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Inflation, Debt, and Default

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We study how the co-movement of inflation and economic activity affects real interest rates and the likelihood of debt crises. First, we show that for advanced economies, periods with procyclical inflation are associated with lower real interest rates. Procyclical inflation implies that nominal bonds pay out more in bad times, making them a good hedge against aggregate risk. However, such procyclicality also increases sovereign default risk when the economy deteriorates, since the government needs to make larger (real) payments. In order to evaluate both effects, we develop a model of sovereign default on domestic nominal debt with exogenous inflation risk and domestic risk-averse lenders. Countercyclical inflation is a *substitute* with default, while procyclical inflation is a *complement* with it, by increasing default incentives. In good times, when default is unlikely, procyclical inflation yields lower real rates. In bad times, as default becomes more material, procyclical inflation can magnify default risk and trigger an increase in real rates.

Keywords: inflation risk, domestic nominal debt, interest rates, sovereign default.

JEL classification codes: E31, F34, G12, H63.

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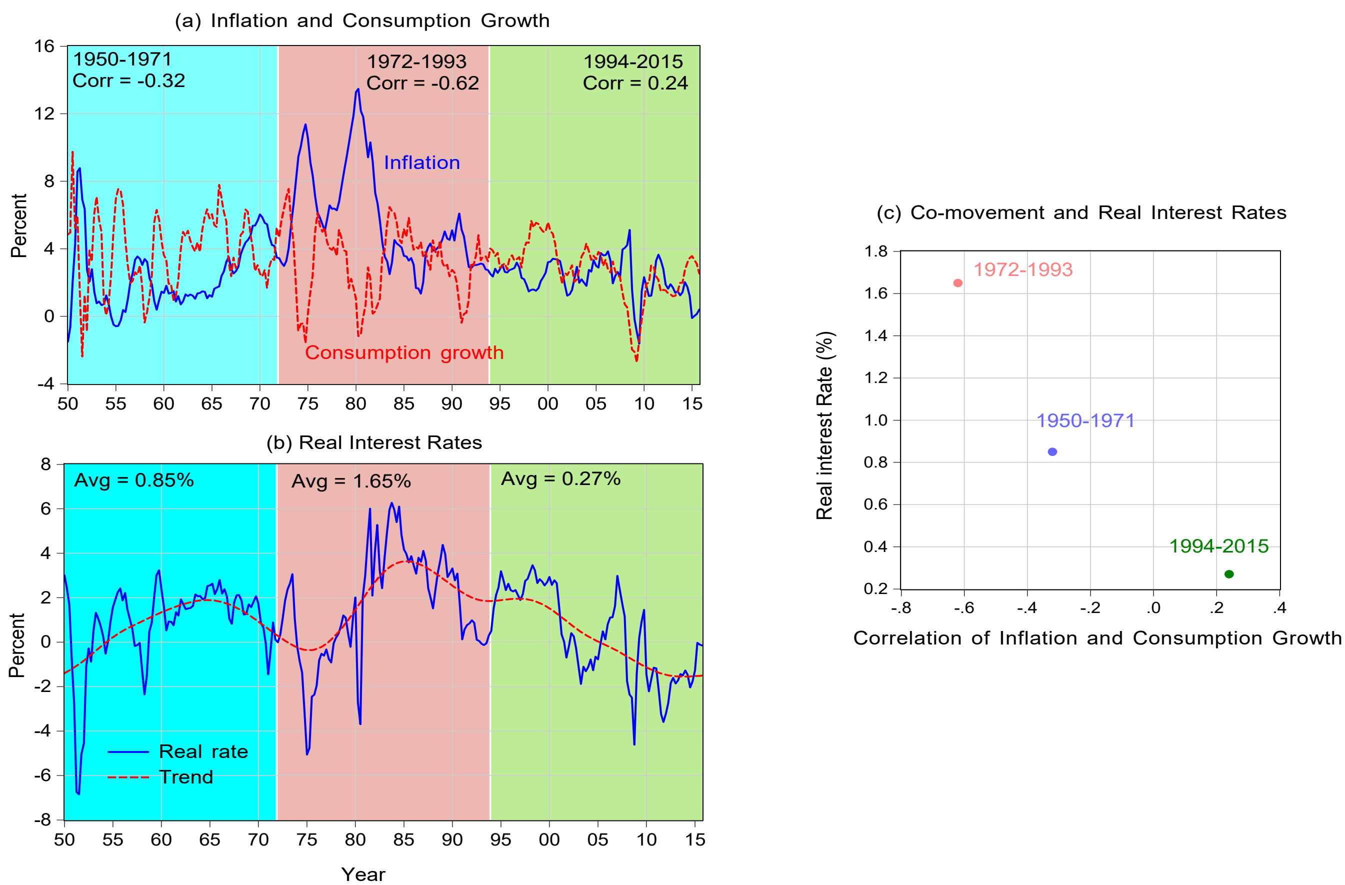
1 Introduction

Over the past half-century, inflation dynamics over the business cycle in advanced economies have changed quite dramatically, as we have observed periods of highly countercyclical inflation, as well as periods of procyclical inflation. The goal of this paper is to study how changes in the co-movement between inflation and economic activity affect real sovereign yields, debt dynamics, and debt crises. To be more concrete, in Figure 1 we provide some motivating evidence for the mechanism we want to highlight.

Figure 1: Inflation, Consumption Growth, and Real Rates in the U.S.



Note: Inflation is the log difference between CPI in quarter t and t-4. Consumption growth is the log difference in real personal consumption expenditures over the same interval. Real interest rates are nominal rates on medium and long term government bonds (from the IMF IFS database) minus expected inflation computed using a linear univariate forecasting model estimated on actual inflation.

Panel (a) plots quarterly time series for year-on-year U.S. inflation and consumption growth from 1950 to 2015. The panel highlights changes in the co-movement of the two series over three equal length sub-samples. It shows how in the first sub-sample (1950–1971), the co-movement between inflation and consumption growth is mildly negative, turns to strongly negative in the second sub-sample (1972–1993), and finally becomes positive in the most recent sample (1994–2015). If inflation co-varies positively with domestic consumption growth, real returns on domestic nominal debt are high when consumption growth is low. This feature makes domestic nominal bonds less risky from a domestic investor's perspective, and thus—if government debt is mostly held domestically, as it is in most developed countries—they should trade, *ceteris paribus*, at a lower real interest rate.[2] The second and third panels in Figure 1 show that this indeed is the case. Panel (b) plots the U.S. real interest rate (along with its trend depicted by the dashed line) over the same sample, while panel (c) plots the average real rate and the average co-movement between inflation and consumption growth in each of the three sub-samples. Notice how the middle sample, which displays the most negative co-movement between inflation and consumption growth, is also the one with the highest real rate. The most recent sample—where the co-movement has turned positive—displays the lowest real rate, while the early sample has intermediate co-movement and an intermediate real rate.

Altogether, the evidence in Figure 1 suggests that the co-movement between inflation and consumption growth is connected with the real yield on government debt. The evidence, however, is not conclusive as there might be a variety of other factors inducing this pattern in the U.S. For this reason, in the first part of this paper, we establish this relationship in a more systematic fashion. In particular, we show that for a large sample of advanced economies, in countries and periods in which the co-movement of inflation with domestic consumption growth is high, real interest rates on government bonds tend to be low, even after controlling for a broad array of macroeconomic variables. Our finding that this co-movement is systematically connected to real interest rates also suggests that changes in this co-movement might be important to understand the secular decline in real interest rates observed in many advanced economies.

[2] For example, as of 2015, the share of public debt held by domestic creditors is 64 percent in the U.S., 69 percent in the United Kingdom, and 78 percent in Canada. Aizenman and Marion (2011) report that the share of U.S. public debt held in Treasury inflation-protected securities (TIPS) was less than 8 percent in 2009.

Notice, though, that the same logic that makes nominal debt more attractive to lenders when inflation is procyclical, also suggests that nominal debt is less attractive to the borrower (the government). Consider, for example, a recession that is also accompanied by deflation. In that state, the lenders are happy to receive a large real payoff on their nominal assets at the time when their consumption is low (recession). As discussed earlier, this pushes down interest rates. Consider now the point of view of the borrower (the government), which has to make larger payments in real terms at exactly the time when its income is low. For this reason, inflation procyclicality tends to make default more likely, and this in turn pushes up interest rates. This discussion suggests that the effect of inflation procyclicality on interest rates depends on how likely default is. If default is not likely, inflation procyclicality should result in a *discount*, but the size of this discount should be smaller when default is more likely. In order to provide evidence for this mechanism, we regress interest rates on procyclicality and on an interaction between procyclicality and dummies that indicates that default is possible.[3] We indeed find that in periods where default is more likely, the *inflation procyclicality discount* is significantly reduced.

In the second part of the paper, we develop a very simple two-period model of debt and default with stochastic inflation where equilibrium outcomes can be characterized using simple diagrams, and we can provide intuition for the relation between inflation cyclicality, the real interest rate, and default. We show that when default is not possible, there is indeed a procyclicality discount. However, when default is possible, procyclicality can lead to increased interest rates, since procyclical inflation tends to make default more likely, as discussed earlier.

In the third part of the paper we develop a richer structural model of debt pricing and default. The goal of the model is to assess, quantitatively, the effects of observed changes in inflation cyclicality (like, for example those documented in figure 1 above) on real interest rates and on default probabilities.

The backbone of our set-up is a standard sovereign debt/default model (like in Arellano 2008), extended along three dimensions. First, it introduces domestic risk-averse lenders, in contrast to the common assumption of foreign risk-neutral lenders. This distinction is impor-

[3]We experiment using both low credit rating and low consumption growth as proxies for the likelihood of default.

tant since a large amount of public debt is held domestically in advanced economies.[4] Second, it introduces exogenous stochastic inflation so that government bond rates reflect both inflation risk and default risk. Inflation dynamics is, in general, driven by both fundamental macroeconomic shocks and the stance of monetary policy. We assume exogenous inflation dynamics to model, in a parsimonious way, changes in the mix of macroeconomic supply and demand shocks and changes in the independence of monetary policy from the fiscal authority in advanced economies.[5] Finally, it assumes that the government and households trade long-term debt, in contrast to the common assumption of one-period debt. Long-term debt is consistent with the fact that a majority of debt issued by governments in advanced economies has a maturity longer than 5 years, and it is important to generate a quantitatively sizeable effect of changes in inflation dynamics on real returns.

We then choose the parameters of the model. Our exercise highlights that several ingredients are necessary to generate quantitative plausible bond prices and, at the same time, replicate our estimated relation between interest rates, inflation cyclicality, and the likelihood of default. In particular, we find that long-term debt, Epstein-Zin preferences of the lender, and very rare occurrences of default are key.

