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TOBIN'S Q, INVESTMENT, AND THE ENDOGENOUS ADJUSTMENT OF FINANCIAL STRUCTURE

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ABSTRACT

This paper asks whether q theory must be modified to take account of financial structure by analyzing a q model of investment in which financial structure affects firm value. The model is a perfect foresight model of general equilibrium similar to that of Brock and Turnovsky (1981), except that there is a debt-related agency cost. The combination of the agency cost and taxes that favor debt yields an interior solution for the debt-to-equity ratio, which varies endogenously. We find that although q is influenced by financial structure, it is still a "sufficient statistic" for investment. However, the model implies that analyses which ignore the endogenous adjustment of financial structure will systematically err in predicting investment.

To illustrate these points, we examine the comparative statics and dynamics of changing the corporate tax rate. An unanticipated increase in the corporate tax rate lowers the cost of capital. The presence of real agency costs, however, may cause the capital stock to decline.
I. Introduction

Analyses of the link between financial markets and the investment decisions of firms have generally differed between the fields of macroeconomics and finance. Macroeconomic analyses of investment now focus on the relation between Tobin's q and the rate of investment. In the field of finance, analyses tend to focus on the relations between the cost of capital, firms' value, and firms' financing decisions in the presence of capital market imperfections. These latter analyses typically describe financing decisions by the leverage or debt-to-equity ratio.¹

Most q models (for example, see Abel and Blanchard [1983], Hayashi [1982], or Von Furstenberg [1977]) assume that neither the market value of a firm nor its cost of capital is affected by the decision of how investment is financed. Analyses of capital market imperfections (see Bradley, Jarrell, and Kim [1984]), on the other hand, fail to provide insight about the dynamic relation between financial structure and investment. In this paper, we investigate whether q theory needs to be modified to take account of financial structure.

The failure of q to perform well empirically is another source of motivation to modify q theory. Although q theory implies that past values of q should not matter in a regression of investment rates on current and past q, most empirical tests (see Abel and Blanchard [1983], Hayashi [1982], or Von Furstenberg [1977]) find that current q has low explanatory power and that residuals are highly correlated. These empirical results suggest that we examine links between financial markets and investment other than q.²
In this paper, the debt-to-equity ratio affects firm value and is determinate, lying between 0 and infinity. The optimal debt-to-equity ratio, chosen by firms, is determined by agency costs of debt together with tax rates favoring debt. The agency cost of debt arises from the presence of bond covenants and other legal or institutional restrictions on firms. Jensen and Meckling (1976) and others have shown how agency problems may be related to financial structure.³ Tax rates favor debt issue, since corporate interest payments are tax-deductible for firms. Differences between the personal income tax rate, the capital gains tax rate, and the corporate tax rate have all been cited as determinants of an optimal financial structure for firms and tax clienteles among investors (see Miller (1977)). Reasonable values for all three tax rates imply that tax rates favor debt over equity.

The model utilizes frameworks developed by Abel and Blanchard (1983) and Brock and Turnovsky (1981). In this dynamic, general equilibrium model of savings and investment, the relation between investors' portfolio decisions and the decisions of firms is clearly exposed. The link between households and firms is the cost of capital, driven by the rates of return required by households. Firms, facing tax rates favoring debt, and a cost of debt, choose the debt-to-equity ratio in order to to minimize the cost of capital. Variations in tax rates or in the interest rate affect the trade-off between debt and equity and, thus, the optimal debt-to-equity ratio.

We find that even if financial structure affects firm value, q is a "sufficient statistic" for investment. Thus, unlike Chirinko (1987), we conclude that financial structure does not explain the poor performance of Tobin's q. Financial structure affects q, however, because q is the present discounted value of after-tax marginal products of capital and because the discount rate (cost of capital) varies with the debt-to-equity ratio. Indeed,
the endogenous adjustment of financial structure has real effects. For example, when interest rates rise, firms offset part of the effect on the cost of capital by increasing the debt-to-equity ratio to take advantage of the interest-deductibility of debt. This implies that models ignoring real effects of financial structure will systematically err in predicting investment.

In addition, one factor that may potentially break the link between marginal q and investment is uncovered. Unanticipated changes in the corporate tax rate may cause marginal q and investment to move in opposite directions. The comparative statics and dynamics of an unanticipated change in the corporate tax rate are analyzed to demonstrate the significance of the endogenous adjustment of financial structure.

II. Description of the Model

The model is a variant of the perfect foresight models of general equilibrium of Abel-Blanchard (A-B) and Brock-Turnovsky (B-T). Following B-T, I indicate the way in which consumers' demand for savings influences the cost of capital. The appropriate form for the cost of capital is determined explicitly from the firm's assumed objective. The incorporation of adjustment costs in the model is in the spirit of A-B. Costly financial structure provides the unique link between financial markets and real decisions.

The model consists of three sectors: consumers, firms, and government. Because all consumers are assumed to be identical, the analysis is conducted in terms of a representative consumer. Similarly, all firms are assumed to be identical, and the corporate sector is aggregated to a single firm. Both the consumer and the firm are infinitely lived and solve explicit maximization problems by making forecasts of variables relevant to these decisions. I
assume that all expectations or forecasts are fulfilled, that all markets
clear, and that all supply and demand functions of firms and households are
derived from maximizing behavior under perfect competition.

The consumer saves in the form of either bonds or equities. Being
concerned only with rates of return on alternative assets, the consumer's
problem implies that if debt and equity are to coexist, then after-tax rates
of return must be equal. Since the returns are equal, the consumer would seem
to be indifferent about the debt-to-equity ratio. The firm, however, is not
indifferent about the debt-to-equity ratio.

The firm maximizes its market value by choosing sequences of notional
demands and supplies. Given a set of financial/accounting constraints, market
value maximization is equivalent to maximization of a particular present
discounted value. The discount rate incorporates the rate of return required
by the consumer, the tax rates facing the consumer and the firm, and the
agency cost of debt. Because of the presence of an agency cost and a tax
structure that favors debt, there is an optimal debt-to-equity ratio which
minimizes the cost of capital and lies between 0 and infinity. Without the
agency cost, the optimal debt-to-equity ratio would be 0 or infinity,
depending on the tax rates.

