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Information Gathering by a Principal

by Ed Nosal

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In the standard principal-agent model, the information structure is fixed. In this paper I allow the principal to choose his level of "informedness" before he contracts with the agent. During the contracting phase, the agent never learns what the principal knows about the state of the world. I examine the cases where the agent observes and does not observe the level of informedness that the principal chooses. The strategic nature of the model environment implies that there are both direct and indirect costs associated with the existence of high quality information. The implications for information gathering, investment and welfare are examined for both cases.

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Ed Nosal is a senior economic advisor at the Federal Reserve of Cleveland. He can be reached at ed.nosal@clev.frb.org or by mail at Research Department, the Federal Reserve Bank of Cleveland, P.O. Box 6387, Cleveland, Ohio 44101-1387. The author would like to thank Jeff Campbell and David de Meza for their comments on an earlier version.

1 Introduction

In the standard principal-agent model (Holmstrom (1979)), a principal designs a contract with the intent to control the behavior of an agent who has an informational advantage. Since the principal is unable to perfectly observe whether a bad outcome is due to bad luck or bad behavior, he designs a contract that exposes the agent to risk in order to provide the agent with an incentive to work hard. The amount of risk that the principal exposes the agent to depends upon the quality of the signals that he receives regarding the effort that the agent puts forth. The more informative the signals that the principal receives, the less risk the agent is exposed to, and the higher the expected payoff to the principal. So, in the standard principal-agent model the agent is exposed to risk because of his informational advantage, but the degree of risk sharing between the principal and agent—and, hence, the expected payoff to the principal—depends positively on the quality of the information that the principal receives.

In this paper I study a principal-agent environment that provides some novel insights into risk-sharing agreements between a principal and an agent, and into the value of high quality information to the principal. I find that the nature of the risk-sharing agreement depends on the informational advantage that the *principal* may (or may) not possess. As well, the possession of higher quality information leads to a worsening in risk-sharing outcomes, and, hence, a lower payoff to the principal. Generally speaking, the insights from my analysis can be applied to any bilateral relationship that is characterized by either private values (Maskin and Tirole (1990)) or common values (Maskin and Tirole (1992)). In a private values problem the principal's private information parameter does not enter the preferences of the agent, while in a common values problem it does. The major difference between the typical principal-agent environment and the environment developed in this paper lies in how information sets are structured. In the typical environment, the information structure is "fixed." For example, in Holmstrom (1979) the principal does not possess any private information and never observes the agent's effort level; in Maskin and Tirole's (1990, 1992) models of the private and common values problem, the principal is privately informed about some important parameter which the agent never observes. This paper examines a private values environment where the information structure is *not* "fixed." Before parties meet to contract with one another, the principal gets to choose whether or not to get informed about an important parameter. So, depending upon the principal's choice, he may or may not possess private information. At the contracting phase, the agent does not observe what the principal observes if he gets informed. And, the agent may or may not observe whether the principal chooses to get informed. My model is, therefore, able to address interesting issues—such whether a principal will over–, under–, or efficiently acquire information—that do not arise in the typical principal-agent model.

Perhaps it would be most instructive to motivate the intuition behind my results within the context of a concrete example. Consider an entrepreneur who has an opportunity to develop and use a new technology for an existing product. The technology is costly and requires the input of an agent, or worker. Since technology is new, there will be some uncertainty associated with it. In particular, the entrepreneur is uncertain about how good the match will be between the technology and the skills of a representative worker. If the match is "good," i.e., the representative worker is well suited to operate the new technology, then both productivity and output will be high. Alternatively, if the match is "bad," i.e., the representative worker is ill-suited to operate the technology, then both productivity and output will be low. Before deciding upon investing in the new technology, the entrepreneur can, at a cost, obtain better information about quality of the match—i.e., the entrepreneur can learn the true state of the match—by performing some tests on existing labor.[1] The obvious benefit associated with entrepreneur learning this kind of information is that he can avoid undertaking a "bad" project if it is found out that the quality of the match between the new technology and existing labor is not very good. The entrepreneur, however, may choose not to get informed about the state of the match if, for example, the cost of getting this better information is "high."

If the entrepreneur invests in the new technology, he then goes to the labor market and attempts to hire a worker. Because of the existence of either agency problems or risk-sharing concerns, it will be optimal for entrepreneur to write and offer an explicit contract to a worker before the work commences. What constitutes an "acceptable" contact from the worker's point of view depends upon what he knows, or what he believes he knows, about the quality of information that the entrepreneur possesses. Suppose that the worker can tell whether or not the entrepreneur has obtained the better information, but doesn't know the state of the match in the event that the entrepreneur "gets informed." Then if the entrepreneur does not get informed, the entrepreneur's (equilibrium) contract offer will correspond to the Pareto optimal contract. If, however, the entrepreneur does get informed, then it will be as if the agent also learns the true state of the match. Since the principal makes the contract offer and is informed, he will attempt to exploit "ignorance" of the worker. But the worker, understanding this, will only accept contracts that guarantee him his reservation utility for each possible state of the match: This contact is identical to the one that would prevail if the worker *could* observe the state of the match. Here, the equilibrium contract will differ from the Pareto optimal contract, which attempts to smooth either the effort levels or consumption over states of the world. Since the value of the equilibrium contract to the "informed entrepreneur" is less than the value of the Pareto optimal contract—and the values of the equilibrium and Pareto optimal contracts are the same to the uninformed entrepreneur—the entrepreneur will, compared to the social optimum, under–acquire the high quality information.

[1]For example, the entrepreneur can select workers from the labor force and pay them to be subjected to some tests that would reveal the quality of the match. Throughout, it is assumed that workers are homogeneous.

As well, in contrast to the intuition that underlies the typical principal-agent model, higher quality information will be associated with lower degrees of risk sharing and lower payoffs to the principal (entrepreneur).

A private information distortion is introduced when the worker does not know whether or not entrepreneur has acquired the higher quality information. Specifically, since the entrepreneur can always claim that he did not acquire the high quality information, the equilibrium contract offer that the *uninformed* entrepreneur makes must be incentive compatible in the sense that the entrepreneur should have no incentive to obtain the high quality information, given that the contract offer will be accepted. As a result, the equilibrium contract for the uninformed entrepreneur may differ from the Pareto optimal contract. Compared to the situation where the worker can observe whether or not the entrepreneur, the value associated with remaining uninformed has now been reduced.[2] Hence, on average, there will be more information acquired when the worker can not tell whether or not the entrepreneur acquires information, compared to when he can. From a social perspective, however, the entrepreneur can either over– or under– acquire information.

The analysis points to a number of costs associated with the existence high quality information. There is, of course, the direct cost. But there are also couple of important indirect costs. First, by getting informed, the entrepreneur can not offer a Pareto optimal contract. Hence, it will be more costly to hire workers for an informed entrepreneur, compared to the social optimum. Second, the private information friction that is introduced when worker can not see whether or not the entrepreneur gets informed distorts the kind of contract offered by an uninformed entrepreneur that the worker finds acceptable, implying that, compared to social optimum, it will be more costly for an uninformed entrepreneur to hire a worker.

One can interpret the social losses that result from the existence of higher quality information as stemming from a lack of commitment on the entrepreneur's behalf. If, for example, the entrepreneur possessed a technology that allowed him to commit to offering a certain contract before he made his information acquisition decision, then social optimal allocation could be implemented as an equilibrium. But within the context of the model—and the real world—it is not at all obvious how an entrepreneur would be able to commit in this manner.

This paper contributes to a small, but growing, literature that seeks to endogenize the information structure within a principal-agent relationship. In contrast to the standard principal-agent model, where the information sets of principal and agent are exogenously specified, this literature gives one party to the principal-agent relationship the opportunity to either learn the state of the world or to remain ignorant, (for example, Crémer and Khalil (1992), Crémer, Khalil and Rochet (1998a and 1998b), Kessler (1998) and Sobel (1993)). The contributions to this literature assume that it is the agent who is given the choice to become informed.[3] Clearly, the nature of the

[2] The value to the entrepreneur of getting informed is the same in both information structures.

[3] In related work, Dewatripont and Maskin (1995) and Crémer (1995), allow the principal to

optimal contract between the principal and agent may be altered by the possibility that the agent is informed. However, since it is the uninformed principal who makes the contract offer, the contract will not *reveal* any information that may have (or may not have) been acquired. Hence, a contract in this literature can be viewed as being a *sorting* device. This paper differs from the existing literature in an important way: here, it is the principal who chooses whether or not to become informed. Should the principal choose to become informed, the contract may now reveal what was learned. An interesting contracting problem arises since, if the principal chooses to become informed, in some circumstances he would like to (somehow) reveal what he learned and in other situations he would like to 'conceal' the information. In contrast to the existing literature, where the contract is a sorting device, in my model the contract can be interpreted as a *signalling* device.

The paper which is closest to mine is Kim (2002). Kim (2002) takes the standard principal-agent model with moral hazard (Holmstrom (1979)), turns it into a common values problem, and examines the principal's information acquisition decision when it is assumed that the agent can observe the principal's decision and when he can not. The results that Kim (2002) produces mirror those contained in this paper. In a way this speaks to the robustness of our insights: Kim's (2002) environment is characterized by common values and moral hazard; my environment is characterized by private values with no moral hazard. There are, however, two major differences between Kim's paper and mine. In Kim's (2002) model, there is no benefit associated with acquiring information: Information acquisition is always socially wasteful. Second, and more important, the most interesting and relevant environment to study is one where the agent does not know whether or not the principal acquires information. This environment is the most relevant because, in practice, it would be hard to detect or verify whether or not someone has acquired some information. This environment is also the most interesting—and the "trickiest" to analyze—because it turns out that the mere existence of high quality information impacts on the structure of an uninformed principal's (equilibrium) contract offer. This latter aspect of the principal's contracting problem, however, disappears when the cost of acquiring information is zero because, in equilibrium, the principal will always get informed. When studying this informational environment, Kim (2002) assumes that the cost of acquiring information is zero and, therefore, is unable to address issues related to the optimal contract for an uninformed principal.

The paper is organized as follows. The next section describes the model. The planner's problem is characterized and solved in Section 3 for two information structures: one where the planner can see if the principal gets informed and another where the planner can not see. Sections 4 and 5 characterize the equilibrium outcomes and examine their implications for welfare. In section 4, it is assumed that the agent

restrict the contingencies that the agent can observe. They find that it may be desirable to limit what the agent can 'see' because this eliminates the possibility of contract renegotiation (and, in their models, renegotiation reduces the value of the relationship).

can observe whether or not the principal gets informed; in section 5 it is assumed that the agent can not observe whether or not the principal gets informed. Section 6 concludes.