We finally use the model to perform our main experiment. We consider two economies, identical in every respect, but which have two different processes for inflation. One in which inflation is countercyclical (resembling U.S. inflation in the 1970s), and one in which inflation is procyclical (like U.S. inflation in recent years). We find that an increase in inflation cyclicality can generate a significant reduction in real rates (85 basis points) when default on government debt is not an issue. On the other hand, when the government is in fiscal trouble and default is a possibility, a more procyclical inflation has very different consequences. We find that it can increase rates by as much as 35 basis points and that it can increase the probability of default by 28 basis points. The logic is again the one discussed earlier. When default is not an issue a procyclical inflation provides more insurance to lenders, which are in turn willing to accept a lower interest rate. When default is an issue

[4] One could also consider the case where foreign risk-averse lenders experience inflation that is correlated with domestic inflation through financial and trade linkages.

[5] See Song (2017) and Campbell et al. (2014) for recent advances in the estimation of New Keynesian DSGE models featuring time-varying inflation risk arising from macroeconomic shocks and regime switches in the stance of monetary policy in the U.S.

though, procyclical inflation, imposing a high repayment burden on the government in bad times, further increases default risk and thus can lead to an increase in the interest rare.

Our paper also has implications for the debate on the costs and benefits of joining or exiting a monetary union. Suppose that the union goes into a recession where some, but not all, members of the union get into fiscal trouble. Then the countries in fiscal trouble would like a more countercyclical monetary policy, while the others don't: the contrast over monetary policy increases in a recession.

Our paper also suggests that changes in the co-movement between inflation and output have contributed to the secular decline in real interest rates while also increasing the likelihood of debt crises during bad times. In that sense, this paper also suggests a new channel through which monetary policy and fiscal policy jointly affect financial stability.

Related literature. Our paper is related to several strands of the literature. On the theoretical side, the backbone of our set-up is a debt default model with incomplete markets as in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), or Arellano (2008). Our paper is especially related to Hatchondo et al. (2016) and Lizarazo (2013), who study default in the context of risk-averse international lenders.[6] While these papers focus on foreign debt, Reinhart and Rogoff (2011) suggest that the connection between default, domestic debt, and inflation is an important one. D'Erasmo and Mendoza (2016), Pouzo and Presno (2014), and Arellano and Kocherlakota (2014) tackle the issue of default on domestic debt but do not include inflation.[7] Araujo et al. (2013), Sunder-Plassmann (2016), Mallucci (2015), and Fried (2017) study how the currency composition of debt interacts with default crises in emerging economies, while Berriel and Bhattarai (2013), Faraglia et al. (2013), and Perez and Ottonello (2016) study nominal debt with inflation, in the absence of default. Kursat Onder and Sunel (2016) and Nuño and Thomas (2016) consider the interaction of inflation and default on foreign investors. Much of the existing literature on debt and inflation has focused on strategic inflation, even hyperinflation, as a countercyclical policy option that governments with limited commitment can use when faced with a high debt burden in bad times. That

[6] Aguiar et al. (2016b) provide an excellent compendium on modeling risk-averse competitive lenders in the sovereign default literature.

[7] Broner et al. (2010) examine the role of secondary asset markets, which make the distinction between foreign and domestic default less stark.

focus is certainly legitimate for emerging economies, but less warranted in the context of advanced economies mainly because of monetary policy independence and monetary union constraints.[8]

On the empirical side, our findings are related to studies on the importance of the inflation risk premium and its variation, as in, for example, Boudoukh (1993), Piazzesi and Schneider (2006), or Ang et al. (2008). Most related to our empirical analysis is the work by Du et al. (2016), who build on the bond-stock return correlation approach of Campbell et al. (2017) to study default risk and debt currency composition when an emerging economy lacks commitment. In contrast, our model of inflation and default risk in advanced economies assumes commitment and independence of the monetary policy authority but limited commitment from the fiscal authority issuing nominal debt. Campbell et al. (2014) quantitatively assess the asset pricing and bond risk premia implications of different monetary policy regimes in the U.S. using a New Keynesian model.[9] Song (2017) also studies the fundamental drivers of time-varying inflation risk in U.S. bond markets by estimating a model with time variations in the stance of monetary policy as well as the co-movement of macroeconomic shocks. The exogenous inflation-output process considered in our model can be rationalized as the process implied by such exogenous macroeconomic shocks, in the absence of default risk.[10]

Our general question is also related to recent work that studies how joining a monetary union can affect the probability of a self-fulfilling crisis in a debt default model (see Aguiar et al. 2015 and Corsetti and Dedola 2016). We complement these papers by highlighting how the cyclicality of inflation impacts fundamental-driven default crises, suggesting a promising extension of existing models of self-fulfilling debt crises.

The paper is structured as follows. Section 2 contains the empirical findings. Sections 3 and 4 contain the simple and quantitative model, respectively. Section 5 discusses the main

[8] In a related paper, Albanesi et al. (2003) study an environment in which the lack of monetary commitment can lead to multiple equilibria with high and low expectation traps. While they suggest that advanced economies have experienced low inflation because they are in a low expectation trap, the authors also argue that the existence of institutions that promote monetary policy commitment and central bank independence can lead to unique equilibria with low inflation.

[9] See also Kang and Pflueger (2015), who document that corporate credit spreads, relative to government yields, are correlated with inflation risk and calibrate a model of defaultable corporate debt to assess the default premium induced by inflation risk. Here, we focus on the underlying sovereign yield.

[10] See also Bianchi (2013) for an estimation of a DSGE model with dynamic beliefs about regimes switches in monetary policy and stochastic volatility.

results of the quantitative model and section 6 concludes.

2 Inflation and Real Interest Rates

In this section, we study the empirical relation between several conditional moments of inflation and real interest rates on government debt. The main novel finding is that higher covariance of inflation with economic activity is significantly associated with lower real interest rates on government debt, especially in good times but not necessarily in bad times.

Our data set includes quarterly observations on real consumption growth, inflation, interest rates on government bonds, and government debt-to-GDP ratios for an unbalanced panel of 19 OECD economies from 1985Q1 to 2015Q4. The countries in the data set are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Korea, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and the United States.

We mainly use quarterly data from the IMF and the OECD to document our empirical findings. We compute inflation as the change in the log GDP deflator using data from the OECD. We use nominal interest rates on government bonds from the IMF's International Financial Statistics (IFS) database. For government debt, we use quarterly series from Oxford Economics on gross government debt relative to GDP, extended with quarterly OECD data on central government debt relative to GDP. Quarterly real consumption is constructed as the sum of private and public real consumption using the data from the OECD.

Using this cross-country quarterly data, we estimate the conditional co-movement between inflation and consumption growth, and derive real interest rates by substracting the expected inflation estimated from nominal yields. To do so, we follow Boudoukh (1993) and formulate the following vector autoregression (VAR) model for inflation and consumption growth:

$$\begin{bmatrix} \pi_{it} \\ g_{it} \end{bmatrix} = \mathbf{A_i} \begin{bmatrix} \pi_{it-1} \\ g_{it-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\pi it} \\ \varepsilon_{git} \end{bmatrix} \tag{1}$$

where π_{it} is inflation, and g_{it} is the change in log consumption in country i in period t, A_i is a country-specific 2-by-2 matrix, and $\varepsilon_{\pi it}$ and ε_{git} are innovations in the two time series.

We then estimate the VAR using standard OLS and construct time series for residuals $\varepsilon_{\pi it}$ and ε_{git} for each country.

We measure the expected inflation as the forward-looking predicted inflation from the VAR, that is, $\mathbf{E}[\pi_{i,t+1}]$. We then derive real rates on government debt as nominal rates less expected inflation. Finally, we measure the conditional co-movement between inflation and consumption growth as the covariance between the two innovations, $\varepsilon_{\pi it}$ and ε_{git}, in overlapping forward-looking country-windows, comprising 40 quarters.[11]

In Figure 2, we plot the path of the conditional correlation for the countries in our sample. Figure 2 illustrates that the co-movement of inflation and consumption growth varies over time and across countries. In many countries, such as Canada, Italy, Norway, the U.S., or the U.K., the co-movement of inflation and consumption growth has clearly increased since the mid-1980s, while it has sharply decreased or fluctuated in other countries, such as Germany.

With this data set, we estimate how the conditional covariance of inflation and consumption growth relates to interest rates faced by governments. All specifications include a full set of country and time fixed effects.

In Table 1, we regress the real interest rate on the conditional covariance of inflation with consumption growth. The main result from Table 1 is that in periods with higher conditional covariance between inflation and consumption growth, governments face lower interest rates. This finding is robust to the inclusion of the level of government debt and average residual inflation and consumption growth (column 2). This association is also robust to the inclusion of the variances of residual inflation and consumption growth as additional regressors (column 3). In the appendix, we also show that the results are robust to using different debt measures and different long-term yield maturities. Interestingly, the results are not robust to using short-term yields, motivating our focus on long maturity debt in our quantitative model in section 5.

Overall, our results show that the co-movement of inflation and consumption growth are associated with lower real interest rates that governments face. We call this the *inflation procyclicality discount*. The magnitude of this discount is economically significant. As an illustration of its magnitude, consider moving from a country/time period in which the

[11] Our results are robust to alternative window definitions.

