James Madison's Monetary Economics

by Bruce D. Smith

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Introduction

James Madison's essay, "Money" (pp. 2–6 as reprinted here), considers issues that are as timely and important today as they were when it was first written. While his concern with an eighteenth-century economy and his focus on an ultimate return to a gold standard may seem to relegate his writings to the history of economic thought, he was in fact wrestling with questions that have been central to monetary theory and policymaking for more than two centuries. Even now, his way of framing these issues creates a wonderful opportunity for contrasting two fundamentally different views of monetary policy's role and function.

Madison, of course, wrote as a member of a Revolutionary government that faced profound fiscal and monetary policy problems. With no established tax base and little ability to borrow in the conventional "capital markets" of the day, it confronted huge wartime expenditures against a great power. With no alternative but to run large deficits and monetize them, this government accomplished the astounding feat of financing 82 percent of its expenditures by printing money.[1]

Needless to say, this achievement had its cost. The massive printing of money was associated with a major inflation, which accelerated over time. In January 1777, $1.25 in Continental currency could purchase $1.00 in gold, reflecting a relatively modest depreciation of 25 percent during the first year and a half of the war. By January 1781, however, $100 Continental was required to purchase $1.00 in gold.

What was the source of this inflation? The conventional answer—with which Madison takes issue—is the quantity of money printed. Madison (p. 6) makes a fairly standard guess that puts the prewar money supply of the 13 colonies at $30,000,000. During the war, the Federal government issued $226,000,000 in Continental currency, and the states issued a similar amount.[2] Was this the "cause" of the

1 This figure, which refers to the period 1775–79, is from Ferguson (1961, pp. 43–44). It contrasts markedly with the fraction of expenditures financed through seignorage revenue in any modern economy.

2 See Ferguson (1961) and Nevins (1927, p. 481).

inflation? Or was it rather that the size of the deficit, combined with the behavior of prices, forced that much money to be printed? And could such a large inflation have been averted?

In analyzing Madison's response, it is important to note carefully how the wartime currency was created. In 1775, the money supply of the 13 colonies consisted of paper printed by the individual colonies plus gold and silver coin.[3] Colonial governments did not redeem their paper currencies in specie, but they did sometimes promise to do so in the future, and they typically made taxes payable in either paper money or specie, accepting paper at a fixed rate in terms of specie. When the Continental Congress began issuing its own money in 1775, it followed the same system. Continental dollars were not redeemable in gold or silver, but the Congress promised postwar redemption of paper into gold on a one-for-one basis, and it also made paper money acceptable for taxes on the same basis as coin.

Madison's writings presumed that the government would honor its promise of a one-for-one redemption. While that faith eventually proved unfounded,[4] one could ask a counterfactual question: What would have happened if the government's promise of redemption had been believed and had been honored?

This question is motivated by much more than pure intellectual curiosity. Over and over again, governments facing large wartime expenditures have suspended gold standards, issued then-irredeemable paper money, and promised to resume gold convertibility at some date after the war's end. Some examples of particular importance in monetary history include Britain during and after the Napoleonic Wars and World War I and the United States during and after the Civil War. In each case, there was some wartime inflation, accompanied either by contemporary or later historical debate over whether "better"—that is, less inflationary—policies were available. Invariably, there was also a postwar deflation before resumption of a gold standard. And indeed, there were substantial deflations in many parts of the United States after the Revolution; in some places, these were just as pronounced as wartime inflation. For example, by 1786 prices in Pennsylvania had returned to their 1773 levels (Bezanson [1951, p. 174]).

The fact that the policies followed by the Revolutionary government—while no doubt necessary for it—have been so widely adopted elsewhere suggests that Madison's concerns are of interest in a far broader context than just that of the Revolutionary War. In fact, he addressed a number of issues that remain basic in monetary theory to this day, including:

i. To what extent is inflation determined by money growth? Is this all that matters, as is often asserted, or does it matter almost not at all, as Madison argued?

ii. If money growth is not all that matters, does the degree of inflation depend on the nature of the government's promises about "backing" its money in the future through gold redemption or some other scheme?

iii. Madison's argument about the lack of inflation resulting from money growth is not based on any commitment to reduce the money stock at some future date. Thus, he asserts that some future redemption in gold—essentially, a commitment to future price-level stabilization or "targeting"—is adequate to prevent money growth from raising the price level today. This seems in conflict with the quantity theory of money, namely, that inflation is always and everywhere a monetary phenomenon.[5] What is the theoretical foundation for Madison's view?

iv. Wisdom that was, at least until recently, "conventional" asserts that it is always less inflationary to finance a deficit by borrowing (issuing bonds) than by printing money. At the end of his essay, Madison denies the validity of this wisdom.

The first three of these issues relate to one of the oldest debates in monetary economics: Does a permanent increase in the money supply necessarily raise the price level? Most adherents of the quantity theory would argue that the answer is yes, and their viewpoint currently prevails in the formulation of monetary policy. However, a competing school of thought argues that permanent increases in the money supply need not be inflationary

3 See Smith (1988) and Rolnick, Smith, and Weber (1994) for a discussion of this period.

4 The ultimate redemption rate was 3 cents on the dollar (Ferguson [1961]).

5 For discussions of the quantity theory of money, see Friedman (1956) and Lucas (1980). Also, while I will follow Madison in using the term "money," note that his arguments apply equally to any expansion in the stock of government liabilities. Viewed from this perspective, Madison's assertions are less obviously in conflict with the quantity theory than they may first appear to be.

if accompanied by appropriate "backing" of the newly created money. The modern intellectual foundations of this idea, which has ancient antecedents,[6] appear in Tobin (1963), Wallace (1981), Sargent (1982), Sargent and Wallace (1982), and Sargent and Smith (1987a,b). Most of the backing discussed in this literature assumes that increases in the money supply are accompanied by government asset acquisitions of equal value; hence, this reasoning can be applied in only a limited way even to temporary deficit monetization. Madison argues that the government can print money to finance temporary deficits, backed by a promise of future redemption (but not retirement), and that the only resulting inflation will be due to the delay in redemption. Moreover, he strongly denies that the behavior of the price level should depend in any way on the quantity of money printed, unless this delays the time to redemption.[7] He also asserts that delays in future redemption put upward pressure on the current price level.

