Tokenomics and Platform Finance

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Digital Platforms and Tokens

• The rise of digital platforms
  • Payment innovation is important, e.g., escrow account on eBay and Alibaba

• Tokens: users’ means of payments on platform
  • Blockchain: preventing double spending, facilitating smart contracts

• Tokens: platforms’ financing instruments
  • Token offerings $ 21 billion in 2018; US VC $ 131 billion
  • Tokens used to gather resources (e.g., engineers, consultants, investors)
  • Tokens enter into circulation gradually (protocol and vesting)

• Tokens: rewards for the founding entrepreneurs
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This Paper

- A dynamic model of platform investment/financing and user activities
  - Tokens are both means of payments for users and also financing instruments for the platform to gather efforts and resources
  - Users’ token demand: transaction and investment value
  - Platform owners’ token supply: reward themselves and pay contributors to improve the platform
  - Token supply is chose to maximize the PV of owners’ rewards (seigniorage)
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Questions

1. A platform can produce tokens with zero cost, so why token supply is finite and value positive?
   - What is the optimal way for platform designers to extract profits via tokens? Vesting schemes are common, but why and how to design?
   - Implications on token inflation/deflation and volatility dynamics

2. What is the key economic inefficiency when tokens serve as both users’ means of payment and platforms’ financing tools?
   - Are users’ and platform designers/founders’ interests aligned?
   - Pitfalls in the value chain? Users → token price → founders’ token payout

3. How can blockchain technology add value?
   - Why platform currencies rise after blockchain technology matures?
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Related Papers

- Platforms without tokens: Rochet and Tirole (2003), Stulz (2019)


- Tokens for users and contributors with exogenous supply: Sockin and Xiong (2018), Pagnotta (2018) among others


- Money: (1) convenience yield in Baumol-Tobin models, Krishnamurthy and Vissing-Jørgensen (2012); (2) demand with inflation expectation in Cagan (1956); (3) financing tools in Bolton and Huang (2016)
Outline

• Introduction

• Model and Solution
  • Franchise Value as Discipline – Durable-Goods Monopoly
  • Token Overhang – Corporate Finance
  • The Value of Commitment – Time Inconsistency

• Conclusion
A **platform** supports a unique set of transactions

**User** _i_ settles transactions in tokens, deriving *convenience yield* from token value

- *Efficient payment, smart contracting* ...

A **platform** supports a unique set of transactions

- Productivity: $A_t$

**User** $i$ settles transactions in tokens, deriving *convenience yield* from token value $x_{i,t} = P_t k_{i,t}$

- Convenience yield: $x_{i,t}^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha dt$
  - Token price: $P_t$
  - Token units: $k_{i,t}$
  - Number of users: $N_t$
  - User heterogeneity: $u_i \sim G_t(u)$
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  - User heterogeneity: \( u_i \sim G_t(u) \)
- Token price appreciation \( k_{i,t} E_t [dP_t] \)

Token price dynamics in equilibrium
\[
\frac{dP_t}{P_t} = \mu_t^p dt + \sigma_t^p dZ_t
\]
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- Token price appreciation $k_{i,t} E_t [dP_t]$
- Participation cost $\phi dt$, if $k_{i,t} > 0$

$$N_t = 1 - G_t(u_t)$$
Objective

\[ \int_{t=0}^{+\infty} e^{-rt} \left[ \max\{0, \text{convenience} + \text{net token return} - \text{participation cost}\} \right] dt \]
Token Demand

\[ k_{i,t} = \frac{F(E_t[dP_t], A_t)}{P_t} u_i \]

\[ \frac{\partial F}{\partial E_t[dP_t]} > 0 \]

\[ \frac{\partial F}{\partial A_t} > 0 \]
Token Market Clearing

\[ M_t = \int_{u=u_t}^{P_t} \frac{F(E_t[dP_t], A_t)}{P_t} udG_t(u) \]
Token Market Clearing

\[ M_t = \frac{F(E_t[dP_t], A_t)}{P_t} \int_{u=u_t}^{u} udG_t(u) \]

- \( P_t \) decreases in supply \( M_t \), increases in \( A_t \)
- 1st, 2nd order derivatives in \( E_t[dP_t] \) by Itô's lemma
  \( \rightarrow \) Differential equation for \( P_t = P(M_t, A_t) \)
A platform supports a unique set of transactions

