The Optimal Response of Bank Capital Requirements to Credit and Risk in a Model with Financial Spillovers

Filippo Occhino
Working papers of the Federal Reserve Bank of Cleveland are preliminary materials circulated to stimulate discussion and critical comment on research in progress. They may not have been subject to the formal editorial review accorded official Federal Reserve Bank of Cleveland publications. The views stated herein are those of the authors and are not necessarily those of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System.

Working papers are available on the Cleveland Fed’s website: https://clevelandfed.org/wp
The Optimal Response of Bank Capital Requirements to Credit and Risk in a Model with Financial Spillovers
Filippo Occhino

This paper studies optimal bank capital requirements in an economy where bank losses have financial spillovers. The spillovers amplify the effects of shocks, making the banking system and the economy less stable. The spillovers increase with banks’ financial distortions, which in turn increase with banks’ credit risk. Higher capital requirements dampen the current supply of banks’ credit, but mitigate banks’ future financial distortions. Capital requirements should be raised in response to both an expansion of banks’ credit supply and an increase in the expected future credit risk of banks. They should be lowered close to one-to-one in response to bank losses.

Keywords: Debt overhang, financial vulnerabilities, financial stability, macroprudential regulation.
JEL Classification Numbers: G20, G28.


Filippo Occhino is at the Federal Reserve Bank of Cleveland (filippo.occhino@clev.frb.org).
1 Introduction

Since the last financial crisis, there has been an increasing focus on financial spillovers—the transmission of risks and losses from bank to bank and the exposure of individual banks to the banking system. Financial spillovers create feedback loops, amplify the effects of shocks and make the economic and financial systems less stable. For instance, several recent papers have proposed to use various indicators of financial spillovers in order to measure the systemic risk of the financial system.\textsuperscript{1}

In parallel, there has been an increasing interest in how to set bank capital requirements depending on credit, financial and economic conditions, in order to enhance financial stability. The 2010 Basel III Accord has introduced a countercyclical capital buffer, varying between 0 and 2.5 percent, with the primary aim of “achieving the broader macroprudential goal of protecting the banking sector from periods of excess aggregate credit growth that have often been associated with the build-up of system-wide risk”. Prudential authorities can adjust the buffer in response to a wide variety of indicators and shocks.\textsuperscript{2}

\textsuperscript{1} The NYU Stern Volatility Institute (V-Lab) estimates the long-run marginal expected shortfall, the expected percent equity loss of a bank when the stock market declines 40 percent in a 6-month period—Acharya, Pedersen, Philippon and Richardson (2010), Acharya, Engle and Richardson (2012) and Brownlees and Engle (2015) describe the expected shortfall and its relationship with systemic risk. Adrian and Brunnermeier (2014) propose the exposure CoVaR, the increase in the value-at-risk of a financial institution given an increase in the value-at-risk of the financial sector—see also Sedunov (2013) and Pagano and Sedunov (2014). Weller (2015) estimates the conditional tail risk for a broad set of factors using high-frequency data on the cross-section of bid-ask spreads. Saldías (2013) estimates the difference between the average distance-to-default of banks and the distance-to-default of the aggregate portfolio of banks, using data from banks’ balance sheets, equity markets and option markets, and proposes this difference as a measure of banks’ interdependence and joint risk of distress.

\textsuperscript{2} The Federal Reserve sets the countercyclical capital buffer taking into account a
This paper studies the optimal response of bank capital requirements to banks’ credit supply and to banks’ credit risk in an economy where bank losses have financial spillovers.

The financial spillovers modeled here work through a worsening of the financial distortions faced by banks, a contraction of banks’ credit supply, a decline of economic activity, a deterioration of firms’ balance sheets, and an additional increase in bank losses (Figure 1). The specific financial distortion used in this paper is the debt-overhang distortion (Myers 1977) on bank lending—banks’ existing liabilities discourage their lending because part of the return on their loans will benefit banks’ existing creditors only. When the balance sheets of a set of banks deteriorate and their credit risk increases, banks’ financial distortions increase and dampen their credit supply; lending, investment and aggregate demand decline; the firms’ balance sheets deteriorate, impairing the firms’ ability to repay their liabilities; the value of loans and securities of other banks declines, and their balance sheets deteriorate as well—Section 2 provides evidence on this channel of financial spillovers.

These financial spillovers generate a feedback loop that is at work in the typical business cycle and was especially evident during the last crisis. As noted by Gertler and Karadi (2011):

range of financial-system vulnerabilities and other factors, including “asset valuation pressures and risk appetite, leverage in the nonfinancial sector, leverage in the financial sector, and maturity and liquidity transformation in the financial sector”, and monitoring “a wide range of financial and macroeconomic quantitative indicators including, but not limited to, measures of relative credit and liquidity expansion or contraction, a variety of asset prices, funding spreads, credit condition surveys, indices based on credit default swap spreads, option implied volatilities, and measures of systemic risk.” In July 2016, the Bank of England cut the countercyclical capital buffer from 0.5 percent to zero to support lending and mitigate the possible adverse economic consequences of the UK’s “Brexit” decision to leave the European Union.
The current crisis has featured a sharp deterioration in the balance sheets of many key financial intermediaries. As many observers argue, the deterioration in the financial positions of financial intermediaries has had the effect of disrupting the flow of funds between lenders and borrowers. Symptomatic of this disruption has been a sharp rise in various key credit spreads as well as a significant tightening of lending standards. This tightening of credit, in turn, has raised the cost of borrowing and thus enhanced the downturn. The story does not end here: the contraction of the real economy has reduced asset values throughout, further weakening intermediary balance sheets, and so on.

First, we examine the determinants and effects of financial spillovers. We measure the spillovers by the size of the effect of a shock to the net-worth of banks on the net-worth of an individual bank that is not hit by the shock. The spillovers increase with banks’ financial distortions, which in turn increase with banks’ credit risk, measured by the credit spread between the yield on bank-issued junior bonds and the risk-free rate. The spillovers amplify the effects of shocks on the banking system and on economic activity.³

³Financial spillovers, combined with debt overhang, can make banks’ lending decisions strategic complements in the sense of Cooper and John (1988)—a bank’s expected discounted marginal profit from lending $l_2$ rises as other banks’ loans rise. This strategic complementarity can amplify small shocks and can generate multiple equilibria, leading to financial crises driven by self-fulfilling expectations. Cooper and John (1988) show that strategic complementarity is necessary and sufficient for multiplier effects, and that strategic complementarity in some region of the state space is necessary for multiple equilibria. Lamont (1995) shows that, in an economy where firms’ investments have positive spillovers, debt overhang on firms’ investments generates strategic complementarities and the potential for multiple equilibria. This type of crises are studied in Occhino (2015,2016).
tortions are large, and financial spillovers are strong. This is consistent with recent evidence that connects the volatility of real GDP growth with financial conditions—for instance, Adrian, Boyarchenko and Giannone (2016) use quantile regressions to show that worse financial conditions are associated with a higher standard deviation of the one-year ahead conditional forecast distribution of GDP growth.

Next, we highlight an intertemporal trade-off faced by the prudential authority: tighter capital requirements dampen the current level of lending and investment activity, by raising banks’ cost of funds, but mitigate the future financial distortions faced by banks, by decreasing banks’ credit risk.

Finally, we study the response of optimal capital requirements, i.e., the capital requirements that maximize the welfare of the representative household. Capital requirements should be raised in response to each of the following fundamental shocks: a risk shock that increases the future credit risk of banks; a shock that increases the net worth of banks; a positive technology shock; a preference shock that increases the household preferences discount factor.

The joint optimal response of capital requirements and credit supply to the various types of shocks is especially noteworthy. In the case of most shocks, capital requirements should be raised as banks’ credit supply expands, a prescription in line with the Basel III indication to raise capital requirements in periods of “excess aggregate credit growth”. However, in the case of a risk shock that increases the future credit risk of banks, capital requirements should be raised as banks’ credit supply contracts, a prescription at odds with the Basel III indication. The intuition for this prescription is that the main effect of a risk shock is to increase future financial distortions, so capital requirements should be raised to mitigate this effect, at the cost of a slightly deeper contraction in banks’ current credit supply. Hence, whether capital requirements and banks’ credit sup-
ply should move in the same direction or in opposite directions depends on the type of shock. This suggests that capital requirements should be set taking into account not only measures of credit growth but also indicators of banks’ future credit risk and financial distortions.

We turn, then, to the optimal response of capital requirements to observable variables. We find that capital requirements should be raised in response to both an expansion of banks’ credit supply and an increase in the expected future credit spread on bank-issued bonds. With our parameter setting, capital requirements should be raised by 0.28 percentage points in response to a 1 percent expansion of banks’ credit supply; and should be raised by 0.32 percentage points in response to a 1 percentage point increase in the expected future credit spread on bank-issued bonds. Furthermore, capital requirements should be lowered by 0.95 percentage points in response to bank losses equivalent to 1 percent of banks’ value, close to one-to-one.

This paper is related to several strands of literature.

There is a rapidly expanding literature that models the aggregate implications of Myers (1977)’ debt-overhang distortion on bank lending. Wilson

---

4 Other papers have studied the effect of related frictions in the banking sector on banks’ credit supply and aggregate economic activity. In Meh and Moran (2010), banks may not properly monitor entrepreneurs because any resulting risk is mostly borne by investors. For this reason, investors require banks to invest their own net worth in entrepreneurs’ projects. Then, the banks’ ability to attract loanable funds and finance entrepreneurs depends on the strength of their balance sheets. Shocks to banks’ balance sheets cause bank lending, investment and output to decline, and banks’ balance sheets amplify and propagate the effects of technology shocks. In Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) bank managers can divert a fraction of assets for their own benefit, defaulting on debt. For this reason, creditors restrict the amount they lend depending on the bank’s balance sheet, which introduces a spread between the loan and deposit rates. During a crisis, this spread widens substantially, raising the cost of credit that nonfinancial borrowers face. As the risk of default rises, this financial friction rises
and Wu (2010), Wilson (2012), Philippon and Schnabl (2013) and Bhat- 
tacharya and Nyborg (2013) study optimal government recapitalization of 
banks that suffer from debt-overhang problems. In Acharya, Drechsler, and 
Schnabl (2014), the financial sector under-invests due to the debt overhang 
problem, while the government may undertake a bailout of the financial sec-
tor, and this generates a feedback loop between sovereign and bank credit 
risk. Occhino (2016) shows that a self-fulfilling-expectations banking crisis 
can arise when the value of banks’ assets is sensitive to economic prospects 
while banks’ liabilities distort their lending.

