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Defining Capital in Growth Models

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Introduction

The definition or implicit measure of capital has, in some respects, been left ambiguous in the literature on economic growth. Most of the theoretical work on economic growth has assumed the existence of an aggregate capital stock that grows at the same rate as output in a balanced-growth equilibrium. This assumes away the aggregation problem by treating different units of capital as homogeneous.

However, empirical researchers on the subject of growth accounting, such as Greenwood, Hercowitz, and Krusell (1992), often must deal with the possibility that technological advance is at least partly embodied in new units of capital. In this case, a theoretically consistent measure of the aggregate capital stock requires the use of efficiency units that give greater weight to *newer* vintages. The question then arises as to the numeraire used to define an efficiency unit of capital.

In the context of a model where both embodied and disembodied technological progress take place at constant rates, this article explores two alternative measures of capital—one using the *newly produced* capital of each year as the numeraire for that year (the conventional measure of net capital, that is, the perpetual-inventory method), and the other using *base-year* capital as the numeraire. These two measures are equivalent for any given year, in that they assign equal *relative* weights to different vintages of capital. However, there are significant differences between the two measures in the context of time series on capital and, hence, in the resulting measures of the growth in capital.

While the base-year measure uses a *consistent* numeraire (that is, the initial year's capital stock) from one year to the next, the current-vintage measure changes the numeraire with each year's vintage. A meaningful comparison of capital stocks across years requires a consistent numeraire; a time series of capital stocks in which the numeraire changes each year would be useless for many empirical problems.

In growth accounting, for example, the Solow residual may be estimated by subtracting the growth rate of capital per worker from the growth rate of output per worker, provided that the former is calculated from a time series in which the capital stock has been aggregated in terms of the base-year numeraire. As we will show, the use of a capital stock series, aggregated in units of each year's vintage, yields an

overestimate of the Solow residual, requiring a correction using an independent estimate of embodied technological advance. Although the use of a current-year numeraire accounts for the heterogeneity of the capital stock *within* each year, it may yield the same upward bias in the estimate of the Solow residual as that which arises when all capital is treated as homogeneous.

The differences between the two aggregates are important conceptually, but they also have broad empirical implications. The magnitude of the difference depends, of course, on the role of technological growth "embodied" in the capital inputs of successive vintages. Gort, Greenwood, and Rupert (1999) show that technological growth in the stock of structures has proceeded at the rate of 1 percent per year. Using a frequently cited estimate of technological growth in equipment of 3 percent per year for the postwar period, they conclude that the stock of structures and equipment has grown at 2.2 percent and 4.4 percent, respectively. In contrast, growth using the conventional perpetual-inventory measures of the Commerce Department's National Income and Product Accounts was 0.75 percent for structures and 2.5 percent for equipment.

Other methods have led to even larger estimates of technological growth for capital goods. Bahk and Gort (1993), for example, estimate the quality change of capital for a large number of manufacturing industries. They conclude that a change in vintage of one year can be expected to lead to a change in output of 2.5 percent to 3.5 percent. Assuming that capital has a weight of roughly one-third in the production function, this implies a rate of quality change in capital of 7.5 percent to 10.5 percent per year. Indeed, when the effect of vintage is taken into account, the measure of residual disembodied technological change goes to zero. While the experience of manufacturing industries may not be characteristic of the economy as a whole, these results show how massive discrepancies can arise from using alternative concepts and their corresponding measures of capital.

The behavior of the capital/output ratio in a balanced-growth equilibrium also depends on the numeraire chosen for aggregating capital. When technological advance is at least partly embodied, the capital/output ratio becomes constant if, and only if, capital is aggregated in units of the current year's vintage; for an aggregate based on the initial year's vintage, the ratio grows toward infinity. The long-run behavior of the interest rate varies accordingly—the return on capital aggregated in current units becomes constant, while the return on capital aggregated in base-year units asymptotically approaches zero.

Empirical estimation of dynamic models often begins by testing the statistical properties of the relevant time series to determine whether their behavior is consistent with that assumed in the model. Since balanced growth implies that capital and output grow at the same rate, an empirical researcher testing the propositions of a balanced-growth model may expect the capital/output ratio to remain stationary over time. Such an expectation would be valid for the capital stock time series aggregated on a current-year numeraire, but not for the base-year aggregate.[1]

I. A Model of Embodied Technological Progress

Assuming constant returns to scale for capital and labor, the per capita production function can be written as

$$(1) \quad y_t = A_t f(k_t),$$

where A_t grows at the rate of disembodied technological progress, μ.