2 The Model

A wealthy entrepreneur, or "principal," has wealth W. The principal has an opportunity to develop a new technology (or investment project) that costs I. The project produces a single cash-flow and requires the use of an agent. The cash flow generated by the project is $\theta\ell$, where $\ell \in [0, \bar{\ell}]$ represents the level of labor input provided by the agent and θ represents the quality of the match between labor and the new technology. Since there is a one-to-one relationship between match quality and productivity, I will refer to θ as the productivity parameter. The productivity parameter is drawn from the set $\{\theta(1), \ldots, \theta(N)\}$, where N is a large number. For convenience, and without loss, let $0 < \theta(1) < \cdots < \theta(N)$. I will say that the state of the world is s if the productivity parameter is $\theta(s)$, $s \in S = \{1, \ldots, N\}$.

If the principal undertakes the investment project and hires an agent, then his payoff from the project is defined to be $\pi \equiv \theta\ell - w - I$, where w represents the total payment received by the agent, who provided ℓ units of labor input. The principal is assumed to be risk-neutral. Note that the payoff to the principal, π, is defined gross of any costs that may be incurred from gathering information.

The agent's preferences are defined over consumption and labor, and are represented by the utility function $u(w, \ell)$, where $u_w > 0$, $u_\ell < 0$ and u is strictly concave, i.e., $u_{ii} < 0$ for $i = w, \ell$ and $u_{ww}u_{\ell\ell} - (u_{w\ell})^2 > 0$. In addition, I assume the following INADA-type conditions: (i) $-u_\ell/u_w \to \infty$ as $\ell \to \bar{\ell}$; (ii) $-u_\ell(w, 0)/u_w(w, 0) < s_1$ for all $w \geq 0$; and (iii) $-u_\ell(0, \ell)/u_w(0, \ell) < s_1$ for all $\ell \in [0, \bar{\ell}]$.[4] Finally, labor (leisure) is assumed to be a normal good, which implies that $-u_{ww}(u_\ell/u_w) + u_{w\ell} < 0$.

The principal's information acquisition, investment, hiring, and labor input choices is best viewed as a multi-date game between the principal and agent. The timing of this game is,

date 1 The principal makes his information acquisition decision: He either remains uninformed or gets informed. It costs $c \geq 0$ to get informed.

* The principal learns the state of the world if he got informed.

date 2 The principal decides whether or not to invest in the project. If the principal does not invest, then the game ends.

[4] These conditions imply that wage-labor input allocations will be in the interior. Condition (i) ensures that $\ell < \bar{\ell}$; condition (ii) ensures that $\ell > 0$; and condition (iii) ensures that $w > 0$.

date 3 If the principal develops the project, then he meets an agent for hire and offers the contract C. The agent accepts contract C if it provides him with a level of expected utility at least equal to his reservation level, $\bar{u}$.

* The state of the world is revealed to everyone. (This assumption implies that contract allocations can depend explicitly on the state of the world.)

date 4 A contract allocation is implemented and everyone consumes. This ends the game.

In terms of what the agent knows at the time when he is offered a contract—at date 3—there are two interesting information structures that I examine. In one structure, which I call a *partial information structure*, it is assumed that the agent knows if the principal got informed or remained uninformed. If, however, the principal gets informed, the agent does not know what the principal learned. In an *asymmetric information structure*, the agent does not know whether the principal is informed or uninformed.[5] Finally, under both information structures the agent can observe whether or not the principal invests in the project.

At date 1, the principal can either remain uninformed or get informed. Although its costs the principal nothing to remain uninformed, it implies that he will continue to be "ignorant" about the state of the world at dates 2 and 3, i.e., when he makes his project investment and hiring decisions. I will assume that the investment project is "*ex ante* profitable" in the sense that there always exists at least one contract, $C = \{(w(s), \ell(s))\}_{s=1}^{N}$, such that: (1) $\sum_{s=1}^{N} p(s)(\theta(s)\ell(s) - w(s) - I) \geq 0$, i.e., the expected payoff to the uninformed principal is greater than zero; and (2) $\sum_{s=1}^{N} p(s)u(w(s), \ell(s)) \geq \bar{u}$, i.e., the contract will be accepted by the agent, where $\{p(s)\}_{s=1}^{N}$ represents the "prior" distribution of states of the world. These two conditions imply that if the principal chooses to remain uninformed, then he has an incentive to invest in the project at date 2 and hire an agent at date 3. For convenience, I will define the "*ex post* state s payoff" to a principal who invests as either $\pi(w(s), \ell(s))$ or $\pi(s)$, where $\pi(w(s), \ell(s)) \equiv \theta_s \ell(s) - w(s) - I$.

Although the project is *ex ante* profitable, it may be case that the if the principal gets informed he may choose *not* to invest in the project because, *ex post,* the project is unprofitable. I will say that the project is *ex post* unprofitable in state s if there does not exist a wage-labor allocation (w, ℓ) such that $u(w, \ell) = \bar{u}$ and $\theta(s)\ell - w - I \geq 0$ in state s. That is, the project is *ex post* unprofitable in state s, if the highest payoff that the principal can obtain from the investment project is negative when the agent is paid his reservation value. I will assume that states $s \in \{1, \ldots, k\} \equiv S^k$, where $k < N$, are *ex post* unprofitable. However, once the investment has been undertaken, and the investment is now a sunk cost, the project is always "profitable" in the sense

[5] In section 4, I also briefly consider a *full information structure*, where the agent not only knows whether or not the principal is informed but, in the event that the principal is informed, he also knows the state of the world.

that there exist wage-labor allocations, (w, ℓ), such that $u(w, \ell) = \bar{u}$ and $\theta(s)\ell - w \geq 0$ for all $s \in S$.

The notion of equilibrium that will be used is that of a perfect Bayesian-Nash equilibrium. Loosely speaking, the equilibrium concept requires that, when called upon, players take optimal actions, that beliefs are generated by equilibrium strategies (whenever possible) and that strategies in each and every subgame are Nash equilibrium strategies.

A note on modeling strategy The basic idea behind this paper is that there is a "trade-off" associated with acquiring information. By getting informed the principal is able to make a "better" investment decision. This is the benefit associated with information acquisition. However, by getting informed it turns out that the set of contracts that are acceptable to the agent will shrink, and this reflects the cost associated with information acquisition. Obviously, in order to examine this trade-off associated with information acquisition, the economic environment under consideration must be one where contracting has real value.[6] In my model, individuals contract for risk-sharing reasons. If it is assumed that leisure has a zero wealth effect, then it will turn out that contracting has no value since the "optimal contract" allocation will coincide with the (no contract) spot market allocation. Hence, in order for contracts to have value in my risk-sharing environment, it must the case that leisure is either normal or inferior. I have assumed the former.

3 The Planner's Problem: Pareto Optimal Allocation

This section characterizes a Pareto optimal allocation. One can image that a planner designs a contract that the principal offers the agent in the event that the investment project is undertaken. When designing the contract and the actions that players undertake, the planner must respect the informational constraints that face the agent. That is, the planner faces the same informational restrictions as the agent. So, for example, in either a partial or asymmetric information structure, the planner can not direct the principal to invest in certain states of the world and not to invest in other states when the principal becomes informed because the planner is unable to observe the state of the world.[7] Hence, the contract that the planner designs must be incentive compatible in the sense that the principal undertakes actions that are consistent with the planner's intent. I will first examine the optimal contract under the partial information structure. For this structure, the planner can direct

[6] By "contracting must have real value," I mean that welfare is higher when agents sign contracts compared to not signing contracts but, instead, negotiating and trading in the *ex post* spot market.

[7] The planner can, however, unconditionally direct the principal to invest in the project at date 2 since date 2 investment is publicly observable.

the principal to get informed or to remain uninformed. I will then examine the asymmetric information structure.

Consider the optimal contract that the planner designs under a partial information structure. Suppose that the principal is uninformed. The optimal contract that the planner designs will specify the set of allocations, $\{(w(s), \ell(s))\}_{s=1}^{N}$, that solves the following problem,

$$\max_{\{(w(s),\ell(s))\}_{s=1}^{N}} \sum_{s=1}^{N} p(s)(\theta(s)\ell(s) - w(s) - I) \tag{1}$$

subject to

$$\sum_{s=1}^{N} p(s)u(w(s), \ell(s)) = \bar{u}. \tag{2}$$

The first order conditions to this problem imply that the wage-labor input allocations are characterized by optimal risk sharing (constant marginal utility of consumption across all states of the world),

$$u_w(w(s), \ell(s)) = 1/\lambda \text{ for all } s \in S \tag{3}$$

where λ represents the multiplier on constraint (2), and *ex post* productive efficiency (marginal rate of substitution equals the marginal product of labor in all states of the world),

$$-\frac{u_\ell(w(s), \ell(s))}{u_w(w(s), \ell(s))} = \theta(s) \text{ for all } s \in S. \tag{4}$$

Denote the set of allocations that solves maximization problem {(1),(2)} as $\{(w_\upsilon^*(s), \ell_\upsilon^*(s))\}_{s=1}^{N} \equiv C_\upsilon^*$. The expected payoff that the principal receives from contract C_υ^*, denoted by Π_υ^*, is

$$\Pi_\upsilon^* = \sum_{s=1}^{N} p(s)[\theta(s)\ell_\upsilon^*(s) - w_\upsilon^*(s) - I] \equiv \sum_{s=1}^{N} p(s)\pi_\upsilon^*(s).$$

Note that in some low states of the world, the principal will be earn negative payoffs, i.e., $\pi_\upsilon^*(s) < 0$ for some $s \in S$.[8] The fact that the principal receives a

[8] To see this first note that the set of contract allocations lie on locus of points that have constant marginal utility of consumption. Along this locus the marginal rate of substitution increases as labor input increases, i.e.,

$$\frac{d(\frac{du_\ell}{du_w})}{d\ell} |_{u_w=\text{constant}} = -u_w(-\frac{(u_{w\ell})^2}{u_{ww}} + u_{\ell\ell}) < 0,$$

since u is strictly concave. Since the optimal contract is characterized by the equality of marginal rates of substitution and marginal products of labor, $\theta(s)$, higher levels of labor input are associated with higher states of the world. Along a locus of constant marginal utility of consumption, utility decreases as the level of labor input increases, i.e.,

$$\frac{du(w,\ell)}{d\ell} |_{u_w=\text{constant}} = -u_w\frac{u_{w\ell}}{u_{ww}} + u_\ell < 0,$$

negative payoff in some states of the world does not necessarily point to an inefficiency in investment: Contract C_v^* attempts to achieve the dual objectives of efficient *ex post* production and efficient risk sharing, and attempting to achieve this latter goal may require that the principal incur negative payoffs in low states of the world. There, of course, exist contracts that simultaneously provides the principal with non-negative payoffs in all states of the world and the agent with his reservation level of utility, $\bar{u}$, but all these contracts will give the principal an expected payoff that is strictly less than Π_v^*.