Figure 2: Conditional correlation between inflation and consumption growth

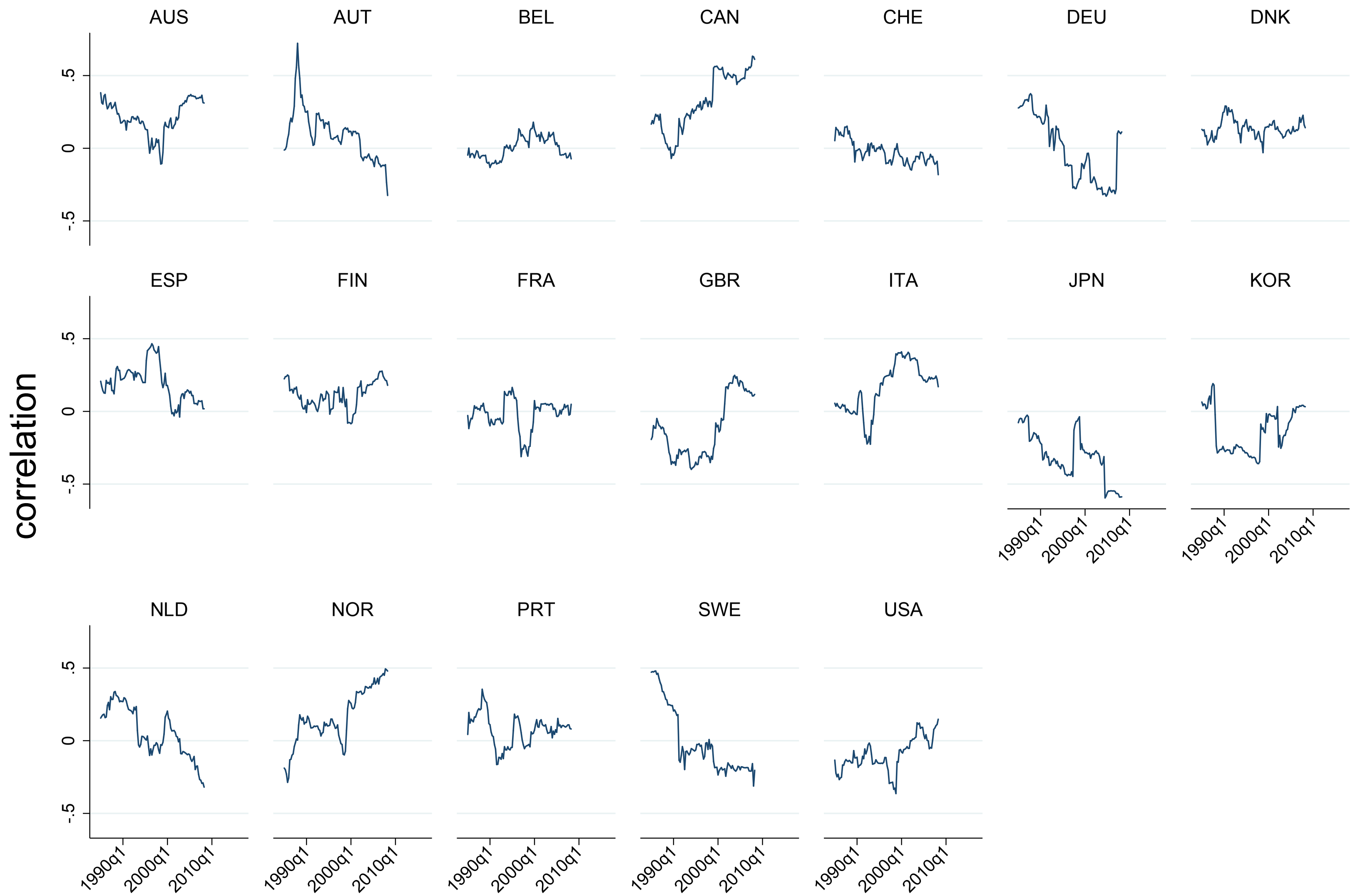


Table 1: Inflation consumption growth co-movement and real interest rates

	Real yield on government debt			
		covariance		correlation
	(1)	(2)	(3)	(4)
Inflation consumption co-movement	-1.89***	-1.64***	-1.80**	-1.06**
	(0.60)	(0.38)	(0.64)	(0.43)
Lagged government debt		0.02***	0.02***	0.02***
		(0.00)	(0.00)	(0.00)
Average inflation residual		2.41**	2.14*	1.91*
		(0.99)	(1.02)	(0.93)
Average cons. growth residual		-1.75	-1.65	-1.52
		(1.07)	(1.04)	(1.08)
Variance of inflation residual			0.30	0.26
			(0.29)	(0.31)
Var. of cons. growth residual			-0.06	0.23*
			(0.18)	(0.12)
standard deviation of co-movement	0.17	0.17	0.17	0.21
adj. R^2	0.87	0.90	0.90	0.90
N	1764	1726	1726	1726

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Standard errors clustered by country. All regressions include country and time fixed effects. The data are a quarterly unbalanced panel from 1985Q1 to 2015Q4 including AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, ITA, JAP, KOR,NLD, NOR, POR, SWE, USA. All variables are computed over a forward-looking 10-year window. The co-movement of inflation and consumption growth is measured as the covariance of residuals within that window: $\mathbf{cov}_t(\varepsilon_{\pi it}, \varepsilon_{git})$. Other regressors are averages and variances of those residuals in the window and lagged debt.

inflation/consumption correlation is around -0.3 (for example, the U.S. in the 1980s) to a sample period in which the correlation is around 0.1 (for example, the U.S. in the 2000s). This roughly corresponds to a change in correlation equal to two times the standard deviation of correlation in our sample. Using the coefficient estimated in column (4) of Table 1, we can see that such an increase in cyclicality correlation is associated with a lowering of real rates by 42 basis points. Similarly, using the coefficient estimated in column (3) of Table 1, a fall in covariance that is twice as large as our sample standard deviation is associated with a decrease in real rates by 61 basis points.

Table 2: Inflation procyclicality discount in good times

	Real yield on government debt		
	(1)	(2)	(3)
Good times measure		cons. growth	credit rating
Inflation consumption covariance	-1.80**		
	(0.64)		
Indicator(good times)		-0.23	0.26
		(0.23)	(0.19)
Interaction term (good times)		-2.99***	-2.70***
		(0.70)	(0.91)
Interaction term (bad times)		-1.16	-1.31
		(0.68)	(0.79)
Other controls	Yes	Yes	Yes
adj. R^2	0.90	0.91	0.92
N	1726	1726	1438

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Standard errors clustered by country. All regressions include country and time fixed effects. The data are a quarterly unbalanced panel from 1985Q1 to 2015Q4 including AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, ITA, JAP, KOR, NLD, NOR, POR, SWE, USA. All variables are computed over a forward-looking 10-year window. The co-movement of inflation and consumption growth is measured as the covariance of residuals within that window: $\mathbf{cov}_t(\varepsilon_{\pi it}, \varepsilon_{git})$. Other regressors are averages and variances of those residuals in the window and lagged debt.

Our second main finding is that the procyclicality discount is a good times discount. This can be seen in Table 2 column (2), which includes an indicator for good times, defined to be a (10-year) window in which the average residual consumption growth is positive, and its interaction with the covariance of inflation and consumption growth. Column (2) shows that the interaction term is negative and statistically significant, while the interaction of covariance and an indicator for bad times (the complement of good times) is not statistically significant, implying that the inflation cyclicality discount is a good times discount. In column (3), we report similar results when the good times indicator is defined as a window in which the average credit rating is AAA, which is the sample median. In both cases, the good times interaction term is negative and statistically significant, while the bad times interaction term is smaller in magnitude and statistically insignificant.

Overall, we find that procyclical inflation episodes are associated with significantly lower real sovereign yields, albeit such an *inflation procyclicality discount* vanishes in bad times. The standard consumption-based asset pricing model suggests that the hedging benefits of procyclical inflation rationalize an inflation procyclicality discount. However, the state-dependent nature of the procyclicality discount suggests that bad times are associated with additional countercyclical credit risk, possibly default risk. From the government's perspective, inflation procyclicality is not desirable in bad times *ceteris paribus* and reduces the government's willingness to pay. In the next section, we develop a simple theory to understand the relation between the covariance of inflation and consumption growth, interest rates, and default.

3 Simple Model

In this section we highlight the main economic mechanism of this paper through a stylized two-period model of inflation and default, where equilibrium outcomes can be characterized using simple diagrams.

3.1 Simple model without default

Consider a two-period, one-good, closed economy with competitive lenders and borrowers. Both borrowers and lenders receive one unit of the good in the first period and an endowment of x in the second period, where x is a random variable with c.d.f. F over X, with finite support $X = [x_{\min}, x_{\max}]$, $\mathbf{E}(x) = \mu$ and $Var(x) = \sigma^2$. The variable x here captures aggregate risk of the economy, to which both lenders and borrowers are exposed. We assume that the only difference between lenders and borrowers (i.e., motive to intertemporal trade) lies in their preferences. In particular, we assume that $\beta_\ell > \beta_b$ are the discount factors of lenders and borrowers, respectively. Lenders and borrowers can trade a nominal bond at price q today, which pays a nominal amount of 1 tomorrow. We normalize the current price level to 1, and assume that the future price level is given by $1 + \pi(x; \kappa) \equiv [1 + \kappa(\mu - x)]^{-1}$, where κ is the key parameter, capturing the cyclicality of inflation. If $\kappa > 0$, prices (and inflation) are procyclical, so the bond pays less in good states of the world (when x is high), while the reverse is true if $\kappa < 0$. We define the real interest rate r as $\mathbf{E}[1/(1 + \pi)]/q - 1$, which is equal to $1/q - 1$.

The borrower solves

$$\max_{b_b} u(1 + qb_b) + \beta_b \int_X v\left(x - \frac{b_b}{1 + \pi(x;\kappa)}\right) dF(x), \tag{2}$$

and the lender solves

$$\max_{b_\ell} u(1 - qb_\ell) + \beta_\ell \int_X v\left(x + \frac{b_\ell}{1 + \pi(x;\kappa)}\right) dF(x), \tag{3}$$

Notice that both borrowers and lenders act competitively, taking bond prices as given. An equilibrium is then simply a bond price and bond quantities of borrowers and lenders such that, given prices, bond quantities are optimal for each agent and the bond market clears.

Theorem 1 shows that, under certain conditions, an inflation cyclicality discount arises from the hedging benefits of inflation procyclicality.

Theorem 1. Inflation procyclicality discount

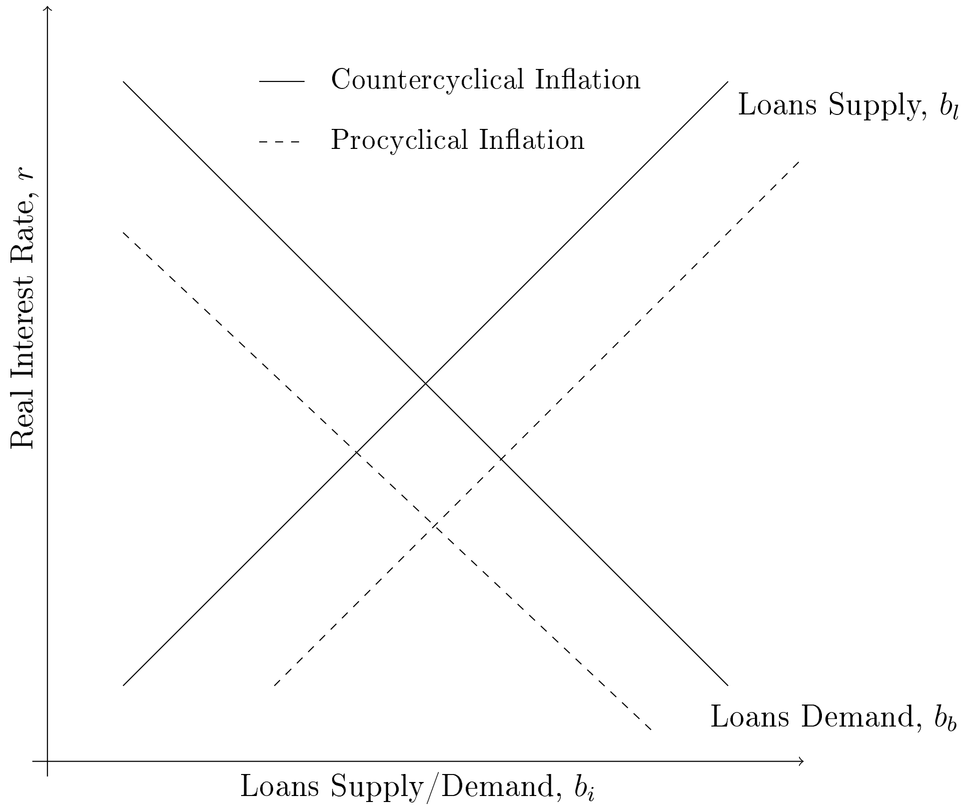
Assume that both borrowers and lenders have quasilinear utility such that $u(c) = Ac$, *and* $v(c) = Ac - \frac{\phi}{2}c^2$ *with* $A > 0, \phi > 0$ *and* $\frac{A}{\phi} > \mu$. *Then, the real interest rate* $r \equiv 1/q - 1$ *features an inflation procyclicality discount. That is,*

$$\frac{\partial r}{\partial \kappa} < 0. \tag{4}$$

Proof: See Appendix B.1.

