The Long-Run Phillips Curve is... a Curve

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1Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank
The question

An old debate: is there any trade-off between inflation and output/unemployment in the long run?

- Phelps (1967), Friedman (1968): Natural rate hypothesis
- "there is no permanent trade-off":
  \[ \Rightarrow \text{the long-run Phillips curve is vertical} \]
- Cornerstone role in macroeconomic theory and practice
- The working assumption of central banks in the implementation of monetary policy
The question

It is surprising to note that:

▶ **Empirically**: There is little econometric work devoted to test the absence of a long-run trade-off.

Some literature: King and Watson (1994); Beyer and Farmer (2007); Berentsen et al. (2011); Haug and King (2014); Benati (2015)

▶ **Theoretically**: Modern macroeconomic sticky price frameworks generally do not imply the absence of a long-run relation

▶ The Generalized NK model delivers a negative relationship between steady state inflation and output. See Ascari (2004); Ascari and Sbordone (2014)
Results

What is the long-run relation between inflation and output?

1. Time series model
   ▶ The LRPC is not vertical, it is negatively sloped (higher inflation is related to lower output in the LR)
   ▶ The key to get this result: model the LRPC as non linear
   ▶ Methodological contribution: a ”convenient” non-linear approach

2. Structural model
   ▶ GNK model (Ascari and Ropele, 2009; Ascari and Sbordone, 2014): higher trend inflation causes lower GDP in the LR
   ▶ The model has the two key features from the statistical analysis: non-linear and negatively sloped LRPC
   ▶ The model is also able to capture the quantitative features of the time series analysis
The time series approach: A time-varying equilibrium VAR

Generalization of Steady State VAR (Villani, 2009; Del Negro et al., 2017; Johannes and Mertens, 2021):

\[ A(L) (X_t - \bar{X}_t) = \varepsilon_t \quad \varepsilon_t \sim N(0, \Sigma_{\varepsilon,t}) \]  

\( X_t \) is a \((n \times 1)\) vector with observed variables at time

\( \bar{X}_t \) is the vector with the long-run values of \( X_t \)

Trend-cycle decomposition:

\[ X_t = \bar{X}_t + \hat{X}_t \]

\( \hat{X}_t \) described by (1): stable component with unconditional expectation equal to zero

\[ \bar{X}_t = h(\theta_t) \]

\[ \theta_t = f(\theta_{t-1}, \eta_t) \quad \eta_t \sim N(0, \Sigma_{\eta}) \]
The model

- Three observables: GDP per capita, inflation and interest rate
- The short-run component: VAR with 4 lags

THE MODEL FOR THE LONG RUN

\[ \bar{y}_t = y_t^* + \delta(\bar{\pi}_t) \]  \text{the equilibrium level of output as function of inflation}
\[ y_t^* = y_{t-1}^* + g_t + \eta_Y^t \]
\[ g_t = g_{t-1} + \eta_G^t \]
\[ \delta(\bar{\pi}_t) : \delta(0) = 0 \]

\[ \bar{\pi}_t = \bar{\pi}_{t-1} + \eta_{\bar{\pi}}^t \]  \text{trend inflation is random walk}

\[ \bar{i}_t = \bar{\pi}_t + cg_t + z_t \]  \text{long-run Fisher equation}
\[ z_t = z_{t-1} + \eta_{z}^t \]
A non-linear long-run Phillips curve

Our choice of \( \delta(\bar{\pi}_t) \) is a piecewise linear function:

\[
\bar{y}_t = y_t^* + \delta(\bar{\pi}_t)
\]

\[
\delta(\bar{\pi}_t) = \begin{cases} 
  k_1 \bar{\pi}_t & \text{if } \bar{\pi}_t \leq \tau \\
  k_2 \bar{\pi}_t + c_k & \text{if } \bar{\pi}_t > \tau 
\end{cases}
\]

- It is simpler to treat: methodological contribution
- It can approximate the kind of non-linearity we have in mind without imposing strong assumptions on a specific functional form
- It is easy to interpret
A piecewise linear approach

The model can be written in state space form:

\[ Y_t = D(\theta_t) + F(\theta_t) \theta_t + \epsilon_t \]  
\[ \theta_t = M(\theta_t) + G(\theta_t) \theta_{t-1} + P(\theta_t) \eta_t \]

(2) 
(3)

where, in particular

\[
(D, F, M, G, P) = \begin{cases} 
(D_1, F_1, M_1, G_1, P_1) & \text{if } \bar{\pi}_t \leq \tau \\
(D_2, F_2, M_2, G_2, P_2) & \text{if } \bar{\pi}_t > \tau 
\end{cases}
\]

(4)

▶ Methodological contribution: we find the likelihood and the posterior distribution of \(\theta_t\) analytically