The remainder of this article constitutes a modern theoretical attempt to formalize and evaluate Madison's views. To a large extent, my conclusions are favorable to his line of argument. The commitment to a future redemption, which is little more than a pledge to stabilize prices in the future, is enough to break the link between money growth and inflation. In addition, his assertion that delayed redemption puts upward pressure on the price level is also strongly supported. However, the analysis does not suggest that price-level behavior is fully independent of changes in the money stock. In this respect, Madison seems to have gone too far.

To be fair to him, a formal theoretical analysis should capture the main economic aspects of his reasoning. I take these to be as follows:

i. Money is held primarily as an asset. If it constitutes a future claim to specie, its current value should be just the discounted present value of that claim.

ii. As a corollary, if other assets earn a higher return than money, this happens only because they involve some "inconvenience," such as being issued in excessively large denominations.

iii. An alternative asset to money exists. The real rate of interest on it is unaffected by government policy (although it need not always be held in positive quantities).

In the subsequent section, I build a model incorporating these features and apply it to some of the issues that concerned Madison. In doing so, I gloss over some other issues that he took up, but the concluding section offers comments on them as well.

I. The Environment

To illustrate Madison's points, I consider a two-period-lived, overlapping-generations model of a particularly simple variety.[8] At each date $t = 0, 1, ...$, a set of N identical agents is born. They are endowed in both periods with some of a single perishable consumption good; let w_j ($j = 1, 2$) denote the age j endowment of a representative agent. I assume throughout that $w_1 > 0$ and $w_2 > 0$.

In addition, agents have access to a *reversible* linear technology which allows one unit of current consumption to be converted into $\phi > 0$ ounces of gold (and back again, if desired). One possible interpretation of the technology is that this is a small open economy operating in a world on a gold standard, so that the consumption good can always be bought or sold abroad at a fixed rate for some amount of gold. Such an interpretation would obviously be fairly appropriate to the economy in which Madison lived.

Once obtained, gold can either be stored in raw form or—if the domestic economy is on a gold standard—it can be coined. In either case, it depreciates at the rate $\delta \in (0,1)$.

Each young agent values age j consumption, denoted c_j, according to the (common) utility function $\ln c_1 + \beta \ln c_2$. Note that gold is then not held by agents for its consumption value; it is held—if at all—only as an asset.

It will be necessary in what follows to allow for the possibility that agents born at certain dates face a government-levied tax. Any direct taxes that the government does levy are lump-sum in nature. I also assume that agents pay these taxes (if at all) only when old.

The following notation will prove useful: Suppose an agent, born at date t, pays a lump-

6 See Mints (1945) for a discussion of this idea, as well as for criticisms.

7 Note that one could also cast this as an increase in the stock of outstanding government debt without affecting the market value of the debt, unless it delayed the time to redemption.

8 The model closely resembles that of Sargent and Wallace (1983).

sum tax of τ_{t+1} when old and faces a gross rate of return on a single asset of r_{t+1} between t and $t+1$. Then, let $s(w_1, w_2 - \tau_{t+1}, r_{t+1})$ denote the savings function of this agent. Given the assumed form of the utility function, clearly

$$(1) \quad s(w_1, w_2 - \tau_{t+1}, r_{t+1}) = \frac{\beta w_1}{1+\beta} - \frac{w_2 - \tau_{t+1}}{(1+\beta)r_{t+1}}.$$

I will assume that agents are willing to save, even if they must do so by storing gold in raw form; I therefore impose an assumption:[9]

Assumption 1. $s(w_1, w_2, 1-\delta) > 0$.

II. The Government

The government that Madison contemplated as he wrote had large wartime spending needs and very limited powers of taxation. The result was a massive government budget deficit that was financed by printing paper money.

In Madison's vision—which essentially eventuated in practice—the war would be followed by a period of peace accompanied by a relatively balanced government budget, or perhaps even one in surplus. During this time, wartime paper money would continue to circulate.[10] After a transitional period, the paper money would be retired, having been converted into gold at some specified rate. Thereafter, the economy would remain on a gold standard, and Madison (as well as others) probably envisioned a purely metallic currency from that point onward.

In consonance with this scenario, I consider a government confronting the following circumstances. For dates $t = 0, 1, ..., T_1$, the government has a real per capita expenditure (and deficit) level of $g > 0$.[11] It finances expenditures solely by printing paper currency that is not then redeemable, but that it promises to convert into gold at some future date.

During this period, the government confronts the budget constraint

$$(2) \quad g = (M_t - M_{t-1})/p_t; \quad t = 0, ..., T_1$$

where M_t is the stock of paper currency outstanding at t, and p_t is the time t price level. The initial money stock M_{-1} is given as an initial condition. For simplicity—and consistency with the realities of a Revolutionary government—I set $M_{-1} = 0$.

The government has no direct expenditures for dates $t > T_1$. I assume that for $t = T_1 + 1, ..., T-1$, it engages in no activity whatsoever, and neither adds to nor subtracts from the existing stock of money. A gold standard has not yet been established, and the money is not yet redeemable. Thus, for $t = T_1, ..., T-1$, $M_t = M_{T_1}$ holds. This corresponds to the transitional period prior to establishment of a full gold standard.

At date T, the government "calls in" the existing stock of paper currency and replaces it dollar for dollar with gold coins which it mints—at its own expense—at that date. Thereafter, it stands ready to coin freely any gold brought to the mint by private agents. The government coins gold dollars and establishes a mint ratio b stating the number of ounces of gold in a newly minted gold dollar. Any subsequent change in the money supply is purely the result of minting and melting activity by the private sector. There are no policy-induced changes in the money supply, nor is there any further government expenditure. I also assume that there is no uncertainty or lack of commitment, so that the transitional dates T_1 and T are known in advance by everyone.

At T, the government must mint enough new coins to redeem the existing money stock and must raise some resources for this purpose. Let n_T^g be the number of new gold coins, in dollars, created by the government at T. Clearly $n_T^g = M_{T_1}$ must hold. Moreover, to mint n_T^g new gold dollars, the government requires $n_T^g(b/\phi)$ units of the consumption good, which it obtains by levying a lump-sum tax on old agents at T. Since there is no other taxation at any date, under the policy described,

$$(3) \quad \tau_T = n_T^g(b/\phi) = M_{T_1}(b/\phi)$$

$$\tau_t = 0; \quad t \neq T.$$

After date $T-1$, the entire stock of money consists of gold dollars. I assume that gold

9 Assumption 1 is equivalent to $\beta w_1(1-\delta) > w_2$.

10 Of course, this did not happen after the Revolution, as the phrase "not worth a Continental" indicates. However, a good deal of new paper money was created, by both state and federal governments, from 1783 to 1789. For an overview of this period, see Rolnick, Smith, and Weber (1994).

