of whether there is an interaction between the "words" (direct communication) and the "actions" (monetary policy decisions). I find that increasing the precision of the communication of one shock makes the interest rate a more precise signal about the other shock. Consequently, this interaction effect makes the welfare implications of communication different from the conventional wisdom: providing more precise information about the efficient shock (natural-rate shock) through central bank communication may reduce welfare, as the private sector also gets more precise information about the inefficient shock (cost-push shock) from the interest rate at the same time.

I extend the analysis to serially correlated shocks to study the dynamic informational effect. In this case, the private agents form beliefs about current shocks by optimally weighing current signals and past beliefs. Therefore, the current interest rate has a lagged effect on future equilibrium through the belief-updating process. I show that the dynamic informational effect makes the equilibrium interest rate have an additional target, which is anchoring expectations. In a numerical example, I show that the size of the gains from commitment crucially depends on the precision of external signals and the serial correlation in shocks.

Relationship to prior work

My analysis connects the growing literature on optimal monetary policy under imperfect information. Papers in this field typically assume that expectations about the state of the economy are exogenous to monetary policy decisions. One of the few exceptions is [Tang \(2015\)](#), which is also the paper most closely related to this one. [Tang \(2015\)](#) characterizes discretionary monetary policy under the assumption that monetary policy has an informational effect. The most important contribution of this paper is to connect the informational effect of monetary policy to the debate over discretion versus commitment.

The field of optimal monetary policy under imperfect information is revived by [Woodford \(2001\)](#), who shows how imperfect information about monetary policy leads to persistent real ef-

path of interest rate responses. Examples include when the central bank faces stabilization bias ([Clarida, Gali, and Gertler \(2000\)](#), [Woodford \(2011\)](#)), and when the current interest rate is bound by the zero lower bound ([Eggertsson and Woodford \(2003\)](#)).

fects through higher-order beliefs. Following [Woodford \(2001\)](#), the majority of papers that study optimal monetary policy under informational frictions assume that beliefs in the private sector are formed independently from monetary policy decisions. [Ball, Mankiw, and Reis \(2005\)](#) assume that information is rigid in the private sector and characterize optimal policy as an elastic price standard. [Adam \(2007\)](#) adds rational inattention and demonstrates that the target of the optimal monetary policy changes from output gap stabilization to price stabilization when private agents choose more precise signals. [Angeletos and La'O \(2011\)](#) show that the flexible-price equilibrium is no longer the first-best when information frictions affect real variables.

The idea that optimal commitment achieves ex-ante welfare improvement has a long tradition dating back to [Kydland and Prescott \(1977\)](#) and [Barro and Gordon \(1983\)](#), although not in the content of informational frictions. The gains from commitment in these papers come from the assumption that the private sector is uncertain about the policy response. In my paper, I assume that the private sector has perfect information about the response of monetary policy in both cases, but has imperfect information on the underlying shocks to which the monetary policy is a response.

There are papers that discuss the gains from policy commitment under imperfect information. [Svensson and Woodford \(2003 2004\)](#) assume that the central bank has imperfect information and show that the optimal policy under commitment displays considerable inertia relative to the discretionary policy, due to the persistence in the learning process. [Lorenzoni \(2009\)](#) and [Paciello and Wiederholt \(2013\)](#) explore the idea that the central bank is able to change the learning process in the private sector if it is able to commit to completely offsetting inefficient shocks. However, the gains from commitment studied in these papers come from the direct effect of monetary policy. In contrast, the emphasis in this paper is the gains from commitment through the informational effect.

The informational effect of monetary policy has increasing support from recent empirical studies, which is also accompanied by the increasing degree of central bank transparency in the U.S. In 1994, the FOMC began to announce its target policy rate. This change in policy is shown to improve private forecasts of interest rates ([Swanson \(2006\)](#)) and to impact private forecasts of economic fundamentals as well. [Romer and Romer \(2000, 2004\)](#) are the first contributions to provide empirical evidence on information asymmetry between the Federal Reserve and the private sector. They show that inflation forecasts by private agents respond to changes in the policy rate after FOMC announcements. [Faust, Swanson, and Wright \(2004\)](#) further confirm that the private sector revises its forecasts in response to monetary policy surprises. In more recent papers, [Campbell et al. \(2012\)](#) show that unemployment forecasts decrease and CPI inflation forecasts increase after a positive innovation to expected future federal funds rates. [Nakamura and Steinsson \(2018\)](#) also show the informational effect of the federal funds rate using high-frequency data. In addition, [Melosi \(2016\)](#) captures this empirical pattern using a DSGE model with dispersed information.

Motivated by these empirical findings, recent papers have begun to study the optimal conduct

of monetary policy when monetary policy provides information about the state of the economy. [Baeriswyl and Cornand \(2010\)](#) note that because monetary policy cannot fully neutralize markup shocks, the central bank alters its policy response to reduce the information revealed about the cost-push shock through monetary policy. [Berkelmans \(2011\)](#) demonstrates that with multiple shocks, tightening policy may initially increase inflation. The paper most closely related to the present work is [Tang \(2015\)](#), who characterizes the optimal discretionary policy when monetary policy has an informational effect. However, to my knowledge, all existing papers that capture the informational effect of monetary policy have only studied the situation in which the central bank optimizes under discretion. I contribute to this literature by showing the gains from commitment that come through the informational channel.

2 Private Sector

In this section, I characterize the equilibrium of the private sector in a standard New Keynesian economy with sticky prices in the style of [Calvo \(1983\)](#) and informational frictions. Fluctuations are driven by two types of shocks: technology shocks and wage markup shocks. I assume that the central bank has perfect information about the two shocks, whereas private agents cannot directly observe the shocks. Private agents have rational expectations and update expectations about the shocks when they observe changes in the interest rate.

2.1 Informational Frictions

I model an "islands economy," following lines similar to [Phelps \(1970\)](#), [Lucas \(1972\)](#), [Woodford \(2001\)](#), and [Angeletos and La'O \(2010\)](#). There is a continuum of islands, indexed by j , and information boundaries are the result of the geographic isolation of islands. There is a representative household, consisting of a consumer and a continuum of workers. At the beginning of each period, each household sends one worker to each island, j . There is a continuum of monopolistic firms, each located on one island and indexed by the island. Each firm demands labor in the local labor market within the island and produces a differentiated intermediate good, j . Information is symmetric within an island, meaning each firm is able to observe its firm-specific shocks. Information is asymmetric across islands, meaning firms are unable to observe shocks or decisions made by other firms.

2.2 Private Sector Optimization Problem

2.2.1 Household

The preferences of the representative household are defined over the aggregate consumption good, C_t , and the labor supplied to each firm, $N_t(j)$, as

$$E_t^H \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t) - \int V(N_t(j)) dj \right\}, \quad (1)$$

where E_t^H denotes the household's subjective expectations conditional on its information set, ω_H . The aggregate good C_t consists a continuum of intermediate goods:

$$C_t = \left(\int_0^1 C_t(j)^{1-\frac{1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (2)$$

where $C_t(j)$ is the consumption of intermediate good j in period t .

The economy is cashless. The household maximizes expected utility subject to the inter-temporal budget constraint:

$$\int_0^1 P_t(j) C_t(j) dj + B_{t+1} \leq \int_0^1 W_t(j) N_t(j) dj + (1 + i_t) B_t + \Pi_t, \quad (3)$$

where B_t is a risk-free bond with nominal interest i_t , which is determined by the central bank. Π_t is the lump-sum component of household income, which includes dividends from ownership of all firms. $W_t(j)$ and $N_t(j)$ are the labor wage and labor supply for firm j , respectively.

The household's optimization problem can be solved in two stages. First, conditional on the level of aggregate consumption, the household allocates intermediate goods consumption to minimize the cost of expenditure conditional on the level of aggregate good consumption. The allocation of intermediate good consumption that minimizes expenditure yields

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} C_t \quad (4)$$

for all $j \in [0, 1]$, and where $P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$.

In the second stage, conditional on the optimal allocation among intermediate products, the household chooses its aggregate consumption, C_t , labor supply to all firms, $N_t(j) \forall j$, and savings in the risk-free bond, B_{t+1} . I assume that the utility of aggregate good consumption and the utility of labor supply take the following forms: $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$, and $V(N_{jt}) = \frac{N_{jt}^{1+\varphi}}{1+\varphi}$, where σ is the inverse of the inter-temporal elasticity of substitution and the parameter φ is the inverse of the Frisch

elasticity of the labor supply.

The inter-temporal consumption decision leads to the following Euler equation:

$$C_t^{-\sigma} = \beta(1 + i_t)E_t^H \left(C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \right). \quad (5)$$

Equation (5) shows that the consumption decision is forward-looking. Specifically, current demand depends of expectations on future real consumption and future changes in the aggregate price level.

The intra-temporal labor supply decision sets the marginal rate of substitution between leisure and consumption equal to the real wage:

$$\frac{N_t^\varphi(j)}{C_t^{-\sigma}} = \frac{W_t}{P_t}. \quad (6)$$

2.2.2 Firms

Firms are intermediate good producers and subject to both price rigidity and informational frictions. As the assumption made in [Calvo \(1983\)](#), in each period, a measure $1 - \theta$ of firms win the Calvo lottery to reset their prices. Other firms charge their previous prices. A firm j that resets its price in period t chooses $P_t^*(j)$ to maximize its own expectation of the sum of all discounted profits while $P_t^*(j)$ remains effective. The profit-optimization problem can be written as follows:

$$\max_{P_t^*(j)} \sum_{k=0}^{\infty} \theta^k E_t^j \left\{ Q_{t,t+k} \left[P_t^*(j) Y_{t+k}(j) - U_{t+k}^w(j) W_{t+k}(j) N_t(j) \right] \right\}, \quad (7)$$

where E_t^j denotes firm j 's expectation conditional on its information set, ω_j . $Q_{t,t+k}$ is the stochastic discount factor given by: $Q_{t,t+k} = \beta^k \frac{U'(C_{t+k})}{U'(C_t)} \frac{P_t}{P_{t+k}}$. $U_{t+k}^w(j)$ denotes the wage markup for firm j .

Following the tradition of the New Keynesian literature, I assume that labor is the only input and each firm produces according to a constant return to scale technology,

$$Y_t(j) = A_t(j)L_t(j), \quad (8)$$

where $A_t(j)$ denotes the technology of firm j .

There are two sources of uncertainty that affect the pricing decisions of each firm: technology shocks and wage markup shocks. I assume that both shocks have an aggregate component and an idiosyncratic component. The idiosyncratic components are drawn independently in every period and log-normally distribute around their aggregate components. Denote $a_t(j) = \log(A_t(j))$ and

$u_t^w = \log(U_t^w(j))$, and it then follows,

$$\begin{aligned} a_t(j) &= a_t + s_t^a(j), & s_t^a(j) &\sim N(0, \sigma_{sa}^2) \\ u_t^w(j) &= u_t^w + s_t^u(j), & s_t^u(j) &\sim N(0, \sigma_{su}^2) \end{aligned}$$

I assume that the aggregate components of both shocks follow AR(1) processes:

$$\begin{aligned} a_t &= \phi^a a_{t-1} + v_t^a, & v_t^a &\sim N(0, \sigma_{va}^2) \\ u_t^w &= \phi^u u_{t-1}^w + v_t^{uw}, & v_t^{uw} &\sim N(0, \sigma_{vuw}^2) \end{aligned}$$

The first-order condition for labor input implies that the nominal marginal cost of production is $U_t(j)W_t(j)/A_t(j)$. Substituting the marginal cost of production into the optimal pricing decision results in

$$P_t^*(j) = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t^j \Sigma(\beta\theta)^k u'(C_{t+k}) P_{t+k}^\varepsilon Y_{t+k} u_{t+k}(j) w_{t+k}(j) A_{t+k}^{-1}(j)}{E_t^j \Sigma(\beta\theta)^k u'(C_{t+k}) P_{t+k}^{\varepsilon-1} Y_{t+k}}. \quad (9)$$

Equation (9) implies that pricing decisions are forward-looking and strategic complements. Specifically, the optimal resetting price of firm j increases with the expectation of a higher firm-specific marginal cost of production and a higher aggregate price level in both the current and all future periods.

2.3 Aggregation and Equilibrium in the Private Sector

Equilibrium variables in the private sector are solved in log deviations from their steady-state values (i.e., $x_t \equiv \ln(X_t/X)$), and denoted by lower-case letters. (See Appendix A for details.)

The Output Gap

Following the New Keynesian tradition, I express output in terms of the output gap, \hat{y}_t , which is defined as the difference between y_t and the natural level of output, y_t^n . The natural level of output is defined as the output level under flexible prices and perfect information. In this situation, y_t^n becomes a linear function of a_t $y_t^n = \frac{\varphi + \sigma}{1 + \varphi} a_t$, and follows an AR(1) process, $y_t^n = \phi y_{t-1}^n + v_t$, where $\phi = \phi^a$, and $\sigma_v = \frac{\varphi + \sigma}{1 + \varphi} \sigma_{va}$.

The output gap is derived as follows:

$$\hat{y}_t \equiv y_t - y_t^n = E_t^H \hat{y}_{t+1} - \frac{1}{\sigma} \left[i_t - \left(\frac{1}{1 - \phi} r_t^n - \frac{\phi}{1 - \phi} E_t^H r_t^n \right) - E_t^H \pi_{t+1} \right], \quad (10)$$

where $E_t^H \hat{y}_{t+1} = E_t^H y_{t+1} - E_t^H y_{t+1}^n = E_t^H y_{t+1} - \phi E_t^H y_{t+1}^n$. r_t^n denotes the natural rate of interest, which measures the expected growth rate of y_t^n , i.e., $r_t^n = \sigma(E_t y_{t+1} - y_t^n) = \sigma(\phi - 1)y_t^n$.³

³The natural-rate shock is mapped from the aggregate component in firm technology shocks in the present model,

Note that equation (10) nests the situation of perfect information, in which case $E_t^H r_t^n = r_t^n$, $E_t^H \hat{y}_{t+1} = E_t \hat{y}_{t+1}$ and $E_t^H \pi_{t+1} = E_t \pi_{t+1}$. The output gap under perfect information is given by,

$$\hat{y}_t = E_t \hat{y}_t - \frac{1}{\sigma} [i_t - r_t^n - E_t \pi_{t+1}] \quad (11)$$

The comparison between equation (10) and equation (11) shows how informational frictions affect the output gap: Suppose there is a positive innovation in r_t^n , meaning the aggregate technology is lower today and higher tomorrow. The equilibrium price should increase and consumption should decrease in the absence of any frictions. Due to price rigidity, the current demand does not decrease sufficiently, which results in a positive output gap. If, in addition to the price rigidity, the household has no information about the change in aggregate technology, it does not adjust demand at all. Consequently, the increase in the output gap is amplified by informational frictions.

This leads to a key implication for optimal monetary policy: After a positive natural-rate shock, the central bank wants to reduce demand by tightening monetary policy, as the direct effect of tightening monetary policy is to increase the household's cost of borrowing. At the same time, however, such policy response also reveals the realization of the natural rate shock, and this informational effect also reduces the output gap. In this case, the informational effect is aligned with the direct effect of the interest rate.