▶ Compromise between efficiency and misspecification
Estimation

- US data, sample from 1960Q1 to 2008Q2
- Bayesian approach

Two sources of non linearity: stochastic volatility and a piecewise linear LRPC

1. "Rao-Blackwellization", thanks to the analytical results on the piecewise linear model
2. Particle filtering also to approximate the posterior distribution of the parameters
   - Particle learning by Carvalho Johannes Lopes and Polson (2010); see also Mertens and Nason (2020)
   - Mixture of Normal distributions as approximation of the posterior of $\tau$ (Liu and West, 2001)

Estimation results - Linear model

A vertical (or flat) long-run Phillips curve

Figure: Posterior distributions of the slope of the LRPC - Linear model.
Estimation results - Non-linear model

Non linear and negatively sloped long-run Phillips curve

**Figure:** Posterior distributions of the slopes of the LRPC - Non-linear model.
Estimation results - Non-linear model

The threshold:

Figure: Posterior distributions of $\tau$ - Non-linear model.
A non-linear, negatively sloped long-run Phillips curve

Figure: LRPC - Non-linear model. Median and 90% probability interval.
Estimation results - non-linear model

Figure: Inflation and trend inflation - Non linear model.
The cost of trend inflation: the long-run output gap

\[ \hat{Y}_t = \frac{Y_t}{\bar{Y}_t} = \frac{Y_t}{Y^*_t} \frac{Y^*_t}{\bar{Y}_t} \]  

(5)

**Figure:** Long-run output gap estimated through the non-linear model.
The structural model

- A variant of Ascari and Ropele (2009), Ascari and Sbordone (2014) GNK model:
  - Inter-temporal Euler equation featuring (external) habit formation in consumption
  - Generalized New Keynesian Phillips curve featuring positive trend inflation
  - Taylor-type monetary policy rule

- Time varying trend inflation $\Rightarrow$ LRPC is:
  - Non-linear
  - Negatively sloped

- When taking decisions the agents consider trend inflation as a constant parameter: anticipated-utility model (Kreps, 1998; Cogley and Sbordone, 2008)

- Stochastic volatility to the four shocks: discount factor, technology, monetary policy and trend inflation
The costs of trend inflation

- Price stickiness $\Rightarrow$ price dispersion and inefficiency in the quantity produced

- Higher trend inflation leads to higher price dispersion and increases output inefficiency

Formally:

$$N_t = \int_0^1 N_{i,t} di = \int_0^1 \left( \frac{Y_{i,t}}{A_t} \right)^{\frac{1}{1-\alpha}} di = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{\frac{-\epsilon}{1-\alpha}} \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}$$

Aggregate output is:

$$Y_t = \frac{A_t}{s_t^{1-\alpha}} N_t^{1-\alpha}$$

with long-run price dispersion: $s_t = g(\bar{\pi}_t)$
Comparing long-run Phillips curves: VAR and GNK

The GNK model measures the costs of trend inflation consistently with the VAR

Figure: Long-run Phillips curve: median (continuous line) and 90% probability interval (dashed lines) - comparison between VAR (blue) and GNK (black) estimates.
Conclusions

- What is the long-run relation between inflation and output?
- A time series model suggests that the LRPC is:
  - Non linear
  - Negatively sloped
- We interpret these findings through the lens of a GNK model
- This model is able to measure the costs implied by the LRPC consistently with the time series model
EXTRA
Econometric strategy

We use a particle filtering strategy to approximate the joint posterior distribution of latent processes and parameters:

**Latent processes:** a "conditional piecewise linear model"

\[
p(\theta_t, \Sigma_{\epsilon,t} | Y_t) = \frac{p(\theta_t | \Sigma_{\epsilon,t}, Y_t)}{p(\Sigma_{\epsilon,t} | Y_t)}
\]

"optimal importance kernel" "blind proposal"

**Parameters:**

- Particle learning by Carvalho Johannes Lopes and Polson (2010); see also Mertens and Nason (2020)
- Mixture of Normal distributions as approximation of the posterior of \( \tau \) (Liu and West, 2001)

A fully adapted particle filter

At \( t - 1 \): \( \{ \theta_{t-1}^{(i)} \} \) approximate \( p(\theta_{t-1}|\psi, X_{1:t-1}) \)

1. Resample
   - Compute \( \tilde{w}_t^{(i)} \propto p\left(X_t|\theta_{t-1}^{(i)}, \psi, X_{1:t-1}\right) \)
   - Resample \( \{ \tilde{\theta}_{t-1}^{(i)} \} \) using \( \{ \tilde{w}_t^{(i)} \} \)

2. Propagate
   - draw \( \theta_t^{(i)} \sim p\left(\theta_t|\tilde{\theta}_{t-1}^{(i)}, \psi, X_{1:t-1}\right) \)

where:
   - \( p\left(X_t|\theta_{t-1}^{(i)}, \psi, X_{1:t-1}\right) \) is a weighted sum of Unified Skew Normal distributions (Arellano-Valle and Azzalini, 2006)
   - \( p\left(\theta_t|\tilde{\theta}_{t-1}^{(i)}, \psi, X_{1:t-1}\right) \) is a weighted sum of multivariate truncated Normal distributions
Household

