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by Andrea Pescatori

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Incomplete Markets and Households' Exposure to Interest Rate and Inflation Risk: Implications for the Monetary Policy Maker

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The present paper studies optimal monetary policy when the representative agent assumption is abandoned and financial wealth heterogeneity across households is introduced. Incomplete market makes households incapable of perfectly insuring against interest rate and inflation risk, creating a trade-off between price level and debt-servicing stabilization. We derive a welfare-based loss function for the policymaker, which includes an additional target related to the cross-sectional distribution of household debt. The extent of deviation from price stability depends on the initial level of debt dispersion. Using U.S. microdata to calibrate the model, we find an optimal inflation volatility equal to almost 20% of the actual volatility of the last 15 years. Finally, the paper studies the design of optimal simple implementable rules. Superinertial rules, which imply a hump-shaped interest rate response to shocks, significantly outperform standard rules.

Key words: optimal monetary policy, household debt, debt servicing, inflation volatility, redistribution, business cycle

JEL codes: E52, E31, C60, D31, D52

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1 Introduction

Since the 1980s, many countries have experienced a sharp increase in household debt. However, aggregate data on the indebtedness of the household sector conceal substantial variation in the distribution of fixed-income assets across households. For example, in 2004, around half of households in the United States had (nominal) mortgage debt, while around 20% of households were holding no debt at all. This implies very different exposures to interest rate and inflation risks, which usually are far from being perfectly hedged. In a recent contribution, Doepke and Schneider (2006) show that a moderate inflation episode would lead to a substantial redistribution of wealth because of changes in the value of nominal assets.

Despite this evidence, much of the recent literature on monetary economics disregards the heterogeneity in households' asset holdings and focuses on designing the optimal response of the monetary authority to business cycle fluctuations in presence of nominal frictions—for example King, Khan, and Wolman (2003) and Rotemberg and Woodford (1997). A distinctive conclusion, recurrent in this framework, is illustrated by the recent work of Schmitt-Grohe and Uribe (2006). They show that, even in a rich medium-scale model with a large variety of frictions, price stability remains the central goal of monetary policy.[1]

The present paper takes a new approach. While still focusing on business cycle and the role of nominal frictions, we depart from the baseline sticky-price model by introducing cross-sectional distribution of household-assets and relax-

[1]There are clearly exceptions, for example when indexation to past inflation or strong wage rigidities are introduced—see Schmitt-Grohe and Uribe (2004) and Erceg, Henderson and Levin (2000).

ing the complete market assumption. This is equivalent to a model in which agents hold heterogeneous portfolios with different exposures to interest rate and inflation risk. The main implication is that the policy maker's welfare-based loss function includes an extra target variable in addition to those typically found in the literature (inflation and output gap): the cross-sectional distribution of household-consumptions. In other words, introducing heterogenous nominal bond holdings entails the central bank's effort to minimize consumption dispersion across households. [2]

This implies a departure from previous results of the literature in two respects. First, thanks to its ability to affect interest payments' volatility, monetary policy has real effects even in a flexible-price-cashless-limit environment. Second, even in a setup with nominal rigidities, price stability is no longer optimal. In other words, introducing debt-burdened households creates a trade off between interest rate reactions meant to stabilize prices and those meant to stabilize the debt-service volatility. In fact, the volatility of interest payments introduces a source of idiosyncratic uncertainty at the household level, which, in turn, is welfare reducing.

In order to assess if our model provides a reasonable description of the data, we perform a calibration exercise using microdata from the Federal Reserve Board of Governors' 2004 Survey of Consumer Finances. Our analysis suggests that an

[2]The study of optimal monetary or fiscal policy when agents hold heterogenous nominal asset positions is not a novelty in the literature. Akyol (2003) finds that, in a model with liquid and illiquid assets, positive inflation can improve risk sharing and, therefore, welfare. Albanesi (2007), studies how taxes and the inflation are set in a political bargaining game between rich and poor households holdings different portfolios of nominal assets. In this case, distributional considerations may determine a positive relation between inflation and income inequality. However, the aforementioned literature fails to put together welfare analysis and business cycle fluctuations. Moreover, it allows no role for monetary policy stemming from nominal rigidities.

equivalent model with symmetric asset positions is not well suited for welfare analysis.[3] In fact, under the optimal policy, the existence of asset heterogeneity would imply an inflation volatility equal to 20% of the observed inflation volatility of the last 15 years. An important implication is that a high dispersion in the initial fixed-income assets distribution does call the price stability goal into question.

Finally, the study examines simple implementable rules and finds that a superinertial rule, i.e., a rule that reacts to lagged interest rate with a coefficient greater than one, is the second-best policy choice. Such a rule allows the monetary authority to have a hump-shaped path for interest rate responses to exogenous shocks. This reduces the volatility of interest rate disbursement but, at the same time, quickly pushes inflation toward zero.

The remainder of the paper is structured as follows: The next section lays out the model and shows the corresponding equilibrium conditions. Section 3 introduces the welfare criterion. Section 4 looks at the problem of the monetary policy maker. Section 5 and 6 contain the calibration and the results, respectively. Section 6 concludes. Proofs are found in the appendices.

2 The Model

The baseline model is a cashless-limit, dynamic, sticky-price model with common factor markets and no capital accumulation (Clarida et al., 1999; Gali, 2002; Rotemberg and Woodford, 1997, 1999).[4] There are households which buy consumption goods, supply factors of production, and can trade in financial markets

[3]Even if, for a given policy rule, it may still constitute a reasonable approximation for studying the behavior of aggregate quantities.

[4]See Woodford (2003) chapter 2, for a discussion of a cashless-limit economy.

for assets. The production side features firms that are imperfect competitors facing infrequent opportunities for price adjustment.

We depart from the standard framework in two respects: Markets are incomplete, and the initial distribution of nominal assets across households is not degenerate.[5]

The two sources of uncertainty are the level of total factor productivity, A, and the level of real government purchases, G. The government can finance the exogenous stream of public consumption with lump-sum taxes, T^G. In period 0, the government is also able to implement a redistributive transfers scheme, $\bar{\tau}$, to favor wealth equality but cannot change it after that period.

The monetary authority controls the short-term nominal interest rate, R, takes the redistributive scheme as given, and can commit to a state-dependent rule. This rule allows the monetary authority to respond to all of the economy's relevant state variables.

In this section, we describe the equilibrium conditions, with households and firms solving dynamic optimization problems for a given transfer scheme and monetary policy rule.

2.1 Households

We assume a continuum of households indexed by $h \in [0,1]$, maximizing the utility

[5] The initial distribution is calibrated using the Federal Reserve Board of Governors, 2004 *Survey of Consumer Finances*, section (5). From a modeling point of view, one could have a non-degenerate distribution of assets across agents, introducing idiosyncratic income or preference shocks at the household level. However, for tractability reasons and because these shocks are irrelevant to the exposition of the main arguments, it is unnecessary to introduce them.

$$U_0^h = E_0 \sum_{t=0}^{\infty} \beta^t \Big[\frac{{C_t^h}^{1-\sigma} - 1}{1-\sigma} - v(N_t^h) \Big].$$

E_0 denotes the expectation operator conditional on the information set at date 0 and β is the intertemporal discount factor, with $\beta \in (0,1)$. Households get utility from the consumption index C^h and disutility from hours worked N^h. Assume that $v(.) : [0, \bar{N}^+) \rightarrow \mathbb{R}$ is twice continuously differentiable with $v' > 0$ and $v'' > 0$; moreover, given some $\delta > 0$, $\varphi \equiv v_{nn} N / v_n$ is at least approximately constant for $N \in I(\bar{N}, \delta)$, where φ can be interpreted as the inverse of the Frisch labor elasticity.[6] The risk-aversion parameter σ is strictly positive.

Consumption index C is defined as a Dixit-Stiglitz aggregate of different goods produced in the economy, with constant elasticity $\theta > 1$:

$$C_t^h = \Big(\int_0^1 c^h(z)^{\frac{\theta-1}{\theta}} dz \Big)^{\frac{\theta}{\theta-1}}.$$

$P_t(z)$ denotes the price of good z and $P_t^{1-\theta} = \int_0^1 P_t(z)^{1-\theta} dz$ defines the aggregate price index that is consistent with the optimal allocation of a given expenditure among the different goods. Optimality implies the following good z demand schedule: $c_t^h(z) = [P_t(z)/P_t]^{-\theta} C_t^h$.

The budget constraint takes the form

$$P_t C_t^h + B_t^h Q_t = B_{t-1}^h + W_t N_t^h + P_t X_t^h. \tag{1}$$

Each household h earns a nominal wage of W_t per hour worked and enters

[6] In a representative agent economy, having no upper bound for hours worked does not represent a serious concern. However, when there is a continuum of heterogenous agents, the possibility, for a single household, of supplying an unbounded number of hours, while leaving the wage unaffected, is not realistic and would place no *natural* limit on debt.

period t with nominal financial wealth B^h_{t-1}. The variable X^h_t is a lump-sum component of income: It summarizes government tax (transfer) T^h_t and profits from firms F^h_t. In the same period, each household h buys (sells), at the market price Q_t, a portfolio of nominal fixed-income assets that pays $1 tomorrow. A value $B^h_t < 0$ means that household h is a net debtor.

In period 0, firms' shares are evenly distributed across households and are not subsequently traded, hence, $F^h_t = F_t$, where F_t is the total amount of profit made in the economy.[7] T^h_t can be divided into an aggregate tax T^G_t, needed to finance current government spending G_t, and a household-specific transfer $\overline{\tau}^h$ that is constant over time. The additive component now reads $X^h_t = \overline{\tau}^h - T^G_t + F_t$.

Assets can be sold short only if they will *almost surely* be repaid. We thus introduce the *natural debt limit*,

$$B^h_t / P_t \geq -\phi^h_b. \tag{2}$$

The value ϕ^h_b can be interpreted as the maximum level of debt a household can repay, allowing the consumption plan $\{C^h_t\}^{\infty}_{t=0}$ and leisure $\{\overline{N}^+ - N^h_t\}^{\infty}_{t=0}$ to be non-negative random sequences.

As a matter of notation, aggregate private-sector demand for a good z is denoted as $c_t(z) \equiv \int_0^1 c^h_t(z)dh$, and aggregate household consumption as $C_t = \int_0^1 C^h_t dh$.

[7]The trading restriction imposed here on stocks may not be innocuous, given the absence of complete financial markets. However, more than one concern has prevented us from introducing this additional feature. First, a sticky-price model is not well suited to describe firms' profit behavior over the business cycle (see, for example, Christiano et al. 1997). Second, it would blur the focus of the analysis on fixed-income assets.

2.2 Firms

We assume a continuum of firms, each producing a differentiated good with a technology

$$y_t(z) = A_t N_t(z), \tag{3}$$

where (log) productivity $\log(A_t)$ follows a Markov-stationary stochastic process.

We define aggregate output as

$$Y_t = \left(\int_0^1 y(z)^{\frac{\theta-1}{\theta}} dz \right)^{\frac{\theta}{\theta-1}}.$$

The government has the same consumption aggregate as the private sector, G, and it demands the same fraction, τ_t^G, of the output of each good produced $g_t(z) = \tau_t^G y_t(z)$, which implies $G_t = \tau_t^G Y_t$. Hence, we can write the resource constraint as

$$C_t + G_t = Y_t, \tag{4}$$

and the demand function for each differentiated good as $y_t^d(z) = [P_t(z)/P_t]^{-\theta} Y_t$.

Employment is subsidized at a constant rate $1-\tau_\mu$. All firms face a common real marginal cost, which in equilibrium is given by $mc_t = \frac{W_t/P_t}{A_t}\tau_\mu$.

Firms are monopolistic competitors and are allowed to change prices with a Calvo probability $1-\psi$.

Firms' objective function is to maximize expected profits discounted by a stochastic discount factor $\Lambda_{t,t+j}$. In general, this is a function of each individual discount factor $\Lambda_{t,t+j}^h$.[8] The optimal pricing policy is

[8] In principle, each household shareholder h would like to have firms maximize discounted profits using its own stochastic discount factor $\Lambda_{t,t+k}^h$. However, if managers have delegated a linear rule then, under the assumption of zero steady-state inflation, the optimal choice of the

$$\sum_{k=0}^{\infty}(\psi\beta)^k E_t \Lambda_{t,t+k} P_{t+k}^{\theta} Y_{t+k}[P_t^{h,\star}(z)/P_{t+k} - \frac{\theta}{\theta-1} mc_{t+k}] = 0. \tag{6}$$

It has a simple interpretation: Firms set prices at a level such that a (suitable) weighted average of anticipated future markups matches the optimal frictionless markup $\theta/(\theta-1)$.[9] A log-linear approximation of the optimal pricing delivers the standard relation between inflation and expected marginal costs that is at the heart of the New Keynesian Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_{mc} mc_t, \tag{7}$$

where $\kappa_{mc} \equiv (1-\psi)(1-\beta\psi)/\psi$ and π_t is the inflation rate.

2.3 The Government and the Monetary Authority

The government runs a balanced budget in each period and government consumption is financed with lump-sum taxes, $G_t = T_t^G$.

A constant redistribution scheme across households, τ, is implemented in period 0 such that we have $\int_0^1 \tau^h dh = 0$.

We assume that the monetary authority can control the (gross) funds rate, R_t. This is perfectly inversely related to the price of the nominal riskless portfolio described in section (2.1),

relative price $P_t^{h,\star}(z)$ is the same for all resetting firms and across all shareholders:

$$\log P_t^{h,\star}(z) = \frac{\theta}{\theta-1} + (1-\psi\beta)\sum_{k=0}^{\infty}(\psi\beta)^k E_t\left[\log(mc_{t+k}P_{t+k})\right]. \tag{5}$$

[9]For a derivation and interpretation of firms' optimality condition, see Woodford (2003) or Gali (2002), among others.

$$Q_t \propto 1/R_t.$$

We also assume that the zero lower bound on the nominal interest rate is never binding under the optimal policy regime.