Inflation

Under the assumptions of [Calvo \(1983\)](#), the current aggregate price level is the composite of the aggregate price in the previous period and the average resetting prices:

$$p_t = \theta p_{t-1} + (1 - \theta) \int p_t^*(j) dj. \quad (12)$$

The integral of resetting prices may potentially lead to the higher-order beliefs problem. As equation (9) shows, $p_t^*(j)$ includes firm j 's expectation about the aggregate price level P_t and, thus, includes other firms' expectations. This leads to the infinite regress problem: Each firm uses its firm-specific shock as a private signal and guesses the private signals observed by other firms.⁴

I abstract from this higher-order beliefs problem by assuming that $\sigma_{sa} = \infty$ and $\sigma_{su} = \infty$. Under this assumption, private signals, $a_t(j)$ and $u_t^w(j)$, become completely uninformative about the aggregate shocks, so firms do not use their firm-specific shocks as signals when forming beliefs about the aggregate economy. The subjective beliefs in the model are homogeneous, which I denote as

but it can also be other types of demand shocks as well, for example, time preference shocks or government spending shocks. As long as the output target in the next period is not known to the household, the expected natural rate affects the output gap in addition to the actual one.

⁴Many papers have shown how higher-order beliefs lead to monetary policy have more persistent effects, for example, [Woodford \(2001\)](#) and [Angeletos and LaO \(2009\)](#). For the solution method to the infinite regress problem, see [Huo and Takayama \(2015\)](#), [Melosi \(2016\)](#) and [Nimark \(2017\)](#).

E_t^s .⁵ Homogeneous beliefs are formed when private agents, including both the household and all firms, use only public signals when forming expectations about aggregate variables.⁶

The aggregation of the individual resetting of prices leads to the New Keynesian Phillips curve under subjective beliefs:⁷

$$\pi_t = \beta \theta E_t^s \pi_{t+1} + (1 - \theta) E_t^s \pi_t + \kappa \theta \hat{y}_t + u_t, \quad (13)$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)(\varphi+\sigma)}{\theta}$, and u_t denotes the cost-push shock, which is linearly mapped from the aggregate wage markup shock as $u_t = (1 - \theta)(1 - \beta \theta)u_t^w$.

Equation (13) nests the situation of perfect information, in which case expected inflation equals actual inflation, and the Phillips curve is given by,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{y}_t + \frac{1}{\theta} u_t \quad (14)$$

The comparison between equation (13) and equation (14) shows how informational frictions affect inflation: After a positive cost-push shock, its effect on inflation is reduced, as θ , the Calvo parameter, is less than 1. To understand the intuition, recall that under perfect information, firms increase prices due to two factors. First, each firm's the cost of production increases due to the firm-specific wage markup shock. Second, as each firm perfectly knows that other firms also increase prices, it further increases its own prices, due to the market structure of monopolistic competition. Under imperfect information and in the absence of public signals, the first factor still exists, as each firm is still able to observe its own firm-specific shocks perfectly. However the second factor, the strategic complementarity in the pricing decisions, does not exist, as each firm does not know that a fraction of its firm-specific shock is an aggregate shock.

This leads to a key implication for optimal monetary policy: After a positive cost-push shock that causes positive inflation in the absence of any policy response, the central bank wants to reduce inflation by tightening monetary policy, as the direct effect of tightening monetary policy is to reduce demand, which reduces inflation. At the same time, however, such an interest rate response also reveals the realization of the cost-push shock, and this informational effect makes firms further increase prices. In other words, the informational effect conflicts with the direct

⁵Note that subjective expectations in this paper refer to the rational expectations formed under imperfect information, rather than any other deviations from rational expectations, for example, least-squares learning as in [Evans and Honkapohja \(2012\)](#).

⁶Another way to model homogeneous beliefs is to assume that firms have the same technology and face the same wage markup but do not observe them when setting prices. This assumption, however, implies that aggregate inflation consists of only the firms' expectations, and does not consist of actual shocks. Consequently, there will be no trade-off between inflation and the output gap due to the lack of actual cost-push shocks, which makes the optimal monetary policy less interesting

⁷See Appendix A for the detailed derivation,

effect.

The equilibrium in the private sector is expressed in terms of the output gap and inflation. The underlying aggregate shocks in the private sector are now written in terms of natural-rate shocks in the output gap and cost-push shocks in inflation. The shock processes are given by

$$\begin{aligned} r_t^n &= \phi r_{t-1}^n + v_t, \\ u_t &= \phi_u u_{t-1} + v_t^u, \end{aligned}$$

where the natural-rate shock and the cost-push shock are mapped from the technology shock and the wage markup shock, correspondingly.⁸

3 Monetary Policy with Serially Uncorrelated Shocks

In this section, I focus on the within-period gains from commitment due to the informational effect of monetary policy. I first characterize the informational effect of the interest rate for a given interest-rate response function, and then compare the equilibrium in which the interest rate policy is chosen under discretion and under commitment.

In this section, I add two assumptions to make the model static in nature. First, I assume that underlying shocks have no serial correlation. Second, I impose the restriction that the central bank cannot commit across periods, which excludes the traditionally studied gains from committing to a delayed response. Under these assumptions, the expected inflation and the output gap in the next period is at their steady state levels.⁹

3.1 The Direct Effect and the Informational Effect of Monetary Policy

Under the assumption that shocks are not serially correlated, future equilibrium variables are expected to be at their steady-state levels. Substitute $\phi = \phi^u = 0$ and $E_t^s \pi_{t+1} = E_t^s \hat{y}_{t+1} = 0$ into equation (11) and (13) and get:

$$\hat{y}_t = -\frac{1}{\sigma} (i_t - r_t^n) \quad (15)$$

$$\pi_t = (1 - \theta) E_t^s \pi_t + \kappa \theta \hat{y}_t + u_t. \quad (16)$$

⁸Specifically, $r_t^n = \frac{\varphi + \sigma}{1 + \varphi} \sigma (\phi - 1) a_t$, and $u_t = (1 - \theta)(1 - \beta \theta) u_t^v$. By construction, r_t^n and u_t have the same auto-coefficients as the aggregate technology process and the wage markup process. Denote the standard deviation of the natural-rate shock and the cost-push shock as σ_r and σ_u . By construction, $\sigma_r = \frac{\varphi + \sigma}{1 + \varphi} \sigma (\phi - 1) \sigma_{va}$, and $\sigma_u = (1 - \theta)(1 - \beta \theta) \sigma_{vuw}$.

⁹Following the conventional New Keynesian literature, the long-run distortion has been eliminated via a Pigouvian tax as an employment subsidy, so that the steady-state levels of the output gap and inflation are all zero.

The output gap is free from expected shocks, because the household is able to observe the current price level and future equilibrium variables are expected to be at steady-state levels. In contrast, inflation is affected by subjective expectations, because each individual firm does not observe the aggregate price level when making its own pricing decision. Rearranging equation (16) leads to the expression of aggregate inflation in terms of expected shocks,

$$\pi_t = \kappa \hat{y}_t + (1 - \theta) \frac{\kappa}{\sigma} (E_t^s r_t^n - r_t^n) + \frac{1 - \theta}{\theta} E_t^s u_t + u_t. \quad (17)$$

Equation (15) and equation (17) show the two effects the interest rate has on the equilibrium in the private sector: The first one is the direct effect, which is the conventionally studied effect on the household's borrowing cost. Through the direct effect, an increase in the interest rate reduces current consumption, as it increases the relative cost of current consumption versus future consumption. The direct effect of an increase in the interest rate is also to reduce inflation, as each firm which gets the Calvo lottery reduces its price due to lower demand. The direct effects of the interest rate on the output gap and inflation are as follows:

$$\frac{\partial \hat{y}_t}{\partial i_t} \Big|_{direct} = -\frac{1}{\sigma}, \quad (18)$$

$$\frac{\partial \pi_t}{\partial i_t} \Big|_{direct} = \frac{\partial \pi_t}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial i_t} = -\frac{\kappa}{\sigma}. \quad (19)$$

The informational effect captures how private agents update beliefs about expected shocks after observing changes in the interest rate. As the output gap is free from subjective beliefs, monetary policy has no informational effect on the output gap. The informational effect of monetary policy on inflation consists of the changes in expectations about both types of shocks. Expected natural-rate shocks increase inflation, because firms increase prices when expecting a higher aggregate demand from the household. Expected cost-push shocks increase inflation, because firms increase prices when they expect the wage markups of other firms to also go up. The informational effects of the interest rate on the output gap and on inflation are as follows:

$$\frac{\partial \hat{y}_t}{\partial i_t} \Big|_{informational} = 0, \quad (20)$$

$$\frac{\partial \pi_t}{\partial i_t} \Big|_{informational} = \frac{\partial \pi_t}{\partial E_t^s r_t^n} \frac{\partial E_t^s r_t^n}{\partial i_t} + \frac{\partial \pi_t}{\partial E_t^s u_t} \frac{\partial E_t^s u_t}{\partial i_t}. \quad (21)$$

where the partial derivatives of inflation on the expected natural-rate and the expected cost-push shocks are defined in equation (17) as: $\frac{\partial \pi_t}{\partial E_t^s r_t^n} = (1 - \theta) \frac{\kappa}{\sigma}$, $\frac{\partial \pi_t}{\partial E_t^s u_t} = \frac{1 - \theta}{\theta}$.

Belief Formation

The informational effect of the interest rate depends on the interest rate response function.

First consider a given interest rate response function that responds linearly to the two aggregate shocks, i.e., $i_t = F_r r_t^n + F_u u_t$. The interest rate becomes *one signal* that simultaneously provides information about *two shocks*. When private agents extract information from the interest rate about one shock, the prior distribution of the other shock becomes the source of noise in this signal.

Agents in the private sector are Bayesian and form best linear forecasts by optimally weighting their prior beliefs (shocks have zero ex-ante mean) and the current signal (the interest rate). Let K_r and K_u denote the optimal weights on the two states after observing interest rate changes, which are determined through the optimal filtering process. Beliefs formed through the Kalman filtering process are given by,

$$\begin{bmatrix} E_t^s r_t^n \\ E_t^s u_t \end{bmatrix} = \begin{bmatrix} 1 - K_r \\ 1 - K_u \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} K_r \\ K_u \end{bmatrix} \hat{i}_t = \begin{bmatrix} K_r F_r & K_r F_u \\ K_u F_r & K_u F_u \end{bmatrix} \begin{bmatrix} r_t \\ u_t \end{bmatrix}, \quad (22)$$

where

$$K_r F_r = \frac{F_r^2 \sigma_r^2}{F_r^2 \sigma_r^2 + F_u^2 \sigma_u^2}, \quad (23)$$

$$K_u F_u = \frac{F_u^2 \sigma_u^2}{F_r^2 \sigma_r^2 + F_u^2 \sigma_u^2}. \quad (24)$$

The above system of equations solve the belief formation process in the private sector. The following lemma provides an interpretation:

Lemma 1: *When the interest rate response function is linear, i.e., $i_t = F_r r_t^n + F_u u_t$, and private agents have model-consistent expectations, expectations about the two shocks have the following properties:*

- *When $\sigma_r = \sigma_u$, $\frac{\partial E_t^s r_t^n / \partial r_t^n}{\partial E_t^s u_t / \partial u_t}$ increases with $\frac{F_r}{F_u}$*
- *When $F_r = F_u$, $\frac{\partial E_t^s r_t^n / \partial r_t^n}{\partial E_t^s u_t / \partial u_t}$ increases with $\frac{\sigma_r}{\sigma_u}$*

Lemma 1 describes how the precision of the informational effect of the interest rate is determined by the interest rate response function and the ex-ante dispersion of the underlying shocks. When the two shocks have the same ex-ante dispersion ($\sigma_r = \sigma_u$), if the interest rate is more sensitive to natural-rate shocks and less sensitive to cost-push shocks ($F_r > F_u$), then after observing a change in the interest rate, agents in the private sector infer that such an interest rate change is less likely to be a response to a cost-push shock. Otherwise, provided that F_u is small, the change in the interest rate has to come from a large cost-push shock, which is less likely to happen given the fact that shocks have same prior distribution. When the interest rate is equally sensitive to both shocks ($F_r = F_u$), agents in the private sector put more weight on the shock that has higher ex-ante dispersion, as the ex-ante mean of the shock has a smaller weight in the belief-formation process.

3.2 Monetary Policy under Discretion

A discretionary central bank chooses the interest rate ex-post, taking two factors as given. The first is the realization of the shocks. The second is the informational effect of the interest rate, i.e., how private agents use the interest rate as a signal to form expectations about the two shocks. As illustrated in Lemma 1, the informational effect of the interest rate, captured in the Kalman gains in the belief-updating process, is determined by the expected interest rate reaction function. Under rational expectations, the actual interest rate response function coincides with the expected interest rate reaction function in equilibrium. Private agents receive one piece of information from the central bank, which is the interest rate decision. They have rational expectations about the interest rate response function in equilibrium. The following figure summarizes the optimization problem of the central bank under discretion.

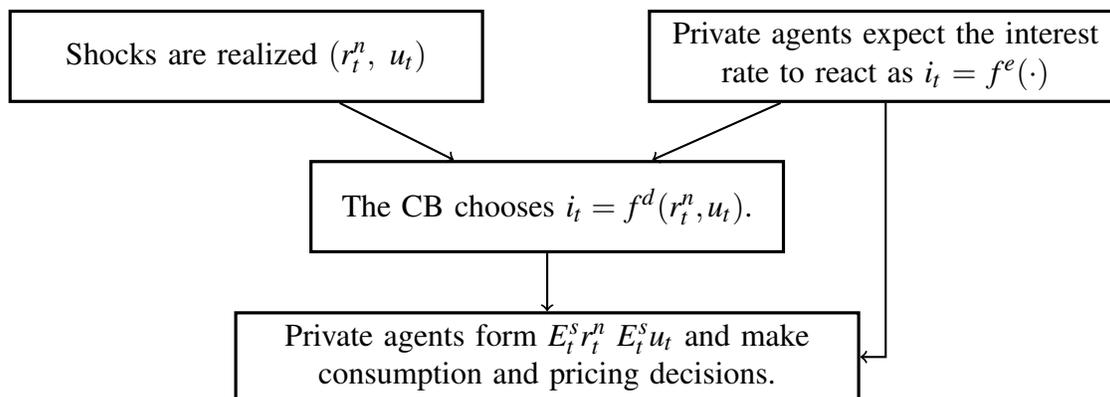


Figure 1: The Sequence of Events under Discretion

As shown in the chart, the central bank takes two factors as given when choosing i_t : the shocks and the expected interest rate reaction function. Private agents receive one piece of information from the central bank, which is the interest rate. An equilibrium exists when $f^d(r_t^n, u_t) = f^e(r_t^n, u_t)$.

3.2.1 The Phillips Curve

The Phillips curve describes the constraint faced by the discretionary central bank when choosing the optimal interest rate under discretion, as it captures the trade-off between output gap stabilization and inflation stabilization. In this section, I use an example to illustrate how the informational effect of the interest rate changes the Phillips curve.

Suppose that the central bank chooses the optimal interest rate as if information is perfect, which is characterized by a linear function, $i_t = F_r^P r_t^n + F_u^P u_t$.¹⁰ In this case, private agents form

¹⁰It is well known that the optimal interest rate under perfect information targets zero inflation after natural rate shocks and partially accommodates cost-push shocks by targeting $\pi_t = -\omega \hat{y}_t$. The optimal interest rate that achieves such targets is: $i_t = r_t^n + \frac{\sigma}{\theta} \frac{1}{\kappa + \omega} u_t$.

expectations on the shocks using the interest rate as a signal, and the belief-updating process is described through equations (22) - (24). Substituting the expected shocks by the interest rate into equation (17) leads to the Phillips curve under imperfect information,

$$\pi_t = \left\{ \kappa - \sigma \left[(1 - \theta) \frac{\kappa}{\sigma} K_r + \frac{1 - \theta}{\theta} K_u \right] \right\} \hat{y}_t + \left\{ (1 - \theta) \frac{\kappa}{\sigma} (K_r - 1) + \frac{1 - \theta}{\theta} K_u \right\} r_t^n + u, \quad (25)$$

Recall that with perfect information, the Phillips curve is exogenous to the interest rate decision. In contrast, with imperfect information, the Phillips curve depends not only on the realization of actual shocks, but also on the expectations about the shocks, which further depends on the expectations about the interest rate reaction function. Mathematically, how the interest rate is expected to respond to both shocks affects the Kalman gains (K_r and K_u) in equation (25). In the following figure, I plot the Phillips curve under perfect information and the Phillips curve under imperfect information after both shocks.

