Unemployment Flows, Participation, and the Natural Rate for Turkey

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This paper measures flow rates into and out of unemployment for Turkey and uses them to estimate the unemployment rate trend, that is, the unemployment rate to which the economy converges in the long run. In doing so, the paper explores the role of labor force participation in determining the unemployment rate trend. We find an inverse V-shaped pattern for Turkey’s unemployment rate trend over time, currently between 8.5 percent and 9 percent, with an increasing labor market turnover. We also find that allowing an explicit role for participation changes the results substantially, initially reducing the “natural” rate but getting closer to the baseline over time. Finally, we show that this parsimonious model can be used to forecast unemployment in Turkey with relative ease and accuracy.

Keywords: unemployment, unemployment flows, labor force participation, Turkey.

JEL Codes: E24; E32; J64

1 Introduction

The long-run rate of unemployment (the underlying trend) has attracted a lot of attention since the Great Recession. In an environment where several developed countries, as well as developing ones, face exceptionally high levels of unemployment, policy makers and economists focused on identifying the unemployment rate that is feasible in the long run, that is, the "natural" rate, to gauge the extent of labor market slack (see, for instance, Daly et al. (2012) and Weidner and Williams (2011)). The Great Recession also drew attention to labor force participation and its role in determining the dynamics of the unemployment rate (Elsby et al. (2013) and Erceg and Levin (2013)).

This paper develops a method that estimates the rate of unemployment in the long run, taking into account changes in the labor force participation rate. We estimate our model using data on Turkey, where the labor force participation rate has increased sharply since 2003 and is three times more volatile than the US participation rate (see Sengul (2014)).

Our analysis builds on Tasci (2012), which estimates the unemployment rate trend using an unobserved-components method and data on flows between employment and unemployment for the US. Extending his work, we estimate the unemployment rate trend for Turkey from 2001 to 2013, taking into account flows from inactivity. In doing so, we also draw on the work of Sengul (2014), which estimates Turkey’s monthly flow rates from 2005 to 2011, including flows from nonparticipation to unemployment. We first estimate quarterly flow rates from 2001 to 2013, following Sengul (2014). Then, using a parsimonious unobserved-components method, as in Tasci (2012), we decompose the flow rates into their trend and cyclical components. Once we infer the trend components, we provide an estimate of the unemployment rate trend, i.e., the natural rate, implied by the steady-state description of the unemployment rate in a standard labor-market search model.
To assess the effect of participation on the unemployment rate, we also estimate the trend unemployment rate while restricting flows to those between employment and unemployment. Our results show a distinct pattern for trend unemployment: The trend unemployment increases during the first two-thirds of the sample period and starts to decline after the 2008-09 recession. This pattern holds, whether or not one explicitly allows for time-varying labor force participation.

However, if an explicit role is given to labor force participation, the estimated unemployment trend stays significantly below the level implied by the model, where we assume a constant participation rate over time. Moreover, we find that this pattern for the natural rate is led by a similar pattern for the inflow rate into unemployment, first increasing and then declining in 2008-09, and a secular rise in the rate of outflow from unemployment over the whole sample. Taken together, these findings imply that Turkish labor markets looked far more dynamic at the end of 2013 than in 2001. We also highlight another potentially useful feature of our framework—improving the accuracy of the unemployment forecast in the short term—even though it is not designed for this purpose. This is an important additional benefit of the framework discussed here, in a country where unemployment data releases have a two-month lag.

Most of the questions about estimating trend unemployment can be addressed by focusing on a variant of a traditional Phillips curve. Even for developed market economies, a stable relationship between inflation and unemployment has been debated in the literature. In developing countries, which go through structural changes and major transitions, relying on such a framework becomes more challenging. Turkey experienced persistently high inflation before the early 2000s. Since then, Turkey has instituted major structural reforms and has undergone continued demographic changes. These conditions make it harder to use a Phillips curve framework to pin down an aggregate unemployment rate that could be sustained, in the long run, in the absence of
high-frequency shocks.

To face this challenge, we need a framework that relies exclusively on labor market features. A recent example of such a framework is Tasci (2012), which approaches the problem by estimating the unemployment rate trend using the underlying flow rates. It relies on data on flows between employment and unemployment and argues, in the context of US labor markets, that this method provides an estimate of the natural rate that has several desirable statistical features and comes very close to the language of the modern theory of unemployment. In this paper, we adopt his methodology to incorporate labor force changes and to estimate the natural rate of unemployment for Turkey.

Moreover, the method developed by Tasci (2012) is flexible enough to be modified to incorporate different labor market structures. Thus, when we implement the same approach for Turkey, we take into account the active role of the participation margin in the labor market. The participation rate’s role in estimating the long-run trend for unemployment becomes very evident in the Turkish data, where the participation rate experienced a sharp increase over the decade. Using flow rates to identify an unemployment rate trend provides us with a way to carefully address the problem for Turkey, in which the persistence in unemployment is quite different from that of a developed country, where labor markets are relatively more dynamic.

We believe that this exercise is valuable, not only in its own right but also because it allows us to highlight the usefulness of our approach in the face of the interesting challenges posed by various structural issues that many economies face. As we argued above, for instance, many developing countries, Turkey included, have a very limited span of data that covers substantial changes in the aggregate economy. Turkey has gone through significant changes in its monetary policy environment, followed by sharply
declining inflation in the early 2000s. The traditional approach of estimating a natural rate by focusing on the relationship between labor market variables and the price level (NAIRU) will not necessarily inform us about the underlying dynamics of the Turkish labor market. Section 4.1 shows that natural rate estimates extracted using the NAIRU method imply an almost invariant level of unemployment, which is the average of the sample period, while our method reveals variation over time. Moreover, our method implies recent values of the natural rate of unemployment that are below the levels implied by NAIRU estimates or the constant labor force model.

The rest of the paper proceeds as follows: In the next section, we lay out the unobserved-components model with the assumption that labor force participation moves over time. Since the model with a constant labor force assumption is nested in our model, we describe that model in the appendix. After describing the methodology for measuring the flow rates and estimating the trends, we present the data and the estimation results for both constant and time-varying labor force models in section 3. A more detailed discussion of the natural rate concept we develop here in conjunction with the more conventional measures of the natural rate used in the literature, including a NAIRU, is in Section 4. We also address the robustness of the estimation in this section. Section 5 presents the forecasting performance of the model. The last section concludes.

2 Model with Participation

We now describe our approach to identifying Turkey’s unemployment trend under the assumption that workers can move between three labor market states: employment,

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1The Central Bank of Turkey implemented implicit inflation targeting from 2002 to 2006, and has been officially targeting inflation since then. Please see Kara (2006) and Kara and Oral (2008), among others, for more information regarding Turkey’s monetary policy.

2For a recent discussion regarding NAIRU see Fitzgerald and Nicolini (2013).

3In our NAIRU estimation, we assume constant parameters. Us (2014) uses time-varying parameters and finds a NAIRU that resembles our natural rate under the constant labor force assumption.
unemployment, and inactivity. To estimate the long run unemployment rate while allowing for variations in the labor force participation rate, we extend the unobserved components model described in Tasci (2012). This extended model nests two-state model of Tasci (2012) as a special case. When we discuss the results, we compare the model with participation to the alternative two-state model.

We compute the long run unemployment rate as the steady-state description of unemployment based on trend inflow and outflow rates. Hence, we need labor market flow rates to estimate the unemployment rate trend. We begin by describing the estimation procedure for the flow rates.