11 As noted above, the Revolutionary government financed about 82 percent of its expenditures by printing money. Thus, the implied abstraction from tax revenue is well founded.

coins circulate by weight,[12] and I let G_t denote the stock of gold dollars—by weight—at t. As before, p_t continues to denote the time t price level.

III. The Behavior of Agents

In this section, I describe the behavior of agents before, during, and after the implementation of a gold standard.

The Paper Money Regime ($t < T - 1$)

For all $t < T$, paper currency is in circulation (although the promise of ultimate redemption is understood and believed). I also focus on the situation where paper money is accepted voluntarily in exchange for private assets.[13] Since agents can choose between holding money and holding raw gold as an asset,[14] clearly money will be held only if it earns a real return as great as that on raw gold.[15] The gross real return on paper currency between t and $t + 1$ is given by p_t / p_{t+1}, and the gross real return on storage of raw gold is $1 - \delta$. Thus,

$$(4) \quad p_t/p_{t+1} \geq 1 - \delta; \quad t = 0, \ldots, T-1$$

must hold.

Let g_t denote the storage of raw gold by a young agent at t, and let m_t denote the accumulation of real balances. Then g_t, m_t, and a consumption profile (c_{1t}, c_{2t}) are chosen to maximize $\ln c_{1t} + \beta \ln c_{2t}$, subject to

$$(5) \quad c_{1t} + m_t + (g_t/\phi) \leq w_1$$

and

$$(6) \quad c_{2t} \leq w_2 + m_t(p_t/p_{t+1}) + (1-\delta)(g_t/\phi).$$

The solution to this problem sets

$$(7) \quad m_t + g_t = s(w_1, w_2, p_t/p_{t+1})$$

and

$$(8) \quad [(p_t/p_{t+1}) - (1-\delta)]g_t = 0.$$

Equation (8) asserts that, if the return on money exceeds that on the storage of raw gold, no raw gold will be stored.

The Transition ($t = T - 1$)

Young agents born at $T - 1$ will live through the transition to a gold standard; when old, they will bear the costs of this transition. They thus bear the lump-sum tax, τ_T, when old.

In addition, when old, these agents will have the opportunity to coin or melt gold. Let n_t^j be the coinage (or melting, if negative) of gold by a representative agent of age j ($j = 1,2$) in period t. Clearly this coinage can be nonzero only for $t \geq T$. Young agents born at $T - 1$ choose a level of real balances, m_{T-1}, a quantity of raw gold storage, g_{T-1}, a consumption profile (c_{1T-1}, c_{2T-1}), and a minting/melting strategy when old n_T^2, to maximize $\ln c_{1T-1} + \beta \ln c_{2T-1}$, subject to

$$(9) \quad c_{1T-1} + m_{T-1} + (g_{T-1}/\phi) \leq w_1$$

and

$$(10) \quad c_{2T-1} \leq w_2 - \tau_T + m_{T-1}(p_{T-1}/p_T) + (1-\delta)(g_{T-1}/\phi) + n_T^2[(1/p_T) - (b/\phi)],$$

where the last term in (10) represents the profit from using (b/ϕ) units of resources to obtain n_T^2 gold dollars, which then have a purchasing power of n_T^2/p_T.

An absence of arbitrage opportunities requires that

$$(11) \quad p_T = \phi/b.$$

When (11) holds, as it must in equilibrium, the total savings of a young agent at $T - 1$ must satisfy

$$(12) \quad m_{T-1} + g_{T-1} = s(w_1, w_2, p_{T-1}/p_T).$$

In addition, $g_{T-1} = 0$ holds if $p_{T-1}/p_T > 1 - \delta$.

12 I could assume equally well that coins circulate according to their face value ("by tale"). Each outcome is an equilibrium, and it makes no qualitative difference to the results which situation obtains. For a discussion of coins that circulate by tale, see Sargent and Smith (1997).

13 In practice, during the Revolution, the army often seized what it needed, offering a choice of paper liabilities or nothing in exchange. Clearly here, when paper money was taken, it was not taken voluntarily. Madison obviously conceived of a situation where agents take money for goods of their own volition.

14 And since there is no uncertainty.

15 Parenthetically, Madison's argument implies that he regarded money as an asset, earning a return competitive with that on relatively close substitutes.

A Gold Standard ($t \geq T$)

For $t \geq T$, the economy is on a gold standard. No further taxes are levied, and all agents, old and young, have the opportunity to mint and melt coins at all dates. As before, agents can select a level of real balances, m_t (now held in the form of gold coins), a quantity of raw gold to store, g_t, a consumption profile, (c_{1t}, c_{2t}), and a minting/melting strategy, (n_t^2, n_t^2), to maximize $\ln c_{1t} + \beta \ln c_{2t}$, subject to

$$(13) \quad c_{1t} + m_t + (g_t/\phi) \leq w_1 + n_t^1[(1/p_t) - (b/\phi)]$$

and

$$(14) \quad c_{2t} \leq w_2 + m_t(1-\delta)(p_t/p_{t+1}) + (g_t/\phi)(1-\delta) + n_{t+1}^2[(1//p_{t+1}) - (b/\phi)].$$

The real balance term in (14) must now be multiplied by $1 - \delta$, since gold coins circulate by weight and depreciate at the rate δ.