2.4 Sources of Inefficiencies

The first source of inefficiency derives from the presence of infrequent price adjustments. All goods enter the utility function symmetrically and are produced with the same technology: Efficient would require allocating the same amount of resources to producing each good. However, when $\psi > 0$, a fraction of firms is committed to satisfying all the demand for a fixed, previously posted price. In the presence of inflation, this induces a misallocation of resources in the economy that can be captured by $\Delta_{p,t} = \int_0^1 [P_t(z)/P_t]^{-\theta} dz$, a measure of relative price dispersion.

To identify the second source of inefficiency, we introduce a term (an analog of $\Delta_{p,t}$) that captures the households' inability to insure their asset position against interest rate and inflation volatility. Taking the stand of debtors, we refer to the latter as volatility in the *refinancing cost*, which is low in period of high inflation and high in periods of high interest rates. When the refinancing cost is high, debtors' available resources decrease and debtors relatively increase their labor supply to smooth consumption over time. Hence, given that all households have the same preferences and ability, a measure of labor supply dispersion is a good candidate as a metric for households' current financial conditions:

$$\Delta_{n,t} \equiv \int_0^1 (N_t^h/N_t)^{-\varphi/\sigma} \geq 1. \tag{8}$$

This term appears in the aggregate consumption/leisure conditions and represents a shift in the labor supply.[10]

It is worth noting that, in the case of complete markets, hours dispersion would be a constant $\Delta_{n,t} = \overline{\Delta}_n$, households' consumptions are perfectly correlated, even though they may have different levels. This makes $\hat{\Delta}_{n,t} = \log(\Delta_{n,t}/\overline{\Delta}_n)$ a good measure to capture the implications of insufficient financial instruments.[11]

We refer to the output prevailing under flexible prices *and* complete markets as the *efficient* output, Y_t^e, to distinguish it from the output prevailing under only flexible prices, Y_t^f, which is monetary-policy dependent. The output gap is defined as the log difference of current output to efficient output, $x \equiv \log(Y_t/Y_t^e)$.

It is also useful to introduce the efficient rate r_t^e which is the ex ante real rate prevailing under flexible prices and complete markets. This is an exogenous process function of technology and government spending shocks.[12]

3 Welfare Analysis

In this section, we lay out the problem of a benevolent policymaker reacting to aggregate exogenous disturbances when the economy is populated by a continuum of households featuring a nondegenerate distribution over nominal asset holdings.

His objective is to maximize a welfare function $\mathbb{W}$ which aggregates agents'

[10]The optimal consumption/leisure choice of each household reads $C_t^h = W_t/P_t{}^{1/\sigma} N_t^{h-\varphi/\sigma}$. Integrating with respect to h and using *hats* for *logs* gives $\hat{W}_t - \hat{P}_t = \varphi\hat{N}_t + \sigma\hat{C}_t - \sigma\hat{\Delta}_{n,t}$

[11]In case of perfect wealth equality, we have $\overline{\Delta}_n = 1$.

[12]It can be easily shown that Y_t^e and Y_t^f have the following expressions: $\hat{Y}_t^e = \frac{\sigma}{\sigma+\varphi}\hat{g}_t + \frac{1+\varphi}{\sigma+\varphi}\hat{A}_t + \frac{\sigma}{\sigma+\varphi}\log\overline{\Delta}_n$ and $\hat{Y}_t^f - \hat{Y}_t^e = \frac{\sigma}{\sigma+\varphi}\hat{\Delta}_{n,t}$; while the efficient rate is $r_t^e = \frac{\sigma^2}{\sigma+\varphi}E_t\Delta\hat{g}_{t+1} + \sigma\frac{1+\varphi}{\sigma+\varphi}E_t\Delta\hat{A}_{t+1}$.

utilities $\mathbb{W}: \mathcal{U} \rightarrow \mathbb{R}$.[13]

$$\mathbb{W}_t = E_t \sum_{k=0}^{\infty} \beta^k \int_0^1 \eta(h)[u(C_t^h) - v(N_t^h)]dh, \tag{9}$$

where $\eta(h) : [0,1] \rightarrow \mathbb{R}^+$ represents a time-invariant weighting function.

When transfers are conveniently chosen (and the long-run inflation target is zero), the model economy oscillates around the *efficient* and *socially desirable allocation* for any arbitrary initial asset distribution and weighting function $\eta(h)$. This is a necessary condition for a direct derivation of a purely quadratic welfare-based loss function. In this case, without loss of generality, we weight every household the same, $\eta(h) = 1$, and choose $\sum_{j=0}^{\infty} \beta^j \overline{\tau}^h = \overline{b}^h$, $\forall h$. This last expression implies that, in the non-stochastic steady-state, the government transfer exactly offsets the asset position of each household, i.e., we impose steady-state wealth equality.

Alternatively, if no transfer scheme can be implemented, there is always a positive weighting function $\eta^{\star}(h)$ that can recover the solution of the complete-markets version of the model, where idiosyncratic risk is perfectly insured and consumptions are perfectly correlated across households.[14] This alternative would deliver the same results as those that we derive in the next sections.

Both approaches would make the central bank accept initial (and long-run)

[13] Qualitatively, the results do not depend on the welfare criterion chosen; in fact, the less utilitarian the welfare function, the stronger the results.

[14] The function $\eta^{\star}(h)$ can be calculated using the inverse of each household's initial marginal utility. We normalize this function such that we can use steady-state consumption, i.e., $\eta(h) = 1/u'(\overline{C}^h)$. To see this let $\tilde{\eta}(h) = 1/u'(C_0^h)$ and use the following normalization:

$$\eta(h) \equiv \tilde{\eta}(h)\frac{u'(C_0^h)}{u'(C^h)} = \tilde{\eta}(h)\frac{u'(C_0)u'(\delta(h))}{u'(C)u'(\delta(h))} = \tilde{\eta}(h)\frac{u'(C_0)}{u'(C)}$$

.

wealth inequality. Loosely speaking, this is equivalent to a monetary authority that accepts the wealth distribution *in statu quo nunc*.

3.1 The Welfare-based Loss Function

We now present the second-order approximation of the policy objective, equation (9), about the deterministic Ramsey steady-state, i.e., the efficient and socially desirable allocation.[15] Details of the derivation can be found in appendix Appendix A; here, we simply claim the result:[16]

$$\mathbb{W}_t \simeq E_t \sum_{k=0}^{\infty} \beta^k \mathbb{L}_t, \tag{10}$$

where

$$\mathbb{L}_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_c \int_0^1 (\tilde{C}_t^h)^2 + o(\|S_{t-1}\|^2). \tag{11}$$

The presence of staggered prices introduces gains from minimizing inflation and the output gap. Relative price dispersion implies a misallocation of resources that is captured by the term $\pi_t^2 + \lambda_x x_t^2$: price rigidities generate no trade-off between output gap and inflation stabilization.[17]

The third *target variable* that enters the loss function is the cross-sectional consumption dispersion. This term induces a trade-off between inflation/output

[15] It can be proved that optimal policy would choose a non stochastic steady-state with $\sum_{j=0}^{\infty} \beta^j \overline{\tau}^h = \overline{b}^h$ and zero relative price dispersion. See Woodford and Benigno (2003) for a proof of the optimality of zero inflation in a sticky-price model.

[16] The approximation error is strictly related to our variables' deviations from their steady-state values, and $\|S_{t-1}\|$ represents a bound on the amplitude of exogenous shocks and the deviations of the time t state.

[17] The relative weight on the output gap is $\lambda_x \equiv (\sigma+\varphi)\frac{\kappa_{mc}}{(1+\theta\varphi)\theta}$. We also notice that the difference between the square of the *natural* and *efficient* output gap is of an order higher than second. Finally, the weight on consumption-dispersion is $\lambda_c \equiv \kappa_{mc}\frac{\sigma}{\theta}(1-\overline{\tau}^G+\sigma/\varphi)$.

gap stabilization and the wealth redistribution effect of monetary policy. As shown in section (2.4), redistribution is captured by the volatility of the labor supply dispersion $\hat{\Delta}_{n,t}$. In fact, the following approximation holds

$$\int_0^1 \tilde{C}_t^{h^2} \simeq 2\frac{\varphi}{\sigma}\hat{\Delta}_{n,t}. \tag{12}$$

Rewriting the loss function in terms of $\Delta_{n,t}$ would imply a weight $\lambda_\Delta \equiv 2\kappa_{mc}[\sigma + (1-\tau^G)\varphi]$. This has a very simple interpretation: The stricter the concavity of the household utility function (higher risk aversion and/or lower labor elasticity) the greater the weight a social planner should put on wealth redistribution effects.[18]

4 Optimal Monetary Policy

In this section, we analyze in greater detail the role played by monetary policy. We show that, even if the monetary authority accepts the initial wealth distribution, the central bank still plays a crucial role in offsetting the potential redistributive impact of *aggregate* shocks on the household budget constraint. Moreover, we will clarify why households' stock of assets becomes a source of idiosyncratic uncertainty, which, in turn, is the source of volatility for our distortion $\Delta_{n,t}$.

4.1 The Private Sector Equilibrium System

Having a purely quadratic loss function, we can evaluate optimal policy using the first-order Taylor approximation of the private sector problem presented in section

[18]The steady-state level of government consumption decreases the weight simply because it lowers the steady-state level of private consumption.

2.[19]

We define assets' deviation from their long-run values as $\tilde{b}_t^h \equiv b_t^h - \overline{b}^h$; the consumption and employment cross-sectional gap is $\tilde{C}_t^h \equiv \log C_t^h / C_t$ and $\tilde{N}_t^h \equiv \log N_t^h / N_t$; exploiting that $\overline{\tau}^h = -\overline{b}^h (1-\beta)$, we can reformulate the agent h budget constraint in deviations from average quantities (see Appendix D) as

$$\kappa_c \tilde{C}_t^h = \tilde{b}_{t-1}^h - \beta \tilde{b}_t^h + \overline{b}^h \xi_t, \tag{13}$$

where $\kappa_c = 1 + \frac{\sigma}{\varphi}$ and $\xi_t \equiv \beta \hat{R}_t - \pi_t$ is (the log-deviation of) the unit real refinancing cost, *extra-refinancing* cost. As we mentioned earlier, it should be read as a cost for a debtor to keep his today's (negative) fixed-income real financial wealth constant when the interest rate rises; note that $\hat{Q}_t = -\beta \hat{R}_t$. For a creditor, it would more properly be read as a reinvestment cash flow.[20]

Hence, deviations of household h consumption from average consumption, $\tilde{C}$, are due either to changes in the real asset positions $\tilde{b}_{t-1}^h - \beta \tilde{b}_t^h$ or to changes in financial income $\overline{b}^h \xi_t$.[21][22] The latter is the channel through which aggregate uncertainty introduces idiosyncratic risk at the household level.

[19] *Caveat emptor*: A local approximation of this model may not be accurate. Prices are not affected by the evolution of the wealth distribution. However, three points make us believe this should not be a major concern: There are no reasons to have kinks in the policy functions—the natural borrowing constraint should never be binding; there are no purely idiosyncratic exogenous shocks—the *ranking* of households across asset holdings is constant over time; and, finally, for the flexible-price case, as we show in the appendix, the solution of the full-fledged model delivers the same outcome as the approximated model. For further readings, see Krusell and Smith (1997), Den Haan (1997) and Young (2005).

[20] More precisely, it would represent the extra real investment income netted from the "reinvestment" necessary to keep the real stock of assets constant.

[21] More precisely, $\tilde{b}_{t-1}^h - \beta \tilde{b}_t^h$ is the real cash flow change derived by changing the real asset position (the dimension of the portfolio), keeping real financial income constant at its long-run value $1-\beta$.

[22] More precisely, ξ refers to real cash flow changes stemming from changing returns but holding the total stock of assets constant at its long-run value. Hence, ξ measures the unit impact of a change in returns.

We conclude the description of the system by taking a log-linear expansion of the households' Euler equation in deviation from aggregate levels,

$$E_t \Delta \tilde{C}^h_{t+1} = -\varphi_b \tilde{b}^h_t. \tag{14}$$

Equation (14) introduces the *ad hoc* term $\varphi_b \tilde{b}^h_t$ with $\varphi_b > 0$. It captures the quasi-random-walk behavior of b_t in the original non linear model, which disappears under a local approximation. Moreover, it can be interpreted as a convex financial adjustment cost that further reduces the households' ability to self-insure. In any case, $\varphi_b > 0$ can be taken arbitrarily small (see section 5 on calibration).[23]

Thanks to linearity, we can close the system using the aggregate Euler equation and the Phillips curve:

$$\sigma E_t \Delta x_{t+1} = \hat{R}_t - E_t \pi_{t+1} - r^e_t \tag{15}$$

and

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{16}$$

where $\kappa \equiv (\sigma + \varphi)\kappa_{mc}$.

4.2 The Monetary Policy Problem

This section examines the policy problem that the monetary authority faces in committing to a state-contingent path for the short-term rate $\{R_t\}_{t=0}^{\infty}$.

[23] Alternatively, this can be microfunded by imposing small quadratic adjustment costs on debt transactions. For a related discussion see Schimtt-Grohe and Uribe (2003). See also Kim, Kim and Kollmann (2005) on barrier methods to convert an optimization problem with borrowing constraints as inequalities into a problem with equality constraints, and then solving the converted model using a local approximation.