Net additions to per capita stocks of capital may be measured in efficiency units,

$$(2) \quad \dot{k}_t = \phi_t i_t - \delta k_t,$$

where i_t is per capita investment at time t; δ is the rate of capital depreciation; and ϕ_t is a measure of the quality of new capital goods, translating one physical unit of investment at time t into equivalent units of base-year capital. Additionally, we assume that $\phi_0 = 1$ and ϕ_t grows at the constant rate γ.

Current investment is related to current output through

$$(3) \quad y_t = i_t + c_t,$$

where c_t is per capita consumption at time t. Equation (3) assumes that investment and consumption goods are perfect substitutes and that quality improvements in new capital goods

1 In practice, econometric tests of growth theory often evade this point by running cross-country regressions. This is done, however, partly because of data availability and partly because of expediency in avoiding modeling problems that would arise from time-series tests. It does not change the fact that time-series econometric models are the proper vehicles for testing the implications of growth theory (for a critique of cross-country regressions, see Levine and Renelt [1992]).

entail no expenditure of current output. This is consistent with the approach taken by Solow (1960), Fisher (1965), and Greenwood, Hercowitz, and Krusell (1992).[2]

An alternative measure of capital is given by

$$(4) \quad \hat{k}_t = \frac{k_t}{\phi_t}.$$

Both k_t and $\hat{k}_t$ measure capital in efficiency units; however, the choice of the numeraire is different. To provide a clearer interpretation of these two measures, it may be helpful to decompose each measure into its underlying investment flow in a discrete-time analog:

$$k_t = (1+\gamma)^t i_t + (1+\gamma)^{t-1}(1-\delta) i_{t-1} + (1+\gamma)^{t-2}(1-\delta) i_{t-2} + \ldots + (1-\delta)^t i_0$$

and

$$\hat{k}_t = i_t + \frac{(1-\delta)}{(1+\gamma)} i_{t-1} + \frac{(1-\delta)^2}{(1+\gamma)^2} i_{t-2} + \ldots + \frac{(1-\delta)^t}{(1+\gamma)^t} i_0.$$

Thus, while k_t aggregates capital by augmenting more recent vintages to reflect their greater efficiency relative to the base-year vintage, $\hat{k}_t$ depreciates past vintages in order to correct for their relative obsolescence. While k_t is consistent across years in its choice of numeraire, $\hat{k}_t$ changes numeraire with each new vintage and reflects the perpetual-inventory method of aggregating each year's capital stock in efficiency units of that year's vintage.

The difference between k_t and $\hat{k}_t$ is potentially large and has drastic implications not only for tests of cointegration among a set of variables including output and capital, but also for estimating models where the capital stock is a key variable. For example, making the conservative assumption that obsolescence represents only half of conventionally measured depreciation, Gort and Wall (1998) estimate that k_t for the aggregate U.S. economy (excluding government and agriculture) grew at an annual rate of 6.06 percent during the 1947–89 period. In contrast, the equivalent measure of growth of $\hat{k}_t$ was only 3.77 percent per year.

Per capita output in terms of $\hat{k}_t$ is given by

$$(5) \quad y_t = A_t f(\phi_t \hat{k}_t),$$

and the rate of change of $\hat{k}_t$ is given by

$$(6) \quad \dot{\hat{k}}_t = i_t - (\delta + \gamma)\hat{k}_t.$$

The measurement of returns to capital, then, depends on whether k_t or $\hat{k}_t$ is used. Let r_t and $\hat{r}_t$ denote the marginal product of an extra physical unit of base-period and new capital, respectively. Thus,

$$(7a) \quad r_t = A_t f'(k_t)$$

$$(7b) \quad \hat{r}_t = A_t \phi f'(\phi_t \hat{k}_t).$$

Multiplying (7a) by ϕ_t, it follows that

$$(8) \quad \hat{r}_t = \phi_t r_t.$$

Note that $\hat{r}_t$ represents the marginal product of an extra unit of *new* capital at time *t*. To the extent that (gross) additions to the aggregate capital stock can take place only through newly produced units of capital, $\hat{r}_t$ (and not r_t) represents the relevant return to physical capital. In the presence of risk-free financial assets, the interest rate would equal $\hat{r}_t$ in equilibrium.

II. Analysis of Growth Paths

The analysis of efficient growth paths usually begins with the Euler equation describing the growth rate of per capita consumption. With embodied technological advance, this is given by

$$(9) \quad \sigma \frac{\dot{c}_t}{c_t} = \hat{r}_t - (\rho + \delta + \gamma),$$

where σ and ρ are preference parameters denoting the elasticity of intertemporal substitution and the rate of time preference, respectively. Solving equation (9) for $\hat{r}_t$ and using equation (8) gives

$$(10) \quad \hat{r}_t = \phi_t r_t = \sigma \frac{\dot{c}_t}{c_t} + \rho + \delta + \gamma.$$

In a balanced-growth equilibrium, per capita consumption grows at a constant rate; therefore, the return to $\hat{k}_t$ becomes constant, while the return to k_t decreases at the rate γ.

