Network Risk and Key Players: A Structural Analysis of Interbank Liquidity

Edward Denbee[◇] Christian Julliard[*†]
Ye Li[‡] Kathy Yuan[*†]

[*]London School of Economics, [†]CEPR [◇]Bank of England [‡]Columbia University

The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England.

- Recent crisis stressed the need of understanding systemic risk generation and exposure in the banking industry.

Our paper: linear quadratic network framework; we can identify:

1. the amplification mechanism, or multiplier, of liquidity shocks;
2. the liquidity level key players (for bailout?);
3. the liquidity risk key players (to regulate?).

Also: we solve the CP problem and have implications for the efficiency of liquidity injections and Quantitative Easing.

The case study: Intraday Liquidity in U.K. Payment System

- a £700bn a day market i.e. UK GDP settled every two days.
- Daily Gross Settlement requires large intraday liquidity buffers.

$\Rightarrow$ Almost all banks in CHAPS have regular intraday liquidity exposures in excess of £1bn to individual counterparties (greater than £3bn for larger banks).

We study banks' intraday liquidity holding decision in the network, and its implications for systemic liquidity risk.

- Recent crisis stressed the need of understanding systemic risk generation and exposure in the banking industry.

Our paper: linear quadratic network framework; we can identify:

1. the amplification mechanism, or multiplier, of liquidity shocks;
2. the liquidity level key players (for bailout?);
3. the liquidity risk key players (to regulate?).

Also: we solve the CP problem and have implications for the efficiency of liquidity injections and Quantitative Easing.

The case study: Intraday Liquidity in U.K. Payment System

- a £700bn a day market i.e. UK GDP settled every two days.
- Daily Gross Settlement requires large intraday liquidity buffers.

⇒ Almost all banks in CHAPS have regular intraday liquidity exposures in excess of £1bn to individual counterparties (greater than £3bn for larger banks).

We study banks' intraday liquidity holding decision in the network, and its implications for systemic liquidity risk.

- Recent crisis stressed the need of understanding systemic risk generation and exposure in the banking industry.

Our paper: linear quadratic network framework; we can identify:

1. the amplification mechanism, or multiplier, of liquidity shocks;
2. the liquidity level key players (for bailout?);
3. the liquidity risk key players (to regulate?).

Also: we solve the CP problem and have implications for the efficiency of liquidity injections and Quantitative Easing.

The case study: Intraday Liquidity in U.K. Payment System

- a £700bn a day market i.e. UK GDP settled every two days.
- Daily Gross Settlement requires large intraday liquidity buffers.

⇒ Almost all banks in CHAPS have regular intraday liquidity exposures in excess of £1bn to individual counterparties (greater than £3bn for larger banks).

We study banks' intraday liquidity holding decision in the network, and its implications for systemic liquidity risk.

- Recent crisis stressed the need of understanding systemic risk generation and exposure in the banking industry.

Our paper: linear quadratic network framework; we can identify:

1. the amplification mechanism, or multiplier, of liquidity shocks;
2. the liquidity level key players (for bailout?);
3. the liquidity risk key players (to regulate?).

Also: we solve the CP problem and have implications for the efficiency of liquidity injections and Quantitative Easing.

The case study: Intraday Liquidity in U.K. Payment System

- a £700bn a day market i.e. UK GDP settled every two days.
- Daily Gross Settlement requires large intraday liquidity buffers.

$\Rightarrow$ Almost all banks in CHAPS have regular intraday liquidity exposures in excess of £1bn to individual counterparties (greater than £3bn for larger banks).

We study banks' intraday liquidity holding decision in the network, and its implications for systemic liquidity risk.

Bank Objective Function

- Bank i decision variables:

$l_i := q_i + z_i$: is the observable liquidity holding of bank i, where:

q_i : liquidity level of bank i absent bilateral effects, given by

$$q_i = q_i(x) := \underbrace{\alpha_i}_{\text{fixed effect}} + \underbrace{\sum_{m=1}^{M} \beta_m x_i^m}_{\text{characteristics}} + \underbrace{\sum_{p=1}^{P} \beta_p x^p}_{\text{common factors}}$$

z_i : the network component of liquidity buffer stock.

Bank Objective Function cont'd

- A quadratic payoff function for buffer stock liquidity z_i

$$u_i(z_i|g) = \underbrace{\tilde{\mu}_i}_{\text{Unit Valuation}} \underbrace{\left(z_i + \psi \sum_{j \neq i} g_{ij} z_j\right)}_{\text{Accesible Liquidity}} - \frac{1}{2}\gamma \left(z_i + \psi \sum_{j \neq i} g_{ij} z_j\right)^2$$

ψ: leakage factor

Network g : characterized by n-square adjacency matrix $\mathbf{G}$ with elements $g_{i,j}$, and $g_{i,i} = 0 \Rightarrow$ directed and weighted network

$g_{i,j \neq i}$: the **fraction of borrowing by Bank i from Bank j**.

$\tilde{\mu}_i = \hat{\mu}_i + \delta \sum_j g_{ij} z_j$ where $\hat{\mu}_i / \gamma = \bar{\mu}_i + \nu_i \sim i.i.d\,(0, \sigma_i^2)$

δ: Information discount, "hair cut"

Bank Objective Function cont'd

- A quadratic payoff function for buffer stock liquidity z_i

$$u_i(z_i|g) = \underbrace{\tilde{\mu}_i}_{\text{Unit Valuation}} \underbrace{\left(z_i + \psi \sum_{j \neq i} g_{ij} z_j\right)}_{\text{Accesible Liquidity}} - \frac{1}{2}\gamma \left(z_i + \psi \sum_{j \neq i} g_{ij} z_j\right)^2$$

ψ: leakage factor

Network g : characterized by n-square adjacency matrix $\mathbf{G}$ with elements $g_{i,j}$, and $g_{i,i} = 0 \Rightarrow$ directed and weighted network

$g_{i,j \neq i}$: the **fraction of borrowing by Bank** i **from Bank** j.

$\tilde{\mu}_i = \hat{\mu}_i + \delta \sum_j g_{ij} z_j$ where $\hat{\mu}_i / \gamma = \bar{\mu}_i + \nu_i \sim i.i.d\,(0, \sigma_i^2)$

δ: Information discount, "hair cut"

(Decentralized) Equilibrium Outcome

$Eq.^{um}$: (Nash) If $|\phi| < 1$

$$z_i^* = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j} z_j + \nu_i$$

$$\phi := \delta/\gamma - \psi$$

If $\phi > 0$ complementarity (reciprocate/herding/leverage stacks e.g. Moore (2011)).

If $\phi < 0$ substitutability (free ride à la Bhattacharya and Gale (1987)).

$$\Rightarrow l_i^* = q_i(x) + z_i^* = q_i(x) + \{\mathbf{M}(\phi, \mathbf{G})\}_{i.}\, \mu$$

where $\mu := \gamma^{-1} [\hat{\mu}_1, ..., \hat{\mu}_n]'$, $\{\}_{i.}$ is the row operator, and

$$\mathbf{M}(\phi, \mathbf{G}) := \mathbf{I} + \phi\mathbf{G} + \phi^2\mathbf{G}^2 + \phi^3\mathbf{G}^3 + ... = \sum_{k=0}^{\infty} \phi^k \mathbf{G}^k.$$

(Decentralized) Equilibrium Outcome

$Eq.^{um}$: (Nash) If $|\phi| < 1$

$$z_i^* = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j} z_j + \nu_i$$

$$\phi := \delta/\gamma - \psi$$

If $\phi > 0$ complementarity (reciprocate/herding/leverage stacks e.g. Moore (2011)).

If $\phi < 0$ substitutability (free ride à la Bhattacharya and Gale (1987)).

$$\Rightarrow l_i^* = q_i(x) + z_i^* = q_i(x) + \{\mathbf{M}(\phi, \mathbf{G})\}_{i.}\, \mu$$

where $\mu := \gamma^{-1} [\hat{\mu}_1, ..., \hat{\mu}_n]'$, $\{\}_{i.}$ is the row operator, and

$$\mathbf{M}(\phi, \mathbf{G}) := \mathbf{I} + \phi\mathbf{G} + \phi^2\mathbf{G}^2 + \phi^3\mathbf{G}^3 + ... = \sum_{k=0}^{\infty} \phi^k \mathbf{G}^k.$$

(Decentralized) Equilibrium Outcome

$Eq.^{um}$: (Nash) If $|\phi| < 1$

$$z_i^* = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j} z_j + \nu_i$$
$$\phi := \delta/\gamma - \psi$$

If $\phi > 0$ complementarity (reciprocate/herding/leverage stacks e.g. Moore (2011)).

If $\phi < 0$ substitutability (free ride à la Bhattacharya and Gale (1987)).

$$\Rightarrow l_i^* = q_i(x) + z_i^* = q_i(x) + \{\mathbf{M}(\phi, \mathbf{G})\}_{i.}\, \mu$$

where $\mu := \gamma^{-1} [\hat{\mu}_1, ..., \hat{\mu}_n]'$, $\{\}_{i.}$ is the row operator, and

$$\mathbf{M}(\phi, \mathbf{G}) := \mathbf{I} + \phi\mathbf{G} + \phi^2\mathbf{G}^2 + \phi^3\mathbf{G}^3 + ... = \sum_{k=0}^{\infty} \phi^k \mathbf{G}^k.$$

Key Players

The total liquidity originating from the network externalities is

$$\sum_{i=1}^{n} z_i^* = \underbrace{\mathbf{1}'\mathbf{M}(\phi, \mathbf{G})\,\bar{\mu}}_{\text{level effect}} + \underbrace{\mathbf{1}'\mathbf{M}(\phi, \mathbf{G})\,\nu}_{\text{risk effect}}$$

where $\bar{\mu} \equiv [\bar{\mu}_1, ..., \bar{\mu}_n]'$, $\nu \equiv [\nu_1, ..., \nu_n]'$

$\Rightarrow$ tradeoff: both terms increasing in ϕ (for $\bar{\mu} > 0$).