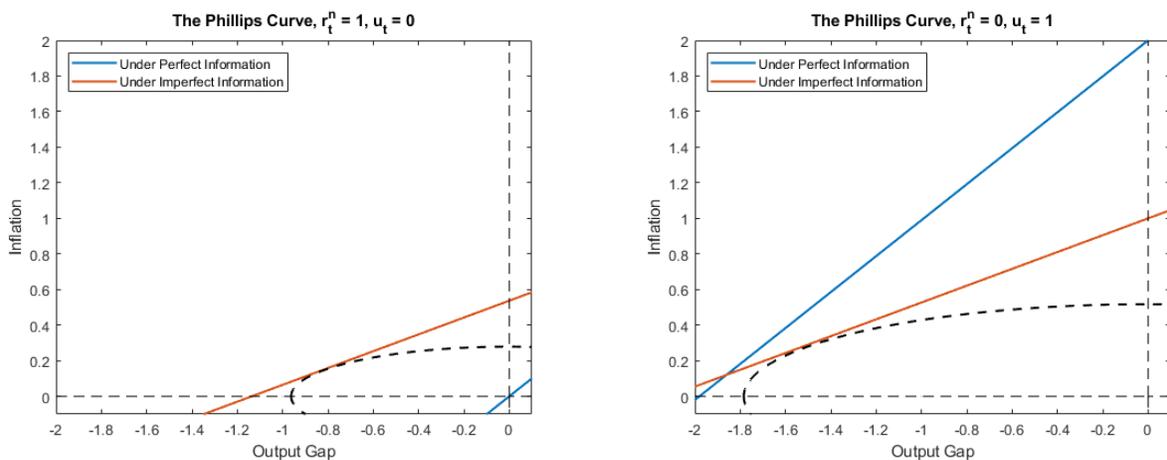


Figure 2: The Phillips Curve under Perfect Information and under Imperfect Information

The above figures plot the Phillips curve after a natural-rate shock (left) and a cost-push shock (right) under perfect information (in blue) and under imperfect information (in red). The dotted ellipse is the indifference curve of the central bank. See Section 3.4 for parameter values.

The above figure shows that the slope of the Phillips curve is reduced under imperfect information, as the informational effect dampens the direct effect of monetary policy. When the interest rate increases, its direct effect increases the household's cost of borrowing of the household, which reduces both the output gap and inflation, as shown in equations (18) and (19). At the same time, the informational effect leads to positive updates of expectations about both shocks, making firms increase prices as they expected a higher aggregate demand and a higher aggregate price level, as shown in equations (20) and (21).

The intercept of the Phillips curve captures the situation when the interest rate responds to close the output gap. After a natural-rate shock, closing the output gap requires the interest rate to respond one-to-one to the realized natural-rate shock. The direct effect of such tightening monetary policy is to reduce inflation, but at the same time, private agents do not know whether the interest rate is responding to a natural-rate shock or a cost-push shock. There are two factors that affect the intercept of the Phillip curve, and their relative size determines the sign of the intercept. First, the expected output gap is lower than the actual output gap, as the expected natural-rate shock is lower than the actual natural-rate shock, which reduces the intercept. Second, as firms positively update their expectations on the cost-push shock, they increase prices as they expect a higher aggregate price level, which increases the intercept.

After a cost-push shock, closing the output gap means the interest rate does not respond at all. Absent any signals, private agents do not update beliefs on both shocks, and firms increase prices only because the increases in their own cost of production. Consequently, inflation is lower under imperfect information.

The changes in the Phillips curve imply the costs and benefits of informational frictions. On the good side, the intercept of the Phillips curve after a cost-push shock is reduced, because each firm increases its price by less when it does not know that the costs of production in other firms also increase. On the bad side, the slope of the Phillips curve is reduced, which means the central bank needs to sacrifice more output as the trade-off to stabilize inflation.

It becomes apparent that the optimal interest rate under perfect information is no longer an optimizing choice due to the changes in the Phillips curve. The optimizing interest rate should achieve the equilibrium (\hat{y}_t, π_t) that is the tangent point between the indifference curve¹¹ and the Phillips curve. As Figure 2 shows, when private agents expect the interest rate to follow $i_t(r_t^n, u_t) = F_r^p r_t^n + F_u^p u_t$, the optimizing discretionary interest rate should respond more to the natural-rate shock ($F_r^d > F_r^p$) and should respond less to the cost-push shock ($F_u^d < F_u^p$). The equilibrium interest rate is found when the expected response coincides with the actual optimizing response, which is solved in the following section.

3.2.2 Optimal Discretionary Monetary Policy

The objective function of a central bank is to minimize the sum of the squared output gap and squared inflation for all periods. Due to the static nature of this benchmark model, the objective function of the discretionary central bank reduces to minimizing current deviations, which is given by:

$$\min_i L(t) = \begin{bmatrix} \pi_t & \hat{y}_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_t \\ \hat{y}_t \end{bmatrix} + \text{indept. terms} \quad (26)$$

¹¹The indifference curve captures the central bank's loss function, $L = \pi_t^2 + \omega \hat{y}_t^2$.

subject to

$$\hat{y}_t = -\frac{1}{\sigma} (i_t - r_t^n) \quad (27)$$

$$\pi_t = \kappa \hat{y}_t + (1 - \theta) \frac{\kappa}{\sigma} (E_t^s r_t^n - r_t^n) + \frac{1 - \theta}{\theta} E_t^s u_t + u_t \quad (28)$$

$$E_t^s r_t^n = K_r i_t \quad (29)$$

$$E_t^s u_t = K_u i_t \quad (30)$$

where ω is a constant that results from the second-order approximation of the household's utility.¹²

Definition: A Markov perfect equilibrium between a discretionary central bank and the private sector with rational expectations can be described in aggregate terms in the following way:

(i) Inflation and the output gap result from the household's optimal consumption choices and firms' optimal price-setting behaviors, which are shown in equations (10) and (13).

(ii) Beliefs in the private sector about the realization of shocks are formed through the Kalman filtering process as shown in equations (22) to (24);

(iii) The interest rate is set by the central bank's optimization problem as specified in (26), subject to the constraints as specified in equations (27) to (30).

As the constraint faced by the central bank is endogenous to the central bank's own optimization problem, further characterization of the discretionary interest rate in equilibrium relies on numerical analysis, which I describe in Section 3.4.

3.3 Monetary Policy under Commitment

A central bank with credible commitment chooses a state-contingent policy rule prior to the realization of shocks and follows the rule to set interest rates after shocks are realized. Private agents receive two pieces of information from the central bank. One is the state-contingent policy rule, and the second is the interest rate. The first piece of information, the policy rule, dictates how to interpret the second piece of information, the interest rate, because how the interest rate is expected to respond to different shocks determines the informational effect of the interest rate. The following figure summarizes the sequence of events for the central bank with credible commitment.

¹²See [Woodford \(2011\)](#) for a general derivation of the second-order approximation of the household's utility under perfect information, and [Adam \(2007\)](#) for the application to imperfect information. Appendix C shows the derivation that applies to the specific assumptions in this paper.

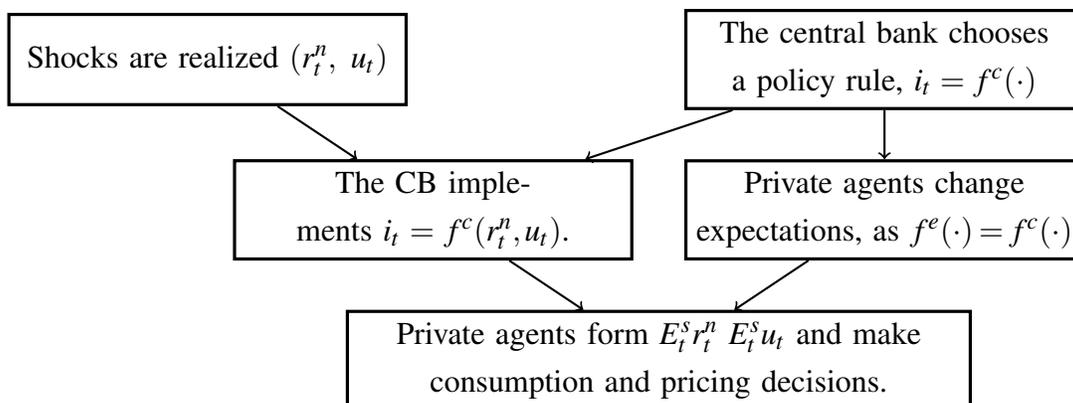


Figure 3: The Sequence of Events under Commitment

f^c denotes the policy rule chosen by the central bank with credible commitment ex-ante. Credible commitment makes the central bank able to change private agents' expectations about the interest rate response function.

The key difference between optimization under discretion (Figure 1) and under commitment (Figure 3) is whether the expected interest rate response function is taken as given (in the case of discretion) or is endogenous to the central bank's policy decision (in the case of commitment). With credible commitment, the central bank can change the informational effect of the interest rate by announcing that it will follow a rule that is different from its best response under discretion. In this way, the central bank chooses a direct mapping from the actual shocks to the expected shocks.

3.3.1 The Phillips Curve

In this section, I illustrate how commitment changes the Phillips curve using an example: Suppose the central bank commits to the interest rate rule that tracks the natural rate one-to-one and does not respond to the cost-push shock, i.e., $i_t = 1 \cdot r_t^n + 0 \cdot u_t$. As the central bank commits to this rule, the interest rate becomes a perfect signal about the natural-rate shock and provides no information about the cost-push shock. Substituting $(F_r, F_u) = (1, 0)$ to equations (23) and (24), it is easy to see that $(K_r, K_u) = (1, 0)$. The following figure plots the Phillips curve under this commitment, in comparison with the equilibrium under discretionary optimization.

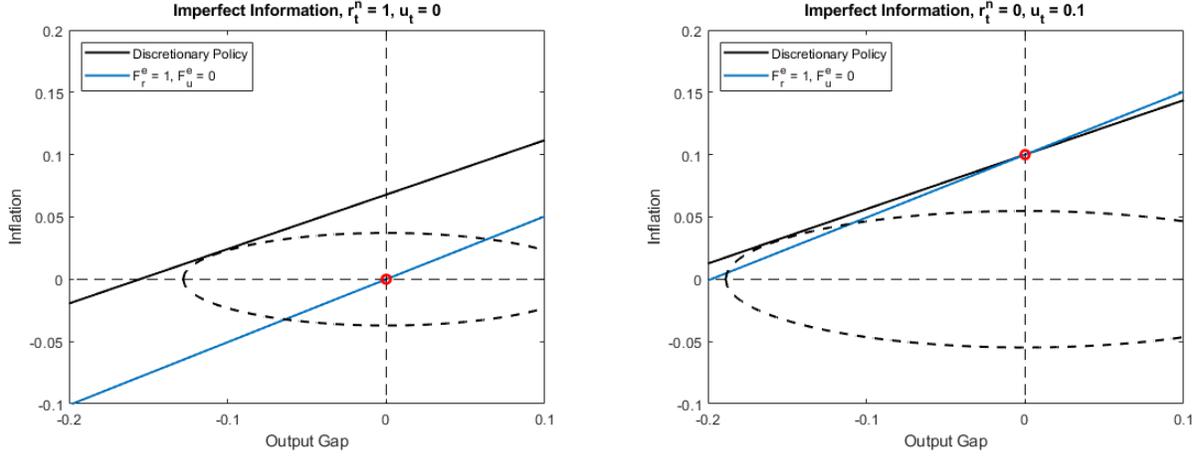


Figure 4: The Phillips Curve under the Rule $i_t = r_t^n$ and under Discretionary Optimization

The black line is the Phillips curve for the discretionary central bank. (The solution for the discretionary interest rate is provided in Section 3.4.) The blue line is the Phillips curve for the central bank, which is expected to implement the rule such that $i_t = r_t^n$. The dotted ellipse denotes the indifference curve of the central bank's loss function.

The first thing to notice is that after a natural-rate shock, the Phillips curve crosses the origin, the same as what would happen under perfect information, which suggests that dual stabilization becomes available under such a commitment. After the positive natural-rate shock, as the interest rate tracks it one-to-one, the direct effect closes the output gap and, at the same time, provides perfect information to private agents. This informational effect makes expected shocks equal to the actual shocks and completely stabilizes inflation as well.

Importantly, the fact that dual stabilization is achieved is due not only to the interest rate's response to the natural-rate shock, but also to the expectation about how the interest rate would react if a cost-push shock is realized. If private agents expect the interest rate to react positively to a cost-push shock as well, the interest rate cannot be a perfect signal of the natural-rate shock.

The second thing to notice is that after either of the two shocks, there can only be one point on the Phillips curve that is consistent with the commitment, which I denote in a red circle. After the natural-rate shock, it is the origin of the plane, i.e., $(\hat{y}_t, \pi_t) = (0, 0)$. Other points capture the situation when private agents have been convinced by the rule, so the informational effect of the interest rate has been fixed, but the central bank deviates from this rule. For example, the points below $\pi_t = 0$ are the equilibrium in which the interest rate acts more strongly than tracking one-to-one with the natural-rate shock.

Third, although this policy rule achieves the first-best after a natural-rate shock, it is not the optimal policy rule. This is because the central bank also cares about the equilibrium after a cost-push shock, in which case being completely inelastic to a cost-push shock results in the equilibrium with a zero output gap and positive inflation, which is clearly not optimal. The optimal policy rule

is solved in Section 3.3.2.

The central bank can shift the Phillips curve by changing the beliefs in the private sector. By making the private agents believe there will be a different interest rate response function, the central bank changes expectations without changing the current interest rate. A similar intuition can be found in papers that study optimal commitment to a delayed response of interest rates, such as [Clarida, Gali, and Gertler \(2000\)](#) and [Eggertsson and Woodford \(2003\)](#). In these papers, the central bank shifts the Phillips curve by committing to future interest rates, which changes the expected future inflation. Due to the forward-looking components in the inflation dynamics, such a commitment changes current inflation without changing the current demand, shifting the Phillips curve. To isolate the informational gains in my model, I impose the restriction that the policy rule only responds to current shocks. The shift of the Phillips curve is still through changes in expected inflation, but it is expected current inflation, not expected future inflation.

3.3.2 Optimal Policy Rule

The central bank with commitment chooses the interest rate feedback rule $i_t = f(r_t^n, u_t, \pi_t, \hat{y}_t)$ prior to the realization of shocks, which becomes $i_t = F_r^c r_t^n + F_u^c u_t$ in equilibrium. The optimal rule is found by choosing F_r and F_u to minimize the central bank's ex-ante loss over the state space:

$$\min_{F_r^c, F_u^c} \int \int \pi_t^2(r_t, u_t) + \omega \hat{y}_t^2(r_t, u_t) dr_t^n du_t, \quad (31)$$

subject to

$$i_t = F_r^c r_t^n + F_u^c u_t, \quad (32)$$

$$\hat{y}_t = -\frac{1}{\sigma} (i_t - r_t^n) \quad (33)$$

$$\pi_t = \kappa \hat{y}_t + (1 - \theta) \frac{\kappa}{\sigma} (E_t^s r_t^n - r_t^n) + \frac{1 - \theta}{\theta} E_t^s u_t + u_t \quad (34)$$

$$E_t^s r_t^n = K_r(F_r^c, F_u^c) F_r^c r_t^n + K_r(F_r^c, F_u^c) F_u^c u_t, \quad (35)$$

$$E_t^s u_t = K_u(F_r^c, F_u^c) F_u^c r_t^n + K_u(F_r^c, F_u^c) F_u^c u_t. \quad (36)$$

Comparing the optimization problem under commitment and the one under discretion, we find that the ability to commit essentially relaxes one constraint: Under discretion, private agents expect the central bank to react with its best response after each shock, i.e., $(F_r^e, F_u^e) = (F_r^d, F_u^d)$, which determines the sensitivity of expected shocks to changes in the interest rate, as measured in (K_r^d, K_u^d) . The central bank cannot change expectations about the interest-rate's behavior, as private agents believe the central bank will optimize in all states. In comparison, the central bank with credible commitment can announce that it will implement a sub-optimal interest rate in some

states, i.e., $(F_r^c, F_u^c) \neq (F_r^d, F_u^d)$. Under rational expectations, the change in the expected interest rate response function changes the sensitivity of expected shocks to the interest rate. In other words, the central bank with commitment can choose a direct mapping from actual shocks to expected shocks, by controlling the informational effect of the interest rate.