As before, an absence of arbitrage opportunities associated with minting and melting requires that

$$(15) \quad p_t = \phi/b; \quad t \geq T.$$

In addition, the price stability revealed in (15) implies that there is never any reason for agents to store raw gold rather than hold gold coins. Hence, without loss of generality, we can take $g_t = 0$; $t \geq T$. Then, agents save entirely in the form of gold, which earns a gross real return of $(1-\delta)(p_t/p_{t+1}) = (1-\delta)$, where the equality follows from (15). Real balances per capita are then given by

$$(16) \quad m_t = s(w_1, w_2, 1-\delta); \; t \leq T.$$

IV. A General Equilibrium

For $t \geq T$, it is clear what must happen in equilibrium. Equation (15) gives the price level. The nominal per capita gold stock at t, G_t, must then obey

$$(17) \quad G_t = p_t s(w_1, w_2, 1-\delta) = (\phi/b)s(w_1, w_2, 1-\delta); \; t \geq T.$$

Since the nominal gold stock (in ounces) is constant, private minting/melting in each period must just replace the depreciated gold stock:

$$(18) \quad (n_t^1 + n_t^2)/2 = \delta G_{t-1}; \; t \geq T+1.$$

In periods before the advent of the gold standard, there is a much richer variety of possible equilibrium outcomes.[16] Here I construct an equilibrium having certain features and then display the restrictions on parameters required for those features to emerge. The equilibrium features I consider are chosen for two reasons. First, they seem illustrative of what Madison had in mind. Second, for much of history, economies have abandoned gold standards in time of war and financed their deficits by printing paper money. With the cessation of hostilities, the government budget is roughly balanced (or even in surplus), although gold convertibility is not immediately resumed. A postwar deflation occurs during this period, terminating with the resumption of gold convertibility. Indeed, such a pattern was observed through much of the United States following both the Revolutionary War and the Civil War, and in the United Kingdom after World War I. For these reasons, I focus on equilibria which display inflation for $t = 0, 1, \ldots, T_1$, followed by a deflation for $t = T_1 + 1, \ldots, T$. This deflation ends with conversion to a gold standard.

The Deflation ($t = T_1 + 1, \ldots, T$)

In this section, I state conditions under which there is an equilibrium satisfying

$$(19) \quad p_t \geq p_{t+1}; \; t = T_1, \ldots, T-1.$$

Note that (19) implies that no agent will wish to store raw gold during the period in question.

At date $T-1$, young agents understand that they will be required to pay for the transition to a gold standard. In addition, since they store no gold when young, the time $T-1$ equilibrium condition in the money market is that

$$(20) \quad M_{T-1}/p_{T-1} = M_{T_1}/p_{T-1} = s(w_1, w_2 - \tau_T, p_{T-1}/p_T) = \beta w_1/(1+\beta) - (w_2 - \tau_T)/(1+\beta)(p_{T-1}/p_T).$$

16 This is not to say that any given economy has multiple possible equilibrium outcomes. Rather, different economies may have equilibria that look quite different from one another.

Substituting (3) and (11) into (20), and solving for M_{T_1}/p_{T-1} yields the equilibrium level of real balances at $T-1$:

(21) $M_{T_1}/p_{T-1} = w_1 - w_2(\phi/b)/\beta p_{T-1}.$

Equation (21) implies that the time $T-1$ price level is given by

(22) $p_{T-1} = (M_{T_1}/w_1) + (\phi/b)(w_2/\beta w_1).$

Notice that $p_{T-1} \geq p_T = (\phi/b)$ holds iff

(23) $M_{T_1} \geq (\phi/b)[w_1 - (w_2/\beta)]$

$$= (\phi/b)\left\{\frac{1+\beta}{\beta}\right\} s(w_1,w_2,1)$$

is satisfied. Below I derive restrictions on parameters implying that (23) holds.

For $t = T_1 + 1, ..., T-2$, young agents will experience no regime transitions and will bear no taxation. In addition, (19) implies that they will store no gold. Hence the equilibrium level of real balances at these dates is given by

(24) $M_t/p_t = M_{T_1}/p_t = s(w_1,w_2,p_t/p_{t+1})$

$$= \beta w_1/(1+\beta) - w_2$$

$$\div (1+\beta)(p_t/p_{t+1}).$$

Equation (24) can be solved for p_t in terms of p_{t+1}; the implied solution is

(25) $p_t = M_{T_1}[(1+\beta)/\beta w_1] + (w_2/\beta w_1)p_{t+1};$

$$t = T_1 + 1, ..., T-2.$$

Equations (22) and (25) describe the evolution of the price level during the transitional period prior to the establishment of a gold standard. Equation (25) implies that $p_t \geq p_{t+1}$ is satisfied iff

(26) $M_{T_1}/p_{t+1} \geq \beta w_1/(1+\beta) - w_2/(1+\beta)$

$$= s(w_1,w_2,1)$$

holds. Thus, by induction, if

(27) $M_{T_1}/p_{T_1+1} \geq s(w_1,w_2,1)$

obtains, so does equation (19). Thus, (27) is sufficient for a sustained postwar deflation to be observed.

The following proposition fully describes the behavior of the price level during this deflation, given the inherited money supply M_{T_1}:

PROPOSITION 1. For $t = T_1 + 1, ..., T-1$, the price level satisfies

(28) $p_t = (\phi/b)(w_2/\beta w_1)^{T-t}$

$$+ M_{T_1}\{w_1^{-1}(w_2/\beta w_1)^{T-(t+1)}$$

$$+ [(1+\beta)/\beta w_1][1 - (w_2/\beta w_1)^{T-(t+1)}]$$

$$\div [1 - (w_2/\beta w_1)]\}.$$

Proposition 1 is easily verified by a comparison of equations (22), (25), and (28).

The Inflation ($t \leq T_1$)

It remains to describe the evolution of the money supply and the price level during the wartime period of positive government expenditure, which was obviously the issue that concerned Madison. In addition, he believed that there was an alternative asset to money that was relevant during this period, that money and this other asset were closely substitutable, and that government policy could not influence the rate of return on the alternative asset. Motivated by Madison's thinking, I proceed as follows in this section: If agents store raw gold, then gold competes with money in agents' portfolios. Moreover, the return on gold, $1-\delta$, is exogenously given. Thus, if agents store gold at any date $t \leq T_1$, this serves the role of Madison's alternative asset.

Of course, gold and paper money can both be held voluntarily at t iff

(29) $p_t/p_{t+1} = 1-\delta.$

I now construct an equilibrium where (29) holds for all $t = 0, 1, ..., T_1 - 1$. In addition, raw gold is (at least potentially) stored at these dates. I also impose

(30) $p_{T_1} > (1-\delta)p_{T_1+1}.$

Equation (30) is consistent with inflation occurring between T_1 and $T_1 + 1$, but raw gold is not stored between these periods. Allowing (29) to be violated at $t = T_1$ eases the construction of the desired equilibrium.

