

Appendix to
“How Well Does the Cleveland Fed’s
Systemic Risk Indicator Predict Stress?” 2020. Ben Craig. Federal Reserve
Bank of Cleveland, *Economic Commentary* 2020-28

This appendix provides technical detail on the model used to calculate the systemic risk index.

Figure A1 illustrates some of the concepts behind the distance to default (DD). In figure A1, the value of the bank’s assets currently is A , and an example of its random future path is shown. Among other possibilities the path could take is that it can dip below the default barrier, D , which does not occur in the path illustrated in figure A1 but could occur in other possible paths. Because the path is random, one can define a probability that D is crossed at some time, T . This probability is a function of the volatility of the process that defines the random path of the asset, σ_A , the value of the bank’s assets, A , and its default barrier, D , the growth of the asset value, r , and the amount of time in the future in which the path has a chance to cross the barrier, T . Under the assumptions of the Merton model, the random path follows a process that implies that probability of being in default at time T is $\Phi(-DD)$, where $\Phi(\cdot)$ is the cumulative standard normal probability distribution function.

There are several complications to calculating the value of DD . As noted in the *Commentary*, the loan holders are the first claimants on the value of the bank, so the price of equity already prices the value due to the lenders in the case of default. This means that the asset value is not the same as the value of the equity price, which is an “option price” that pays off to the equity holders in the future only when the value of the assets is larger than the default barrier, D . The true value of the asset, A , is unobserved, as is the value of the volatility, σ_A . Further, assumptions have to be made about the process that A will follow in the future so that a probability distribution at a given time can be computed. Merton used some fairly standard assumptions about the nature of the process to compute the values that we use for the SRI. In particular, he relates the value of the equity, E , and its volatility, σ_E , (which we estimate from options

prices) with two equations to the value of the assets, A , and its volatility, σ_A . Merton's model (which is the standard model for distance-to-default) assumes that the process follows a geometric Brownian motion process. Given this, he shows that the value of the equity, E , can be computed from two nonlinear equations in two unknowns:

$$(A1) \quad E = A\Phi(d_1) - e^{-rT}D\Phi(d_2),$$

$$(A2) \quad E\sigma_E = A\sigma_A\Phi(d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{A}{D}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma_A\sqrt{T},$$

and where $\Phi(\cdot)$ is the cumulative standard normal probability distribution function as before. In the case of the SRI, option values on equity prices, along with the daily equity value for E and its volatility, σ_E , are used to calculate from equations A1 and A2 the values of A and σ_A , for each bank and the total portfolio. This calculation gives a forward-looking estimate, rather than estimates based upon the GARCH estimates of σ_E , as suggested by Merton.

This approach allows us to solve for A and σ_A , from which we can compute

$$DD_i = \frac{\ln\left(\frac{A}{D}\right) + \left(r - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}},$$

where r is the 10-year constant maturity US Treasury yield, D is the default barrier, and T is debt maturity in years, which is set to 1. The default barrier is defined by Merton (as well as much of the subsequent literature that uses distance to default) as short-term debt plus one-half long-term debt and can be computed from the bank balance sheets. This is a convention that is used by the literature and, like many

of the assumptions of the Merton model, is unrealistic. The Merton model of *DD* does not provide an exact interpretation of the SRI in terms of a bank's behavior but rather it places the SRI in the context of a large literature that financial economists are familiar with. For another example of a departure from realism in the Merton model, the geometric Brownian motion assumption often abstracts from shocks that will give a non-normal distribution to the probability of crossing the barrier.

The portfolio distance to default (*PDD*) is computed by first aggregating individual banks in the KBE index and then applying the equation:

$$PDD = \frac{\ln\left(\frac{A^P}{D^P}\right) + \left(r - \frac{1}{2}\sigma_p^2\right)T}{\sigma_p\sqrt{T}},$$

where D^P is the weighted average of individual distress barriers across all banks in the index, A^P is the value of the asset portfolio, and σ_p is computed from σ_{pE} , the implied volatility of the option on a KBE banking index, by solving the equations (A1) and (A2). In other words, the *PDD* uses options on the KBE banking index, which takes a basket of major US banks and aggregates them using the KBE set of weights, w_i . These same weights are also used to calculate the average distance to default (*ADD*), which is a simple average of the banks' DD_i 's:

$$ADD_t = \sum_{i=1}^N w_{it} DD_{it},$$

Figure 1A. Distance to Default (DD)