Proposition 1: *When $\sigma_r > 0$ and $\sigma_u > 0$, the ex-ante welfare under commitment is strictly greater when the central bank commits to the optimal policy rule rather than optimizing interest rate decisions under discretion.*

It is very easy to understand that the ex-ante welfare under optimal commitment is always *weakly* better than that under discretion, because the solution to the discretionary optimization is within the choice set of the optimization problem under commitment. Therefore, the question then becomes whether the gains are strictly positive, which holds true as long as one of the marginal informational effects (defined as $\frac{\partial K_r}{\partial F_r}, \frac{\partial K_r}{\partial F_u}, \frac{\partial K_u}{\partial F_r}, \frac{\partial K_u}{\partial F_u}$) is not zero. Details of the proof are provided in Appendix C.

3.4 Numerical Illustration

Since solving the optimization problem for the central bank both under discretion and under commitment requires numerical methods, in this section, I provide a numerical example to illustrate properties of the equilibrium interest rate under discretion, and the gains from committing to the optimal policy rule. Parameter values are adopted to align with the traditional New Keynesian literature. Specifically, I set $\varphi = 1$ and $\sigma = 1$, assuming a unitary Frisch elasticity of labor supply and log utility of consumption. I use $\beta = 0.99$, which implies a steady-state real return on financial assets of 4 percent. For price rigidity, I set θ , the price stickiness parameter, to be 0.5, which is indicated by the average price duration from macro and micro empirical evidence.¹³ For the parameter that governs the elasticity of substitution between intermediate goods, I set $\varepsilon = 4$, which implies a steady-state price markup of one-third of revenue.

The solution of the equilibrium interest rate under discretion involves a circularity problem, as the constraint that the central bank faces depends on the optimal decision of the interest rate under rational expectations. I follow the method of [Svensson and Woodford \(2003\)](#) to find the solution of the optimizing interest rate. First, I conjecture an interest rate reaction function, $i_t = F_r^0 r_t^n + F_u^0 u_t$, which determines the Phillips curve. Then, the central bank chooses the interest rate to maximize its objective function under the constraint of the Phillips curve. The equilibrium interest rate under rational expectations is found as the fixed point between the conjectured interest rate function and the solution to the optimization problem.¹⁴

¹³Sources: [Bils and Klenow \(2004\)](#), [Galí and Gertler \(1999\)](#), and [Nakamura and Steinsson \(2010\)](#)

¹⁴A detailed derivation for solving for the equilibrium optimizing interest rate is provided in Appendix B.

The solutions of the equilibrium interest rate, the output gap and inflation as functions of state variables are found to be:

$$i_t = [1.1074, 1.5878] [r_t^n, u_t]', \quad (37)$$

and

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} -0.1074 & -1.5878 \\ 0.0210 & 0.3060 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix}. \quad (38)$$

As a comparison, recall that under perfect information, the equilibrium interest rate, the output gap and inflation under discretion are given by:

$$i_t = [1, 1.8279] [r_t^n, u_t]', \quad (39)$$

and

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0 & -1.8279 \\ 0 & 0.1538 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix}. \quad (40)$$

The following result summarizes the key differences between the discretionary equilibrium under imperfect information and under perfect information.

Result 1: *When shocks are not serially correlated, the discretionary central bank targets a negative ratio between inflation and the output gap, and the absolute value of the targeted ratio between inflation and the output gap is higher under imperfect information than under perfect information.*

The intuition of this result is as follows: The discretionary optimization problem yields that the optimal combination of inflation and the output gap is the tangent point between the indifference curve of the central bank's loss function and the Phillips curve. As shown in equation (25), the slope of the Phillips curve is reduced by the informational effect of the interest rate as long as the interest rate counteracts the effects of both shocks in equilibrium, i.e., both F_r and F_u are positive. Because the Phillips curve is flattened due to the informational effect, the vector of (\hat{y}_t^*, π_t^*) becomes steeper in equilibrium.

More precisely, the targeted ratio between inflation and the output gap results from the first-order condition of the central bank's optimization problem, which is given by:

$$\pi_t = - \left(\frac{\partial \pi_t}{\partial i_t^*} \right)^{-1} \frac{\partial \hat{y}_t}{\partial i_t^*} \omega \hat{y}_t \equiv -R \hat{y}_t. \quad (41)$$

Under full information, the absolute value of the targeted ratio between inflation and the output gap is given by:

$$R_{perfect\ info} = \left(\frac{\partial \pi_t}{\partial i_t^*} \right)^{-1} \frac{\partial \hat{y}_t}{\partial i_t^*} \omega = \left(-\frac{\kappa}{\sigma} \right)^{-1} \left(-\frac{1}{\sigma} \right) \omega. \quad (42)$$

With informational frictions, the targeted ratio becomes:

$$R_{imperfect\ info} = \left(\frac{\partial \pi_t}{\partial i_t^*} \right)^{-1} \frac{\partial \hat{y}_t}{\partial i_t^*} \omega = \left(-\frac{\kappa}{\sigma} + (1 - \theta) \frac{\kappa}{\sigma} K_r + \frac{1 - \theta}{\theta} K_u \right)^{-1} \left(-\frac{1}{\sigma} \right) \omega. \quad (43)$$

Since $F_r > 0$, $F_u > 0$ in this numerical example, the Kalman gains ($K_r > 0$, $K_u > 0$) are both positive, which yields that $R_{imperfect\ info} > R_{perfect\ info}$.

To solve the optimal policy rule under commitment, I numerically search for the linear interest rate rule, $i_t = F_r^c r_t^n + F_u^c u_t$, such that the central bank's ex-ante loss is minimized. The optimal policy rule, the equilibrium output gap and inflation under commitment are found to be:

$$i_t = [1.1965, 1.6902] [r_t^n, u_t]', \quad (44)$$

and

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} \begin{bmatrix} -0.1963 & -1.6902 \\ -0.0633 & 0.1972 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} \quad (45)$$

The gains from commitment can be analyzed through the lens of the changes in the Phillips curve, which is the trade-off between inflation stabilization and the output gap stabilization. When the central bank has the ability to commit, it wants to improve the trade-off from the discretionary equilibrium, which translates into 1) steepening the slope and 2) reducing the intercept after a natural-rate shock. Equation (25) shows that these two goals are aligned when the Phillips curve has a positive intercept after a natural-rate shock under discretion, which is satisfied under the parameter specifications in this numerical example.

Result 2: *In comparison with the equilibrium interest rate under discretion, the optimal policy rule reduces the degree to which the informational effect dampens the direct effect of the interest rate, which is given by:*

$$\sigma \left[(1 - \theta) \frac{\kappa}{\sigma} K_r + \frac{1 - \theta}{\theta} K_u \right]. \quad (46)$$

Equation (37) and equation (44) show how the optimal policy rule reduces the informational effect of the interest rate. First, $\frac{F_r}{F_u}$ is greater under optimal commitment. As indicated by Lemma 1, the optimal policy rule reduces the sensitivity of expected cost-push shocks to actual cost-push shocks ($K_u \cdot F_u$). Second, both F_r and F_u are higher under the optimal policy rule. For both shocks, the sensitivity of expected shocks to interest rate changes (K) is smaller for a given sensitivity of expected shocks to actual shocks ($K \cdot F$). Intuitively, for a given change in the interest rate, the private agents believe that the realized shock is of a smaller size when they know the interest rate is very sensitive. I demonstrate the change in the Phillips curve in the following graph.

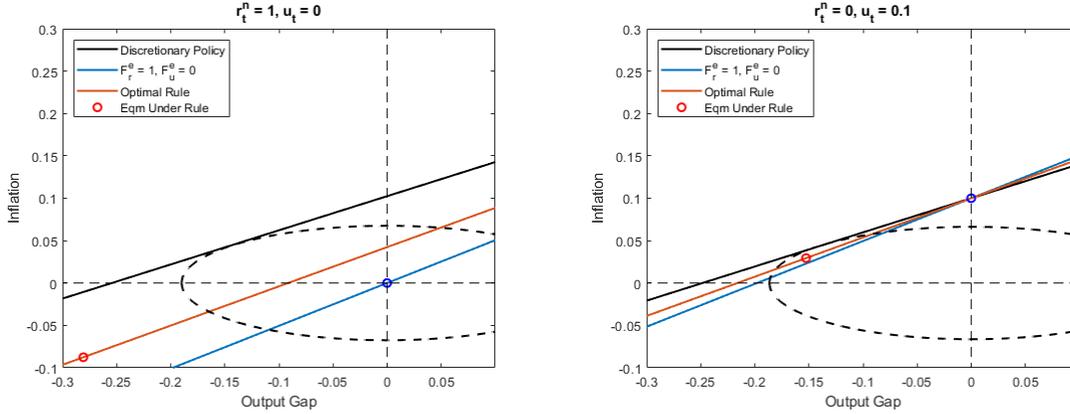


Figure 5: The Gains from Commitment and the Problem of Time Inconsistency

The black line is the Phillips curve when the central bank is expected to be discretionary. The red and blue lines are the Phillips curves when the central bank is expected to follow (1) the optimal policy rule and (2) the rule such that $(F_r, F_u) = (1, 0)$, respectively. The red and blue circles are the equilibrium consistent with such a commitment. The dotted ellipse is the indifference curve for the central bank.

First, compared with the Phillips curve under discretion (black line), the Phillips curve under the optimal policy rule (red line) has a steeper slope after both shocks, which shows that the degree to which the informational effect dampens the direct effect of the interest rate is reduced. Second, the intercept of the Phillips curve after the natural-rate shock is reduced under optimal commitment for the same reason. The intercept after the cost-push shock stays the same, because the intercept refers to the equilibrium in which the interest rate does not respond to the cost-push shock. In this case, there is no informational effect from the interest rate.

Associated with the gains from commitment is the time inconsistency problem. After both shocks, the equilibrium under the optimal policy rule is not the one that is tangent to any indifference curves, meaning the central bank *commits to be sub-optimal* after both shocks. If the central bank decides to deviate after a natural-rate shock, it wants to reduce its committed interest rate response, hoping to achieve a higher output gap without increasing inflation. However, this can only be achieved if the Phillips curve stays the same, in which case the private sector is fooled by the central bank. If private agents anticipate such a deviation, they will change their expectations accordingly, making the Phillips curve shift back to the one under discretion.

In addition, the optimal policy rule does not achieve better outcomes in all states. Rather, the central bank with optimal commitment sacrifices the outcome after the natural-rate shock to gain after a cost-push shock. The central bank balances outcomes across all states and achieves welfare gains ex-ante.¹⁵

¹⁵Notice that in the right panel (the cost-push shock), the unit of the y-axis is chosen to be 0.1 instead of 1. This is to make both figures in the same size while having all different Phillips curves observable.

4 External Information

In practice, central banks are not restricted by the dimension of signals. Instead, in recent decades, central banks have been treating communication as an important policy tool to improve policy effectiveness. However, the question is: what is the optimal communication strategy?

Without the informational effect of the interest rate, the optimal communication strategy is straight forward. The central bank wants to reveal full information about the natural-rate shock, because doing so also reveals the fact that it is able to achieve dual stabilization using the direct effect of monetary policy. By contrast, after a cost-push shock, the optimal monetary policy does not completely stabilize inflation. Therefore, the central bank wants to withhold information about the cost-push shock, as it does not want the private sector to update its expectations about inflation. In summary, welfare is maximized when the central bank provides a perfectly precise signal about the efficient shock (the natural-rate shock) and a completely uninformative signal about the inefficient shock (the cost-push shock).¹⁶ In my model, the informational effect of the interest rate interacts with direct communication, which complicates the optimal communication strategy.

4.1 Interaction between the Informational Effect of Monetary Policy and Central Bank Direct Communication

Direct communication is modeled as the central bank providing additional signals independent of the interest rate as a signal, and controlling the precision of the additional signals. Unlike the informational effect through the interest rate, which is restricted by the signal dimension, central bank direct communication is not bounded by the signal dimension.

Denote the external signals sent through the central bank communications as m_t^r and m_t^u , which are distributed log normally around the actual shocks, r_t^n and u_t . The signals received by private agents consist of both the interest rate and the external signals from central bank direct communication. Signals are summarized as follows:

$$\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} = \begin{bmatrix} F_1 & F_3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^r \\ \varepsilon_t^u \end{bmatrix} \quad (47)$$

As the interest rate is one signal about two shocks, its informational effect interacts with the direct communication, which makes the central bank unable to separately control the information revealed about one shock. Specifically, for a given response function of the interest rate, increasing

¹⁶For a more general discussion on the value of information without the informational effect of monetary policy, see [Morris and Shin \(2002\)](#), [Angeletos and Pavan \(2007\)](#), [Angeletos, Iovino, and La'O \(2016\)](#), for examples. See [Blinder et al. \(2008\)](#) for a survey of literature on the central bank communication.

the precision of communication about one shock also increases the precision of information about the other shock. The intuition is that when the central bank communicates more precisely about one shock, the interest rate becomes a less precise signal about that shock, but at the same time, the interest rate becomes a more precise signal about the other shock. For example, suppose that the central bank provides perfect information about the r_t^n shock. Then suppose that a positive cost-push shock is realized and the natural rate stays at zero. The private agents know that $r_t^n = 0$ from the central bank's direct communication about the natural-rate shock. In addition, the private agents also observe that the interest rate responds positively, and they can then know for sure that the increase in the interest rate is due to the positive realization of a cost-push shock.

4.2 Value of (External) Information

In the first row of Figure 6, I plot the expected loss for the central bank at varying levels of precision of central bank communication under discretion (left) and under commitment (right). It shows that when communication about the cost-push shock becomes more precise, which is modeled by a lower σ_{eu} , the ex-ante loss increases. This is consistent with the conventional wisdom that more precise information about the inefficient shock is welfare reducing. However, when the precision of central bank communication about natural-rate shocks increases, the ex-ante loss also increases, which contradicts the conventional wisdom.

An important underlying assumption drives the above results: The only source of noise is the ex-ante distribution of the two shocks. In the literature, it is often thought that monetary policy has two components: One is the monetary authority's response to fluctuations in the private sector, and the other is a random variable that is usually referred to as a shock to monetary policy.¹⁷ Allowing for such randomness in monetary policy makes the interest rate less informative, and the optimal communication strategy may change. The second row of Figure 6 plots this case, with a relatively large standard deviation of the implementation error ($\sigma_e = 0.5$). In this situation, the ex-ante loss is minimized when the information on the natural-rate shock is perfectly precise and information on the cost-push shock is completely zero.

¹⁷The previous literature has given several interpretations of the policy shock. [Christiano, Eichenbaum, and Evans \(1999\)](#) summarize three types of interpretations, including (1) shocks to the preferences of the members of the FOMC, (2) fluctuations of private agents' expectations to which the monetary authority reacts, and (3) measurement error in the preliminary data available at the time it makes policy decisions.