The government sets the tax rates $\tau_y$, $\tau_p$, and $\tau_c$, and consumes
consumption goods, but issues no money or debt. The government income
statement identity requires that the lump-sum tax $\tau$ vary endogenously.

III. The Structure

A. The Consumer

The utility of the consumer is a function only of consumption: there is no
disutility associated with labor. The firm determines employment. Individual
consumers view themselves as unable to influence the size of the lump sum tax. The household's objective is to choose the sequence 

\( \{c_t, b_t^d, E_t^d\}, t = \infty, \infty \) 

to solve

\[
(1) \quad \max_{c_t} \int_0^\infty e^{-\alpha t} U(c_t) \, dt 
\]

subject to

\[
(2) \quad c_t + b_t^d + z_t E_t^d = w_t \ell_t + s_t b_t^d + \tau c [w_t \ell_t + s_t b_t^d + d_t E_t^d] - \tau_y [w_t \ell_t + s_t b_t^d + d_t E_t^d] - \tau \] 

\[
(3) \quad \lim_{t \to \infty} e^{\alpha t} b_t > 0, \lim_{t \to \infty} e^{\alpha t} z_t E_t > 0, 
\]

and

\[
(4) \quad b_t^d = 0, E_t^d = 0. 
\]

c_t = real private consumption  
\( \ell_t = \) real employment  
\( b_t^d = \) demand for real corporate bonds  
\( E_t^d = \) number of shares of equity demand  
\( z_t = \) relative price of equity in terms of output  
\( w_t = \) real wage rate  
\( s_t = \) real interest rate on private bonds  
\( \tau_c = \) tax rate on capital gains  
\( \tau_y = \) tax rate on wage and interest income  
\( \tau = \) lump sum tax determined as residual from government income identity  
\( d = \) dividend payout rate: \( = D/zE \)  
\( D = \) real dividends  
\( \beta = \) rate of time discount  
\( \Theta = \) rate of return on consumption, defined below
The utility function \( U(c_t) \) has the usual concavity properties: \( U'(c_t) > 0, U''(c_t) < 0, U'(0) = \infty \). Expression (2) is the intertemporal budget constraint facing the consumer. Income is received as labor income, interest income, dividend income, and capital gains. Labor, interest, and dividend income are taxed at the rate \( \tau_y \). Capital gains are taxed at the rate \( \tau_c \). All capital gains and losses on equity are immediately realized. With this income, the household consumes or purchases bonds or equities. Expression (3) states that the values of debt and equity are bounded. All bonds mature instantaneously. Expression (4) states initial conditions on the stock of debt and equities. The outcome of this optimization problem will be sequences of notional demands, where unambiguous \( t \) subscripts are omitted.

This problem leads to the formulation of the following Hamiltonian:

\[
H = e^{-\Theta t} U\left\{ \left( \omega d + s b_d + d z E_d \right) (1-\tau_y) - \tau_c z E_d - b_d - z E_d - \tau \right\} + a(b_d) + \xi(E_d)
\]

where \( a \) and \( \xi \) are the costate variables associated with \( b \) and \( E \), respectively.

If both debt and equity are to be held by the household, then after-tax rates of return must be equal. The existence of both debt and equity will be optimal for the firm and, hence, both assets will be held by the household. This implies that condition (6) will hold:

\[
(1-\tau_y) = \Theta = (1-\tau_c) z / z + d(1-\tau_y).
\]

\( \Theta \) is the rate of return on consumption: \( \Theta = \beta - U_{c,c}/U_c \).

**B. The Firm**

The firm maximizes the market value of debt plus equity at time \( t = 0 \), \( V_{t=0} = b_{t=0} + z_{t=0} E_{t=0} \). Given a set of financial and production constraints, and numerical values for \( K_0, b_0, \) and \( E_0 \), maximizing \( V_{t=0} \)
is equivalent to maximizing a particular present discounted value. This equivalence is demonstrated in appendix A. The appropriate discount rate or cost of capital is determined by this equivalence. These constraints are:

(7) \( y^s = F(K^d, \bar{I}^d) \)

(8) \( \pi = y^s - w\varepsilon^d - \psi(I,K^d) \)

(9) \( \pi = sb^s + \tau_p [y^s - w\varepsilon^d - sb^s - \psi(I,K^d)] + RE + a(\lambda)b^s + D \)

(10) \( I = RE + b^s \)

(11) \( I = K^d + 6K^d \)

(12) \( K_{t=0}^d = K_0, E_{t=0}^s = E_0, b_{t=0}^s = b_0 \)

where

\( \bar{I}^d = \) real labor demand by the firm

\( K^d = \) real capital demand by the firm

\( \pi = \) real gross profits

\( 6 = \) exogenous rate of depreciation

\( b^s = \) real supply of corporate bonds

\( \lambda = b/zE, \) the debt-to-equity ratio

\( y^s = \) real output, the supply of goods

\( \psi(\cdot) = \) adjustment cost of investing

\( a(\lambda)b = \) cost of maintaining the bond portfolio

\( RE = \) retained earnings

\( E^s = \) real supply of equities

\( \tau_p = \) corporate tax rate

Expression (7) is a production function with positive but diminishing marginal productivities and constant returns to scale. Expression (8) defines real gross profits as revenue minus the wage bill and adjustment costs associated with gross investment. Expression (9) indicates that gross profits go to bondholders as interest, to the government as taxes, to stockholders as
dividends, into retained earnings, or are absorbed by the agency cost.\textsuperscript{4} Dividends are paid out at a fixed rate \(d\). Interest payments are deducted from taxable income. Expression (10) states that all investment must be financed through retained earnings or debt issue. There is no equity issue, although there is an initial stock of equity, \(E_0\). Expression (11) states that all investment is net investment or replacement investment where depreciation occurs at a constant rate \(6\). There is no deduction for depreciation. Expression (12) states the initial conditions for capital, equity, and debt.

The adjustment cost \(\psi(I,K)\) is deducted from taxable income, since increases in the rate of investment draw resources of the firm away from productive activities.\textsuperscript{5} The adjustment cost function \(\psi(I,K)\) is assumed to be linearly homogenous so that it can be written as \(h(I/K)I\) with the properties \(h(0) = 0, h'(0) > 0, 2h'(0) + (I/K)h''(0) > 0\). This is similar to formulations of adjustment costs proposed by Hayashi (1982) and others.