We assume that the central bank has full information in setting its instrument. The information available at time t is captured by the all-relevant-time-t state of the economy: The exogenous process r_t^e (which captures all exogenous uncertainty), the set of endogenous state $(\tilde{b}_{t-1}^h)_{h\in[0,1]}$, and a set of costates, denoted $\mathcal{L}_t$, associated with the constraints introduced for satisfying equilibrium conditions dated $t < 0$.

Hence, the central bank's problem is to choose processes $\{\pi_t, \hat{R}_t, (\tilde{C}_t^h)_{h\in[0,1]}, (\tilde{b}_t^h)_{h\in[0,1]}\}_{t\geq 0}$ to minimize (10) subject to constraints (A-29), (14), (15), (16) for every $t \geq 0$, given initial conditions $(\tilde{b}_{-1}^h)_{h\in[0,1]}$ and the evolution of the exogenous shock $\{r_t^e\}_{t\geq 0}$.[24]

$$
\begin{aligned}
\min \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \lambda_x x_t^2 + \lambda_c \int_0^1 \tilde{C}_t^{h^2} \right) \\
s.t. \quad & \pi_t = \beta E_t \pi_{t+1} + \kappa x_t \\
& \sigma E_t \Delta x_{t+1} = \hat{R}_t - E_t \pi_{t+1} - r_t^e \\
& \kappa_c \tilde{C}_t^h = -\beta \tilde{b}_t^h + \tilde{b}_{t-1}^h + \overline{b}^h (\beta \hat{R}_t - \pi_t), \ \ \forall h \in [0,1] \\
& E_t \Delta \tilde{C}_{t+1}^h = -\varphi_b \tilde{b}_t^h, \ \ \forall h \in [0,1] \\
& (\tilde{b}_{-1}^h)_{h\in[0,1]} \ \text{ given.}
\end{aligned}
$$

The system of necessary conditions is shown in the appendix. Here, it is worth introducing the following definitions:

[24] We do not necessarily need to assume that the optimal policy honors commitments made in the past—referred to as *timeless perspective*. Whenever $(\tilde{b}_{-1}^h)_{h\in[0,1]} = (0)_{h\in[0,1]}$, there is no advantage that monetary authority wants to exploit at time-0. In a closely related setup, Khan et al. (2003) introduce, in the standard (unconstrained) Ramsey problem, lagged Lagrange multiplier corresponding to the forward-looking constraints in the initial period, making the problem stationary. The initial values are chosen to be the steady-state values. For a discussion, see also Benigno and Woodford (2005).

- the initial debt dispersion: $\zeta_b^2 \equiv \int_0^1 \bar{b}^{h^2} dh$
- the consumption-debt covariance: $w_t \equiv \int_0^1 \bar{b}^h \tilde{C}_t^h dh$
- additional debt dispersion: $z_t \equiv \int_0^1 \bar{b}^h \tilde{b}_t^h dh$.

We can express the consumption-debt covariance in terms of its correlation $w_t = \rho_t^{\tilde{c}\bar{b}} \zeta_b \sqrt{Var_t(\tilde{C})}$ or, as in section 3.1, in terms of hours dispersion $w_t \simeq \rho_t^{\tilde{c}\bar{b}} \zeta_b \sqrt{2\varphi\hat{\Delta}_{n,t}/\sigma}$. Furthermore, notice that, given the nature of our "idiosyncratic" shocks, the household *ranking* across assets has no reason to change over time. This means that, at each point in time, there is a monotonic relation between $\tilde{C}$ and $\bar{b}$. The relation takes a positive (negative) sign when the realized and expected refinancing costs, ξ, are penalizing the group of debtors (creditors). Hence, we may want to write $w_t/\zeta_b \simeq sign(\rho_t^{\tilde{c}\bar{b}}) \sqrt{2\varphi\hat{\Delta}_{n,t}/\sigma}$. This tells us that the volatility of w_t is an important statistic for the welfare impact of changes in the refinancing costs, ξ_t, and we are going to show that these two variables are strictly related.

Taking a linear combination of budget constraints (A-29) and Euler equations (14), and using the previous definitions, we can write

$$\kappa_c w_t = -\beta z_t + z_{t-1} + \zeta_b^2 \xi_t, \tag{17}$$

$$E_t \Delta w_{t+1} = -\varphi_b z_t. \tag{18}$$

The system can be solved forward

$$w_t/\zeta_b = (1-\alpha\beta)[z_{t-1}/\zeta_b + \zeta_b \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \xi_{t+1+j}], \tag{19}$$

$$z_t/\zeta_b = \alpha[z_{t-1}/\zeta_b - \zeta_b \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \Delta\xi_{t+1+j}], \tag{20}$$

where $\alpha \leq 1$ is a function of the structural parameters.[25]

The above equations also show that the systematic component of monetary policy has redistributive effects on welfare. Monetary policy may affect household wealth through current and future changes in the refinancing cost, ξ. In particular, the lower is $1-\alpha\beta$ (ability to self-insure) and the higher is ζ_b (initial asset dispersion), the larger is the impact of ξ's volatility on w_t/ζ_b.

The variable z_t captures how the asset distribution is changing over time with respect to the initial one. A positive (negative) value for z_t means that on average, households that started with a debt (credit) have been worsening (improving) their asset position even further. Moreover, $1-\alpha\beta > 0$ implies that z_{t-1} is positively correlated with w_t, i.e., households that have accumulated a stock of debt that exceeds their long-run average are experiencing below average consumption.

4.3 A Special Case: The Flexible-Price Environment

When prices are perfectly flexible, $\psi = 0$, there is no relative price dispersion, $\Delta_{p,t} = 1$; hence, inflation and output gap drop out of the loss function, which simplifies to $L_t = \int_0^1 \tilde{C}_t^{h^2}$.

The Phillips curve (16) is no longer well defined, given that marginal costs

[25] We also have that $\lim_{\varphi_b \to 0} \alpha = 1$ and $\partial\alpha/\partial\varphi_b < 0$

are constant. The deviation of flexible-price output from the efficient is relevant for welfare but not for first-order dynamics. This means that the IS equation (15) simply says that the ex ante real rate is exogenous: $\hat{R}_t - E_t\pi_{t+1} = r_t^e$.

We state the following proposition (a proof is given in the appendix):

Proposition 1 *In a flexible-price environment, where the only distortion is created by wealth dispersion, optimal monetary policy is given by a state-contingent path for inflation*

$$\pi_t = \beta E_t \sum_{j=0}^{\infty} \beta^j r_{t+j}^e + \frac{z_{t-1}}{\zeta_b^2}, \ \forall\, t \geq 0, \tag{21}$$

which implies a targeting rule [26]

$$\hat{R}_t = \frac{\pi_t}{\beta} + \frac{z_t - z_{t-1}/\beta}{\zeta_b^2}, \ \forall\, t \geq 0. \tag{22}$$

The optimal interest rate reaction is a function of inflation, π_t, and debt dispersion, z_t. The coefficient on inflation, being of order 1.01, satisfies the Taylor principle but is much smaller than standard prescriptions. The potentially redistributive effects of persistent expected increases in the exogenous real rate r_t^e are offset by a rise in current inflation, which reduces the real stock of debt. The *persistence* and *volatility* of r_t^e determine the magnitude of the rise in inflation.

Current inflation also optimally reacts to the additional-debt-dispersion z_{t-1}/ζ^2. A positive value would rise inflation aimed to reduce the "excess" real stock of debt. [27] This means that the central bank, looking at z_t, is able to identify which of those two groups, debtors or creditors, is experiencing a wealth increase.[28]

[26] For a definition of targeting rules, see Svensson (1999) or Giannoni and Woodford (2002).

[27] Recalling the definition of z_t, the steady-state dispersion ζ_b^2 in the policy rule can be interpreted as a scaling parameter.

[28] We also notice that the optimal rule, given any z_{-1}, would always imply $z_t = w_t = 0\ \forall t \geq 0$.

5 Calibration

In this section, we calibrate the structural parameters of the model.

The first part of this section is focused on determining the initial distribution of nominal fixed-income assets across households, which our model takes as given. The objective is twofold: to determine the net asset position of each household necessary to evaluate the welfare loss and to calibrate the asset-dispersion parameter ζ_b.

The rest of the parameters represent preferences and technology, and are calibrated following the conventional approach of the business cycle literature.

5.1 Household-Level Data

We calibrate the debt dispersion parameter $\zeta_b^2 \equiv \int_0^1 \overline{b}_{-1}^{h^2} dh$ using microdata from the Federal Reserve Board's *Survey of Consumer Finances* (SCF) for 2004. We calculate the net asset position for each household in the survey. The gross credit position is computed as the sum of the following items: Money market accounts, saving and call accounts, CDs, directly held pooled investment funds, saving bonds, directly held bonds, and quasi-liquid retirement accounts.[29] On the other hand, we proxy the liabilities as the sum of debt secured by primary residence and other residential property, other lines of credit, credit card balances after last payment, installment loans, and other debt.[30] The net debt is given by the algebraic difference between the credit and debit gross positions. The inclusion of

[29]The SCF codes are MMA, SAVING, CALL, CDS, NMMF, SAVBND, BOND, and RETQLIQ, respectively.

[30]The SCF codes are NH_MORT, HELOC, RESDBT, OTHLOC, CCBAL, INSTALL, and ODEBT, respectively.

Table 1. Second moment of the net fixed-income assets to yearly income distribution

Year	SCF weights ζ_b^2	Sampling ζ_b^2
2004	18.31	15.77
2001	13.12	11.24
1998	17.12	13.94
1995	15.67	12.73
1992	11.75	8.25
1989	6.99	6.65

In the first column we use directly the weights attached to each observation given by the SCF. In the second column we sample 1,000 households using the normalized weights as probabilities. All zero income observations have been dropped.

quasi-liquid retirement accounts substantially increases the first moment of the distribution but it does not fundamentally alter its shape.

Consistently with the model, we divide the net asset position by the total household income.[31] Table 1 reports the second moment of the net-assets-to-income distribution during the last six survey years. This is calculated by using both the weights assigned to each household by the SCF (first column) and by sampling 1,000 households from the survey, using the normalized weights as probabilities (second column). Because the total sample of the survey is around 4,000 observations, the second method smoothes outliers.[32] In any case, the dispersion increases across years. This is described in greater detail in table 2, where the percentiles of the asset-income distribution are shown across years.

[31] We dropped zero-income observations, which ranged from 0.5% to 1% of all respondent households during the survey years.

[32] All the tables show the stock of assets over *yearly* income, whereas the model's simulations and exercises consistently use assets over *quarterly* income.

Table 2. Percentiles of the net fixed-income assets to yearly income distribution over time

Percentiles	2004	2001	1998	1995	1992	1989
10%	-2.58	-1.77	-1.82	-1.86	-1.60	-1.41
20%	-1.43	-0.91	-0.98	-0.96	-0.87	-0.76
30%	-0.68	-0.40	-0.49	-0.45	-0.39	-0.36
40%	-0.28	-0.12	-0.16	-0.17	-0.13	-0.10
50%	-0.06	0.00	0.00	-0.01	-0.01	0.00
60%	0.01	0.06	0.03	0.02	0.01	0.01
70%	0.18	0.38	0.30	0.20	0.17	0.13
80%	0.68	0.93	0.90	0.61	0.59	0.50
90%	2.34	2.99	2.43	1.97	1.82	1.74
99%	12.21	10.98	10.31	10.12	8.50	8.43

We sample 1,000 households using normalized weights as probabilities. All zero-income observations have been dropped. Households surveyed for 2004 numbered 4,498, excluding those with zero income.

5.2 Preferences and Technology Parameters

As is common in the business cycle literature, we let the relative risk aversion and the inverse of the Frisch elasticity parameters take values in the following range: $\sigma \in [1,5]$ and $\varphi \in [0,3]$.

The time is one quarter, and we assign a value of 0.9902 to the subjective discount factor β, which is consistent with an annual real interest rate of 4% (see Prescott, 1986).

We set the steady-state share of government purchases at $\overline{\tau}^G = 20\%$, matching the U.S. historical experience in the postwar period. Following Sbordone (2002) and Clarida, Gali and Gertler (1999), we assign a value of 2/3 to ψ, the fraction of firms that cannot change their price in any given quarter. This value implies that on average, firms change prices every three quarters. The price elasticity of the demand θ is set to 11 such that the steady-state markup is 10%.

We set the financial adjustment cost parameter to $\varphi_b = 10^{-6}$. A possible functional form for the adjustment cost is quadratic $\varphi_b(b_t - b_{-1})^2/2$. In this case, a household with an initial (yearly) debt-to-income ratio of 4 that decides to repay all its debt in a given period would incur a financial cost of about 0.0004% of its income. For the median-income household, this would mean about \$0.12. For $\varphi_b = 10^{-4}$ this is about \$12. We believe those numbers are still relatively small.

We estimate the parameters of the the driving processes A_t and g_t, fitting an AR(1) process for labor productivity and real government consumption expenditure quarterly data series from 1990 to 2006.[33] We find the point estimates of the persistence parameters to be $\rho_a = .89$ and $\rho_g = .94$, respectively, while the standard deviations of the correspondent innovations are $\sigma_a = .00670$ and $\sigma_g = .00164$. The two processes are assumed to be uncorrelated. Table 1 summarizes all the parameters just described.

6 Results

6.0.1 Model Dynamics under the Optimal Rule

We now analyze the optimal responses to a transitory disturbance in the level of productivity and government spending, which is summarized in the reaction of the *efficient* rate, r_t^e.

In the baseline sticky-price model without asset dispersion, the stabilization policy would be straightforward: tracking the natural rate and closing all the gaps.

[33] More precisely, the labor productivity is the SA nonfarm business output per hour, while government expenditure is a fraction of total consumption. The productivity and government spending series are detrended using a band-pass filter (6,32) and a cubic trend, respectively. The 90% confidence bands are relatively tight.