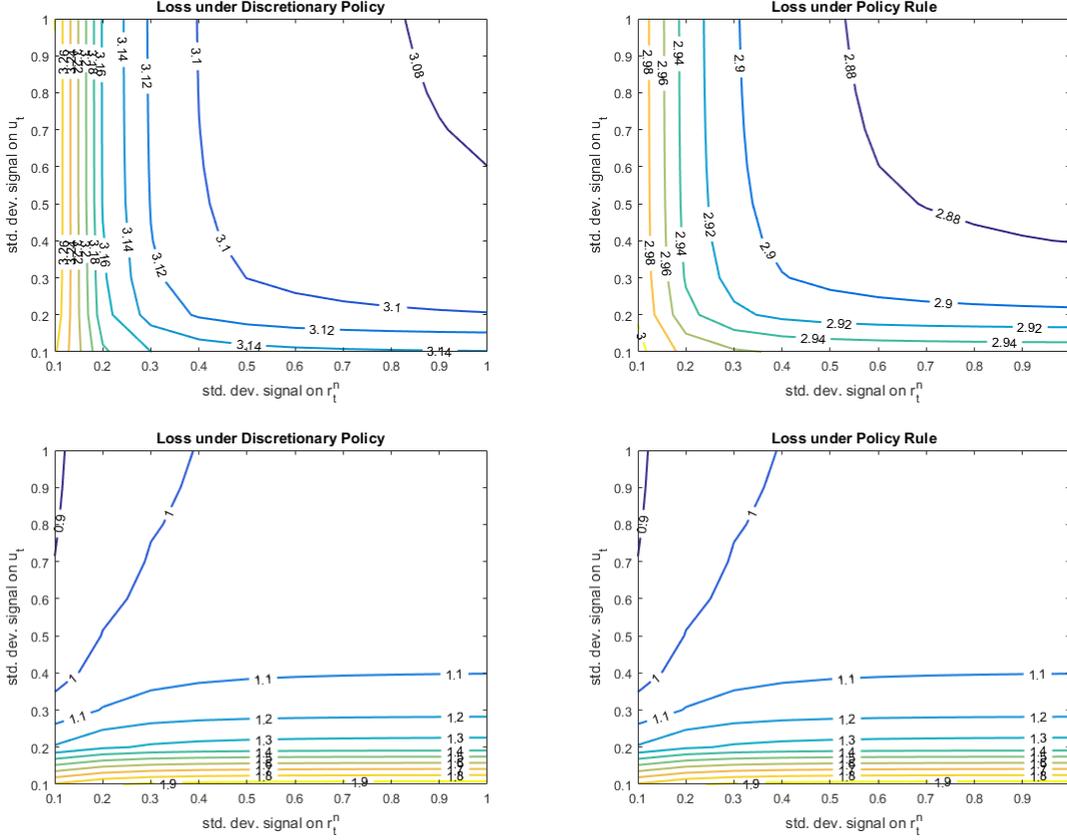


Figure 6: The Value of (External) Information

The first row shows the ex-ante loss of the central bank at varying precision of external signals. In the second row, an implementation error is added to the interest rate, i.e., $i_t = i_t^* + e_t$, with $\sigma_e = 0.5$ and i_t^* being either the equilibrium interest rate under discretion or the optimal policy rule under commitment.

5 Dynamic Informational Effect

With serially correlated shocks, the learning process becomes persistent, which leads to the dynamic informational effect of interest rates. This section shows how the dynamic informational effect changes the equilibrium interest rate under discretion and gains from commitment.

5.1 The Equilibrium in the Private Sector

The process of the actual shocks is given by:

$$\begin{bmatrix} r_t^n \\ u_t \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} r_{t-1}^n \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_t^r \\ v_t^u \end{bmatrix}. \quad (48)$$

The information set of the private sector includes the values of all parameters and the entire

history of interest rates upon t . Due to the serial correlation, private agents optimally weigh current signals and past beliefs (priors) when forming beliefs about current shocks. Due to the persistent belief-updating process, the current equilibrium is affected by past beliefs as well. Therefore, when the interest rate is set to minimize deviations of the current output gap and inflation, it should react to beliefs in the past period as well.

I conjecture that the equilibrium interest rate under discretion is a linear function of all pre-determined state variables in period t , which includes both the actual shocks at time t and beliefs in period $t - 1$.

$$i_t = F_1 r_t^n + F_2 E_{t-1}^s r_{t-1}^n + F_3 u_t + F_4 E_{t-1}^s u_{t-1}. \quad (49)$$

The private agents have perfect memory of their beliefs in the past. They are able to distinguish the fraction of the interest rate that reacts to current shocks from the fraction of the interest rate that reacts to past beliefs. Let \hat{i}_t denote the fraction of i_t that reacts to current shocks, which is given by:

$$\hat{i}_t \equiv i_t - F_3 E_{t-1}^s r_{t-1}^n - F_4 E_{t-1}^s u_{t-1} = F_1 r_t^n + F_3 u_t. \quad (50)$$

\hat{i}_t becomes a signal that simultaneously provides information on both shocks.

Beliefs Formation

The private sector forms expectations about current states through the Kalman filtering process. Denote the unobserved state variables as

$$z_t = \Phi z_{t-1} + v_t \quad (51)$$

where $z_t = [r_t^n, u_t]'$, $\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix}$, and $v_t = [v_t^r, v_t^u]'$ with white noise of variance Q .

Denote the observable signal as

$$s_t = D z_t \quad (52)$$

where $s_t = \hat{i}_t$, and $D = [F_1, F_3]'$.

The Kalman filtering process makes beliefs about the current state variables be the optimal combination of prior beliefs and signals in the current period, which is given by

$$E_t^s z_t = \Phi E_{t-1}^s z_{t-1} + K (s_t - D \Phi E_{t-1}^s z_{t-1}), \quad (53)$$

where the optimal weight, K , is determined by the Ricatti iteration as follows,

$$K = P D' (D P D')^{-1}, \quad (54)$$

$$P = \Phi (P - P D' (D P D')^{-1} D P) \Phi + Q. \quad (55)$$

Solution in the Private Sector under Arbitrary Policy Coefficients

The equilibrium in the private sector is described by the system of equations summarizing private-sector optimization decisions in aggregate variables (equations 10 and 12), the evolution process of the actual shocks (equation 48), the interest rate reaction function (equation 49), and the belief-updating process characterized in equation (53).

To deal with the expectations about future equilibrium variables, I use the method of undetermined coefficients. I first conjecture that \hat{y}_t and π_t are linear functions of the state variables in period t . To economize the use of notations, I use z_t from now on to denote the vector of pre-determined state variables, i.e., $z_t = [r_t^n, u_t, E_{t-1}^s r_{t-1}^n, E_{t-1}^s u_{t-1}]$. The equilibrium output gap and inflation can be written as:

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} = \Gamma z_t \quad (56)$$

This conjecture allows me to write the expected future equilibrium variables in terms of the beliefs about current shocks, $E_t^s r_t^n$ and $E_t^s u_t$ as:

$$\begin{bmatrix} E_t^s \hat{y}_{t+1} \\ E_t^s \pi_{t+1} \end{bmatrix} = \Gamma E_t^s z_{t+1} \quad (57)$$

where $E_t^s z_{t+1} = [\phi E_t^s r_t^n, E_t^s r_t^n, \phi^u E_t^s u_t, E_t^s u_t]$.

Substituting the expression of the expected future output gap and inflation into the IS and the Phillips curve results in expressions of \hat{y}_t and π_t as functions of the actual shocks $[r_t^n, u_t]$ and the associated expectations, $[E_t^s r_t^n, E_t^s u_t]$. Applying the belief-updating process yields the expressions as functions that consist only of predetermined states. (See Appendix B for the detailed derivation.)

5.2 Discretionary Monetary Policy

Due to the persistent learning process, the current interest has a lagged effect. The central bank has *objective expectations* on the lagged effect, as the central bank has perfect information on the realization of shocks and the entire history of beliefs in the private sector. Denote objective expectations by E_t . The information set of the central bank at t includes the entire history of natural-rate and cost-push shocks upon t and the beliefs formed in the private sector upon $t - 1$, i.e.,

$$I_t = \{r_T^n, E_{T-1}^s r_{T-1}^n, u_T, E_{T-1}^s u_{T-1} \quad \forall T = 0 \dots t\}.$$

The central bank's objective expectations about future equilibrium are given by:

$$\begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} = \Gamma E_t z_{t+j}. \quad (58)$$

The evolution of $E_t z_{t+j}$ includes the evolution process of the actual shocks and the expected shocks through the learning process, which is given by

$$\begin{bmatrix} E_t r_{t+j}^n \\ E_t E_{t+j-1}^s r_{t+j-1}^n \\ E_t u_{t+j} \\ E_t E_{t+j-1}^s u_{t+j-1} \end{bmatrix} = \begin{bmatrix} \phi & 0 & 0 & 0 \\ K_{11}F_1 & \phi - K_{11}F_1\phi & K_{11}F_3 & -K_{11}F_3\phi^u \\ 0 & 0 & \phi^u & 0 \\ K_{21}F_1 & -K_{21}F_1\phi & K_{21}F_3 & \phi^u - K_{21}F_3\phi^u \end{bmatrix} \begin{bmatrix} E_t r_{t+j-1}^n \\ E_t E_{t+j-2}^s r_{t+j-2}^n \\ E_t u_{t+j-1} \\ E_t E_{t+j-2}^s u_{t+j-2} \end{bmatrix} \equiv \Lambda E_t z_{t+j-1} \quad (59)$$

where the second row and the fourth row come from the Kalman filtering process.

Combining the auto-regressive process of $E_t z_{t+j}$ and the expectations about the future equilibrium output gap and inflation as functions of $E_t z_{t+j}$ results in:

$$\begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} = \Gamma \Lambda^{j-1} E_t z_t. \quad (60)$$

The current interest rate affects expectations about realization of current shocks, $E_t^s r_t^n$ and $E_t^s u_t$. Due to the Bayesian learning process, the expectations about current shocks affect expectations about future state variables. Consequently, the current interest rate has a lagged effect on the future equilibrium. This mechanism is summarized in the following lemma.

Lemma 2 *With serially correlated shocks, current interest rates affect the future equilibrium through the persistent learning process in the private sector.*

We now turn to how the consideration of the dynamic informational effect changes the equilibrium interest rate under discretion. A discretionary central bank minimizes the expected output gap and inflation deviations in all periods. The central bank's optimization problem can be written as follows:

$$E_t L(t) = E_t [\pi_t^2 + \omega \hat{y}_t^2] + \beta E_t (L(t+1)) \quad (61)$$

$E_t (L(t+1))$ includes inflation and the output gap in all future periods:

$$\begin{aligned} E_t (L(t_1)) &= \sum_{j=1}^{\infty} \beta^j E_t \left\{ \begin{bmatrix} \pi_{t+j} & \hat{y}_{t+j} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_{t+j} \\ \hat{y}_{t+j} \end{bmatrix} \right\} \\ &= \sum_{j=1}^{\infty} \beta^j \left\{ \begin{bmatrix} E_t \pi_{t+j} & E_t \hat{y}_{t+j} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} + \text{indep. terms} \right\} \end{aligned} \quad (62)$$

As the central bank considers the dynamic informational effect of its current interest rate decisions, the objective function can no longer be reduced to the one-period loss function. The following proposition characterizes the equilibrium interest rate under discretionary optimization.

Proposition 2: *With a dynamic informational effect, the optimizing discretionary monetary*

policy is dynamically "leaning against the wind" as it targets a negative correlation between current and future deviations of the output gap and inflation.¹⁸ The consideration of the dynamic informational effect makes the equilibrium interest rate target beliefs in addition to targeting current inflation and the output gap.

To understand this proposition, first express the first-order condition of the central bank's objective function:

$$\left\{ \frac{\partial E_t \pi_t}{\partial i_t^*} E_t \pi_t + \omega \frac{\partial E_t \hat{y}_t}{\partial i_t^*} E_t \hat{y}_t \right\} = -\frac{1}{2} \sum_{j=1}^{\infty} \beta^j \left\{ \frac{\partial E_t \pi_{t+j}}{\partial i_t^*} E_t \pi_{t+j} + \omega \frac{\partial E_t \hat{y}_{t+j}}{\partial i_t^*} E_t \hat{y}_{t+j} \right\} \quad (63)$$

As Lemma 2 indicates, the effect of current interest rates on future equilibrium inflation and the output gap comes entirely on its dynamic informational effect. The first order condition can be written as:

$$\left\{ \frac{\partial E_t \pi_t}{\partial i_t^*} E_t \pi_t + \omega \frac{\partial E_t \hat{y}_t}{\partial i_t^*} E_t \hat{y}_t \right\} + \frac{1}{2} \sum_{j=1}^{\infty} \beta^j \Delta (j-1) = 0 \quad (64)$$

where Δ captures how the current interest rate affects future deviations through its informational effect on $[E_t^s r_t^n, E_t^s u_t]'$. (See the Appendix for the derivations.)

The consideration of the dynamic informational effect consists of two parts. First, as both consumption and pricing decisions are forward-looking, the discretionary central bank takes into account that as it changes current beliefs, it changes expectations about the future equilibrium, which in turn changes the current equilibrium. The first part is captured in the first two terms in equation (64). Second, the central bank also takes into account that by changing current beliefs, it changes the state variables for future periods, which is in addition to stabilizing the current economy. Further characterization of the equilibrium interest rate depends on a numerical solution, which I provide in Section 5.4.

5.3 Monetary Policy Rule

As private agents are forward-looking, committing to a path-dependent policy rule is able to change expectations about the future equilibrium, which leads to the traditionally studied gains from commitment. Different from the static model in which I shut down this traditionally studied gains from commitment, I now allow it to interact with the informational gains from commitment.¹⁹

¹⁸As long as there are shocks that the central bank is unable to completely offset, optimal policy can be described as "leaning against the wind" - seeking a contemporary negative correlation between the output gap and inflation. For discussion about the conventional within-period "leaning against" policy that is caused by informational frictions, see [Adam \(2007\)](#), [Angeletos and La'O \(2011\)](#), and [Tang \(2015\)](#), among others.

¹⁹I need to make an assumption that current interest rates only react to past beliefs and do not directly react to past actual shocks. Otherwise it complicates the information revealed by the current interest rate. Instead, the path dependence is modeled by responding to past beliefs.

Potentially, the policy rule can react to current cost-push shocks to a lesser extent and commits to a large response in later periods. In doing so, not only does the interest rate reveal less information about the current realization of the cost-push shock, but it also decreases expected future inflation through committing to a future tightening policy. The traditional gains from committing to a delayed response strengthen the gains from the informational effect.

The objective function for the committed central bank is the same as that for the discretionary central bank. In equilibrium, the optimal rule follows the same functional form as the discretionary interest rate, i.e., $i_t = F_1 r_t^n + F_2 E_{t-1}^s r_{t-1}^n + F_3 u_t + F_4 E_{t-1}^s u_{t-1}$. The coefficients of the optimal rule, $[F_1, F_2, F_3, F_4]$ are selected to minimize the ex-ante loss from the steady state.²⁰

$$\min_{F_1, F_2, F_3, F_4} E_t L(t) = \int \int (\pi_t^2 + \omega \hat{y}_t^2 + \beta E_t L(t+1)) dr_t^n du_t \quad (65)$$

where the output gap follows equation (10), inflation follows equation (12), actual shocks evolve as equation (34), and beliefs are formed using the Kalman filtering process as specified in equations (39) to (41). Finding the optimal policy rule involves numerical methods. I characterize the optimal policy rule and compare it with the equilibrium interest rate under discretion, in a numerical example in the following section.