This paper is also related to the growing literature on the effect of bank 
capital requirements on the aggregate economy and on the use of capital 
requirements for macroprudential purposes. Our paper differs from this 
literature for its focus on financial spillovers—what generates them, what 
determines their size, and how capital requirements should be set in the 
presence of spillovers.

and Nguyen (2015), focus on the effect of the level of capital requirements 
on welfare. Zhu (2008) argues that a risk-sensitive capital regime that 
makes capital requirements higher for riskier banks and lower for less risky 
banks is welfare-improving. Dib (2010) shows that financial frictions in the 
interbank and bank capital markets amplify and propagate the effects of 
shocks; however, capital requirements attenuate the real impacts of aggre-
gate shocks and reduce macroeconomic volatilities.

Some papers focus on the cyclicality of capital requirements. Covas 
and Fujita (2010) show that output is more volatile and household wel-
fare is smaller when capital requirements are procyclical. Repullo and 
Suarez (2013) point out that the optimal capital requirements are lower 
and more cyclically varying if the social cost of bank failure is low, and 
and the credit supply declines.
they are higher and less cyclically varying if it is large. Clerc at al. (2015) find that higher capital requirements make the economy less vulnerable to shocks, and that countercyclical capital requirements enhance stability when bank capital is high, but amplify the effects of shocks when bank capital is low. Malherbe (2016) shows that, in economies where banks do not fully internalize the social costs of default, there is aggregate over-investment, and capital requirements should be tighter when the aggregate bank capital is higher. Davydiuk (2016) proposes that capital requirements should be procyclical in a model where capital requirements mitigate excessive risk-taking among banks but restrict bank loan and liquidity provision.

Other papers focus on the optimal response of capital requirements to shocks and variables. Repullo (2013) shows that capital requirements should be lowered after an exogenous negative shock to bank capital, to mitigate the reduction in aggregate investment. Rubio and Carrasco-Gallego (2016) propose that capital requirements should respond to deviations of credit from its steady state.

The rest of the paper is organized as follows. Section 2 surveys the evidence on the type of financial spillovers modeled in the paper. Section 3 describes the model. Section 4 contains the main results on the determinants and effects of financial spillovers and on the optimal response of capital requirements to shocks and observable variables. Section 5 concludes.

2 Evidence on financial spillovers

The financial spillovers modeled in this paper are the result of two effects: the effect of banks’ balance sheets on banks’ credit supply and economic activity; and the effect of economic activity on banks’ balance sheets.

Several studies document that weak banks’ balance sheets have a nega-

Other studies suggest that a contraction in credit supply, identified using the Senior Loan Officer Opinion Survey on bank lending practices, has a negative effect on economic activity. Asea and Blomberg (1998) show that changes in lending standards can amplify economic fluctuations. Lown and Morgan (2006) document that shocks to lending standards explain business loans and real economic activity. Bassett, Chosak, Driscoll, and Zakrajsek (2014) find that a tightening shock to a credit supply indicator leads to a substantial decline in output. Specifically, an adverse credit supply shock of one standard deviation is associated with a decline in the level of real GDP of about 0.75 percent 2 years after the shock, while the capacity of businesses and households to borrow from the banking sector falls more than 4 percent over the same period. Such disruptions in the credit-intermediation process also lead to a substantial rise in corporate bond credit spreads. Using the euro area Bank Lending Survey, Altavilla, Paries and Nicoletti (2015) document that loans’ tightening shocks explain a large fraction of the contraction in real activity.

The effect of economic activity on banks’ balance sheets is most evident from recent panel studies of firms’ default rates and banks’ credit risk. Simons and Rolwes (2009) find a significant negative relation between GDP growth and the default rate of Dutch firms (but not Austrian firms). Using a panel of Portuguese firms, Bonfim (2009) document that, in
addition to firm-specific characteristics, macroeconomic conditions play an important role in explaining firms’ default probabilities. For Italian banks, Marcucci and Quagliariello (2009) highlight that the effects of the business cycle on credit risk are more pronounced during downturns and the cyclicality is higher for those banks with riskier portfolios. Louzis, Vouldis and Metaxas (2012) show that macroeconomic variables, including the real GDP growth rate, have a strong effect on the level of non-performing loans in the Greek banking system.

The dynamics of variables over the business cycle are consistent with the effects modeled in this paper. Table 1 shows the correlations of various indicators with real GDP growth, while Figure 2 plots the times series of these indicators with recession bars. Although these correlations are the result of many effects and shocks, it is worth noting that they are consistent with the effects of economic activity on banks’ balance sheets and banks’ credit supply: a contraction of economic activity is correlated with a drop in firms’ profits, a deterioration of firms’ balance sheets, a surge in banks’ loan losses, a deterioration of banks’ balance sheets, a contraction of banks’ credit supply, and a decline of banks’ lending and firms’ investment.

3 Model

Before describing the model in detail, it is helpful to highlight the main features and events.

In the model, there are households, banks, intermediate-goods firms, final-goods firms, and a prudential authority. All types of agents have measure equal to one, so sums across agents are equal to averages. Banks are owned by households—the initial number of bank shares owned by households is normalized to 1. Firms are owned by banks—each bank owns a continuum of intermediate-goods firms, but only one final-goods
There are three periods. The key events occur in the first two periods—no choice is made in the third period. In the first period, after the prudential authority sets bank capital requirements, banks sell new equity and senior debt to households and lend to intermediate-goods firms. In the second period, intermediate-goods firms sell intermediate goods to final-goods firms and repay their bank loans, while banks sell junior bonds to households and lend to final-goods firms. In the third period, final-goods firms sell final goods and repay their bank loans, while banks repay the payoff of their senior debt and junior bonds. Bank lending in the second period is distorted by the debt-overhang distortion. In contrast, there are no financial frictions at the level of firms—both types of firms borrow from banks with an optimal contract.

Let \( \omega \) and \( \xi \) be, respectively, the idiosyncratic shocks to the production functions of the intermediate-goods firms and the final-goods firms. The two shocks are log-normally distributed, with \( \mathbb{E}_\omega \{ \omega \} = 1 \) and \( \mathbb{E}_\xi \{ \xi \} = 1 \), where \( \mathbb{E}_\cdot \{ \cdot \} \) are the expectations over \( \omega \) and \( \xi \). Let \( \sigma_\xi \) and \( \sigma_\omega \) be, respectively, the standard deviation of \( \ln(\xi) \) and \( \ln(\omega) \). The first-period aggregate state, \( \Theta_1 \equiv \{ \tau_1, \theta_1, \beta, \sigma_\xi \} \), is made of a shock \( \tau_1 \) to banks’ net-worth (a first-period transfer from households to banks), a technology shock \( \theta_1 \), the households’ preferences discount factor \( \beta \), and the volatility \( \sigma_\xi \). The second-period aggregate state, \( \Theta_2 \equiv \{ \tau_2, \theta_2 \} \), is made of a shock to banks’ net-worth \( \tau_2 \) (a second-period transfer from households to banks), and a technology shock \( \theta_2 \). There is no aggregate shock in the third period. Let \( \mathbb{E}_1 \{ \cdot \} \) be the expectation taken in the first period with respect to the second-period aggregate state \( \Theta_2 \).

Banks begin with initial real funds \( m_0 \) and with a given level of existing liabilities (old deposits), \( d_0 \), due to the households at the beginning of the first period. Households receive given endowments of goods in each of the
three periods.

In the first period, first, the aggregate state $\Theta_1$ is revealed. Then, the prudential authority sets the bank capital requirements, $\kappa$. Households provide funds to banks by purchasing bank senior debt (new deposits) and newly-issued bank equity shares. Banks use these funds to repay their existing liabilities $d_0$ and to lend to intermediate-goods firms. Banks’ equity value is required to be greater than a fraction $\kappa$ of banks’ total value, the sum of the equity value and the senior debt value. Intermediate-goods firms use the funds borrowed from banks to purchase investment goods from households.

In the second period, first, the aggregate state $\Theta_2$ is revealed. Then, the idiosyncratic shock $\omega$ is revealed, intermediate-goods firms produce and sell intermediate goods to final-goods firms, repay their debt to banks, and distribute their profits to banks. Households provide additional funds to banks by purchasing junior bonds. Banks use the funds obtained from households and from intermediate-goods firms to lend to final-goods firms—this lending decision is distorted by debt overhang. The final-goods firms use the funds borrowed from banks to purchase investment goods from households and intermediate goods from intermediate-goods firms.

In the third period, the idiosyncratic shock $\xi$ is revealed, final-goods firms produce and sell final goods to households, repay their debt to banks, and distribute their profits to banks. Banks use the funds obtained from the final-goods firms to repay the payoff of their senior debt and junior bonds to households, and distribute their profits to households. Lump-sum taxes are levied on households in the third period to guarantee banks’ senior debt (deposit insurance).

We now turn to a more detailed description of the agents’ problems.
3.1 Households

The households’ objective function is

\[ W(c_1, c_2, c_3) \equiv u(c_1) + \beta u(c_2) + \beta^2 u(c_3) \]  

(1)

where \( \beta \in (0, 1) \), the utility function satisfies \( u'(c) \equiv c^{-\gamma} \) with \( \gamma > 0 \), while \( c_1, c_2 \) and \( c_3 \) are the consumption levels in the three periods.