Risk Key Player: (the one to worry about...)

$$\max_i \frac{\partial \mathbf{1}' z^*}{\partial \nu_i} \sigma_i = \max_i \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i} \sigma_i \rightarrow \text{outdregree centrality}$$

Level Key Player: (the one you might want to bailout...)

$$\max_i E\left[\mathbf{1}' z^* - \mathbf{1}' z^*_{\setminus i}\right] = \max_i \{\mathbf{M}(\phi, \mathbf{G})\}_{i.} \bar{\mu} + \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i} \bar{\mu}_i - m_{i,i}\bar{\mu}_i$$

$\rightarrow$ indegree centrality + shock analogous − correct double counting

Key Players

The total liquidity originating from the network externalities is

$$\sum_{i=1}^{n} z_i^* = \underbrace{\mathbf{1}'\mathbf{M}(\phi, \mathbf{G})\,\bar{\mu}}_{\text{level effect}} + \underbrace{\mathbf{1}'\mathbf{M}(\phi, \mathbf{G})\,\nu}_{\text{risk effect}}$$

where $\bar{\mu} \equiv [\bar{\mu}_1, ..., \bar{\mu}_n]'$, $\nu \equiv [\nu_1, ..., \nu_n]'$

$\Rightarrow$ tradeoff: both terms increasing in ϕ (for $\bar{\mu} > 0$).

Risk Key Player: (the one to worry about...)

$$\max_i \frac{\partial \mathbf{1}' z^*}{\partial \nu_i} \sigma_i \;=\; \max_i \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i} \sigma_i \rightarrow \text{outdregree centrality}$$

Level Key Player: (the one you might want to bailout...)

$$\max_i E\left[\mathbf{1}' z^* - \mathbf{1}' z^*_{\setminus i}\right] \;=\; \max_i \{\mathbf{M}(\phi, \mathbf{G})\}_{i.} \bar{\mu} + \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i} \bar{\mu}_i - m_{i,i}\bar{\mu}_i$$

$\rightarrow$ indegree centrality + shock analogous − correct double counting

Key Players

The total liquidity originating from the network externalities is

$$\sum_{i=1}^{n} z_i^* = \underbrace{\mathbf{1}'\mathbf{M}(\phi, \mathbf{G})\,\bar{\mu}}_{\text{level effect}} + \underbrace{\mathbf{1}'\mathbf{M}(\phi, \mathbf{G})\,\nu}_{\text{risk effect}}$$

where $\bar{\mu} \equiv [\bar{\mu}_1, ..., \bar{\mu}_n]'$, $\nu \equiv [\nu_1, ..., \nu_n]'$

$\Rightarrow$ tradeoff: both terms increasing in ϕ (for $\bar{\mu} > 0$).

Risk Key Player: (the one to worry about...)

$$\max_i \frac{\partial \mathbf{1}' z^*}{\partial \nu_i} \sigma_i \;=\; \max_i \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i} \sigma_i \rightarrow \text{outdregree centrality}$$

Level Key Player: (the one you might want to bailout...)

$$\max_i E\left[\mathbf{1}' z^* - \mathbf{1}' z^*_{\setminus i}\right] \;=\; \max_i \{\mathbf{M}(\phi, \mathbf{G})\}_{i.} \bar{\mu} + \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G})\}_{.i} \bar{\mu}_i - m_{i,i}\bar{\mu}_i$$

$\rightarrow$ indegree centrality + shock analogous − correct double counting

Empirical Model

SEM: the theoretical framework is matched by a Spatial Error Model

$$l_{i,t} = \alpha_i + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \beta_p^{time} x_t^p + z_{i,t}$$

$$z_{i,t} = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j,t} z_{j,t} + \nu_{i,t}, \quad \nu_{i,t} \sim iid\left(0, \sigma_i^2\right),$$

where $g_{i,j,t}$, $x_{i,t}^m$ and x_t^p are predetermined at time t.

Note:
1. Network as a shock propagation mechanism
   $\Rightarrow$ (average) Network Multiplier: $1/(1-\phi)$
2. Total liquidity, $L_t \equiv \mathbf{1}'[l_{1,t}, ..., l_{n,t}]$, is heteroskedastic.
3. Can perform Q-MLE (ϕ overidentified if $rank(\mathbf{M}(\phi, \mathbf{G}_t)) > 2$)
4. (true) network impulse-response to a shock to bank i

$$NIRF_i(\phi, \mathbf{G}_t, \sigma_i) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.i} \sigma_i$$

Empirical Model

SEM: the theoretical framework is matched by a Spatial Error Model

$$l_{i,t} = \alpha_i + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \beta_p^{time} x_t^p + z_{i,t}$$

$$z_{i,t} = \bar{\mu}_i + \phi \sum_{j=1}^{n} g_{i,j,t} z_{j,t} + \nu_{i,t}, \quad \nu_{i,t} \sim iid\left(0, \sigma_i^2\right),$$

where $g_{i,j,t}$, $x_{i,t}^m$ and x_t^p are predetermined at time t.

Note:
1. Network as a shock propagation mechanism
   $\Rightarrow$ (average) Network Multiplier: $1/(1-\phi)$
2. Total liquidity, $L_t \equiv \mathbf{1}'[l_{1,t}, ..., l_{n,t}]$, is heteroskedastic.
3. Can perform Q-MLE (ϕ overidentified if $rank(\mathbf{M}(\phi, \mathbf{G}_t)) > 2$)
4. (true) network impulse-response to a shock to bank i

$$NIRF_i(\phi, \mathbf{G}_t, \sigma_i) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.i} \sigma_i$$

Network and Other Data Description

Sample: from Feb 2006 to Sept 2010, daily data.

Network Banks: all CHAPS members ▸ vd ▸ cl ▸ www

(non CHAPS banks must channel their payments through these banks)

Network Proxy: $g_{i,j,t}$ = the fraction of bank i's loans borrowed from bank j ▸ e-v (computed as monthly averages in previous month)

Dependent Variable: liquidity available at the beginning of the day (account balance plus posting of collateral)

Macro Controls: (aggregate risk proxies, lagged) LIBOR; Interbank Rate; Intraday Volatility of Liquidity Available; Turnover Rate in Payment System; Right Kurtosis of Aggregate Payment Time; time trend.

Banks Characteristics: (lagged) Borrowing Rate; Right Kurtosis of Payment (Out) Time; Right Kurtosis of Payment (In) Time; Intraday Volatility of Liquidity Available; Total Intraday Payments; Liquidity Used; Repo liability to Total Asset Ratio; Cumulative Change in Retail Deposit to Total Asset Ratio; Total Lending and Borrowing in Interbank Market; Stock Return; CDS.

Network and Other Data Description

Sample: from Feb 2006 to Sept 2010, daily data.

Network Banks: all CHAPS members ▸ vd ▸ cl ▸ www

(non CHAPS banks must channel their payments through these banks)

Network Proxy: $g_{i,j,t}$ = the fraction of bank i's loans borrowed from bank j ▸ e-v (computed as monthly averages in previous month)

Dependent Variable: liquidity available at the beginning of the day (account balance plus posting of collateral)

Macro Controls: (aggregate risk proxies, lagged) LIBOR; Interbank Rate; Intraday Volatility of Liquidity Available; Turnover Rate in Payment System; Right Kurtosis of Aggregate Payment Time; time trend.

Banks Characteristics: (lagged) Borrowing Rate; Right Kurtosis of Payment (Out) Time; Right Kurtosis of Payment (In) Time; Intraday Volatility of Liquidity Available; Total Intraday Payments; Liquidity Used; Repo liability to Total Asset Ratio; Cumulative Change in Retail Deposit to Total Asset Ratio; Total Lending and Borrowing in Interbank Market; Stock Return; CDS.

SEM Estimation

Two types of estimation:

1. Subsample estimations:

(good times) Pre Hedge Fund Crisis/ Northern Rock
(fin. crisis) Hedge Fund Crisis – Asset Purchase Program Announcement
(Q.E.) Post Asset Purchase Program Announcement ▸ Agg. Liq.