The cost \(a(\lambda)b\) is assumed not to reduce output by drawing on productive resources. While only the interest rate on debt is tax-deductible, the gross cost of debt to the firm is \([s + g(\lambda)]b\). The size of the firm does not affect this cost directly, but the firm's value does. If the firm's value increases due to increases in equity share prices, the debt-to-equity ratio falls. For a given firm value, increases in debt lead to an increase in the agency cost.

This cost is best viewed as an agency cost. Smith and Warner (1979) point out that the existence of bond covenants can be viewed as a method of controlling the conflict between bondholders and stockholders. It is assumed here that bond covenants succeed in eliminating the conflict, and thus firm-value maximization, rather than equity-value maximization, is the proper firm objective.
Bond covenants typically restrict debt issue. Without such restrictions, stockholders would issue more debt and incur greater agency costs. For example, it may be that the incentive for stockholders to shift to higher-variance investment projects increases with the face value of debt (see Barnea, Haugen, and Senbet [1985]). If such a shift decreases the total value of the firm, then bondholders, anticipating such actions, would demand higher interest rates. The increase in the interest rate demanded by bondholders is a type of agency cost. Here I assume that bond covenants are negotiated to restrict the level of debt, for a given value of equity. The higher the debt-to-equity ratio, the more likely that the covenant will be violated, resulting in restrictions on investment activities and a decrease in firm value. Thus, the cost of issuing $b$ units of debt increases with the debt-to-equity ratio. I assume that $a(0) > 0$, $a'(1) > 0$, and $a''(1) > 0$.

Appendix A demonstrates how expressions (7)-(12) can be used to derive expression (13):

$$ V_{t=0} = \int_0^\infty Y(t) dt $$

$Y$, the cash flow, is defined as

$$ Y = (y - w \ell - \psi(I,K))(1 - \tau_p) - I. $$

$I$, the cost of capital, is defined as

$$ I = \Theta + \frac{[(\tau_v - \tau_p)\Theta + a(\lambda)]}{(1 - \tau_v)} \frac{\lambda}{1 + \lambda} + \frac{[\Theta c + d(\tau_v - \tau_c)]}{(1 - \tau_c)} \frac{1}{1 + \lambda} $$

Now the firm's problem is to choose the sequence $\{K_t, \ell_t^*, b_t^*\}$ so as to

$$ \max_{\ell, b} \int_0^\infty \int_0^\infty Y(t) dt $$

subject to $K_{t=0} = K_0$, $b_{t=0} = b_0$. 
Note that \( Y \) is solely a function of "real" variables \( K_t \) and \( \ell_t \), whereas \( \Gamma \) is a function of only "financial" variables summarized by \( \lambda_t \). These considerations imply that the firm can optimize in the following sequence: first, choose \( K_t^d \) and \( \ell_t^d \) to maximize \( Y(t) \), then choose \( \lambda_t \) to minimize \( \Gamma(t) \).

If \( a(\lambda) = 0 \), the firm would choose \( \lambda \) as follows:

\[
\begin{align*}
(17) \text{ set } \lambda &= \infty \text{ if } (1 - \tau_v) > (1 - \tau_p)(1 - \tau_e) \\
(18) \text{ set } \lambda &= 0 \text{ if } (1 - \tau_v) < (1 - \tau_p)(1 - \tau_e)
\end{align*}
\]

where \( \tau_e = \frac{d\tau_v + (z/z)\tau_e}{d + z/z} \) is the marginal tax rate on equity income.

Expressions (17) and (18) imply that if the after-tax income from bonds exceeds the after-tax income from equity, investors prefer debt finance. Given reasonable values of \( d \), \( \tau_v \), \( \tau_p \), and \( \tau_e \), (17) is likely to be the case and will be assumed. Condition (17) in conjunction with a cost to debt \( a(\lambda)b \) results in an optimal \( \lambda: 0 < \lambda < \infty \).

The firm's problem leads to the following Hamiltonian formulation:

\[
H = \frac{t}{t} - \int \Gamma(\tau) dt
\]

where \( q' \) and \( Q \) are the costate variables associated with \( K \) and \( b \), respectively. Expressions (20) through (23) are the optimality conditions for the firm.

\[
\begin{align*}
(20) \quad F_{\lambda} &= w \\
(21) \quad q &= 1 + (1 - \tau_p)\psi \\\n(22) \quad \dot{q} &= (\Gamma + \delta)q - (1 - \tau_p)(F_{K} - \psi_K) \\
(23) \quad a(\lambda) + a'(\lambda)\lambda(1 + \lambda) = \Theta\tau_p \\
(24) \quad (TVC) \lim_{t \to \infty} \rho_t q_t K_t = 0 = \lim_{t \to \infty} \rho_t \Omega_t b_t \\
\end{align*}
\]

where \( \rho_t = e \)
Expression (21) states that marginal q differs from one by the after-tax decline in real cash flow due to installation costs and implies that the investment rate is an increasing function of marginal q. Expression (22) is the arbitrage condition that the shadow return from holding capital must equal the required return on capital, \((\Gamma + \delta)q\). Expression (22) can be integrated subject to the transversality condition (24) to obtain

\[
q_t = \int_t^\infty e^{-\int_t^\tau \Gamma(\tau')d\tau} (1 - \tau_p)(F_K - \psi_K)ds.
\]

Thus, q equals the present discounted sum of after-tax marginal products of a unit of capital installed at time t. Since \(\Gamma\) depends on tax rates and the debt cost, so does q. Expression (23) states the relation between \(\lambda\), \(\theta\), and \(\tau_p\) when the firm chooses \(\lambda\) so as to minimize \(\Gamma\). Since there is no equity issue, \(\lambda\) is adjusted by varying the b versus RE financing mix. Having chosen the path of the capital stock, employment, and the debt-to-equity ratio, the firm has maximized \(V_{t=0}\), and the resulting initial share price is determined by the condition \(b_{t=0} + z_{t=0}E_{t=0} = V_{t=0}\).