In the first period, households receive an endowment \( e_1 \), they receive from banks the face value of their existing deposits \( d_0 \), and are subject to the banks’ net-worth shock \( \tau_1 \). They purchase newly-issued equity shares \( \tilde{s} \) from each of the banks at the price \( q_s \), bank senior debt (new deposits) with face value \( d \) due at the beginning of the third period at the price \( q_d \), and consumption goods \( c_1 \). The new deposits are fully insured, so they are risk-free. Their first-period budget constraint is

\[ c_1 + q_d d + q_s \tilde{s} = e_1 + d_0 - \tau_1 \]  

(2)

In the second period, households receive an endowment \( e_2 \), and are subject to the banks’ net-worth shock \( \tau_2 \). They purchase \( b \) junior bonds from each of the banks at the price \( q_b \), and consumption goods \( c_2 \). Their second-period budget constraint is

\[ c_2 + q_b b = e_2 - \tau_2 \]  

(3)

In the third period, households receive an endowment \( e_3 \), the face value of the fully-insured senior debt \( d \), the payoff of the junior bonds \( \Pi_b \), and the payoff of the equity shares \( \Pi_s \). They pay lump-sum taxes \( T \) and purchase consumption goods \( c_3 \). Their third-period budget constraint is

\[ c_3 + T = e_3 + d + b \Pi_b + (1 + s) \Pi_s \]  

(4)
The representative household’s stochastic discount factors are

\[ A_1 = \frac{\beta u'(c_2)}{u'(c_1)} \]  \hspace{1cm} (5) \\
\[ A_2 = \frac{\beta u'(c_3)}{u'(c_2)} \]  \hspace{1cm} (6)

The first-order conditions for senior debt and junior bonds are:

\[ q_d = \mathbb{E}_1\{A_1 A_2\} \]  \hspace{1cm} (7) \\
\[ q_b = A_2 \Pi_b \]  \hspace{1cm} (8)

To make equity funding more expensive than debt, we introduce in an ad-hoc way agency costs that depress the households’ demand for equity. Myers and Majluf (1984) highlight that an adverse selection problem leads banks with better prospects not to issue equity, so issuing equity contains a signal that equity is overvalued, which depresses the demand for newly-issued equity and its price, making equity funding more expensive. To proxy for these agency costs, we introduce in the optimality condition for the households’ equity choice a wedge between the equity price and the expected discounted value of the equity payoff:

\[ q_s \left( 1 + C \frac{s}{1+s} \right) = \mathbb{E}_1\{A_1 A_2 \Pi_s\} \]  \hspace{1cm} (9)

where \( C > 0 \), and \( s \) is the supply of newly-issued shares. The wedge increases with the number of newly-issued shares \( s \), so, in equilibrium, as banks increase the number of newly-issued shares, the wedge increases and depresses the equity price relative to the expected discounted value of the equity payoff, raising the cost of equity funding.

The risk-free rates are defined by

\[ r_1 \equiv 100 \cdot \left( 1/\mathbb{E}_1\{A_1\} - 1 \right) \]  \hspace{1cm} (10) \\
\[ r_2 \equiv 100 \cdot \left( 1/A_2 - 1 \right) \]  \hspace{1cm} (11)
while the yield and spread of the banks’ junior bonds are defined by

\[ i_b \equiv 100 \cdot (1/q_b - 1) \] (12)

\[ X_b \equiv i_b - r_2 \] (13)

### 3.2 Intermediate-goods firms

In the first period, intermediate-goods firms borrow \( l_1 \) real resources from banks at the interest rate \( R_1 \). To abstract from financial frictions at the firms’ level, we assume that \( l_1 \) and \( R_1 \) are chosen with an optimal contract between firms and banks. Since firms are owned by banks, firms and banks share the same objective function, so \( l_1 \) and \( R_1 \) are chosen to maximize the objective function of banks, as detailed below in Section 3.4.

Intermediate-goods firms do not make any further decision. They use \( l_1 \) to produce intermediate goods \( z_\omega \):

\[ z_\omega = \omega \theta_1 h(l_1) \] (14)

where \( h(l) \equiv Bl^\mu, \ B > 0, \ \mu \in (0, 1), \ \omega \) is a firm’s idiosyncratic shock realized in the second period, and \( \theta_1 \) is an aggregate shock revealed at the beginning of the first period (so \( l_1 \) depends on \( \theta_1 \) but not on \( \omega \)).

In the second period, the intermediate-goods firms sell \( z_\omega \) to the final-goods firms at the price \( p \), return the loan payoff \( \min \{R_1 l_1, pz_\omega \} \) to the banks, and distribute the remaining profits \( \pi^f_\omega \) to the banks:

\[ \pi^f_\omega \equiv pz_\omega - \min \{R_1 l_1, pz_\omega \} = \max \{pz_\omega - R_1 l_1, 0\} \] (15)

### 3.3 Final-goods firms

In the second period, final-goods firms borrow \( l_2 \) real resources from banks at the interest rate \( R_2 \). Similarly to the case of intermediate-goods firms, we assume that \( l_2 \) and \( R_2 \) are chosen with an optimal contract between firms
and banks. Since firms are owned by banks, firms and banks share the same objective function, so $l_2$ and $R_2$ are chosen to maximize the objective function of banks, as detailed below in Section 3.4.

Final-goods firms, then, use $l_2$ to purchase $n$ intermediate goods from intermediate-goods firms at the price $p$, and invest the rest in investment goods $k$:

$$pn + k = l_2$$  \hspace{1cm} (16)

After that, they produce final goods $y_\xi$:

$$y_\xi = \xi \theta_2 f(n, k)$$  \hspace{1cm} (17)

where $f(n, k) = A (n^\alpha k^{1-\alpha})^\nu$, $A > 0$, $\alpha \in (0, 1)$, $\nu \in (0, 1)$, $\xi$ is a firm’s idiosyncratic shock realized in the third period, and $\theta_2$ is an aggregate shock revealed at the beginning of the second period (so $l_2$ depends on $\theta_2$ but not on $\xi$).

In the third period, the final-goods firms sell $y_\xi$ to the households, return the loan payoff $\min\{R_2 l_2, y_\xi\}$ to the banks, and distribute the remaining profits $\pi^F_\xi$ to the banks:

$$\pi^F_\xi \equiv y_\xi - \min\{R_2 l_2, y_\xi\} = \max\{y_\xi - R_2 l_2, 0\}$$  \hspace{1cm} (18)

For given loans $l_2$ and interest rate $R_2$, the inputs $n$ and $k$ are chosen by final-goods firms to maximize the expected profits $E_\xi\{\pi^F_\xi\}$:

$$\max_{\{n,k\}} E_\xi\{\max\{\xi \theta_2 f(n, k) - R_2 l_2, 0\}\}$$  \hspace{1cm} (19)

subject to: $pn + k = l_2$

Equivalently, $n$ and $k$ solve:

$$g(l_2, p) \equiv \max_{\{n,k\}} f(n, k)$$  \hspace{1cm} (20)

subject to: $pn + k = l_2$
Notice that the levels of \( n \) and \( k \) would be the same if they were chosen optimally in the contract between the banks and the final-goods firms.

The first-order condition is

\[
fn(n, k) = f_k(n, k)p
\]

\[
\alpha \nu An^{\nu - 1}k^{(1-\alpha)\nu} = (1 - \alpha)\nu An^{\nu}k^{(1-\alpha)\nu - 1}p
\]

\[
\frac{pn}{k} = \frac{\alpha}{1 - \alpha}
\]

Using this first-order condition together with the constraint \( pn + k = l_2 \),

\[
n = \frac{\alpha l_2}{p}
\]

\[
k = (1 - \alpha)l_2
\]

Then,

\[
g(l_2, p) = f(\alpha l_2/p, (1 - \alpha)l_2)
\]

\[
= A \left( (\alpha l_2/p)^\alpha ((1 - \alpha)l_2)^{1-\alpha} \right)^\nu
\]

\[
= A(\alpha^\alpha (1 - \alpha)^{1-\alpha} l_2 / p^\alpha)^\nu
\]

At the optimizing values of \( n \) and \( k \),

\[
y_\xi = \xi \theta_2 g(l_2, p)
\]

\[\text{(21)}\]

### 3.4 Banks

Banks begin with initial real funds \( m_0 \) and receive a net-worth shock \( \tau_1 \) from the households. Let

\[
a_1 \equiv m_0 + \tau_1
\]

\[\text{(22)}\]

be the first-period bank’s assets. Then, they raise new funds by issuing and selling to the households new equity shares \( s \) at the price \( q_s \), and bank senior debt with face value \( d \) due at the beginning of the third period at the
price $q_d$. With these resources, they repay the households the face value of their existing deposits $d_0$, and they lend $l_1$ real resources to intermediate-goods firms at the interest rate $R_1$. Their first-period budget constraint is

$$l_1 + d_0 = a_1 + q_s s + q_d d$$  \hspace{1cm} (23)

Let

$$E \equiv q_s (1 + s)$$  \hspace{1cm} (24)
$$V \equiv q_s (1 + s) + q_d d$$  \hspace{1cm} (25)

be, respectively, the bank equity value and the bank total value (the sum of the values of equity and senior debt). Banks are subject to capital requirements that the ratio of the equity value to the total value is greater than a constant $\kappa \in (0, 1)$:

$$\frac{E}{V} = \frac{q_s (1 + s)}{q_s (1 + s) + q_d d} \geq \kappa$$
$$(1 - \kappa) q_s (1 + s) \geq \kappa q_d d$$  \hspace{1cm} (26)

In the second period, banks receive the return from their first-period loans $E \min \{R_1 l_1, p_{z_\omega}\}$. Let the loan loss rate be

$$L_l = 100 \cdot \left(1 - \frac{E \min \{R_1 l_1, p_{z_\omega}\}}{R_1 l_1}\right)$$  \hspace{1cm} (27)

They also receive the profits of the intermediate-goods firms $E \pi^I_\omega$, and a net-worth shock $\tau_2$ from the households. Let

$$a_2 = E \min \{R_1 l_1, p_{z_\omega}\} + E \pi^I_\omega + \tau_2$$
$$= E \min \{R_1 l_1, p_{z_\omega}\} + E \max\{p_{z_\omega} - R_1 l_1, 0\} + \tau_2$$
$$= E p_{z_\omega} + \tau_2$$
$$= E p_{z_\omega} \theta_1 h(l_1) + \tau_2$$
$$= p \theta_1 h(l_1) + \tau_2$$  \hspace{1cm} (28)
be the second-period bank’s assets. Then, they sell $b$ junior bonds to the households at the price $q_b$. With these funds, they lend $l_2$ real resources to the final-goods firms at the interest rate $R_2$. Their second-period budget constraint is

$$l_2 = a_2 + q_b \ldots \tag{29}$$

In the third period, banks receive the return from their second-period loans $\min\{R_2 l_2, y_\xi\}$, and the profits of the final-goods firms $\pi^F_\xi = \max\{y_\xi - R_2 l_2, 0\}$—the sum of the two is equal to $y_\xi$. With these funds, they pay the senior debt payoff $\min\{y_\xi, d\}$. Then, with the residual, they pay the junior bonds payoff $\min\{\max\{y_\xi - d, 0\}, b\}$. Finally, they distribute to the equity shareholders the remaining profits:

$$\pi^R_\xi \equiv \max\{y_\xi - d - b, 0\} = \max\{\xi \theta_2 g(l_2, p) - d - b, 0\} \ldots \tag{30}$$

The objective of the bank is to maximize the expectation of the profits distributed to the initial shareholders $\mathbb{E}_\xi \{\pi^R_\xi/(1 + s)\}$. This objective takes into account the dilution cost of issuing new equity $s$ sustained by the initial shareholders. Other than for the choice of $s$, this objective is equivalent to maximizing the expectation of the overall profits $\mathbb{E}_\xi \{\pi^R_\xi\}$.