	Period 1	*Period 2*	*Period 3*
Network Effect: $\hat{\phi}$	0.8137	0.3031	−0.1794
	(21.47)	(1.90)	(−4.96)
R^2	69.11%	89.71%	85.54%
(average) Network Multiplier	5.3677	1.4349	0.8479
	(4.92)	(4.37)	(32.61)

SEM Estimation

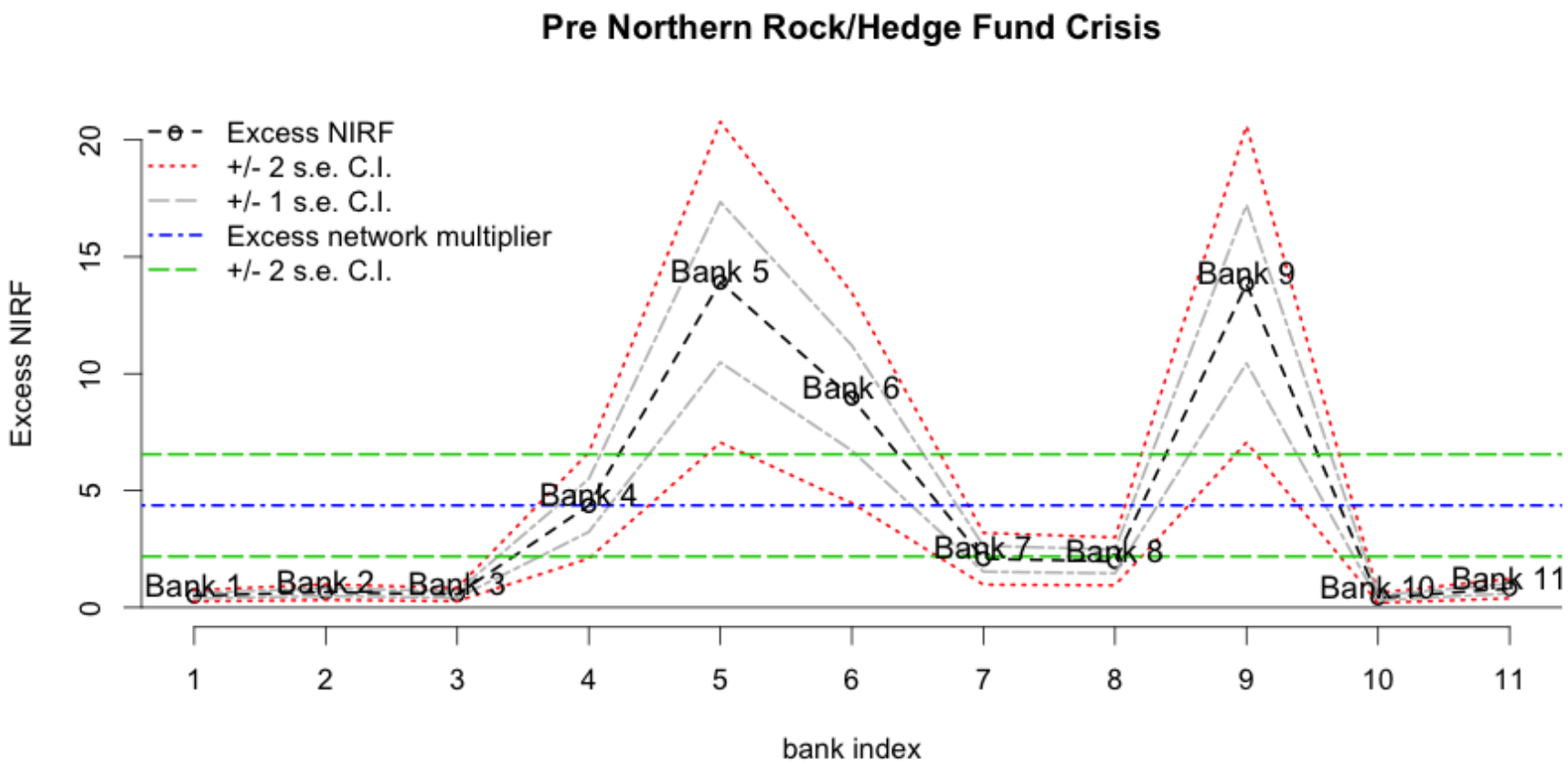
Two types of estimation:

1. Subsample estimations:

(good times) Pre Hedge Fund Crisis/ Northern Rock
(fin. crisis) Hedge Fund Crisis – Asset Purchase Program Announcement
(Q.E.) Post Asset Purchase Program Announcement ▸ Agg. Liq.

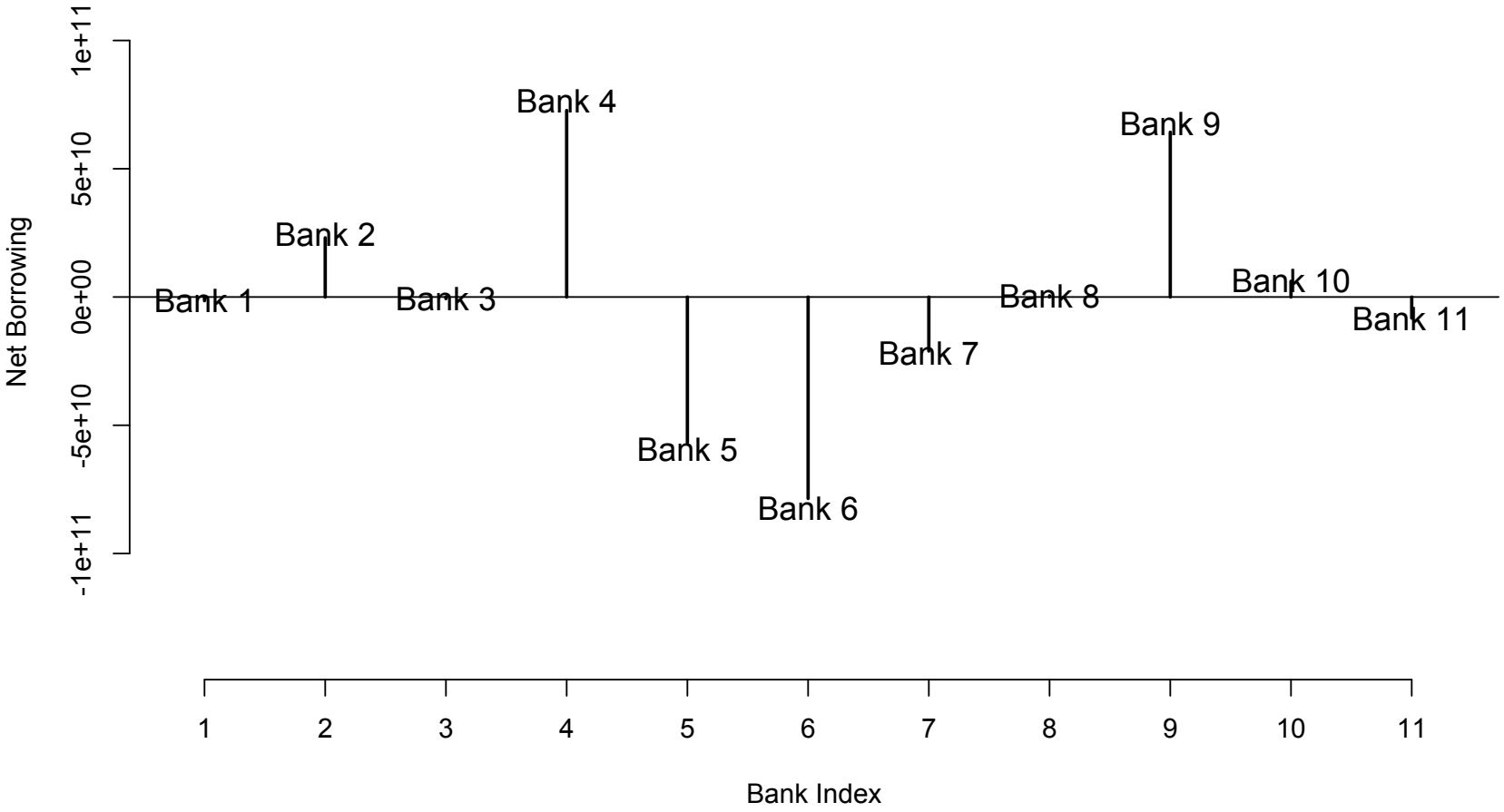
	Period 1	*Period 2*	*Period 3*
Network Effect: $\hat{\phi}$	0.8137	0.3031	−0.1794
	(21.47)	(1.90)	(−4.96)
R^2	69.11%	89.71%	85.54%
(average) Network Multiplier	5.3677	1.4349	0.8479
	(4.92)	(4.37)	(32.61)