2.1 Measurement of Flow Rates

To measure flow rates, we rely on the quarterly aggregate data. There is now an extensive literature on measuring flow rates using aggregate data. Most of this literature uses a simple measurement based on unemployment duration data to infer these rates (i.e., Shimer (2012), Elsby et al. (2009), Fujita and Ramey (2009), Elsby et al. (2013), Petrongolo and Pissarides (2008)). We follow the method used in Sengul (2014), which extends the method used by Elsby et al. (2013), Shimer (2012) and Elsby et al. (2009), to allow for changes in the labor force between two consecutive periods.

In what follows, we assume that time is continuous, and the data is available at discrete months $t$. Hence, “period $t$” refers to the interval $[t, t + 1)$. Let $N_{t+\tau}$ be the population and let the population grow at a rate $\rho_t$ and the participation rate (the ratio of the labor force to the population) grow at a rate $G_t$. The laws of motion for the population and the participation rate are $\dot{N}_{t+\tau} = \rho_t N_{t+\tau}$ and $\dot{P}_{t+\tau} = G_t P_{t+\tau}$, respectively. $P_{t+\tau}$ is the participation rate, computed as the ratio of the labor force to the population ($P_{t+\tau} = L_{t+\tau}/N_{t+\tau}$), where $L_{t+\tau}$ is the number of people in the labor force.
Furthermore, let $U_{t+\tau}$, and $U^{<1}_{t+\tau}$ be the number of unemployed and the number of unemployed for less than five weeks at time $t + \tau$, respectively, while $A_t$ is the fraction of the inactive population $(N_{t+\tau} - L_{t+\tau})$ that decide to look for a job.

People become unemployed because they separate from their employment or enter labor force as unemployed; they leave unemployment because they either find a job or leave the labor force. Let $S_t$ and $F_t$ be the job-separation and unemployment exit (outflow) rates during period $t$. We can write the law of motion for unemployment as follows:

$$\dot{U}_{t+\tau} = (L_{t+\tau} - U_{t+\tau})S_t - U_{t+\tau}F_t + (N_{t+\tau} - L_{t+\tau})A_t. \tag{1}$$

We are limited in our ability to distinguish between exits from unemployment into employment or into inactivity in the data because data are not available at a high frequency. Hence, $F_t$ absorbs exits from unemployment, regardless of their destination. Since some of the outflow may result from inactivity, $F_t$ is the unemployment exit rate, not necessarily the job-finding rate. This way of modeling does not affect our analysis because the focus of the paper is to measure and estimate the flows into and out of unemployment. We do not need specific information about the nature of the exit from unemployment per se. Note that if the labor force is constant, then we have $\rho_t = 0$ and $G_t = 0$ and $A_t = 0$. In this case, the equation above is the same as the equation of Elsby et al. (2013), which assumes a constant labor force between two consecutive periods.\footnote{We derive flow equations under this assumption in the appendix.}

We solve equation (1) and iterate it three months to get the evolution of the unemployment rate based on observed data in discrete intervals as\footnote{We use quarterly data.}

$$u_t = u_{t-3}(1 - \lambda_t) + \frac{\lambda_t(S_t - A_t)}{S_t + F_t + \rho_t + g_t} + \frac{A_t(1 - e^{-3(S_t + F_t + \rho_t)})}{P_t(S_t + F_t + \rho_t)}, \tag{2}$$
where $\lambda_t = (1 - e^{-3(S_t + F_t + \rho_t + G_t)})$ is the quarterly convergence rate. Note that the effect of participation on the law of motion for unemployment has two channels. First, we need to account for the fraction of the inactive population who start looking for a job and become unemployed, i.e. $A_t$. Second, we have to consider the fact that the size of the labor force changes with participation, as well as with population growth.

One can use equation (2) and write the steady state unemployment rate as

$$u_{t}^{ss} = \frac{(S_t - A_t)}{S_t + F_t + \rho_t + G_t} + \frac{A_t(1 - e^{-3(S_t + F_t + \rho_t)})}{P_t(S_t + F_t + \rho_t)\lambda_t}. \quad (3)$$

To compute the flow rates, we also need the law of motion for the short-term unemployed, i.e. those unemployed for less than five weeks:

$$\dot{U}_t^{<1}(\tau) = (L_{t+\tau} - U_{t+\tau})S_t - U_t^{<1}(\tau)F_t + (N_{t+\tau} - L_{t+\tau})A_t. \quad (4)$$

The change in the number of short-term unemployed consists of workers entering unemployment, workers separating from their jobs and workers who became unemployed after the last time data was available and did not leave unemployment, respectively.

Subtracting equation (4) from equation (1) results in

$$\dot{U}_t + \tau = \dot{U}_t^{<1}(\tau) - (U_t + \tau - U_t^{<1}(\tau))F_t. \quad (5)$$

We do not observe $A_t$ in (5) explicitly. This is not because participation does not affect the law of motion for unemployment, but because the fraction of inactive population entering to unemployment is already present in $U_t^{<1}$ and $F_t$ also covers unemployed workers leaving the labor force.

Solving the differential equation above and the laws of motion for the population...
and the participation rate (and rewriting the equation in terms of rates) yields:

\[ u_t = e^{-F_t - \rho_t - G_t} u_{t-1} + u_{t<1}. \]  \hspace{1cm} (6)

where \( u_t \) denotes the unemployment rate in period \( t \).

Assuming that unemployment exit occurs with a Poisson process with parameter \( F_t \), the probability of exiting unemployment within a month is \( \hat{F}_t = 1 - e^{-F_t} \). Therefore, equation (6) can be rewritten as

\[ \hat{F}_t = 1 - \frac{u_t - u_{t<1}}{e^{-G_t - \rho_t u_{t-1}}}. \]  \hspace{1cm} (7)

The intuition behind (7) is that we infer the average outflow probability by measuring the size of the decline in the unemployment pool who are not short-term unemployed. Notice that \( \rho_t + G_t \) is the labor force growth rate, as labor force varies due to changes in population and the participation decisions. The equation above takes into account the change in the size of the labor force to get the average outflow probability.

The monthly outflow probability relates to the associated monthly outflow hazard rate for the short-term unemployed, \( F_{t<1} \), through the equation \( F_{t<1} = -ln(1 - \hat{F}_{t<1}) \).

Equation (7) works well to estimate outflow probability in labor markets for which the flow rate out of unemployment is high (duration of unemployment is low). For countries with longer durations, like Turkey, there are relatively few people in \( u_{t<1} \) at any time since exit rates are low. Hence, the variance of the estimate will be higher (that is, \( \hat{F} \) will be noisy). We follow Elsby et al. (2013) and Sengul (2014) and use additional duration data to increase the precision of the estimate of \( \hat{F}_t \). Using the unemployment data by duration, we can calculate the probability that an unemployed
worker exits unemployment within $d$ months as

$$
\hat{F}_t^d = 1 - \frac{u_t - u_t^{<d}}{e^{-\sum_{j=0}^{d-1}(G_{t-j} + \rho_{t-j})u_{t-j}}}.
$$

(8)

Subsequently, we can calculate the outflow rates as $F_t^{<d} = -\ln(1 - \hat{F}_t^d)/d$ for different durations, $d = 1, 3, 6, 9, 12$. This rate is interpreted as the rate at which an unemployed worker exits unemployment within the subsequent $d$ months.