Table 3. Structural Parameters

Parameter	Value	Description
β	.9902	Subjective discount factor (quarterly)
σ	2	Relative risk aversion
φ	.1	Frisch elasticity
θ	11	Price elasticity of demand for a good-variety
μ	.10	Firms' markup
ψ	.75	Fraction of non-resetter firms
$\overline{\tau}^G$	.20	Steady-state value of government consumption over GDP
ζ_b	16.66	Fixed-income asset dispersion
ρ_A	.89	Serial correlation of (log) of technology process
ρ_G	.94	Serial correlation of (log) of government spending process
σ_A	.00670	Std. dev. innovation to (log) of technology
σ_G	.00164	Std. dev. innovation to (log) of government consumption
ϕ_b	1E-6	Bonds adjustment costs

However, as figure 1 shows, the higher the debt dispersion ζ_b, the bigger the deviation from price stability.[34]

When a positive persistent shock to r_t^e hits the economy, indebted households, anticipating higher real rates in the future, reduce their consumption below the average, $w_t > 0$. To mitigate this effect, optimal policy aims to reduce the impact-response of real *extra-refinancing* cost ξ_t by reducing the funds rate and letting inflation raise. In fact, at the time of the shock's impact, the nominal interest rate does not move together with the natural rate; the reaction is much smaller. We have an inversion of sign for the baseline calibration, i.e., for $\zeta_b \in [15.77, 18.31]$: The nominal rate decreases at the time of the shock (in both cases the interest rate gap $R_t - r_t^e$ is negative). On the other hand, the (log) price level drifts away and

[34]We let the debt-dispersion parameter take two values: Our calibrated value 16.66 (high) and a "trimmed" value 2.34 (low), which are the solid lines and diamonds, respectively, in figure 1. The low value is determined by trimming the tails (the lowest and highest 5%) of the distribution.

converges at a higher value, implying a permanent effect on the real stock of debt.

The initial reduction in the refinancing cost generates a windfall financial income that allows the debt-covariance to be negative at impact, $z_0 < 0$: Indebted households reduce their stock of real debt. However, given that price dispersion is a social cost, this favorable condition lasts for one period; in order to smooth their consumption, debtors will start accumulating new debt in an effort to reduce the impact of higher interest payments on consumption.

Figure 2 plots simulated series for the refinancing cost and efficient rate, together with the disaggregate series for debt deviations from the long-run level, $\tilde{b}_t$, debt in level b_t and consumption log-deviations from average consumption $\tilde{C}_t$, for a subset of the households surveyed in the 2004 SCF.[35] At the disaggregate level the previous results are also confirmed: Periods with a positive extra-refinancing cost imply a further accumulation of debt (credit) for debtors (creditors) and, given market incompleteness, a relatively lower (higher) consumption.

In fact, the distribution of net assets and consumption across households flattens during periods of higher refinancing costs. Figure 3 shows how the estimated density functions of the simulated series (figure 2) vary over time in periods of high refinancing costs.[36] Both distributions' tails increase remarkably.

In figure 2, the refinancing cost shows much less persistence than the efficient rate, even though it is more volatile. The monetary authority tries to strike a balance between stabilizing prices and reducing debt-servicing volatility, ξ; it

[35] More precisely, we draw 1200 households using as probabilities the normalized weights that the survey attributes to each entry. We then plot a subset of 30 households. Plotting a higher number of households would not add further insights.

[36] For clarity, we take a window of time between period 140 and period 250. In that range, the average refinancing cost is close to 0.005. This is equivalent to an annual real rate of 5%, which is 2% higher than the 3% long-run rate. For these plots, we use all the 1,200 households sampled.

may well be that reducing the volatility of ξ is too costly in terms of inflation stabilization. In this case, it is still possible to reduce the *persistence* of ξ and increase welfare (see also equation 19). This result also appears in table 2, where the estimated autocorrelation coefficient for ξ is almost zero, while under price stability it would be as high as 0.86. The reason, as we know from the incomplete markets literature, is that a lower persistence of idiosyncratic shocks enhances households' ability to self-insure.[37] Hence, optimal policy achieves a Pareto improvement through a drastic reduction in the autocorrelation of the refinancing cost, improving households' ability to smooth consumption over time.

6.0.2 Optimal Simple Rules

To give practical, implementable policy advice, we also study optimal simple rules, restricting monetary policy rules to a class of "simple" functional forms

$$\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t. \tag{23}$$

The above rule dictates a reaction of the nominal rate to the lagged nominal rate and inflation, both of which are easy to observe.

We maximize with respect to the coefficients of the rule, η_r and η_p, over a grid.[38] Under the baseline calibration, the optimal simple rule has a *superinertial* component, i.e., $\eta_r > 1$, while still showing a strong reaction to inflation (see figure 4A). The same is true when we increase the cost of renegotiating the ini-

[37] See Hugget (1993) and Marcet and Singleton (1999), among others.

[38] The initial grid is $[0,3] \times [1,300]$ for η_r and η_p, respectively. For that range, no indeterminacy issues arise. We subsequently have a finer grid on a neighborhood of the optimum previously found. We distinguish for two cases: the baseline $\phi_b = 1E-6$ and $\phi_b = 1E-4$. The finer grids are $[0.90, 1.25] \times [3.00, 6.00]$ and $[0.90, 1.25] \times [1.80, 4.00]$, respectively.

tial debt, ϕ_b; however, the reaction to inflation is smaller. The higher the cost of changing the asset position, the greater the burden of readjustment borne by consumption relative to bond holdings. Table 3 summarizes the results.

Thanks to superinertiality, the optimal simple rule can closely replicate the system's response to a real rate shock under the optimal rule (see figure 1 and figure 5). This is also confirmed by the welfare analysis of the next section. A reaction greater than one to the previous-period rate, R_{t-1}, allows the monetary authority to have a hump-shaped path for the interest rate. This reduces the volatility of interest-rate disbursement and, at the same time, promising even more diverging rates in the future, can quickly push inflation toward zero.

Contrary to the optimal rule, the best simple rule maximizes welfare through a reduction in the refinancing cost's *volatility* (see table 2). In fact, no simple functional form can reduce the autocorrelation coefficient of ξ, which now is even higher than the efficient rate.

6.0.3 Welfare Comparison

The following section compares alternative policy rules in terms of *unconditional* expected welfare.[39] Table 4 defines the rules compared; tables 5 and 6 rank each rule according to its welfare score.[40]

[39] To compute it, we generate 200 paths for the endogenous variables over 220 quarters, discarding the first 100 (i.e., 40 years) and then compute the average loss across all simulations. All initial aggregate-state variables are set to zero. To calculate the consumption dispersion, we draw 1,200 households' initial net-asset position from the 2004 SCF, and then center the distribution on zero.

[40] In principle, we could have drawn a sample equal in size to the U.S. household population. This has not been done for two reasons: One is simply computational; the second is that a number of households, possibly higher than that of the SCF sample, would increase the probability of having outliers in the welfare calculations. Hence, selecting a higher number of households would most probably strengthen the results. On the other hand, using too few households for welfare calculations (say, less than 200) has been found to strongly underestimate the welfare loss related

Under the baseline calibration (see table 5), the optimal rule (GOMP) and the optimal simple rule (OSR) give similar results: The percentage loss of the estimated rule with respect to the optimal is around 25%. Pure inflation targeting would be fairly suboptimal, with a loss 50% higher than the optimal. In this case, inflation and the output gap are perfectly stabilized (see the relevant rows in table 5), but at the cost of a much larger variation in w_t and z_t, which in turn represent the $var_h(\tilde{C}_t)$ and so the welfare loss from consumption dispersion. On the other hand, a rule showing a standard reaction of 1.5 to inflation and of 0.5 to output gap (TR) would imply less redistribution but too much inflation variability. In this case, the welfare loss would be several times the optimal.

The magnitude of the losses can be approximately expressed in terms of steady-state aggregate consumption (see second row of table 5). If we take aggregate consumption to be $9 trillions for a population of 300 million, the loss of business cycles under the optimal rule would be about $2 per capita, whereas under the IT rule it would be about $3. Only under the TR rule could it reach something near $25.

6.0.4 Optimal Inflation Volatility

We try to summarize all the results calculating the implied optimal inflation volatility that we would obtain under the baseline calibration and optimal policy. As shown in table 1, the model is able to generate almost 20% of the inflation volatility of the last 15 years. This number is further increased to 30% when financial frictions are relatively higher, $\phi_b = 1E-4$.

to consumption dispersion.

Table 1: Optimal Inflation Volatility

	Model	Data	Ratio
GOMP ($\varphi_b = 1E-4$)	0.1184	0.4342	27.26%
GOMP ($\varphi_b = 1E-6$)	0.0832		19.17%
OSR ($\varphi_b = 1E-4$)	0.0784		18.05%
OSR ($\varphi_b = 1E-6$)	0.0503		11.59%

The inflation volatility in the data has been calculated taking the standard deviation of quarterly, annual rate, data on the Cleveland Federal Reserve Bank's trimmed-mean CPI inflation rate from 1992 to 2006. For the same period, the CPI all-items inflation volatility was 1.52

7 Conclusion

Most of the results in the recent monetary policy literature have been derived under the assumption of a representative household or complete markets. The present paper relaxes these assumptions and shows how market incompleteness renders households vulnerable to changes in interest rates and inflation when they different types of portfolios of nominal fixed-income assets.

An implication is that business cycle fluctuations—in aggregate economic activity and in the price level—endogenously introduce idiosyncratic uncertainty at the household level. In other words, economywide aggregate disturbances generate unwarranted redistributive patterns across agents that we are welfare reducing.

In this new scenario, we show that the standard recommendation of price stability is no longer optimal. In fact, in the presence of zero inflation, the nominal rate would absorb all business cycle fluctuations, implying a highly volatile and persistent process for interest payments. The main result is that systematic monetary policy can achieve a Pareto improvement by reducing either interest payments volatility or persistency, or both.

We calculate the magnitude of the deviation from price stability through a calibration exercise and we show that the optimal inflation volatility would be equal to almost 20% of the observed inflation volatility of the last 15 years.

Finally, when simple-implementable-rules are analyzed, superinertial rules outperform other rules. In fact, superinertial rules can generate hump-shaped interest rate responses to shocks that, in turn, reduce interest rate and interest payments volatility.

References

[1] A. Akyiol. Optimal monetary policy in an economy with incomplete markets and idiosyncratic risk. *Journal of Monetary Economics*, 51:349–398, 2004.

[2] S. Albanesi. Inflation and inequality. *Journal of Monetary Economics*, 54:4, May 2007.

[3] P. Benigno and M. Woodford. Inflation stabilization and welfare: The case of a distorted steady state. *Journal of the European Economic Association*, 3(4):1–52, December 2005.

[4] M. Doepke and M. Schneider. Redistribution of nominal wealth. *Journal of Political Economy*, 114(6), December 2006.

[5] J. Gali. New perspectives on monetary policy, inflation, and the business cycle. *NBER WP8767*, 2002.

[6] R. Clarida J. Gali M. Gertler. The science of monetary policy: A new keynesian perspective. *Journal of Economic Literature*, Vol. XXXVII, 1999.

[7] M. Giannoni and M. Woodford. Optimal interest-rate rules: I. general theory. *NBER Working Paper no. 9419*, December 2002.

[8] M. Giannoni and M. Woodford. Optimal interest-rate rules: Ii. applications. *NBER Working Paper no. 9420*, December 2002.

[9] W. Den Haan. Solving dynamics models with aggregate shocks and heterogenous agents. *Macroeconomics Dynamics*, 1997.

[10] M. Huggett. The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, 17(5-6):953–69, Sept.-Nov. 1993.

[11] M. Iacoviello. House prices, borrowing constraints and monetary policy in the business cycle". *American Economic Review*, 95(3):739–764, June 2005.

[12] M. Iacoviello. Household debt and income inequality, 1963-2003. mimeo Boston College, 2006.

[13] P. Krusell A. Smith Jr. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5).

[14] A. Khan, R. King, and A. Wolman. Optimal monetary policy. *Review of Economic Studies*, 70:825–860, 2003.

[15] R. E. Jr. Lucas and N. Stokey. Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, 12:55–93, 1983.

[16] A. Marcet and K. Singleton. Equilibrium equilibrium asset prices and savings of heterogeneous agents in the presence of incomplete markets and portfolio constraints. *Macroeconomic Dynamics*, 3(Vintage Article):243–76, June 1999.

[17] B. McCallum. Monetary policy analysis in models without money. *Federal Reserve Bank of St. Luis*, pages 145–160, 2001.

[18] J. Miao. Competitive equilibria of economies with a continuum of consumers and aggregate shocks. *Journal of Economic Theory*, In press.

[19] J. Kim R. Kollmann and S. Kim. Applying perturbation methods to incomplete market models with exogenous borrowing constraints. working paper, February 2005.

[20] J. Rotemberg and M. Woodford. *in B. Bernanke and J. Rotemberg (eds.), NBER Macroeconomics Annual 1997*, chapter An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy. MIT Press, Cambridge, MA, 1997.

[21] J. Rotemberg and M. Woodford. *in J.Taylor (ed.), Monetary Policy Rules*, chapter Interest Rate Rules in an Estimated Sticky Price Model. University of Chicago Press, Chicago, IL, 1999.

[22] A. Sbordone. Prices and unit labor costs: A new test of price stickiness. *Journal of Monetary Economics*, 49(2):265–92, March 2002.

[23] S. Schmitt-Grohe and Martn Uribe. Closing small open economy models. *Journal of International Economics*, 61:163–185, 2003.

[24] L. Svensson. Inflation targeting as a monetary policy rule,”. *Journal of Monetary Economics*, 43(607-54), 1999.