Figure 3 provides some visual intuition for this result. The lines in the figure represent the desired supply of loans from the lender (increasing in the interest rate) and the desired demand for loans by the borrower (decreasing in the interest rate). The solid lines are supply and demand with countercyclical inflation, while the dashed lines are supply and demand with procyclical inflation. Note that as inflation goes from counter- to procyclical, the supply of loans increases. Intuitively, with procyclical inflation, for every level of interest rate a risk-averse lender wants to save more, as saving yields higher returns in states of the world where its income is low. While procyclical inflation makes debt more attractive for the lender, the

Figure 3: Interest rates and cyclicality of inflation without default



opposite is true for the borrower, and this results in the decrease in the demand for loans. Since supply increases and demand falls, the equilibrium interest rate unequivocally falls, while the equilibrium level of debt can move in either direction.

3.2 Simple model with default

Now consider the possibility that the nominal contract can be defaulted on. In particular, the borrower can default on its bond payments, and if it does so, no payments are made and it incurs a cost $C(x) = \psi(x - x_{\min})^2$. As in Dubey et al. (2005), we keep the assumption of competitive borrowers, so they do not perceive that their borrowing decision affects the equilibrium interest rate they face. In this environment, there will be equilibrium default when default costs are below repayment; hence, the default set $\widehat{X}(\kappa, b_b)$ is given by

$$\widehat{X}(\kappa, b_b) = \left\{ x \in [x_{\min}, x_{\max}] \;:\; C(x) < \frac{b_b}{1+\pi(x;\kappa)} \right\} \tag{5}$$

which typically is an interval, i.e., default happens when income is low enough and when debt is high enough. The key observation is that in a world with default, the cyclicality of inflation can change the default set, thereby altering the hedging properties of bonds. Theorem 2 shows that, under certain regularity conditions, the default set $\widehat{X}$ increases with the level of debt (b_b) and the cyclicality of inflation (κ).

Theorem 2. Inflation procyclicality and default

Assume that $-(\mu - x_{\min})^{-1} < \kappa < (x_{\max} - \mu)^{-1}$. *For* ψ *large enough, there exists a unique threshold* $\widehat{x}(\kappa, b_b) \in [x_{\min}, \mu]$ *such that default occurs if and only if* $x \in [x_{\min}, \widehat{x}]$. *Furthermore, the default threshold is increasing in debt* (b_b) *and the cyclicality of inflation* (κ). *That is,*

$$\frac{\partial \widehat{x}(\kappa, b_b)}{\partial b_b} > 0 \tag{6}$$

$$\frac{\partial \widehat{x}(\kappa, b_b)}{\partial \kappa} > 0. \tag{7}$$

Proof: See Appendix B.2.

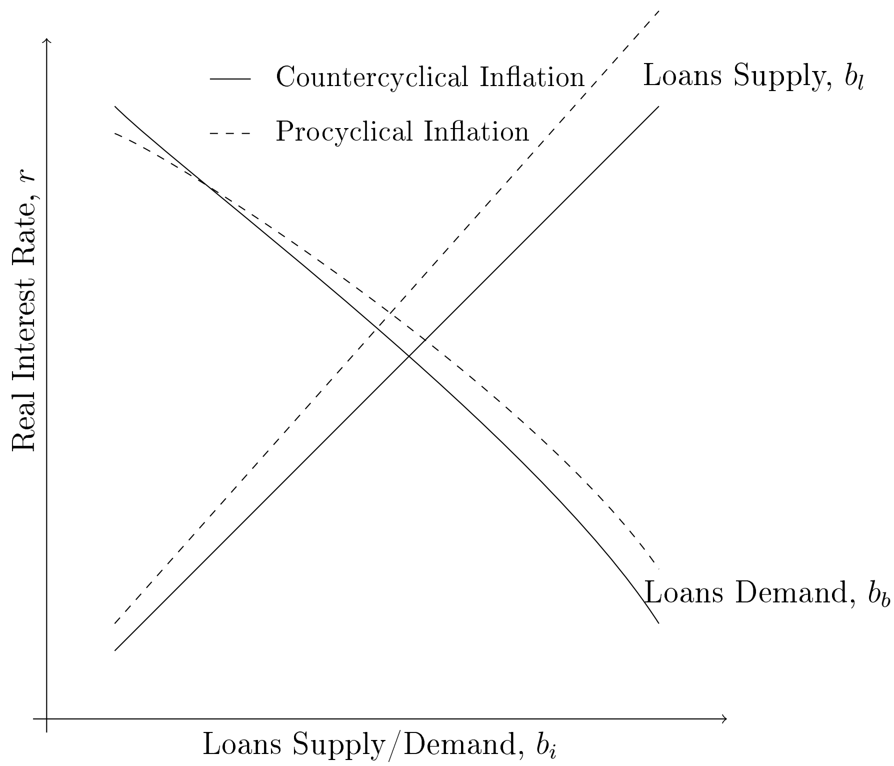
Given this result we can then write the problem of the borrower as

$$\max_{b_b} u\left(1 + q b_b\right) + \beta_b \left(\underbrace{\int_{\widehat{x}(b_b,\kappa)}^{x_{\max}} v\left(x - \frac{b_b}{1+\pi(x)}\right)}_{\text{Repayment}} + \underbrace{\int_{x_{\min}}^{\widehat{x}(b_b,\kappa)} v\left(x - C(x)\right)}_{\text{Default and suffer cost}} \right) dF\left(x\right) \tag{8}$$

The lender, taking as given the default threshold $\widehat{x}$, solves

$$\max_{b_\ell} u\left(1 - q b_\ell\right) + \beta_\ell \left(\underbrace{\int_{\widehat{x}}^{x_{\max}} u\left(x + \frac{b_\ell}{1+\pi(x)}\right)}_{\text{Repayment}} + \underbrace{\int_{x_{\min}}^{\widehat{x}} u\left(x\right)}_{\text{Defaulted on}} \right) dF\left(x\right). \tag{9}$$

Figure 4: Interest rates and cyclicality of inflation with default



In the model with default, changes in covariance lead to changes not only to quantities but also to the default threshold, complicating the analysis. Thus, to gain further intuition, we use a simple numerical illustration. Figure 4 shows that, unlike the model without default in which higher inflation procyclicality unequivocally reduced interest rates, in the model with default, higher inflation procyclicality can increase real rates.

To understand why this is the case consider the savings curve with and without default. In the absence of default in Figure 3, as inflation goes from countercyclical to procyclical, the saving curve shifts to the right: lenders are willing to accept a lower interest rate because of the hedging properties of inflation. In Figure 4 instead, the curve shifts to the left due to default risk. This is because countercyclical inflation, which implies low repayments in bad states, *substitutes* default, while procyclical inflation, which implies high repayments in bad states, *complements* default. Thus, in this example, a move from counter- to procyclical inflation causes an increase in default risk, which shifts the credit supply to the left and causes an increase in the interest rate.

This simple model highlights a fundamental relation between inflation cyclicality, interest rates, and default. It shows that when default is not a concern, a more procyclical inflation results in lower rates. However, it cannot be used to assess how large of an interest rate differential can be explained by different inflation co-movement properties, nor to assess how much a change in inflation cyclicality can affect default risk. For these questions we now turn to a standard quantitative model of default, augmented with nominal long-term debt and inflation.

4 Quantitative Model

We extend the standard sovereign default model of Eaton and Gersovitz (1981) and Arellano (2008) along three dimensions: exogenous *inflation*, domestic *risk-averse lenders*, and long-term debt. Note that risk-averse lenders are important to capture the impact of inflation cyclicality on the the pricing of nominal bonds, while long-term debt is important to generate a quantitatively relevant impact of inflation cyclicality on returns to nominal debt.

Environment We consider a closed economy inhabited by a continuum of (relatively patient) risk-averse lenders and a (relatively impatient) government. Both government and lenders are exposed to the same aggregate risk and, in equilibrium, the difference in patience results in the government borrowing from lenders. Importantly, the government has the option of defaulting on debt obligations to lenders, and if it does so, aggregate output in the economy is reduced. Time is discrete and indexed by $t = 0, 1, 2, ...$ and we let s_t denote the state of the world in period t. In each period the economy receives a stochastic endowment $y(s_t)$. The government receives a fraction τ of the endowment, net of default costs, and lenders receive the remaining fraction $1 - \tau$.

Preferences The government uses its fraction of output plus proceeds from borrowing to finance public spending $g(s_t)$, which is valued according to

$$E_0 \sum_{t=0}^{\infty} \beta_g^t \frac{g(s_t)^{1-\gamma_g}}{1-\gamma_g} \tag{10}$$

where $0 < \beta_g < 1$ is the government's discount factor and γ_g is the risk aversion of the government.[12]

Lenders evaluate payments in two states of the world s_t and s_{t+1} using a stochastic discount factor $m(s_t, s_{t+1})$, and thus value a sequence of payments $\{x(s_t)\}_{t=0}^{\infty}$ as

$$E_0 \sum_{t=0}^{\infty} m(s_0, s_t) x_t \tag{11}$$

where $m(s_0, s_t) = \Pi_{j=0}^{t-1} m(s_j, s_{j+1})$.

Following the recent work that focuses on long-term interest rates with default risk (see, for example, Bocola and Dovis 2016, and Hatchondo et al. 2016) we assume that $m(s_t, s_{t+1})$ takes the form

$$m(s_t, s_{t+1}) = \beta_\ell \left(\frac{y(s_{t+1})}{y(s_t)} \right)^{-1} \left(\frac{W(s_{t+1})^{1-\gamma_\ell}}{\mathbf{E}_t \left[W(s_{t+1})^{1-\gamma_\ell} \right]} \right) \tag{12}$$

where β_ℓ and γ_ℓ can be interpreted as the lender's discount factor and risk aversion, respectively, and $W(s_t)$ is defined recursively as

$$\log W(s_t) = (1 - \beta_\ell) \log y(s_t) + \frac{\beta_\ell}{1 - \gamma_\ell} \log \left(E_t \left[W(s_{t+1})^{1-\gamma_\ell} \right] \right). \tag{13}$$

Thus, the lender's stochastic discount factor is derived from recursive preferences as in Epstein and Zin (1989) and Weil (1989) where the intertemporal elasticity of substitution has been set to 1. Note that we assume that the lender's discount factor depends on total endowment $y(s_t)$, and not on the lender's consumption, which is its fraction of endowment minus the lending. This assumption greatly simplifies the computation of equilibria in this economy.[13]

[12] An alternative interpretation is that the government uses its revenues to finance and smooth the consumption of another class of 'median' agents who are poorer and have no access to financial markets.

