Money and Spending Multipliers with HA-IO

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Beyond representative agent, one sector

- Heterogeneous agents + input-output network
  - workers consume different bundles of goods
  - firms hire different bundles of workers (+ fixed factors)

- Heterogeneous nominal and real rigidities
  - sticky wage, work for sticky sector...
  - employer (or his customers...) relies on fixed factors
  - more or less elastic labor supply

- New questions:
  - how does policy redistribute across agents?
  - aggregate response to policy same as with rep agent?
Money multiplier

$(\mathbb{L}_M)_h = \frac{\partial \log l_h}{\partial \log M}$

- Cross section:
  - nominal rigidity $\uparrow$, real rigidity $\downarrow \iff$ price volatility $\downarrow$, employment volatility $\uparrow$

- Aggregate
  - substitute towards agents with more nominal rigidity / less real rigidity $\rightarrow$ more non-neutrality
Spending multiplier

\[(\mathbb{L}G)_{hi} = \frac{\partial \log l_h}{\partial \log G_i}\]

- Spending affects relative demand for different workers
  - direct towards agents with more nominal rigidity / less real rigidity → larger multiplier
  - replicate aggregate consumption → “as if” rep agent
  - flex prices, no fixed factors, uniform labor supply elasticity → composition irrelevant for aggregate employment
Literature

**HA-IO:** Baqee and Farhi (2018), Flynn, Patterson, Sturm (2020)

**Monetary/fiscal policy with heterogeneous agents:**


**Monetary policy with input-output:**


**Spending multipliers:** Bouakez, Rachedi, Santoro (2020), Cox, Muller, Pasten, Schoenle, Weber (2020)

Roadmap

- Setup
- Demand & supply blocks at high level
  - general expression for multipliers
  - “as if” results
- Specific structural model
  - break “as if” results
  - examples for intuition
Outline

Setup

Multipliers

Examples

Empirics

Conclusion
Environment

- $H$ worker types, $K$ fixed factors, $N$ production sectors
- Agents
  - consume different bundles of goods
  - own different shares of sectors and fixed factors
  - have different wage rigidity and labor supply elasticity
- Sectors
  - hire different bundles of workers and fixed factors
  - have different position in the input-output network
  - have different price rigidity and demand elasticity
- Log-linearized model
  - evolution described by measurable steady-state shares and elasticities
Consumers

- Type-$h$ preferences:

\[
\frac{C_h(x_1, \ldots, x_N)^{1-\gamma_h}}{1 - \gamma_h} - \frac{L_h^{1+\varphi_h}}{1 + \varphi_h}
\]

- Parameters:
  - wealth effects: $\Gamma \equiv \text{diag} \ (\gamma_1, \ldots, \gamma_H)$
  - Frish elasticities: $\Phi \equiv \text{diag} \ (\varphi_1, \ldots, \varphi_H)$
  - consumption shares $\beta = (\beta_{i,h})$
Consumers

- **Type-\(h\) budget constraint:**

\[
P_h C_h = \underbrace{W_h L_h}_{\text{labor}} + \sum_k \underbrace{Z_{kh} R_k K_k}_{\text{fixed factors}} + \sum_i \underbrace{\Theta_{ih} \Pi_i}_{\text{profits}} - \underbrace{T_j}_{\text{lump-sum tax}}
\]

- **Factor income shares:**

\[
\varsigma_h \equiv \frac{W_h L_h}{GDP}, \quad \varsigma_k \equiv \frac{R_k K_k}{GDP}
\]

- **Agents’ income shares:**

\[
s_h \equiv \frac{P_h C_h}{GDP} = \varsigma_h + \sum_j Z_{kh} \varsigma_k
\]
Producers

- CRS sectoral production functions:
  
  \[ Y_i = A_i F_i \left( L_{ih}, K_{ik}, \{ x_{ij} \} \right) \]
  
  - Hicks-neutral shifter
  - Factor shares \( \alpha = \begin{pmatrix} \alpha_{ih} & \alpha_{ik} \end{pmatrix} \), input shares \( \Omega = \Omega_{ij} \)
  - Domar weights: \( \lambda^T \equiv \beta^T (I - \Omega)^{-1} \)
  - Elasticities of substitution
Producers

