

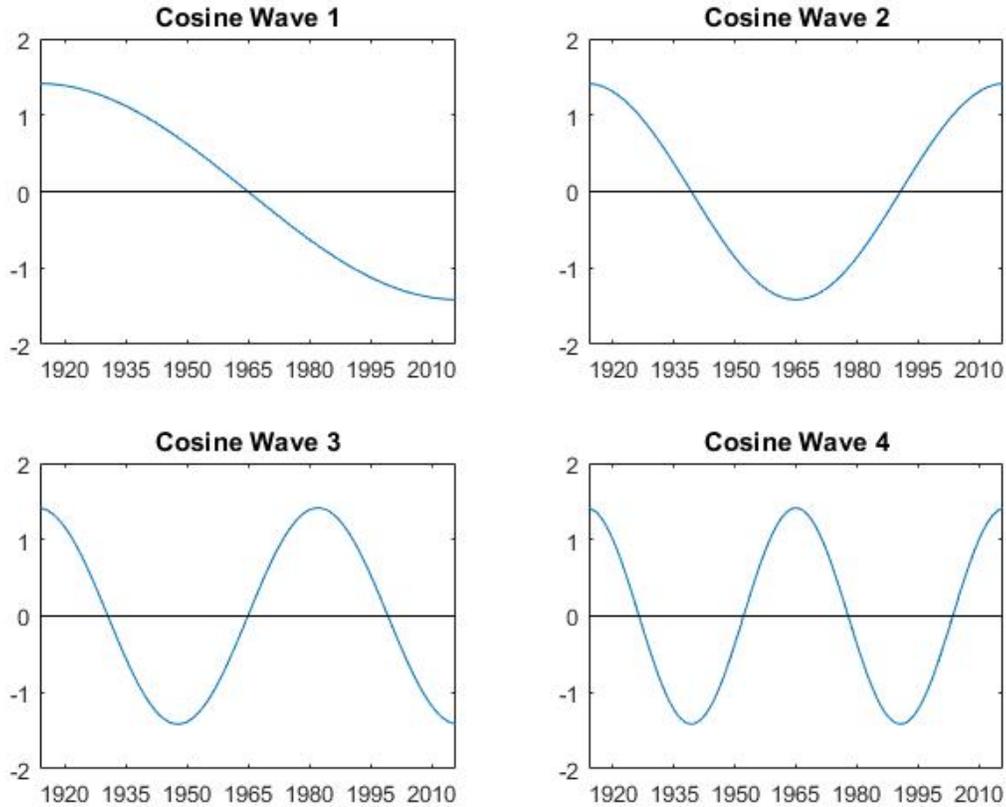
Appendix to “Productivity Growth and Real Interest Rates in the Long Run” by Kurt G. Lunsford

This appendix accompanies the Federal Reserve Bank of Cleveland *Economic Commentary* entitled “Productivity Growth and Real Interest Rates in the Long Run” by Kurt G. Lunsford. This appendix provides details for filtering the data to preserve long-run patterns. It also discusses estimates of and confidence intervals for correlations produced from the computation files of Müller and Watson (2017).

The filtered data is based on the following linear regression

$$y_t = \alpha_0 + \alpha_1 x_{1,t} + \dots + \alpha_q x_{q,t} + e_t,$$

where y_t is either the real interest rate or productivity growth. Following Müller and Watson (2008, 2015), $x_{1,t}, \dots, x_{q,t}$ is a sequence of cosine waves. These cosine waves are given by $x_{j,t} = \sqrt{2} \cos(\pi j r_t)$ where $r_t = (t - 1/2)/T$, and T denotes the sample size, which is 103 for the 1914 to 2016 sample and 69 for the 1948 to 2016 sample. The variable j indexes the period of cosine wave for $j = 1, \dots, q$. The first cosine wave, $j = 1$, completes half of a cycle over the whole sample, giving it a period of $2T$ years. The second wave, $j = 2$, completes one cycle over the whole sample, giving it a period of T years. The third wave, $j = 3$, completes one and a half cycles over the whole sample, giving it a period of $(2/3)T$ years, and so on. Following this pattern, each cosine wave has a shorter period than the one before it. The following figure displays the first four cosine waves.



The first four cosine waves used in the linear regression.

I estimate the above regression with least squares. The estimated parameters are denoted by $\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_q$, and the filtered data is given by $\hat{y}_t = \hat{\alpha}_0 + \hat{\alpha}_1 x_{1,t} + \dots + \hat{\alpha}_q x_{q,t}$. Based on the above pattern of cosine waves, I use $q = 20$ for the 10-year filtered data, $q = 13$ for the 15-year filtered data and $q = 10$ for the 20-year filtered data with the 1914 to 2016 sample. For the 1948 to 2016 sample, I use $q = 13$ for the 10-year filtered data, $q = 9$ for the 15-year filtered data and $q = 6$ for the 20-year filtered data.

The confidence intervals for the filtered data in Tables 1 and 2 of the *Commentary* assume that the underlying data are stationary. To check the robustness of this result, I use the computation files for Müller and Watson (2017) to re-estimate the correlations and their confidence intervals for Tables 1 and 2. As described in Müller and Watson (2017), these computation files make no *ex ante* assumption about stationarity. Further, they are robust against data that have unit roots, data that are fractionally integrated, and data that follow other models of persistence. One short-coming of these files is that they only produce results for $q = 6$, $q = 12$, or $q = 18$. Hence, they don't perfectly align with the 10-year, 15-year and 20-year cut-offs in the *Commentary*, which is why they are only used for robustness.

For the 1914 to 2016 sample, I use the $q = 12$ and $q = 18$ results, which correspond to filters with cut-offs of about 17 years and 11 years, respectively. These time periods are roughly comparable to those in Table 1. I do not use the $q = 6$ results, which correspond to a cut-off of about 34 years. For $q = 12$, the estimated correlation is -0.48 with a confidence interval of (-0.78,-0.01). This aligns with the 15-year correlation in Table 1 remarkably well. For $q = 18$, the estimated correlation is -0.35 with a confidence interval of (-0.71,0.05). This is a slightly smaller correlation than the 10-year correlation in Table 1. It is also a modestly wider confidence interval, which includes zero. However, it is similar to Table 1 and suggests that the negative correlations in Table 1 are not artifacts of the stationarity assumption.

For the 1948 to 2016 sample, I use the $q = 6$ and $q = 12$ results, which correspond to filters with cut-offs of about 23 years and 11 years, respectively. I do not use the $q = 18$ results, which correspond to a cut-off of about 7 years. For $q = 6$, the estimated correlation is -0.08 with a confidence interval of (-0.60,0.49). This aligns well with the 20-year correlation in Table 2. For $q = 12$, the estimated correlation is -0.003 with a confidence interval of (-0.46,0.46), which aligns well with the 10-year correlation in Table 2.

References

- Müller, Ulrich K., and Mark W. Watson, 2008. "Testing Models of Low-Frequency Variability." *Econometrica*, 76(5): 979-1016.
- Müller, Ulrich K. and Mark W. Watson, 2015. "Low-Frequency Econometrics," NBER Working Paper, No. 21564.
- Müller, Ulrich K. and Mark W. Watson, 2017. "Long-Run Covariability," NBER Working Paper, No. 23186.