Consider now the optimal contract that the planner designs when the principal gets informed. Since the planner can not observe the state of the world when the principal gets informed, the contract that the planner designs has to provide the correct incentives for the principal to invest when it is socially optimal to do so and not to invest otherwise. It will *never* be socially optimal for the principal to invest when the true state of the world s is *ex post* unprofitable, i.e., if $s \in S^k$. Recall that when it is known that $s \in S^k$, the principal will earn a negative payoff from the investment if he invests and provides the agent with a payoff of $\bar{u}$. Hence, it will be optimal for the planner to design the contract in such a manner that when the principal gets informed and observes state $s \in S^k$ he will have no incentive to undertake the investment. It is actually quite simple to design such a contract. For example, the contract can specify the wage-labor allocation $(w, 0)$, where $w > 0$, for all states $s \in S^k$. If state $s \in S^k$ occurs *and* the principal invests and hires an agent, his payoff will be $-w - I < 0$. Clearly, the principal will have no incentive to invest in these states of the world.

Now let's consider contract allocations for states $s \in S^{\sim k} = \{s+1, \ldots, N\}$. For these states of the world, it is socially optimal for the principal to invest in the project. The principal will invest in the project when he observes state $s \in S^{\sim k}$ only if his payoff to the investment is non-negative, i.e., only if $\theta(s)w(s) - \ell(s) - I \geq 0$. Therefore, the optimal contract allocations for states $s \in S^{\sim k}$ solves the following problem,

$$\max_{\{(w(s),\ell(s))\}_{s=k+1}^{N}} \sum_{s=k+1}^{N} p'(s)(\theta(s)\ell(s) - w(s) - I) \tag{5}$$

subject to

$$\sum_{s=1}^{N} p'(s)u(w(s), \ell(s)) = \bar{u}. \tag{6}$$

and

$$\theta(s)\ell(s) - w(s) - I \geq 0 \text{ for all } s \in S^{\sim k}, \tag{7}$$

since labor (leisure) is a normal good. Hence, the level of utility decreases as the state of the world increases. Since the optimal contract gives the agent a level of expected utility equal to $\bar{u}$, it must be the case that $u(w^*(1), \ell^*(1)) > \bar{u}$. But since $\theta(1)\ell - w - I < 0$ for any (w, ℓ) that satisfies $u(w, \ell) = \bar{u}$, it must be the case that $\theta(1)\ell^*(1) - w^*(1) - I < 0$.

where

$$p'(s) = \frac{p(s)}{\sum_{j=k+1}^{N} p'(j)} \text{ for } s \in S^{\sim k}. \tag{8}$$

One may wonder why constraints (7) are required since for all $s \in S^{\sim k}$ the investment project is "*ex post* profitable." The reason is that perfect risk sharing, i.e., constant marginal utility of consumption over *all* states in which the principal invests, may be incompatible with non-negative *ex post* payoffs to principal in "low" states of the world. More specifically, if constraints (7) are ignored, then it may very well be the case that the *ex post* payoff to the principal is negative in, say, state $k+1$. The implication of this is that if the principal observes state $k+1$ when he gets informed, he will not invest in this state since his payoff is negative. But it is socially optimal for the principal to invest in this state.

The first order conditions to maximization problem {(5),(6),(7)} are:

$$p'(s)\theta(s) + \lambda p'(s)u_\ell(w(s), \ell(s)) + \beta(s)\theta(s) = 0 \text{ for } s \in S^{\sim k}$$

and

$$-p'(s) + \lambda p'(s)u_w(w(s), \ell(s)) - \beta(s) = 0 \text{ for } s \in S^{\sim k},$$

where λ represents the multiplier for constraint (6) and $\beta(s)$ is the multiplier for constraint (7) in state s. The first order conditions can be simplified to read,

$$-\frac{u_\ell(w(s), \ell(s))}{u_w(w(s), \ell(s))} = \theta(s) \text{ for all } s \in S^{\tilde{}k} \tag{9}$$

and

$$u_w(w(s), \ell(s)) = 1/\lambda + \beta(s) \text{ for all } s \in S^{\tilde{}k}. \tag{10}$$

As in contract C_υ^*, the contract allocations here are characterized by efficient *ex post* production (in states $s \in S^{\sim k}$), i.e., equation (9). In addition, the contract will be characterized by efficient risk sharing if $\beta(k+1) = 0$. If, however, $\beta(s) > 0$ for $s \in \{k+1, ..., j\}$, then the optimal contract does not attempt to share risks in these states: The contract allocations will be determined by the zero *ex post* payoff condition in these states. For states $s \in \{j+1, ..., N\}$, contract allocations will be characterized by constant marginal utility of consumption for the agent. I will denote the contract that the planner designs for the principal when he gets informed by $C_\iota^* = \{(w_\iota^*(s), \ell_\iota^*(s))\}_{s=1}^N$, where $(w_\iota^*(s), \ell_\iota^*(s)) = (w, 0)$, $w > 0$, for $s \in S^k$ and $\{(w_\iota^*(s), \ell_\iota^*(s))\}_{s=k+1}^N$ is the solution to maximization problem {(5),(6),(7)}. The expected payoff to the principal, gross of the cost of getting informed, is denoted by Π_ι^*, and is equal to[9]

$$\Pi_\iota^* = \sum_{s=k+1}^{N} p(s)[\theta(s)\ell_\iota^*(s) - w_\iota^*(s) - I] \equiv \sum_{s=k+1}^{N} p(s)\pi_\iota^*(s).$$

[9]Throughout, I will maintain the convention that the subscript 'υ' refers to an uninformed principal and the subscript 'ι' refers to an informed principal.

If the principal remains uninformed his expected payoff will be Π_{υ}^*; if he gets informed his expected payoff will be $\Pi_{\iota}^* - c$. Hence, if $\Pi_{\upsilon}^* \geq \Pi_{\iota}^* - c$, the planner will instruct the principal to remain uninformed at date 1, to invest in the project at date 2 and offer contract C_{υ}^* at date 3. If $\Pi_{\iota}^* - c > \Pi_{\upsilon}^*$, the planner will instruct the principal to get informed at date 1 and offer contract C_{ι}^* at date 3 if he undertook the investment at date 2. Note that contract C_{ι}^* is designed in a manner so that if the state of the world is *ex post* unprofitable, i.e., $s \in S^k$, the principal will not invest in the project at date 2; otherwise the principal will invest in the project at date 2.

Note that there is a cost associated with the fact that the planner is asymmetrically informed *vis á vis* the principal at date 3. If the planner *could* observe the state of the world at date 1, then the contract that the planner would design for the principal would be the solution to problem $\{(5), (6)\}$, i.e., constraint (7) would not be needed. Denote the expected payoff the principal receives in this environment, net of the cost of getting informed, by Π_{ι}^{**}. The planner would instruct the principal to invest in the project at date 2 if $s \in S^{\tilde{}k}$, and offer the contract that solves $\{(5), (6)\}$ at date 3. The expected payoff to the principal in this environment, $\Pi_{\iota}^{**} - c$, is greater than $\Pi_{\iota}^* - c$. The difference in the expected payoffs reflects the cost of providing the principal with an incentive to invest in states $s \in S^{\tilde{}k}$ when the planner can not observe the state of the world when the principal gets informed.

Suppose now that the planner is operating in the asymmetric information environment. That is, now the planner can not observe whether or not the principal gets informed. Remarkably, this reduction in the planner's information set does not affect what he is able to implement. For example, if $\Pi_{\upsilon}^* \geq \Pi_{\iota}^* - c$, then the planner can instruct the principal to invest in the project at date 2 and offer contract C_{υ}^* at date 3. Note that the principal has no incentive to get informed since his expected payoff will be $\Pi_{\upsilon}^* - c < \Pi_{\upsilon}^*$ if he gets informed, i.e., it costs the principal c to get informed but he always invests in the project and offers contract C_{υ}^* to the agent. Similarly, if $\Pi_{\iota}^* - c > \Pi_{\upsilon}^*$, then the planner can instruct the principal to offer contract C_{ι}^* to the agent at date 3 if he (the principal) invests in the project. If the wage payments for the *ex post* unprofitable states of the world in contract C_{ι}^* are "sufficiently large," then the principal will have an incentive to get informed at date 1, i.e., by making the wage payments for the *ex post* unprofitable states of the world sufficiently large, the payoff to the principal associated with *not* getting informed and offering contract C_{ι}^* can be made less than $\Pi_{\iota}^* - c$. When the principal gets informed he will only invest in states $s \in S^{\tilde{}k}$.

The solution to the planner's problem allows us to think about and characterize the various dimensions of efficiency associated with the four date game.

1. Because the state of the world is revealed to all in between dates 3 and 4, production is always *ex post* efficient in the sense that the agent's marginal rate of substitution always equals the marginal product of labor.

2. When the principal remains uninformed, the optimal contract is characterized

by "perfect" risk sharing, i.e., the contract gives the agent a constant marginal utility of consumption over all states of the world. However, when the principal gets informed, the optimal contract may be characterized by "imperfect" risk sharing. This imperfection arises from the informational asymmetry that exists between the planner and principal at the time when investment is undertaken. In this latter situation, the optimal contract must induce the principal to invest in states $s \in S^{\tilde{k}}$, and for "low" states of the world in $S^{\tilde{k}}$ this may imply that the must contract abandon its risk-sharing objectives over some states of the world.

3. When the principal gets informed, the investment decision is always socially optimal in that he does not invest in *ex post* unprofitable states, S^k, and always invests when $s \in S^{\tilde{k}}$.

4. The socially efficient level of information acquisition can be defined by the comparing the various expected payoffs that the principal can obtain. Specifically, it is socially efficient for the principal to get informed if $\Pi_\iota^* - c > \Pi_\upsilon^*$; otherwise it is socially efficient for the principal to remain uninformed.

5. The solution to the planner's problem is invariant to the information structure that he faces. That is, the planner is able to implement the same (Pareto optimal) allocation under both the partial information structure and the more constrained asymmetric information structure.