- Continuum of firms within sectors, CES bundle
  - fraction $\delta_i$ of producers adjust price after seeing $A$
  - notation: $\Delta = \text{diag}(\delta_1...\delta_N)$

- Sticky wages: add labor unions with sticky price

- Optimal input subsidies ($\tau_i$), log-linearize around efficient equilibrium
Policy instruments

- Government spending: $G = (G_1 \ldots G_N)^T$, normalize $G^* = 0$
- Money supply ($\leftrightarrow$ nominal GDP), normalize $M^* = 1$

\[
\sum_h P_h C_h + \sum_i G_i = M
\]

- Budget constraint:

\[
\sum_h T_h = \sum_i (G_i + \tau_i m c_i y_i)
\]

- For this presentation:

\[
T_h = \sum_i \left[ \left( (I - \Omega)^{-1} \alpha \right)_{hi}^T G_i + \Theta_{ih} \tau_i m c_i y_i \right]
\]
Supply: \( I = L(w, G) \)

- Prices and profits:
  \[
  \pi = \Delta (I - \Omega \Delta)^{-1} \alpha w, \quad \Pi = - (I - \Delta) (I - \Omega \Delta)^{-1} \alpha w
  \]

- Consumption:
  \[
  c = rw + \overbrace{\Xi w_K}^{\text{fixed factors}} + \overbrace{\hat{\Theta}^T \Pi}^{\text{profits}} - T(G), \quad rw = w_L - \delta_{\beta}(\alpha) w
  \]

- Consumption-leisure tradeoff:
  \[
  \Gamma c + \Phi l = rw \rightarrow l = L(w, G)
  \]
Demand

- Aggregate GDP:

\[ \delta \bar{\beta} (\alpha) w + \varsigma^T l = d \log M \]

- Factor income shares:
  - direct effect \((w \uparrow, l \uparrow \Rightarrow \varsigma \uparrow)\)
  - change in wages/prices → substitution → factor demand
  - change in private incomes, spending → factor demand

\[ S_w w + S_l l = S_G G \]

- \(\varsigma^T S = 0\)
Equilibrium

- Aggregate demand:

\[
\left( \delta \bar{\beta} (\alpha) + \varsigma^T \varsigma_L \right) w = d \log M - \varsigma_L^T \mathcal{L}_g G
\]

- Relative demand:

\[
- \left( \varsigma_w + \varsigma_I \mathcal{L}_w \right) w = (\varsigma_G + \varsigma_I \mathcal{L}_g) G
\]

\[
\equiv \varsigma_w
\]
Equilibrium

- Aggregate demand:

\[
\left( \delta \beta (\alpha) + \varsigma_{L}^{T} \mathcal{L}_{w} \right) \omega = d \log M
\]

- Relative demand:

\[
- (S_{w} + S_{L} \mathcal{L}_{w}) \omega = S_{G} G
\]

- Decomposition:

\[
S_{w} = S^{XS} \left( I - 1\varsigma^{T} \right) - \bar{S}_{\varsigma}^{T}
\]
Money multiplier

- Full symmetry, no fixed factors \[ \bar{S} = 0 \]

\[
W_m = \frac{1}{\delta (\bar{\alpha}) + \frac{1}{\gamma+\varphi} (1 - \delta (\bar{\alpha}))} d \log M, \quad L_m = \frac{1}{\delta (\bar{\alpha}) + \frac{1}{\gamma+\varphi} (1 - \delta (\bar{\alpha}))} \frac{1}{\gamma+\varphi} (1 - \delta (\bar{\alpha}))
\]

- Proportional increase
- Satisfy CIA constraint
- Balance excess demand

\[
W_m = \frac{1 + S^{xS-1} \bar{S}}{\mathcal{E}^T [1 + S^{xS-1} \bar{S}]} d \log M, \quad L_m = L_w W_m
\]
Spending neutrality