[13] The reason for this is that lender's consumption depends on equilibrium bond prices, which in turn depends on the stochastic discount factor. Therefore, computing an equilibrium where the lenders' discount factor depends on the lender's consumption involves computing a fixed point of higher dimensionality. To check on potential problems stemming from this assumption, in simplified versions of our economy we have computed equilibria using both types of stochastic discount factors, and found that the quantitative properties of the equilibria were similar. This is because the aggregate endowment and the lender's consumption are strongly correlated.

Market structure The government issues nominal long-term non-contingent bonds to the domestic lenders. Payouts of the bonds are nominal because they are subject to inflation risk. In particular, a nominal payout in state s_t, $x(s_t)$, is worth $\frac{x(s_t)}{1+\pi(s_t)}$, where $\pi(s_t)$ follows an exogenous Markov process, possibly correlated with the process for $y(s_t)$. Bonds have a fixed coupon payment of r and mature in each period with probability δ, as in Arellano and Ramanarayanan (2012), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2013). Setting $\delta = 1$ corresponds to the model with one-period debt and $\delta = 0$ corresponds to the model with consols.

Default choices The government enters the period with outstanding assets B and, upon realization of the state of the world, it decides to default on its obligations or not. We define the value of the government at this point as $V^o(B, s)$, which satisfies

$$V^o(B,s) = \max_d \left\{(1-d)V^c(B,s) + dV^d(B,s)\right\} \tag{14}$$

where V^c is the value of not defaulting, V^d is the value of default, and $d \in \{0,1\}$ is a binary variable capturing the default choice.

When the government defaults, it defaults on all existing debt, in which case the government is excluded from debt markets for a stochastic number of periods, and during those periods, the value of the endowment for the economy is lower. During this time, no debt payments are paid. Upon re-entry after k periods, the government's debt obligation is $-\lambda^k B$, where $1-\lambda$ is the rate at which the government's debt obligation decays each period. This tractable way of modeling partial default is also consistent with the fact that longer default episodes are associated with lower recovery rates, as documented by Benjamin and Wright (2009). Setting $\lambda = 0$ corresponds to the model with full default.

The government's value of default is then given by

$$\begin{aligned} V^d(B,s) = {} & u_g\left(\tau(y(s) - \phi^d(s))\right) \\ & + \beta_g \mathbf{E}_{s'|s}\left[\theta V^o\left(\frac{\lambda B}{1+\pi(s')}, s'\right) + (1-\theta)V^d\left(\frac{\lambda B}{1+\pi(s')}, s'\right)\right] \end{aligned} \tag{15}$$

where $0 < \theta < 1$ is the probability that the government will regain access to credit markets, and $\phi^d(s)$ is the loss in income during default. In particular, we assume a quadratic function

$$\phi^d(s) = d_1(s) \max\left\{0, \frac{1}{d_0} y(s) + \left(1 - \frac{1}{d_0}\right) y(s)^2\right\}, \tag{16}$$

similar to Chatterjee and Eyigungor (2013), except that the expression has been written such that $d_1(s)$ is the default cost at mean output ($y = 1$) and d_0 determines the output threshold above which the default costs are positive. Note that, similar to Aguiar et al. (2016a), default costs move stochastically according to the Markov process $d_1(s)$. So in this set-up there are three possible exogenous shocks that increase the likelihood of default. The first (present in most standard models) is a low realization of the endowment $y(s)$, which raises the marginal value of current resources and makes repayment more costly. The second is a low realization of the default costs $d_1(s)$, which obviously makes default more attractive. The last, and specific to our set-up, is a low realization of inflation $\pi(s)$, which increases the real value of the government's repayment, and thus makes default a more attractive option. It turns out that all three of these forces play an important role in our quantitative results.

The value of not defaulting is given by

$$V^c(B, s) = \max_{B' \leq 0} \left\{ \begin{array}{l} u\left(\tau y - q(s, B')\left(B' - (1-\delta)B\right) + B(r+\delta)\right) \\ +\beta_g \mathbf{E}_{s'|s}\left[V^o\left(\frac{B'}{1+\pi(s')}, s'\right)\right] \end{array} \right\} \tag{17}$$

where $B(r + \delta)$ represents the payment the government needs to make to lenders (maturing bonds plus coupon), and $q(s, B')$ is the price schedule that the government faces on its new issuance, $(B' - (1 - \delta)B)$. Note that the real return on government debt is stochastic, even in the absence of default, due to inflation risk.

In this environment, the bond price schedule satisfies

$$\begin{aligned} q(s, B') \;\; = \;\; & \mathbf{E}_{s'|s}\left[\frac{1-d'}{1+\pi(s')}\left(r + \delta + (1-\delta) q\left(s', B''\right)\right) m(s, s')\right] \\ & + \mathbf{E}_{s'|s}\left[\frac{d'}{1+\pi(s')} q^{def}\left(\frac{B'}{1+\pi(s')}, s'\right) m(s, s')\right] \end{aligned} \tag{18}$$

where d' and B'' are the optimal default and debt decisions given the state $(\frac{B'}{1+\pi(s')}, s')$, and

q^{def} is the value of a bond in default and is given by

$$\begin{aligned} q^{def}(B,s) &= \lambda \mathbf{E}_{s'|s}\left[\frac{\theta(1-d')}{1+\pi(s')}\left(r+\delta+(1-\delta)q\left(s',B''\right)\right)m(s,s')\right] \\ &\quad +\lambda \mathbf{E}_{s'|s}\left[\frac{1-\theta+\theta d'}{1+\pi(s')}q^{def}\left(\frac{\lambda B}{1+\pi(s')},s'\right)m(s,s')\right]. \end{aligned} \tag{19}$$

where d' and B'' are the optimal default and debt decisions given the state $(\frac{\lambda B}{1+\pi(s')}, s')$,

Recursive equilibrium A Markov perfect equilibrium for this economy is defined as value functions for the government $\left\{V^o, V^c, V^d\right\}$, the associated policy functions $\{B', d\}$, and bond pricing functions $\left\{q, q^{def}\right\}$ such that: (a) given $\left\{q, q^{def}\right\}$, $\left\{V^o, V^c, V^d, B', d\right\}$ solve the government's recursive problem in (14), (15), and (17); and (b) given the government policy functions $\{B', d\}$, the bond pricing functions $\left\{q, q^{def}\right\}$ satisfy (18) and (19).

Real bond price and spread It is convenient to define the real bond price as

$$\begin{aligned} \widehat{q}(s,B') &= \mathbf{E}_{s'|s}\left[(1-d')\frac{1+\bar{\pi}(s)}{1+\pi'}\left(r+\delta+(1-\delta)\widehat{q}\left(s',B''\right)\right)m(s,s')\right] \\ &\quad +\mathbf{E}_{s'|s}\left[d'\frac{1+\bar{\pi}(s)}{1+\pi(s')}\widehat{q}^{def}\left(\frac{B'}{1+\pi(s')},s'\right)m(s,s')\right] \end{aligned} \tag{20}$$

where lenders adjust for expected inflation, defined as $1+\bar{\pi}(s) \equiv 1/\mathbf{E}_{s'|s}\left[\frac{1}{1+\pi(s')}\right]$. As before, d' and B'' are the optimal default and debt decisions given the state $\left(\frac{B'}{1+\pi'}, s'\right)$, and the real price of a bond in default $\widehat{q}^{def}$ is similarly defined as

$$\begin{aligned} \widehat{q}^{def}(B,s) &= \lambda \mathbf{E}_{s'|s}\left[\theta(1-d')\frac{1+\bar{\pi}(s)}{1+\pi(s')}\left(r+\delta+(1-\delta)\widehat{q}\left(s',B''\right)\right)m(s,s')\right] \\ &\quad +\lambda \mathbf{E}_{s'|s}\left[(1-\theta+\theta d')\frac{1+\bar{\pi}(s)}{1+\pi(s')}\widehat{q}^{def}\left(\frac{\lambda B}{1+\pi(s')},s'\right)m(s,s')\right] \end{aligned} \tag{21}$$

where d' and B'' are the optimal default and debt decisions given the state $\left(\frac{\lambda B}{1+\pi(s')}, s'\right)$.

In the special case where $\lambda = 0$ and $\delta = 1$, we can express the equilibrium spread as

$$\begin{aligned}
\mathrm{spr}(B,s) \equiv \frac{q^{RF}(s) - \widehat{q}(B,s)}{q_t^{RF}(s)} = \; & \underbrace{\Pr\left[d' = 1\right]}_{\text{default premium}} \\
& + \mathbf{cov}\left[\frac{m(s,s')}{\bar{m}(s)}, d'\right] \\
& + \mathbf{cov}\left[\frac{1+\bar{\pi}(s)}{1+\pi(s')}, d'\right] \\
& \underbrace{- \Pr\left[d' = 0\right] \mathbf{cov}_t \left[\frac{m(s,s')}{\bar{m}(s)}, \frac{1+\bar{\pi}(s)}{1+\pi(s')}\right]}_{\text{procyclicality discount}}.
\end{aligned} \tag{22}$$

where the risk-free price is defined as the price of a non-defaultable real bond, which is $q^{RF}(s) \equiv \mathbf{E}_{s'|s}\left[\left(\delta + r + (1-\delta)q^{RF}(s')\right) m(s,s')\right]$, and $\bar{m}(s) \equiv \mathbf{E}_{s'|s}\left[m(s,s')\right]$. The first two terms add to the spread and reflect the probability of default and the compensation for countercyclical default—effects that are standard but are now endogenous to the cyclicality of inflation. The third term lowers the spread as default (low returns for lender) is negatively correlated with surprise disinflation (high returns for lender). Finally, the last term can be either positive or negative depending on the conditional co-movement between surprise inflation and surprise output growth, and is positive in the procylical inflation regime.

Overall, equation (22) elicits the intuition from the simple model: the cyclicality of inflation in a model with domestic default entails various endogenous channels including, but not limited to, an endogenous default risk and the standard hedging argument. The interplay between these channels also varies over the cycle: inflation procyclicality is likely to be associated with a discount when default risk is low, but not in bad times as default motives increase with inflation procyclicality. We turn to a quantitative analysis of these forces in the next section and use the model to assess the implications of the *inflation procyclicality discount* we documented.

5 Quantitative Analysis

In this section, we use a calibrated version of the model to investigate the role of the inflation process on the dynamics of interest rates, debt, and default crises. We first calibrate the model with zero covariance and then compare and contrast the models with procyclical and countercyclical inflation to assess the differential impact of inflation cyclicality on interest rates, debt dynamics, and default crises. See Table 3 for a summary of our parameters.