Equation (29) implies that

(31) $p_t = (1-\delta)^{T_1 - t}p_{T_1}; \quad t = 0, ..., T_1$

In addition, the government budget constraint (2) requires the money supply to evolve according to

$$(32)\quad M_t = M_{t-1} + gp_t$$
$$= M_{t-1} + gp_{T_1}(1-\delta)^{T_1 - t};\ t \le T_1,$$

with $M_{-1} = 0$ given as an initial condition. The following proposition then describes the evolution of the real and nominal money supplies:

PROPOSITION 2.

a) For $t = 0, 1, ..., T_1$, the nominal money supply satisfies

$$(33.a)\quad M_t = M_0 + (g/\delta)(1-\delta)^{T_1 - t} \times [1-(1-\delta)^t]p_{T_1}$$

with

$$(33.b)\quad M_0 = gp_0 = g(1-\delta)^{T_1}p_{T_1}.$$

b) For $t = 0, 1, ..., T_1$, the real money supply satisfies

$$(34)\quad M_t/p_t = (g/\delta)[1-(1-\delta)^{t+1}].$$

Part a of the proposition can be verified directly by substituting (33.a) into (32). Part b is immediate from (31) and (33.b).

Of course, the construction of equilibrium just undertaken is predicated on gold being stored at all dates prior to T_1 and on (30). Raw gold is stored for $t \le T_1 - 1$ if

$$(35)\quad M_t/p_t = (g/\delta)[1-(1-\delta)^{t+1}] < s(w_1,w_2,1-\delta)$$

is satisfied for all such dates. Clearly this condition is equivalent to

$$(35')\quad M_{T_1-1}/p_{T_1-1} = (g/\delta)[1-(1-\delta)^{T_1}] < s(w_1,w_2,1-\delta).$$

Also, in order for (30) to hold,

$$(36)\quad M_{T_1}/p_{T_1} = (g/\delta)[1-(1-\delta)^{T_1+1}] > s(w_1,w_2,1-\delta)$$

must obtain.

It remains to determine the price level and money stock at time T_1. Since there is no raw gold storage at T_1, money market clearing requires that

$$(37)\quad M_{T_1}/p_{T_1} = s(w_1,w_2,p_{T_1}/p_{T_1+1})$$
$$= \beta w_1/(1+\beta)$$
$$- w_2/(1+\beta)(p_{T_1}/p_{T_1+1}).$$

Solving (37) for p_{T_1} yields

$$(38)\quad p_{T_1} = M_{T_1}[(1+\beta)/\beta w_1] + (w_2/\beta w_1)p_{T_1+1}.$$

It is then immediate from (38), (25), and proposition 1 that

$$(39)\quad p_{T_1} = (\phi/b)(w_2/\beta w_1)^{T-T_1}$$
$$+ M_{T_1}\{w_1^{-1}(w_2/\beta w_1)^{T-(T_1+1)}$$
$$+ [(1+\beta)/\beta w_1]$$
$$\times [1-(w_2/\beta w_1)^{T-(T_1+1)}]$$
$$\div [1-(w_2/\beta w_1)].$$

Equation (36) can be rewritten as

$$(40)\quad M_{T_1} = (g/\delta)[1-(1-\delta)^{T_1+1}]p_{T_1}.$$

Equations (39) and (40) then determine M_{T_1} and p_{T_1}. Once those values have been obtained, all other equilibrium price levels can be deduced from (28) and (31).

It will now be useful to introduce some notation. Define x by the relation

$$(41)\quad s(w_1,w_2,x) \equiv (g/\delta)[1-(1-\delta)^{T_1+1}].$$

A comparison of (36), (37), and (41) will indicate that $x = p_{T_1}/p_{T_1+1}$, and x is clearly an exogenous variable. Condition (30) requires that $x > 1-\delta$ hold. In addition, define ψ_1 and ψ_2 by

$$(42)\quad \psi_1 \equiv s(w_1,w_2,x)(w_2/\beta w_1)^{T-T_1}$$

and

$$(43)\quad \psi_2 \equiv (\beta/w_2)\psi_1\{1+[(1+\beta)/\beta]$$
$$\times [(\beta w_1/w_2)^{T-(T_1+1)} - 1]$$
$$\div [1-(w_2/\beta w_1)]\}.$$

The following result is then immediate:

PROPOSITION 3. Suppose that

(44) $\psi_1 \geq (1 - \psi_2)(1 + \beta)s\,(w_1, w_2, 1)/\beta > 0$

and

(45) $g(1 - \delta)^{T_1 - 1} \geq w_2[x - (1 - \delta)]/(1 + \beta)x$

$> \delta\beta w_1/(1 + \beta).$

Then, an equilibrium satisfying (19), (29), *and* (30) *exists. This equilibrium has*

(46) $M_{T_1} = (\phi/b)\,\psi_1/(1 - \psi_2)$

and

(47) $p_{T_1} = (\phi/b)(w_2/\beta w_1)^{T - T_1}/(1 - \psi_2).$

The proof of proposition 3 appears in appendix A. The first inequality in (44) implies that (23) is satisfied and hence that $p_{T-1} \geq p_T$ holds. The second inequality in (44) is required for $M_{T_1} > 0$ and $p_{T_1} > 0$ to hold. Finally, (45) implies that (35), (36), $x > 1 - \delta$, and (27) are satisfied. Satisfaction of (27), of course, implies that $p_t \geq p_{t+1}$ holds for all $t = T_1 + 1, ..., T - 1$.

It remains to describe conditions under which the inequalities in (45) are satisfied. These conditions are stated in the following:

PROPOSITION 4.
a) The relations in (44) *hold iff*

(48) $[(w_1 + w_2)/w_1](w_2/\beta w_1)^{T - (T_1 + 1)}$

$> w_2(x - 1)/x\,s\,(w_1, w_2, x)$

$\geq (w_2/\beta w_1)^{T - (T_1 + 1)}$

is satisfied.

b) Suppose that w_1, w_2, β, T_1 *and* T *satisfy* $\beta w_1 > w_2$, $T \geq T_1 + 2$, *and*

(49) $1/(1 + T_1) > w_2/\beta w_1.$

Then, there exists a nonempty interval, $[\underline{x}, \overline{x})$, *with* $\underline{x} > 1$, *such that* (48) *holds iff* $x \in [\underline{x}, \overline{x})$. *In addition, for all* $x \in [\underline{x}, \overline{x})$, (1.*a*.) *and* (45) *hold if* δ *is sufficiently close to zero.*

Proposition 4, which is proved in appendix B, asserts that parameter values can always be chosen so that the construction of equilibrium performed here is valid. In the next section, I examine some properties of this equilibrium.