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- Productivity: \( \frac{dA_t}{A_t} = L_t dH_t \)
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- Contributor resource: endogenous \( L_t \)
A platform supports a unique set of transactions

- **Productivity:** \( \frac{dA_t}{A_t} = L_t dH_t \)
- **Contributor resource:** *endogenous* \( L_t \)
- **Entrepreneur contribution:** \( dH_t = \mu^H dt + \sigma^H dZ_t \)
A platform supports a unique set of transactions

- Productivity: \[
\frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t)
\]
- Platform investment: endogenous \(L_t\)
A **platform** supports a unique set of transactions

- Productivity:  
  \[
  \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t)
  \]
- Platform investment:  
  \[
  \text{endogenous } L_t
  \]

**Tokens paid**  
\[
\frac{F(L_t, A_t) dt}{P_t}
\]

**Token Supply**  
\[
dM_t = \frac{F(L_t, A_t) dt}{P_t}
\]
A **platform** supports a unique set of transactions

- **Productivity:**
  \[
  \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t)
  \]

- **Platform investment:**
  endogenous \(L_t\)

- **Tokens paid to owner (cumulative):**
  \(D_t\)

**Token Supply**

\[
dM_t = \frac{F(L_t, A_t) dt}{P_t}
\]
A **platform** supports a unique set of transactions

- **Productivity:**
  \[ \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t) \]

- **Platform investment:**
  *endogenous* \( L_t \)

- **Tokens paid to owner:**
  \[ dD_t > 0 \]

- **Tokens burnt by owner:**
  \[ dD_t < 0 \]

\[ Token \ Supply \]

\[ dM_t = \frac{F(L_t, A_t)dt}{P_t} + dD_t \]
A platform supports a unique set of transactions

- Productivity: \( \frac{dA_t}{A_t} = L_t(\mu^H dt + \sigma^H dZ_t) \)
- Platform investment: endogenous \( L_t \)
- Tokens paid to owner: \( dD_t > 0 \)
- Tokens burnt by owner: \( dD_t < 0 \)

\[
\text{Token Supply} \\
\quad dM_t = \frac{F(L_t, A_t) dt}{P_t} + dD_t
\]
\[
\max_{\{L_t, d_{Dt}\}} \int_{t=0}^{+\infty} e^{-rt} P_t dD_t \left[I\{d_{Dt} \geq 0\} + (1 + \chi)I\{d_{Dt} < 0\}\right] dt
\]

- Token buy-back financing cost: \( \chi \)
$$\max\{L_t, dD_t\} \int_{t=0}^{+\infty} e^{-rt} P_t dD_t \left[ I\{dD_t \geq 0\} + (1 + \chi) I\{dD_t < 0\} \right] dt$$

- $V_t = V(M_t, A_t)$, $\frac{\partial V}{\partial M} < 0$, $\frac{\partial V}{\partial A} > 0$
- HJB is differential equation for $V(M_t, A_t)$

$$dM_t = \frac{F(L_t, A_t) dt}{P_t} + dD_t \quad \frac{dA_t}{A_t} = L_t (\mu^H dt + \sigma^H dZ_t)$$
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• $V_t = V(M_t, A_t)$, $\frac{\partial V}{\partial M} < 0$ $\frac{\partial V}{\partial A} > 0$

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A platform supports a unique set of transactions

- **Productivity:**
  \[
  \frac{dA_t}{A_t} = L_t(\mu dt + \sigma dZ_t)
  \]

- **Contributor resource:**
  \[
  \text{Payment} \quad \frac{endogenous L_t}{F(L_t, A_t)dt}
  \]

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**Objective**
\[
\int_{t=0}^{+\infty} e^{-r t} P_t dD_t [I_{\{dD_t \geq 0\}} + (1 + \chi)I_{\{dD_t < 0\}}] dt
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- **\(V_t = V(M_t, A_t)\), \(\frac{\partial V}{\partial M} < 0\) \(\frac{\partial V}{\partial A} > 0\)**

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\[
M_t = \frac{F(E_t[dP_t], A_t)}{P_t} \int_{u=u_t} udG_t(u)
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**Token Price**
\[
\frac{dP_t}{P_t} = \mu_t dt + \sigma_t dZ_t
\]
Transform the State Space