C. Government

The government is characterized by an income statement identity:

\[
\tau_v \left[ wI + sD + dZ \right] + \tau_c \dot{Z}E + \tau_p \left[ y - wI - \psi(I,K) - sb \right] + \tau = g.
\]

The government sets \(\tau_v\), \(\tau_c\), \(\tau_p\), and g. The lump sum tax \(\tau\) varies so as to satisfy the identity. Although \(\tau\) adjusts endogenously, the impact of a change in \(\tau\) is ignored in the dynamic analysis that follows because individual consumers take \(\tau\) as given parametrically.
IV. Perfect Foresight Equilibrium

A perfect foresight equilibrium is a sequence of prices and quantities for which notional demands and supplies for bonds and equities are equal in the present and future. Given the optimality conditions of consumers and firms and the government income statement identity, expressions (27)-(32) determine the sequence of variables, \( \{K_t, c_t\}, t=0, \infty \).

(27) \( \theta = \beta - U_{cc} \hat{c}/U_c \)

(28) \( F(K, \theta) = c + g + \dot{K} + \delta K + h(I/K)I + a(\lambda)\lambda qK/(1 + \lambda) \)

(29) \( q = 1 + (1-\tau_p)[h(I/K) + (I/K)h'(I/K)] \)

(30) \( \dot{q} = (\Gamma-\delta)q - (1-\tau_p)[F_K + (I/K)^2h'(I/K)] \)

(31) \( a(\lambda) + a'(\lambda)\lambda(1+\lambda) = \theta \tau_p \)

(32) \( \Gamma = \theta + [a(\lambda) - \tau_p \theta] \frac{\lambda}{1+\lambda} \)

Here I have simplified the model by setting \( \tau_y = \tau_c = 0 \). I also assume that employment is given exogenously. Together with expression (20), this is analogous to assuming that the real wage adjusts to equilibrate the labor market. So, condition (20) is not needed below. Expression (28) is a restatement of the material balance constraint which uses the facts: \( b = [\lambda/(1 + \lambda)]V, V = qK. \)

Expression (31) gives the optimal debt-to-equity ratio and indicates that a higher interest rate or corporate tax rate means a higher debt-to-equity ratio as the value of the interest deduction rises. To see that (31) implies an optimal \( \lambda \), derive expression (33) from expression (32) for given \( \tau_p \) and \( \lambda \).

(33) \( \frac{d\Gamma}{d\lambda} = \frac{a(\lambda) - \tau_p \theta + \lambda a'(\lambda)(1 + \lambda)}{(1 + \lambda)^2} \)
Figure 1 shows how expression (33) implies that an increase in $\tau_p$ from $\tau_{p0}$ to $\tau_{p1}$ leads to an increase in the optimal $\lambda$ from $\lambda_0^*$ to $\lambda_1^*$.

An increase in $\tau_p$ makes a higher $\lambda$ optimal. Expressions (31) and (32) together show that the minimized cost of capital is equal to

$\Gamma = \theta - a'(\lambda)\lambda^2$.  

The decrease in the cost of capital due to an increase in $\tau_p$ is indicated by the $\tau_{p1}$ line in figure 1. However, expression (28) indicates that an increase in the debt-to-equity ratio, which reduces the cost of capital, may increase the debt cost. The change in $a(\lambda)b$ depends on the change in $X$, the response of $q$, and the response of $K$. These relationships are examined below.

V. Steady State and Comparative Results

In the steady state, $\dot{K} = 0$ and $\dot{c} = 0$. The first condition implies that $\delta = \delta K$ and thus $I/K = 6$. The second implies that $\theta = \beta$: the steady state interest rate is the rate of time preference of consumers. The condition that $\theta = \beta$, together with (30), implies that $q = 0$. The debt-to-equity ratio must also be constant in the steady state. This implies that $d = \beta$. Expressions (35)-(38) describe the steady state:

\[
\begin{align*}
(35)\quad & [\Gamma + \delta][1+ (1-\tau_p)(h(\delta)+\delta h'(\delta))] = (1-\tau_p)[F_K + \delta^2 h(\delta)] \\
(36)\quad & F(K,\theta) = c + g + \delta K + h(\delta)\delta K + \frac{a(\lambda)\lambda}{1+\lambda} [1 + (1-\tau_p)(h(\delta) + \delta h'(\delta))]K \\
(37)\quad & \Gamma = \beta - a'(\lambda)\lambda^2 \\
(38)\quad & a(\lambda) + a'(\lambda)\lambda(1 + \lambda) = \beta \tau_p
\end{align*}
\]
Figure 1

The Effect of an Increase in $\tau_p$ on the Cost of Capital ($\Gamma$)
The impact of increasing the corporate tax rate is given by:

\[
\begin{align*}
\frac{dK}{d\tau_p} &= \frac{F_k + \delta h'(\delta) + (\Gamma + \delta) dq + q \frac{d\Gamma}{d\tau_p}}{(1-\tau_p)F_{kk}} \\
\frac{dc}{d\tau_p} &= \left[F_k - \delta - h(\delta)\delta - A(\lambda)H(\delta, \tau_p)\right] \frac{dK}{d\tau_p} - \frac{dA}{d\tau_p} \frac{dq}{d\tau_p} - A(\lambda)K \frac{dq}{d\tau_p}
\end{align*}
\]

(39)  

(40)  

where \[dq = -[h(\delta) + \delta h'(\delta)] < 0, \frac{d\Gamma}{d\tau_p} = d\Gamma d\lambda \frac{d\lambda}{d\tau_p}, \] and 

\[\frac{d\lambda}{d\tau_p} > 0, A(\lambda) = a(\lambda)\lambda/(1 + \lambda).\]