5.4 Numerical Illustration

In this section, I provide a numerical example to illustrate how the consideration of the dynamic informational effect changes the equilibrium interest rate under discretion and the gains from committing to the optimal policy rule. In addition, I add external signals that captures the central bank direct communication and implementation errors to the interest rate.

For the evolution of underlying shocks, I set the auto-correlation of natural-rate shocks to be 0.9, with a standard deviation of 3 percent, as measured by [Laubach and Williams \(2003\)](#). There is less consensus on the persistence and volatility of cost-push shocks, as they stem from various sources, and I set the auto-correlation for cost-push shocks to be 0.3. In addition, I set the standard deviation of cost-push shocks and interest rate implementation errors to be the same as the standard deviation of the natural-rate shock. I set the standard deviations in the noise of the external signals to be 0.1 in the impulse response calculation, and use various numbers when quantifying the gains from commitment.

²⁰In the steady state, $E_{t-1}^s r_{t-1}^n = 0$, and $E_{t-1}^s u_{t-1} = 0$.

5.4.1 Impulse Response Analysis

The numerical solution for the dynamic model yields the following results for the equilibrium interest rate under discretion and the optimal policy rule:

$$i_{discretionary} = 1.9658r_t^n - 0.8692E_{t-1}^s r_{t-1}^n + 0.3071u_t + 0.3038E_{t-1}^s u_{t-1}, \quad (66)$$

$$i_{rule} = 1.9308r_t^n - 0.8044E_{t-1}^s r_{t-1}^n + 0.0009u_t + 1.0156E_{t-1}^s u_{t-1}. \quad (67)$$

The associated equilibrium output gap and inflation are shown in the following impulse response to (a) a natural-rate shock, (b) a cost-push shock, and (c) a policy implementation error.

First, the equilibrium responses to the natural rate shock are similar between discretion and commitment. In both cases, the interest rate overshoots the natural rate in the first period, and then decreases in the next period. To understand the intuition, notice that as suggested in the IS curve, the serial correlation enlarges the positive output gap caused by r_t^n , and the $E_t^s r_t^n$ reduces the positive output gap. As a result, the interest rate responds more than one-to-one to r_t^n and responds negatively to $E_t^s r_t^n$.

The responses to cost-push shocks are significantly different between discretion and commitment. In both cases, the interest rate exhibits inertia after the cost-push shocks, which is reflected by a positive response to $E_{t-1}^s u_{t-1}$. However, the degree of inertia is much higher under commitment. In the impulse response figure, the interest rate under commitment has a humped shape - the initial interest rate response is very small, and the central bank commits to a future path of higher interest rates.

In this case, the traditionally studied gains from committing to a delayed response reinforce the informational gains. After a cost-push shock, because the central bank is committed to responding by less in the first period, the interest rate reveals less information about the realization of the shock. In addition, by committing to higher interest rates in future periods, the central bank decreases expected future inflation. As pricing decisions are forward-looking, expectations of lower future inflation decreases current inflation.

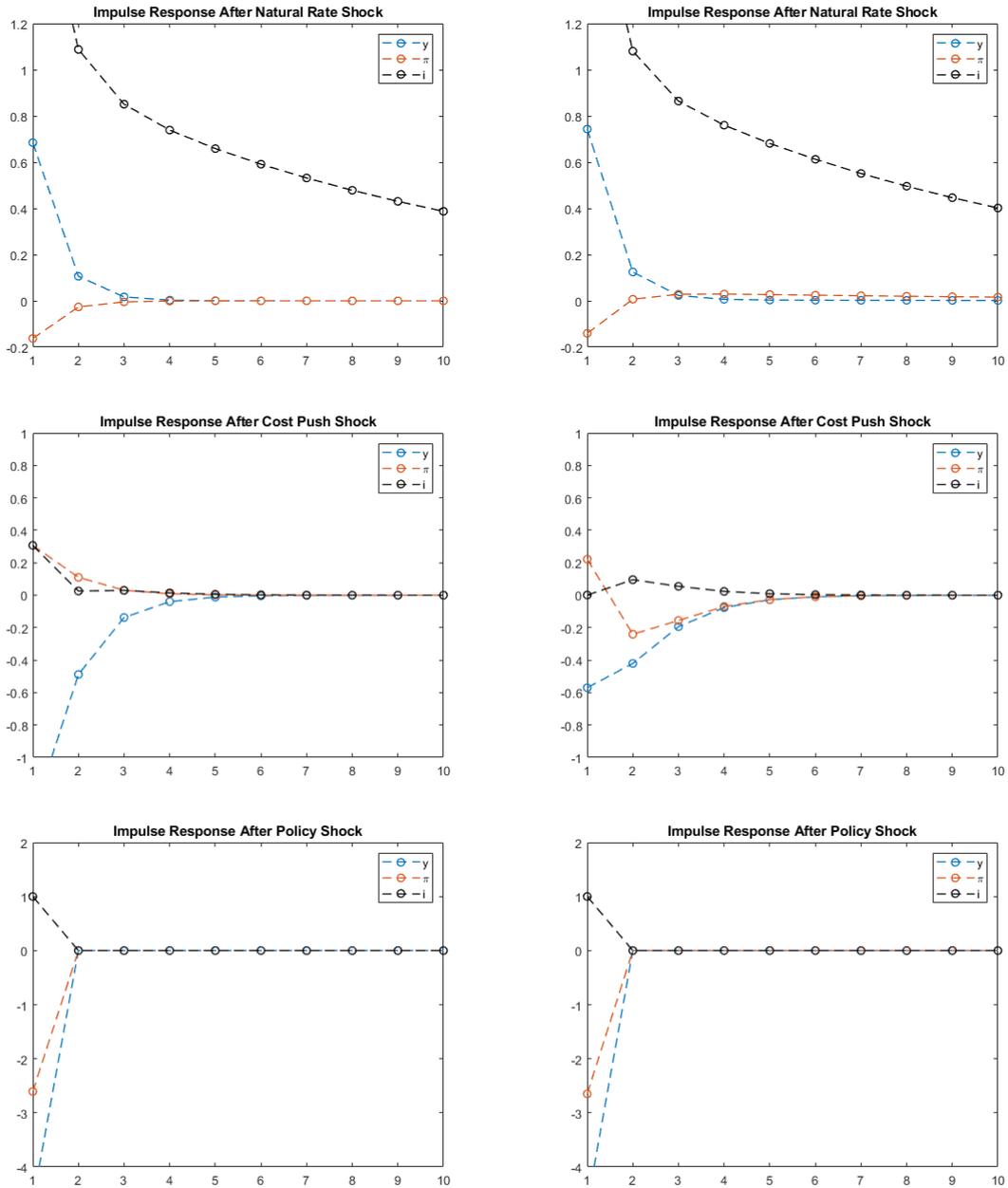


Figure 7: Impulse Response under Discretion (Left) and under Commitment (Right)

5.4.2 The Size of the Gains from Commitment

As Section 4 shows, the gains from commitment depend on the interaction between the informational effect of the interest rate and the information conveyed through external signals. I stop short of calibrating the precision of external signals. Instead, I vary the precision of external signals and calculate the ex-ante loss, the variance of inflation, and the variance of the output gap. Results are shown in the following table.

	Discretionary			Rule		
	$\sigma_\varepsilon = 0.03$	$\sigma_{\varepsilon r} = 1$	$\sigma_\varepsilon = 10$	$\sigma_\varepsilon = 0.03$	$\sigma_{\varepsilon r} = 1$	$\sigma_\varepsilon = 10$
Ex-ante Loss	5.87	3.75	3.74	3.81	3.62	3.60
$var(\pi_t)$	1.67	1.01	1.00	0.76	0.93	0.94
$var(\hat{y}_t)$	28.64	18.71	18.69	14.46	17.75	17.55

Table 1: Gains from Commitment at Varying Levels of Precision of External Information

The ex-ante loss is calculated as the objective function of the central bank. The numbers are denoted in percentage points.

From Table 1, it is apparent that the size of the gains from commitment depends on the precision of external signals. When external signals are very precise (as in the example of $\sigma_\varepsilon = 0.03$), the difference in the ex-ante loss between discretion and commitment is very large. In contrast, when external signals are less precise (as the example of $\sigma_\varepsilon = 10$), the gains from commitment are very small.

6 Conclusion

This paper studies the optimal conduct of monetary policy when the interest rate has an informational effect. This applies when the private sector has imperfect information about the realization of shocks, whereas the central bank has perfect information when making policy decisions.

In this economy, monetary policy has dual effects: The first is the traditionally studied direct effect on the household's cost of borrowing. The second is the informational effect. The value of the informational effect depends on whether the direct effect of the interest rate can achieve the "divine coincidence": Information on the natural-rate shock is beneficial, and information on the cost-push shock is welfare reducing. However, the complication of the model lies in the fact that as the interest rate responds to both shocks, the informational effect on the expected natural-rate shock and the expected cost-push shock cannot be separable.

I studied two types of central bank. A central bank that optimizes the interest rate under discretion takes as given the informational effect of the interest rate. In comparison, a central bank with credible commitment sets a policy rule ex-ante and takes into account that by committing to this rule, it changes the informational effect of the interest rate. It is shown that the optimal policy rule reduces the degree to which the informational effect dampens the direct effect of the interest rate.

The analysis is extended in two ways. First, central bank direct communication is modeled as additional signals independent of the interest rate. The interaction between the information conveyed through communication and the informational effect of the interest rate complicates the welfare implications of central bank communication. More precise communication about the natural-

rate shock might be welfare-reducing, because it makes the interest rate a more precise signal of the natural-rate shock. Second, serially correlated shocks lead to the dynamic informational effect. In this case, the current interest rate affects the future equilibrium through the persistent learning process. Both factors matter in determining the size of the gains from commitment.

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Appendices

A Log-Linearization and Aggregation

From the household's first order conditions, we first do a log-linear approximation to the Euler equation in (A.6) by

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) \quad (\text{A.1})$$

The log-linear approximation to the labor supply of equation (A.7) is $\varphi n_t(j) + \sigma y_t = w_t(j)$ where w_t denotes the log approximated real wage, $\log(W_t/P_t)$. Recall that the resource constraint implies that $c_t^j = y_t^j \forall j$, which further implies $c_t = y_t$. We can then write the labor supply as follows:

$$\varphi n_t(j) + \sigma y_t = w_t(j) \quad (\text{A.2})$$

Next, we want to relate the individual firm's real marginal cost of production to aggregate output. To do this, first integrate equation (A.13):

$$\int w_t(j) = \varphi \int n_t(j) dj + \sigma y_t \quad (\text{A.3})$$

Then, substitute the log-linear approximation of the individual good demand, i.e., $y_t(j) - y_t = -\varepsilon (p_t(j) - p_t)$, which results in:

$$\int n_t(j) dj = y_t + \int (-\varepsilon)(p_t(j) - p_t) - \int a_t(j) = y_t - a_t \quad (\text{A.4})$$

Substitute this into $\int w_t(j)$, and then deduct a_t from both sides:

$$\int w_t(j) - a_t(j) = (\varphi + \sigma)y_t - (1 + \varphi)a_t \quad (\text{A.5})$$

Define the natural level of output as the equilibrium output level without price rigidity and under perfect information, which makes y_t^n a linear function of the aggregate technology. Then, write the above equation in terms of output gap:

$$\int w_t(j) - a_t(j) = (\varphi + \sigma)(y_t - y_t^n) \quad (\text{A.6})$$

We now move on to the firm's side. Taking the log-linear approximation of the individual firm's optimal resetting prices:

$$p_t^*(j) = (1 - \beta\theta)E_t^j \left\{ \sum (\beta\theta)^k [p_{t+k} + u_{t+k}(j) + w_{t+k}(j) - a_{t+k}(j)] \right\} \quad (\text{A.7})$$

The Calvo assumption implies that the aggregate price index is an average of the price charged by the fraction of $1 - \theta$ of firms that reset their prices at t , and the fraction of θ of firms whose prices remain the same as the last period prices. Thus, the log-linear approximation of the aggregate price in period t becomes:

$$p_t = \theta p_{t-1} + (1 - \theta) \int p_t^*(j) dj \quad (\text{A.8})$$

Subtract p_{t-1} from both sides to express in terms of inflation:

$$\pi_t = (1 - \theta) \left(\int p_t^*(j) - p_{t-1} \right) \quad (\text{A.9})$$

As explained in Section 2.3, assume homogeneous subjective beliefs in order to abstract from the higher-order beliefs problem in aggregating prices. This assumption allows me to write the individual resetting prices as:

$$p_t^*(j) = (1 - \beta\theta) (E_t^s p_t + u_t(j) + w_t(j) - a_t(j)) + (1 - \beta\theta) \sum_{k=1}^{\infty} (\beta\theta)^k E_t^s (p_{t+k} + u_{t+k}(j) + w_{t+k} - a_{t+k}) \quad (\text{A.10})$$

Integrate over j :

$$\int p_t^*(j) dj = (1 - \beta\theta) (E_t^s p_t + u_t + w_t - a_t) + (1 - \beta\theta) \sum_{k=1}^{\infty} (\beta\theta)^k E_t^s (p_{t+k} + w_{t+k} - a_{t+k}) \quad (\text{A.11})$$

To write in the difference equation, first calculate:

$$\beta\theta \int E_t^s p_{t+1}^*(j) dj = (1 - \beta\theta) \sum_{k=1}^{\infty} E_t^s (p_{t+k} + u_{t+k} + w_{t+k} - a_{t+k}) = \beta\theta E_t^s p_{t+k}^* \quad (\text{A.12})$$

The second equation holds due to homogeneous beliefs.

Subtract equation (A. 23) from equation (A. 22):

$$\begin{aligned} \int p_t^*(j) dj - \beta\theta E_t^s p_{t+1} &= (1 - \beta\theta) E_t^s p_t + (1 - \beta\theta) u_t + (1 - \beta\theta) (\varphi + \sigma) \hat{y}_t \quad (\text{A.13}) \\ \int p_t^*(j) dj - p_{t-1} &= \beta\theta (E_t^s p_{t+1}^* - E_t^s p_t) + E_t^s p_t - p_{t-1} + (1 - \beta\theta) u_t + (1 - \beta\theta) (\varphi + \sigma) \hat{y}_t \\ \pi_t &= \beta\theta E_t^s \pi_{t+1} + (1 - \theta) E_t^s \pi_t + (1 - \theta) (1 - \beta\theta) u_t + (1 - \beta\theta) (1 - \theta) (\varphi + \sigma) \hat{y}_t \end{aligned}$$

In the last equation, I assume that the aggregate price is observable after one period, i.e., $p_{t-1} = E_t^s p_{t-1}$

Write inflation as:

$$\pi_t = \beta\theta E_t^s \pi_{t+1} + (1 - \theta) E_t^s \pi_t + \kappa\theta \hat{y}_t + u_t \quad (\text{A.14})$$

where $\kappa = \frac{(1 - \beta\theta)(1 - \theta)(\varphi + \sigma)}{\theta}$, and $u_t = (1 - \theta)(1 - \beta\theta) u_t$

B Solution to the Markov Perfect Equilibrium under Discretionary Monetary Policy

In this section, I first solve the model with serially uncorrelated shocks and then solve the model with serially correlated shocks. For both cases, I solve for the fixed point where the beliefs by people in the private sector about the best response of the interest rate in any state match the optimizing discretionary interest rate. This means that, in equilibrium, people have rational expectations.