State space: \((M_t, A_t) \rightarrow (m_t, A_t)\), where \(m_t = \frac{M_t}{A_t}\)
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\[ V(M_t, A_t) = A_t \nu(m_t), \text{ and } P(M_t, A_t) = P(m_t) \]

Solve ODEs of \(\nu(m_t)\) and \(P(m_t)\)
Transform the State Space

State space: \((M_t, A_t) \rightarrow (m_t, A_t)\), where \(m_t = \frac{M_t}{A_t}\)

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Solve ODEs of \(\nu(m_t)\) and \(P(m_t)\)

\[
\frac{\partial V}{\partial M_t} = \nu'(m_t) < 0 \quad P'(m_t) < 0
\]
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- Model and Solution
- Franchise Value as Discipline
- Token Overhang
- The Value of Commitment
- Conclusion
Coase (1972):  • Producers of durable goods are always tempted to meet the residual demand until the product price falls to marginal cost
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  - Tokens are durable - $dD_t > 0$ permanently increases $M_t$ - and no resources are needed to produce tokens (MC = 0)
Coase (1972):  

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  - Tokens are durable - $dD_t > 0$ permanently increases $M_t$ - and no resources are needed to produce tokens ($MC = 0$)
- Consumers wait for the lowest price
Coase (1972):

• Producers of durable goods are always tempted to meet the residual demand until the product price falls to marginal cost
  ▪ Tokens are durable - $dD_t > 0$ permanently increases $M_t$ - and no resources are needed to produce tokens ($MC = 0$)

• Consumers wait for the lowest price
  ▪ Consumers rationally form expectation of token price
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  - Tokens are durable - $dD_t > 0$ permanently increases $M_t$ - and no resources are needed to produce tokens ($MC = 0$)
- Consumers wait for the lowest price
  - Consumers rationally form expectation of token price
- Producers sell all goods immediately at price equal to MC
Coase (1972):

- Producers of durable goods are always tempted to meet the residual demand until the product price falls to marginal cost
  - Tokens are durable - $dD_t > 0$ permanently increases $M_t$ - and no resources are needed to produce tokens ($MC = 0$)
- Consumers wait for the lowest price
  - Consumers rationally form expectation of token price
- Producers sell all goods immediately at price equal to $MC$
  - Producers sell $\infty$ tokens immediately at price equal to $0$?
Difference:  • Token demand is *not stationary* – $A_t$ grows geometrically, so future demand is stronger – users cannot expect $P_t$ falls to 0
  ▪ Bulow (1982), Stokey (1981)
**Difference:**

- Token demand is *not stationary* – $A_t$ grows geometrically, so future demand is stronger – users cannot expect $P_t$ falls to 0
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- Real option concern: $A_t$ grows stochastically, and increasing token supply can only be reversed costly due to $\chi$
Difference: • Token demand is not stationary – $A_t$ grows geometrically, so future demand is stronger – users cannot expect $P_t$ falls to 0
  ▪ Bulow (1982), Stokey (1981)
• Real option concern: $A_t$ grows stochastically, and increasing token supply can only be reversed costly due to $\chi$

Platform resists excess supply

$$m_t = \frac{M_t}{A_t} \in [\underline{m}, \overline{m}]$$

*Incentive to buyback and burn tokens*
Optimal Platform Payout and Buy-back (burn) $dD_t$
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\[
\frac{m}{m_t} = \frac{m}{\bar{m}}
\]

\[
dD_t < 0
\]

\[-\frac{\partial V}{\partial M_t} = -v'(m_t) = P_t (1 + \chi)\]
Luxury brands including Burberry burn stock worth millions
Optimal Platform Payout and Buy-back (burn) $dD_t$

\[
\begin{align*}
\frac{m}{m_t} & \quad \frac{m_t}{\bar{m}} \\
\text{if } dD_t > 0 & \quad \text{if } dD_t < 0
\end{align*}
\]

\[
\begin{align*}
- \frac{\partial V}{\partial M_t} &= -v'(m_t) = P_t \\
- \frac{\partial V}{\partial M_t} &= -v'(m_t) = P_t(1 + \chi)
\end{align*}
\]
Optimal Platform Payout and Buy-back (burn) $dD_t$

\[ m_t \]

\[ dD_t > 0 \]

\[ dD_t < 0 \]

\[ - \frac{\partial V}{\partial M_t} = -v'(m_t) = P_t \]

\[ - \frac{\partial V}{\partial M_t} = -v'(m_t) = P_t (1 + \chi) \]