Generally speaking, there are two distinct costs associated with getting informed. There is, of course, the direct cost, $c \geq 0$, of getting informed. If $\beta(k+1) = 0$, then the direct cost c is the only cost associated with getting informed. In this situation $\Pi_i^* > \Pi_\upsilon^*$, which implies that if the direct cost of getting informed is "small," for example $c = 0$, then it is optimal for the principal to always get informed. If, however, $\beta(k+1) > 0$, then there is an indirect cost associated with getting informed. This cost is associated with the inability of the principal to fully share risk with the agent over all states of the world for which the principal invests in the project. Here, the optimal contract must be distorted away from one of optimal risk sharing in order to give the principal an incentive to invest in all *ex post* profitable states of the world $s \in S^{\tilde{k}}$. Hence, it is quite possible to have $\Pi_\upsilon^* > \Pi_i^*$, which implies that even if the direct cost of getting informed is zero, it is optimal for the principal not to get informed.[10]

One can interpret the planner's solution in terms of *commitment* in the following sense: If the principal could commit to offering certain contracts at date 3, then he would be able to implement the planner's solution as an equilibrium. To see this,

[10]If there was no informational asymmetry between planner and principal, then there will be no indirect cost associated with getting informed. As a result, if direct cost is small, principal will get informed because $\Pi_i^{**} > \Pi_\upsilon^*$.

consider, for example, the situation where $\Pi_v^* \geq \Pi_\iota^* - c$. Here, the principal can implement the planner's solution for both the partial and asymmetric information structures if he can commit to offer contract C_v^* at date 3. If he can credibly commit to contract C_v^*—meaning that he must unconditionally offer contract C_v^* at date 3—then he would have no incentive to get informed at date 1 and would has an incentive to invest in the project at date 2. Similarly, if $\Pi_\iota^* - c > \Pi_v^*$, then the principal will be able to implement the planner's solution for both information structures if he can commit to offer contract C_ι^* at date 3. Hence, if it turns out that the equilibria to the four date game do *not* correspond to the planner's solution, then one can interpret the principal's inability to commit to future actions as the source of the failure to implement the Pareto optimal allocation.

4 Equilibrium Contracts Under Partial Information

Before we characterize the equilibrium outcome to the four date game under the partial information structure, it will turn out to be both useful and insightful to analyze a much simpler informational structure. Suppose that the four date game is characterized by *full information*, in the sense that the agent can observe everything that the principal observes. Specifically, if the principal gets informed at date 1, the full information assumption means that the agent knows that the principal got informed *and* knows what the state of the world is when he meets the principal at date 3, the contracting date. If the principal does not get informed at date 1, then neither the principal nor agent knows the true state at date 3.

Suppose that there is an equilibrium to this game where the principal chooses not to get informed at date 1. Then, in equilibrium, the principal will invest at date 2 and will offer the agent contract C_v^* at date 3. Any other proposed equilibrium contract offer, say $\tilde{C}_v \neq C_v^*$, that is acceptable to the agent will necessarily give the principal a payoff that is less than Π_v^*. As a result, if the (proposed) equilibrium asks the principal to offer contract $\tilde{C}_v \neq C_v^*$ to the worker at date 3, then the principal will defect from proposed equilibrium play at date 3 and will, instead, offer contract C_v^*. Since the agent knows that the principal is uninformed, he will accept such a contract offer. Therefore, any contract $\tilde{C}_v \neq C_v^*$ can not be an equilibrium contract offer.

Now suppose that the principal gets informed at date 1 and invests in the project at date 2. At date 3, when the principal and agent meet, the agent knows the true state of the world, say s. The "best" acceptable contract offer that the principal can make to the agent solves the following problem,

$$\max_{(w(s),\ell(s))} \theta(s)\ell(s) - w(s) \tag{11}$$

subject to

$$u(w(s), \ell(s) = \bar{u}. \tag{12}$$

Any other contract offer will either be unacceptable to the agent or will give the principal a payoff less than the allocation that solves maximization problem {(11),(12)}. Denote the wage-labor input allocation that solves maximization problem {(11),(12)} by $(\bar{w}(s), \bar{\ell}(s))$ and define $\bar{C} = \{(\bar{w}(s), \bar{\ell}(s))\}_{s=1}^{N}$ and $\bar{\pi}(s) = \theta(s)\bar{\ell}(s) - \bar{w}(s)$. Hence, if there is an equilibrium where the principal gets informed, it is characterized by the principal investing in the project at date 2 if $s \in S^{\tilde{}k}$ and offering the contract $\bar{C}$ at date 3, which the agent accepts. At date 4 allocation $(\bar{w}(\tilde{s}), \bar{\ell}(\tilde{s})) \in \bar{C}$ is implemented if the state of the world is $\tilde{s}$. (The principal will not invest in the project at date 2 if $s \in S^k$.) Note that since the agent can observe the state of the world at date 3, there does not exist any other contract that can provide the principal with a higher payoff, i.e., the principal has no incentive to defect from equilibrium play of offering contract $\bar{C}$ to the worker at date 3.

The equilibrium characterization of the four date game under full information can be summarized as follows. If the principal remains uninformed, he will invest in the project at date 2 and will offer contract C_{υ}^{*} at date 3, which the agent accepts. The principal's expected payoff associated with this strategy is Π_{υ}^{*}. If the principal gets informed at date 1, he will invest in the project at date 2 only if $s \in S^{\tilde{}k}$ and offers contract $\bar{C}$ at date 3, which the agent accepts. The *expected* payoff to the principal associated with this strategy[11], denoted $\bar{\Pi}_{\iota}$, is equal to

$$\bar{\Pi}_{\iota} = \sum_{s=k+1}^{N} p(s)(\theta(s)\bar{\ell}(s) - \bar{w}(s) - I) \equiv \sum_{s=k+1}^{N} p(s)\bar{\pi}(s).$$

Therefore, the principal will get informed at date 1 if $\bar{\Pi}_{\iota} - c > \Pi_{\upsilon}^{*}$; otherwise he remains uninformed.

One may wonder why the full information structure was examined when focus of interest in this section is the partial information structure. The reason is that if the agent can observe that the principal gets informed, then, in equilibrium, it is as if the agent can also observe the state of the world. To see this suppose that, in equilibrium, the principal gets informed, invests in the project, and at date 3 makes the (equilibrium) contract offer $\tilde{C}_{\iota} = \{(\tilde{w}_{\iota}(s), \tilde{\ell}_{\iota}(s))\}_{s=1}^{N}$. Without loss, assume that for all $s \in S$ contract $\tilde{C}_{\iota}$ is characterized by the equality of the agent's marginal rate of substitution with the marginal product of labor.[12] Define S^I as the set of states for which principal undertakes the investment project. For the states in which the principal invests, i.e., for all $s \in S^I$, it can not be the case that $\tilde{\pi}_{\iota}(s) < \bar{\pi}(s)$ for any $s \in S^I$, since in those states the principal can always defect from equilibrium play and offer contract $\bar{C}$ at date 3, which the agent will accept. Hence, any equilibrium

[11] The expectation is taken just prior to the principal becoming informed.

[12] An implication of this assumption is that if the principal's payoff is equal to (greater than, less than) $\bar{\pi}(s)$ in state s, then the agent's utility will be equal to (less than, greater than) $\bar{u}$ in state s.

contract offer will be characterized by $\tilde{\pi}_\iota(s) \geq \bar{\pi}(s)$ for the states in which the principal invests. As well, it cannot be the case that for any $s \in S^I$, $\tilde{\pi}_\iota(s) > \bar{\pi}(s)$. If this was the case, then the agent would reject the contract offer since the expected utility associated with contract $\tilde{C}_\iota$ would be less than $\bar{u}$.[13] Since any equilibrium contract offer is characterized by $\tilde{\pi}_\iota(s) \geq \bar{\pi}(s)$ and not $\tilde{\pi}_\iota(s) > \bar{\pi}(s)$ for all $s \in S^I$, it must be the case that $\tilde{\pi}_\iota(s) = \bar{\pi}(s)$ for all $s \in S^I$. This implies that the equilibrium date 3 contract offer must be $\bar{C}$. Since $\bar{\pi}(s) < 0$ for all $s \in S^k$, the principal will only invest in states $s \in S^{\tilde{\ }k}$, i.e., $S^I = S^{\tilde{\ }k}$. Hence, if the principal gets informed, then, in equilibrium, the principal will undertake the investment in states $s \in S^{\tilde{\ }k}$ at date 2 and, in date 3, will offer contract $\bar{C}$ to the agent, which he accepts.[14]

If, on the other hand, the principal remains uninformed at date 1, then the equilibrium strategies will be identical to the equilibrium strategies from the full information game when the principal remains uninformed: The principal will invest in the project at date 2 and at date 3 will offer contract C^*_υ, and the agent accepts any contract that provides an expected payoff greater than or equal to $\bar{u}$.

When the principal is choosing between acquiring information or not, he is effectively choosing between having everyone know the state of the world and having nobody know the state of the world. Because the informed principal knows the true state of the world and makes all of the contract offers, he will only offer a contract that is different from one that guarantees the agent his reservation utility only because he (the principal) is trying to give the agent something less than his reservation utility. In such a situation the agent will reject the contract offer. The "best" contract offer that the informed principal can make is $\bar{C}$, which is the equilibrium contract offer in the full information structure. If the principal chooses not the get informed, then obviously, nobody will learn the state of the world before the contract offer and the equilibrium contract offer is C^*_υ.

In deciding upon whether or not to get informed, the principal merely compares the (equilibrium) payoffs associated with getting informed and remaining uninformed. If $\bar{\Pi}_i - c > \Pi^*_\upsilon$, then the payoff to getting informed exceeds that of remaining uninformed, so the principal will get informed at date 1. If $\Pi^*_\upsilon \geq \bar{\Pi}_i - c$, then the principal will remain uninformed.

4.1 Discussion: Efficiency

Because information is symmetric after date 3, *ex post* allocations are always efficient in the sense that the marginal product of labor equals the agent's marginal rate of substitution between consumption and leisure in all states of the world. If the

[13] In states in which the principal receives $\bar{\pi}(s)$, the agent's payoff will be $\bar{u}$; in states in which the principal receives strictly more than $\bar{\pi}(s)$, the agent's payoff will be strictly less than $\bar{u}$. Hence, the agent's expected payoff will be less than $\bar{u}$.

[14] A formal proof that contract $\bar{C}$ is the equilibrium contract offer when the principal gets informed is provided in appendix I.

principal gets informed, then his investment decision will be socially optimal in the sense that the principal does not invest in the project in the *ex post* unprofitable states $s \in S^k$ and always invests in states $s \in S^{\tilde{}k}$. When the principal remains uninformed, then there is "perfect" risk sharing. However, when he gets informed, the optimal contract effectively abandons any notion of risk sharing. This abandonment of risk sharing when the principal get informed—which arises for incentive reasons—has implications for the efficiency of information acquisition and for the (unconditional) efficiency of investment.