[25] M. Woodford. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, 2003.

[26] E. Young. Approximate aggregation. University of Virginia, mimeo, 2006.

Appendices

Appendix A Some Results

In a second-order approximation, for any variable $x \in \mathbb{R}_+$ and $\overline{x} \in \mathbb{R}_+$

$$\frac{x-\overline{x}}{\overline{x}} \simeq \hat{x} + .5\hat{x}^2 \tag{A-1}$$

$$\left(\frac{x-\overline{x}}{\overline{x}}\right)^2 \simeq \hat{x}^2, \tag{A-2}$$

where $\hat{x} = \log(x/\overline{x})$.

Given a function of the kind $f(x,y) = xg(y)$, with $y \in \mathbb{R}_+$, $g(.)$ twice differentiable and $\overline{x} = 0$,

$$f_y(\overline{x},y) = \overline{x}g'(y) = 0,$$

$$f_{yy}(\overline{x},y) = \overline{x}g''(y) = 0.$$

This means that if we take the second-order approximation of f about $(\overline{x},\overline{y})$ $\forall\overline{y}$, we find that

$$f(x,y) \simeq g(\overline{y})x + g'(\overline{y})x(y-\overline{y}) = g(\overline{y})x + \overline{y}g'(\overline{y})x\hat{y}. \tag{A-3}$$

To calculate the effect of price and output dispersion on overall output, we use the following result for a household or firm variable, say $x(h)$, in deviation from its average value, $x \equiv \int_0^1 x(h); dh$:

$$\int_0^1 \log(x_t(h)/x_t) \simeq -0.5 \int_0^1 \left(\frac{x_t(h) - x_t}{x_t}\right)^2. \tag{A-4}$$

This also means that

$$\int_0^1 \hat{x}_t(h) - \hat{x}_t \simeq -0.5 \int_0^1 (\hat{x}_t(h) - \hat{x}_t)^2. \tag{A-5}$$

We note that the first-order effect is zero.

In relation with the previous result, if we let $x_t(h) = X_t(h)/X_t$ and we have $\bar{x} = 1$ and $\int_0^1 x_t(h) = 1$, then $\Delta_{x,t} = \int_0^1 x_t^{\alpha}(h)$ can be approximated as

$$\hat{\Delta}_{x,t} = \log \Delta_{x,t} \simeq -0.5\alpha \int_0^1 \hat{x}_t^2(h). \tag{A-6}$$

Appendix B Recursive Equilibrium

Let $Z_t = (A_t, \tau_t^G)$ be the vector of exogenous economy wide stochastic processes and Φ_t be the measure (cumulative distribution) of households over asset holdings at time t. The law of motion concerning Φ_t is described by the function $f(.)$, such that $\Phi_t = f(\Phi_{t-1}, Z_t)$.

Also let

$$\Delta_{p,t} = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\theta} dz \tag{A-7}$$

represent the price dispersion in the economy. In the case of infrequent possibilities of readjusting prices, $\Delta_{p,t-1}$ becomes a state for our economy.

We can now introduce the aggregate state vector for this economy, $\omega_t = (Z_t, \Phi_t, \Delta_{p,t-1}, \mathcal{L}_t)$, and the individual state vector, $s_t^h = (b_{t-1}^h, X_t^h, \omega_t)$. The role of the aggregate state is to allow agents to predict future prices and monetary authority actions. The household problem can be recast in the recursive form

$$V(s,\omega) = \max\Big[u(C) - v(N) + \beta E\, V(s',\omega')\Big] \tag{A-8}$$

$$s.t.$$

$$c + b'/R(\omega) = b/\Pi(\omega) + w(\omega)N + X(s,\omega)$$

$$\Phi' = f(\Phi, Z, Z')$$

$$b \geq -\phi_b.$$

The policy function for asset investment is $b' = b(s)$.

For a given monetary policy and transfer scheme $\left(R(\omega), \Phi^{\tau}\right)$ and an initial condition ω_0, a *recursive imperfectly competitive equilibrium* is a law of motion $f(.)$, value and policy functions V and b and pricing functions $\left(w(\omega), \Pi(\omega), (p(z))(\omega)_{z\in[0,1]})\right)$ such that i)V and b solve (**??**); ii) the pricing functions, together with a law of motion for the price level, solve the resetting firm problem; iii) there is consistency between aggregate variables and summing up of agents' optimal choices, i.e., Φ generates bond market clearing $\int_0^1 b' d\Phi = 0$ and labor market clears.[41]

Appendix C Output Gap

We have defined the efficient rate of output Y^e as the one prevailing with complete markets, equal initial wealth distribution, and flexible prices. In this case, it is easy to show that

$$\hat{Y}_t^e = \frac{\sigma}{\sigma+\varphi}\hat{g}_t + \frac{1+\varphi}{\sigma+\varphi}\hat{A}_t + \frac{\sigma}{\sigma+\varphi}\hat{\Delta}_n. \tag{A-9}$$

[41] A formal proof of the existence of an equilibrium for an economy very similar ours can be found in Miao (2005).

The introduction of nominal rigidities does not alter any fundamental relation except the markup determination. So we still have that $\hat{mc}_t = \hat{W}_t^r - \hat{A}_t$, from the consumption/leisure choice $\hat{W}_t^r = \sigma\hat{C}_t + \varphi\hat{N}_t$, from the resource constraint $\hat{Y}_t - \hat{g}_t = \hat{C}_t$. However it does alter output aggregation of the production functions $Y_t = A_t N_t / \Delta_{p,t}$, such that consumption (in *logs*) is given by

$$\hat{C}_t = \hat{A}_t + \hat{N}_t + \hat{g}_t - \hat{\Delta}_{p,t}. \tag{A-10}$$

So we can write

$$\hat{mc}_t = (\sigma + \varphi)x_t + \varphi\hat{\Delta}_{p,t}. \tag{A-11}$$

For our market structure, we cannot exploit the aggregate consumption/leisure relation directly. However, even in the sticky-price case, the aggregate consumption-labor relation found in section (**??**) must hold:

$$\hat{W}_t^r = \varphi\hat{N}_t + \sigma\hat{C}_t - \sigma\hat{\Delta}_{n,t}. \tag{A-12}$$

Moreover, it is always true that $\hat{W}_t^r = \hat{mc}_t + \hat{A}_t$ and that aggregate consumption is related to output as in equation (**??**). Combining those two relations with (**??**) and using the output gap definition $x_t \equiv \hat{Y}_t - \hat{Y}_t^e$, we get

$$\hat{mc}_t = (\sigma + \varphi)x_t + \varphi\hat{\Delta}_{p,t} - \sigma(\hat{\Delta}_{n,t} - \hat{\Delta}_n), \tag{A-13}$$

as in equation (**??**) of the text.

Appendix D The Optimal Deterministic Steady State

Here we show the existence of an optimal steady state, i.e., of a solution to the recursive policy problem defined in section (2.5), which involves (under appropriate initial conditions) constant values for all variables, in the case of no stochastic disturbances: $A_t \equiv \bar{A}$ and (without loss of generality) $G_t \equiv \bar{G} = 0$.

To prove the result, we split the problem into two stages. In the first stage, the government sets and commits to a redistributive policy Φ^τ, taking as given inflation, price dispersion, and total production–i.e., $R_t = R^\star$, $\Pi_t = \Pi^\star$, $Y_t = Y^\star$ $w_t = w^\star$. Using the consumption-leisure condition, we can write $N_t^h = v_n^{-1}(w^\star/u_c(C_t^h))$. We accordingly redefine the momentary utility

$$u(C_t^h) - v(N_t^h) = \tilde{u}(C_t^h) \tag{A-14}$$

and the wage income

$$w_t N_t^h = g(C_t^h). \tag{A-15}$$

We can now formulate the deterministic version of the Ramsey problem for a given (and at the moment arbitrary) initial distribution of households over debt Φ_{-1}

$$\max \sum_{t=0}^{\infty} \beta^t \int_0^1 \tilde{u}(C_t^h) dh \tag{A-16}$$

$s.t.$

$$C_t^h + b_t^h / R^\star = b_{t-1}^h / \Pi^\star + \overline{\tau}^h + g(C_t^h) \; \forall h \in [0,1]$$
$$\int_0^1 \overline{\tau}^h dh = 0$$
$$\int_0^1 C_t^h dh = Y^\star.$$

(A-17)

We denote the associated set of Lagrange multipliers $\{(\varphi_t^h)_{h\in[0,1]}, \varphi_{1,t}, \varphi_{2,t}\}$. The first order necessary conditions (FONC) for optimal consumption allocation reads

$$\tilde{u}_c(C_t^h) = \varphi_t^h g'(C_t^h) + \varphi_{2,t} \; \forall h \in [0,1]. \tag{A-18}$$

On the other hand, we have the relation $\varphi_t^h = \varphi_{1,t}$. Putting together the two equations, we realize that individual consumption must be equalized

$$C_t^h = \overline{C}_t \; \forall h \in [0,1]. \tag{A-19}$$

The intuition is straightforward: For a given amount of available resources and (strictly) concave utilities, the previous solution is a necessary and sufficient condition, which tells us that a social planner will (strictly) prefer to equalize marginal utilities of consumption across agents.

The induced transfer system, denoted $\Phi^{\tau^\star}(\overline{\tau})$, can be recovered from the intertemporal household budget constraint and will be proportional to the initial debt dispersion $\Phi^{\tau^\star}(\bar{\tau}) \propto \Phi_{-1}$, with the constant of proportionality function of $R^\star$ and $\Pi^\star$. In fact, for each household we have

$$\bar{\tau}^h = \bar{b}^h_{-1}(1/\Pi^\star - 1/R^\star)\ \forall h \in [0,1]. \tag{A-20}$$

In the second stage, we take $\Phi^{\tau^\star}(\bar{\tau})$ as given and we wish to find an initial degree of price dispersion Δ_{-1}, such that the recursive problem involves a constant policy.

However, under the optimal transfer scheme, we have shown that households consume and work the same; this means that our second stage boils down to the same problem solved in Benigno and Woodford (2004) which shows that zero price dispersion (i.e., zero inflation) is the optimal long-run monetary policy. Given no price dispersion, $\bar{\Delta}_p = 0$, we have

$$1 = \bar{p}(z) = \mu\bar{mc} = \mu\bar{w}/\bar{A}. \tag{A-21}$$

Because employment is subsidized at a rate τ_μ, which exactly offsets the monopolistic distortion, we have

$$\bar{W}^r = \frac{\bar{A}}{\mu\tau_\mu} = \bar{A}. \tag{A-22}$$

Thus, output is at its efficient level

$$\bar{Y} = \bar{A}^{\frac{1+\varphi}{\sigma+\varphi}}(1-\bar{\tau}^g)^{-\frac{\sigma}{\sigma+\varphi}}. \tag{A-23}$$

Appendix E Loss Function

We recall that the resource constraint implies that $C_t = Y_t - G_t = Y_t(1-\tau_t^G)$ at all times. We start with the utility of consumption

$$u(C_t^h) = u(\frac{C_t^h}{C_t}(Y_t - G_t)) \simeq \tag{A-24}$$

$$\simeq \overline{u} + u_c(\overline{Y} - \overline{G})(\tilde{C}_t^h + .5\tilde{C}_t^{h^2}) + u_c\overline{Y}(\hat{Y}_t + .5\hat{Y}_t^2) + .5u_{cc}\overline{Y}^2[(1-\overline{\tau})^2\tilde{C}_t^{h^2} + \hat{Y}_t^2]$$

$$+(u_c\overline{Y} + u_{cc}\overline{Y}(\overline{Y} - \overline{G}))\tilde{C}_t^h\hat{Y}_t - u_{cc}\overline{Y}\overline{G}\hat{Y}_t\hat{G}_t - [u_c + (\overline{Y} - \overline{G})u_{cc}]\overline{G}\hat{G}_t\tilde{C}_t^h + t.i.p.$$

where $\tilde{C}_t^h \equiv \widehat{C_t^h/C_t}$. Rearranging and integrating with respect to households and using the fact that $\int_0^1 \tilde{C}_t^h = -.5\int_0^1 \tilde{C}_t^{h^2} + h.s.o.$, we get[42]

$$\int_0^1 u(C_t^h) = \tag{A-25}$$

$$= t.i.p. + u_c\overline{Y}\hat{Y}_t - .5u_c\overline{Y}[\sigma(1-\overline{\tau}^G)\int_0^1 \tilde{C}_t^{h^2} - (1-\sigma)\hat{Y}_t^2] + u_c\overline{Y}\sigma\overline{\tau}^G\hat{Y}_t\hat{G}_t =$$

$$= t.i.p. + u_c\overline{Y}[\hat{Y}_t + (1-\sigma)\hat{Y}_t^2 + \sigma\overline{\tau}^G\hat{Y}_t\hat{G}_t - .5\sigma(1-\overline{\tau}^G)\int_0^1 \tilde{C}_t^{h^2}].$$

We define $\tilde{N}_t^h \equiv \hat{N}_t^h - \hat{N}_t$ and make use of the following two facts: $\int_0^1 \tilde{N}_t^h \simeq -.5\int_0^1 \tilde{N}_t^{h^2}$; and, from the labor supply conditions, we realize that in a second-order approximation, $\tilde{N}_t^{h^2} \simeq \frac{\sigma^2}{\varphi^2}\tilde{C}_t^{h^2}$. Using this last fact we write the quadratic

[42] In the text, we made use of labor supply dispersion $\Delta_{n,t}$. However, in the derivation of the loss function, we prefer to work with $\tilde{C}_t^h$. It is nonetheless not difficult to see how $\Delta_{n,t}$ would enter into the loss function derivation: Just note that we can write $C_t^h = (Y_t - G_t)(N_t^h/N_t)^{-\varphi/\sigma}/\Delta_{n,t}$.

approximation of the disutility of labor:

$$\int_0^1 v(N_t^h)dh = \tag{A-26}$$

$$= \int_0^1 v(\frac{N_t^h}{N_t}N_t)dh = \int_0^1 v(\frac{N_t^h}{N_t}\frac{1}{A_t}\int_0^1 y(z)dz)dh \simeq t.i.p+$$

$$+v_n\frac{\bar{Y}}{\bar{A}}[\int_0^1 \hat{y}_t(z)dz + .5(1+\varphi)\int_0^1 \hat{y}_t^2(z)dz - (1+\varphi)\hat{A}_t\int_0^1 \hat{y}_t(z)dz + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}dh] = t.i.p+$$

$$+v_n\frac{\bar{Y}}{\bar{A}}[E_z\hat{y}_t(z) + .5(1+\varphi)[(E_z\hat{y}_t(z))^2 + V_z\hat{y}_t(z)] - (1+\varphi)\hat{A}_tE_z\hat{y}_t(z) + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}],$$

where we define $\varphi \equiv \varphi/\bar{A}$ and use $E_z[\hat{y}_t(z)^2] = (E_z\hat{y}_t(z))^2 + V_h\hat{y}_t(z)$.