Franchise (continuation) value \rightarrow Resistance against over-supply
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**Conflict of Interest and Under-investment**

Investment paid by new tokens ➔ User convenience ↑
Conflict of Interest and Under-investment

Investment paid by new tokens → User convenience ↑ → Can platform seize all surplus via token price ↑ ?
**Conflict of Interest and Under-investment**

Investment paid by new tokens $\rightarrow$ User convenience $\uparrow$ $\rightarrow$ Can platform seize all surplus via token price $\uparrow$ ? $\textit{NO!}$

User heterogeneity $+$

One token price integrated market

High $u_i$ keep surplus; only the marginal user breaks even
Conflicts of Interest and Under-investment

Investment paid by new tokens $m_t = \frac{M_t}{A_t}$ ↓ if negative shock

User convenience ↑

User heterogeneity

One token price integrated market

Can platform seize all surplus via token price ↑? \textbf{NO!}

High $u_i$ keep surplus; only the marginal user breaks even

Closer to $\overline{m}$

costly buy-back

Platform pays $\chi$ and cannot share it with users
**Conflict of Interest and Under-investment**

- **Investment paid by new tokens** → **User convenience ↑** → **Can platform seize all surplus via token price ↑?** *NO!*

- **User heterogeneity** + **One token price integrated market** → **High \( u_i \) keep surplus; only the marginal user breaks even**

- **\( m_t = \frac{M_t}{A_t} \downarrow \text{if negative shock} \) → **Closer to \( \bar{m} \) costly buy-back** → **Platform bears \( \chi \) and cannot share it with users**
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Time Inconsistency

A rule of investment set at $t = 0 \rightarrow \text{higher } V \text{ in every state}$

\[
\frac{dM_t}{M_t} = \mu^M dt \quad \text{at } m_t \in (m, \bar{m}), \text{ s.t., } \tilde{L}(m_t) > L_t
\]

Higher token value dominates the cost of more frequent token burning
Value Function: Discretion vs. Commitment

A: Platform Owner Value – Discretion

B: Platform Owner Value – Commitment
Time Inconsistency

* A rule of investment set at \( t = 0 \) \( \rightarrow \) higher \( V \) in every state

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\frac{dM_t}{M_t} = \mu^M dt \text{ at } m_t \in (\underline{m}, \bar{m}), \text{s.t., } \tilde{L}(m_t) > L_t
\]

Commitment via Blockchain
Conclusion: Token-Based Digital Ecosystem

- A model of token-based ecosystem
  - Endogenous token supply and platform development
  - Endogenous token price and user-base formation

1. Platform franchise value → discipline on token supply ("dilution")
   - Durable-good problem, because of endogenous platform development
   - Token burning contributes to token price stability; stablecoin without collateral-backing (in the paper)

2. Token overhang
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   - Blockchain enables token as means of payment and financing tools
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  - Endogenous token supply and platform development
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Optimal Platform Investment $L_t$

\[
\frac{\partial V}{\partial A_t} A_t \mu^H + \frac{\partial^2 V}{\partial A_t^2} A_t^2 (\sigma^H)^2 L_t = \frac{\partial F}{\partial L_t} \left( \frac{\partial V / \partial M_t}{P_t} \right)
\]

**Marginal contribution to $V$**

**Marginal cost**

*Marginal cost of investment:*

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**Marginal contribution to $V$**

**Marginal cost**

**Marginal cost of investment:** $\frac{\partial F}{\partial L_t}$

Dynamic token issuance cost: $-\frac{\partial V/\partial M_t}{P_t} > 1$, at $\bar{m}$, $-\frac{\partial V}{\partial M_t} = P_t (1 + \chi)$

Underinvestment!
Token Overhang

A: Dynamic Token Issuance Cost

\[-\frac{V_M}{P_t}\]

B: Platform Investment

\[L_t\]