Period 1: $NIRF^e\left(\phi, \bar{\mathbf{G}}, 1\right)$ – Risk Key Players



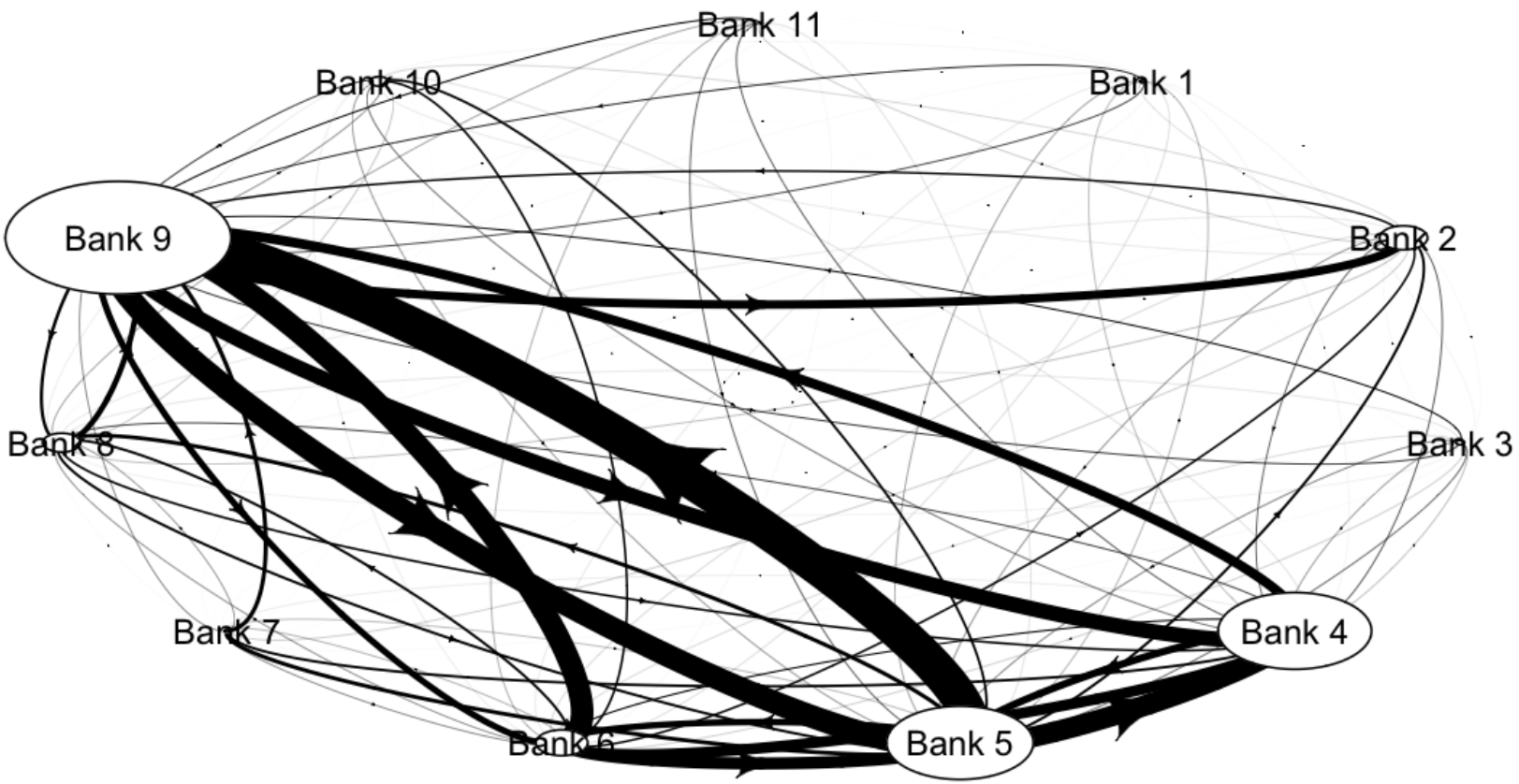
NIRF P2 & P3

Period 1: Net Borrowing



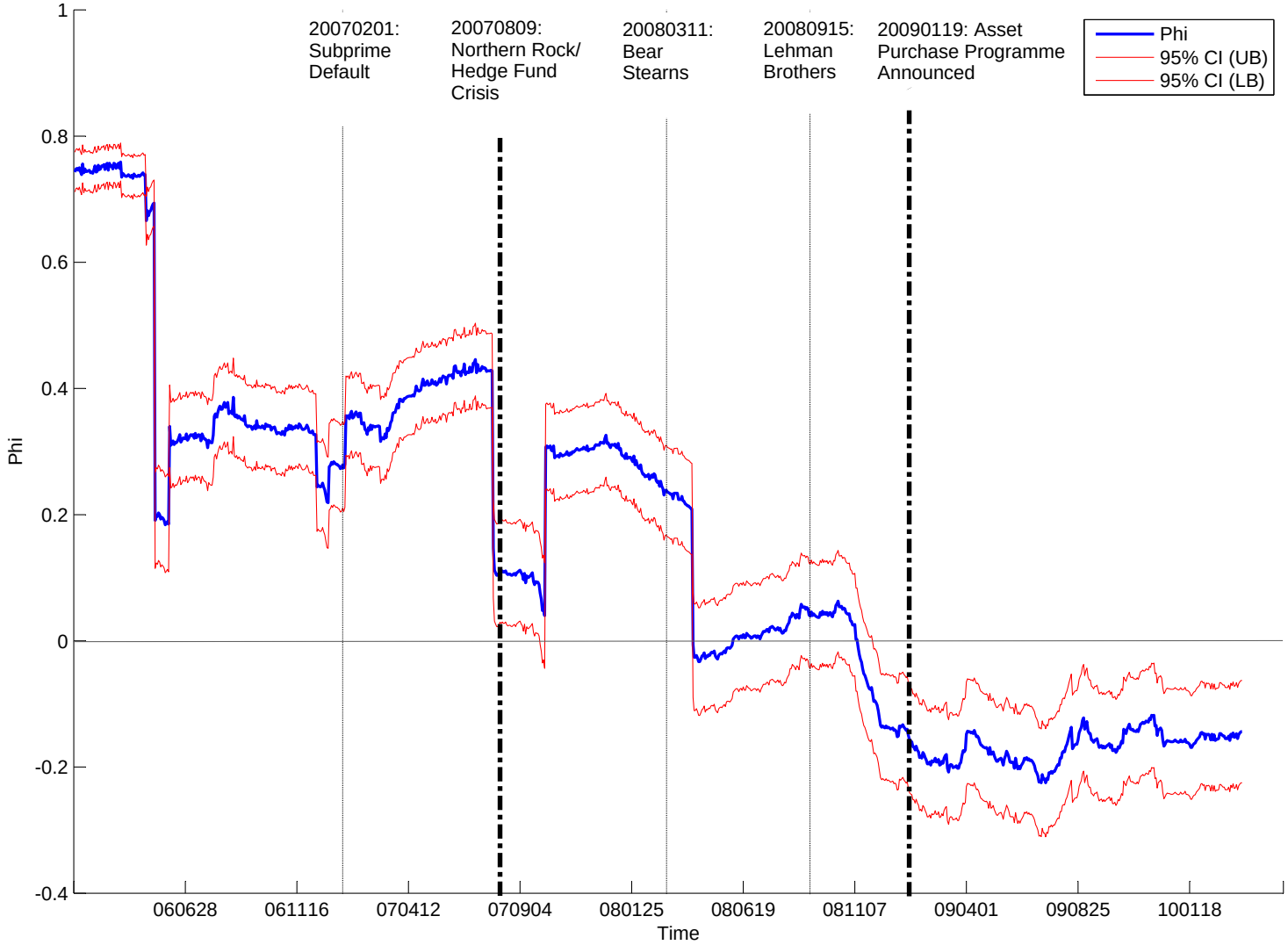
Borrowing P2 & P3

Period 1: Network Borrowing/Lending Flows



Flows P2 & P3
SEM

$\hat{\phi}$: SEM Rolling Estimation (6-month window)



Conclusions

We provide:

- an implementable approach to assess interbank network risk:
  1. network shocks multiplier
  2. risk, and level, key players identification
  3. network impulse-response functions

Empirical Findings:

1. First estimation of network risk multiplier ⇒ a significant shock propagation mechanism for liquidity
2. The network multiplier and risk:
   - vary significantly over time, and can be very large.
   - implies complementarity (and high risk) before the crisis.
   - it's basically zero post Bearn Stearns ⇒ rational reaction.
   - indicates free riding on the liquidity provided by the Quantitative Easing.
3. most of the systemic risk is generated by a small subset of key players (and not necessarily the obvious ones).

Conclusions

We provide:

- an implementable approach to assess interbank network risk:
  1. network shocks multiplier
  2. risk, and level, key players identification
  3. network impulse-response functions

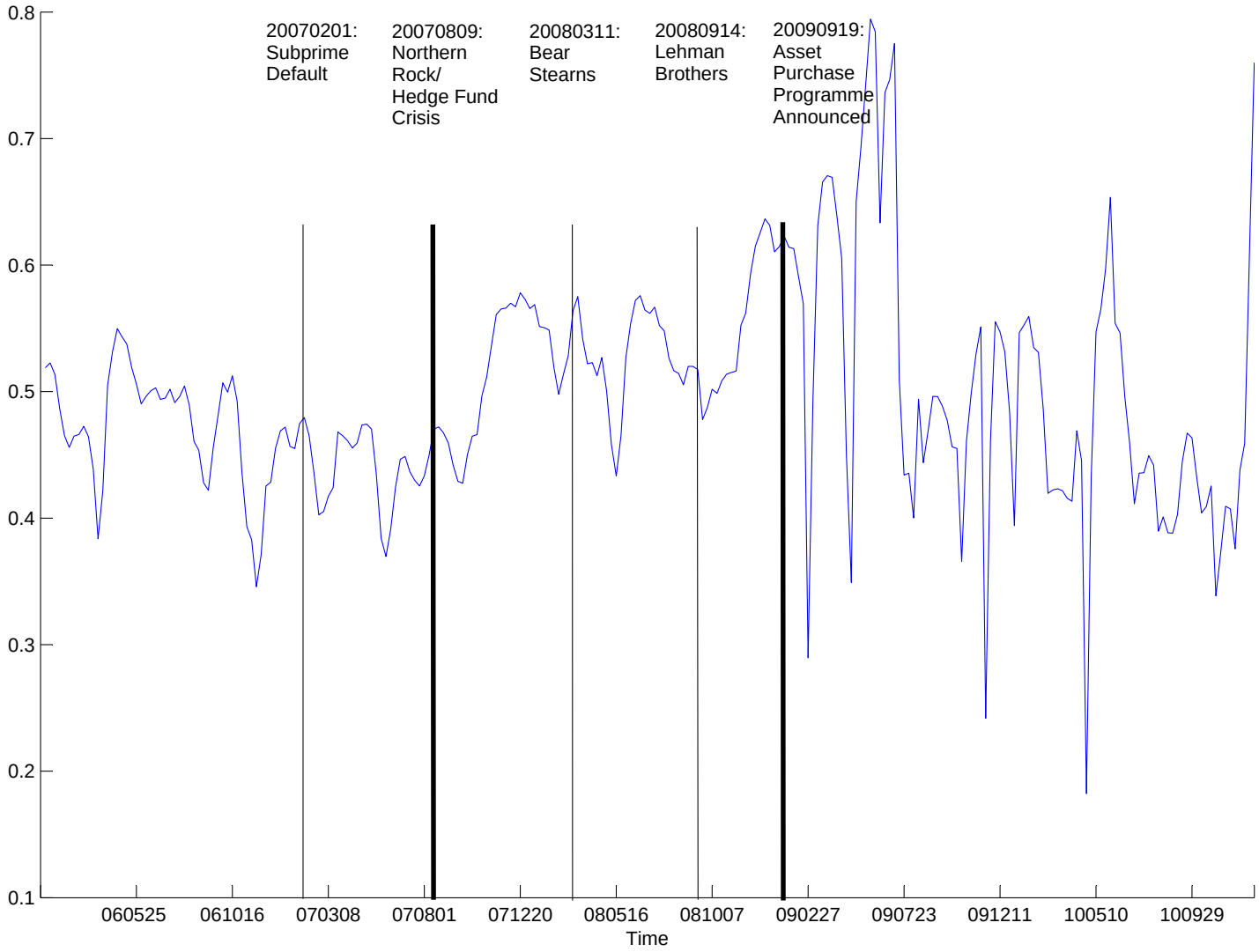
Empirical Findings:

1. First estimation of network risk multiplier $\Rightarrow$ a significant shock propagation mechanism for liquidity
2. The network multiplier and risk:
   - vary significantly over time, and can be very large.
   - implies complementarity (and high risk) before the crisis.
   - it's basically zero post Bearn Stearns $\Rightarrow$ rational reaction.
   - indicates free riding on the liquidity provided by the Quantitative Easing.
3. most of the systemic risk is generated by a small subset of key players (and not necessarily the obvious ones).