The bank’s second-period problem is

$$H(a_2, q_b, \theta_2, p, d, s) = \max_{l_2, b} \mathbb{E}_\xi \left\{ \frac{\max\{\xi \theta_2 g(l_2, p) - d - b, 0\}}{1 + s} \right\} \ldots \tag{31}$$

subject to $l_2 = a_2 + q_b \ldots$

As mentioned in Section 3.3, $l_2$ is chosen with an optimal contract between the final-goods firms and banks. Since firms are owned by banks, firms and banks share the same objective function, so $l_2$ is chosen to maximize the objective function of banks.

Appendix A shows that the first-order condition for $l_2$ is

$$N(\delta_1) \theta_2 \frac{\partial g(l_2, p)}{\partial l_2} = N(\delta_2) \frac{1}{q_b} \ldots \tag{32}$$
where \(N(\cdot)\) is the cumulative distribution function of a standard normal,
\[
\delta_1 \equiv \frac{\ln(y/(d + b))}{\sigma_\xi} + \sigma_\xi/2
\]
and
\[
\delta_2 \equiv \delta_1 - \sigma_\xi
\]
are distances to default, \(\sigma_\xi\) is the standard deviation of \(\ln(\xi)\), and
\[
y \equiv E_\xi\{y_\xi\} = \theta_2 g(l_2, p)
\]
Disregarding the two factors \(N(\delta_1)\) and \(N(\delta_2)\), one can see the debt-overhang distortion on lending. The marginal product of loans \(\theta_2 \partial g(l_2, p)/\partial l_2\), is inversely related to the price of junior bonds \(q_b\), which implies a direct relationship between \(q_b\) and \(l_2\). As the credit risk of a bank increases—due, for instance, to a higher level of senior debt—the credit spread on junior bonds increases, the price of junior bonds \(q_b\) decreases, the cost of funding increases, and this discourages the bank’s lending \(l_2\).

Using \(\partial g(l_2, p)/\partial l_2 = \nu g(l_2, p)/l_2\), the first-order condition (32) becomes
\[
N(\delta_1)E_\xi\{\xi\} \theta_2 \frac{\nu g(l_2, p)}{l_2} = N(\delta_2) \frac{1}{q_b}
\]
\[
N(\delta_1) \nu \frac{y}{l_2} = N(\delta_2) \frac{1}{q_b}
\]
The bank’s first-period problem is
\[
\max_{l_1, d, s, a_2} E_1\{H(a_2, q_b, \theta_2, p, d, s)\}
\]
subject to (23), (26) and (28).

As mentioned in Section 3.2, \(l_1\) is chosen with an optimal contract between intermediate-goods firms and banks. Since firms are owned by banks, firms and banks share the same objective function, so \(l_1\) is chosen to maximize the objective function of banks.

Appendix B shows that the first-order condition for \(l_1\) is
\[
E_1 \left\{ \frac{N(\delta_2)}{q_b} \rho_1 h'(l_1) \right\} = (1 - \kappa) \frac{E_1\{N(\delta_2)\}}{q_d} + \kappa \frac{E_1\{H(a_2, q_b, \theta_2, p, d, s)\}}{q_s}
\]
which equates the expected marginal benefit of lending in the first period and using the proceeds (both the loan payoff and the profits received from intermediate-goods firms) to issue fewer junior bonds to the expected marginal cost of funding the loan with a fraction $\kappa$ of new equity and a fraction $1 - \kappa$ of senior debt.

Using $h'(l_1) = \mu h(l_1)/l_1$, the first-order condition (37) becomes

$$
\mathbb{E}_1 \left\{ \frac{N(\delta_2)}{q_b} p \theta_1 \frac{\mu h(l_1)}{l_1} \right\} = (1 - \kappa) \mathbb{E}_1 \left\{ N(\delta_2) \right\} + \kappa \mathbb{E}_1 \left\{ H(a_2, q_b, \theta_2, p, d, s) \right\} / q_s
$$
$$
\mathbb{E}_1 \left\{ \frac{N(\delta_2)}{q_b} \frac{pz}{l_1} \right\} = (1 - \kappa) \mathbb{E}_1 \left\{ N(\delta_2) \right\} + \kappa \mathbb{E}_1 \left\{ H(a_2, q_b, \theta_2, p, d, s) \right\} / q_s
$$

where

$$
z \equiv \mathbb{E}_w \{ z_w \} = \theta_1 h(l_1)
$$

(38)

The interest rate $R_1$ is set so that the expected average rate of return is equal to to the expected marginal opportunity cost of funds, which, in turn, is equal to the expected marginal return on $l_1$, so

$$
\mathbb{E}_1 \left\{ \mathbb{E}_w \{ \min \{ R_1 l_1, p z_w \} \} \right\} / l_1 = \mathbb{E}_1 \left\{ p \theta_1 h'(l_1) \right\}
$$
$$
\mathbb{E}_1 \left\{ \mathbb{E}_w \{ \min \{ R_1 l_1, p z_w \} \} \right\} / l_1 = \mu \mathbb{E}_1 \left\{ p \right\} z
$$
$$
\mathbb{E}_1 \left\{ \mathbb{E}_w \{ \min \{ R_1 l_1, p z_w \} \} \right\} = \mu \mathbb{E}_1 \left\{ p \right\} z
$$

Using an analytical result holding for log-normally distributed random variables (for a proof, see the Appendix of Chapter 13 in Hull 2005),

$$
\mathbb{E}_1 \left\{ N(\zeta_2) R_1 l_1 + (1 - N(\zeta_1)) p z \right\} = \mu \mathbb{E}_1 \left\{ p \right\} z
$$

(39)

$$
\zeta_1 \equiv \frac{\ln(pz/(R_1 l_1))}{\sigma_w} + \sigma_w / 2
$$

(40)

$$
\zeta_2 \equiv \zeta_1 - \sigma_w
$$

(41)

where $\sigma_w$ is the volatility of $\ln(\omega)$.

The interest rate $R_2$ is set similarly to how $R_1$ is set—we omit the details because $R_2$ does not play any role in the paper.
3.5 Equilibrium

In equilibrium, the lump-sum tax paid by the households is equal to the cost of the deposit insurance:

\[ T = d - \mathbb{E}_\xi \{ \min\{ y_\xi, d \} \} \]
\[ = \mathbb{E}_\xi \{ \max\{ d - y_\xi, 0 \} \} \]

Using an analytical result holding for log-normally distributed random variables (for a proof, see the Appendix of Chapter 13 in Hull 2005),

\[ T = (1 - N(\rho_2))d - (1 - N(\rho_1))y \quad (42) \]
\[ \rho_1 \equiv \ln(y/d) + \sigma_\xi/2 \quad (43) \]
\[ \rho_2 \equiv \rho_1 - \sigma_\xi \quad (44) \]

The payoff of the equity shares is equal to the sum of the per-share banks’ profits:

\[ \Pi_s = \mathbb{E}_\xi \left\{ \frac{\pi_\xi^B}{1 + s} \right\} \]
\[ = \mathbb{E}_\xi \{ \max\{ y_\xi - d - b, 0 \} \} \]
\[ = \frac{N(\delta_1)y - N(\delta_2)(d + b)}{1 + s} \quad (45) \]

and the payoff of the junior bonds is equal to sum of the minimum between their face value and the banks’ revenue net of the senior debt:

\[ \Pi_b = \mathbb{E}_\xi \{ \min\{ y_\xi - d, b \} \} \]
\[ = \frac{b}{b} = \frac{(1 - N(\delta_1))y - (1 - N(\delta_2))d + N(\delta_2)b}{b} \quad (46) \]

Finally, the household demand for newly-issued equity shares is equal to the bank supply:

\[ \tilde{s} = s \quad (47) \]
and the demand for intermediate goods by the final-goods firms equals the supply by intermediate-goods firms:

\[ n = \mathbb{E}_\omega \{ z_\omega \} = z \]  

(48)

The variables \( \{d_0, m_0, e_1, e_2, e_3, \sigma_\omega, \kappa\} \) are given and can be treated as parameters. The first-period aggregate state, \( \Theta_1 \equiv \{\tau_1, \theta_1, \beta, \sigma_\xi\} \), is revealed at the very beginning of the first-period. The equilibrium is a function of \( \Theta_1 \). However, for clarity, we do not make explicit the dependence of the equilibrium on \( \Theta_1 \), so we treat \( \Theta_1 \) as a set of parameters.