If the exit rate from unemployment is independent of the duration of unemployment, then $F_t^{<d}$ for different values of $d$ does not vary much, and we have the monthly outflow hazard rate as $F_t^{<1}$. However, if the exit rate from unemployment depends on the duration of unemployment, then the $F_t^{<1}$ rate would not be a consistent estimate of the average outflow rate. We formally test the duration dependence by testing the hypothesis that $F_t^{<1} = F_t^{<3} = F_t^{<6} = F_t^{<9} = F_t^{<12}$. The approach is generally to derive the asymptotic distribution of unemployment rates for different durations, and then to apply the Delta method to compute the joint asymptotic distribution of the outflow rate estimates. For Turkey, the hypothesis that there is no duration dependence (i.e., the hypothesis that $F_t^{<d}$ is the same for all $d$) can be rejected at a 95 percent confidence level. We use the asymptotic distribution to compute an optimally weighted estimate of outflow rate that minimizes the mean squared error of the estimate. We discuss computation of the $A_t$ series below when we describe the data, since we infer the series directly from the data.

Given $F_t$, $u_t$ and $A_t$ series, equation [2] gives us the separation rate data that we need to proceed with our estimation.

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6Formal details of the test can be found in Elsby et al. (2013) with the only difference being that this paper has an extra term, the duration $d < 9$. 
2.2 Estimating The Trend Unemployment Rate

After discussing the measurement of flow rates, we now present our approach for identifying an unemployment trend in the presence of varying labor force participation. In doing so, we extend the model used in Tasci (2012) to include participation. Tasci (2012) uses a simple reduced form unobserved components model that incorporates the comovement of flows between employment and unemployment into previous attempts at estimating the natural rate, such as Clark (1987, 1989) and Kim and Nelson (1999).

Our reduced form model assumes that real GDP - the measure of the aggregate business cycle we use - has both a stochastic trend and a stationary cyclical component, where only real GDP is observed by the econometrician. The stochastic trend follows a random walk while the cyclical component is an autoregressive process. More specifically, let $Y_t$ be log real GDP, $\bar{y}_t$ be a stochastic trend component, and $y_t$ be the stationary cyclical component. Then we consider an unobserved components model starting with the process for the GDP as,

$$Y_t = \bar{y}_t + y_t,$$

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon^{yc}_t,$$

$$\bar{y}_t = r_{t-1} + \bar{y}_{t-1} + \varepsilon^{yn}_t,$$

$$r_t = r_{t-1} + \varepsilon^r_t,$$

where $r_t$ is a drift term in the stochastic trend component of output, which is also a random walk, and the cyclical component of output follows an AR(2) process, as in Ozbek and Ozlale (2005).

The model also assumes that rates for both unemployment exit and job separation ($F_t$ and $S_t$) have a stochastic trend as well as a stationary cyclical component. Furthermore, the stochastic trend in these flow rates follows a random walk while their
cyclical component depends on the cyclical component of real GDP. Let \( \bar{f}_t \) and \( \bar{s}_t \) be the stochastic trend components, and \( f_t \) and \( s_t \) be the stationary cyclical components of \( F_t \) and \( S_t \), respectively. The time series behavior of these flow rates takes the form

\[
F_t = \bar{f}_t + f_t, \quad \bar{f}_t = \bar{f}_{t-1} + \varepsilon_{fn}^f,
\]

(10)

\[
f_t = \tau_1 y_t + \tau_2 y_{t-1} + \tau_3 y_{t-2} + \varepsilon_{fc}^f,
\]

and

\[
S_t = \bar{s}_t + s_t, \quad \bar{s}_t = \bar{s}_{t-1} + \varepsilon_{sn}^s,
\]

(11)

\[
s_t = \theta_1 y_t + \theta_2 y_{t-1} + \theta_3 y_{t-2} + \varepsilon_{sc}^s.
\]

We assume that all the error terms are independent white noise processes.

As equations (10) and (11) show, we also assume that the cyclical component of the outflow and separation rates moves with the aggregate cycle. This idea captures the empirical pattern that recessions are times when a substantial number of matches dissolve because they cease to be productive enough and significantly fewer new matches are formed because firms do not demand as much labor as before. Hence, a priori, we expect a negative co-movement between the cyclical components of the flow rates, \( s_t \) and \( f_t \) and the cyclical component of real GDP.\(^7\) This basic description of the co-movement between flow rates and the aggregate cycle can easily be reconciled with extensions of the basic labor market search model with endogenous separations, as in Mortensen and Pissarides (1994). Tasci (2012) argues that the low-frequency movements in the trends, \( \bar{f}_t \) and \( \bar{s}_t \), will capture the effects of institutions, demographics, tax structure, labor market rigidities, and the long-run matching efficiency of the labor

\(^7\)We are agnostic about the existence of co-movement, if any, among the trends of the flow rates, as long as they are not correlated with the aggregate output. Even though such interaction is possible, we abstract from it. Given the short sample we are working with, any further complication in the form of another latent variable will substantially reduce the precision of the estimates we get in this unobserved components model.
markets, which will be more important in determining the steady state of unemployment.

After modeling unemployment exit and separation rates, we are left with the time series behavior of the participation rate, inactivity-to-unemployment flow rate and population. Due to the length of our sample and the additional number of parameters that arise when we introduce another variable into our unobserved-components model, we cannot fully model all the flow rates that determine the steady state unemployment rate. Hence, we need to make some assumptions. We begin by assuming that population growth $\rho_t$ has a trend and a cycle that are independent of the GDP, and we identify these components using HP filter.\footnote{We also fit an AR process to population growth and see that the trend we extract does not change much.} We also subject $A_t$ series to the same procedure. Even though one expects the cyclical component of flows from inactivity to unemployment to depend on the overall cycle (in GDP), we cannot model it together with the participation rate and its growth because we run out of degrees of freedom. Since $A_t$ is measured indirectly, we think including $P_t$ and $G_t$ in our model can be more informative.

Given these assumptions, we model participation as having a cyclical component and potentially a stochastic growth component in its trend:

\begin{align*}
P_t &= \bar{p}_t + p_t \\
p_t &= \mu_1 y_t + \mu_2 y_{t-1} + \mu_3 y_{t-2} + \varepsilon_{pc}^t \\
\bar{p}_t &= \bar{p}_{t-1} + g_{t-1} + \varepsilon_{pn}^t \\
g_t &= g_{t-1} + \varepsilon_g^t
\end{align*}

The model described by equations (9) through (12) can be represented in a state-space form in the following way:
\[
\begin{bmatrix}
Y_t \\
F_t \\
S_t \\
P_t
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \tau_1 & \tau_2 & \tau_3 & 0 & 0 & 0 & 1 & 0 \\
0 & \theta_1 & \theta_2 & \theta_3 & 0 & 0 & 0 & 0 & 1 \\
0 & \mu_1 & \mu_2 & \mu_3 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
y_t \\
y_t-1 \\
y_t-2 \\
r_t \\
g_t \\
\bar{p}_t \\
\bar{f}_t \\
\bar{s}_t
\end{bmatrix} + \begin{bmatrix}
0 \\
\varepsilon_t^e \\
\varepsilon_t^sc \\
\varepsilon_t^pc
\end{bmatrix}, \quad (13)
\]

\[
\begin{bmatrix}
\bar{y}_t \\
y_t \\
y_t-1 \\
y_t-2 \\
r_t \\
g_t \\
\bar{p}_t \\
\bar{f}_t \\
\bar{s}_t
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\bar{y}_t-1 \\
\bar{y}_t-1 \\
\bar{y}_t-2 \\
\bar{r}_t-1 \\
\bar{g}_t-1 \\
\bar{p}_t-1 \\
\bar{f}_t-1 \\
\bar{s}_t-1
\end{bmatrix} + \begin{bmatrix}
\varepsilon_t^m \\
\varepsilon_t^r \\
\varepsilon_t^fc \\
\varepsilon_t^sc \\
\varepsilon_t^g \\
\varepsilon_t^pc \\
\varepsilon_t^fn \\
\varepsilon_t^pc
\end{bmatrix}. \quad (14)
\]

where all error terms come from an i.i.d. normal distribution with zero mean and variance \(\sigma_i\), such that \(i = \{yn, yc, r, g, fn, fc, sn, sc, pn, pc\}\).