Appendix

4. Additional Data Info
   - Second Largest Eigenvalue of $\mathbf{G}_t$
   - Average Clustering Coefficient
   - Other Variables
5. Additional Estimation Result
   - Full SEM Results
   - Specification Test
   - Central Planner vs. Market Equilibria
6. Network Evolution
   - NIRFs
   - Net Borrowing and Flows

The Second Largest Eigenvalue of $\mathbf{G}_t$



Cohesiveness

Q: How cohesive is this network?

A: Average Clustering Coefficient (Watts and Strogatz, 1998)

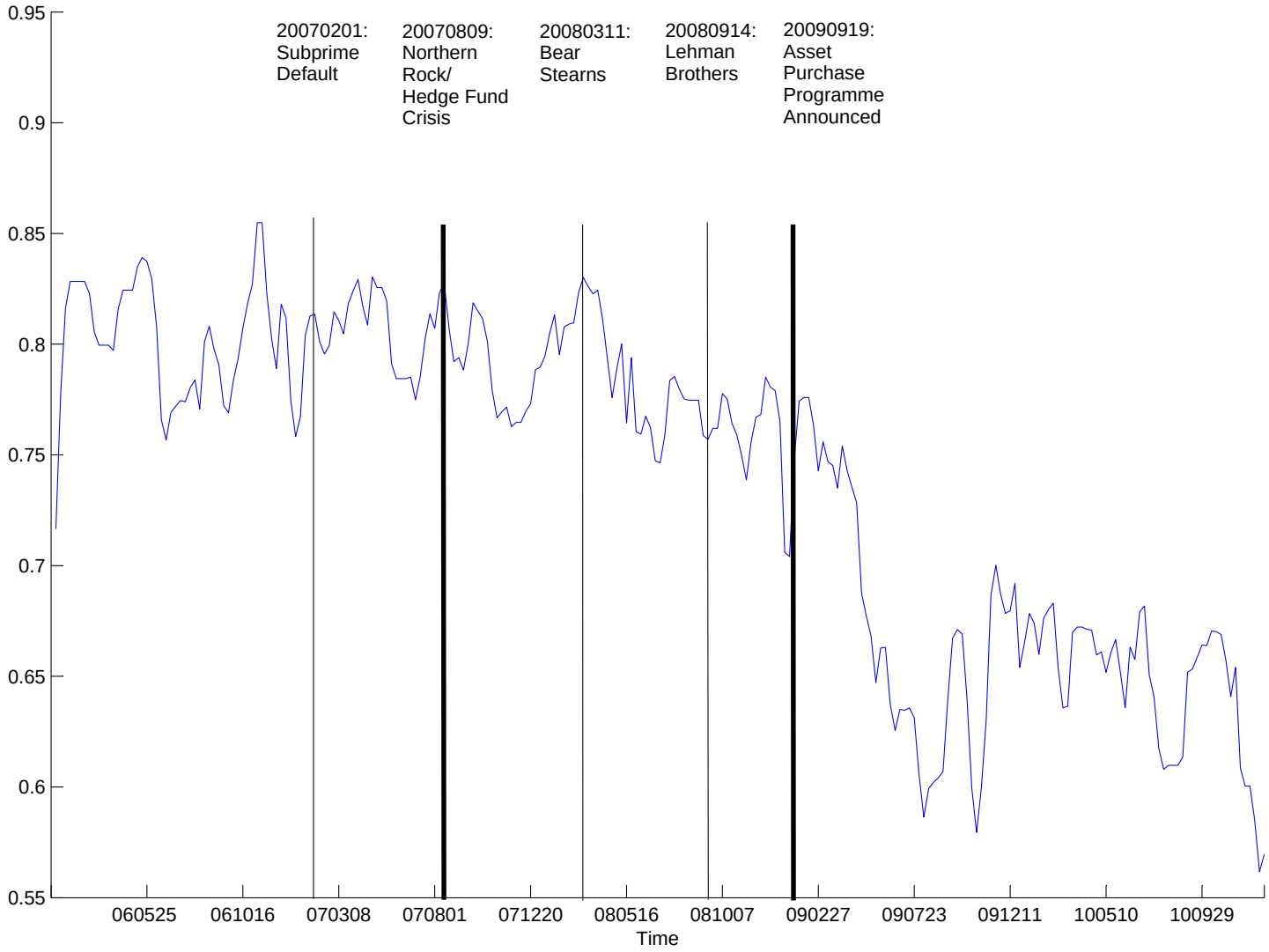
$$
\begin{aligned}
ACC &= \frac{1}{n}\sum_{i=1}^{n} CL_i(\mathbf{G}), \\
CL_i(G) &= \frac{\#\{jk \in \mathbf{G} \mid k \neq j,\, j \in n_i(\mathbf{G}),\, k \in n_i(\mathbf{G})\}}{\#\{jk \mid k \neq j,\, j \in n_i(\mathbf{G}),\, k \in n_i(\mathbf{G})\}}
\end{aligned}
$$

where n is the number of members in the network and $n_i(\mathbf{G})$ is the set of players between whom and player i there is an edge.

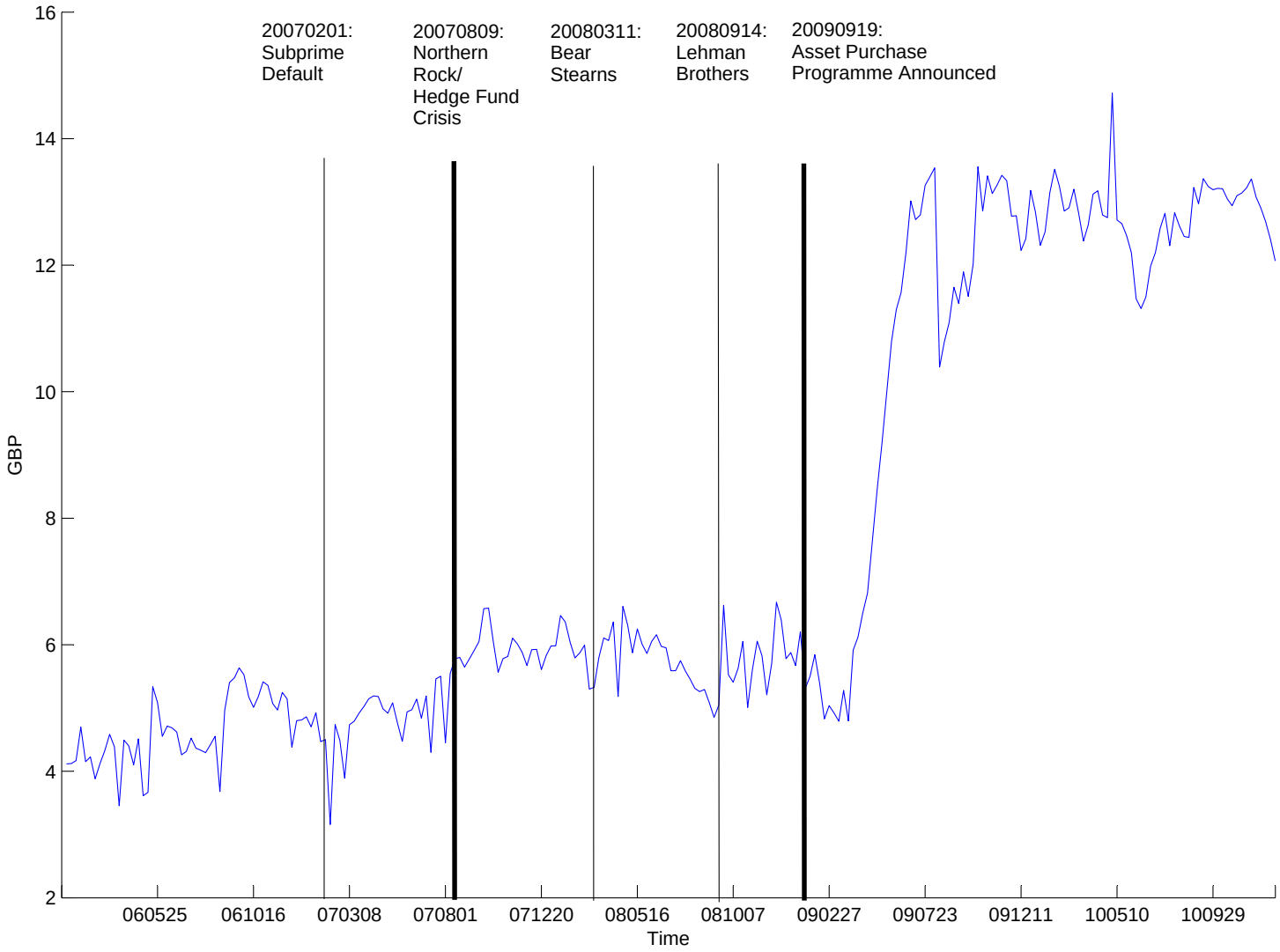
Numerator: # of pairs of banks linked to i that are also linked to each other