The sign of \(dK/d\tau_p\) is unclear because of two competing effects. First, the increase in \(\tau_p\) reduces the cost of capital by making a greater \(\lambda\) optimal. This is clear from expression (34). Second, the reduction in the after-tax return from a unit of capital reduces steady state \(q\). The sign of \(dc/d\tau_p\) is also unclear. In the absence of the agency cost, \(dc/d\tau_p\) would be \(-[F_k - \delta - h(\delta)] dK/d\tau_p\). Then the requirement for steady state \(c\) to change in the same direction as \(K\) would be: \(F_k - \delta - h(\delta)\delta < 0\). Here, the analogous version is \(F_k - \delta - h(\delta)\delta - A(\lambda)H(\delta, \tau_p) < 0\), where \(A(\lambda)H(\delta, \tau_p)\) is the per-unit of capital agency cost. This condition, however, is not sufficient for steady state \(K\) and \(c\) to move in the same direction, since the increase in \(\tau_p\) also affects \(X\), \(q\), and \(K\). Note that \(dc/d\tau_p\) can be written as:

\[
\frac{dc}{d\tau_p} = [F_k - \delta h(\delta)\delta] \frac{dK}{d\tau_p} - \frac{dA}{d\tau_p} \frac{dq}{d\tau_p} - A(\lambda)K \frac{dq}{d\tau_p} - A(\lambda)q \frac{dK}{d\tau_p}
\]

(41)  

where the last three terms equal \(d[a(\lambda)b]/d\tau_p\). Although \(\lambda\) will increase, total agency costs may fall, since the value of the firm may fall.
VI. Dynamic Behavior

As is well known, in dynamic rational-expectations models, endogenous variables may overshoot the new steady state. Consequently, it is useful to distinguish between short-run and long-run responses to policy changes. In this section I describe the movement of the system about the steady state in response to an unanticipated increase in $\tau_p$. The movement can be described by changes in the rate of investment, $x = I/K$, and the capital stock. This simplification recognizes that the cost of capital varies with the interest rate and the debt-to-equity ratio, both of which vary with the rate of change of consumption and marginal utility. The following system is derived in appendix B by expanding the system described by expressions (27)-(32) about the steady state:

\[
\begin{bmatrix}
 x^*-x^* \\
 K^*-K^*
\end{bmatrix} =
\begin{bmatrix}
 \pi_1 & \pi_2 \\
 K & 0
\end{bmatrix}
\begin{bmatrix}
 x-x^* \\
 K-K^*
\end{bmatrix}
\]

(42)

\[\pi_1 = B_2/B_1, \quad \pi_2 = B_3/B_1\]

(43)

\[B_1 = 1 - \frac{H(\delta, \tau_p)I^* U_{cc}}{H'(\delta, \tau_p)C_1 U_c} \left[1 + h(\delta) + \delta h'(\delta) + A(\lambda)H'(\delta) + A' qK H'(\delta) \right]
\]

(44)

\[B_2 = \frac{1}{H'(\delta, \tau_p)} \left\{ (\Gamma + \delta) H'(\delta, \tau_p) - (1-\tau_p)(\delta_n^2(\delta) + 2\delta h'(\delta)) \right\}
\]

(45)

\[B_3 = \frac{1}{(1-\tau_p)F_{KK}}
\]

(46)

\[C_1 = 1 + \frac{H(\delta, \tau_p)KA'qK U_{cc}}{zE U_c}
\]

(47)

Here I have made use of the following definitions: 1) $H() = 1+(1-\tau_p)\psi_1 = q$; 2) $H'(\delta) = dH(\delta)/d\delta$; 3) $\Gamma' = d\Gamma/d\theta$; 4) $A'(\delta) = dA(\lambda)/d\lambda$; and 5) $h'(\delta) = dh(\delta)/d\delta$. 
Since investment \( (I) \) can change instantaneously, \( x \) is the "jump" variable for which discontinuities can occur. Since \( K \) is predetermined while \( x \) depends on the future path of the system, in order for the system to be stable, there must be one positive and one negative eigenvalue for the matrix in (42). This condition assures me of a jump onto the stable arm of the system in its movement toward the new steady state. To assure this stability, I assume that:

\[
(48) \quad \det \begin{vmatrix} \pi_1 & \pi_2 \\ K & 0 \end{vmatrix} < 0, \quad -K\pi_2 < 0, \quad \pi_2 > 0.
\]

From (43)-(47), it is clear that expression (49) is sufficient for (48) to hold.

(49) \( C_1 > 0 \)

The second term in \( C_1 \) reflects that an increase in consumption accompanies a rising interest rate and an increase in equity values. An increase in equity values reduces the rate of change of the debt-to-equity ratio, thus reducing the rate of change of the debt cost. Condition (49) states that the direct effect of rising consumption on aggregate demand outweighs the indirect effect operating through the debt cost. So, to assure \( C_1 > 0 \), I assume that the rate of change of demand for goods varies positively with the rate of change of consumption.

Note that for the stable eigenvalue \( \Omega < 0 \) and the corresponding eigenvector, \( (X_1, X_2)' \),

\[
(50) \quad \begin{vmatrix} \pi_1 - \Omega & \pi_2 \\ K & -\Omega \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}.
\]
Expression (50) implies that the movement of \( x \) and \( K \) from their steady state values can be described by expression (51):

\[
(51) \quad x_t - x^* = Y_1 \pi_2 e^{\alpha t}, \quad K_t - K^* = Y_1 (\Omega - \pi_1) e^{\alpha t}.
\]

\( Y_1 \) is a constant determined from the fact that \( K \) remains fixed at the time of the announcement and implementation of an unanticipated change in \( \tau_p \). Having determined \( Y_1 \) as described in appendix B, the movements of \( x \) and \( K \) about the steady state are described by expressions (52) and (53).

\[
(52) \quad x_t - x^* = (-\frac{dK}{d\tau_p}) d\tau_p \pi_2 e^{\alpha t}/(\Omega - \pi_1)
\]

\[
(53) \quad K_t - K^* = (-\frac{dK}{d\tau_p}) d\tau_p e^{\alpha t}
\]

These imply \( x_t - x \geq 0 \) as \( dK/d\tau_p \geq 0 \). In other words, if the steady state capital stock increases, investment is above its steady state value during the adjustment period. In fact, \( x \) must immediately increase at time \( t = 0 \). The jump is computed as:

\[
(54) \quad x_t - x^*|_{t=0} = [-\pi_2 \frac{dK}{d\tau_p}]/(\Omega - \pi_1).
\]

**VII. Dynamic Relationships Between Tobin's \( q \), the Debt-to-Equity Ratio, and the Rate of Investment**

Because the movements of \( q \) and \( x \) are closely related by expression (21), on the stable adjustment path, \( \text{sign}\left[\dot{x}\right] = \text{sign}\left[\dot{q}\right] \). The new steady state value of \( q \) will be lower, since \( \tau_p \) has increased.