We use the Kalman filter to filter the unobserved components and write the log-likelihood function to estimate the model via maximum likelihood. Since we are interested in the unobserved stochastic trend and cyclical components, once we estimate the model, we use the Kalman smoother to infer them over time. Then, we obtain the unemployment rate trend using the flow steady state equation and evaluate at the current trend levels of the variables:

\[
\bar{u}_t = \frac{(\bar{s}_t - \bar{a}_t)}{\bar{s}_t + \bar{f}_t + \bar{p}_t + \bar{g}_t} + \bar{a}_t \left(1 - e^{-3(\bar{s}_t+\bar{f}_t+\bar{p}_t)}\right) \frac{1}{\bar{p}_t(\bar{s}_t + \bar{f}_t + \bar{p}_t)\lambda_t}, \quad (15)
\]

where \(\lambda_t = 1 - e^{-3(\bar{s}_t+\bar{f}_t+\bar{p}_t+\bar{g}_t)}\). Recall that \(\bar{p}_t\) and \(\bar{a}_t\) are not estimated through the model, but computed separately as the trend implied by the HP filter.

Tasci (2012) interprets the unemployment rate trend expressed in (15) as the steady
state unemployment rate implied by the current trend estimates of the flow rates. Note that, since trend flow rates are random walks, current trend estimates are also the best estimates for future trend values. Hence, we interpret this rate as the rate of unemployment in the long run, to which the actual unemployment rate would converge in the limit.

3 Data and Estimation

Before proceeding to the estimation, we describe our data sources and the treatments we have to implement to address some concerns. Then we present the estimation results for the flow rates and the long-run unemployment rate for Turkey implied by these rates. To better understand the contribution of incorporating changes in the labor force to the unemployment rate trend, we also estimate the model with the assumption that the labor force does not change between two consecutive quarters. Since our mode already nests the constant labor force model, we put the explicit discussion of the model without a variable labor force in the appendix.

The data used to estimate the flow rates is from the Turkish Statistical Agency (TurkStat).

9 We have quarterly data from 2000:Q1 to 2013:Q4 on the population, the number of workers in the labor force, and people who are unemployed for less than $d$ months, where $d \in \{1, 3, 6, 9, 12\}$.

10 The raw data requires some adjustments because of breaks prior to construction of the flow hazard rates, $F_t$ and $S_t$. First, there is a break in the 2005:Q1 data, due to a change in population projection methods. TurkStat updated quarterly data until

9For more information go to http://www.tuik.gov.tr.

10$d = 1$ corresponds to the number of workers unemployed for less than five weeks and this data is provided by TurkStat upon request.

11In 2007, Turkey implemented an address-based population registration system (ADNKYS), which allows yearly data for population. Turkstat was using population numbers based on projections from census data prior to this change, and it realized a discrepancy between the projections and the actual
2005:Q1 and yearly data until 2004. To correct the data prior to 2005, we make use of the availability of unadjusted quarterly and adjusted annual values for 2004. Thus, we update the unadjusted quarterly values for 2004 such that quarterly growth rates in 2004 are the same for adjusted and unadjusted series and the average of the new quarterly data for 2004 is the same as the adjusted annual value reported by TurkStat. Once we adjust the quarterly series for 2004, we also update the data before 2004 so that the quarterly growth rates are the same as in the unadjusted series.

In addition, there is a break in 2004 in the data for unemployed with different durations. To correct for this, we assume that the growth rate of the share of unemployed with a duration of $d$ months among all unemployed from 2003:Q4 to 2004:Q1 is the average of the growth rate of the same quarter of the two previous and the following years’ shares. Then, we back up the new shares for periods prior to 2003:Q4 from the new growth rates, and readjust all duration data so that the shares add up to 1. We adjust the number of people unemployed for less than one month, so that their share among people unemployed for less than three months (in the unadjusted series) remains the same. All these treatments are unfortunately dictated by our concerns about data breaks, survey redesign, and methodological changes. However, the fact that no major aggregate economic shock hit the economy around this time reassures us that the impact of our treatments on the estimation results will be nonsubstantial. Finally, we also use the aggregate real GDP data from TurkStat.

Apart from the data described above, we also make use of the data on unemployment by reason to construct $A_t$ series. An ideal computation would require data on the labor numbers delivered by ADNKYS.

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12This break may result from sample redesign in 2004, which may have allowed a better measurement of unemployment with different durations.

13There was also an anomaly in the unemployed for 6-7 months data for 2003:Q2 and 2003:Q3, which generated a level shift in the seasonally adjusted data. We replace the growth rates of shares from 2003:Q1 to 2003:Q2 and from 2003:Q2 to 2003:Q3 with the average of the growth rate of the same quarter of the two previous and the following years’ shares.

14Expenditure based, in 1998 prices.
market transitions of entrants who will be unemployed for less than one month. The ratio of this pool to the inactive population would be $A_t$. However, data on the number of unemployed for less than one month by reason of unemployment is not available. Thus, we use data on people unemployment by reason for a duration less than three months and assume that the fraction of entrants among those unemployed for less than three months (the shortest duration for which we have data) is the same as the fraction of entrants among those unemployed for less than one month. This assumption implies that $\frac{U_{e,<1}}{U_t} \approx \frac{U_{e,<3}}{U_t}$, where $U_{e,<d}$ denotes labor market entrants who are unemployed for less than $d$ months. Note that $U_{e,<1} \approx U_t \frac{U_{e,<3}}{U_t}$ and we have data for the right-hand side of this approximation. Hence, we compute $A_t$ as $U_t \frac{U_{e,<3}}{U_t} / (N_t - L_t)$.

Before proceeding to the estimation and results, we would like to discuss an issue that must be tackled in estimating the model. The model, as spelled out in equations (13)-(14), has four observable series and ten shock parameters that need estimating; consequently, it is subject to a potential identification problem. The solution involves normalizing the standard deviation of the cyclical component of a variable relative to its trend component, thereby reducing the number of parameters to estimate. We address this issue in more detail and describe the process in section 3.2.

### 3.1 Results for the Constant Labor Force Model

Once we make necessary adjustments to the data, we compute the aggregate flow rates following our discussion in the preceding section. We start with the restricted model which assumes that the labor force is constant, which is effectively equivalent to the assumption that we have $\rho_t = 0$ and $G_t = 0$ and $A_t = 0$ in our extended model. We start with this simplified case to present an easier benchmark. Table 1 presents the basic moments of the data. Average unemployment in Turkey was about 10.5 percent over our sample period, rising from around 7.5 percent to more than 14 percent in the middle
of the last recession. We are in a sense fortunate to have unemployment move around this much, as it helps to identify the movements in the trend and cycle components of the flow rates, even within a short sample. The flow rate levels shown in Table I show that the Turkish labor market also features very low turnover rates, similar to those of some OECD countries. Thus our approach of using more duration data to compute the average flow hazards is clearly warranted. Like the pattern in other countries, the outflow hazard, $F_t$, is at least six times more volatile than the inflow hazard, $S_t$.