**Definition 1. (Equilibrium).** An equilibrium is a set of first-period values \( \{c_1, \tilde{s}, q_d, q_s, a_1, l_1, d, s, z, R_1\} \) and a set of functions of the second-period aggregate state \( \{c_2, c_3, A_1, A_2, q_b, n, k, a_2, l_2, b, y, \delta_1, \delta_2, \zeta_1, \zeta_2, T, \rho_1, \rho_2, \Pi_s, \Pi_b, p\} \) such that: \( \{c_1, c_2, c_3, A_1, A_2, q_d, q_s, q_b\} \) satisfy equations (2)–(9); \( \{n, k\} \) solve problem (20); \( \{a_1\} \) satisfies equation (22); \( \{l_2, b\} \) solve problem (31); \( \{y, \delta_1, \delta_2\} \) satisfy equations (33)–(35); \( \{l_1, d, s, a_2\} \) solve problem (36); \( \{z, R_1, \zeta_1, \zeta_2, T, \rho_1, \rho_2, \Pi_s, \Pi_b, \tilde{s}, p\} \) satisfy equations (38)–(48).

To be clear, while the first-period variables are values, the second-period and third-period variables are functions of the second-period aggregate state \( \Theta_2 \)—for instance, \( c_2 = c_2(\Theta_2) \), \( b = b(\Theta_2) \), and so forth.

## 4 Results

In this section, we study the financial spillovers that take place in the second period, and the optimal setting of capital requirements in the first period.

### 4.1 Parameter values

Table 2 lists the parameter values as well as the steady state values of some selected variables. The parameter values are jointly set so that the
steady-state of the model describes the U.S. economy in a stylized way, with
the aim of studying the determinants of financial spillovers and the effect
of capital requirements. In general, matching each moment or feature is
the result of all the parameterization, and not of a single parameter value.
However, in what follows, to gain intuition, we associate each moment or
feature with the subset of parameters that affect it the most.

One period is one year. The household preferences’ discount factor,
$\beta = 0.99$, and relative risk aversion, $\gamma = 2$, are set to standard values in
the literature.

The exponents of the production functions, $\mu = 0.8$ and $\nu = 0.8$ are set
lower than one, but relatively close to one—those exponents are constrained
to be lower than one for the bank’s problem to be concave.

Capital requirements $\kappa = 0.10$ are set to approximate the average banks’
equity-assets ratio. The initial bank funds $m_0$ are normalized to 1, while the
initial bank liabilities $d_0$ are set so that $s = 0.05$, so the ratio of newly-issued
shares to existing shares is equal to 5 percent. The model’s prediction are
the same if $m_0$ is raised by an arbitrary constant, while simultaneously $d_0$
is raised by the same constant and $e_1$ is lowered by the same constant. The
dilution cost parameter $C$ is set so that $\kappa = 0.10$ is optimal in the steady
state, i.e. it maximizes the households welfare.

The first-period production function parameter, $B$, is set so that the
intermediate goods $z$ are equal to 1. The other first-period production
function parameter, $\sigma_\omega$, is set so that the spread between the loan interest
rate $R_1$ and the expected return on loans $E_\omega \{\min\{R_1 l_1, pz_\omega\}\}/l_1$ is equal
to 5 percent, so the loan loss rate $L_l$ is, approximately, equal to 5 percent
as well.

The second-period production function parameters, $A$, $\alpha$ and $\sigma_\xi$, are
set so that $p = 1$, the credit spread on banks’ junior bonds $X_b$ is equal to
5 percent, and investment $k$ is 15 percent of output $y$, to approximate the
average investment share of GDP.

The aggregate real resources available in the second period, \( e_2 \), are set equal to 10 times the amount of lending in the second period, \( l_2 \), to approximate the share of lending to GDP. Given \( e_2 \), the third-period endowment, \( e_3 \), is set so that \( A_2 = 0.98 \), so the second-period risk-free rate, \( r_2 \), is, approximately, equal to 2 percent. Similarly, the first-period endowment, \( e_1 \), is set so that \( A_1 = 0.98 \), so the first-period risk-free rate, \( r_1 \), is, approximately, equal to 2 percent.

Finally, we assume that \( \tau_1 = 0 \) and \( \theta_1 = 1 \), while \( \tau_2 \) is distributed uniformly in \([-0.05, 0.05]\) and \( \theta_2 \) is distributed uniformly in \([0.95, 1.05]\). In what follows, we list the values of second-period variables for the values \( \tau_2 = 0 \) and \( \theta_2 = 1 \), unless indicated otherwise.

### 4.2 Financial spillovers

To describe the spillovers formally and to study their determinants, we define the spillovers as the exposure of an individual bank to an exogenous shock to the net worth of all other banks. We measure the size, \( \Psi \), of financial spillovers as the effect of a net-worth shock \( \tau_2 \) to the assets of the representative bank on the assets of an individual bank that is not hit by the shock:

\[
\Psi = \frac{d(pz)}{d\tau_2} \tag{49}
\]

where the symbol \( d \) indicates the total derivative.

Table 3 and Figure 3 display the effects of a positive shock \( \tau_2 \) to the net-worth of a representative bank. As a result of this favorable shock, the credit spread \( X_b \) on the banks’ junior debt decreases, the debt-overhang distortion mitigates and the banks’ credit supply \( l_2 \) expands. In equilibrium, final-goods firms borrow more and increase their demand for intermediate goods. The price of intermediate goods \( p \) rises, the revenue of intermediate-
goods firms $p z$ rises, the amount they repay to banks as loan payoff and as dividends increases. The banks’ loan loss rate $L_i$ drops and their assets $a_2$ rise. Figure 3 shows that these effects are non-linear, larger for more negative values of the shock $\tau_2$. The shock benefits all banks, even if they are not hit by the shock. The size of financial spillovers is 0.40, i.e., a 1 percent positive shock to the net-worth of the representative bank increases the net-worth of other individual banks, not hit by the shock, by 0.4 percent.

The size of financial spillovers $\Psi$ rises with the size of financial distortions, which in turn rises with the risk of banks’ default, proxied by the credit spread between the bank bond yield and the risk free rate—there are no financial spillovers unless there is some credit risk and distortion. The first row of Figure 4 shows that, as $\sigma_\xi$ increases, the risk of banks’ default increases, the credit spread $X_b$ increases, the size of the debt-overhang distortion increases, and financial spillovers $\Psi$ become stronger. When $\sigma_\xi = 0.1277$, 50 percent greater than in the benchmark case where $\sigma_\xi = 0.0851$, the credit spread $X_b$ increases from 5 percent to 17.3 percent, and the spillovers $\Psi$ increase from 0.40 to 0.77.

4.2.1 Financial spillovers amplify the effect of shocks

As the size of financial spillovers increases, the effects of the shock are amplified. The greater the financial spillovers $\Psi$, the greater the total increase in the bank asset value as a result of a positive net-worth shock: $d a_2/d\tau_2 = d(p z + \tau_2)/d\tau_2 = 1 + \Psi$. As a result, the greater the credit spread, the greater the financial distortions and spillovers, the greater the effect of the bank net-worth shock on loans $l_2$, investment $k$ and output $y$, as shown in the last two rows of Figure 4. Table 3 and Figure 5 show that, when $\sigma_\xi$ is 50 percent greater than in the benchmark case, the effects on lending, investment, output more than double. All the effects are zero when there is no risk—if there is no risk, the firm lends the same amount,
while borrowing less by an amount equal to $\tau_2$.

Financial spillovers amplify the effect of other aggregate shocks as well. For instance, Table 4 and Figure 6 display the effect of a positive 1 percent technology shock $\theta_2$. As a result of the shock, the final-goods firms’ demand for loans, investment and intermediate goods rises, leading to higher bank lending $l_2$, higher investment $k$, and higher price of intermediate goods $p$. The intermediate-goods firms’ revenue $pz$ rises, the banks’ loan loss rate $L_l$ drops and banks’ assets $a_2$ rise. The banks’ credit spread $X_b$ drops, and the size of financial spillovers decrease. Figure 6 shows that these effects are non-linear, larger for more negative values of the shock $\theta_2$. As $\sigma_\xi$ rises, the credit spread increases, the debt-overhang distortion increases financial spillovers rise and the effect of the technology shock on the economic and banking system increases. Table 4 and Figure 7 show that, when $\sigma_\xi$ is 50 percent greater, the effects on lending, investment and normalized output, $y/\theta_2$, almost double.

The fact that financial spillovers amplify the effects of aggregate shocks has two interesting implications. First, as the size of financial spillovers increases, ex-ante volatilities of aggregate variables increase. This is consistent with the literature referenced in footnote 1 that connects systemic risk with the exposure of individual banks to financial system conditions. Second, as the size of financial spillovers increases, ex-ante correlations between individual banks’ variables increase—as financial spillovers gets larger, the effect of aggregate shocks becomes more important relative to the effect of idiosyncratic shocks, raising ex-ante correlations.

Collecting some of the previous results, when the credit risk of banks is high, financial distortions are large, financial spillovers are strong, the effect of aggregate shocks is amplified, and ex-ante volatilities of aggregate variables are large. In other words, ex-ante volatilities of aggregate variables are larger when banking conditions are weaker. As highlighted in the intro-
duction, this is consistent with recent evidence that connects the volatility of real GDP growth with financial conditions (Adrian, Boyarchenko and Giannone 2016).

4.3 Capital requirements

The prudential authority faces an intertemporal trade-off, as shown in Table 5 and Figure 8. If it raises capital requirements \( \kappa \), in the first period, banks shift their funding from senior debt \( q_{sd} \) to equity \( E \). Since equity is more expensive, the banks’ cost of funds increases, and the banks’ credit supply \( l_1 \) decreases, so in equilibrium lending and investment activity in the first period decrease. In the second period, however, banks carry over less senior debt from the first period, so their credit risk, credit spread \( X_b \), debt-overhang distortion and financial spillovers \( \Psi \) are smaller. The effects on second-period variables are non-linear, larger for smaller capital requirements, as shown in Figure 8.

Since financial spillovers amplify the effects of shocks, the higher the capital requirements \( \kappa \), the lower the financial spillovers \( \Psi \), the smaller the effect of shocks on the economic and financial system. For instance, in the case that \( \kappa = 15\% \), the effect of a 1 percent positive technology shock \( \theta_2 \) on lending, investment and normalized output, \( y/\theta_2 \), is about 3/5 relative to the case that \( \kappa = 10\% \) (Table 6 and Figure 9).