We make use of the fact that $\hat{Y}_t = E_z\hat{y}_t(z) + .5(1-1/\theta)V_z\hat{y}_t(z)$ and $(E_z\hat{y}_t(z))^2 = \hat{Y}_t^2$ and also that $\hat{A}_tE_z\hat{y}_t(z) = \hat{A}_t\hat{Y}_t$ (being terms of order higher than the second). Hence, we can write:

$$\int_0^1 v(N_t^h)dh \simeq$$

$$t.i.p + v_n\frac{\bar{Y}}{\bar{A}}[\hat{Y}_t + .5(1+\varphi)\hat{Y}_t^2 + .5(1/\theta+\varphi)V_z\hat{y}_t(z) - (1+\varphi)\hat{A}_t\hat{Y}_t + \frac{\sigma^2}{\varphi}\int_0^1 \tilde{C}_t^{h^2}].$$

Using the steady-state relation $u_c = v_n/\bar{A}$, we can combine both the expressions we have found (up to a multiplicative constant) to define the loss function we were looking for:

$$L_t = \tag{A-27}$$

$$= (\sigma+\varphi)\hat{Y}_t^2 - 2(\sigma+\varphi)\hat{Y}_t\hat{Y}_t^N + (1/\theta+\varphi)V_z\hat{y}_t(z) + \sigma(1-\bar{\tau}^G+\sigma/\varphi)\int_0^1 \tilde{C}_t^{h^2} =$$

$$= (\sigma+\varphi)x_t^2 + (1/\theta+\varphi)V_h\hat{y}_t(h) + \sigma(1-\bar{\tau}^G+\sigma/\varphi)\int_0^1 \tilde{C}_t^{h^2}.$$

We have used the fact that $(\sigma+\varphi)\hat{Y}_t^N = (1+\varphi)\hat{A}_t + \sigma\bar{\tau}^G\hat{G}_t$ and the output gap definition $x_t \equiv \hat{Y}_t - \hat{Y}_t^N$. Then, knowing that $V_z\hat{y}_t(z) = \theta^2V_z\hat{p}_t(z)$ and following

Woodford (2003) we write

$$L_t = \pi_t^2 + \lambda_x x_t^2 + \lambda_c \int_0^1 \tilde{C}_t^{h2}, \tag{A-28}$$

where $\kappa \equiv (1-\psi)(1-\beta\psi)(\sigma+\varphi)/\psi/(1+\varphi\theta)$ is the Phillips curve parameter, while $\lambda_x \equiv \frac{\kappa}{\theta}$ and $\lambda_c \equiv \frac{(1-\psi)(1-\beta\psi)}{(1+\varphi\theta)\psi}\sigma(1-\overline{\tau}^G+\sigma/\varphi)$.

If we wanted to use $\eta(h)$ we would simply observe that when $\eta(h) = 1/u_c(C^h)$ we have $\eta(h)v_n(N^h)/A = \eta(h)u_c(C^h) = 1$. Hence, all the results would hold up to a multiplicative constant.

Appendix F Discussion of Aggregation

Krusell and Smith (1998) shows that in an economy with incomplete market, idiosyncratic income shocks, and an asset (capital) available for partial self-insurance, an *approximate* aggregation result holds. In their words, "...all aggregate variables–consumption, the capital stock and relative prices–can be almost perfectly described as a function of two simple statistics: the mean of the wealth distribution and the aggregate productivity shock". Moreover, the marginal propensity to save out of current wealth is almost completely independent of the levels of wealth and labor income (even with leisure choice).

Den Haan (1997), in a setup similar to ours, shows that, without tight borrowing constraints, policy functions are almost linear and changes of asset distribution on prices have much smaller effects than those implied by aggregate shocks. For example, even if the stationary level of interest is shifted by wealth heterogeneity (as Hugget 1993 shows in relation to the low risk-free puzzle), the percentage changes during business cycle fluctuations are mainly driven by aggregate shocks.

The previous results suggested my conjecture that variations in the cross-sectional distribution of assets are of minor order with respect to variations in the other endogenous state variables. In this model, in fact, the first moment of the asset distribution–which is a ”sufficient statistics” in Krusell and Smith–is constant by construction. Second and higher moments do affect endogenous variables; however, their stationary levels rather than their oscillations around those levels, are what matters most in my welfare analysis.

Appendix G The Natural Debt Limit

Imposing $C_t^h \geq 0$ and $N_t^h \leq \bar{N}^+$ implies the emergence of what Aiyagari (1994), in a slightly simpler context, calls a *natural debt limit*. We want to solve the budget constraint forward, imposing the "worst possible scenario" for repaying a contracted debt. Over all possible realizations, let $\underline{\beta} = \min R_{t+k}$, $\underline{T}^G = \min T_t^G$ and $\underline{w} = \min W_t^r$. Also set $\Pi_t = 1 \; \forall t \geq 0$. Call $\underline{y} = \underline{w}\bar{N}^- \underline{T}^G$.

The budget constraint can now be written as

$$-b_{t-1}^h (1 - \underline{\beta} L^{-1}) = \underline{y} + \bar{\tau}^h. \tag{A-29}$$

Let $\phi_b \equiv \underline{y}/(1 - \underline{\beta})$ and recall that $\bar{\tau}^h = -\bar{b}^h (1 - \beta)$ and $\tilde{b}_t^h = b_t^h - \bar{b}^h$. We can now write

$$\tilde{b}_{t-1}^h \geq -\phi_b \geq -\phi_b - \bar{b}^h \left(\frac{\beta - \underline{\beta}}{1 - \underline{\beta}} \right). \tag{A-30}$$

So taking $-\phi_b$ as the natural borrowing limit means that everybody has the same limit when the problem is formulated in deviation from the steady state, and that the mass of agents hitting the limit in the stationary equilibrium is zero.

Appendix H The Complete Markets Case

We assume a continuum of households indexed by $h \in [0,1]$ maximizing the utility

$$U_0^h = E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t^h) - v(N_t^h) \right].$$

The budget constraint takes the form

$$P_t C_t^h + E_t B_t^h Q_{t,t+1} = B_{t-1}^h + W_t N_t^h + P_t X_t^h, \tag{A-31}$$

where B_t is a set of state-contingent securities that pays \$1, while $Q_{t,t+1}$ is the pricing kernel.

From the Euler equations, we have that

$$\frac{C_{t+1}^h}{C_t^h} = \frac{C_{t+1}^{h^o}}{C_{t+1}^{h^o}}, \ \forall (h, h^o) \in [0,1]^2. \tag{A-32}$$

In the next proposition, we claim that there exists an average household.

Proposition 2 *For any continuous initial distribution of wealth, $\exists h^o \in [0,1]$ such that $C_t^{h^o} = C_t \ \forall t \geq 0$*

Proof. Given any continuous initial distribution of wealth, $\exists h^o \in [0,1]$ $C_0^{h^o} = \int_0^1 C_0^h dh$.

Then from the Euler we have that

$$C_t^{h^o} = \frac{C_0^{h^o}}{C_0^h} C_t^h = \frac{C_0}{C_0^h} C_t^h. \tag{A-33}$$

So

$$C_t^{h^o}\int_0^1 C_0^h dh = C_0 \int_0^1 C_t^h dh, \tag{A-34}$$

which shows the above proposition. ■

We can now introduce a metric for deviations of households' consumption from the average consumption in the economy:

$$C_t^h = \frac{C_0^h}{C_0} C_t = \delta(h) C_t \tag{A-35}$$

$$\Delta_{n,t}^{CM} = \left(\int_0^1 \delta(h)^{-\varphi/\sigma}\right)^{\varphi/\sigma}. \tag{A-36}$$

Thus, under complete markets, $\Delta_{n,t}$ is constant.

To determine the value of this constant, we must specify the initial wealth, hence the transfer scheme.

We can always find a transfer scheme such that $\delta(h) = 1 \; \forall h \in [0,1]$.

This would also be the optimal scheme that a benevolent government would implement, weighting all households the same.

To find this transfer scheme, we write the intertemporal budget constraint and impose that $C_t^h = C_t \; \forall h \in [0,1], t \geq 0$. Then the intertemporal budget constraint is

$$B_{-1}^h = \sum_{t=0}^{\infty} E_0 Q_{0,t} [W_t N_t^h + P_t \overline{\tau}^h - P_t C_t^h]. \tag{A-37}$$

Given that $C_t^h = C_t$, it must also be that $N_t^h = N_t$, so that (considering that the profits equal the taxes for subsidies[43]) we have that $C_t = W_n N_t$. Thus, the budget

[43] The subsidy rate is constant, but total subsidies are not and are always equal to profits.

constraint reduces to

$$B_{-1}^{h} = \overline{\tau}^{h} \sum_{t=0}^{\infty} E_0 Q_{0,t} P_t \tag{A-38}$$

or

$$\overline{\tau}^{h} = -\frac{b_{-1}^{h}/\Pi_0}{\sum_{t=0}^{\infty} E_0 Q_{0,t} P_t / P_0} = -\frac{b_{-1}^{h}/\Pi_0}{\sum_{t=0}^{\infty} \beta^t E_0 u_{c,t}/u_{c,0}}. \tag{A-39}$$

If we define $(1-\beta^*) = \sum_{t=0}^{\infty} \beta^t E_0 u_{c,t}/u_{c,0}$, we can write

$$\overline{\tau}^{h} = -\frac{b_{-1}^{h}}{\Pi_0}(1-\beta^*) \tag{A-40}$$

Given that $\Pi_0 = 1$ is optimal in the case of no initial relative price distortion, we set

$$\overline{\tau}^{h} = -b_{-1}^{h}(1-\beta^*). \tag{A-41}$$

Appendix I Optimal Monetary Policy, Flex Case

Here we give a proof of proposition (1).

Proof. We write the Lagrangian for the policy problem

$$\mathcal{LG} = E_0 \sum_{t=0}^{\infty} \beta^t \lambda_c \int_0^1 \tilde{C}_t^{h2} + \int_0^1 \lambda_{1,t}^{h} \left(\kappa_c \tilde{C}_t^h - \tilde{b}_{t-1}^h + \beta \tilde{b}_t^h - \overline{b}^h (\beta \hat{R}_t - \pi_t)\right) +$$
$$+ \int_0^1 \lambda_{2,t}^{h} \left(\Delta \tilde{C}_{t+1}^h + \varphi_b \tilde{b}_t^h\right) + \lambda_{3,t}\left(\hat{R}_t - \pi_{t+1} - r_t^n\right) + \int_0^1 \lambda_{2,-1}^{h} \tilde{C}_0^h/\beta - \lambda_{3,-1}\pi_0/\beta.$$

Necessary conditions read (substituting out the interest rate $\hat{R}_t$):

$$\kappa_c \tilde{C}_t^h - \tilde{b}_{t-1}^h + \beta \tilde{b}_t^h - \overline{b}^h (\beta r_t^n + \beta E_t \pi_{t+1} - \pi_t) = 0 \forall h \in [0,1] \qquad \text{(A-42)}$$

$$\Delta \tilde{C}_{t+1}^h + \varphi_b \tilde{b}_t^h = 0 \forall h \in [0,1] \qquad \text{(A-43)}$$

$$\lambda_c \tilde{C}_t^h = \kappa_c \lambda_{1,t}^h - \lambda_{2,t}^h + \beta^{-1} \lambda_{2,t-1}^h \quad \forall h \in [0,1] \qquad \text{(A-44)}$$

$$\beta (E_t \lambda_{1,t+1}^h - \lambda_{1,t}^h) = \varphi_b \lambda_{2,t}^h \qquad \forall h \in [0,1] \qquad \text{(A-45)}$$

$$\int_0^1 \overline{b}^h \lambda_{1,t}^h = \lambda_{3,t-1} / \beta \qquad \text{(A-46)}$$

$$\beta \int_0^1 \overline{b}^h \lambda_{1,t}^h = \lambda_{3,t} \qquad \text{(A-47)}$$

We multiply the first four equations by $\overline{b}^h$ and we integrate up with respect to h. Using the definitions given in the text, we can write a bloc of the system as

$$\kappa_c w_t - z_{t-1}^h + \beta z_t^h - \overline{b}^h (\beta \hat{R}_t - \pi_t) = 0 \qquad \text{(A-48)}$$

$$E_t \Delta w_{t+1} + \varphi_b z_t = 0 \qquad \text{(A-49)}$$

$$\lambda_c w_t = \kappa_c \lambda_{1,t} - \lambda_{2,t} + \beta^{-1} \lambda_{2,t-1} \qquad \text{(A-50)}$$

$$E_t \lambda_{1,t+1} - \lambda_{1,t} = \varphi_b \lambda_{2,t} / \beta \qquad \text{(A-51)}$$

$$\lambda_{1,t} = \lambda_{1,t-1}. \qquad \text{(A-52)}$$

If our solution is correct, then $\zeta_b (\beta \hat{R}_t - \pi_t) = \beta z_t - z_{t-1}$. So equation A-12 can be written (for every $t \geq 0$) as $\kappa_c w_t = -\beta z_t + z_{t-1} + \beta z_t - z_{t-1} = 0$. Given that $w_t = 0 \; \forall t \geq 0$, then from equation (A-13) we also have $z_t = 0 \; \forall t \geq 0$.