We estimate the unobserved components model with constant labor force via maximum likelihood using the flow rates described above. The potential identification issue discussed above appears not to be a major one for the data at hand. The log-likelihood function turns out to be well behaved and quite variable, such that we can avoid the normalization for the GDP components that Tasci (2012) relies on for the U.S. data. The same is not true for the flow rates, which implies that we estimate the process for both $\varepsilon^m_t$ and $\varepsilon^c_t$, but we resort to normalization for the flow rates. Our estimation results suggest that the drift term for the trend output for this time-period in Turkey was constant, that is $\sigma_r = std(\varepsilon^r_t) = 0$. Hence, we impose this restriction in our estimation, obtaining $r = 0.012$ for the sample period. This rate translates into a 4.9 percent average annualized quarterly growth rate for the trend output. The normalization we find to be optimal for the flow rates in this restricted model estimation implies that $\gamma_f = \frac{\sigma_{f_m}}{\sigma_{f_c}} = 0.75$ and $\gamma_s = \frac{\sigma_{s_m}}{\sigma_{s_c}} = 0.75$. The procedure to choose parameter values for $\gamma_s$ and $\gamma_f$ is explained in detail in section 4.

In our estimation, we rely on the Kalman filter to generate the log-likelihood function and to obtain the smoothed unobserved states. Because we have several variables following a random walk, initializing the Kalman filter requires starting with a diffuse prior, which requires us to exclude some of the quarters at the beginning of the sample.

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15 This version of the model is expressed in (A.25)-(A.26) in the appendix.
We exclude the first eight quarters of the data in our estimation. We discuss the potential effects of this exclusion restriction in Section 4.

In Figure 1, we plot the estimated unobserved trend components as well as the data on the flow rates, unemployment rate, and the rate of convergence, $\lambda_t$. The upper panel of Figure 1 shows interesting changes in the underlying trends for the flow rates. In particular, the outflow rate, rate at which an average unemployed would find a job in a given month, has increased over the course of the decade by essentially doubling from 0.06 to 0.12, implying a monthly probability of roughly 11 percent by the end of the sample. In a somewhat similar fashion, the inflow rate also trended up over the sample period, tripling from its 0.005 level to 0.015. Since the end of the last recession, the trend changed course and has started to decline towards a level of 0.012.

These trend changes together imply a relatively stable pattern for the unemployment rate trend early on in the sample period, with the exception of the first recessionary episode. Then, trend unemployment gradually declines from its recession era highs of 12 percent to around 9 percent at the end of the sample. In the first part of the sample, trend changes in $F$ and $S$ offset each other to some extent as they push trend unemployment in opposing directions. However, since the end of the last recession, changes in direction of the trend behavior of $S$ reinforced the decline in the unemployment rate trend that is implied by the gradual increase in the outflow rate, $F$, over time.

A more important observation is that overall reallocation in the labor markets have experienced a steady increase in Turkey. The picture on the lower-right panel plots the reallocation measure we look at, $\lambda_t$, which governs the rate at which unemployment approaches its flow steady state. The magnitude of the changes over time implies that the half-life of a cyclical gap in the unemployment rate declined from more than five quarters in early 2000s to around three quarters by the end of the sample. Hence, our results not only suggest a declining trend for the unemployment rate, but also more
churning in the labor market implying faster adjustments in response to cyclical changes in the unemployment rate.

3.2 Results for the Variable Labor Force Model

Next we turn to the case with variable labor force participation, which is our main focus. We begin with the descriptive statistics for flow rates under the assumption that measurement takes into account variation in the labor force participation over time. Table 1 shows the average levels of flow rates for both cases, with constant and varying labor force assumptions. We observe that relaxing constant labor force assumption affects both the levels and the standard deviations of flow rates.

The results for the estimation of the extended model with participation are displayed in Table 3. Some of the individual parameter estimates lose significance, however, overall the model is preferable to the one with these parameters excluded and to the model with no participation, because the improvement in log-likelihood is significant. Contrary to the stochastic growth rate for the output trend, labor force participation does indeed have a time-varying growth rate in its trend. Consistent with the cyclical behavior of $F$ and $S$, we observe that $\tau_1$ is positive whereas $\theta_1$ is negative. We see that, although $\tau_3$ is not independently significant, the model is preferable to the one without $\tau_3$.

Our estimates of the model with a varying labor force suggest that the impact on the unemployment rate could be substantial. Figure 2 plots the unemployment rate trend from the restricted model together with the estimated trend from the extended model of this section. According to our estimates, for most of the early part of the sample, the difference between the two models is quite substantial, but it shrinks towards the end of the sample. For instance, we observe a difference of as much as a 2.5 percentage point between two trend estimates in the middle of the sample period and a 0.5 percentage point difference at the end. The main reason the two alternative trend estimates diverge
early in the sample is the behavior of the flows from the inactive population directly into the unemployment pool, $A_t$. Our measurement of $A_t$ implies a level of 0.0016 at the beginning of the sample, tumbling by more than 75 percent over the next 12 years, mostly in the first five quarters. One possible interpretation is that at the early parts of the sample period there is a movement from inactivity to unemployment, which implies a natural rate with variable participation rate that is very different from the one with constant participation. As $A_t$ declines, that is, as flows from inactivity to unemployment slow, we see a narrowing in the gap between two natural rates. However, we suspect that part of the decline we observe in $A_t$ could result from a measurement problem in the household survey, or from nonparticipants’ extraordinary response to the first major recession in our sample. We have no convincing way to isolate one or the other. In any case, the absence of abnormal behavior in $A_t$ later on and the apparent convergence between the two alternatives suggest that this channel has become less important. In addition, the implied natural rate with a varying participation rate is lower than the one implied by the restricted model. However, their overall patterns throughout the sample, including the turning points, align very closely with one another.

Figure 3 displays all of the important unobserved components for the extended model with a variable labor force participation rate. Even though the implied trend estimates for $F$ and $S$ change somewhat, results confirm the secular trends we obtained from the restricted model. More importantly, the participation rate trend implied by the estimation (right-hand figure in the middle panel) shows that there has been an important trend growth change. The participation rate grew in Turkey over this period, and our model identifies part of this as a trend increase. This is not unlike the behavior in the U.S. where participation responds little, if at all, to the business cycle. Taken together, the convergence rate now reflects the added impact of an increasing growth rate in labor force participation, which is pictured in the lower panel.
4 Discussion and Robustness

We have proposed and estimated a natural rate for Turkey using a relatively parsimonious model that relies purely on the flow rates in and out of unemployment. We view this concept consistent with Tasci (2012) and perceive it as the steady state unemployment rate that is implied by the current trend estimates of the flow rates. Practically, this means that it is the rate of unemployment in the long-run, to which the actual unemployment rate would converge.

This view offers a stark contrast to the alternatives that the literature focuses on, such as Gordon (1997) and Staiger et al. (1997, 2001). These studies are concerned with a natural rate concept that relates price pressures to a level of unemployment that is consistent with constant inflation rate. As we argued in the introduction, some of the structural changes in the case of Turkey, render such a concept uninformative. In this section, we address this issue and compare our estimates to some alternatives, including a NAIRU. Furthermore, we address some of the robustness issues of the underlying estimation we employed, such as the normalization implied by $\gamma_s$ and $\gamma_f$, as well as the exclusion restrictions for the early part of the sample in the maximum likelihood estimation.

4.1 Alternative Natural Rates and Filters

In this section, we present a basic comparison between our measures of the natural rate and some alternatives proposed in the literature. One of these alternatives is a NAIRU. Taking a different approach, one can also use an unobserved components method without using the flow rates, but instead focusing on the unemployment rate. We will refer to this alternative as the bivariate unobserved components model with unemployment rate (UC-UR). Finally, we will address the question of whether purely
statistical filters could be good substitutes for our proposed natural rate. We view our approach as an alternative that relies solely on data from labor markets and the real economy in determining long-run trend for the unemployment rate. There is a widespread use of similar terminology in the literature, as Rogerson (1997) discusses thoroughly. However, getting into the details of this discussion would exceed the scope of this paper.