The principal will, "on average," underinvest in information acquisition compared to what is socially optimal when operating in the partial information structure. To see this, note that it is socially optimal for the principal to get informed if $\Pi_i^* - c > \Pi_\upsilon^*$ or if

$$\Pi_i^* - \Pi_\upsilon^* > c. \tag{13}$$

However, under the partial information structure, the principal will *not* get informed if $\Pi_\upsilon^* \geq \bar{\Pi}_i - c$ or if

$$\bar{\Pi}_i - \Pi_\upsilon^* \leq c. \tag{14}$$

Since $\Pi_i^* > \bar{\Pi}_i$ it is possible for inequalities (13) and (14) to be simultaneously satisfied, which means that the principal may not get informed even though it is socially optimal to do so. The principal will, however, always remain uninformed when it is socially optimal to do so. The principal will remain uninformed if $\Pi_\upsilon^* - c \geq \bar{\Pi}_i$ and it is socially optimal for the principal to remain uninformed if $\Pi_\upsilon^* - c \geq \Pi_i^*$; the latter inequality implies the former because $\Pi_i^* > \bar{\Pi}_i$.

The principal underinvests in information acquisition because it is "costly" for him to do so. Compared to the planner's solution, where contract C_ι^* is able to achieve some degree of risk sharing when the principal gets informed, the notion of risk sharing is effectively abandoned when the principal gets informed and offers contract $\bar{C}$. (Contract $\bar{C}$ is designed to ensure that the agent gets a level of utility $\bar{u}$ is every state of the world in which the principal undertakes the investment project.) Hence, compared to the planner's outcome, it is more costly for the principal to get informed and, as a result, the principal will on average underinvest in information acquisition. A direct implication of the principal's reluctance to get informed is that on average too many "bad" projects will be undertaken compared to the social optimum. That is, when inequalities (13) and (14) both hold, it is socially optimal for the principal to get informed, but he will choose to remain uninformed. As a result, the principal will invest in the project *and* will end up producing in states $s \in S^k$, states of the world in which the investment would not have been undertaken had the principal got informed.

If the principal could somehow commit—say, prior to deciding whether or not the get informed—to offering a particular date 3 contract, then both his information acquisition and project investment decisions would be socially optimal. For example, if it is socially optimal to get informed and the principal can commit to offer contract

C_ι^* at date 3, then he will, in fact, get informed. However, if the principal cannot credibly commit to future actions, then he will be unable to offer contract C_ι^* in any equilibrium since he will only offer this contract if it is advantageous to him; otherwise he will offer contract $\bar{C}$, i.e., there exists states s where $\pi_\iota^*(s) < \bar{\pi}(s)$. So the social loss associated with principal's information acquisition and investment decision can be linked to his inability to commit to future actions.

5 Equilibrium Contracts Under Asymmetric Information

In the previous section, the agent knows whether or not the principal got informed when the contract is offered. In this section, it is assumed that the agent does not know whether or not the principal gets informed. As it turns out, the change in the information structure does not affect the equilibrium contract offer when the principal gets informed, but, in general, will affect the equilibrium contract offer when he stays uninformed. Since the payoff to the informed principal is unchanged and the payoff to the uninformed principal may change, the efficiency associated with both information gathering and investment may be different under the two information structures. Hence, unlike the case of the planner (or that of a principal who can commit), equilibrium outcomes will generally *not* be invariant to the information structure of the economy.

I will start with the straightforward case in which the equilibrium features the principal getting informed.[15] The equilibrium strategies here are identical to those under the partial information structure when the principal gets informed: At date 1 the principal gets informed; he undertakes the investment project at date 2 if $s \in S^{\tilde{}k}$ and; at date 3, offers the agent contract $\bar{C}$. The agent will accept any contract $\tilde{C}$, at date 3, that is characterized by $u(\tilde{w}(s), \tilde{\ell}(s)) \geq \bar{u}$ for all $s \in S^I$; otherwise the agent will reject the contract. The intuition that underlies these equilibrium strategies is the same as in the partial information structure. When the principal gets informed, it is as if the agent also observes the state of the world. As a result, the "best" acceptable contract that the principal can offer is one that pays the agent his reservation utility in each state of the world.

Now suppose now that there exists an equilibrium to the game where at date 1 the principal chooses to stay uninformed, invests in the project at date 2 and, at date 3, offers the contract $\tilde{C}_\upsilon = \{(\tilde{w}_\upsilon(s), \tilde{\ell}_\upsilon(s))\}_{s=1}^N$, which the agent accepts. Denote the equilibrium payoff that the principal receives as $\tilde{\Pi}_\upsilon = \sum_{s=1}^N p(s)\tilde{\pi}_\upsilon(s)$, where $\tilde{\pi}_\upsilon(s) = \theta(s)\tilde{\ell}_\upsilon(s) - \tilde{w}_\upsilon(s) - I$. If this is indeed an equilibrium, then the principal should have no incentive to get informed. A *necessary* condition that ensures that the principal will have no incentive to get informed is that the equilibrium payoff to the principal must exceed the payoff associated with the principal getting informed

[15] The conditions under which this equilibrium exists will be provided below.

and, on the basis of what he learns, choosing one of the following actions: (i) do not undertake the investment at date 2; (ii) undertake the investment at date 2 and offer contract $\tilde{C}_v$ at date 3; or (iii) undertake the investment at date 2 and offer contract $\bar{C}$ at date 3. Note that the agent will always accept either contract offer $\tilde{C}_v$ or contract offer $\bar{C}$ because contract $\tilde{C}_v$ is the (proposed) equilibrium contract offer and contract $\bar{C}$ guarantees the agent his reservation utility in each state of the world.

At this point it will be helpful to define three disjoint sets of states of the world,

1. Let S^{NI} represent the sets of states where the principal does not undertake the investment. The principal would not under take the investment in state s if both contracts $\tilde{C}_v$ and $\bar{C}$ generate a negative payoff, i.e.,

$$S^{NI} = \{s \mid \tilde{\pi}_v(s) < 0 \text{ and } \bar{\pi}(s) < 0\}$$

2. Let $\tilde{S}^I$ represent the set of states where the principal undertakes the investment and offers contract $\tilde{C}_v$. The principal offers contract $\tilde{C}_v$ instead of contract $\bar{C}$ in state s because the former contract generates a higher payoff, i.e.,

$$\tilde{S}^I = \{s \mid \tilde{\pi}_v(s) \geq \bar{\pi}(s)\} \text{ and } \tilde{\pi}_v(s) \geq 0\}$$

3. Let $\bar{S}^I$ represent the set of states where the principal undertakes the investment and offers contract $\bar{C}$. The principal offers contract $\bar{C}$ instead of contract $\tilde{C}_v$ in state s because the former contract generates a higher payoff, i.e.,

$$\bar{S}^I = \{s \mid \bar{\pi}(s) > \tilde{\pi}_v(s)\} \text{ and } \bar{\pi}(s) > 0\}$$

Note that $S^{NI} \cup \tilde{S}^I \cup \bar{S}^I = S$. If the principal defects from proposed play and becomes informed, then—conditional on offering either contract $\tilde{C}_v$ or $\bar{C}$ in the event that he invests in the project—the *optimal defection strategy* in terms of date 2 investment and the date 3 contract offer is,

1. do not invest at date 2 if $s \in S^{NI}$.
2. invest in the project at date 2 and offer contract $\tilde{C}_v$ if $s \in \tilde{S}^I$.
3. invest in the project at date 2 and offer contract $\bar{C}$, if $s \in \bar{S}^I$.

If the expected payoff associated with the optimal defection strategy exceeds the principal's proposed equilibrium payoff, then the proposed equilibrium can not be an equilibrium. Hence, a *necessary* condition for the principal to stay uninformed and offer contract $\tilde{C}_v$ is,

$$\tilde{\Pi}_v \geq \sum_{s \in \tilde{S}^I} p(s)\tilde{\pi}_v(s) + \sum_{s \in \bar{S}^I} p(s)\bar{\pi}(s) - c, \tag{15}$$

where the right-hand represents the payoff from the optimal defection strategy. I will define the set of contracts that simultaneously satisfy inequality (15) and (2), the agent's participation constraint, by $\mathcal{C}$. Without loss, assume that all contracts in $\mathcal{C}$ are characterized by the equality of the agent's marginal rate of substitution with the marginal product of labor.

Clearly, if $\mathcal{C} = \emptyset$, then the unique equilibrium to the game is characterized by the principal getting informed, investing in the project if $s \in S^{\tilde{}k}$ and offering contract $\bar{C}$ to the agent.

I now consider what the equilibrium outcome will be when $\mathcal{C} \neq \emptyset$. A *sufficient* (but not necessary) condition for $\mathcal{C} \neq \emptyset$ is that $\bar{\Pi}_\upsilon > \bar{\Pi}_\iota - c$. When $\bar{\Pi}_\upsilon > \bar{\Pi}_\iota - c$, condition (15) will be satisfied for $\tilde{C}_\upsilon = \bar{C}$ and, by construction, contract $\bar{C}$ provides the agent with utility $\bar{u}$, i.e., $\bar{C} \in \mathcal{C}$. In fact, when $\bar{\Pi}_\upsilon > \bar{\Pi}_\iota - c$, there exist contracts $\tilde{C}_\upsilon \neq \bar{C}$ that belong to the set $\mathcal{C}$.[16] When condition $\bar{\Pi}_\upsilon > \bar{\Pi}_\iota - c$ is satisfied, there can not exist an equilibrium where the principal gets informed. If there did exist such an equilibrium, then the equilibrium would be characterized by the principal offering contract $\bar{C}$ in the event that he invests in the project at date 2. But in this situation the principal could always make himself better off by simply *not* getting informed at date 1, investing in the project at date 2 and offering contract $\bar{C}$ at date 3. Hence, when $\bar{\Pi}_\upsilon > \bar{\Pi}_\iota - c$, any equilibrium must be characterized by the principal choosing to remain uninformed at date 1.

Since there exists more than one contract in $\mathcal{C}$ when $\bar{\Pi}_\upsilon \geq \bar{\Pi}_\iota - c$, the issue of multiple equilibria arises. I will use the Cho-Kreps refinement—and the belief structure that is implied it—in an attempt to reduce the number of potential equilibrium. It turns, however, that the Cho-Kreps refinement does not eliminate all but one contract from the set $\mathcal{C}$, i.e., there exist multiple equilibria if one uses the Cho-Kreps refinement. The discussion and demonstration of this result is relegated to appendix II. I will define $\mathcal{C}^{ck} \subseteq \mathcal{C}$ as the set of contracts that survive the Cho-Kreps refinement.