Initially, however, \( q \) and \( x \) may jump in different directions. The increase in the rate at which current returns are taxed tends to decrease \( q \) at the time of the announcement. The movement of \( q \) along the stable arm can be described by expression (56), where \( q^*_t \) is the new steady state value:

\[
(56) \quad q_t - q^*_t = (1 - \tau_p)[2h'(\delta) + h''(\delta)][-\pi_2/(\Omega - \pi_1)] \frac{dK}{d\tau_p} d\tau_p e^{\alpha t}.
\]
The jump in $q$ at $t = 0$ is

$$q_t - q^*_t |_{t=0} = (1 - \tau_p)\left[2h'(\delta) + h''(\delta)\right] \frac{dK}{d\tau_p}$$

$$- \left[h(\delta) + \delta h'(\delta)\right]d\tau_p.$$

If steady state $K$ increases, $q$ may either rise or fall initially. However, if steady state $K$ falls, $q$ must immediately fall. These results are depicted in figure 2.

The initial movement in $X$ can be inferred directly from the jump in $q$ using the following relations:

1. $\lambda = b/zE$
2. $V = b + zE$
3. $V = qK$

The variables $b$, $E$, and $K$ are predetermined. These imply that the jump in $\lambda$ is opposite in sign to that of $q$. An increase in the market value of existing capital will be immediately reflected in a higher price of equity and a lower debt-to-equity ratio.

Once this initial adjustment has been made, however, subsequent movements in $\lambda$ are governed by relation (31): $\theta \tau_p = a(\lambda) + a'(\lambda)\lambda(1 + \lambda)$. In fact, the initial reaction of the interest rate is also governed by (31). Thus, $\lambda$ and $\theta$ may initially jump in opposite directions. To demonstrate the dynamics of these relations, two possible cases are examined below.

Case I: $dK/d\tau_p > 0$.

As noted above, $x$ will immediately increase while the movement of $q$ is unclear. If $q$ immediately increases, then the debt-to-equity ratio drops. Then, from (31), in order to be on the stable path, the interest rate must initially fall, given the higher $\tau_p$. When the interest rate is below $\beta$, consumption is falling. This is seen from expression (27):

$$\dot{c} = (U_c/U_{cc})(\beta - \theta).$$
The Effect of an Increase in $r_\theta$ on $q$

\[ \frac{dk}{dr_\theta} > 0 \]

\[ 0 > x \]

\[ t \]

\[ 0 < 0 = \sqrt{b^2 - b} \]

\[ b \]

\[ \text{Figure 2} \]
As the interest rate rises toward $\beta$, $\lambda$ rises toward its higher steady state level. The sign of the initial change in $c$ is indeterminate, as is the sign of the comparative static result, because of the presence of debt costs that may move in the opposite direction of the capital stock. The former case is depicted in figure 3.

Case II: $dK/d\tau_p < 0$.

Here the initial impact on $q$ is clear. Both the drop in investment and the higher tax rate require $q$ to fall. In this case, it is clear that the new steady state firm value will be lower. As a result of the change in $\tau_p$, the relationship between $\lambda$ and $\Theta$ has been altered, and the direction of the jump in $\Theta$ is indeterminate. Two possibilities are pictured in figure 4. The ambiguity regarding the path of $\lambda$ and $\Theta$ in figure 4 can be resolved by restricting the response of the system to the change in $\tau_p$.

Expressions (B7) and (B8) in appendix B imply expression (58):

\[
(c - c^*) = \frac{(x - x^*)}{C_1} K (F_x - \delta(1 + h(\delta)) - H(\lambda) + A'qK\frac{1}{K})
\]

\[
- \frac{(K - K^*)}{C_1} \pi_2 K [1 + h(\delta) + \delta h'(\delta) + A(\lambda)H'() + A'qK'H'()]
\]

Here $\pi_1 > 0$ and $C_1 > 0$ imply that if $dK/d\tau_p > 0$, as $K$ increases toward the new steady state, $c^* > 0$. The sign of $\pi_1 = B_2/B_1$ can be determined through inspection of $B_2$. $B_2$ is the response of $q$ to an increase in $x$. If an increase in the level of investment decreases $x$, $q$ will also decrease. This is clear from expression (29) and will be assumed, implying $\pi_1 < 0$. 
The Effect of an Increase in $\tau_p$ on $q$, $\lambda$, $x$, and $\theta$

When $\frac{dK}{d\tau_p} > 0$ and $q_t - q^*|_{t=0} > 0$
The Effect of an Increase in $\tau_p$ on $q$, $A$, $x$, and $\Theta$

When $dK/d\tau_p$ and $q_t - q^*_t |_{t=0} > 0$
The first term in the coefficient for \((x - x^*)\) in expression (58) represents the response of excess supply of goods to an increase in \(K\). This will be assumed positive, assuring us that \((c - c^*)\) varies positively with \((x - x^*)\). Thus, (58) implies that if \(dK/d\tau_p < 0\), \((c - c^*) > 0\). This, in turn, tells us that path \(A\) is the appropriate path from figure 4.

VIII. The Distinction Between the Cost of Capital and the Interest Rate

This model predicts that the interest rate and the debt-to-equity ratio are positively correlated. However, this is not the result of bankruptcy risk, but rather the interest deduction for debt. While the cost of capital is positively correlated with the debt-to-equity ratio, the ratio between the cost of capital and the interest rate is given by expression (34).

(34) \( \Gamma = 8 - a'(\lambda)\lambda^2 \).

Since the debt-to-equity ratio rises with the interest rate, the gap between the cost of capital and the interest rate also rises with \(8\). This implies that models that use the interest rate as the discount rate overstate the cost of capital by an increasing amount as the interest rate rises. We would expect that these models would understate investment.