The NAIRU estimation takes a simple form, relating current inflation to lagged inflation and the “unemployment gap” (Gordon (1997)), using quarterly changes in headline CPI at an annualized rate for the measure of inflation. The bivariate model we have in mind is similar to the flow model, but only uses data on the actual unemployment rate and real output as in Clark (1987, 1989) and Kim and Nelson (1999). In both frameworks, one can use the Kalman filter to infer unobserved trends in the unemployment rate much like we do for unobserved trends in the flow rates. Our comparison relies on these unobserved trends, which are interpreted as alternative natural rates.

Figure 4 presents these alternatives along with the flow-based estimates of the natural rate from the restricted and the extended models. Both estimated NAIRU and UC-UR are almost constant at around 10.5 percent over the entire sample period. There is virtually no variation at all.

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16 More specifically, we assume that, $\pi_t = \beta_{\pi_t} \pi_{t-1} + \beta_u [u_t - \bar{u}_t] + \varepsilon_{\pi_t}$, where $\pi_t$ and $u_t$ denote actual inflation and unemployment rate, respectively. The natural rate, $\bar{u}_t$, follows a random walk, whereas the “unemployment gap”, $u_t^c = u_t - \bar{u}_t$, is assumed to follow an AR (2) process; $u_t^c = \theta_1 u_{t-1}^c + \theta_2 u_{t-2}^c + \varepsilon_{u_t}$. 

17 Output is modeled as in equation (9). The observed unemployment has cyclical and trend components such that the trend component follows a random walk and the cyclical component depends on the cyclical component of the real output, much like the flow rates.

18 Both alternative models are estimated using maximum likelihood estimation and results are available upon request.

19 Though our NAIRU estimation assumes time-invariant parameters, we do not restrict the natural rate itself to be constant over time.

20 Us (2014) estimates NAIRU for Turkey using time-variant parameters, her findings is inline with our estimations under the constant labor force assumption.
this is the case. Turkey experienced a sharp drop in consumer inflation over the early part of the sample period, as a result of aggressive efforts by the newly independent central bank that effectively instituted an inflation target. This will undoubtedly affect the statistical relationship between inflation and the unemployment rate, which any NAIRU estimate will rely on. Inflation tumbled from more than 60 percent per year to single digits in a relatively short period, while unemployment increased only modestly and stayed at those levels for some time. This, in turn, renders the relative variation in inflation with respect to unemployment uninformative. Thus, we obtain a flat NAIRU. It is important to keep in mind that our empirical estimation does not restrict the unobserved NAIRU to be constant in our empirical estimation.

The bivariate model, UC-UR, also implies an almost constant natural rate over our sample period. This model exploits the variation in observed unemployment relative to cyclical changes in the real GDP to identify the natural rate. First, we observe that there are two major episodes of business cycle contractions in our sample; one occurs within the first year of the sample by 6 percent; the other one (about 15 percent) coincides with the global recession. Even though the output contractions were significantly different, unemployment rate increases were almost identical, by about 70 percent, in both episodes. Moreover, the unemployment rate did not decline at all after the first recession, demonstrating considerable persistence. These factors imply a constant natural rate in the UC-UR case. Our method, on the other hand, addresses the persistence in the unemployment rate without implying a constant natural rate; we focus on the underlying flow rates, thereby easily accommodating the non-linearities.

Note that the first recession actually started right before the beginning of our sample, in 2000:Q4, with an overall peak-to-trough decline of 10 percent in real GDP. See Ceritoğlu et al. (2012) for more on the comparison of the unemployment in two recessions. Tasci (2012) also compares a variant of our baseline model with flows to these alternatives on some other dimensions, such as the precision of estimates, required retrospective revisions with additional data, and prediction accuracy for inflation and concludes that the flow-based approach has several desirable properties along those dimensions as well.
One might argue that if our objective is to derive an empirically useful unemployment rate trend, a purely statistical unemployment rate trend might be more practical, if unemployment flows do not seem to provide us with any additional information. In order to address this issue, we focus on different statistical filtering methods with and without unemployment flows to distinguish the role they play. For the sake of exposition, we focus on the restricted model.

Taking an HP-filter of the unemployment rate itself has been one approach used in the literature to identify an unemployment rate trend in the context of the natural rate discussion (see Rogerson (1997)). We compare our estimate of the long-run trend for the unemployment rate with those that could be obtained using an HP or a bandpass filter. Figure 5 presents the results of this exercise. When we omit the information on unemployment flows and filter the quarterly unemployment rate (top panel), we find considerable variation in the trend and significant diversion across different filters. For instance, applying an HP-filter with a high smoothing parameter (1600) gives a relatively smooth trend that moves closely with the preferred trend from the flow model. However, a bandpass filter or an HP-filter with a smaller smoothing parameter (98) produces much more variation in the trend. The top panel also shows the well-known problem related to the end points of the sample in one-sided filters.

A different picture emerges if we include information on unemployment flows and impute an unemployment rate trend, as we did above, based on the trends of these underlying flows. As the lower panel of figure 5 shows, unemployment trends imputed this way do not vary much across different filters and are much smoother than trend estimates based solely on unemployment rate information. Moreover, the flow model, which imposes far more structure on the co-movement of flows and real output, produces a trend that moves closely with these other filters. We interpret this result as evidence that unemployment flows are important in understanding the unemployment rate trend.
over the long run. There is an obvious discrepancy between various estimates of the
trend with different filters when flows data are ignored; this makes it harder to get an
empirically consistent, and otherwise useful, measure.

4.2 The Robustness of the Estimation

When computing our estimate for the trend unemployment rate, we rely on equation
(15) where we substitute the HP filter of the variables $A_t$ and $\rho_t$. We resort to this
solution because of data availability, but are mindful of its potential impact on our
results. Therefore, we conduct a robustness check where we model the process that
governs $A_t$ and $\rho_t$ in a linear AR process and analyze the effect on the trend unemploy-
ment. Note that this exercise is still confined to the same model with participation,
but the process that determines the trend components of $A_t$ and $\rho_t$ are assumed to be
a product of a process other than a basic HP filter. We back out the actual estimate of
the trend assuming an AR process yields virtually the same result. We do not report
them separately to save space here.\[24\]

In principle, the results of our estimation could be sensitive to the exact values of $\gamma_f$
and $\gamma_s$ that we use. In the benchmark estimation, we use values of 0.75 for both. These
parameters control the relative variation in the cyclical components of the flow rates
with respect to their estimated trends. Hence, it is reasonable to have different implied
unemployment rate trends with different values. To pin down the exact numbers, we fol-
low the approach proposed in Tasci (2012). This essentially means that we re-estimate
the model over a fine grid for both $\gamma_f$, and $\gamma_s$; $\gamma_f = \{0.25, 0.375, 0.5, ..., 3.375, 3.5\}$
and $\gamma_s = \{0.5, 0.625, 0.75, ..., 3.875, 4\}$. We target two moments to match: one is the
maximum log-likelihood over this combination of points, the other is the maximum
correlation between the implied natural rate from the estimation and the trend of the

\[24\]Results are available upon request.
observed unemployment rate, calculated using a bandpass filter. Since we do not use the actual unemployment rate in the estimation, we are trying to impose some discipline on the estimation by not letting it diverge too much from the data\textsuperscript{25}. The objective here is to maximize the likelihood of the model without getting an implied unemployment trend that is far from a statistical trend obtained by the bandpass filter. We implement this robustness check exercise using the restricted model with constant labor force. The effects on the log-likelihood and the overall objective function respond the same way in the extended model.