4.3.1 Optimal response of capital requirements to shocks

We now turn to the optimal capital requirements, i.e., the capital requirements that maximize the household welfare \( \mathbb{E}_1 \{ W(c_1, c_2, c_3) \} \). It should be noticed that, due to the presence of financial spillovers, bank capital requirements can raise welfare. An increase in a bank’s capital decreases its future risk of default, lowers the associated financial distortions, encour-
ages its lending, stimulates economic activity and has positive spillovers on other banks’ net worth. Since the bank does not internalize this effect, without capital requirements the bank would choose a level of capital lower than socially optimal.

We solve the optimization problem numerically, restricting attention to the case of no aggregate uncertainty in the second-period, i.e., setting \( \tau_2 = 0 \) and \( \theta_2 = 1 \) (so the expectation \( \mathbb{E}_t \{ \cdot \} \) is actually a perfect forecast).

The first column of Figure 10 shows the optimal response of capital requirements to first-period shocks. Capital requirements should be raised in response to each of the following fundamental shocks: a risk shock (a positive shock to \( \sigma_\zeta \)) that increases the future credit risk of banks and associated financial distortions; a shock \( \tau_1 \) that increases the net worth of banks; a positive technology shock \( \theta_1 \); a preference shock increasing the household preferences discount factor \( \beta \).

Some intuition for these optimal responses to shocks follows from the intertemporal trade-off faced by the prudential authority: higher capital requirements decrease current lending but mitigate the future financial distortions faced by banks.

First, consider the optimal response to a risk shock (first row of Figure 10). Without a change in capital requirements, the main effect of the shock would be to increase the future financial distortions faced by banks, while the negative effect on current lending would be relatively minor. Then, capital requirements should be raised to mitigate the increase in future financial distortions, at the cost of a larger contraction in current lending.

Next, consider the optimal response to a bank net-worth shock \( \tau_1 \) raising bank capital by 1 percent of the bank’s total value \( V \) (second row of Figure 10). First, suppose that capital requirements did not change: the main effect of the shock would be to expand the current credit supply,
while the mitigating effect on future financial distortions would be relatively minor; then, capital requirements should be raised to further mitigate future financial distortions, at the cost of a smaller current credit expansion. Second, suppose that capital requirements increased by the same amount of the shock, i.e., 1 percent of the bank’s total value $V$: the main effect of the shock would be to decrease future financial distortions, while the positive effect on current lending would be relatively minor; then, capital requirements should be raised a little less than 1 percent, to raise the current credit supply at the cost of a smaller improvement in future financial distortions. Collecting these two results, capital requirements should be raised in response to a bank net-worth shock, but less than one-to-one. This prescription is consistent with Repullo (2013), who points out that capital requirements should be lowered after an exogenous negative shock to bank capital, to mitigate the reduction in aggregate investment.

Finally, consider the optimal response to a positive technology shock (third row of Figure 10). Without a change in capital requirements, the main effect of the shock would be to expand the current credit supply, while the mitigating effect on future financial distortions would be relatively minor. Hence, capital requirements should be raised to further mitigate future financial distortions, at the cost of a smaller current credit expansion. As an implication, if the business cycle were mainly driven by technology shocks, then the bank capital requirements should be raised in expansions and lowered in recessions.

The intuition for the optimal response to a preference shock increasing the household preferences discount factor $\beta$ is similar to the one for a positive technology shock (fourth row of Figure 10).
4.3.2 Optimal response of capital requirements to observable variables

The relation between the responses of capital requirements $\kappa$ and credit $l_1$ to the various types of shocks is especially interesting. In the case of most shocks, capital requirements should be raised as the banks’ credit supply expands (first and second columns of Figure 10). However, in the case of a risk shock, capital requirements should be raised even though the banks’ credit supply contracts. This suggests that setting capital requirements based solely on measures of credit growth may not be optimal. Furthermore, since the credit spread $X_b$ responds sizeably to the risk shock (third column of Figure 10), it seems reasonable to set capital requirements taking into account not only measures of credit growth but also indicators of banks’ credit risk.

Motivated by these observations, we let capital requirements respond to four observable variables, the credit supply $l_1$, the banks’ expected future credit spread $\mathbb{E}_t\{X_b\}$, the banks’ assets $a_1$, and the risk-free rate $r_1$. We find that the optimal local rule is:

$$\Delta \kappa = 0.28 \cdot \Delta \log(l_1) + 0.0032 \cdot \Delta \mathbb{E}_t\{X_b\} + 0.7174 \cdot \Delta a_1 - 0.00015 \cdot \Delta r_1$$

where $\Delta$ indicates the difference relative to steady state values. In words, capital requirements should be raised by 0.28 percentage points in response to an expansion of the banks’ credit supply by 1 percent; and should be raised by 0.32 percentage points in response to an increase in the expected future credit spread on bank-issued bonds by 100 basis points. When the bank’s assets increase by 0.01319, an amount that corresponds to 1 percent of the bank’s total value $V$, capital requirements should increase by $(0.01319 \cdot 0.7174) = 0.0095$, close to 1 percentage point. In other words, capital requirements should be lowered by 0.95 percentage points in response to bank losses equivalent to 1 percent of the bank’s total value, close to
one-to-one. When the risk-free rate increases by 100 basis points, capital requirements should decrease by a negligible amount, 0.015 percentage points only.

5 Conclusion

This paper has modeled the spillovers associated with banks’ financial distortions. We have shown that the credit risk of banks increases the financial distortions faced by banks and the associated spillovers, which in turn amplify the effects of shocks on the economy and the banking system. The volatilities of aggregate variables increase with banks’ credit risk and financial distortions. Higher capital requirements decrease the expected future financial distortions and spillovers and enhance the expected future financial stability, at the cost of a lower current credit supply. With our parameter setting, capital requirements should be raised by 0.28 percentage points in response to a 1 percent expansion of banks’ credit supply; and should be raised by 0.32 percentage points in response to a 1 percentage point increase in the expected future credit spread on bank-issued bonds. They should be lowered by 0.95 percentage points in response to bank losses equivalent to 1 percent of banks’ value, close to one-to-one.

In response to the financial crisis, a growing literature is investigating whether monetary policy should respond to risks to financial stability and how monetary policy and prudential policy should be jointly set taking into account those risks—Adrian and Liang (2016) provide a broad overview of these issues and the literature. It would be very interesting to incorporate our debt-overhang mechanism in a standard monetary policy model and characterize the optimal response of monetary policy and prudential policy in a model where financial distortions and financial spillovers play an important role.
A Bank’s second-period problem

After substituting the expression for $b$ from the constraint into the objective function, the bank’s second-period problem (31) becomes

$$H(a_2, q_b, \theta_2, p, d, s) = \max_{l_2} \mathbb{E}_\xi \left\{ \max \left\{ \frac{\xi \theta_2 g(l_2, p) - d - (l_2 - a_2)/q_b, 0} {1 + s} \right\} \right\}$$

$$= \max_{l_2} \mathbb{E}_\xi \left\{ \max \left\{ \frac{\xi \theta_2 g(l_2, p) - d - (l_2 - a_2)/q_b, 0} {1 + s} \right\} \right\}$$

Using an analytical result holding for log-normally distributed random variables (for a proof, see the Appendix of Chapter 13 in Hull 2005),

$$H(a_2, q_b, \theta_2, p, d, s) = \max_{l_2} \left\{ \frac{N(\delta_1)S - N(\delta_2)K} {1 + s} \right\}$$

where

$$S \equiv \mathbb{E}_\xi \{ \xi \theta_2 g(l_2, p) \}$$

$$K \equiv d + (l_2 - a_2)/q_b$$

$N(\cdot)$ is the cumulative distribution function of a standard normal,

$$\delta_1 \equiv \frac{\ln(S/K)} {\sigma_\xi} + \sigma_\xi/2$$

$$\delta_2 \equiv \delta_1 - \sigma_\xi$$

are distances to default, and $\sigma_\xi$ is the volatility of $\ln(\xi)$.

The first-order condition for $l_2$ is

$$\frac{\partial} {\partial l_2} \left\{ N(\delta_1)S - N(\delta_2)K \right\} = 0$$

$$\left[ N'(\delta_1)S \frac{\partial \delta_1} {\partial l_2} - N'(\delta_2)K \frac{\partial \delta_2} {\partial l_2} \right] + N(\delta_1) \frac{\partial S} {\partial l_2} - N(\delta_2) \frac{\partial K} {\partial l_2} = 0$$

The term in square brackets is equal to zero because $\partial \delta_1/\partial l_2 = \partial \delta_2/\partial l_2$, 

33
and $N'(\delta_1)S = N'(\delta_2)K$, as the following steps show:

$$N'(\delta_2)K = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \frac{s^2}{\xi^2}} K$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} (\delta_1 - \sigma_\xi)^2} K$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} (\delta_1 - \sigma_\xi + 2\delta_1 \sigma_\xi) K}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \delta_1^2 - \frac{1}{2} \sigma_\xi^2 + \delta_1 \sigma_\xi K}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \delta_1^2 + \ln(S/K) + \frac{1}{2} \sigma_\xi^2 K}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \delta_1^2 + \ln(S/K) K}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \delta_1^2} \frac{S}{K} K$$

$$= N'(\delta_1)S$$

Then, the first-order condition for $l_2$ becomes

$$N(\delta_1) \frac{\partial S}{\partial l_2} - N(\delta_2) \frac{\partial K}{\partial l_2} = 0$$

$$N(\delta_1) \mathbb{E}_\xi \{ \xi \} \theta_2 \frac{\partial g(l_2, p)}{\partial l_2} - N(\delta_2) \frac{1}{q_b} = 0$$

which is equivalent to the first-order condition (32).

Using (29) and (35), it follows that $S = y$ and $K = d + b$, so

$$\delta_1 \equiv \frac{\ln(y/(d + b))}{\sigma_\xi} + \sigma_\xi/2 \quad \delta_2 \equiv \delta_1 - \sigma_\xi$$

which are the same as equations (33) and (34).