Using expressions (23), (34), and (58), it is straightforward to show how the gap between \(\Gamma\) and \(8\) varies. Expressions (23) and (34) imply:

\[
(59) \quad d[\Gamma - \theta] = \frac{\lambda}{1+\lambda} \frac{U_{cc}}{U_c} \tau_p (c - c^*).
\]

Together with (58), (59) implies:

\[
(60) \quad d[\Gamma - \theta] = \frac{\lambda}{1+\lambda} \frac{U_{cc}}{U_c} \tau_p \frac{K_1}{C_1} \left[ F_k - \delta(1 + h(\delta)) - H(\lambda) + A'(\lambda) + A'qK \right] (x - x^*)
- \frac{\lambda}{1+\lambda} \frac{U_{cc}}{U_c} \frac{\pi}{C_1} \left[ 1-h(\delta) + \delta h'(\delta) + A(\lambda) H'(\delta) + A'(\lambda) qK H'(\delta) \right] (K - K^*).
\]
Given the assumptions above, if \( \frac{dK}{d\tau} > 0 \), \( d[\Gamma - \Theta] < 0 \). Since \( \Gamma - \Theta = -a'(\lambda)\lambda^2 < 0 \), if the capital stock is rising, the absolute value of the gap between the cost of capital and the interest rate is rising.

IX. Summary

Previous q models of investment have implicitly assumed that financial decisions have no impact on the cost of the capital. Here we show how financial structure affects the cost of capital and the time paths of investment, \( q \) and \( X \). In this paper, the presence of a debt cost together with a tax code that favors debt results in an optimal debt-to-equity ratio that covaries with the interest rate. We find that \( q \) is still a "sufficient statistic" for investment, although financial structure has real effects. In addition, the gap between the cost of capital and the interest rate covaries with interest rates and financial structure. As the debt-to-equity ratio rises, however, the increase in the cost of debt reduces the amount of output available for consumption or investment.

The analysis implies that models that ignore the endogenous adjustment of financial structure will systematically err in predicting the response of investment to changes in tax rates or interest rates. For example, a change in the corporate tax rate affects both the cost of capital and the gap between the cost of capital and the interest rate. In addition, the presence of real costs to financial structure may imply that the capital stock would decline in spite of a lower cost of capital.
Appendix A

Here I derive the appropriate form for the cost of capital through the use of the constraints (7)-(12) and the assumed firm objective: maximization of $V(0)$. First, use (8) and (9) to write

\[
A \quad y - w - \psi(I,K)(1 - \tau_p) = (s(1 - \tau_p) + a(\lambda))b + dz + RE.
\]

Note that $\dot{V} = b + \dot{z}E$ implies

\[
(A2) \quad \dot{V} = \dot{b} + \dot{z}E.
\]

Expressions (10) and (B1) imply (B3):

\[
(A3) \quad y - w - \psi(I,K)(1 - \tau_p) - I + \dot{z}E = (s(1 - \tau_p) + a(\lambda))b + dz - \dot{b} - \dot{z}E
\]

\[
(A4) \quad y - w - \psi(I,K)(1 - \tau_p) - I - V = (s(1 - \tau_p) + a(\lambda))b + dz + \dot{z}E
\]

where \((y - w) - \psi(I,K)(1 - \tau_p) - \dot{f} k\) is defined as \(Y\), the real cash flow of the firm. Using (6), (A4) can be rewritten as

\[
(A5) \quad Y + V = \frac{[(1 - \tau_p)\theta + a(\lambda)]b + [\theta + d(\tau_v - \tau_c)]zE}{(1 - \tau_v)}.
\]

Using the definition of \(\lambda\), (A5) can be written as

\[
(A6) \quad Y + V = \frac{\theta + [(\tau_v - \tau_p)\theta + a(\lambda)]}{(1 - \tau_v)} \frac{\lambda}{1+\lambda} + \frac{\theta\tau_c + d(\tau_v - \tau_c)}{(1 - \tau_c)(1+\lambda)} V.
\]

Finally, defining the cost of capital \(\Gamma\) as

\[
\Gamma = \theta + \frac{[(\tau_v - \tau_c)\theta + a(\lambda)]}{(1 - \tau_v)} \frac{\lambda}{1+\lambda} + \frac{\theta\tau_c + d(\tau_v - \tau_c)}{(1 - \tau_c)(1+\lambda)} \frac{1}{1+\lambda}
\]

(A6) can be written as

\[
(A7) \quad Y + \dot{V} = \Gamma V
\]

which is a linear differential equation in \(V\) that can be integrated to show that \(\Gamma\) is the discount factor which maintains the equality between the integral in expression (13) and \(b_{t=0} + z_{t=0}E_{t=0}\).
Appendix B

Here I derive expressions (42)-(47) of the text by expanding the system (27)-(32) about the steady state. First note that (29) implies

\[ \dot{q} = H'(x,\tau_p)\dot{x} \]

where \( H(x,\tau_p) = 1 + (1-\tau_p) [h(x) + h'(x)] \) and \( H'(x,\tau_p) = dH(x)/dx \).

Together with (30), this implies

\[ \dot{\tau} = (1-\tau_p) [F_k + x^2 h'(x)]. \]

Since \( \theta = \beta - U_{cc}c/U_c \), in order for (B2) to yield a relation for \( \dot{\tau} \), I require an expression for \( \dot{\tau} \). An expression for \( \dot{\tau} \) can be derived from (28).

Take the time derivative of (28):

\[ \ddot{\tau} = \ddot{c} + \dot{K} + xK(x-\delta) + h(x)\dot{x}K + h(x)\dot{K}x + xKh'(x)\dot{x} + A(\lambda)H(\dot{\tau})k(x-\delta) \]

\[ + A(\lambda)KH'(\dot{\tau})\dot{x} + H(\dot{\tau})KA' \frac{qK}{Z} \left[ \frac{h'(\dot{\tau})}{H'(\dot{\tau})} \right] \]

Here I have used the fact that \( K = K(x-\delta) \) and \( \dot{K} = xK \). The last term on the right is an expression for \( qKd[A(\lambda)]/dt \) that makes use of the following facts:

(a) \( A' = \frac{dA(\lambda)}{d\lambda} \)

(b) \( \lambda = \frac{b}{zE} = \frac{V - zE}{zE} \)

(c) \( V = qK, \lambda = (qK - zE)/zE = qK - 1 \)

(d) \( \dot{\lambda} = qK(\dot{K} + K - z) \frac{zE}{qK} \frac{z}{K} \)

(e) \( \dot{\theta} = H'(x,\tau_p)\dot{x} \)