Token Supply / Platform Productivity \(m_t\)
Users and Token Demand

- Price-taking, in equilibrium \( dP_t = P_t \mu_t^P dt + P_t \sigma_t^P dZ_t \)
- Maximize the NPV, given \( r \), the cost of capital

\[
\mathbb{E} \left[ \int_{t=0}^{\infty} e^{-rt} dy_{i,t} \right],
\]

where

\[
dy_{i,t} = \max \left\{ 0, \max_{k_{i,t}>0} \left[ (P_t k_{i,t})^{1-\alpha} (N_t^\gamma A_t u_i)^\alpha \right. \right.
\]

convenience

\[
\left. \left. \left. \left. + k_{i,t} \mathbb{E}_t [dP_t] - \phi dt - P_t k_{i,t} r dt \right] \right\} \right\}
\]

- Deadweight access cost \( \phi dt \): cognitive, application integration etc.
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\]

- Deadweight access cost \( \phi dt \): cognitive, application integration etc.
Users and Token Demand (con’t)

- Agent $i$’s optimal holding of tokens is given by

$$k_{i,t}^* = \frac{N_t \gamma A_t u_i}{P_t} \left( \frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \quad (2)$$

It has the following properties:
1. $k_{i,t} \uparrow$ in $N_t$, user base.
2. $k_{i,t} \downarrow$ in token price $P_t$.
3. $k_{i,t} \uparrow$ in $A_t$, platform usefulness, and agent-specific $u_i$.
4. $k_{i,t} \uparrow$ in the expected token price change, $\mu_t^P$.

- Determine $N_t$: if profits > 0, agents participate

- Adoption: maximized profit $N_t \gamma A_t u_i \alpha \left( \frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1 - \alpha}{\alpha}} > \phi$
  - A threshold value of $u_i$ above which users adopt
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Token Valuation

- Users' aggregate transaction need: \( U_t := \int_{u \geq u_t} u g(u) \, du \), where \( u_t \) is the indifference threshold.

- Token market clearing, \( M_t = \int_{i \in [0,1]} k_{i,t}^* \, di \).

- The equilibrium token price is given by
  \[
P_t = \frac{N_t A_t}{M_t} \left( \frac{1 - \alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}.
  \]  
  (3)

- \( \mu_t^P \) is the expectation of risk-adjusted token appreciation.
Token Valuation

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- Token market clearing, 
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- The equilibrium token price is given by
  \[ P_t = \frac{N^\gamma_t U_t A_t}{M_t} \left( \frac{1 - \alpha}{r - \mu^P_t} \right)^{\frac{1}{\alpha}}. \] (3)

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  \]

- \( \mu_t^P \) is the expectation of risk-adjusted token appreciation.
Optimal Token Supply

- Two controls: $L_t$ (investment) and $D_t$ (payout/buy-back)
- Two state variables: $M_t$ and $A_t$

$$V_t = \max_{\{L_t,D_t\}_{s \geq t}} \int_{s=t}^{+\infty} \mathbb{E}_t \left[ e^{-r(s-t)} P_s dD_s \left( \mathbb{I}_{\{dD_s \geq 0\}} - (1 + \chi) \mathbb{I}_{\{dD_s < 0\}} \right) \right],$$

- Continuation value: the present value of seigniorage
**Calibration**

### Panel A: Key Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Model</th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $\alpha$</td>
<td>0.3</td>
<td>Comovement: $N_t$ &amp; $P_t$</td>
<td>Cong, Li, and Wang (2018a)</td>
</tr>
<tr>
<td>(2) $\mu^H$</td>
<td>50%</td>
<td>Productivity growth</td>
<td>Cong, Li, and Wang (2018a)</td>
</tr>
<tr>
<td>(3) $\sigma^H$</td>
<td>200%</td>
<td>Productivity volatility</td>
<td>Cong, Li, and Wang (2018a)</td>
</tr>
<tr>
<td>(4) $\theta$</td>
<td>1e4</td>
<td>Investment variation</td>
<td>Illustrative purpose</td>
</tr>
<tr>
<td>(5) $\zeta$</td>
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<td>The Distribution of $u_i$</td>
<td>Illustrative purpose</td>
</tr>
<tr>
<td>(6) $\kappa$</td>
<td>0.8</td>
<td>The Distribution of $u_i$</td>
<td>Illustrative purpose</td>
</tr>
<tr>
<td>(7) $\theta$</td>
<td>5e5</td>
<td>The Distribution of $u_i$</td>
<td>Comparative Statics – Competition Effects</td>
</tr>
<tr>
<td>(8) $\chi$</td>
<td>20%</td>
<td>Token buyback cost</td>
<td>Comparative Statics – Financial Frictions</td>
</tr>
<tr>
<td>(9) $\gamma$</td>
<td>1/8</td>
<td>$N_t$ in total productivity</td>
<td>Parameter restriction</td>
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</table>