5.1 Functional forms and calibration

Income and inflation processes Endowments y and inflation π follow a joint process:

$$\begin{bmatrix} \log y' \\ \pi' \end{bmatrix} = \begin{bmatrix} \rho_{y,y} & \rho_{\pi,y} \\ \rho_{y,\pi} & \rho_{\pi,\pi} \end{bmatrix} \begin{bmatrix} \log y \\ \pi \end{bmatrix} + \begin{bmatrix} \epsilon_y \\ \epsilon_\pi \end{bmatrix} \tag{23}$$

where

$$\begin{bmatrix} \epsilon_y \\ \epsilon_\pi \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_y^2 & \sigma_{\pi,y} \\ \sigma_{\pi,y} & \sigma_\pi^2 \end{bmatrix} \right).$$

Note that since we consider a closed economy environment, output in our model is equal to consumption. We set the persistence of output $\rho_{y,y}$ to 0.8, the persistence of inflation $\rho_{\pi,\pi}$ to 0.8, the spillover terms $\rho_{y,\pi}$ and $\rho_{\pi,y}$ to zero, and both variance terms σ_y and σ_π to 0.010 based on the parameters estimated for the cross section of OECD economies in our data set. Table 10 in Appendix A contains the detailed estimates by country. We consider two values for the covariance of inflation and output $\sigma_{\pi,y}$: $+0.255e^{-4}$ and $-0.255e^{-4}$, which respectively generate a 1.5 standard deviation above and below the median covariance of inflation and consumption residuals computed at 10-year windows, which is close to zero.

Default cost regime switching Next, we calibrate the default cost regime switching process. We assume the default cost regimes $d_1(s)$ follow a Markov switching process with a transition matrix P

$$P = \begin{bmatrix} p_H & 1 - p_H \\ 1 - p_L & p_L \end{bmatrix}.$$

We estimate the persistence parameters using spreads relative to German bonds from the sub-sample of our data set covering Eurozone countries between 1999 and 2015. We estimate p_H and p_L as the persistence of low spreads and high spreads, respectively.

We define the cutoff for high and low spreads to capture material changes in the nature of sovereign credit risk. In our benchmark calibration, low spreads and high spreads are defined to be below and above 200 basis points, respectively. The estimation yields $p_H = 0.992$ and $p_L = 0.909$. We present the robustness of our results to changes in this definition in the appendix.

Preferences We set the discount factor β_ℓ of the lender to be 0.99 to match an annual risk-free rate of 4 percent. We set the lender's risk aversion γ_ℓ to be 59, following Hatchondo et al. (2016) and Piazzesi and Schneider (2006). This higher level of risk aversion of the lender is also common in the finance and equity premium puzzle literature (for example, see Bansal and Yaron 2004 and Mehra and Prescott 1985). We set the government's risk aversion γ_g to be 2, as is standard in the macro and sovereign debt literature.

Jointly calibrated parameters We jointly choose the mean income loss parameters $d_1(H) = 0.20$ and $d_1(L) = 0.16$ along with the government's discount factor $\beta_g = 0.9875$ to match the cyclical properties of default risk across the two default cost regimes.

Specifically, we choose these parameters so that the acyclical economy has (i) an unconditional default probability of 0.2 percent, (ii) a conditional default probability of 0.5 percent in the low default cost regime L, and (iii) a conditional default probability of 0.0 percent when output is above average across both default cost regimes.

The unconditional default probability of 0.2 percent implies that defaults, on average, occur once every 500 years, which is the average frequency at which the countries in our data set have defaulted between 1900 and 2015, excluding the two world wars, according to the default and debt rescheduling episodes reported by Reinhart and Rogoff (2009). Since all four of these default and debt rescheduling episodes occurred during the midst of the Great Depression, we set the probability of default in tranquil times (above mean output) to 0.0 percent.

Finally, we set the low default cost regime default probability to be 0.5 percent, which is

Table 3: Calibration – Baseline economy with acyclical inflation

Parameters	Values	Targets / Source
Gov't discount factor β_g	0.988	Unconditional default probability: 0.2 percent
Default cost at mean $d_1(H)$	0.200	Default probability in good times: 0.0 percent
Default cost at mean $d_1(L)$	0.160	Default probability in $L-$regime: 0.5 percent
Lender discount factor β_ℓ	0.990	Risk-free rate: 4 percent
Lender risk aversion γ_ℓ	59	Hatchondo et al. (2016)
Gov't risk aversion γ_g	2	Hatchondo et al. (2016)
Default cost threshold d_0	-0.028	Sensitivity analysis in Appendix C
Probability of re-entry θ	0.100	Average exclusion: 10 quarters†
Recovery parameter λ	0.960	Average recovery rate: 50 percent‡
Tax rate τ	0.193	Government consumption (percent GDP)
Debt maturity δ	0.054	OECD average maturity: 4.6 years
Persistence of $H-$regime p_H	0.992	Persistence of low Eurozone spreads (1999–2015)
Persistence of $L-$regime p_L	0.909	Persistence of high Eurozone spreads
Persistence $\rho_{y,y} = \rho_{\pi,\pi}$	0.800	VAR estimates (OECD cross section)
Spillovers $\rho_{\pi,y} = \rho_{y,\pi}$	0.000	VAR estimates
Volatility $\sigma_y = \sigma_\pi$	0.010	VAR estimates
Covariance of innovations $\sigma_{\pi,y}$	0.000	Acyclical baseline $\pm$1.5 s.d. = $\pm$0.255e-4

† : See Richmond and Dias (2008). ‡ : See Benjamin and Wright (2009).

between the unconditional default probability of 0.2 percent and the higher unconditional default probabilities typically used in the literature for emerging economies, which is around 2 percent.[14] We discuss the sensitivity of our main findings in section 5.2.

Other externally calibrated parameters We set the default cost parameter $d_0 = -0.0275$, which corresponds to an output threshold of 1.5 standard deviations below mean, above which the default cost is positive. We show in Table 11 of Appendix C that the main results are robust to alternative values.

We set δ to be 0.054 to match the average domestic debt maturity of 4.6 years in our sample (1999–2010). We set the tax rate τ to be 19 percent to match the government consumption share of GDP in OECD economies between 1985 and 2015.

The probability of re-entry θ is set to match the average exclusion of 10 quarters as documented by Richmond and Dias (2008) and the recovery parameter λ is set to be consistent with the average recovery rate of 50 percent reported by Benjamin and Wright (2009). To compute the average recovery rate, we consider a default to be over when the government regains access to credit and we discount the payment in exclusion back to the period of default at an annualized interest rate of 10 percent, as in Benjamin and Wright (2009).

5.2 Quantitative results

Using the calibrated model, we contrast two default cost regimes and two inflation regimes: a procyclical economy and a countercyclical economy. The main goal of this exercise is to quantitatively explore the differences between the two inflation regimes and how the differential impact of inflation cyclicality changes when default risk becomes more material.[15]

The unconditional inflation procyclicality discount First, we present the results from our calibrated benchmark model. In Table 4, we show the average equilibrium interest rates, debt, and default risk across inflation regimes.

We find that, relative to its countercyclical counterpart, the procyclical economy faces spreads that are 39 basis points lower. Such *inflation procyclicality discount* represents

[14] See, for example, Aguiar et al. (2016b) for a benchmark calibration for emerging economies.
[15] See the computational appendix for a description of our solution algorithm and the model simulation.

Table 4: Baseline results on the procyclicality discount, default risk and debt

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Spreads (percent)	1.25	1.64	–0.39
Default probability (percent)	0.24	0.15	+0.09
Public debt (percent of tax receipts)	63.9	69.7	–5.79

about 40 percent of the discount implied by the regression estimates. Despite such a sizable discount, the procyclical economy is very marginally more prone to debt crises and sustains lower debt burdens compared with the countercyclical economy.

These results are also qualitatively consistent with the intuition given in the spread decomposition equation (22) and the simple model in section 3: spreads feature an inflation procyclicality hedging discount in addition to an inflation procyclicality default premium.

The procyclicality discount: A tranquil-times effect Moreover, the procyclicality discount is state-contingent as in the data. To show this, we first focus on the high default cost regime and report spreads and default probabilities, conditional on safe and crisis times, defined as when output is above and below average, respectively.[16] We summarize our findings in Table 5.

In good times, default risk is immaterial—default probabilities are near-zero in both inflation regimes—and the conditional *inflation procyclicality discount* is about 85 basis points, about 45 basis points larger than the unconditional procyclicality discount.

In bad times, both the procyclical-inflation economy and the countercyclical-inflation economy feature countercyclical default risk. However, in bad times, default risk spikes more in the procyclical economy compared to the countercyclical economy. Even though both economies feature near-zero default risk in good times, default risk spikes to 0.40 percent in bad times in the procyclical economy, about 0.15 percentage points more than its countercyclical counterpart.

This result highlights the 'complementarity' of inflation and default in the procyclical

[16] The goal of this exercise is to contrast crisis-prone and tranquil states. Conditioning on output allows us to perform a cleaner comparison across exogenous states, since default is endogenous to inflation cyclicality.

Table 5: State-contingent procyclicality discount and debt crises under high default costs

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Spreads (high default cost regime)			
in good times (pct)	0.58	1.43	–0.85
in bad times (pct)	1.79	1.77	+0.02
Default prob. (high default cost regime)			
in good times (pct)	0.00	0.00	–0.00
in bad times (pct)	0.40	0.26	+0.15

economy, as opposed to the 'substitutability' of inflation and default under countercyclical inflation. Altogether, as default risk becomes material in bad times, the procyclicality discount vanishes. In fact, relative to its countercyclical counterpart, spreads are 2 basis points higher in the procyclical economy.

Inflation procyclicality: A double-edged sword The previous results highlighted the offsetting nature of the procyclicality hedging discount and default incentives. To more sharply put these mechanisms in perspective, we now focus on the low default cost regime in which default motives are higher *ceteris paribus*. We summarize our findings in Table 6.

We find that the offsetting effect of countercyclical default risk under procyclical inflation is magnified when default risk is more material. The 'complementarity' between inflation procyclicality and default is stronger in the sense that default risk spikes even more in the procyclical regime under low default costs. In bad times, under low default costs, default risk is 0.92 percent in the procyclical economy and 0.64 percent in the countercyclical economy, as opposed to a bad-times default risk of 0.40 percent and 0.26 percent, respectively, under high default costs.

Overall, under low default costs, this excess bad-times default risk in the procyclical-inflation economy leads to a *bad-times procyclicality premium* of 35 basis points, while the *good-times procyclicality discount* remains large, at 83 basis points.

Discussion In summary, we find that: (i) in good times, default risk is immaterial, and the procyclical economy enjoys a large discount, regardless of the default cost regime; (ii)

Table 6: State-contingent procyclicality discount and debt crises under low default costs

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Spreads (low default cost regime)			
in good times (pct)	0.86	1.68	–0.83
in bad times (pct)	2.68	2.33	+0.35
Default prob. (low default cost regime)			
in good times (pct)	0.00	0.00	0.00
in bad times (pct)	0.92	0.64	+0.28

however, in bad times, the procyclical economy faces a larger increase in default risk, and its spreads can be higher relative to its countercyclical counterpart, especially in the low default cost regime.