Figure 6 shows how these two moments change across $\gamma_f$ and $\gamma_s$. The preferred benchmark values maximize the objective of high log-likelihood and high correlation, which is clearly shown in figure 6. For instance, we do not improve the likelihood of the model for higher values of $\gamma_f$, whereas smaller values do not result in any reduction. The likelihood value seems more concave in $\gamma_f$, and the preferred value of 0.75 is close to its global maximum. As $\gamma_s$ declines, the trend of the separation converges to a straight line; hence, the natural rate will be determined more by the trend of the job-finding rate. The opposite is true when $\gamma_f$ is small and its trend is close to a straight line. Hence, when one flow has a constant trend imposed (low $\gamma_i$), and the other flow has very little cyclical variation (high $\gamma_j, j \neq i$), we miss the low-frequency movements in the observed unemployment rate by a significant margin. Any increase in $\gamma_s$ sharply reduces the correlation of the statistical filter with the trend estimate to the extent that the correlation may change sign. The objective function determines the optimal trade-off between these two dimensions by putting more weight on the more informative moment, that is, by using the inverse of the covariance matrix as the weighting matrix. Finally, for almost all of the values of $\gamma_f$ and $\gamma_s$, the natural rate implied by the model varies between 9.5 percent and 11 percent at the end of the sample.

\textsuperscript{25}Note that with the flow rates themselves, the unemployment rate does not give any more information for our model, hence, it is not part of it.
Another robustness issue arises with respect to the exclusion restrictions. Recall that, since we model most of the trend variables as random walks, we had to start with a diffuse prior for the Kalman filter. For the first few periods, the impact of the diffuse prior can sometimes be substantial as the Kalman filter does not converge on a reasonable unconditional variance for the unobserved states. This is usually handled by ignoring the initial several periods in the actual estimation - by not considering its contribution to the log-likelihood. Since we have a very short sample, this might be somewhat tricky and we are concerned about potentially losing useful information that the Kalman filter can infer from the likelihood function for the initial data points, which in this case coincide with a recession. The tradeoff is between losing valuable information from the first several quarters versus getting potentially noisy estimates for the unconditional variance due to the diffuse prior.

In order to address this, we have re-estimated the model several times, each time excluding a higher number of quarters from the initial part of the sample. Our results suggest that after 8 quarters, the estimates for the unconditional variance behave well. Figure 7 plots the estimated natural rates corresponding to each exclusion case and shows that with the exception of the excluded part of the sample, our results do not change much. The estimated parameters reported in Table 3 correspond to the case where the likelihood function ignores the first 8 quarters. Note that this does not mean that the smoothed unobserved variables we present do not include them. They include the first 8 data points, but the parameter estimates are only estimated using the rest of the data.
5 Forecasting Performance

Flow rates provide a measure of the natural rate for the Turkish economy, which in turn can help policymakers gauge labor market slack. Beyond providing a simple way to measure the unemployment rate trend in a theoretically meaningful way, another useful feature of this framework has recently been highlighted by Meyer and Tasci (2013): its forecasting accuracy. Meyer and Tasci (2013) argue that by essentially disciplining the long-run trends with the unobserved-components method, this modeling framework does a remarkable job of forecasting the unemployment rate’s evolution in the short- and medium run. Since the framework relies more heavily on the flow rates than the unemployment rate itself, it is very flexible in capturing the non-linearities around the turning points in the business cycle. We suspect that this is even more of a concern for Turkey, where reallocation rates are much lower than US levels. Moreover, the absence of high-frequency, timely information about the unemployment rate motivates us to develop a good forecasting framework for Turkey.26

To evaluate the forecast performance of the framework, we estimate both the baseline model and the extended version with the participation rate over time, starting from the fourth quarter of 2007 and repeating the exercise for every quarter until the end of 2013. For every estimation sample, we produce two-period-ahead forecasts of the unemployment rate, using its predicted flows and observed initial condition. Note that the models produce forecasts of the flow rates internally. However, for the respective equation of motion for the unemployment rate, we use equations (A.23) and (2). In order to gauge the framework’s forecasting performance, we report one- and two-periods ahead root mean squared forecast errors (RMSFE) relative to those generated from a simple time series process for the measured unemployment rate. In particular,

\footnote{Turkish Statistical Institute only releases unemployment rate data with more than two months of lag.}
we choose an AR(2) process. It is important to remember that we are not running this numerical exercise with real-time data. Given the changes in data collection and methodology over the sample period and the sheer length of the data span (or lack thereof), repeating this experiment in real time seems to be a futile effort.

Table 4 reports RMSFEs for one and two-quarter ahead forecasts from the two models we used in the paper and the AR process that does not rely on flow rates at all. As forecast errors suggest, both models produce more accurate unemployment rate forecasts relative to the time series model for the forecast sample period we considered, especially at one-quarter ahead forecast horizon. This relative improvement in forecast accuracy over the near-term could provide a useful tool for policymakers in Turkey.

Having established a relative improvement in forecasting the unemployment rate with the unobserved components models we used in the paper, we finally provide the predictions of them conditional on the data we have for the whole sample; 2001:Q1-2013:Q4. Figure presents the forecast paths for the extended model as well as the restricted model with constant labor force. Regardless of the model we use, we predict a gradual decline in the unemployment rate beyond 2013. Recall that the model with participation implies a lower natural rate in the long-run, therefore yielding a lower path beyond 2015 relative to the restricted model. Unfortunately, Turkish Statistical Institute implemented a methodological change beginning in 2014, which made the data prior incomparable. Nevertheless, the first quarter data which is available confirms the forecast qualitatively, pointing a decline in the aggregate unemployment rate.

\[ u_t = \kappa_1 u_{t-1} + \kappa_2 u_{t-2} + \epsilon_t, \] where data is quarterly.
6 Conclusion

We use a parsimonious unobserved-components model with unemployment flow rates, similar to the one used by Tasci (2012) for the US, to estimate a time-varying unemployment rate trend for Turkey that is grounded in the modern theory of labor market search. We believe that the specific challenges presented by the Turkish data makes it a compelling case. One of these challenges concerns participation rate behavior, which we handled by extending the basic model to incorporate time-varying labor force participation. Our results suggest that by the end of 2013, the natural rate, or underlying trend, for unemployment is hovering around 9 percent for Turkey. Models with and without the participation margin imply substantially different estimates at the earlier parts of the sample period, and the gap narrows over time, with the extended model featuring a participation rate predicting a level slightly below 9.5 percent. This is due to a slowdown in the rate of flows from inactivity to unemployment.

More importantly, we find that the reallocation rate, the sum of the inflow and outflow rates, has been gradually trending up for Turkey, which suggests an increasingly dynamic labor market. Finally, we argue that the modeling framework we provide here can be used for near-term forecasting of the unemployment rate with relative ease and accuracy.

We are mindful of our paper’s main caveat: the sample size. Our data covers only 13 years at a quarterly frequency. However, the considerable variation in the variables of interest over the sample period reassures us that the lack of longer time-series data does not undermine the usefulness of our approach. In future work, it would be interesting to focus on understanding the secular increase in the reallocation rate over time.
References


Table 1: Flow Rates

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Constant Labor Force

| 0.089 | 0.011 | - |
| (0.022) | (0.003) | - |

Note: Standard deviations are in parentheses.

Table 2: Estimation Results: 2001:Q1-2013:Q4

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Notes: Log-likelihood is 492.1310, $\gamma_f = 0.75$, and $\gamma_s = 0.75$. Standard deviations are in parentheses.
Figure 1: Estimation Results (Constant Labor Force)

Note: Dashed lines are trend and solid lines are actual data.