From equation (A-16) we have that the first multiplier must be constant $\lambda_{1,t} = \lambda_{1,-1}$ and $\lambda_{3,t} = \beta \lambda_{1,-1}$. Using equation (A-15), this means that $\lambda_{2,t} = 0 \; \forall t \geq 0$.

Hence, by the last unused equation (A-14), we must also have that for all $t \geq 1$

$$0 = \lambda_c w_t = \kappa_c \lambda_{1,-1} - \lambda_{2,t} + \lambda_{2,t-1}/\beta = \kappa_c \lambda_{1,-1},$$

which implies $\lambda_{1,-1} = 0$. Now it is also straightforward to see that $\lambda_{2,-1} = 0$. So the system is satisfied and the initial values of the cross-Lagrange multipliers consistent with our solution are exactly $\lambda_{i,-1} = 0$. ∎

Appendix J Non-linear Solution Flex Case

The wage becomes exogenous $w_t = A_t$. The policy problem is

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 u(C_t^h) - v(N_t^h) \tag{A-53}$$

s.t.

$$C_t^h + b_t^h/R_t = b_{t-1}^h/\Pi_t + A_t N_t^h + \tau^h \tag{A-54}$$

$$\beta R_t E_t \frac{u_{c,t+1}^h}{u_{c,t}^h \Pi_{t+1}} = 1 \tag{A-55}$$

$$(C_t^h)^\sigma (N_t^h)^\phi = A_t \tag{A-56}$$

$$\int_0^1 b_t^h = 0 \ \text{ given } b_{-1}^h = \overline{b}^h \forall h \in [0,1]. \tag{A-57}$$

We attach the following Lagrange multiplier from top-down constraints λ_1^h, λ_2^h, λ_3^h, μ. Given a transfer scheme $\tau^h = -\gamma_h \overline{b}^h$ and assuming $\lambda_{2,-1}^h = 0$, it can be verified that any solution of the problem assumes the following form:[44]

$$R_t = \frac{\Pi_t}{1-\gamma_h \Pi_t}, \tag{A-58}$$

$$b_t^h = \overline{b}^h,\; C_t^h = C_t,\; N_t^h = N_t, \tag{A-59}$$

$$\lambda_{2,t}^h = \lambda_{3,t}^h = 0,\; \lambda_{1,t}^h = C_t^{-\sigma}. \tag{A-60}$$

When $\gamma_h = (1-\beta)$, we have $R_t = \frac{\Pi_t}{1-(1-\beta)\Pi_t}$, which, in a first-order approximation is $\hat{R}_t \simeq \frac{1}{\beta}\pi_t$—the result of section 4.3 for $z_{-1} = 0$.

If $b_{-1}^h \neq \overline{b}^h$, then there are not enough instruments

$$R_0 = \frac{\Pi_0}{\frac{b_{-1}^h}{\overline{b}^h} - \gamma_h \Pi_0}. \tag{A-61}$$

Appendix K Budget Constraint Derivation

The resource constraint for our economy without capital accumulation, which is simply $C_t = (1-\tau_t^G)Y_t$, can be written as

$$C_t = W_t^r N_t + F_t - G_t. \tag{A-62}$$

Thus, we can subtract the resource constraint (A-26) from the household h budget constraint equation (1):

$$C_t^h - C_t + \tilde{b}_{t-1}^h / R_t = \tilde{b}_{t-1}^h / \Pi_t + W_t^r (N_t^h - N_t) + \overline{b}^h \left(\frac{\beta R_t - 1}{R_t} - \frac{\Pi_t - 1}{\Pi_t}\right). \tag{A-63}$$

[44]First order conditions are not shown for brevity.

Taking a linear expansion of this equation around the steady state of the deterministic model, we have

$$\tilde{C}_t^h + \beta \tilde{b}_t^h = \tilde{b}_{t-1}^h + \bar{W}^r \bar{N} \tilde{N}_t^h + \bar{b}^h(\beta \hat{R}_t - \pi_t). \quad \text{(A-64)}$$

Recalling the result of section (2.4), $C_t = W_t^{r\,1/\sigma} N_t^{-\varphi/\sigma} \Delta_{n,t}$, we can find the relation between that relates consumption and hours deviations: $\frac{\varphi}{\sigma}\tilde{N}_t^h = -\tilde{C}_t^h + \hat{\Delta}_{n,t}$.

In the case of sufficiently small exogenous disturbance, the term $\Delta_{n,t}$ will be either small or constant for ranking welfare. In fact, we have $\Delta_{n,t} \simeq .5\frac{\varphi}{\sigma} Var_h \tilde{N}_t^h$. Thus, we can substitute $\tilde{C}^h$ for $\tilde{N}^h$ in equation (A-28) neglecting $\Delta_{n,t}$ [45] to get

$$\kappa_c \tilde{C}_t^h = \tilde{b}_{t-1}^h - \beta \tilde{b}_t^h + \bar{b}^h(\beta \hat{R}_t - \pi_t), \quad \text{(A-65)}$$

where $\kappa_c = 1 + \frac{\sigma}{\varphi}$.

Appendix L The System of First Order Necessary Conditions

In what follows we write the system of first-order necessary conditions of the policy problem of section (4) (the Lagrange multipliers associated with the con-

[45] We recall that in the steady state, we have offset the monopolistic distortion such that $\bar{W}^r \bar{N} = \bar{A}\frac{\tau_\mu}{\mu}\bar{Y}/\bar{A} = \bar{Y}$. We further normalize the output to $\bar{Y} = 1$.

straints are μ_1, μ_2 λ_1^h λ_2^h, respectively):

$$\lambda_x x_t + \kappa\mu_{1,t} + \sigma\mu_{2,t} - \sigma\beta^{-1}\mu_{2,t-1} = 0 \tag{A-66}$$

$$\pi_t + \mu_{1,t-1} - \mu_{1,t} - \beta^{-1}\mu_{2,t-1} - \lambda_{1,t} = 0 \tag{A-67}$$

$$\mu_{2,t} + \beta\lambda_{1,t} = 0 \tag{A-68}$$

$$\lambda_c \tilde{C}_t^h = \kappa_c \lambda_{1,t}^h - \lambda_{2,t}^h + \beta^{-1}\lambda_{2,t-1}^h \tag{A-69}$$

$$\beta(E_t\lambda_{1,t+1}^h - \lambda_{1,t}^h) = \tilde{\varphi}_b \lambda_{2,t}^h. \tag{A-70}$$

Using the definitions introduced in the paper (plus the analog for λ_1^h λ_2^h), the "aggregate" block of the system is

$$\lambda_x x_t + \kappa\mu_{1,t} + \sigma\mu_{2,t} - \sigma\beta^{-1}\mu_{2,t-1} = 0 \tag{A-71}$$

$$\pi_t + \mu_{1,t-1} - \mu_{1,t} - \beta^{-1}\mu_{2,t-1} - \lambda_{1,t} = 0 \tag{A-72}$$

$$\mu_{2,t} + \beta\lambda_{1,t} = 0 \tag{A-73}$$

$$\lambda_c w_t = \kappa_c \lambda_{1,t} - \lambda_{2,t} + \beta^{-1}\lambda_{2,t-1} \tag{A-74}$$

$$\beta(E_t\lambda_{1,t+1} - \lambda_{1,t}) = \tilde{\varphi}_b \lambda_{2,t} \tag{A-75}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{A-76}$$

$$\sigma E_t \Delta x_{t+1} = R_t - E_t \pi_{t+1} - r_t^e \tag{A-77}$$

$$\kappa_c w_t = -\beta z_t + z_{t-1} + \zeta_b^2(\beta \hat{R}_t - \pi_t) \tag{A-78}$$

$$E_t w_{t+1} = w_t - \tilde{\varphi}_b z_t. \tag{A-79}$$

Figures and Tables

Table 2: Efficient Rate vs Refinancing Cost

	GOMP	OSR	IT
$Std(\xi)$	1.259	.617	.742
$Std(r^e)$	.742	.742	.742
$AutoCorr(\xi)$	-0.013	.953	0.863
$AutoCorr(r^e)$	0.863	0.863	0.863

Volatility and autocorrelations of the efficient rate r^e and the refinancing cost ξ under the optimal policy and the baseline calibration.

Table 3: Optimal Simple Rules

Parameter	$\phi_b = 10^{-6}$	$\phi_b = 10^{-4}$
η_r	1.047	1.047
η_p	4.241	2.331

The functional form used is $\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t$. The initial grid is $[0,3] \times [1,300]$ for η_r and η_p, respectively. In that range, no indeterminacy issues arises. We subsequently use a finer grid on the neighborhood of the optimum previously found: $[0.90, 1.25] \times [3.00, 6.00]$, for $\phi_b = 10^{-6}$ and $[0.90, 1.25] \times [1.80, 4.00]$, for $\phi_b = 10^{-4}$ with 20×40 points.

Table 4: Monetary Policy Rule

Rule Code	η_r	η_p	η_x			
GOMP	-	-	-	-	-	Optimal Rule
OSR($\phi_b = 10^{-6}$)	1.047	4.24	0			
OSR($\phi_b = 10^{-4}$)	1.047	2.33	0			
TR	0	1.5	.5			
IT	-	∞	-	-	-	

Rules used for the welfare comparison. For OSR, TR, and IT, the functional form is: $\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t + \eta_x x_t$.

Table 5: Welfare Comparison. Baseline Calibration

Losses	GOMP	OSR	TR	IT
Ratios	1	1.25	4.69	1.49
Levels	6.046e-5	7.538e-5	1.053e-3	9.018e-05
Inflation	9.306e-6	5.346e-6	9.979e-4	0
Output Gap	2.943e-4	4.741e-5	3.001e-4	0
Cons. Disp.	1.908e-3	2.967e-3	2.055e-3	3.882e-03
$\phi_b = 10^{-6}$				

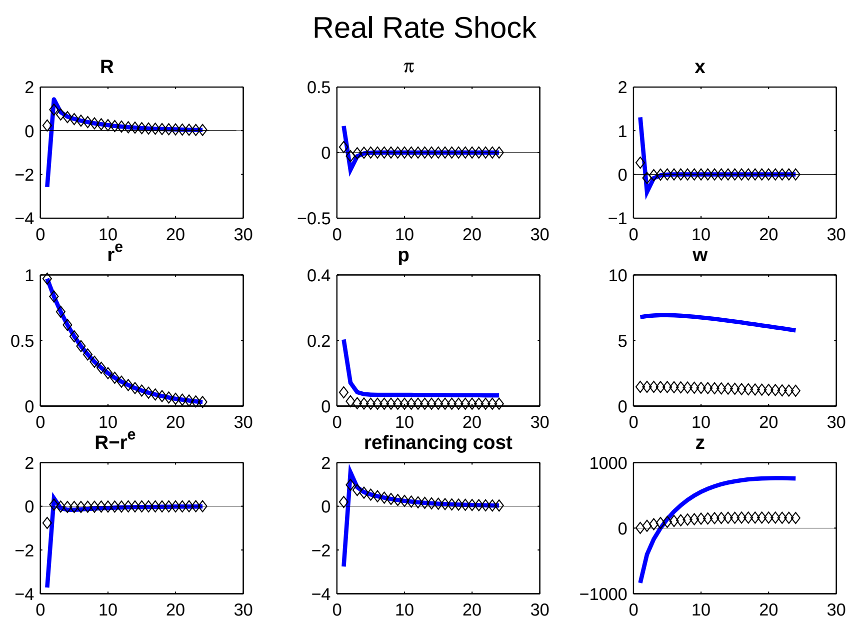
The welfare loss is expressed in percentage terms with respect to the optimal rule, "Ratios", and in steady-state consumption, "Levels". The last three rows show the discounted expected volatility of the targets: inflation, output gap, and consumption dispersion. Once appropriately multiplied by the loss function weights λ_x and λ_c, the sum of the targets gives the loss in levels.