- $\Gamma = \emptyset$ OR uniform $\gamma$, $\varphi$ and no fixed factors
- No effect on relative demand $\iff$ replicate aggregate consumption basket

$$\mathbb{S}_G G = 0 \iff G \propto \bar{\beta}$$

- Multiplier $\approx$ one sector, representative agent:

$$\mathbb{L}_g \bar{\beta} = \mathbb{L}_m + (1 - \mathbb{L}_m) \frac{\gamma}{\gamma + \varphi}$$
Spending multiplier

\[ L_g \bar{\beta} = L_m + (1 - L_m) \frac{\gamma}{\gamma + \varphi} \]

- Wealth effect in labor supply
- Satisfy CIA constraint
- Balance excess demand

\[ L_g = L_m 1^T + \left( I - L_m s_L^T \right) L_g + \left[ L_w - L_m \mathcal{E}^T \right] S^{XS^{-1}} S_G \]
Irrelevance of composition

- Flex prices, no fixed factors, uniform $\gamma$ and $\varphi$

$$I = \mathcal{L}(w, G) = \frac{1 - \gamma}{\gamma + \varphi} (I - \lambda^T \alpha) w + \frac{\gamma}{\gamma + \varphi} v \sum_i G_i$$

$$\Rightarrow \bar{I}_G = \frac{\gamma}{\gamma + \varphi} \sum_i G_i$$

- Aggregate real wages unaffected by spending
- Same labor supply elasticity for all agents
Cross-section: $I \downarrow$ for sticky workers in a contraction

$$I_{\text{sticky}} - I_{\text{flex}} \propto \frac{\varphi \theta \delta}{1 + \varphi \theta \delta} (\delta_{\text{flex}} - \delta_{\text{sticky}}) d \log M$$

Substitution $\rightarrow$ more non-neutrality:

$$\Pi_m = \frac{1 + \left(\frac{\delta_{\text{flex}} - \delta_{\text{sticky}}}{1 - \delta}\right)^2 \frac{\varphi \theta}{1 + \varphi \theta \delta}}{1 + \varphi \frac{\delta}{1 - \delta} - (\varphi - 1) \left(\frac{\delta_{\text{flex}} - \delta_{\text{sticky}}}{1 - \delta}\right)^2 \frac{\varphi \theta}{1 + \varphi \theta \delta}}$$
Spending increases agg employment iff directed to sticky sector:

\[ \bar{L}_g = \frac{\delta_{\text{flex}} - \delta_{\text{sticky}}}{1 + \varphi \theta \bar{\delta}} (G_{\text{sticky}} - G_{\text{flex}}) \]

Substitution → smaller XS multiplier

\[ l_1 - l_2 = \left[ 1 - \frac{\varphi \theta \bar{\delta}}{1 + \varphi \theta \bar{\delta}} \right] (G_1 - G_2) \]
Labor supply elasticity

- Expansion benefits elastic workers ($\varphi_E < \varphi_I$):

$$l_E - l_I = (\varphi_I - \varphi_E) \frac{\theta \delta}{1 + \bar{\varphi} \theta \delta} \bar{L}_m$$

- Substitution $\rightarrow$ larger aggregate multiplier:

$$\bar{L}_m = \frac{1}{1 + \bar{\varphi} \delta} - \frac{\theta \delta}{1 - \delta} \frac{1}{1 + \bar{\varphi} \theta \delta} (\varphi_I - \varphi_E)^2$$

- Spending increases $\bar{I}$ iff directed to elastic workers:

$$\bar{L}_g \propto \frac{\varphi_I - \varphi_E}{\bar{\varphi} + \varphi_E \varphi_I \theta \delta} (G_E - G_I)$$
Input-output linkages

\[ L_1 \quad L_2 \]

\[ I \quad \alpha \]

\[ 1 - \alpha \]

\[ F \]

\[ HH \]

- Longer chain \( \sim \) stickier wage
- Replace

\[ \delta_F - \delta_I = \delta - \delta^2 \]
Chain-weighted ES

\[
L_1 \quad L_2 \quad L_1 \quad L_2
\]

\[
\alpha_1 \quad 1 - \alpha_1 \quad 1 - \alpha_2 \quad \alpha_2
\]

\[
C_1 \quad HH \quad C_2
\]