The main finding of a stronger procyclical discount in good times is qualitatively robust to alternative preferences, risk aversion, debt maturity, and regime switching assumptions. However, these assumptions are important quantitatively. In particular, the tranquil-times inflation procyclicality discount is increasing in risk aversion and debt maturity, and is smaller under a single default cost regime (Tables 12, 13, and 14 in Appendix C). We also show that the tranquil-times procyclicality discount is robust to using a constant relative risk-aversion utility, but in this case, the calibrated economy features very volatile risk-free rates (Table 15 in Appendix C).

Finally, we show that while the tranquil-times discount is robust to higher default probabilities, obtained by lowering the patience of the government, the unconditional discount vanishes under this parameterization (Table 16 in Appendix C). In other words, the unconditional inflation procyclicality discount does not materialize when default probabilities are on the order of magnitude of default risk in emerging economies.

5.3 When is procyclicality preferred?

Since the effect of inflation cyclicality on default and interest rates is both sizable and state-dependent, it is useful to discuss when the government prefers a procyclical inflation regime. In Table 7 we report which cyclicality regime the government prefers, measured in

Table 7: Government preferences for procyclicality regime

	Consumption equivalent (percent)
Overall	0.04
Good times	0.12
Bad times	-0.03
Very bad times	-0.21

consumption equivalents, across different states.

Table 7 reveals that the government typically prefers the procyclical regime, especially in good states of the world, when the endowment realization is above the mean. During good times, the government can borrow at lower interest rates and is not at risk of defaulting. However, in bad states of the world, when output is below the mean, the government has a slight preference for countercyclicality. In very bad states, when output is below average and the probability of default exceeds 10 percent, the government has a strong preference for countercyclicality. This is consistent with the endogenous state- and regime-dependent default premium present in this model and the implied debt pricing.

As discussed above, a procyclical inflation regime is more likely to have disinflation, possibly deflation, increasing its real debt burden during recessions, leading to this reversal in preferences.[17] In that sense, our findings are relevant for the debate on the costs and benefits of joining or exiting a monetary union, since countries within a union that are in fiscal trouble would prefer a countercylical monetary policy, while the others would not: the contrast over monetary policy increases in a recession.

6 Conclusion

This paper has investigated how inflation cyclicality affects borrowing costs, and debt and default dynamics. Empirically, we documented that the co-movement of inflation innovations and consumption growth innovations fluctuates over time across a large number of

[17] In the simple case of CRRA lender preferences, the lenders always prefer the procyclical inflation regime. However, with Epstein-Zin preferences, differences across inflation regimes in intertemporal debt dynamics and state-contingent default risk make the lenders' ranking of inflation regimes less trivial.

advanced countries. Moreover, we find that increased co-movement of inflation and consumption growth is associated with lower borrowing costs, especially in tranquil times. Theoretically, we showed that the inflation processes—especially inflation cyclicality—can be important in explaining interest rates and the dynamics of default. In particular, in our benchmark calibration, the procyclical inflation economy faces lower borrowing costs, even as default is more likely. However, when the economy deteriorates, the procyclical economy faces a much higher likelihood of facing a debt crisis, because it is more likely to face lower inflation, possibly even deflation, and thus an increasing real debt burden. Inflation procyclicality not only induces a hedging discount but it also acts as a 'complement' to default that magnifies countercyclical default risk and interest rate spikes in bad times.

Our findings have implications for the debate on the costs and benefits of joining or exiting monetary unions. Our findings also suggest a new channel—the interaction of monetary policy and interest rates in the presence of sovereign credit risk—that can help us understand the secular decline in real rates and its implications. All throughout the paper we have modeled inflation as an exogenous process. In reality, many studies—starting with Sargent and Wallace (1981)—showed that the process for inflation and its co-movement with output are the result of explicit monetary policy choices, of the interaction between the monetary policy and the fiscal authority, all in response to different types of shocks. We think that including the link between inflation cyclicality, debt pricing, and debt crises highlighted by this paper in a study of optimal monetary and fiscal responses to shocks is an interesting and policy-relevant direction for future research.

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Appendix

A Additional Tables

Table 8: Sensitivity to yield maturities

	Real yield on government debt			
	(1)	(2)	(3)	(4)
Yield Source	IFS	Haver 10-year	Haver 5-year	Haver 3-month
Inflation consumption covariance	-1.80^{**}	-1.76^{**}	-2.09^{**}	-1.12
	(0.64)	(0.70)	(0.87)	(0.84)
Other controls	Yes	Yes	Yes	Yes
adj. R^2	0.90	0.92	0.93	0.95
N	1726	1620	1280	1134

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Standard errors clustered by country. All regressions include country and time fixed effects. The data are a quarterly unbalanced panel from 1985Q1 to 2015Q4 including AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, ITA, JPN, KOR, NLD, NOR, PRT, SWE, USA. All variables are computed over a forward-looking 10-year window. The co-movement of inflation and consumption growth is measured as the covariance of residuals within that window: $\mathbf{cov}_t(\varepsilon_{\pi it}, \varepsilon_{git})$. Other regressors are averages and variances of those residuals in the window and lagged debt.

Table 9: Sensitivity to debt measure

	Real yield on government debt			
	(1)	(2)	(3)	(4)
Debt Source	Oxford+OECD	OECD	Oxford	OECD+Oxford
Inflation consumption co-movement	-1.80**	-1.35	-1.82***	-1.67**
	(0.64)	(1.59)	(0.56)	(0.64)
Other controls	Yes	Yes	Yes	Yes
adj. R^2	0.90	0.82	0.91	0.91
N	1726	918	1556	1731

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses. Standard errors clustered by country. All regressions include country and time fixed effects. The data are a quarterly unbalanced panel from 1985Q1 to 2015Q4 including AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, ITA, JPN, KOR, NLD, NOR, PRT, SWE, USA. All variables are computed over a forward-looking 10-year window. The co-movement of inflation and consumption growth is measured as the covariance of residuals within that window: $\mathbf{cov}_t(\varepsilon_{\pi it}, \varepsilon_{git})$. Other regressors are averages and variances of those residuals in the window and lagged debt.

Table 10: VAR results

country	$\rho_{\pi\pi}$	$\rho_{c\pi}$	$\rho_{\pi c}$	ρ_{cc}	σ_c	σ_π	$\sigma_{\pi,c}$
USA	0.93	0.06	-0.10	0.86	0.17	0.34	0.00
AUS	0.82	0.10	-0.02	0.67	0.67	0.54	0.07
AUT	0.82	0.04	-0.10	0.65	0.27	0.43	0.00
BEL	0.85	0.02	-0.04	0.77	0.33	0.33	0.00
CAN	0.75	0.18	-0.02	0.72	0.63	0.42	0.06
CHE	0.90	0.09	-0.02	0.83	0.27	0.29	0.01
DEU	0.85	0.10	-0.15	0.49	0.32	0.53	0.02
DNK	0.56	-0.05	-0.25	0.71	0.56	0.66	0.02
ESP	0.87	0.01	-0.04	0.91	0.34	0.59	0.01
FIN	0.67	0.12	-0.01	0.87	0.65	0.73	0.05
FRA	0.89	0.10	-0.18	0.67	0.22	0.32	-0.01
GBR	0.83	0.09	-0.11	0.83	0.56	0.51	-0.06
ITA	0.67	-0.03	-0.01	0.88	0.61	0.44	-0.01
JPN	0.92	0.10	-0.26	0.48	0.37	0.70	-0.11
KOR	0.69	0.10	-0.30	0.81	0.97	1.24	-0.32
NLD	0.67	0.04	-0.05	0.85	0.53	0.44	0.00
NOR	0.81	0.14	-0.02	0.68	1.79	0.80	-0.02
PRT	0.88	-0.04	0.02	0.89	0.68	0.71	-0.02
SWE	0.75	-0.12	-0.02	0.75	0.72	0.52	0.09
average	0.80	0.06	-0.09	0.75	0.56	0.56	-0.01
median	0.82	0.09	-0.04	0.77	0.52	0.56	0.00
min	0.56	-0.12	-0.30	0.48	0.29	0.17	-0.32
max	0.93	0.18	0.02	0.92	1.24	1.79	0.09

The data are a quarterly panel from 1985Q1 to 2015Q4.

B Proofs

B.1 Proof of Theorem 1

Theorem 1. Inflation procyclicality discount

Assume that both borrowers and lenders have quasilinear utility, i.e. $u(c) = Ac$ *and* $v(c) = Ac - \frac{\phi}{2}c^2$ *with* $\frac{A}{\phi} > \mu$. *There is an inflation procyclicality discount. That is,*

$$\frac{dr(\kappa)}{d\kappa} < 0.$$

Proof: Notice first that since $r(\kappa) \equiv \frac{1}{q(\kappa)} - 1$, $\dfrac{dr(\kappa)}{d\kappa} < 0 \Leftrightarrow \dfrac{dq(\kappa)}{d\kappa} > 0$

Lender. The lender's first-order condition is given by

$$-qu'(1-qb) + \beta_\ell \mathbf{E}\left[v'\left(x + \frac{b}{1+\pi(x;\kappa)}\right)\frac{1}{1+\pi(x;\kappa)}\right] = 0 \tag{24}$$

which can be written as

$$qA = \beta_\ell\left[A - \phi(\mu+b) + \phi\kappa\sigma^2 - \phi b\kappa^2\sigma^2\right]. \tag{25}$$

Rearranging terms in equation (25) yields the optimal debt supply:

$$b_\ell(q;\kappa) = \frac{-\frac{A}{\phi}q + \beta_\ell\left(\frac{A}{\phi} - \mu + \kappa\sigma^2\right)}{\beta_\ell\left(1+\kappa^2\sigma^2\right)} \tag{26}$$

Borrower. The borrower's first-order condition is given by

$$qu'(1+qb) + \beta_b \mathbf{E}\left[u'\left(x - \frac{b}{1+\pi(x;\kappa)}\right)\frac{1}{1+\pi(x;\kappa)}\right] = 0 \tag{27}$$

which can be written as

$$qA = \beta_b\left[A - \phi(\mu-b) + \phi\kappa\sigma^2 + \phi b\kappa^2\sigma^2\right]. \tag{28}$$

Hence, the optimal debt demand is given by

$$b_b(q;\kappa) = \frac{\frac{A}{\phi}q - \beta_b\left(\frac{A}{\phi} - \mu + \kappa\sigma^2\right)}{\beta_b\left(1+\kappa^2\sigma^2\right)}. \tag{29}$$

Inflation Procylicality Discount. The market clearing condition is

$$b_\ell(q;\kappa) = b_b(q;\kappa). \tag{30}$$

Substituting equations (26) and (29), and rearranging terms, we obtain

$$q = \frac{\phi}{A}\frac{2\beta_b\beta_\ell}{\beta_b+\beta_\ell}\left(\frac{A}{\phi} - \mu + \kappa\sigma^2\right) \tag{31}$$

Finally, taking the derivative of q with respect to κ yields the desired result. □

B.2 Proof of Theorem 2

Theorem 2. Inflation procyclicality and default

Assume that $-(\mu - x_{\min})^{-1} < \kappa < (x_{\max} - \mu)^{-1}$. *For* ψ *large enough, there exists a unique threshold* $\widehat{x}(\kappa, b_b) \in [x_{\min}, \mu]$ *such that default occurs if and only if* $x \in [x_{\min}, \widehat{x}]$. *Furthermore, the default threshold is increasing in debt* (b_b) *and the cyclicality of inflation* (κ), *ceteris paribus. That is,*

$$\frac{\partial \widehat{x}(\kappa, b_b)}{\partial b_b} > 0 \tag{32}$$

$$\frac{\partial \widehat{x}(\kappa, b_b)}{\partial \kappa} > 0. \tag{33}$$

Proof: The borrower defaults when the cost of default is less than the cost of repayment, i.e., when

$$C\left(x\right) \leq b_b[1 + \pi\left(x; \kappa\right)]^{-1}$$

or

$$C\left(x\right)\left[1 + \pi\left(x; \kappa\right)\right] \leq b_b. \tag{34}$$

The proof proceeds in the following steps. First, we show that if a solution exists, it is unique. Second, we show that the unique threshold is increasing in debt and the cyclicality of inflation.