Denominator: # of pairs of banks linked to i

Average Clustering Coefficient of the Network

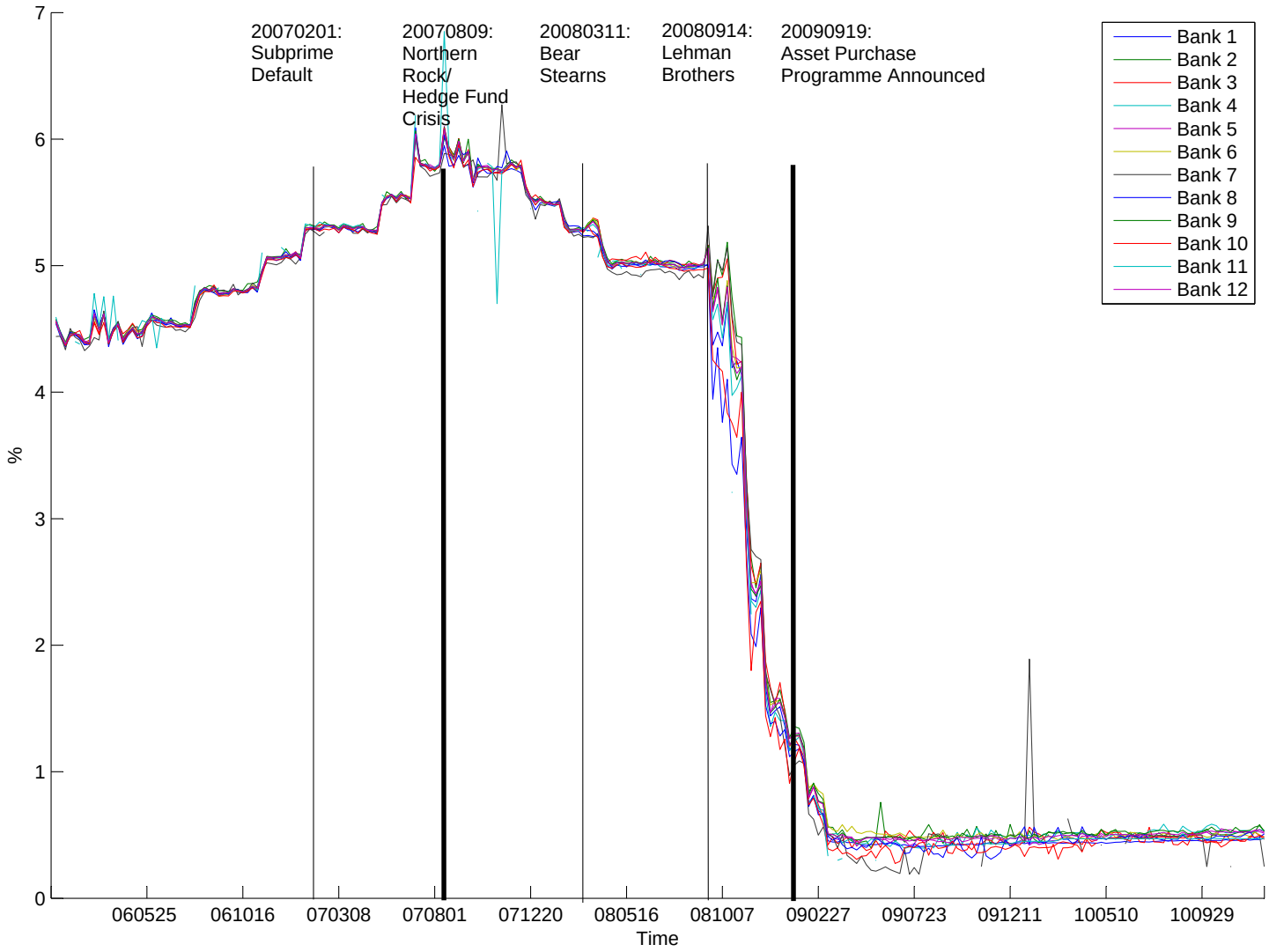


Aggregate Liquidity Available at the Beginning of a Day

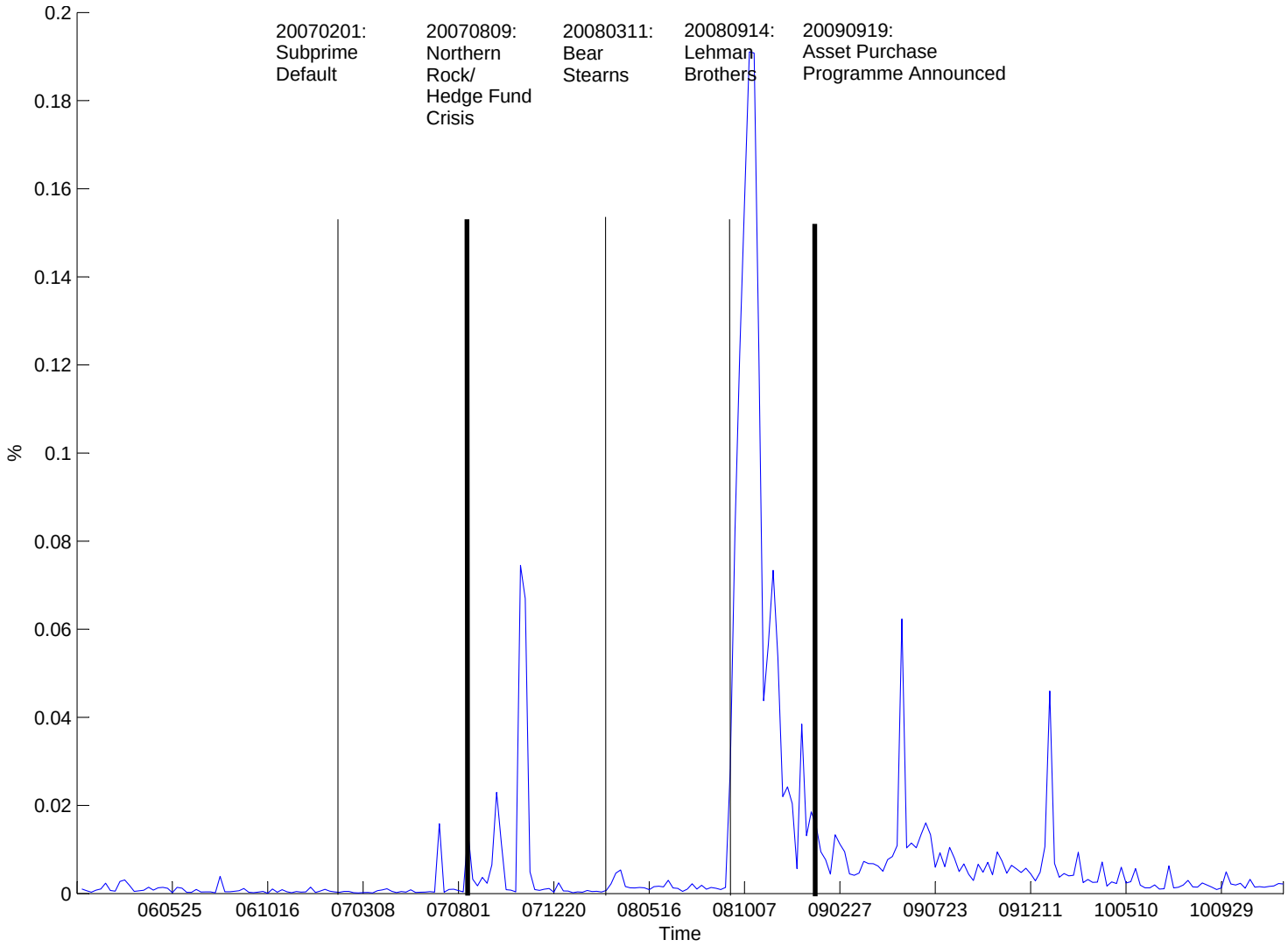


estimation