V. Madison's Assertions

It is now possible to use the construction of sections III and IV to investigate the validity of some of Madison's main assertions, which I take to be as follows:

i. For a government following the kind of policy outlined in section II, the simple quantity theory of money fails, even before the transition to a gold standard.
ii. That is, the rate of inflation and the growth rate of the money stock are not the same, and it is easy for the money growth rate to substantially exceed the inflation rate.
iii. More strongly, the behavior of the price level is independent of the quantity of money printed, so long as there is no uncertainty about the date of transition to a gold standard.
iv. While this may reflect my own reading, Madison seems to suggest that the behavior of the price level does not depend on the size of the government deficit, so long as there is no uncertainty about *T*. Anything that delays the transition to a gold standard acts to raise prices, at least up to date T_1.

I now investigate each of these propositions.

It is the case here that the rate of growth of the money stock does exceed the rate of inflation in all periods prior to *T*. Indeed, in some periods the difference can be quite substantial. I now state the following result:

PROPOSITION 5. For all $t < T$, *it is true that* $M_{t+1}/M_t > p_{t+1}/p_t$ *holds. Indeed,*

a) for $t \leq T_1 - 1$,

(50) $p_{t+1}/p_t = (M_{t+1}/M_t)[1 - (1 - \delta)^{t+1}]$

$\div [1 - (1 - \delta)^{t+2}] < (M_{t+1}/M_t).$

b) For $t = T_1, ..., T - 1$,

$$(51) \quad p_{t+1}/p_t \leq (1/x)(M_{t+1}/M_t) < M_{t+1}/M_t .$$

The proof of proposition 5 appears in appendix C. For small values of δ, $[1-(1-\delta)^{t+1}] \div [1-(1-\delta)^{t+2}]$ is approximately equal to $(t+1)/(t+2)$. Thus, early in the period of deficit finance, the price level can rise far more slowly than the money supply. In addition, since the money supply is constant for $t = T_1, ..., T-1$, and since deflation is under way during this time, the money supply grows faster than the price level here as well. Indeed, since x can be fairly large, equation (51) implies that the difference between the rate of inflation and the rate of money creation can again be quite great. Thus, Madison's first assertion is borne out.

As is apparent from proposition 1 and equation (31), however, the model supports Madison's second assertion less well. The price level at all dates can be viewed as depending—and, moreover, depending proportionally—on M_{T_1}. Since $M_{-1} = 0$, M_{T_1} is the total quantity of paper money printed during the period of deficit finance. The entire time path of prices, up to date T, depends on M_{T_1}, although the rate of inflation does not.

Similarly, the analysis suggests that the total size of the deficit financed through date T_1 affects the *price level* at all dates and the rate of inflation/deflation at all dates $t = T_1, ..., T-1$. However, it does *not* affect the rate of inflation for $t < T$, which is simply $1/(1-\delta)$. To see the first point, notice from equations (41), (42), and (43) that a higher government budget deficit raises ψ_2 and hence—by equation (47)—raises p_{T_1}. From equation (41), an increase in g also raises x; this clearly raises p_{T_1}/p_{T_1+1}; indeed, it raises p_t/p_{t+1} for all $t = T_1, ..., T-1$. Thus, a larger deficit raises the price level for all dates up to and including T_1; but a larger deflation also ensues when deficit spending ceases.

It remains to investigate Madison's last assertion, namely, that a delay in the transition to a gold standard implies a higher price level for all $t \leq T_1$. This is, in fact, accurate, as the next proposition asserts. Its proof is given in appendix D.

PROPOSITION 6. Consider two economies that are identical in all respects except their dates of transition to a gold standard. Let $\tilde{T}$ (T) denote the transition date in the first (second) economy, and let $\tilde{p}_t(p_t)$ be the date t price level in the first (second) economy. Suppose that $\tilde{T} > T$ and $x \geq \underline{x}$ hold. Then, $\tilde{p}_t > p_t$ for all $t \leq T_1$.

Thus, other things equal, a more rapid movement to a gold standard implies less upward pressure on the price level, exactly as Madison argued. It is also easy to show that it implies less money will be printed.

VI. Conclusion

Madison's essay, "Money," challenges the belief in a necessary connection between money growth and inflation that underlies much of the quantity theory of money. He obviously considered the circumstances of a government that was engaged in monetizing a temporary budget deficit, issuing inconvertible paper money, and promising to establish a gold standard and redeem its paper currency at some future time. If honored, as this (and Madison's) analysis assumes, such a promise would constitute a type of future "backing" of money issues that he thought would limit inflation and break the connection between inflation and the rate of money growth. Moreover, his concerns have universal application; many other governments at other times have confronted similar circumstances and conducted similar policies.

The model constructed here can—under circumstances that have been described—give rise to equilibria that mimic general observations about what occurs when governments follow these kinds of policies. There is inflation during wars, but deflation begins when the government's wartime spending ceases. This deflation permits resumption to begin as scheduled, even if the government does nothing to contract the money supply. The latter point is of some interest: Friedman and Schwartz (1963), for example, argue that it was a purely "accidental" consequence of the postwar deflation that the United States was able to resume gold convertibility after the Civil War and that very little active policy was conducted to restore it. The analysis here, however, suggests that resumption of convertibility was no accident.

I have argued that Madison's views have much theoretical validity. Indeed, the kind of policy he describes allows the inflation rate to be very different from the rate of money growth, and the time to redemption has potentially great importance in determining the behavior of the price level. However, the link between the behavior of the money supply and the behavior of the price level is not completely broken, as he asserts it should be.