For use in Appendix B, we list below three envelope conditions:

$$H_a \equiv \frac{\partial H(a_2, q_b, \theta_2, p, d, s)}{\partial a_2} = \frac{N(\delta_2)/q_b}{1 + s}$$

$$H_d \equiv \frac{\partial H(a_2, q_b, \theta_2, p, d, s)}{\partial d} = -\frac{N(\delta_2)}{1 + s}$$

$$H_s \equiv \frac{\partial H(a_2, q_b, \theta_2, p, d, s)}{\partial s} = \frac{-\{N(\delta_1) \mathbb{E}_\xi \{ \xi \} \theta_2 g(l_2, p) - N(\delta_2)[d + (l_2 - a_2)/q_b]\}}{(1 + s)^2}$$

$$= \frac{-H(a_2, q_b, \theta_2, p, d, s)}{1 + s}$$

34
In deriving $H_a$ and $H_d$ we have used $\partial \delta_1/\partial a_2 = \partial \delta_2/\partial a_2$, $\partial \delta_1/\partial d = \partial \delta_2/\partial d$, and $N'(\delta_1)S = N'(\delta_2)K$, in a way similar to the previous derivation of the first-order condition for $l_2$.

**B Bank’s first-period problem**

After substituting the expression for $a_2$ from the last constraint into the objective function, the bank’s first-period problem (36) becomes:

$$
\max_{l_1, d, s} \mathbb{E}_1 \{ H(p\theta_1 h(l_1) + \tau_2, q_b, \theta_2, p, d, s) \}
$$

subject to: $l_1 + d_0 = a_1 + q_s s + q_d d$

$$(1 - \kappa)q_s (1 + s) = \kappa q_d d$$

where the capital requirements constraint is binding because of the assumption that equity funding is more expensive than senior debt.

The first-order conditions are:

$$
\mathbb{E}_1 \{ H_d p \theta_1 h'(l_1) \} = \lambda_1 \\
\mathbb{E}_1 \{ H_d \} = -\lambda_1 q_d - \lambda_2 \kappa q_d \\
\mathbb{E}_1 \{ H_s \} = -\lambda_1 q_s + \lambda_2 (1 - \kappa) q_s
$$

where $H_a$, $H_d$ and $H_s$ are given by the envelope conditions (50), (51) and (52).

Multiplying the last two first-order conditions by, respectively, $d$ and $(1 + s)$, and summing side by side:

$$
\mathbb{E}_1 \{ H_d \} d = -\lambda_1 q_d d - \lambda_2 \kappa q_d d \\
\mathbb{E}_1 \{ H_s \} (1 + s) = -\lambda_1 q_s (1 + s) + \lambda_2 (1 - \kappa) q_s (1 + s) \\
\mathbb{E}_1 \{ H_d \} d + \mathbb{E}_1 \{ H_s \} (1 + s) = -\lambda_1 q_d d - \lambda_2 \kappa q_d d - \lambda_1 q_s (1 + s) + \lambda_2 (1 - \kappa) q_s (1 + s) \\
\mathbb{E}_1 \{ H_d \} d + \mathbb{E}_1 \{ H_s \} (1 + s) = -\lambda_1 q_d d - \lambda_1 q_s (1 + s) + \lambda_2 (1 - \kappa) q_s (1 + s) - \kappa q_d d
$$

35
The term in square brackets is equal to zero because of the capital requirements constraint, so

\[
\begin{align*}
\mathbb{E}_1\{H_d\}d + \mathbb{E}_1\{H_s\}(1 + s) &= -\lambda_1[q_d d + q_s (1 + s)] \\
\lambda_1 &= \frac{\mathbb{E}_1\{-H_d\}d + \mathbb{E}_1\{-H_s\}(1 + s)}{q_d d + q_s (1 + s)} \\
\lambda_1 &= \frac{\mathbb{E}_1\{-H_d\}d}{q_d} + \frac{\mathbb{E}_1\{-H_s\}(1 + s)}{q_s} \\
\lambda_1 &= (1 - \kappa) \frac{\mathbb{E}_1\{-H_d\}}{q_d} + \kappa \frac{\mathbb{E}_1\{-H_s\}}{q_s}
\end{align*}
\]

where we have used \( q_d d/(1 - \kappa) = q_d d + q_s (1 + s) \), and \( q_s (1 + s)/\kappa = q_d d + q_s (1 + s) \), which are implied by the capital requirements constraint.

Substituting the first first-order condition:

\[
\mathbb{E}_1\{H_a p \theta_1 h'(l_1)\} = (1 - \kappa) \frac{\mathbb{E}_1\{-H_d\}}{q_d} + \kappa \frac{\mathbb{E}_1\{-H_s\}}{q_s}
\]

Using the envelope conditions (50), (51) and (52):

\[
\begin{align*}
\mathbb{E}_1\left\{ N(\delta_2)/q_b p \theta_1 h'(l_1) \right\} &= (1 - \kappa) \frac{\mathbb{E}_1\left\{ N(\delta_2) \right\}}{q_d} + \kappa \frac{\mathbb{E}_1\left\{ H(a_2, q_b, \theta_2, p, d, s) \right\}}{q_s} \\
\mathbb{E}_1\left\{ N(\delta_2)/q_b p \theta_1 h'(l_1) \right\} &= (1 - \kappa) \frac{\mathbb{E}_1\left\{ N(\delta_2) \right\}}{q_d} + \kappa \frac{\mathbb{E}_1\left\{ H(a_2, q_b, \theta_2, p, d, s) \right\}}{q_s}
\end{align*}
\]

which is the first-order condition (37).
References


Brownlees, C., and Engle, R., 2015. A Conditional Capital Shortfall Index for
Systemic Risk Measurement. SSRN eLibrary 1611229.


38


Table 1: Correlations with real GDP growth rate over the 1985-2013 period (or over the shorter period for which data are available: since 1991 for S&P 500 financial, and since 1990:Q2 for Banks tightening credit standards and for Banks increasing spreads), quarterly data. Variables: Corporate profits with inventory valuation and capital consumption adjustments; Standard & Poor’s 500 Composite, stock price index; Moody’s US all corporates trailing 12-month issuer default rates; C&I loan charge-off rates, all commercial banks; Standard & Poor’s 500 Financial, stock price index; Net percentage of banks tightening standards on C&I loans for large and middle-market firms, Senior Loan Officer Opinion Survey on bank lending practices; Net percentage of banks increasing spreads of C&I loan rates over cost of funds for large and middle-market firms, Senior Loan Officer Opinion Survey on bank lending practices; Total loans, all commercial banks; Real business fixed investment.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate profits growth rate</td>
<td>0.1942</td>
</tr>
<tr>
<td>S&amp;P 500 growth rate</td>
<td>0.5038</td>
</tr>
<tr>
<td>Corporate default rate</td>
<td>-0.6248</td>
</tr>
<tr>
<td>C&amp;I loan charge-off rate</td>
<td>-0.5187</td>
</tr>
<tr>
<td>S&amp;P 500 financial growth rate</td>
<td>0.4789</td>
</tr>
<tr>
<td>Banks tightening credit standards</td>
<td>-0.5280</td>
</tr>
<tr>
<td>Banks increasing spreads</td>
<td>-0.5024</td>
</tr>
<tr>
<td>Loan growth rate</td>
<td>0.3866</td>
</tr>
<tr>
<td>Investment growth rate</td>
<td>0.7550</td>
</tr>
</tbody>
</table>
Parameters and steady-state values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Steady-state values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.9900</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>2.0000</td>
</tr>
<tr>
<td>( B )</td>
<td>1.1961</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.8000</td>
</tr>
<tr>
<td>( \sigma_\omega )</td>
<td>0.2749</td>
</tr>
<tr>
<td>( A )</td>
<td>2.0332</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.8000</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.8000</td>
</tr>
<tr>
<td>( \sigma_\xi )</td>
<td>0.0851</td>
</tr>
<tr>
<td>( e_1 )</td>
<td>11.9874</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>12.5000</td>
</tr>
<tr>
<td>( e_3 )</td>
<td>10.6836</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>0.1000</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>1.3938</td>
</tr>
<tr>
<td>( m_0 )</td>
<td>1.0000</td>
</tr>
<tr>
<td>( C )</td>
<td>1.0150</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Values of parameters and selected variables in the steady state.
Effect of $\tau_2$ for different $\sigma_\xi$

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_\xi=0.0851$</th>
<th></th>
<th>$\sigma_\xi=0.1277$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_2=0$</td>
<td>$\tau_2=0.01$</td>
<td>Percent difference</td>
</tr>
<tr>
<td>$X_b$</td>
<td>5.0003</td>
<td>4.3162</td>
<td></td>
</tr>
<tr>
<td>$q_b$</td>
<td>0.9342</td>
<td>0.9399</td>
<td>0.6138</td>
</tr>
<tr>
<td>$q_b b$</td>
<td>0.2500</td>
<td>0.2409</td>
<td>-3.6334</td>
</tr>
<tr>
<td>$b$</td>
<td>0.2676</td>
<td>0.2563</td>
<td>-4.2213</td>
</tr>
<tr>
<td>$l_2$</td>
<td>1.2500</td>
<td>1.2546</td>
<td>0.3665</td>
</tr>
<tr>
<td>$k$</td>
<td>0.2500</td>
<td>0.2509</td>
<td>0.3665</td>
</tr>
<tr>
<td>$y$</td>
<td>1.6288</td>
<td>1.6297</td>
<td>0.0586</td>
</tr>
<tr>
<td>$p$</td>
<td>1.0000</td>
<td>1.0036</td>
<td>0.3665</td>
</tr>
<tr>
<td>$p z$</td>
<td>1.0000</td>
<td>1.0037</td>
<td>0.3665</td>
</tr>
<tr>
<td>$L_t$</td>
<td>4.7530</td>
<td>4.6579</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.0000</td>
<td>1.0137</td>
<td>1.3665</td>
</tr>
<tr>
<td>$c_2$</td>
<td>12.2500</td>
<td>12.2491</td>
<td>-0.0075</td>
</tr>
<tr>
<td>$c_3$</td>
<td>12.3124</td>
<td>12.3133</td>
<td>0.0077</td>
</tr>
<tr>
<td>$r_2$</td>
<td>2.0417</td>
<td>2.0728</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Effect of a bank net-worth shock $\tau_2$ for different values of the volatility $\sigma_\xi$. 