The economy is populated by a representative agent with utility

\[ E_0 \sum_{t=0}^{\infty} \beta^t d_t \left[ \ln (C_t - hC_{t-1}) - d_n \frac{N_t^{1+\phi}}{1+\phi} \right] \]

Budget constraint is given by

\[ P_tC_t + R_t^{-1}B_t = W_tN_t + D_t + B_{t-1} \]

\(d_t\) is a discount factor shock which follows an AR(1) process

\[ \ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t} \]
Final good firm

Perfectly competitive final good firms combine intermediate inputs

\[ Y_t = \left[ \int_0^1 Y_{i,t}^\frac{\varepsilon-1}{\varepsilon} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \varepsilon > 1 \]

Price index is a CES aggregate of intermediate input prices

\[ P_t = \left[ \int_0^1 P_{i,t}^\frac{1}{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}} \]

The demand schedule for intermediate input

\[ Y_{i,t} = \left[ \frac{P_{i,t}}{P_t} \right]^{-\varepsilon} Y_t \]
Intermediate good firm

Each firm $i$ produces according to the production function

$$Y_{i,t} = A_t N_{i,t}^{1-\alpha}$$

where $A_t$ denotes the level of technology and its growth rate $g_t \equiv A_t / A_{t-1}$ follows

$$\ln g_t = \ln \bar{g} + \epsilon_{g,t}$$
Price setting

Firms adjust prices $P_{i,t}^*$ to maximize expected discounted profits with probability $0 < 1 - \theta < 1$

$$
E_t \sum_{j=0}^{\infty} \theta^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[ \frac{P_{i,t}^*}{P_{t+j}} Y_{i,t+j} - \frac{W_{t+j}}{P_{t+j}} \left[ \frac{Y_{i,t+j}}{A_{t+j}} \right]^{\frac{1}{1-\alpha}} \right]
$$

subject to the demand schedule

$$
Y_{i,t+j} = \left[ \frac{P_{i,t}^*}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j},
$$

where $\lambda_t$ is the marginal utility of consumption.
The first order condition for the optimized relative price $x_t(=\frac{P_{i,t}^*}{P_t})$ is given by

$$(x_t)^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \frac{\varepsilon}{(\varepsilon - 1)(1 - \alpha)} \frac{E_t \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j} \frac{W_{t+j}}{P_{t+j}} \left[ \frac{Y_{t+j}}{A_{t+j}} \right]^{\frac{1}{1-\alpha}} \pi_{t|t+j}^{(\alpha)}}{E_t \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j} \pi_{t|t+j}^{\varepsilon-1} Y_{t+j}}.$$ 

where $\pi_{t|t+j} = \frac{P_{t+1}}{P_t} \times \ldots \times \frac{P_{t+j}}{P_{t+j-1}}$ for $j \geq 1$ and $\pi_{t|t} = \pi_t$. 

Back
Price setting contd.

Aggregate price level evolves according to

\[
P_t = \left[ \int_0^1 P_{i,t}^{1-\epsilon} \, di \right]^{\frac{1}{1-\epsilon}} \Rightarrow \\
\chi_t = \left[ \frac{1 - \theta \pi_t^{\epsilon-1}}{1 - \theta} \right]^{\frac{1}{1-\epsilon}}.
\]

Finally, price dispersion \( s_t \equiv \int_0^1 (\frac{P_{i,t}}{P_t})^{-\epsilon} \, di \) can be written recursively as:

\[
s_t = (1 - \theta) \chi_t^{-\epsilon} + \theta \pi_t^{\epsilon} s_{t-1}
\]
Monetary policy

\[
\frac{R_t}{R_t} = \left( \frac{R_{t-1}}{R_t} \right)^{\rho} \left[ \left( \frac{\pi_t}{\overline{\pi}_t} \right)^{\psi_\pi} \left( \frac{Y_t}{Y^n_t} \right)^{\psi_x} \left( \frac{g_t^y}{\bar{g}} \right)^{\psi_{\Delta y}} \right]^{1-\rho} e^{\epsilon_{r,t}}
\]

\[\ln \overline{\pi}_t = \ln \overline{\pi}_{t-1} + \epsilon_{\overline{\pi},t}\]

where \(\overline{\pi}_t\) denotes trend inflation, \(Y^n_t\) is the flex-price output and \(g^y_t\) is growth rate of output.
# Estimates of the parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Density</th>
<th>Mean</th>
<th>Prior St Dev</th>
<th>Posterior</th>
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<tbody>
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<td>$\psi_\pi$</td>
<td>Gamma</td>
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<td>2.05</td>
<td>[1.83 2.3]</td>
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<td>[0.71 0.77]</td>
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<td>[0.36 0.46]</td>
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<td>[0.46 0.56]</td>
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<td>$\rho_d$</td>
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<td>0.1</td>
<td>0.79</td>
<td>[0.74 0.83]</td>
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<table>
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Posterior median and 90% credibility interval in brackets.