But why isn't this link broken? Tobin (1963), Wallace (1981), and Sargent and Smith (1987a,b) describe circumstances under which appropriately conducted increases in the money supply—that is, increases which are appropriately "backed"—have no price level consequences. Madison's policy backs current money creation with a promise of future gold redemption; here, this promise requires that the government run future surpluses to raise the resources required for redemption. Why don't these resources constitute the backing required by Tobin, Wallace, and Sargent and Smith? The answer is that Madison's scheme assigns the redemption cost to a specific generation; that is, it redistributes resources among generations. This prevents the kind of policy he discusses from being irrelevant to price-level behavior.

Madison's analysis nonetheless raises a host of fascinating issues which remain unaddressed here. For example, is there an "optimal" speed of transition to a gold standard? Mitchell (1897) maintained that the United States took too long to resume gold convertibility after the Civil War; Keynes argued that Britain resumed too quickly after World War I, causing an excessively large postwar deflation. An analysis of the "correct" length of time to redemption would definitely be interesting in light of these discussions.

Madison also challenged the common notion that borrowing to finance a deficit is less inflationary than monetizing the same deficit. His particular concern was that the implied interest payments on the government debt simply add to the government's financial burden, exacerbating inflation. The same concern is reflected in Sargent and Wallace's (1981) work on "unpleasant monetarist arithmetic," which describes conditions under which Madison's reservations are well founded. Indeed, the Sargent–Wallace conditions can be weakened substantially, as shown by Bhattacharya, Guzman, and Smith (1995).

However, Madison's analysis of money versus bond financing of a government budget deficit raises an even subtler issue. His final paragraph discusses government bonds that bear interest only because their large denominations make them costly to use in many transactions. Adding this feature to the others implicit in his description would yield a model similar to that of Bryant and Wallace (1979), in which bond finance is always more inflationary than money finance because it increases the costs of trade. However, Bryant and Wallace did not consider a government confronting some of the other conditions that concerned Madison. An integration of these considerations would also be extremely interesting.

Appendix A

Proof of Proposition 3. Equations (46) and (47) are immediate from the equilibrium conditions (39) and (40), and from the definitions of ψ_1 and ψ_2. It then remains to verify that the solution sequence $\{p_t\}$ implied by (46), (47), (38), (31), and (28) satisfies the maintained hypotheses of the construction. These hypotheses are that equations (19), (29), and (30) are satisfied, as are (35) and (36). Equation (29) is clearly satisfied by construction for $t \leq T_1 - 1$.

The first equality in (44) implies that (23) is satisfied; as noted in the text, satisfaction of (23) is equivalent to $p_{T-1} \geq p_T$. $p_{T_1} > (1-\delta)p_{T_1+1}$ is equivalent to $x > 1-\delta$. Iff (27) is satisfied, $p_t \geq p_{t+1}$ holds for $t = T_1+1, ..., T-1$. In view of (30), a sufficient condition for (27) is that

$$\text{(A.1)} \quad (1-\delta)M_{T_1}/p_{T_1} = (1-\delta)(g/\delta) \times [1-(1-\delta)^{T_1+1}] = (1-\delta)s(w_1,w_2,x) \geq s(w_1,w_2,1)$$

be satisfied. (A.1) is easily shown to be equivalent to the second inequality in (45). This inequality also implies that $x > 1-\delta$. Thus, the first inequality in (44) and the second inequality in (45) imply that (19) and (30) are satisfied.

As already noted, (36) holds iff $x > 1-\delta$, which is implied by the second inequality in (44). Moreover, by the definition of x, (35′) is equivalent to

$$\text{(A.2)}\quad (g/\delta)[1-(1-\delta)^{T_1+1} - 1 + (1-\delta)^{T_1}]$$
$$= g(1-\delta)^{T_1} \geq s(w_1,w_2,x)$$
$$- s(w_1,w_2,1-\delta)$$
$$= w_2[x-(1-\delta)]/(1+\beta)(1-\delta)x.$$

But this is obviously the first inequality in (45), establishing the proposition. ■

Appendix B

Proof of Proposition 4. The second inequality in (44) obviously holds iff $\psi_2 < 1$. It is easy to verify that

$$\text{(A.3)}\quad \psi_2 \equiv [s(w_1,w_2,x)/s(w_1,w_2,1)]$$
$$\times \{1 - (w_2/\beta w_1)^{T-(T_1+1)}$$
$$\times [1 - s(w_1,w_2,1)/w_1]\}.$$

Then, $\psi_2 < 1$ holds iff

$$\text{(A.4)}\quad [s(w_1,w_2,x) - s(w_1,w_2,1)]$$
$$\div s(w_1,w_2,x) < [1 - s(w_1,w_2,1)/w_1]$$
$$\times (w_2/\beta w_1)^{T-(T_1+1)}$$

is satisfied. Rearranging terms in (A.4) yields the first inequality in (48).

To obtain the second inequality in (48), note that the first inequality in (44) holds iff

$$\text{(A.5)}\quad [\beta/(1+\beta)]s(w_1,w_2,x)(w_2/\beta w_1)^{T-T_1}$$
$$\geq s(w_1,w_2,1) - s(w_1,w_2,x)$$
$$+ s(w_1,w_2,x)[1 - s(w_1,w_2,1)/w_1]$$
$$\times (w_2/\beta w_1)^{T-T_1-1}.$$

Rearranging terms in (A.5) yields the second inequality in (48).

To establish part (b) of the proposition, notice that

$$\text{(A.6)}\quad w_2(x-1)/x\,s(w_1,w_2,x)$$
$$= (1+\beta)w_2(x-1)/\beta w_1[x-(w_2/\beta w_1)].$$

and

$$\text{(A.7)}\quad \lim_{x\to\infty} w_2(x-1)/x\,s(w_1,w_2,x)$$
$$= (1+\beta)w_2/\beta w_1$$

are satisfied. Moreover, $T \geq T_1 + 2$ implies that

$$\lim_{x\to\infty} w_2(x-1)/x\,s(w_1,w_2,x)$$
$$> (w_2/\beta w_1)^{T-(T_1+1)}$$

holds. Thus, the condition

$$\text{(A.8)}\quad w_2(\underline{x}-1)/\underline{x}\,s(w_1,w_2,\underline{x})$$
$$\equiv (w_2/\beta w_1)^{T-(T_1+1)}$$

has a solution $\underline{x} > 1$. Likewise, if the condition

$$\text{(A.9)}\quad w_2(x-1)/x\,s(w_1,w_2,x)$$
$$= [(w_1+w_2)/w_1](w_2/\beta w_1)^{T-(T_1+1)}$$

has a solution, let $\bar{x}$ denote it. If (A.9) has no solution, let $\bar{x} = \infty$. Then, the inequalities in (48) are clearly satisfied iff $x \in [\underline{x},\bar{x})$.