Existence and uniqueness. If a solution exists, it is unique if the left-hand side of (34) is strictly increasing,

$$C_x\left[1 + \pi\left(x; \kappa\right)\right] + C\left(x\right)\pi_x\left(x; \kappa\right) > 0. \tag{35}$$

We know that

$$\begin{aligned}
\pi\left(x;\kappa\right) &= \frac{-\kappa\left(\mu-x\right)}{1+\kappa\left(\mu-x\right)} \\
\Rightarrow \pi_x\left(x;\kappa\right) &= \frac{\kappa+\kappa\pi\left(x;\kappa\right)}{1+\kappa\left(\mu-x\right)} \\
&= \kappa[1+\pi\left(x;\kappa\right)]^2
\end{aligned}$$

Condition (35) then becomes

$$C_x > -C\left(x\right)\kappa\left[1+\pi\left(x;\kappa\right)\right]$$

which holds since

$$\begin{aligned}
& C_x > -C\left(x\right)\kappa\left[1+\pi\left(x;\kappa\right)\right] \\
\Leftrightarrow & 2\psi\left(x-x_{\min}\right) > -\psi(x-x_{\min})^2\kappa\left[1+\pi\left(x;\kappa\right)\right] \\
\Leftrightarrow & 2\left[1+\kappa\left(\mu-x\right)\right] > -\left(x-x_{\min}\right)\kappa \\
\Leftrightarrow & \kappa\left(\mu-\frac{x+x_{\min}}{2}\right) > -1 \\
\Leftarrow & \frac{-1}{\mu-x_{\min}} < \kappa < \frac{1}{x_{\max}-\mu}
\end{aligned}$$

Hence if a solution exists, it is unique. Since $C(x)$ is continuous, by the intermediate value theorem, a solution exists in $x \in [x_{\min}, \mu]$ if

$$C\left(x_{\min}\right)\left[1+\pi\left(x_{\min};\kappa\right)\right] \leq 0,$$

which holds since $C(x_{\min}) = 0$, and

$$C\left(\mu\right)\left[1+\pi\left(\mu;\kappa\right)\right] \geq b_b$$

which holds for ψ large enough.

Hence, there exists an output threshold

$$\hat{x} \in [x_{\min}, \mu]$$

such that the borrower defaults if and only if $x \leq \hat{x}$.

Comparative Statics. Let $G(\hat{x};\kappa,b_b) = C(\hat{x}) - b_b(1+\pi(\hat{x};\kappa))^{-1} = 0$. By the implicit function theorem,

$$\frac{\partial G(\hat{x};\kappa,b_b)}{\partial \hat{x}}\frac{d\hat{x}}{db_b} + \frac{\partial G(\hat{x};\kappa,b_b)}{\partial b_b} = 0$$

and

$$\frac{\partial G(\hat{x};\kappa,b_b)}{\partial \hat{x}}\frac{d\hat{x}}{d\kappa} + \frac{\partial G(\hat{x};\kappa,b_b)}{\partial \kappa} = 0.$$

Hence

$$\begin{aligned}
\frac{d\hat{x}}{db_b} &= -\frac{-(1+\pi(\hat{x};\kappa))^{-1}}{C_x(\hat{x}) + b_b(1+\pi(\hat{x};\kappa))^{-2}\pi_x(\hat{x};\kappa)} \\
&= \frac{1}{C_x(\hat{x})[1+\pi(\hat{x};\kappa)] + b_b[1+\pi(\hat{x};\kappa)]^{-1}\pi_x(\hat{x};\kappa)} \\
&= \frac{1}{C_x(\hat{x})[1+\pi(\hat{x};\kappa)] + C(\hat{x})\pi_x(\hat{x};\kappa)} > 0
\end{aligned}$$

since

$$C_x[1+\pi(x;\kappa)] + C(x)\pi_x(x;\kappa) > 0$$

from (35). We also have

$$\begin{aligned}
\frac{d\hat{x}}{d\kappa} &= -\frac{b_b[1+\pi(\hat{x};\kappa)]^{-2}\pi_\kappa(\hat{x};\kappa)}{C_x(\hat{x}) + b_b(1+\pi(\hat{x};\kappa))^{-2}\pi_x(\hat{x};\kappa)} \\
&= -\frac{b_b[1+\pi(\hat{x};\kappa)]^{-1}\pi_\kappa(\hat{x};\kappa)}{C_x(\hat{x})[1+\pi(\hat{x};\kappa)] + b_b[1+\pi(\hat{x};\kappa)]^{-1}\pi_x(\hat{x};\kappa)} \\
&= -\frac{b_b[1+\pi(\hat{x};\kappa)]^{-1}\pi_\kappa(\hat{x};\kappa)}{C_x(\hat{x})[1+\pi(\hat{x};\kappa)] + C(\hat{x})\pi_x(\hat{x};\kappa)} > 0
\end{aligned}$$

since

$$\pi(x;\kappa) = \frac{-\kappa(\mu - x)}{1+\kappa(\mu - x)} \tag{36}$$

$$\Rightarrow \pi_\kappa(\hat{x};\kappa) = \frac{-(\mu-\hat{x}) - (\mu-\hat{x})\,\pi(\hat{x};\kappa)}{1+\kappa(\mu-\hat{x})} \tag{37}$$

$$= \frac{-(\mu-\hat{x})(1+\pi(\hat{x};\kappa))}{1+\kappa(\mu-\hat{x})} \tag{38}$$

$$= -(\mu-\hat{x})\left[1+\pi(\hat{x};\kappa)\right]^2 < 0 \tag{39}$$

This concludes the proof of Theorem 2. □

C Sensitivity Analyses

Table 11: Robustness to default cost threshold d_0

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Lower output threshold ($d_0 = -0.035$)			
Spreads (pct)	1.24	1.63	–0.40
Spreads in good times (pct)	0.57	1.44	–0.87
Spreads in bad times (pct)	1.81	1.80	+0.02
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.44	0.24	+0.19
Higher output threshold ($d_0 = -0.020$)			
Spreads (pct)	1.29	1.62	–0.32
Spreads in good times (pct)	0.64	1.44	–0.80
Spreads in bad times (pct)	1.97	1.80	+0.16
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.41	0.32	+0.09

Table 12: Robustness to lender's risk aversion

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Lower risk aversion ($\gamma_\ell = 8$)			
Spreads (pct)	1.36	1.41	–0.05
Spreads in good times (pct)	0.79	1.18	–0.39
Spreads in bad times (pct)	1.90	1.62	+0.28
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.43	0.34	+0.09
Higher risk aversion ($\gamma_\ell = 120$)			
Spreads (pct)	1.07	1.96	–0.89
Spreads in good times (pct)	0.36	1.80	–1.44
Spreads in bad times (pct)	1.74	2.11	–0.38
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.43	0.20	+0.23

Table 13: Robustness to debt maturity

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Shorter debt maturity (4 years)			
Spreads (pct)	0.94	1.37	–0.43
Spreads in good times (pct)	0.39	1.19	–0.80
Spreads in bad times (pct)	1.46	1.54	–0.08
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.38	0.24	+0.14
Longer debt maturity (6 years)			
Spreads (pct)	2.18	2.39	–0.21
Spreads in good times (pct)	1.30	2.19	–0.89
Spreads in bad times (pct)	3.03	2.58	+0.45
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.51	0.31	+0.21

Table 14: Robustness to single default cost regime

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
High default cost regime ($p_h = 1$)			
Spreads (pct)	1.31	1.61	–0.30
Spreads in good times (pct)	0.63	1.43	–0.80
Spreads in bad times (pct)	1.97	1.79	+0.18
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.40	0.23	+0.17
Low default cost regime ($p_l = 1$)			
Spreads (pct)	1.65	1.86	–0.22
Spreads in good times (pct)	0.87	1.60	–0.80
Spreads in bad times (pct)	2.39	2.11	+0.28
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.56	0.39	+0.17

Table 15: Robustness to the lender's utility function

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Epstein-Zin ($\gamma_\ell = 8$)			
Spreads (pct)	1.36	1.41	–0.05
Spreads in good times (pct)	0.79	1.18	–0.39
Spreads in bad times (pct)	1.90	1.62	+0.28
Def. prob. in good times (pct)	0.00	0.00	–0.00
Def. prob. in bad times (pct)	0.43	0.34	+0.09
CRRA ($\gamma_\ell = 8$)			
Spreads (pct)	1.49	2.05	–0.56
Spreads in good times (pct)	1.48	2.38	–0.90
Spreads in bad times (pct)	1.51	1.74	–0.23
Def. prob. in good times (pct)	0.00	0.01	–0.01
Def. prob. in bad times (pct)	0.46	0.37	+0.09

Table 16: Robustness to government discount factor

	Positive co-movement (+1.5 s.d.)	Negative co-movement (–1.5 s.d.)	Difference
Lower patience ($\beta_g = 0.985$)			
Spreads (pct)	3.68	3.77	–0.09
Spreads in good times (pct)	2.37	3.28	–0.91
Spreads in bad times (pct)	4.94	4.24	+0.70
Def. prob. in good times (pct)	0.00	0.01	–0.00
Def. prob. in bad times (pct)	1.10	0.89	+0.21
Higher patience ($\beta_g = 0.989$)			
Spreads (pct)	0.30	0.86	–0.55
Spreads in good times (pct)	–0.03	0.79	–0.82
Spreads in bad times (pct)	0.62	0.92	–0.29
Def. prob. in good times (pct)	0.00	0.00	0.00
Def. prob. in bad times (pct)	0.20	0.07	+0.12