I will now examine the case where $\bar{\Pi}_\iota - c \geq \bar{\Pi}_\upsilon$ and $\mathcal{C} \neq \emptyset$.[17] Note that when

[16] Consider, for example, a contract, $\tilde{C}_\upsilon$, that provides the agent with a level of utility slightly greater than $\bar{u}$ in state j, slightly less than $\bar{u}$ in state $j+i$, where $i \geq 1$, and exactly $\bar{u}$ in all other states. As well, the expected utility associated with contract $\tilde{C}_\upsilon$ is $\bar{u}$. The contract allocations in states j and $j+i$ for contract $\tilde{C}_\upsilon$ are constructed in a manner so that allocations$(\tilde{w}_\upsilon(j), \tilde{\ell}_\upsilon(j))$ and $(\tilde{w}_\upsilon(j+i), \tilde{\ell}_\upsilon(j+i))$ have better risk sharing features than allocations $(\bar{\omega}(j), \bar{\ell}(j))$ and $(\bar{\omega}(j+i), \bar{\ell}(j+i))$. If contract $\tilde{C}_\upsilon$ is constructed so that $j, j+i \in S^k$ and the *ex post* payoff to the entrepreneur is negative in states j and $j+1$, then it will be the case that $\tilde{\Pi}_\upsilon > \bar{\Pi}_\upsilon$. As well, an informed entrepreneur would never have a strict incentive to offer contract $\tilde{C}_\upsilon$, even if this contract would be accepted by the agent. Hence, $\tilde{C}_\upsilon \in \mathcal{C}$. If, alternatively, contract $\tilde{C}_\upsilon$ is constructed so that $j = 1$ and $j+i = S$, then, in contrast to the above constructed contract, we will have $j \in S^k$ and $j+i \in S^{\tilde{}k}$. Hence, as long as allocation $(\tilde{w}_\upsilon(S), \tilde{\ell}_\upsilon(S))$ is not "too far" from allocation $(\bar{w}(S), \bar{\ell}(S))$, it will be the case that contract $\tilde{C}_\upsilon \in \mathcal{C}$.

[17] Both of these conditions can hold simultaneously. The argument here is similar to that provided in footnote 15. Suppose that $\bar{\Pi}_\iota - c = \bar{\Pi}_\upsilon$. Now, consider a contract, $\tilde{C}_\upsilon$, that provides the agent with expected utility $\bar{u}$, where the level of utility slightly greater than $\bar{u}$ in state j, slightly less than $\bar{u}$ in state $j+1$, and exactly $\bar{u}$ in all other states. States j and $j+1$ are chosen from the set S^k. The

the first condition holds with a strict inequality, $\bar{C} \notin \mathcal{C}$. Also note that the principal prefers to stay uniformed and offer a contract $\tilde{C}_\upsilon \in \mathcal{C}$ as opposed to getting informed and offering contract $\bar{C}$ since

$$\tilde{\Pi}_\upsilon \geq \sum_{s \in \tilde{S}^I} p(s)\tilde{\pi}_\upsilon(s) + \sum_{s \in \bar{S}^I} p(s)\bar{\pi}(s) \geq \sum_{s \in S^{\tilde{\ } k}} p(s)\bar{\pi}(s) - c = \bar{\Pi}_\iota - c$$

The issue of multiple equilibria also arises when $\mathcal{C} \neq \emptyset$ and $\bar{\Pi}_\iota - c \geq \bar{\Pi}_\upsilon$. But now added to the possibilities is a potential equilibrium that is characterized by the principal getting informed and offering contract $\bar{C}$ if he undertakes the investment. It turns out, however, that such an equilibrium does not survive the Cho-Kreps refinement, (see appendix II).

We are now in a position to summarize and characterize the equilibria that emerge under the asymmetric information structure.

1. If out-of-equilibrium beliefs are generated by the Cho-Kreps refinement, then an equilibrium will be characterized by the principal remaining uninformed if and only if $\mathcal{C} \neq \emptyset$. Any contract $\tilde{C}_\upsilon \in \mathcal{C}^{ck}$ can be an equilibrium contract, where the equilibrium strategies are: (i) the principal remains uninformed at date 1 and offers contract $\tilde{C}_\upsilon \in \mathcal{C}^{ck}$ at date 2; and (ii) the agent accepts any contract offer $\hat{C}$ as long as $u(\hat{w}(s), \hat{\ell}(s)) \geq u(\tilde{w}_\upsilon(s), \tilde{\ell}_\upsilon(s))$ for $\hat{\pi}(s) > 0$.[18]

2. When $\mathcal{C} = \emptyset$, then the equilibrium is unique and is characterized by the principal getting informed. A necessary, but not sufficient, condition for $\mathcal{C} = \emptyset$ is that $\bar{\Pi}_\iota - c > \bar{\Pi}_\upsilon$. The equilibrium strategies are: (i) the principal gets informed at date 1, invests in the project at date 2 if $s \in S^{\tilde{\ } k}$, and offers contract $\bar{C}$ to the worker at date 3 if the investment was undertaken; and (ii) the agent accepts any contract offer that provides a payoff of at least $\bar{u}$ in states $s \in S^{\tilde{\ } k}$.

Finally, note that if $c = 0$, then $\mathcal{C} = \emptyset$ since for any contract $\tilde{C}_\upsilon$ that satisfies

contract allocations in states j and $j+1$ for contract $\tilde{C}_\upsilon$ are constructed so that (i) the allocations $(\tilde{w}_\upsilon(j), \tilde{\ell}_\upsilon(j))$ and $(\tilde{w}_\upsilon(j+1), \tilde{\ell}_\upsilon(j+1))$ have better risk sharing features than allocations $(\bar{\omega}(j), \bar{\ell}(j))$ and $(\bar{\omega}(j+1), \bar{\ell}(j+1))$ and (ii) the *ex post* payoff to the entrepreneur is negative in both states, i.e., $\tilde{\pi}_\upsilon(j), \tilde{\pi}_\upsilon(j+1) < 0$. These conditions imply, respectively, that (i) $\tilde{\Pi}_\upsilon > \bar{\Pi}_\upsilon$, and (ii) if the entrepreneur gets informed, he would never offer contract $\tilde{C}_\upsilon$ if this contract would be accepted by the agent. Hence, $\mathcal{C} \neq \emptyset$ when $\bar{\Pi}_\iota - c = \bar{\Pi}_\upsilon$. By continuity, if $\bar{\Pi}_\iota - c$ is only slightly greater than $\bar{\Pi}_\upsilon$ it is possible to construct a contract, $\tilde{C}_\upsilon$, in the manner described above that gives an expected payoff to the entrepreneur which is greater than $\bar{\Pi}_\iota - c$ and an informed entrepreneur would never offer $\tilde{C}_\upsilon$ even if the agent would accept it, i.e., $\mathcal{C} \neq \emptyset$ when $\bar{\Pi}_\iota - c > \bar{\Pi}_\upsilon$. Note that it *can not* be the case that contract $\tilde{C}_\upsilon$ is characterized by $\tilde{\pi}_\upsilon(s) > \bar{\pi}(s)$ and $\tilde{\pi}_\upsilon(s) > 0$ for some $s \in S$ since such a contract would violate inequality (15).

[18] When the contract offer survives the Cho-Kreps refinement, then the only out-of-equilibrium contract offers that the agent would accept are those that provide a higher state contingent payoff, compared to the equilibrium contract offer, in states where the informed principal would have an incentive to invest.

$\sum_{s\in S} u(\tilde{w}_v(s), \tilde{\ell}_v(s)) = \bar{u}$, it will be the case that

$$\sum_{s\in\tilde{S}^I} \tilde{\pi}_v(s) + \sum_{s\in\bar{S}^I} \bar{\pi}(s) > \tilde{\Pi}_v. \tag{16}$$

If $\tilde{C}_v = \bar{C}$, then inequality (16) reduces to $\bar{\Pi}_\iota > \bar{\Pi}_v$, which is a valid one. If $\tilde{C}_v \neq \bar{C}$, then $\bar{S}^I \neq \emptyset$, which means that inequality (16) is valid. Hence, if the cost of getting informed is sufficiently small, then the principal will always get informed under the asymmetric information structure.[19]

5.1 Discussion: Multiple Equilibria and Efficiency

In some contexts the existence of multiple equilibria is problematic because it implies that the model lacks predictive content. In the context of the present model, however, it is rather important to emphasize that all of the potential equilibrium contracts when the principal remains uninformed, i.e., all contracts in $\mathcal{C}^{ck}$, make the same (qualitative) predictions in regard to the efficiency properties associated with both information acquisition and investment. Hence, the possible existence of multiple equilibria does not diminish the insights provided by the model. The efficiency properties associated with information acquisition and investment will now be documented.

The asymmetric information structure shares some of the same implications for efficiency as the partial information structure. In both information structures *ex post* allocations are always efficient. As well, when the principal gets informed, his investment decision will be *ex post* socially optimal—i.e., the principal invests only states $s \in S^{\tilde{}k}$—but the optimal contract under both information structures, $\bar{C}$, abandons any notion of risk sharing.

When the principal remains uninformed, the contract that he offers the agent will generally differ between the two information structures. Recall that under the partial information structure, the contract that the uninformed principal offers the agent, C_v^*, is characterized by perfect risk sharing and corresponds to the contract that a planner would design. Under the asymmetric information structure, the principal will remain uninformed if only if $\mathcal{C} \neq \emptyset$ and any contract $\tilde{C}_v \in \mathcal{C}^{ck}$ can be an equilibrium contract offer. If contract C_v^* does not satisfy inequality (15), i.e., $C_v^* \notin \mathcal{C}^{ck}$, then there does not exist an equilibrium contract that will be characterized by perfect risk sharing. In this situation, it will necessarily be the case that $\tilde{\Pi}_v < \Pi_v^*$ for all $\tilde{C}_v \in \mathcal{C}^{ck}$. If, however, $C_v^* \in \mathcal{C}^{ck}$, then there will exist an equilibrium in the asymmetric information structure that is identical to the (unique) equilibrium in the partial equilibrium structure. Since the Cho-Kreps refinement is not able to eliminate all but the "good" equilibrium, all that can be said in terms of comparing

[19] Kim (2002, Proposition 7) finds that in an asymmetric information structure, the principal will *always* get informed. But his results hinges on imposing the condition that $c = 0$.

the expected payoffs of the uninformed principal under the two information structures is that $\tilde{\Pi}_\upsilon \leq \Pi^*_\upsilon$.