Figure 2: Unemployment Rate Trends - Impact of the Variable Participation
Table 3: Estimation Results: 2001:Q1-2013:Q4

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<td>(0.6756)</td>
<td>$\sigma_{pm}$</td>
<td>0.0041</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>-0.5556</td>
<td>(0.3266)</td>
<td>$\sigma_{fn}$</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_{sn}$</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Note: Log likelihood is 659.17. Standard deviations are in parentheses. $\gamma_f = 0.75$, and $\gamma_s = 0.75$

Table 4: Forecast Performance: RMSFEs for 2009:Q1-2013:Q4

<table>
<thead>
<tr>
<th></th>
<th>AR (2) in UR</th>
<th>Restricted Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t + 1$</td>
<td>0.6447</td>
<td>0.5089</td>
<td>0.5040</td>
</tr>
<tr>
<td>$t + 2$</td>
<td>1.1001</td>
<td>0.9477</td>
<td>0.9784</td>
</tr>
</tbody>
</table>

Figure 3: Estimation Results (Variable Labor Force)

Note: Dashed lines are trend and solid lines are original series.
A Appendix

In this section we lay out the model which abstracts from the variation in the labor force in detail. This is essentially nested in the benchmark model we present in the main text with the restriction that $\rho_t = 0$, $G_t = 0$ and $A_t = 0$.

**Flow Rates in Restricted Model:** Let us start with the flow rates. Since there is no change in the labor force, all flows are between unemployment and employment in this restricted model. Recall that we need the law of motion for unemployment and the short term unemployment to compute the flow rates. The law of motion for unemployment becomes:

$$
\dot{U}_{t+1} = (L_{t+1} - U_{t+1})S_t - U_{t+1}F_t.
$$

**Figure 4: Alternative Natural Rates**

Note that the only difference between the equation above and equation is that the former lacks the term with $A_t$. The law of motion for short-term unemployed, unemployed
Figure 5: Alternative Filters - The Role of Flows

Alternative Trending Methods with Flows

Alternative Trending Methods without Flows
Figure 6: Robustness for $\gamma_f$, and $\gamma_s$

Figure 7: Robustness for Exclusion Restrictions
Figure 8: Forecasting Performance of Both Models

\[ \dot{U}_t^{<1}(\tau) = (L_{t+\tau} - U_{t+\tau})S_t - U_t^{<1}(\tau)F_t. \]  \hspace{1cm} (A.17)

As mentioned earlier, this equation is not affected by the assumption regarding labor force directly, it is the same as equation [4]. However, here \( F \) is the job finding rate.

Subtracting equation (A.17) from equation (A.16) yields:

\[ \dot{U}_{t+\tau} = \dot{U}_t^{<1}(\tau) - (U_{t+\tau} - U_t^{<1}(\tau))F_t. \]  \hspace{1cm} (A.18)

Solving the differential equation above provides us with a simple measurement equation for the outflow hazard:

\[ u_t = e^{-F_t}u_{t-1} + u_t^{<1}, \]  \hspace{1cm} (A.19)

where \( u_t \) denotes the unemployment rate in period \( t \). This equation is the same as [6].

for less than five weeks is:

\[ \dot{U}_t^{<1}(\tau) = (L_{t+\tau} - U_{t+\tau})S_t - U_t^{<1}(\tau)F_t. \]  \hspace{1cm} (A.17)
with $\rho_t = 0$ and $G_t = 0$.

Regardless of the assumption on labor force, if unemployment exit occurs with a Poisson process with parameter $F_t$, then the probability of exiting unemployment within a month is $\hat{F}_t = 1 - e^{-F_t}$. Therefore, equation (A.19) can be rewritten as

$$\hat{F}_t = 1 - \frac{u_t - u_{t<1}}{u_{t-1}}. \quad (A.20)$$

The monthly outflow probability relates to associated monthly outflow hazard rate, $F_{t<1}$, through the following equation: $F_{t<1} = -\ln(1 - \hat{F}_t)$.

We rely on additional duration data to estimate $\hat{F}_t$. Based on the unemployment data by duration, we can calculate the probability that an unemployed worker exits unemployment within $d$ months as

$$\hat{F}_t^d = 1 - \frac{u_t - u_{t<d}}{u_{t-d}}. \quad (A.21)$$

As before, we can calculate the outflow rates as

$$F_{t<^d} = -ln(1 - \hat{F}_t^d)/d, \quad (A.22)$$

for different durations, $d = 1, 3, 6, 9, 12$. We follow the procedure described in section 2.1 to estimate $\hat{F}_t$.

Solving equation (??) and iterating it three months, we get the evolution of unemployment rate in the data, observed in discrete intervals, as:

$$u_t = u_{t-3}(1 - \lambda_t) + \lambda_t \frac{S_t}{S_t + \hat{F}_t}, \quad (A.23)$$

where $\lambda_t = (1 - e^{-3(S_t+F_t)})$ is the quarterly convergence rate. Note that this is the
original equation of [Elsby et al. (2013)]. Solving this equation for the steady state leads to the definition of the flow steady state unemployment as follows

\[ u_t^{ss} = \frac{S_t}{S_t + F_t}. \]  

(A.24)

**Unobserved Components in Restricted Model:** Having the flow rates, we now turn to unobserved components model. Note that modeling labor force does not affect the process the real output follows. It is given by equation 9. Furthermore, the time series behavior of \( F \) and \( S \) are also not directly affected from the assumption regarding the labor force, and hence are given by 10 and 11, respectively. However, note that now interpretation of \( F \) is different. \( F \) is the unemployment exit rate (which could be exit to employment or to inactivity) in the benchmark model while \( F \) in restricted model is job finding rate as all exits from unemployment must go to employment. Since we have \( \rho_0, G_t = 0, \) and \( A_t = 0 \) we are left with no other flow rate.

We can express the empirical in a convenient state-space representation as

\[
\begin{bmatrix}
Y_t \\
F_t \\
S_t
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \tau_1 & \tau_2 & \tau_3 & 0 & 1 & 0 \\
0 & \theta_1 & \theta_2 & \theta_3 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\tilde{y}_t \\
y_t \\
y_{t-1} \\
y_{t-2} \\
r_t \\
\tilde{f}_t \\
\tilde{s}_t
\end{bmatrix} +
\begin{bmatrix}
0 \\
\varepsilon_{fc}^t \\
\varepsilon_{sc}^t
\end{bmatrix},
\]  

(A.25)
\[
\begin{bmatrix}
\ddot{y}_t \\
y_t \\
y_{t-1} \\
y_{t-2} \\
r_t \\
\ddot{f}_t \\
\ddot{s}_t
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\ddot{y}_{t-1} \\
y_{t-1} \\
y_{t-2} \\
y_{t-3} \\
r_{t-1} \\
\ddot{f}_{t-1} \\
\ddot{s}_{t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_t^{yn} \\
\varepsilon_t^{yc} \\
\varepsilon_t^{r} \\
\varepsilon_t^{fn} \\
\varepsilon_t^{s} \\
\varepsilon_t^{sn}
\end{bmatrix},
\tag{A.26}
\]

where all error terms come from an i.i.d. normal distribution with zero mean and variance \( \sigma_i \), such that \( i = \{yn, yc, r, fn, fc, sn, sc\} \).

As in the extended model with variable participation, we use the Kalman filter to filter the unobserved components and write the log-likelihood function to estimate the model via maximum likelihood. Once we estimate the model, we use the Kalman smoother to infer unobserved stochastic trend and cyclical components over time. These time-varying trend estimates for the flow rates, \( \ddot{f}_t \) and \( \ddot{s}_t \), determine the unobserved unemployment rate trend over time. More specifically, our definition of the long-run trend for the unemployment rate is given by

\[
\bar{u}_t = \frac{\ddot{s}_t}{\ddot{s}_t + \ddot{f}_t},
\tag{A.27}
\]

which is consistent with the search theory of the labor market.