The rate at which per capita quantities grow along a balanced-growth path can be solved using a "sources of growth" equation, obtained by totally differentiating the production function with respect to time. Given the two measures of

■ **2** An alternative approach, taken by Hulten (1992), is to adjust each year's output to reflect the increased quality of investment goods. Thus, $y_t = \phi_t i_t + c_t$. This approach, however, leads to netting out the effects of embodied technological progress along the balanced-growth path.

capital, the sources of growth in the relevant equation can be expressed as

$$\text{(11a)} \quad \frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{A_t} + \theta_t \frac{\dot{k}_t}{\dot{k}_t} = \mu + \theta_t \frac{\dot{k}_t}{k_t}$$

and

$$\text{(11b)} \quad \frac{\dot{y}_t}{y_t} = \frac{\dot{A}_t}{A_t} + \theta_t \left[\frac{\dot{\hat{k}}_t}{\hat{k}_t} + \frac{\dot{\phi}_t}{\phi_t} \right] = \mu + \theta_t \gamma + \theta_t \frac{\dot{\hat{k}}_t}{\hat{k}_t},$$

where $\theta_t \equiv r_t k_t / y_t$ is capital's share in output at any time t.

In calculating the Solow residual, $\frac{\dot{A}}{A}$, attention must be paid to the distinction between $\hat{k}_t$ and k_t. Mistakenly using data for $\hat{k}_t$ in equation (11a), as if it represented measures of k_t, would lead to an upward bias in the estimated contribution of disembodied technological progress. However, even if the distinction between the two measures of capital is kept in mind, $\hat{k}_t$ cannot be used by itself to get at $\frac{\dot{A}}{A}$, without independent estimates of $\frac{\dot{\phi}}{\phi}$ as in equation (11b).

In a balanced-growth equilibrium, the capital/output ratio remains constant. With differing growth rates between the two measures of capital, however, the capital/output ratio cannot stay constant for both measures. Solving equation (11a) under the conjecture that k_t grows at the same rate as y_t yields the common growth rate $(\mu / 1 - \phi_t)$. This is unacceptable for two reasons: First, the rate of embodied technological progress, γ, does not appear in the solution for the final growth rate. Second, since r_t declines over time, so will θ_t, and the long-run growth rate itself will be a declining function of time.

Is a balanced growth path with constant long-run growth consistent with $\hat{k}_t$ growing at the same rate as y_t? If so, the output share of capital, θ_t, would also remain constant because $\hat{r}_t$ is constant in the balanced-growth equilibrium, and θ_t may be written as $\hat{r}_t \hat{k}_t / y_t$. With this construction, the long-run rate of balanced growth may be derived using equation (11b). Denoting the common growth rate of $\hat{k}_t$ and y_t by g, equation (11b) implies

$$\text{(12)} \quad \frac{\dot{y}_t}{y_t} = \frac{\dot{\hat{k}}_t}{\hat{k}_t} = g = \frac{\mu + \theta\gamma}{1 - \bar{\theta}},$$

where $\bar{\theta}$ denotes the value of θ_t in long-run equilibrium. Thus, in a balanced-growth equilibrium, y_t and $\hat{k}_t$ both grow at the common rate g, while the capital stock measured in base-period units grows at the higher rate of $(g + \gamma)$.

Indeed, since the two measures of capital grow at different rates whenever embodied progress occurs, they cannot be expected to be cointegrated. Thus, in studying cointegration of the capital stock and other economic variables, the hypotheses to be tested should be constructed in light of the difference between them.

In testing the propositions of balanced growth, theory suggests the use of $\hat{k}_t$. This also happens to be the conventional measure of perpetual-inventory net stocks on the assumption, of course, that the depreciation rate used to construct such stocks correctly captures *both* obsolescence and physical decay. However, $\hat{k}_t$ requires an independently derived measure of $\frac{\dot{\phi}}{\phi}$ in order to calculate disembodied technological progress, $\frac{\dot{A}}{A}$.

III. Conclusion

When technological progress is embodied in capital, measuring the capital stock is problematic because capital is not homogeneous across vintages, especially in the context of time series. The choice of the measure of the capital stock is dictated by the question being addressed. It has been shown that the relevant measure differs for balanced growth and growth decomposition models. In particular, when aggregating heterogeneous capital across vintages, a numeraire must be consistent with the underlying theory. A further complication is that little progress has been made in obtaining independent estimates of embodied technological progress—a necessary step in the empirical implementation of a broad range of growth models.

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