Table 6: Welfare Comparison. Higher bond adjustment costs

Losses	GOMP	OSR	TR	IT
Ratios	1	1.4	11.78	1.88
Levels	8.567e-5	1.244e-4	1.009e-3	1.611e-04
Inflation	1.872e-5	1.281e-5	9.979e-4	1.110e-11
Output Gap	5.919e-4	9.552e-5	3.004e-4	3.365e-12
Cons. Disp.	2.290e-3	4.707e-3	3.667e-3	6.934e-03
$\phi_b = 10^{-4}$				

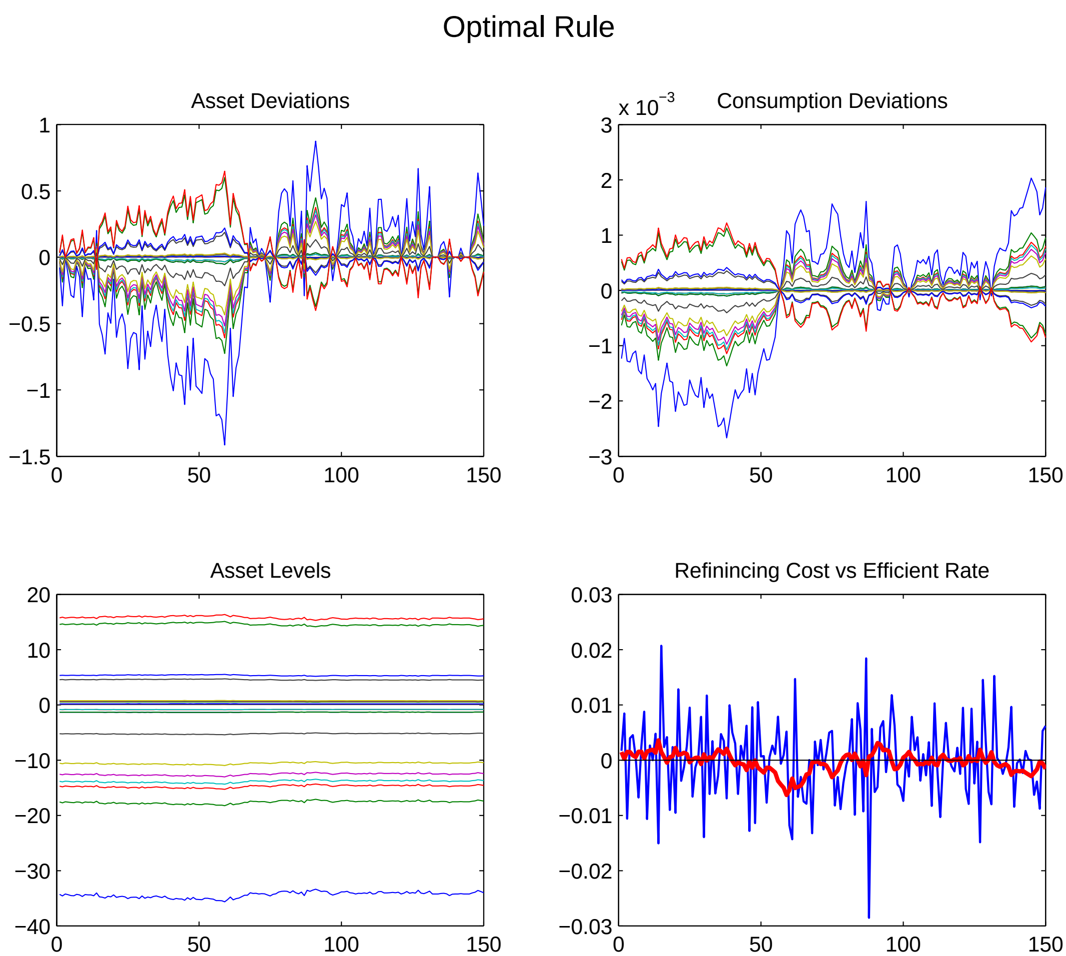
The welfare loss is expressed in percentage terms with respect to the optimal rule, “Ratios”, and in steady-state consumption, “Levels”. The last three rows show the discounted expected volatility of the targets: inflation, output gap, and consumption dispersion. Once appropriately multiplied by the loss function weights λ_x and λ_c, the sum of the targets gives the loss in levels.

Figure 1: Impulse Response Functions



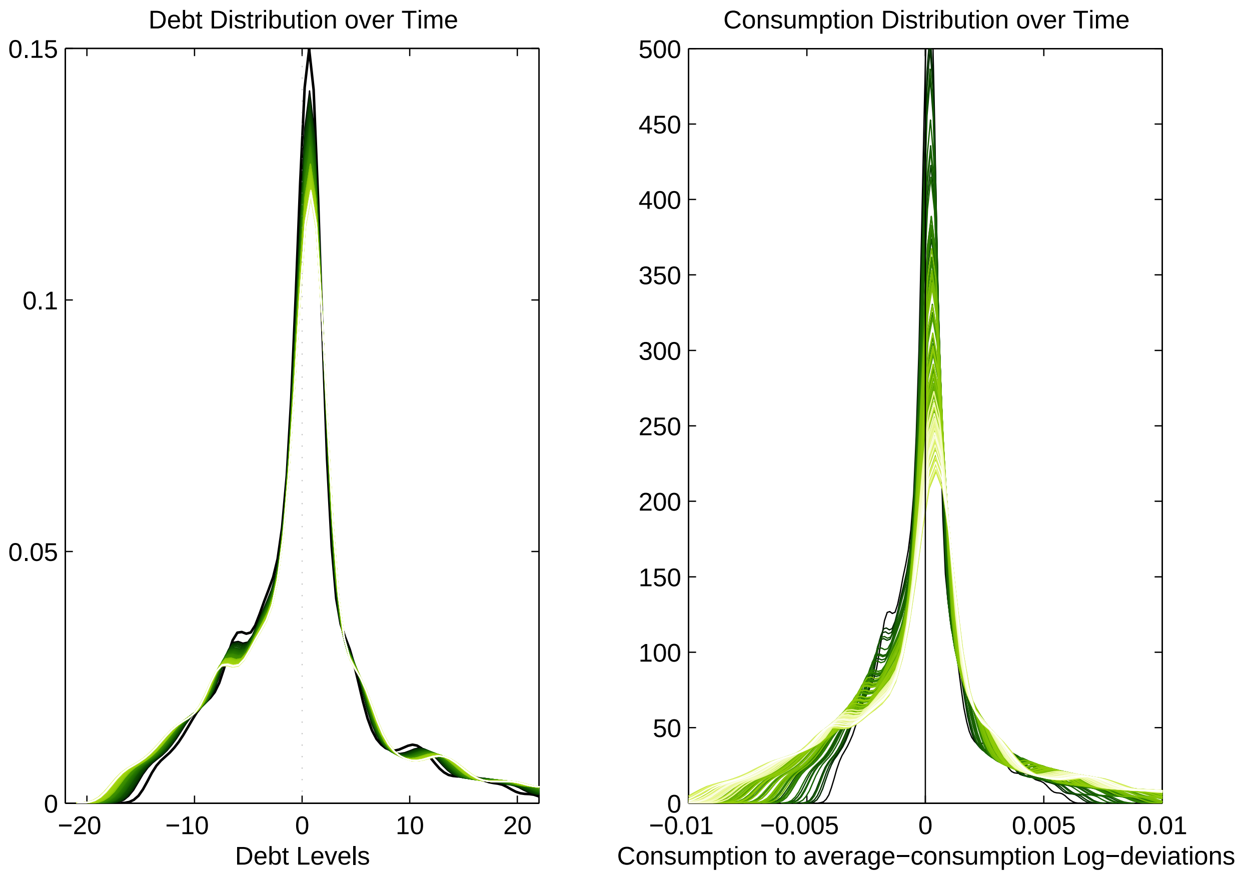
Impulse response functions to a positive 100% shock to the *efficient* real rate r_t^e. The Solid line and diamonds represent a debt variance parameter, ζ_b^2, of 18.31 and 2.34, respectively.

Figure 2: Simulated Series



Simulated disaggregate series, under the baseline calibration, using 1,200 randomly-drawn households; 24 of them selected for plots. Initial values at steady-state values. No initial periods discarded. Top left: debt deviations from long-run level $\tilde{b}_t^h$ for selected households, over time. Top right: consumption to average consumption log-deviations $\tilde{C}_t^h$ for selected households, over time. Bottom left: debt levels b_t^h for selected households, over time. Bottom right: unitary real refinancing cost $\beta\hat{R}_t - \pi_t$ (blue line) and efficient rate r_t^e (red line), over time.

Figure 3: Distributions over Time



Density functions over time t. Left panel: debt levels. Right panel: log-deviations of households' consumption over average consumption $\tilde{C}^h$. Total periods $T = 110$. Solid black line initial distribution $t = 0$; colors fade from dark green to light green as t approaches T. In the right panel, the initial distribution is degenerate (vertical black line). Densities are estimated using a normal kernel-smoother function.

Figure 4A: Optimal Simple Rule

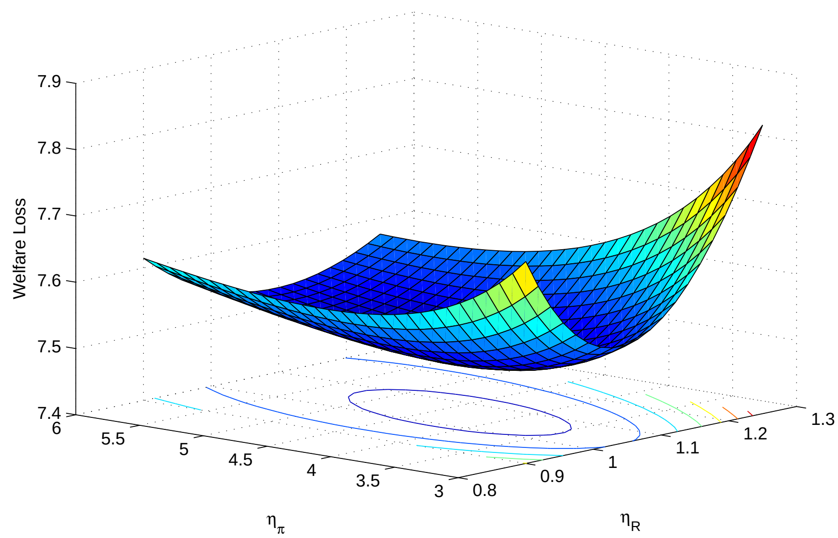
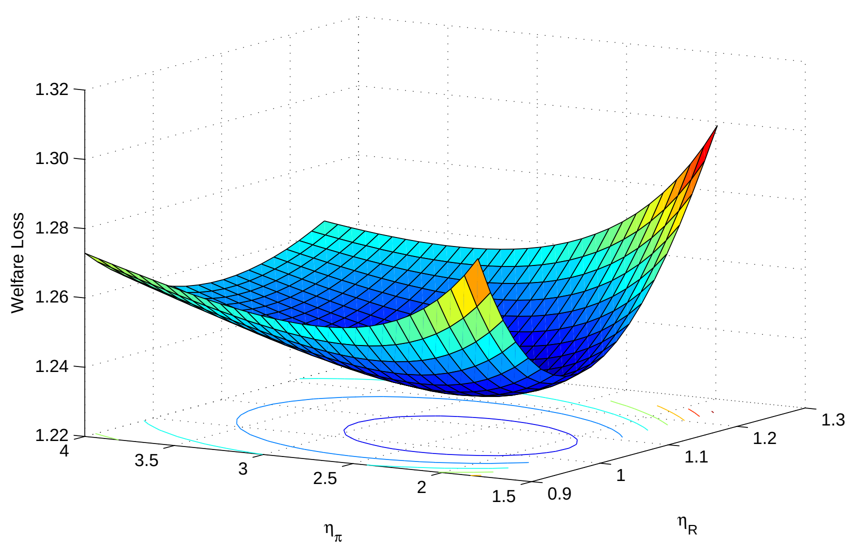
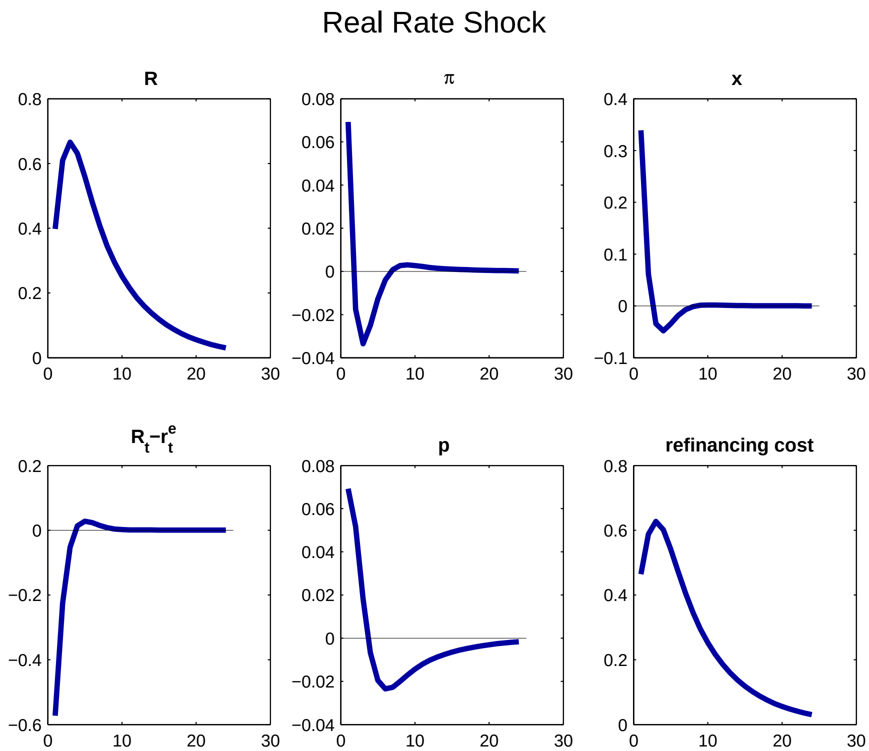


Figure 4B: Optimal Simple Rule



Rule functional form: $\hat{R}_t = \eta_r \hat{R}_{t-1} + \eta_p \pi_t$. Panel A: baseline calibration. Panel B: higher financial adjustment costs $\phi_b = 10^{-4}$. For panels A and B, grids are $[0.90, 1.25] \times [3.00, 6.00]$ and $[0.90, 1.25] \times [1.80, 4.00]$ with 20×40 points, respectively.

Figure 5: Optimal Simple Rule



Impulse response functions to a positive 100% shock to the *efficient* real rate of interest r_t^e. Baseline calibration. Simple rule: $\hat{R}_t = 1.0474\hat{R}_{t-1} + 4.2414\pi_t$.