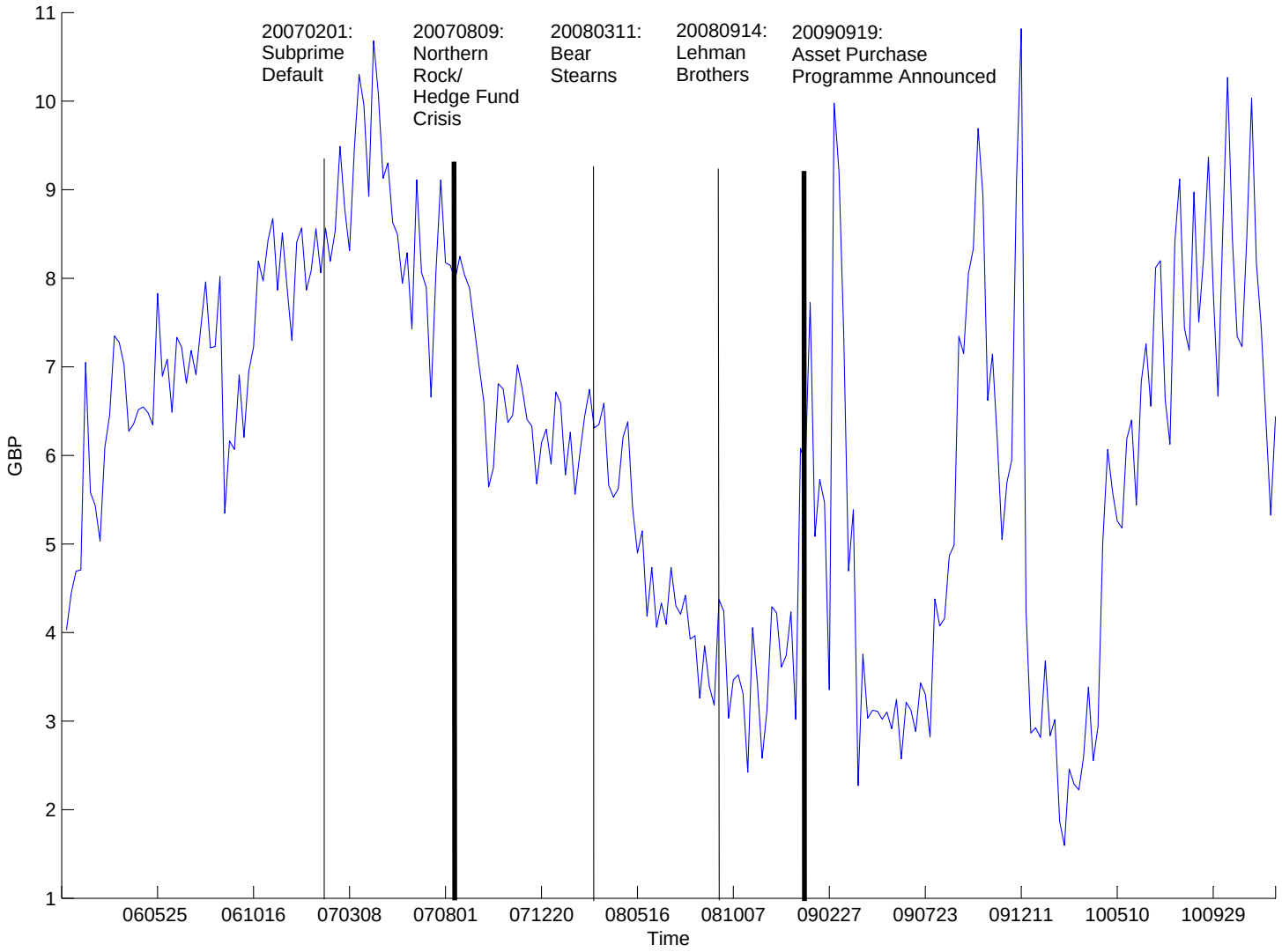
Interest Rate in Interbank Market



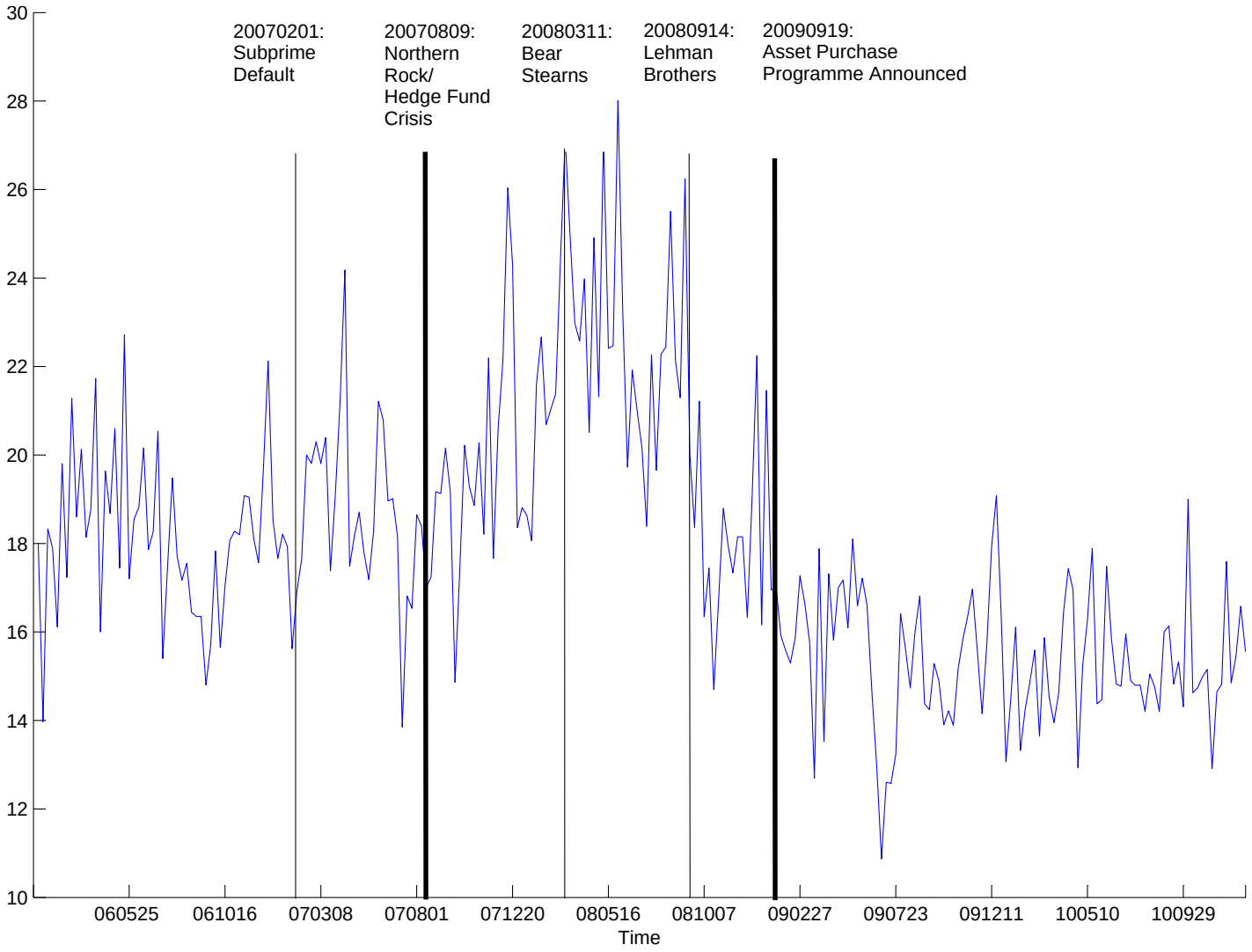
Cross-Sectional Dispersion of Interbank Rate



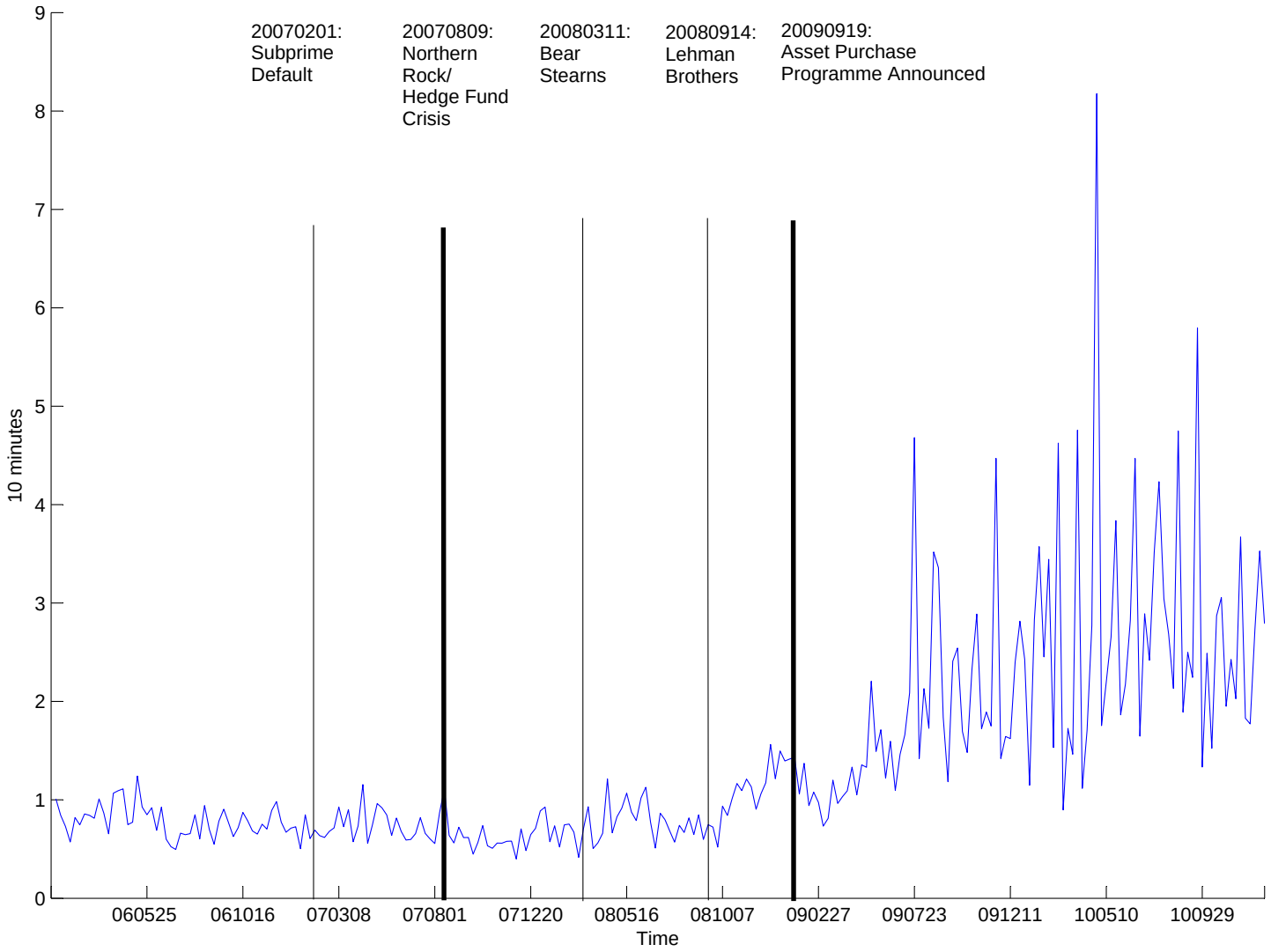
Intraday Volatility of Aggregate Liquidity Available