B.1 Equilibrium Optimizing Discretionary Policy with Serially Uncorrelated Shocks

Summary of the iteration process:

1. I conjecture that the interest rate reacts linearly to both shocks, i.e., $i_t = F_r^0 r_t^n + F_u^0 u_t$.
2. With this interest rate, I solve for the beliefs formed about natural-rate shock and cost-push shock in the private sector as functions of i_t .
3. With beliefs formed in the private sector, $E_t^s r_t^n$ and $E_t^s u_t$, the actual shocks, r_t^n and u_t , I solve for \hat{y}_t and π_t as a function of i_t , r_t^n and u_t .
4. Solve for i_t that minimizes the loss function, $L_t = \pi_t^2 + \omega \hat{y}_t$, and express the interest rate as actual shocks, $i_t = F_r r_t^n + F_u u_t$.
5. Check if $F_r = F_r^0$ and $F_u = F_u^0$. If not, go back to step 1 and update the values of F_r^0 and F_u^0 in the conjectured function. Iterate the process until convergence.

Details are given as follows:

In step 1, $i_t = F_r^0 r_t^n + F_u^0 u_t$.

In step 2, beliefs about underlying shocks follow:

$$E_t^s r_t^n = K_r i_t \tag{B.1}$$

$$E_t^s u_t = K_u i_t \tag{B.2}$$

where $K_r F_r^0 = \frac{F_r^{02} \sigma_r^2}{F_r^{02} \sigma_r^2 + F_u^{02} \sigma_u^2}$, and $K_u F_u^0 = \frac{F_u^{02} \sigma_u^2}{F_r^{02} \sigma_r^2 + F_u^{02} \sigma_u^2}$.

In step 3, write out the expression of the output gap and inflation as functions of the interest rate:

$$\hat{y}_t = -\frac{1}{\sigma} (i_t - r_t^n) \tag{B.3}$$

$$\pi_t = \kappa \hat{y}_t + (1 - \theta) \frac{\kappa}{\sigma} (E_t^s r_t^n(i_t) - r_t^n) + \frac{1 - \theta}{\theta} E_t^s u_t(i_t) + u_t \tag{B.4}$$

In step 4, I first write out the first-order conditions of the interest rate:

$$\pi_t \frac{\partial \pi_t}{\partial i_t} + \omega \hat{y}_t \frac{\partial \hat{y}_t}{\partial i_t} = 0 \quad (\text{B.5})$$

Substitute π_t and \hat{y}_t by equation (B.3) and (B.4):

$$\left\{ (1 - \theta) \frac{\kappa}{\sigma} (E_t^s r_t^n - r_t^n) + \frac{1 - \theta}{\theta} E_t^s u_t + u_t \right\} \frac{\partial \pi_t}{\partial i_t} + \left(\omega \frac{\partial \hat{y}_t}{\partial i_t} + \kappa \frac{\partial \pi_t}{\partial i_t} \right) \left\{ -\frac{1}{\sigma} (i_t - r_t^n) \right\} = 0 \quad (\text{B.6})$$

Substituting $E_t^s r_t^n$ and $E_t^s u_t$ as i_t leads to:

$$\lambda_1 r_t^n + \lambda_2 u_t + \lambda_3 i_t = 0 \quad (\text{B.7})$$

where

$$\begin{aligned} \lambda_1 &= \left\{ \left(\kappa \frac{\partial \pi_t}{\partial i_t} + \omega \frac{\partial \hat{y}_t}{\partial i_t} \right) \frac{1}{\sigma} - \frac{\partial \pi_t}{\partial i_t} (1 - \theta) \frac{\kappa}{\sigma} \right\} \\ \lambda_2 &= \frac{\partial \pi_t}{\partial i_t} \\ \lambda_3 &= \frac{\partial \pi_t}{\partial i_t} (1 - \theta) \frac{\kappa}{\sigma} K_{11} + \frac{\partial \pi_t}{\partial i_t} \frac{1 - \theta}{\theta} K_{21} - \left(\kappa \frac{\partial \pi_t}{\partial i_t} + \omega \frac{\partial \hat{y}_t}{\partial i_t} \right) \frac{1}{\sigma} \end{aligned}$$

and partial derivatives are given by:

$$\begin{aligned} \frac{\partial \hat{y}_t}{\partial i_t} &= -\frac{1}{\sigma} \\ \frac{\partial \pi_t}{\partial i_t} &= -\frac{\kappa}{\sigma} + (1 - \theta) \frac{\kappa}{\sigma} K_r + \frac{1 - \theta}{\theta} K_u \end{aligned}$$

Rearranging the above equation to get:

$$i_t = F_1 r_t^n + F_3 u_t \quad (\text{B.8})$$

where $F_1 = -\frac{\lambda_1}{\lambda_3}$, and $F_3 = -\frac{\lambda_2}{\lambda_3}$.

In step 5, update the initial conjectured policy function and iterate the above process until $F_r = F_r^0$ and $F_u = F_u^0$.

B.2 Equilibrium Optimizing Discretionary Policy with Serially Correlated Shocks

In this section, I solve for the general version of the model where I have serially correlated shocks, external signals that capture central bank direct communication and implementation errors.

Due to the dynamic learning process, expectations about the future equilibrium play a role in current decisions. I use the method of undetermined coefficients to solve the subjective expectations formed by private agents. I am then able to express current inflation and the output gap in terms of actual shocks and beliefs in the current period. The solution of the equilibrium interest rate under discretion follows a process similar to that for the static case, in which I first conjecture the policy function for the interest rate, and then find the fixed point between the initial guess and the interest rate found in the central bank's optimization problem. The summary of the iteration process is given as follows:

1. I conjecture that the interest rate reacts linearly to predetermined state variables, which include current actual shocks and beliefs in the last period, i.e., $i_t = F_1^0 r_t^n + F_2^0 E_{t-1}^s r_{t-1}^n + F_3^0 u_t + F_4^0 E_{t-1}^s u_{t-1}$.
2. With this interest rate, I solve for the beliefs about the natural-rate shock and the cost-push shock in this period, $E_t^s r_t^n$ and $E_t^s u_t$.
3. (Undetermined Coefficient) I conjecture that the output gap and inflation are linear functions of current state variables, which include actual shocks and past beliefs. Due to the serial correlation in shocks and the conjectured linear relationship, I am able to express the expected future output gap and inflation as functions of $[E_t^s r_t^n, E_t^s u_t]$.
4. I solve for \hat{y}_t and π_t as a function of i_t and other predetermined state variables.
5. Solve for i_t that minimizes the loss function, $L_t = \pi_t^2 + \omega \hat{y}_t$, and express the interest rate as actual shocks, $i_t = F_r r_t^n + F_u u_t$.
6. Check if $F = F_0$. If not, go back to step 1 and update the initial guess of the coefficients in the policy function. Iterate the process until convergence.

Specifically, in step 1, I conjecture that $i_t = F_1 r_t^n + F_r E_{t-1}^s r_{t-1}^n + F_3 u_t + F_4 E_{t-1}^s u_{t-1}$.

In step 2, to solve the beliefs formed in the private sector, I first specify the evolution of actual shocks:

State:

$$\begin{bmatrix} r_t^n \\ u_t \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u u_t \end{bmatrix} + \begin{bmatrix} v_t \\ v_t^u \end{bmatrix} \quad (\text{B.9})$$

which I denote as $z_t = \Phi z_{t-1} + v_t$, where $\Phi = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix}$ and $v_t = [v_t, v_t^u]$ with the white noise of variance Q .

Signals

As private agents have perfect memory of beliefs they have had in the past, they are able to back out the part of the interest rate that reacts to current shocks, which I denote as

$$\hat{i}_t \equiv i_t - F_2 E_{t-1}^s r_{t-1}^n - F_4 E_{t-1}^s u_{t-1} \quad (\text{B.10})$$

All signals are summarized as

$$\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} = \begin{bmatrix} F_1 & F_3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_t^n \\ u_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_t \\ \varepsilon_t^r \\ \varepsilon_t^u \end{bmatrix}, \quad (\text{B.11})$$

which I denote as $s_t = Dz_t + R_t$

Beliefs

People in the private sector are Bayesian, and update their beliefs through the Kalman filtering process, in which they optimally weigh between all current signals and past beliefs by their precision. The beliefs follow:

$$\begin{bmatrix} E_t^s r_t^n \\ E_t^s u_t \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} E_{t-1}^s r_{t-1}^n \\ E_{t-1}^s u_{t-1} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \end{bmatrix} \left(\begin{bmatrix} \hat{i}_t \\ m_t^r \\ m_t^u \end{bmatrix} - \begin{bmatrix} F_1 & F_3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \phi & 0 \\ 0 & \phi^u \end{bmatrix} \begin{bmatrix} E_{t-1}^s r_{t-1}^n \\ E_{t-1}^s u_{t-1} \end{bmatrix} \right) \quad (\text{B.12})$$

Write out the expression for \hat{i}_t and collect terms:

$$\begin{aligned} E_t^s r_t^n &= (K_{11}F_1 + K_{12})r_t^n + \phi(1 - K_{11}F_1 - K_{12})E_{t-1}^s r_{t-1}^n \\ &\quad + (K_{11}F_3 + K_{13})u_t + \phi^u(-K_{11}F_3 - K_{13})E_{t-1}^s u_{t-1} + K_{12}\varepsilon_t^r + K_{13}\varepsilon_t^u + K_{11}e_t \end{aligned} \quad (\text{B.13})$$

$$\begin{aligned} E_t^s u_t &= (K_{21}F_1 + K_{22})r_t^n + \phi(-K_{21}F_1 - K_{22})E_{t-1}^s r_{t-1}^n \\ &\quad + (K_{21}F_3 + K_{23})u_t + \phi^u(1 - K_{21}F_3 - K_{23})E_{t-1}^s u_{t-1} + K_{22}\varepsilon_t^r + K_{23}\varepsilon_t^u + K_{21}e_t \end{aligned} \quad (\text{B.14})$$

Denote the above equations as $E_t^s r_t^n = \Psi(1)r_t^n + \Psi(2)E_{t-1}^s r_{t-1}^n + \Psi(3)u_t + \Psi(4)E_{t-1}^s u_{t-1} + \Psi(5)\varepsilon_t^r + \Psi(6)\varepsilon_t^u + \Psi(7)e_t$, and $E_t^s u_t = \Psi(8)r_t^n + \Psi(9)E_{t-1}^s r_{t-1}^n + \Psi(10)u_t + \Psi(11)E_{t-1}^s u_{t-1} + \Psi(12)\varepsilon_t^r + \Psi(13)\varepsilon_t^u + \Psi(14)e_t$. I will use this notation in solving the equilibrium in the private sector by the method of undetermined coefficients.

In step 3, I first write out the the forward-looking output gap and inflation as:

$$\hat{y}_t = E_t^s \hat{y}_{t+1} - \frac{1}{\sigma} \left[i_t - \left(\frac{1}{1-\phi} r_t^n - \frac{\phi}{1-\phi} E_t^s r_t^n \right) - E_t^s \pi_{t+1} \right] \quad (\text{B.15})$$

$$\pi_t = \beta \theta E_t^s \pi_{t+1} + (1-\theta) E_t^s \pi_t + \kappa \theta \hat{y}_t + u_t \quad (\text{B.16})$$

Following the method of undetermined coefficients, I first need to conjecture that equilibrium variables are linear functions to current state variables, which include current actual shocks (r_t^n, u_t) , past beliefs $(E_{t-1}^s r_{t-1}^n, E_{t-1}^s u_{t-1})$, and noise in current signals $(\varepsilon_t^r, \varepsilon_t^u, e_t)$.

$$\begin{bmatrix} \hat{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ \gamma_8 & \gamma_9 & \gamma_{10} & \gamma_{11} \end{bmatrix} \begin{bmatrix} r_t^n \\ E_{t-1}^s r_{t-1}^n \\ u_t \\ E_{t-1}^s u_{t-1} \end{bmatrix} + \begin{bmatrix} \gamma_5 & \gamma_6 & \gamma_7 \\ \gamma_{12} & \gamma_{13} & \gamma_{14} \end{bmatrix} \begin{bmatrix} \varepsilon_t^r \\ \varepsilon_t^u \\ e_t \end{bmatrix} \quad (\text{B.17})$$

Next, substitute this conjecture into the forward-looking variables, $E_t \hat{y}_{t+1}$ and $E_t^s \pi_{t+1}$. Notice that the noise in all signals is temporary, so is expected to be zero in the future period.

$$\begin{bmatrix} E_t^s \hat{y}_{t+1} \\ E_t^s \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \gamma_1 \phi + \gamma_2 & \gamma_3 \phi^u + \gamma_4 \\ \gamma_8 \phi + \gamma_9 & \gamma_{10} \phi^u + \gamma_{11} \end{bmatrix} \begin{bmatrix} E_t^s r_t^n \\ E_t^s u_t \end{bmatrix} \quad (\text{B.18})$$

First substitute this into the output gap expression:

$$\hat{y}_t = \left[(\gamma_1 \phi + \gamma_2) + \frac{1}{\sigma} (\gamma_8 \phi + \gamma_9) - \frac{1}{\sigma} \frac{\phi}{1-\phi} \right] E_t^s r_t^n \quad (\text{B.19})$$

$$+ \left[(\gamma_3 \phi^u + \gamma_4) + \frac{1}{\sigma} (\gamma_{10} \phi^u + \gamma_{11}) \right] E_t^s u_t - \frac{1}{\sigma} i_t + \frac{1}{\sigma} \frac{1}{1-\phi} r_t^n \quad (\text{B.20})$$

Next work on π_t , as the actual inflation also includes the expected current inflation, and expected current inflation includes the expected current output gap, I first need to calculate:

$$E_t^s \hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma} [i_t - E_t^s r_t^n - E_t^s \pi_{t+1}] \quad (\text{B.21})$$

$$E_t^s \pi_t = \beta E_t^s \pi_{t+1} + \kappa \left\{ E_t^s \hat{y}_{t+1} - \frac{1}{\sigma} [i_t - E_t^s r_t^n - E_t^s \pi_{t+1}] \right\} + \frac{1}{\theta} E_t^s u_t \quad (\text{B.22})$$

Substitute $E_t^s \pi_t$ into π_t :

$$\begin{aligned} \pi_t = & \beta \theta E_t^s \pi_{t+1} + (1 - \theta) \left\{ \beta E_t^s \pi_{t+1} + \kappa E_t^s \hat{y}_t + \frac{1}{\theta} E_t^s u_t \right\} + \kappa \theta \hat{y}_t + u_t & \text{(B.23)} \\ & + \left\{ (1 - \theta) \kappa (\gamma_3 \phi^u + \gamma_4) + \frac{1 - \theta}{\theta} + \left(\beta + (1 - \theta) \frac{\kappa}{\sigma} \right) (\gamma_{10} \phi^u + \gamma_{11}) \right\} E_t^s u_t - (1 - \theta) \frac{\kappa}{\sigma} i_t + \kappa \theta \hat{y}_t + u_t \end{aligned}$$

The values of γ can be solved in the following matrix:

In step 5, in order to solve for the optimizing interest rate, I first need to specify the central bank's objective function.