Compared to the partial information structure, there will be "on average" a greater level of information acquisition under the asymmetric information structure. To see this, note that under the asymmetric information structure if $\mathcal{C}^{ck} \neq \emptyset$, then the principal will remain uninformed and will receive a payoff of $\tilde{\Pi}_\upsilon$, where $\tilde{\Pi}_\upsilon > \bar{\Pi}_\iota - c$. Under the partial information structure, the principal will remain uninformed if $\Pi^*_\upsilon \geq \bar{\Pi}_\iota - c$. Since, $\Pi^*_\upsilon \geq \tilde{\Pi}_\upsilon$, if the principal remains uninformed under the asymmetric information structure, then he will also remain uninformed under the partial equilibrium structure. If $\bar{\Pi}_\iota - c > \Pi^*_\upsilon$, then necessarily, $\mathcal{C} = \emptyset$. Hence, when the principal gets informed under the partial information structure, he will also get informed under the asymmetric information structure. However, it may very well be the case that $\Pi^*_\upsilon > \bar{\Pi}_\iota - c$ and $\mathcal{C} = \emptyset$. This could happen if, for example, c is "small." This configuration implies that under the asymmetric information structure, the equilibrium outcome will have the principal getting informed, while under the partial information structure the equilibrium outcome will have the principal remaining uninformed. Therefore, on average, there will be more information acquisition under the asymmetric information structure than under the partial information structure. The intuition behind why there will be greater information acquisition under the asymmetric information structure is rather straightforward. Under both information structures the payoff to the entrepreneur is $\bar{\Pi}_\iota$ if he gets informed. When the entrepreneur remains uninformed, his payoff will be Π^*_υ under the partial information structure and $\tilde{\Pi}_\upsilon$ under the asymmetric information structure, where $\Pi^*_\upsilon \geq \tilde{\Pi}_\upsilon$. This latter inequality results from the fact that the contract under the asymmetric information structure has to be designed in order to ensure that the principal does not have an incentive to (secretly) get informed. As a result of this, the benefit of remaining uninformed diminishes under the asymmetric information structure and as a result the principal will, on average, acquire more information, compared to the partial information structure.

Under the asymmetric information structure their may be either an over–provision or under–provision of information acquisition from a social perspective. For example, suppose that $\Pi^*_\iota - c > \Pi^*_\upsilon$; hence it is socially optimal for the principal to get informed. It is, however, quite possible that the inequality $\bar{\Pi}_\upsilon > \bar{\Pi}_\iota - c$ also holds, which implies that under the asymmetric information structure the principal will remain uninformed. Similarly, when $\Pi^*_\upsilon > \Pi^*_\iota - c$, it is optimal from a social perspective for the principal to remain uninformed. But is it quite conceivable that $C = \emptyset$, which implies that the principal will get informed under the symmetric information structure. Intuitively, compared to the social optimum, the payoff declines for both the informed and uninformed principal. Depending on the magnitude of the differences in the declines, it is possible to get either an over- or under-provision of information acquisition. And, unlike the partial information structure where, from a social perspective, their will be a tendency for "overinvestment," under the symmetric information structure, depending upon the model parameters, there can be either "overinvestment" or "underinvestment."

As in the case of the partial information structure, if the principal could somehow commit to offering a contract before he makes his information acquisition decision, then he could implement the Pareto optimal allocation and, compared to the equilibrium outcome, can make himself better off. But such commitment is problematic. In a related vein, if, in certain circumstance, the principal could commit to not to get informed or could some how prove that he does not know the state of the world, then, compared to the equilibrium outcome, he can make himself better off. In this case, he would be able to replicate the equilibrium in the partial information structure. But, again, this kind of commitment or proof may be difficult to fulfill in practice.

6 Conclusion

The decision to acquire information involves both costs and benefits. The benefit of acquiring information is that unproductive investments can be avoided. There are two kinds of costs associated with acquiring information: one is the direct cost of getting the information and the other is indirect costs. The indirect costs manifest themselves as restrictions on the kinds of contracts that will be offered in equilibrium. In many circumstances the indirect costs can dominate the direct costs.

An implication of the existence of these indirect costs is that information acquisition may be suboptimal from a social perspective. When agents can tell whether or not the principal got informed, the contract that the informed principal offers the agent is distorted away from the Pareto optimal contract because of the information that the principal possesses. As a result of this there will be an underprovision of information acquisition, since, compared to the social optimum, it is more costly for the principal to get informed. When agents can not tell whether or not the principal got informed, the mere possibility of the principal getting informed distorts the contracts of both informed and uninformed principal. In general, information acquisition will be suboptimal, but depending on the relative magnitudes of these costs, there can be either "too much" or "too little" information acquisition.

The costs that the economy incurs as a result of information acquisition can be linked to the inability of the principal to commit to future actions. If the principal could somehow commit to offer certain contracts, then information structure—partial or asymmetric—that the principal faces would be of no real consequence. Furthermore, the principal would be able to implement the Pareto optimal allocation. It is not at all obvious, however, how the principal would be able to commit to future actions.

Finally, in the text I have examined two polar cases: Either the agent receives a "perfect signal" about the state of the principal's informedness—the partial information structure—or receives "no signal" at all—the asymmetric information structure. Perhaps the relevant case is one where the agent receives a noisy signal pertaining to whether or not the entrepreneur is informed, where the signal is correlated with entrepreneur's actual "information state." By appealing to the results of Bagwell (1995), we can conclude that the equilibrium outcome when the agent receives a noisy signal is identical to the equilibrium outcome under the asymmetric information structure. The intuition behind this result is straightforward. If the equilibrium calls for the entrepreneur to remain uninformed and the agent receives a signal, before the contract offer, that the entrepreneur is informed, then the agent would not conclude that the entrepreneur deviated from equilibrium play because observing the an "incorrect" signal is consistent with equilibrium play, i.e., the signal is noisy. Hence, the principal can defect from proposed play—by, for example, getting informed when the equilibrium calls for him to remain uninformed—and this defection would not be detected by the agent. As a result, unless the signal is perfectly correlated with state of the principal's informedness, in equilibrium, the signal will not provide the agent with any useful information.

Appendix I: Proof that $\bar{C}$ is equilibrium contract offer for an informed principal under the partial information structure.

Suppose that the principal gets informed at date 1 and invests in the project at date 2. If $\bar{C}$ is the date 3 equilibrium contract offer, then one must demonstrate that the principal has no incentive to defect from proposed equilibrium play by offering some other contract at date 3, i.e., one must describe what happens both on and off the equilibrium path. Suppose that the principal gets informed at date 1, invests at date 2 and defects from equilibrium play by offering contract $\hat{C}_\iota \neq \bar{C}$ at date 3. Will agent accept or reject contract offer $\hat{C}_\iota$? If $u(\hat{w}_\iota(s), \hat{\ell}_\iota(s)) \geq \bar{u}$ for all $s \in S$, then, by a strict dominance argument, the agent will accept contract $\hat{C}_\iota$. But, clearly, the principal would never have an incentive to make such a contract offer since the payoff to contract $\hat{C}_\iota$ is less than the equilibrium payoff. If, instead, contract $\hat{C}_\iota$ is characterized by $u(\hat{w}_\iota(s), \hat{\ell}_\iota(s)) < \bar{u}$ for some state(s) $s \in S^I$ and $u(\hat{w}_\iota(s), \hat{\ell}_\iota(s)) \geq \bar{u}$ for other state(s) $s \in S^I$, then agent must form some (out-of-equilibrium) beliefs in order to be able to either accept or reject the contract offer $\hat{C}_\iota$. I assume that the agent form believes in a manner that is implied by the Cho-Kreps refinement. Specifically, the agent first determines, relative to the proposed equilibrium contract offer, which type of principal would have an incentive to offer contract $\hat{C}_\iota$ if the agent were to accept contract $\hat{C}_\iota$. Let S^d represent the set of principal types that have an incentive to offer contract $\hat{C}_\iota$, i.e., $S^d = \{s | \hat{\pi}_\iota(s) > \bar{\pi}(s)\}$. According to the Cho-Kreps procedure, the agent will place *some* probability distribution over the set S^d, $\{p^d(s)\}_{s \in S^d}$, where $p^d(s) \geq 0$ and $\sum_{s \in S^d} p^d(s) = 1$, and will reject the contract offer if

$$\sum_{s \in S^d} p^d(s) u(\hat{w}_\iota(s), \hat{\ell}_\iota(s)) < \bar{u} \tag{17}$$

But *any* probability distribution $\{p^d(s)\}_{s \in S^d}$ will satisfy inequality (17) since $u(\hat{w}_\iota(s), \hat{\ell}_\iota(s)) < u((\bar{w}(s), \bar{\ell}(s))$ for all $s \in S^d$. Therefore, if the principal defects from equilibrium play (of offering contract $\bar{C}$ at date 3) and offers contract $\hat{C}_\iota$, where $u(\hat{w}_\iota(s), \hat{\ell}_\iota(s)) < \bar{u}$ for some s, the agent will reject the contract offer. Hence, the principal will have no incentive to defect from equilibrium play of offering contract $\bar{C}$ at date 3 since his payoff will be less than the equilibrium payoff.

Appendix II: The Cho-Kreps Refinement

Refining the set of contracts in $\mathcal{C}$

Suppose that the proposed equilibrium has the principal remaining uninformed and offering contract $\tilde{C}_v \in \mathcal{C}$ at date 3. Instead of offering contract $\tilde{C}_v$ at date 3, the principal defects from proposed equilibrium play and offers contract $\hat{C}$. What is the agent to think? *If* the agent accepts this contract, then an *uninformed* principal would have an incentive to make such an offer if

1. $\hat{\Pi}_v > \tilde{\Pi}_v$.

Similarly, an *informed* principal of type $s \in \hat{S}^I_d$ would have an incentive to make this offer if

2. $\sum_{s\in\hat{S}^I_d} p(s)\hat{\pi}(s) + \sum_{s\in\tilde{S}^I_d} p(s)\tilde{\pi}_v(s) + \sum_{s\in\bar{S}^I_d} p(s)\bar{\pi}(s) - c > \tilde{\Pi}_v$, where

   (i) $\hat{S}^I_d = \{s|\hat{\pi}(s) > 0, \hat{\pi}(s) > \tilde{\pi}_v(s), \text{and } \hat{\pi}(s) > \bar{\pi}(s)\}$

   (ii) $\tilde{S}^I_d = \{s|\tilde{\pi}_v(s) \geq 0, \tilde{\pi}_v(s) \geq \hat{\pi}(s), \text{and } \tilde{\pi}_v(s) \geq \bar{\pi}(s)\}$, and

   (iii) $\bar{S}^I_d = \{s|\bar{\pi}(s) > 0, \bar{\pi}(s) > \tilde{\pi}_v(s), \text{and } \bar{\pi}(s) > \hat{\pi}(s)\}$.