- XS spending multiplier:

\[
l_2 - l_1 = \frac{\varphi \beta_1 (1 - \beta_1) (\alpha_1 - \alpha_2) \left( \frac{G_1}{\beta_1} - \frac{G_2}{1 - \beta_1} \right)}{1 + \varphi \left[ \frac{\beta_1 (1 - \beta_1) (\alpha_1 - \alpha_2)^2}{s_1 (1 - s_1)} \sigma \delta + \left( 1 - \frac{\beta_1 (1 - \beta_1) (\alpha_1 - \alpha_2)^2}{s_1 (1 - s_1)} \right) \theta \right]}
\]
Real Estate

- Price stickiness vs labor share
  - \( \theta < \frac{\delta}{1-\delta} \) → real wage ↑ less → smaller multiplier
  - \( \theta > \frac{\delta}{1-\delta} \) → real wage ↑ more → larger multiplier

\[
\bar{L}_m = \frac{1 - \frac{1-\alpha}{1-\alpha+\varphi\theta}}{1 + \varphi \frac{\delta}{1-\delta} - (1 + \varphi \theta) \frac{1-\alpha}{1-\alpha+\varphi\theta}}
\]
NY or Boise?

$\nL_{NY} \quad Land_{NY} \quad Land_B \quad L_B \n$

$\alpha_{NY} \quad 1 - \alpha_{NY} \quad 1 - \alpha_B \quad \alpha_B$

$\nC_1 \quad HH \quad C_2 \n$

- Locate construction projects in Boise $\iff \theta < \frac{\delta}{1-\delta}$

$$\bar{L}_G \propto \varphi \theta \left( \frac{\delta}{1-\delta} - \theta \right) (\alpha_B - \alpha_{NY}) (G_B - G_{NY})$$

- Geographic mobility:
  - $\sigma \delta < \theta$: must live where you work $\rightarrow$ construction ↑ in NY
  - $\sigma \delta > \theta$: work from home $\rightarrow$ construction ↑ in Boise

$$l_B - l_{NY} \propto \theta (\sigma \delta - \theta) (\alpha_B - \alpha_{NY}) d \log M$$
Data

- I’m looking into:
  - ACS → employment shares
  - CEX → consumption bundles
  - BEA → capital shares
  - ADP → wage rigidity

- Suggestions?
Outline

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Conclusion

- Monetary expansion:
  - cross-section: nominal rigidity $\uparrow$, real rigidity $\downarrow \iff$ price volatility $\downarrow$, employment volatility $\uparrow$
  - aggregate: substitution $\rightarrow$ more non-neutrality

- Government spending changes demand composition
  - larger multiplier iff target workers with more nominal / less real rigidity
  - “as if” representative agent $\iff$ replicate private consumption basket

- Spending vs transfers: TBD
Timing

One-period model

- Period 0: prices are pre-set
- Period 1: money supply and spending shock
  - only a fraction of producers can adjust prices
  - production and consumption take place
  - the world ends
Seignorage

- Consumers need to purchase new money issuances
  - agent $h$ buys share $v_h$
- Revenues are fully rebated through lump-sum transfers
- Budget constraint:

$$P_h C_h + v_h dM = \text{income}_h - T_h + v_h dM$$

money purchase

seignorage rebate
Shares

- Change in shares

\[
\left(1 - \frac{\partial \log \text{demand}}{\partial \log \text{income}}\right) \partial \log \varsigma = \left(\frac{\partial \log \text{demand}}{\partial \log w} + \frac{\partial \log \text{profits}}{\partial \log w}\right) w + \frac{\partial \log \text{profits}}{\partial \log \text{income}}.
\]

- Definition of factor shares

\[
\left(1 - \frac{\partial \log \text{demand}}{\partial \log \text{income}}\right) \partial \log \varsigma = \left(1 - \frac{\partial \log \text{demand}}{\partial \log \text{income}}\right) (w + l).
\]