### Panel B: Other Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Model</th>
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</thead>
<tbody>
<tr>
<td>(10) $r$</td>
<td>5%</td>
<td>Risk-free rate</td>
</tr>
<tr>
<td>(11) $\phi$</td>
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<td>Scaling effect on $A_t$</td>
</tr>
<tr>
<td>(12) $\rho$</td>
<td>1</td>
<td>Shock correlation: SDF &amp; $A_t$</td>
</tr>
<tr>
<td>(13) $\eta$</td>
<td>1</td>
<td>Price of risk</td>
</tr>
</tbody>
</table>
Parametric Assumption of $u_i$ Distribution

- $u_i$ follows a Pareto distribution on $[U_t, +\infty)$ with c.d.f.

$$G_t(u) = 1 - \left( \frac{U_t}{u} \right)^{\xi},$$

(4)

where $\xi \in (1, 1/\gamma)$ and $U_t = 1/(\omega A_t^\kappa), \omega > 0, \kappa \in (0, 1)$.

- The cross-section mean of $u_i$ is $\frac{\xi U_t}{\xi - 1}$

- $U_t$ decreases in $A_t$: (1) to capture competition effects; (2) for analytical convenience
Endogenous User Base

Proposition

Given $\mu_t^P$, we have a unique non-degenerate solution for $N_t$ under the Pareto distribution of $u_i$ given by Equation (4):

$$N_t = \left( \frac{A_t^{1-\kappa}}{\omega \varphi} \right)^{\frac{\xi}{1-\xi \gamma}} \left( \frac{1 - \alpha}{r - \mu_t^P} \right)^{\left( \frac{\xi}{1-\xi \gamma} \right) \left( \frac{1-\alpha}{\alpha} \right)},$$

(5)

if $A_t^{1-\kappa} \left( \frac{1-\alpha}{r - \mu_t^P} \right)^{\frac{1-\alpha}{\alpha}} \leq \frac{\omega \varphi}{\alpha}$; otherwise, $N_t = 1$.

- Why hold token? (1) Usage $A_t$. (2) Investment $\mu_t^P$. 
Optimal Control

HJB equation:

$$rV(M_t, A_t) \, dt = \max_{L_t, dD_t} \left[ \underbrace{V_{M_t} \left[ \frac{F(L_t, A_t)}{P_t} \right]}_{\text{Insider's value}} dt + dD_t \right] + V_{A_t} A_t \mu^H dt$$

$$+ \frac{1}{2} V_{A_t} A_t^2 L_t^2 \sigma^2 dt + P_t dD_t \left[ I\{dD_t \geq 0\} - (1 + \chi) I\{dD_t < 0\} \right],$$

with

$$dM_t = \frac{F(L_t, A_t)}{P_t} \, dt + dD_t,$$

and

$$\frac{dA_t}{A_t} = \left( \mu^L dt + \sigma^L dZ_t \right) L_t.$$

Proposition

The optimal token supply is given by (1) the optimal choice of $L_t$,

$$L^*_t = \frac{V_{A_t} \mu^H + V_{M_t} \frac{1}{P_t}}{-V_{M_t} \frac{\theta}{P_t} - V_{A_t} A_t \sigma^2},$$

and (2) the optimal choice of $dD_t$ – the platform pays out token dividends ($dD^*_t > 0$) if $P_t \geq -V_{M_t}$, and the insiders buy back and burn tokens out of circulation ($dD^*_t < 0$) if $-V_{M_t} \geq P_t (1 + \chi)$.
Risk-Neutral to Physical Measure

- SDF: \( \frac{d\Lambda_t}{\Lambda_t} = -rdt - \eta d\hat{Z}_t^\Lambda \)

- Risk-neutral measure: \( dZ_t^\Lambda = d\hat{Z}_t^\Lambda + \eta dt \).

- \( \rho = \text{corr}(dZ_t^\Lambda, dZ_t^A) \)

- Calibrate the model to the speed of \( N_t \) growth in data
  - Drift of \( A_t \) under physical measure: \( \mu^A + \eta \rho \sigma^A \)