43
Effect of $\theta_2$ for different $\sigma_\xi$

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_\xi=0.0851$</th>
<th></th>
<th>$\sigma_\xi=0.1277$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_2=1$</td>
<td>$\theta_2=1.01$</td>
<td>Percent difference</td>
</tr>
<tr>
<td>$l_k$</td>
<td>1.2500</td>
<td>1.2689</td>
<td>1.5105</td>
</tr>
<tr>
<td>$k$</td>
<td>0.2500</td>
<td>0.2538</td>
<td>1.5105</td>
</tr>
<tr>
<td>$y$</td>
<td>1.6288</td>
<td>1.6490</td>
<td>1.2426</td>
</tr>
<tr>
<td>$q_k b$</td>
<td>0.2500</td>
<td>0.2538</td>
<td>1.5105</td>
</tr>
<tr>
<td>$b$</td>
<td>0.2676</td>
<td>0.2698</td>
<td>0.8364</td>
</tr>
<tr>
<td>$p$</td>
<td>1.0000</td>
<td>1.0151</td>
<td>1.5105</td>
</tr>
<tr>
<td>$p_k z$</td>
<td>1.0000</td>
<td>1.0151</td>
<td>1.5105</td>
</tr>
<tr>
<td>$L_l$</td>
<td>4.7370</td>
<td>4.3568</td>
<td>1.5105</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.0000</td>
<td>1.0151</td>
<td>1.5105</td>
</tr>
<tr>
<td>$X_b$</td>
<td>4.9980</td>
<td>3.8882</td>
<td></td>
</tr>
<tr>
<td>$q_b$</td>
<td>0.9342</td>
<td>0.9405</td>
<td>0.6686</td>
</tr>
<tr>
<td>$c_2$</td>
<td>12.2500</td>
<td>12.2462</td>
<td>-0.0308</td>
</tr>
<tr>
<td>$c_3$</td>
<td>12.3124</td>
<td>12.3326</td>
<td>0.1644</td>
</tr>
<tr>
<td>$r_2$</td>
<td>2.0419</td>
<td>2.4408</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Effect of a technology shock $\theta_2$ for different values of the volatility $\sigma_\xi$. 

44
### Effect of $\kappa$

<table>
<thead>
<tr>
<th></th>
<th>$\kappa=0.1$</th>
<th>$\kappa=0.15$</th>
<th>Percent difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>0.1319</td>
<td>0.1940</td>
<td>47.0365</td>
</tr>
<tr>
<td>$V$</td>
<td>1.3192</td>
<td>1.2931</td>
<td>-1.9757</td>
</tr>
<tr>
<td>$q_{ad}$</td>
<td>1.1873</td>
<td>1.0992</td>
<td>-7.4215</td>
</tr>
<tr>
<td>$d$</td>
<td>1.2362</td>
<td>1.1257</td>
<td>-8.9385</td>
</tr>
<tr>
<td>$l_1$</td>
<td>0.7994</td>
<td>0.7509</td>
<td>-6.0742</td>
</tr>
<tr>
<td>$c_1$</td>
<td>12.1880</td>
<td>12.2365</td>
<td>0.3984</td>
</tr>
<tr>
<td>$r_1$</td>
<td>2.0408</td>
<td>1.2523</td>
<td>-7.4215</td>
</tr>
<tr>
<td>$X_b$</td>
<td>5.0000</td>
<td>1.3039</td>
<td></td>
</tr>
<tr>
<td>$q_b$</td>
<td>0.9342</td>
<td>0.9760</td>
<td>-4.4767</td>
</tr>
<tr>
<td>$q_{bb}$</td>
<td>0.2500</td>
<td>0.2488</td>
<td>-0.4791</td>
</tr>
<tr>
<td>$b$</td>
<td>0.2676</td>
<td>0.2549</td>
<td>-4.7434</td>
</tr>
<tr>
<td>$l_2$</td>
<td>1.2500</td>
<td>1.2440</td>
<td>-0.4791</td>
</tr>
<tr>
<td>$k$</td>
<td>0.2500</td>
<td>0.2488</td>
<td>-0.4791</td>
</tr>
<tr>
<td>$y$</td>
<td>1.6287</td>
<td>1.5761</td>
<td>-3.2319</td>
</tr>
<tr>
<td>$p$</td>
<td>1.0000</td>
<td>1.0464</td>
<td>4.6373</td>
</tr>
<tr>
<td>$p_z$</td>
<td>1.0000</td>
<td>0.9952</td>
<td>-0.4791</td>
</tr>
<tr>
<td>$L_l$</td>
<td>4.7588</td>
<td>4.7588</td>
<td></td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.0000</td>
<td>0.9952</td>
<td>-0.4791</td>
</tr>
<tr>
<td>$c_2$</td>
<td>12.2500</td>
<td>12.2512</td>
<td>0.0098</td>
</tr>
<tr>
<td>$c_3$</td>
<td>12.3123</td>
<td>12.2597</td>
<td>-0.4275</td>
</tr>
<tr>
<td>$r_2$</td>
<td>2.0408</td>
<td>1.1504</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Effect of an increase in capital requirements $\kappa$. 

45
### Effect of $\theta_2$ for different $\kappa$

<table>
<thead>
<tr>
<th></th>
<th>$\kappa=0.1$</th>
<th></th>
<th>$\kappa=0.15$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_2=1$</td>
<td>$\theta_2=1.01$</td>
<td>Percent difference</td>
<td>$\theta_2=1$</td>
</tr>
<tr>
<td>$l_2$</td>
<td>1.2500</td>
<td>1.2689</td>
<td>1.5105</td>
<td>1.2440</td>
</tr>
<tr>
<td>$k$</td>
<td>0.2500</td>
<td>0.2538</td>
<td>1.5105</td>
<td>0.2488</td>
</tr>
<tr>
<td>$y$</td>
<td>1.6288</td>
<td>1.6490</td>
<td>1.2426</td>
<td>1.5760</td>
</tr>
<tr>
<td>$q_b$</td>
<td>0.2500</td>
<td>0.2538</td>
<td>1.5105</td>
<td>0.2488</td>
</tr>
<tr>
<td>$b$</td>
<td>0.2676</td>
<td>0.2698</td>
<td>0.8364</td>
<td>0.2549</td>
</tr>
<tr>
<td>$p$</td>
<td>1.0000</td>
<td>1.0151</td>
<td>1.5105</td>
<td>1.0464</td>
</tr>
<tr>
<td>$pz$</td>
<td>1.0000</td>
<td>1.0151</td>
<td>1.5105</td>
<td>0.9952</td>
</tr>
<tr>
<td>$L_t$</td>
<td>4.7370</td>
<td>4.3568</td>
<td></td>
<td>4.7532</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1.0000</td>
<td>1.0151</td>
<td>1.5105</td>
<td>0.9952</td>
</tr>
<tr>
<td>$X_b$</td>
<td>4.9980</td>
<td>3.8882</td>
<td></td>
<td>1.3034</td>
</tr>
<tr>
<td>$q_b$</td>
<td>0.9342</td>
<td>0.9405</td>
<td>0.6686</td>
<td>0.9761</td>
</tr>
<tr>
<td>$c_2$</td>
<td>12.2500</td>
<td>12.2462</td>
<td>-0.0308</td>
<td>12.2512</td>
</tr>
<tr>
<td>$c_3$</td>
<td>12.3124</td>
<td>12.3326</td>
<td>0.1644</td>
<td>12.2597</td>
</tr>
<tr>
<td>$r_2$</td>
<td>2.0419</td>
<td>2.4408</td>
<td></td>
<td>1.1500</td>
</tr>
</tbody>
</table>

Table 6: Effect of a technology shock $\theta_2$ for different capital requirements $\kappa$. 
Financial spillovers

Balance sheets of a set of banks deteriorate → Credit risk of banks increases and financial distortions worsen

Banks’ credit supply contracts and economic activity declines

Other banks’ loan losses surge and their balance sheets deteriorate ← Firms’ profits drop and their balance sheets weaken

Figure 1: Financial spillovers modeled in this paper, transmitting risks and losses from a set of banks to another set of banks.
Time Series

Figure 2: Time series of economic and banking variables. The vertical bars indicate the three recessions.
Figure 3: Effect of a bank net-worth shock $\tau_2$. 

Effect of $\tau_2$

- $X_b$
- $l_2$
- $k$
- $y$
- $L_l$
- $a_2$
Credit spread, financial spillovers and sensitivities to $\tau_2$

Figure 4: Banks’ credit spread $X_b$, financial spillovers $\Psi$ and sensitivities of economic variables to a bank net-worth shock $\tau_2$ as functions of the volatility $\sigma_\xi$. 
Effect of $\tau_2$ for different $\sigma_\xi$

Figure 5: Effect of a bank net-worth shock $\tau_2$ for different values of the volatility $\sigma_\xi$. 
Effect of $\theta_2$

![Graphs showing the effect of $\theta_2$ on various variables: $X_b$, $l_2$, $k$, $y$, $L_1$, and $a_2$.](image)

Figure 6: Effect of a technology shock $\theta_2$. 

52
Effect of $\theta_2$ for different $\sigma_\xi$

Figure 7: Effect of a technology shock $\theta_2$ for different values of the volatility $\sigma_\xi$. 

$\sigma_\xi = 0.08515$

$\sigma_\xi = 0.1277$
Figure 8: Intertemporal trade-off, faced by the authority setting capital requirements $\kappa$, between banks’ current credit supply $l_1$ and banks’ future credit spread $X_b$ and financial spillovers $\Psi$. 

Trade-off between $l_1$ and $\Psi$
Effect of $\theta_2$ for different $\kappa$

$\Delta X_b$

$\Delta \log(l_2)$

$\Delta \log(k)$

$\Delta \log(y)$

$\Delta L_l$

$\Delta \log(a_2)$

Figure 9: Effect of a technology shock $\theta_2$ for different capital requirements $\kappa$. 

55
Figure 10: Optimal response of capital requirements $\kappa$ to a shock to $\sigma_\xi$, a bank net-worth shock $\tau_1$, a technology shock $\theta_1$ and a shock to the household preferences discount factor $\beta$. 