(f) \( K = K(x - \delta) \)

(g) \( \dot{z} + d = \theta = \beta - \frac{U_{cc}c}{U_c} \frac{z}{z} \)
These yield the following expression for $\dot{c}$:

$$\dot{c} = \frac{1}{(1 + H)KA'qK U_{cc}} \left\{ [K(x-\delta)(1+h(x)) - A(\lambda)H(\cdot) - H(\cdot)A'qK] \frac{zE}{X} \right\}$$

Now linearize (2) about the steady state to obtain an expression for $(x - x^*)$:

$$(x - x^*) = \frac{A_1 dA_2 - A_2 dA_1}{A_1^2}$$

where $A_1 = (x, \tau_p)$, $A_2 = (\tau + \delta)H(x, \tau_p) - (1 - \tau_p)(F_K + x^2h'(x))$

and, since all coefficients are evaluated at the steady state, $A_2 = q = 0$. So we have:

$$(x - x^*) = \frac{1}{H'(x, \tau_p)} \left\{ (\tau + \delta)H'(x, \tau_p)(x - x^*) - (1 - \tau_p)F_K(K - K^*) \right\}$$

From (B4) we derive:

$$(c - c^*) = \frac{(x - x^*)}{(1 + H)KA'qK U_{cc}} \frac{zE}{X} K(1 + (1+h(x)) - H(\cdot)[A(\lambda) + A'qK \frac{zE}{X} K])$$

This expression makes use of the fact that in the steady state $x-\delta = x = d-8 = 0$. $d-8 = 0$ stems from the requirement that in the steady state, $\lambda = 0$, and thus $z/z = 0$. This implies that $\theta = \beta = d$. The resulting expression for $(x - x^*)$ is:

$$(x - x^*) = \pi_1(x - x^*) + \pi_2(K - K^*),$$

where $\pi_1$ and $\pi_2$ are defined as in the text.

In this section the determination of the constant $Y_1$ in expression (51) in the text is described. Expressions (B9)-(B11) are implied by the
fact that capital stock remains fixed at the time of the announced change, but eventually changes by \((dK/d\tau_p)\, d\tau_p\).

(B9) \[ K_t - K_1 \big|_{t=0} = K^* - K_1^* = (-dK/d\tau_p)\, d\tau_p \]

(B10) \[ K_t - K_1 \big|_{t=0} = Y_1 (\Omega - \pi_1) = (-dK/d\tau_p)\, d\tau_p \]

(B11) \[ Y_1 = (-dK/d\tau_p)\, d\tau_p / (\Omega - \pi_1) \]
Footnotes

1. As an example of more recent developments, Bernanke and Gertler (1986) have incorporated financial structure into a stochastic general equilibrium model.

2. Chirinko (1987) and Hayashi (1985) have incorporated financial structure into q frameworks. Chirinko, however, ignores tax effects and investigates whether the debt versus equity choice can explain the poor performance of q, defined as the market value of equity divided by the replacement value of capital. The analysis of this paper, as do others, defines q as the market value of debt and equity divided by the replacement cost of capital. Utilizing a partial equilibrium framework, Hayashi (1985) shows how the firm's choice of financial policy depends on the level of profits relative to investment. He finds that only when incremental investment is entirely debt-financed is the link between q and investment broken. Chirinko and King (1985) take into account the role of debt in the response of investment by equity-maximizing firms to changes in inflation.

3. A series of articles (see Auerbach [1983] for a review) has concluded that bankruptcy risk and tax rates that vary across investors and types of income can lead to a determinate debt-to-equity ratio and a cost of capital that varies with the debt-to-equity ratio. However, if with bankruptcy risk a firm's decisions influence the implicit prices of the Arrow-Debreu state contingent commodities, the cost of capital is not well-defined.

4. If $a(\lambda)b$ were treated as a deduction from taxable income or as a cost that reduced taxable revenue, the form for the optimal cost of capital would be somewhat different. To see this, replace (14) by the following:

\[(13)' \quad \pi = y^s - w^t - \psi(I,K) - a(\lambda)b\]

and proceed as in appendix B. The resulting expression for $\Gamma$ is

\[(15)' \quad \Gamma = [(\theta + a(\lambda))(1-\tau_p)\lambda + \theta]/(1 + \lambda).\]

At the optimal debt-to-equity ratio, the condition that $d\Gamma/d\lambda = 0$ implies that

\[(23) \quad (1-\tau_p) [\theta + a(\lambda) + (1+\lambda) \lambda a'(\lambda)] = 0\]

which yields $\Gamma = \theta - (1-\tau_p)\lambda^2 a'(\lambda)$. It can be shown that an increase in $\tau_p$ still will decrease the steady state cost of capital.

5. The formulation of the adjustment cost implies a production function of the form $G(e,K,I)$ with $G_e > 0$, $G_K < 0$, $G_I < 0$, $G_{K\lambda} < 0$, $G_{\lambda\lambda} < 0$ and $G(e,K,I)$ homogeneous of degree one. Then with the assumption that $G_{\lambda I} = 0$, $G(e,K,I)$ can be written as $G(e,K,I) = F(K,e) - \psi(I,K)$. 
6. Chirinko (1982) estimates $\tau_c$ at .033 when rates applicable to various investor classes are weighted by transactions shares. The estimate when weights reflect SEC market value data is .031. Feldstein, Poterba, and Dicks–Mireaux (1981) estimate $\tau_c$ at .083. While Gordon and Malkiel (1981) argue that the rate on dividend and interest income should be approximately equal for relevant investors, other researchers disagree. Feldstein, Poterba, and Dicks–Mireaux estimate the dividend tax rate at .349 in 1979, and estimate the tax rate on interest income at .317. Chirinko estimates the dividend tax rate at .168 with transactions weights, and .278 when rates are weighted by market shares. Gordon and Malkiel estimate the dividend and interest income tax rate at .44. The federal corporate tax rate is .48. Feldstein, Poterba, and Dicks–Mireaux estimate the corporate tax rate inclusive of state and local government corporate taxes at .472.
References


Auerbach, Alan J. "Taxation, Corporate Financial Policy and the Cost of Capital," *Journal of Economic Literature*, vol. 21, no. 3 (September 1983), 905–940.