It remains to show that δ can be selected to satisfy (a.1) and (45). To begin, rewrite (45) as

$$\text{(A.10)}\quad g(1-\delta)^{T_1-1} \equiv \delta(1-\delta)^{T_1-1}$$
$$\times s(w_1,w_2,x)/[1-(1-\delta)^{T_1+1}]$$
$$\geq w_2[x-(1-\delta)]/(1+\beta)x$$
$$> \delta\beta w_1/(1+\beta).$$

Clearly, $\beta w_1 > w_2$ implies that, for δ near zero, (a.1) and the second inequality in (A.10) are satisfied. Moreover, the first inequality in (A.10) can be written in the form

$$\text{(A.11)}\quad \delta(1-\delta)^{T_1-1}/[1-(1-\delta)^{T_1+1}]$$
$$\geq (w_2/\beta w_1)[x-(1-\delta)]$$
$$\div [x-(w_2/\beta w_1)].$$

For δ satisfying assumption 1 and for all $x > 1$, we clearly have

$$(w_2/\beta w_1)[x-(1-\delta)]/[x-(w_2/\beta w_1)]$$
$$< (w_2/\beta w_1).$$

Moreover, by L'Hopital's rule,

$$\lim_{\delta \to 0} \delta(1-\delta)^{T_1 - 1}/[1-(1-\delta)^{T_1+1}]$$

$$= 1/(T_1 + 1).$$

Thus, condition (49) implies that, for all $x \in [\underline{x}, \bar{x})$, both inequalities in (45) are satisfied whenever δ is chosen sufficiently small. ■

Appendix C

Proof of Proposition 5.

a) For $t \leq T_1 - 1$, equation (50) follows immediately from (34). Moreover, by L'Hopital's rule,

$$\lim_{\delta \to 0} [1-(1-\delta)^{t+1}]/[1-(1-\delta)^{t+2}]$$

$$= (t+1)/(t+2).$$

Hence, the assertion in the text following the proposition.

b) Equation (34), the definition of x, and $M_t/p_t = s(w_1, w_2, p_t/p_{t+1})$, $t = T_1, \ldots, T-1$, imply that $p_{T+1}/p_{T_1} = x = x(M_{T_1+1}/M_{T_1})$, since the money supply is constant for all $t \geq T_1$. Moreover, for $t = T_1 + 1, \ldots, T-1$, the conditions $p_{t+1} \leq p_t$ and $M_t/p_t = M_{T_1}/p_t = s(w_1, w_2, p_t/p_{t+1})$ are satisfied. Therefore, $M_t/p_t \geq M_t/p_{t-1} \geq \cdots \geq M_{T_1}/p_{T_1} = s(w_1, w_2, x)$ must hold. It is then immediate that, for all such t, $p_t/p_{t+1} > x$, and $p_{t+1}/p_t < (1/x)(M_{t+1}/M_t) = 1/x$ obtain. ■

Appendix D

Proof of Proposition 6. Define $\tilde{\psi}_2(\psi_2)$ by

$$\text{(A.12)} \quad \tilde{\psi}_2 \equiv (g/\delta)[1-(1-\delta)^{T_1+1}]$$

$$\{w_1^{-1}(w_2/\beta w_1)^{\tilde{T}-(T_1+1)}$$

$$+ [(1+\beta)/\beta w_1][1-(w_2/\beta w_1)^{\tilde{T}-(T_1+1)}]$$

$$\div [1-(w_2/\beta w_1)]\}$$

and

$$\text{(A.13)} \quad \psi_2 \equiv (g/\delta)[1-(1-\delta)^{T_1+1}]$$

$$\times \{w_1^{-1}(w_2/\beta w_1)^{T-(T_1+1)}$$

$$+ [(1+\beta)/\beta w_1][1-(w_2/\beta w_1)^{T-(T_1+1)}]$$

$$\div [1-(w_2/\beta w_1)]\},$$

respectively. Then,

$$\text{(A.14)} \quad \tilde{p}_{T_1} = (\phi/b)(w_2/\beta w_1)^{\tilde{T}-T_1}/(1-\tilde{\psi}_2)$$

$$\text{(A.15)} \quad p_{T_1} = (\phi/b)(w_2/\beta w_1)^{T-T_1}/(1-\psi_2)$$

hold, and $\tilde{p}_{T_1} > p_{T_1}$ iff

$$\text{(A.16)} \quad (w_2/\beta w_1)^{\tilde{T}-T}(1-\psi_2) > 1-\tilde{\psi}_2$$

is satisfied.

Now, straightforward manipulation establishes that

$$\text{(A.17)} \quad (w_2/\beta w_1)^{\tilde{T}-T}\psi_2 = \tilde{\psi}_2 - (g/\delta)$$

$$\times [1-(1-\delta)^{T_1+1}][(1+\beta)/\beta w_1]$$

$$\times [1-(w_2/\beta w_1)^{\tilde{T}-T}]/[1-(w_2/\beta w_1)].$$

Substituting (A.17) into (A.16) and rearranging terms, one obtains that $\tilde{p}_{T_1} > p_{T_1}$ holds iff

$$\text{(A.18)} \quad \tilde{\psi}_2 - (w_2/\beta w_1)^{\tilde{T}-T}\psi_2$$

$$= (g/\delta)[1-(1-\delta)^{T_1+1}][(1+\beta)/\beta w_1]$$

$$\times [1-(w_2/\beta w_1)^{\tilde{T}-T}]/[1-(w_2/\beta w_1)]$$

$$> 1-(w_2/\beta w_1)^{\tilde{T}-T}.$$

When $\tilde{T} > T$ obtains, (A.18) is equivalent to

$$\text{(A.18}') \quad (g/\delta)[1-(1-\delta)^{T_1+1}][(1+\beta)/\beta w_1]$$

$$\div [1-(w_2/\beta w_1)] \equiv s(w_1, w_2, x)$$

$$\div s(w_1, w_2, 1) > 1.$$

Since $\underline{x} > 1$, (A.18′) clearly holds for all $x \geq \underline{x}$. ■

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