Central Bank Objective Function

As the current interest rate has persistent effect through the dynamic learning process, the central bank also considers how the current interest rate affects the future equilibrium. Consequently, the loss function includes the output gap and inflation in the current and all future periods.

$$E_t L(t) = [\pi_t^2 + \omega \hat{y}_t^2] + \beta E_t(L(t+1)) \quad (\text{B.24})$$

where

$$\begin{aligned} E_t(L(t+1)) &= \sum_{j=1}^{\infty} \beta^j E_t \left\{ \begin{bmatrix} \pi_{t+1} & \hat{y}_{t+j} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \pi_{t+j} \\ \hat{y}_{t+j} \end{bmatrix} \right\} \\ &= \sum_{j=1}^{\infty} \beta^j \left\{ \begin{bmatrix} E_t \pi_{t+1} & E_t \hat{y}_{t+j} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} + \text{indep. terms} \right\} \end{aligned} \quad (\text{B.25})$$

The central bank's expectation is *objective*, denoted by E_t , in the sense that it observes all past shocks, and expects all future shocks to be zero. The information set of the central bank at period t is:

$$I_t = \{r_T^n, u_T, \forall T = 0 \dots t\}$$

Let $z_t = [r_t^n, E_{t-1}^s r_{t-1}^n, u_t, E_{t-1}^s u_{t-1}]'$ denote the persistent state variables. So the central bank's objective expectation of the future period output gap and inflation becomes a linear function of $E_t z_{t+j}$:

$$\begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} = \begin{bmatrix} \gamma_8 & \gamma_9 & \gamma_{10} & \gamma_{11} \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \end{bmatrix} E_t z_{t+j} \equiv \Gamma E_t z_{t+j} \quad (\text{B.26})$$

$E_t z_{t+j}$ follows:

$$\begin{bmatrix} E_t r_{t+j}^n \\ E_t E_{t+j-1}^s r_{t+j-1}^n \\ E_t u_{t+j} \\ E_t E_{t+j-1}^s u_{t+j-1} \end{bmatrix} = \begin{bmatrix} \phi & 0 & 0 & 0 \\ K_{11}F_1 + K_{12} & \phi(1 - K_{11}F_1 - K_{12}) & K_{11}F_3 + K_{13} & -\phi^u(K_{11}F_3 + K_{13}) \\ 0 & 0 & \phi^u & 0 \\ K_{21}F_1 + K_{22} & -\phi(K_{21}F_1 + K_{22}) & K_{21}F_3 + K_{23} & \phi^u(1 - K_{21}F_3 - K_{23}) \end{bmatrix} \begin{bmatrix} E_t r_{t+j-1}^n \\ E_t E_{t+j-2}^s r_{t+j-2}^n \\ E_t u_{t+j-1} \\ E_t E_{t+j-2}^s u_{t+j-2} \end{bmatrix} \quad (\text{B.27})$$

$$\begin{bmatrix} E_t \pi_{t+j} \\ E_t \hat{y}_{t+j} \end{bmatrix} = \Gamma \Lambda^{j-1} E_t z_{t+1} \quad (\text{B.28})$$

Substitute into $E_t(L(t+1))$:

$$\sum \beta^j E_t z'_{t+1} (\Lambda^{j-1})' \Gamma' \Omega \Gamma \Lambda^{j-1} E_t z_{t+1} \equiv \sum \beta^j E_t z'_{t+1} \Theta_{j-1} E_t z_{t+1} \quad (\text{B.29})$$

Take the first-order condition on i_t^* of $E_t L(t+1)$:

$$\left\{ \frac{\partial E_t \pi_t}{\partial i_t^*} E_t \pi_t + \omega \frac{\partial E_t \hat{y}_t}{\partial i_t^*} E_t \hat{y}_t \right\} + \frac{1}{2} \sum_{j=1}^{\infty} \beta^j \Delta(j-1) = 0 \quad (\text{B.30})$$

where

$$\begin{aligned} \Delta_{j-1} &= (\Theta_{j-1}^{21} + \Theta_{j-1}^{12}) \phi r_t^n \frac{\partial E_t^s r_t^n}{\partial i_t} + (\Theta_{j-1}^{32} + \Theta_{j-1}^{23}) \phi^u u_t \frac{\partial E_t^s r_t^n}{\partial i_t} + (\Theta_{j-1}^{42} + \Theta_{j-1}^{24}) E_t^s u_t \frac{\partial E_t^s r_t^n}{\partial i_t} \\ &+ \Theta_{j-1}^{22} \cdot 2 E_t^s r_t^n \frac{\partial E_t^s r_t^n}{\partial i_t} + (\Theta_{j-1}^{41} + \Theta_{j-1}^{14}) \phi r_t^n \frac{\partial E_t^s u_t}{\partial i_t} + (\Theta_{j-1}^{43} + \Theta_{j-1}^{34}) \phi^u u_t \frac{\partial E_t^s u_t}{\partial i_t} \\ &+ (\Theta_{j-1}^{42} + \Theta_{j-1}^{24}) E_t^s r_t^n \frac{\partial E_t^s u_t}{\partial i_t} + \Theta_{j-1}^{44} \cdot 2 E_t^s u_t \frac{\partial E_t^s u_t}{\partial i_t} \\ &\equiv \Delta_{j-1}(1) r_t^n + \Delta_{j-1}(2) u_t + \Delta_{j-1}(3) E_t^s u_t + \Delta_{j-1}(4) E_t^s r_t^n \\ &+ \Delta_{j-1}(5) r_t^n + \Delta_{j-1}(6) u_t + \Delta_{j-1}(7) E_t^s r_t^n + \Delta_{j-1}(8) E_t^s u_t \end{aligned}$$

To solve for the first-order condition on the interest rate, first write the equilibrium variables in terms of i_t :

Beliefs:

$$E_t^s r_t^n = (\phi(1 - K_{11} F_1 - K_{12}) - K_{11} F_2) E_{t-1}^s r_{t-1}^n - (K_{11} F_4 + \phi^u (K_{11} F_3 + K_{13})) E_{t-1}^s u_{t-1} + K_{12} r_t^n + K_{13} u_t + K_{11} i_t \quad (\text{B.31})$$

$$E_t^s u_t = (\phi^u (1 - K_{21} F_3 - K_{23}) - K_{21} F_4) E_{t-1}^s u_{t-1} - (\phi (K_{21} F_1 + K_{22}) + K_{21} F_2) E_{t-1}^s r_{t-1}^n + K_{22} r_t^n + K_{23} u_t + K_{21} i_t \quad (\text{B.32})$$

$$+ K_{22} r_t^n + K_{23} u_t + K_{21} i_t \quad (\text{B.33})$$

Output gap:

$$\hat{y}_t = \Xi(1) E_t^s r_t^n + \Xi(2) E_t^s u_t - \frac{1}{\sigma} i_t + \frac{1}{\sigma} \frac{1}{1 - \phi} r_t^n \quad (\text{B.34})$$

Inflation:

$$\pi_t = \kappa \theta \hat{y}_t + \Xi(3) E_t^s r_t^n + \Xi(4) E_t^s u_t - (1 - \theta) \frac{\kappa}{\sigma} i_t + u_t \quad (\text{B.35})$$

Substitute the above endogenous variables into the first order condition on i_t :

$$\lambda_1 E_t^s r_t^n + \lambda_2 E_t^s u_t + \lambda_3 r_t^n + \lambda_4 u_t + \lambda_5 i_t = 0 \quad (\text{B.36})$$

where

$$\lambda_1 = \left(\kappa\theta \frac{\partial \pi_t}{\partial i_t} + \omega \frac{\partial \hat{y}_t}{\partial i_t} \right) \Xi(1) + \frac{\partial \pi_t}{\partial i_t} \Xi(3) + \frac{1}{2} \Sigma \beta^j (\Delta_{j-1}(4) + \Delta(7)) \quad (\text{B.37})$$

$$\lambda_2 = \left(\kappa\theta \frac{\partial \pi_t}{\partial i_t} + \omega \frac{\partial \hat{y}_t}{\partial i_t} \right) \Xi(2) + \frac{\partial \pi_t}{\partial i_t} \Xi(4) + \frac{1}{2} \Sigma \beta^j (\Delta_{j-1}(3) + \Delta(8)) \quad (\text{B.38})$$

$$\lambda_3 = \left(\kappa\theta \frac{\partial \pi_t}{\partial i_t} + \omega \frac{\partial \hat{y}_t}{\partial i_t} \right) \frac{1}{\sigma} \frac{1}{1-\phi} + \frac{1}{2} \Sigma \beta^j (\Delta_{j-1}(1) + \Delta(5)) \quad (\text{B.39})$$

$$\lambda_4 = \frac{\partial \pi_t}{\partial i_t} + \frac{1}{2} \Sigma \beta^j (\Delta(2) + \Delta(6)) \quad (\text{B.40})$$

$$\lambda_5 = \left(\kappa\theta \frac{\partial \pi_t}{\partial i_t} + \omega \frac{\partial \hat{y}_t}{\partial i_t} \right) \left(-\frac{1}{\sigma} \right) + \frac{\partial \pi_t}{\partial i_t} \left(-(1-\theta) \frac{\kappa}{\theta} \frac{1}{\sigma} \right) \quad (\text{B.41})$$

and partial derivatives are derived as:

$$\frac{\partial E_t^s r_t^n}{\partial i_t} = K_{11} \quad (\text{B.42})$$

$$\frac{\partial E_t^s u_t}{\partial i_t} = K_{21} \quad (\text{B.43})$$

$$\frac{\partial \hat{y}_t}{\partial i_t} = \Xi(1) \frac{\partial E_t^s r_t^n}{\partial i_t} + \Xi(2) \frac{\partial E_t^s u_t}{\partial i_t} - \frac{1}{\sigma} \quad (\text{B.44})$$

$$\frac{\partial \pi_t}{\partial i_t} = \kappa\theta \frac{\partial \hat{y}_t}{\partial i_t} + \Xi(3) \frac{\partial E_t^s r_t^n}{\partial i_t} + \Xi(4) \frac{\partial E_t^s u_t}{\partial i_t} - (1-\theta) \frac{\kappa}{\sigma} \quad (\text{B.45})$$

Further substitute $E_t^s r_t^n$ and $E_t^s u_t$:

$$\begin{aligned} 0 = & \lambda_1 \{ (\phi(1 - K_{11}F_1 - K_{12}) - K_{11}F_2) E_{t-1}^s r_{t-1}^n - (K_{11}F_4 + \phi^u(K_{11}F_3 + K_{13})) E_{t-1}^s u_{t-1} + K_{12}r_t^n + K_{13}u_t + K_{11}i_t \} \\ & + \lambda_2 \{ (\phi^u(1 - K_{21}F_3 - K_{23}) - K_{21}F_4) E_{t-1}^s u_{t-1} - (\phi(K_{21}F_1 + K_{22}) + K_{21}F_2) E_{t-1}^s r_{t-1}^n + K_{22}r_t^n + K_{23}u_t + K_{21}i_t \} \\ & + \lambda_3 r_t^n + \lambda_4 u_t + \lambda_5 i_t \end{aligned}$$

The above equation solves the optimal nominal interest rate. Comparing it with the guessed form yields the solution of $[F_1, F_2, F_3, F_4]$.

$$F_1 = -\frac{\lambda_1 K_{12} + \lambda_2 K_{22} + \lambda_3}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \quad (\text{B.46})$$

$$F_2 = -\frac{\lambda_1 (\phi(1 - K_{11}F_1 - K_{12}) - K_{12}F_2) - \lambda_2 (\phi(K_{21}F_1 + K_{22}) + K_{21}F_2)}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \quad (\text{B.47})$$

$$F_3 = -\frac{\lambda_1 K_{13} + \lambda_2 K_{23} + \lambda_4}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \quad (\text{B.48})$$

$$F_4 = -\frac{-\lambda_1 (K_{11}F_4 + \phi''(K_{11}F_3 + K_{13})) + \lambda_2 (\phi''(1 - K_{21}F_3 - K_{23}) - K_{21}F_4)}{\lambda_1 K_{11} + \lambda_2 K_{21} + \lambda_5} \quad (\text{B.49})$$

I iterate the process until the conjectured interest rate function matches the above solution.

C Proofs

C.1 Second-Order Approximation of the Household's Utility Function

Follow [Woodford \(2011\)](#), [Galí \(2015\)](#), [Walsh \(2017\)](#) to prove that maximizing the utility of the household is equivalent, up to a second-order approximation, to

$$W = -\frac{1}{2}E_0\Sigma\beta^t \left((\varepsilon^{-1} + \varphi)\varepsilon^2 \text{var}_j(p_t(j)) + (\sigma + \varphi)\hat{y}_t^2 \right) \quad (\text{C.1})$$

The next step is to prove the relationship between $\text{var}_j(p_t(j))$ with $\text{var}(\pi_t)$. Denote $\Delta_t = \text{var}_j[\log p_{jt}]$. Since $\text{var}_j\bar{P}_{t-1} = 0$, we have

$$\begin{aligned} \Delta_t &= \text{var}_j[\log p_{jt} - \bar{P}_{t-1}] \\ &= E_j[\log p_{jt} - \bar{P}_{t-1}]^2 - [E_j \log p_{jt} - \bar{P}_{t-1}]^2 \\ &= E_j[\log p_{jt-1} - \bar{P}_{t-1}]^2 + (1 - \theta) \left(\int p_{tj}^* - \bar{P}_{t-1} \right)^2 - (\bar{P}_t - \bar{P}_{t-1})^2 \end{aligned} \quad (\text{C.2})$$

As noted in Appendix A, $\bar{P}_t = (1 - \theta) \int p_{tj}^* + \theta \bar{P}_{t-1}$, we have $(1 - \theta) \int \log p_{tj}^* + \theta \bar{p}_{t-1}$, which implies that $(1 - \theta) \int \log p_{tj}^* - (1 - \theta) \bar{p}_{t-1} = \bar{p} - \bar{p}_{t-1}$. So, we have:

$$\int \log p_{tj}^* = \left(\frac{1}{1 - \theta} \right) (\bar{p} - \bar{p}_{t-1}) \quad (\text{C.3})$$

Substitute this into (D.2) and get $\Delta_t = \theta \omega_{t-1} + \left(\frac{\theta}{1 - \theta} \right) (\bar{p} - \bar{p}_{t-1})^2$. Applying the definition of inflation results in:

$$E_t \Sigma \beta^t \Delta_t = \frac{\theta}{(1 - \theta)(1 - \theta \beta)} E_t \Sigma \beta^t \pi_t^2 + t.i.p. \quad (\text{C.4})$$

C.2 Proposition 1

To show that the gains from commitment are *strictly* positive, one needs to show that (F_r, F_u) chosen under commitment is different from the solution to the discretionary optimization problem. (If they are equal, then the size of the gains from commitment is zero.)

To show this, write out the first-order condition on F_r after the r_t^n shock as:

$$\underbrace{-\frac{\kappa}{\sigma}}_{\text{direct}} + \underbrace{\Omega_r K_r + \Omega_u K_u}_{\text{informational}} + \underbrace{\Omega_r \frac{\partial K_r}{\partial F_r} F_r + \Omega_u \frac{\partial K_u}{\partial F_r} F_r}_{\text{change of informational effect}} = \frac{\omega \hat{y}_t}{\sigma \pi_t}, \quad (\text{C.5})$$

where $\Omega_r = (1 - \theta) \frac{\kappa}{\sigma}$, and $\Omega_u = \frac{1 - \theta}{\theta} K_u$. Similarly, the first-order condition on F_u after the u_t shock

is

$$\underbrace{-\frac{\kappa}{\sigma}}_{\text{direct}} + \underbrace{\Omega_r K_r + \Omega_u K_u}_{\text{informational}} + \underbrace{\Omega_r \frac{\partial K_r}{\partial F_u} F_u + \Omega_u \frac{\partial K_u}{\partial F_u} F_u}_{\text{change of informational effect}} = \frac{\omega \hat{y}_t}{\sigma \pi_t} \quad (\text{C.6})$$

In comparison, rewriting the first-order condition for the discretionary central bank in the same form yields:

$$\underbrace{-\frac{\kappa}{\sigma}}_{\text{direct}} + \underbrace{\Omega_r K_r + \Omega_u K_u}_{\text{informational}} = \frac{\omega \hat{y}_t}{\sigma \pi_t} \quad (\text{C.7})$$

The differences between equations (C.5) and (C.6) and equation (C.7) show that the central bank with commitment internalizes the change in the informational effect of the interest rate, whereas the discretionary central bank takes the informational effect as exogenous.