The left hand side of the inequality represents the expected payoff to a principal who, *before he learns the state of the world,* defects from proposed equilibrium play by getting informed and offering contract $\hat{C}$ if his type turns out to be $s \in \hat{S}^I_d$. Note that if $s \notin \hat{S}^I_d$, then the principal: (i) will not invest at all if he is of type $s \in (\hat{S}^I_d \cup \tilde{S}^I_d \cup \bar{S}^I_d)\backslash S$; (ii) will invest and offer contract $\tilde{C}_v$ if he is of type $s \in \tilde{S}^I_d$; and (iii) will invest and offer contract $\bar{C}$ if he is of type $s \in \bar{S}^I_d$.

If condition 2. does not obtain and condition 1 holds, then the agent must conclude that the principal is uninformed. According to the Cho-Kreps refinement, the agent must believe with probability one that the principal is uninformed. Hence, the agent will accept contract offer $\hat{C}$ only if $\sum_{s\in S} p(s)u(\hat{w}(s), \hat{\ell}(s)) \geq \bar{u}$. If the agent accepts contract $\hat{C}$, then the proposed equilibrium contract offer $\tilde{C}_v$ will be eliminated as a potential equilibrium contract offer. If, however, contract $\hat{C}$ satisfies condition 2.—and may or may not satisfy condition 1.—and the informed principal offers contract $\hat{C}$, then the agent will reject contract $\hat{C}$ since the Cho-Kreps refinement allows him to put all of his probability weight on those types of principals that belong to the set $\hat{S}^I_d$. (Note that $u(\hat{w}(s), \hat{\ell}(s)) < \bar{u}$ for all $s \in \hat{S}^I_d$, where $(\hat{w}(s), \hat{\ell}(s)) \in \hat{C}$.) Denote the set of contacts that satisfy the Cho-Kreps refinement as $\mathcal{C}^{ck} \subseteq \mathcal{C}$.

The Cho-Kreps refinement can eliminate some contracts as possible equilibria from the set $\mathcal{C}$. Suppose that for contract $\tilde{C}^1_v \in \mathcal{C}$, where $\tilde{C}^1_v \neq C^*_v$, (15) holds as a strict inequality. Consider an alternative contract $\hat{C}$ whose state contingent wage–labor allocations are "very close" to that of $\tilde{C}^1_v$ but have better risk-sharing properties. As well, contract $\hat{C}$ is characterized by $\sum_{s\in S} p(s)u(\hat{w}(s), \hat{\ell}(s)) = \bar{u}$. Since contract $\hat{C}$ has better risk-sharing properties, inequality 1. above will hold. On the other hand, since contract $\hat{C}$ is "very close" to contract $\tilde{C}_v^1$ inequality 2. will not obtain. If the principal defects from proposed equilibrium play, of offering contract $\tilde{C}_v^1$ at date 3, and instead offers contract $\hat{C}$, the agent must conclude that the principal is uninformed and will accept the contract. As a result, contract $\tilde{C}_v^1$ is eliminated as a possible equilibrium contract, i.e., $\tilde{C}_v^1 \notin \mathcal{C}^{ck}$. Hence, for all $\tilde{C}_v \in \mathcal{C}^{ck}$, where $\tilde{C}_v \neq C_v^*$, (15) must hold as a strict equality.

Every contract that belongs to the set $\mathcal{C}$ and satisfies (15) with a strict equality may not necessarily survive the Cho-Kreps refinement. To see this, suppose contract $\tilde{C}_v^1 \in \mathcal{C}$ satisfies (15) with an equality and that there exists another contract $\hat{C} \in \mathcal{C}$ that satisfies (15) with an equality that has better risk-sharing features than contract $\tilde{C}_v^1$. In particular, the state contingent wage–labor allocations are such that in states of the world where the principal's payoff is non-negative, the payoff to the principal is never greater under contract $\hat{C}$, i.e., $\tilde{\pi}_v^1(s) \geq \hat{\pi}(s)$ or $\bar{\pi}(s) \geq \hat{\pi}(s)$ for all s where $\tilde{\pi}_v^1(s) \geq 0$ and/or $\bar{\pi}(s) \geq 0$, and payoffs to the principal under contract $\hat{C}$ may be greater when payoffs are negative, i.e., there exist some s such that $\hat{\pi}(s) > \tilde{\pi}_v^1(s)$) and $\hat{\pi}(s) > \bar{\pi}(s)$, where $\hat{\pi}(s) < 0$. Here the principal has no incentive to defect from equilibrium play and get informed and offer contract $\hat{C}$ because his expected payoff will be no greater than and possibly less than the payoff associated with the proposed equilibrium, i.e., condition 2. above does not hold.[20] But an uninformed principal has an incentive to offer contract $\hat{C}$ because contract $\hat{C}$ has better risk-sharing properties than contract $\tilde{C}_v^1$, i.e., condition 1. above does hold. Therefore, contract $\tilde{C}_v^1$ can not be an equilibrium contract offer, i.e., $\tilde{C}_v^1 \notin \mathcal{C}^{ck}$.

Finally, the Cho-Kreps refinement cannot eliminate the following contract, $\tilde{C}_v^1$. Suppose contract $\tilde{C}_v^1 \in \mathcal{C}$ satisfies (15) with an equality and that there exists another contract $\hat{C} \in \mathcal{C}$ that satisfies (15) with an equality but has better risk-sharing features than contract $\tilde{C}_v^1$. However, unlike above, there exist states s where $\hat{\pi}(s) > \tilde{\pi}_v^1(s)$ and $\hat{\pi}(s) > \bar{\pi}(s)$, where $\hat{\pi}(s) > 0$, i.e., $\hat{S}_d^I \neq \emptyset$. Condition 2. above holds for contract $\hat{C}$, i.e., if contract $\hat{C}$ is accepted by the agent, the principal will defect from equilibrium play and get informed. Hence, if contract $\hat{C}$ is offered, the agent can belief that the principal is informed and that his type comes from the set $\hat{S}_d^I$. But the payoff that the agent receives from any $s \in \hat{S}_d^I$ is strictly less than $\bar{u}$, meaning that the agent will reject the contract offer. Hence, $\tilde{C}_v^1 \in \mathcal{C}^{ck}$.

In summary, except for contract C_v^*, all $\tilde{C}_v \in \mathcal{C}^{ck}$ satisfy (15) with a strict equality and for any pair $\tilde{C}_v^1, \tilde{C}_v^2 \in \mathcal{C}^{ck}$, it must be the case that there exists states s^1 and s^2 such that, (i) $\tilde{\pi}_v^1(s^1) > \tilde{\pi}_v^2(s^1)$ and $\tilde{\pi}_v^1(s^1) > \bar{\pi}(s^1)$, where $\tilde{\pi}_v^1(s^1) > 0$ and (ii) $\tilde{\pi}_v^2(s^2) > \tilde{\pi}_v^1(s^2)$ and $\tilde{\pi}_v^2(s^2) > \bar{\pi}(s^2)$, where $\tilde{\pi}_v^2(s^2) > 0$.

Getting informed when $\mathcal{C} \neq \emptyset$

When $\mathcal{C} \neq \emptyset$ and $\bar{\Pi}_\iota - c \geq \bar{\Pi}_v$, there exists a potential equilibrium where the principal gets informed at date 1 and offers contract $\bar{C}$ at date 3 in the event that

[20] Another way of putting it is that for contracts $\tilde{C}_v^1$ and $\hat{C}$, $\hat{S}_d^I = \emptyset$.

he undertakes the investment. Such an equilibrium does not survive the Cho-Kreps refinement. To see this, note that for all $\tilde{C}_v \in \mathcal{C}$, $\tilde{\Pi}_v > \bar{\Pi}_\iota - c$ and, as pointed out in footnote 15, any contract $\tilde{C}_v \in \mathcal{C}$ is characterized, in part, by $\tilde{\pi}_v(s) > \bar{\pi}(s)$ only if $\tilde{\pi}_v(s) < 0$. The proposed equilibrium has the principal getting informed and offering contract $\bar{C}$ if he undertakes the investment. Suppose, instead, the principal offers a contract $\tilde{C}_v \in \mathcal{C}$. An uninformed principal would have an incentive to offer this contract. Since the state contingent payoffs for contract $\tilde{C}_v$ are greater that the state contingent payoffs for contract $\bar{C}$ only in states for which an informed principal would not undertake the investment, an informed principal would have no incentive to offer contract $\tilde{C}_v$. According to the Cho-Kreps refinement, the agent must believe with probability one that the principal is uninformed. Hence, the agent will accept contract $\tilde{C}_v$, which implies that the proposed equilibrium is not an equilibrium.

References

1. Bagwell, K., 1995, "Commitment and observability in games," *Games and Economic Behavior*, pp. 271-280.

2. Crémer, J., 1995, "Arm's length relationships," *Quarterly Journal of Economics*, pp. 275-295.

3. Crémer, J. and F. Khalil, 1992, "Gathering information before signing a contract," *American Economic Review*, pp. 566-578.

4. Crémer, J., F. Khalil, J.-C. Rochet, 1998, "Strategic information gathering before a contract is offered," *Journal of Economic Theory*, pp. 163-200.

5. Dewatripont, M. and E. Maskin, "Credit and efficiency in centralized and decentralized economies," *Review of Economic Studies*, pp. 541-555.

6. Holmstrom, B., 1979, "Moral Hazard and Observability," *Bell Journal of Economics*, pp. 74-91.

7. Kessler, A., 1998, "The value of ignorance," *RAND Journal of Economics*, pp. 339-354.

8. Kim, S. K., 2002, "Should principals have information ex-ante or ex-post?" mimeo, Seoul National University.

9. Maskin, E., and J. Tirole, 1990, "The principal-agent relationship with an informed principal: The case of private values,"*Econometrica*, pp. 379-410.

10. Maskin, E., and J. Tirole, 1992, "The principal-agent relationship with an informed principal II: Common values," *Econometrica*, pp. 1-42.

11. Sobel, J., 1993, "Information control in the principal-agent problem," *International Economic Review*, pp. 259-269.

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