Turnover Rate in the Payment System



Right Kurtosis of Aggregate Payment Time



SEM Estimation

	Period 1	*Period 2*	*Period 3*
$\hat{\phi}$	0.8137^* (21.47)	0.3031^* (1.90)	-0.1794^* (-4.96)
$1/\left(1-\hat{\phi}\right)$	5.3677^* (4.92)	1.4349^* (4.37)	0.8479^* (32.61)
Macro Control Variables			
rK_{t-1}	0.1845 (1.30)	0.0084 (0.55)	-0.0032^* (-3.88)
ln $VolPay_{t-1}$	-0.4451 (-1.00)	0.0308 (1.17)	0.0291 (1.72)
TOR_{t-1}	0.0166 (1.80)	0.0007 (0.69)	0.0018 (1.75)
LIBOR	0.2378 (0.27)	0.0928 (1.28)	0.5800^* (2.52)
Interbank Rate Premium	3.8845 (1.61)	-0.0405 (-0.33)	0.6973^* (3.00)

SEM Estimation cont'd

Bank Characteristics			
Interbank Rate	−0.2081	−0.0473	−0.0880
	(−0.98)	(−1.03)	(−1.92)
ln $LevPay_{i,t-1}$	−0.0235	0.0802*	0.0808*
	(−0.62)	(3.29)	(5.09)
$rK^{in}_{i,t-1}$	0.0010	−0.0086	0.0045
	(0.14)	(−0.63)	(1.03)
$rK^{out}_{i,t-1}$	0.0090	0.0320*	−0.0061
	(0.92)	(3.62)	(−1.32)
ln $VolPay_{i,t-1}$	0.0129*	0.0039	0.0196*
	(4.59)	(1.92)	(5.96)
ln $LU_{i,t-1}$	−0.0038*	−0.0039*	−0.0027*
	(−2.86)	(−3.41)	(−3.79)
$\frac{\text{Repo Liability}}{\text{Assets}}$	−5.5625*	0.0282	−0.3057
	(−3.61)	(0.43)	(−1.45)
Total Assets (log)	1.2590*	0.6328*	1.0170*
	(5.39)	(10.31)	(18.92)
$\frac{\Delta\text{Deposit}}{\text{Assets}}$	−0.0014	0.0149*	0.0481*
	(−0.20)	(5.15)	(11.76)
Total Lending and Borrowing (log)	−0.1882*	0.0612*	−0.0025
	(−5.57)	(2.95)	(−1.27)
CDS (log)	0.0051	−0.1212*	−0.0383*
	(0.13)	(−6.61)	(−4.00)
Stock Return (Inc. Dividend)	−0.5667	0.1927	0.2574
	(−0.88)	(1.49)	(1.88)

Specification Test

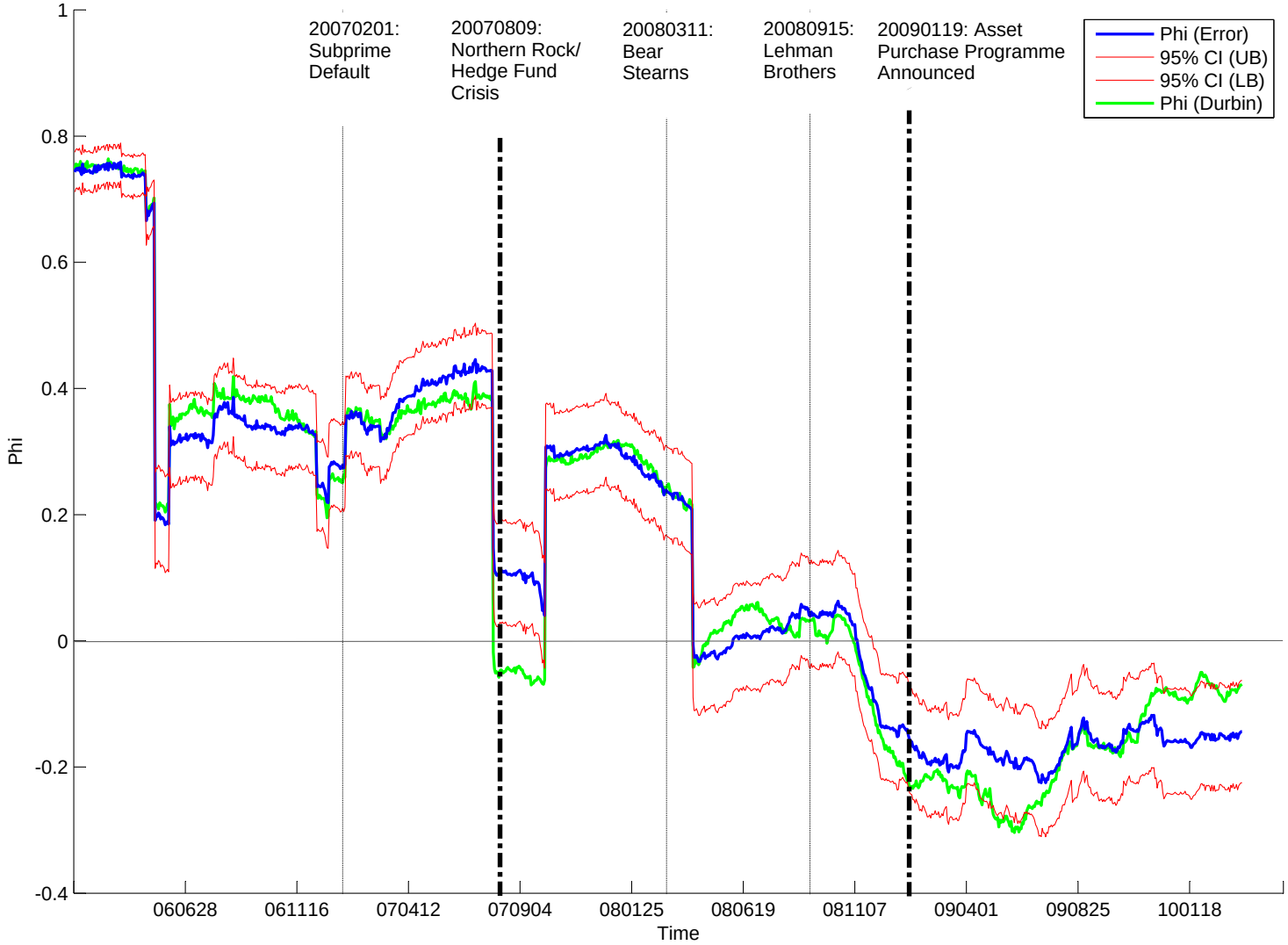
SDM: For robustness, we also consider a direct network effect of banks observable characteristic, liquidity decisions, and possible match specific control variables, $x_{i,j,t}$ (Spatial Durbin Model)

$$\begin{aligned} l_{i,t} &= \bar{\alpha}_i + \sum_{m=1}^{M} \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^{P} \gamma_p^{time} x_t^p \\ &\quad + \rho \sum_{j=1}^{n} g_{i,j,t} l_{j,t} + \sum_{j=1}^{n} g_{i,j,t} x_{i,j,t} \theta + v_{i,t} \end{aligned}$$

Note: if $x_{i,j,t} := vec(x_{j \neq i,t}^m)'$, $\rho = \phi$, $\theta = -\phi vec(\beta_m^{bank})$, $\gamma_p^{time} = (1-\phi)\beta_p^{time} \forall p \Rightarrow$ back to SEM

$\Rightarrow$ this more general spatial structure provides a specification test for our model.

$\hat{\phi}$ and $\hat{\rho}$: SEM and SDM Rolling Estimation (6-month window)



Central Planner's Problem

A planner chooses $z_i, i = 1, , \ldots n$ to maximize

$$\sum_{i=1}^{n} u_i \left(z_i | g_i \right)$$

FOC:

$$z_i = \underbrace{\mu_i + \phi \sum_{j \neq i} g_{ij} z_j}_{\text{decentralized f.o.c.}} + \underbrace{\psi \sum_{j \neq i} g_{ji} \mu_j}_{\substack{\text{neighbors' idiosyncratic} \\ \text{valuations of own liquidity}}} + \underbrace{\phi \sum_{j \neq i} g_{ji} z_j}_{\substack{\text{neighbors' indegree} \\ \text{i.e. own outdegree}}} \underbrace{- \psi \left(\psi - \frac{2\delta}{\gamma} \right) \sum_{j \neq i} \sum_{m \neq j} g_{ji} g_{jm} z_m}_{\substack{\text{volatility of neighbors'} \\ \text{accessible network liquidity}}}$$

$\Rightarrow$ wedge btw market and CP solutions

Central Planner's Problem

A planner chooses $z_i, i = 1, , \ldots n$ to maximize

$$\sum_{i=1}^{n} u_i \left(z_i | g_i \right)$$

FOC:

$$z_i = \underbrace{\mu_i + \phi \sum_{j \neq i} g_{ij} z_j}_{\text{decentralized f.o.c.}} + \underbrace{\psi \sum_{j \neq i} g_{ji} \mu_j}_{\text{neighbors' idiosyncratic valuations of own liquidity}} + \underbrace{\phi \sum_{j \neq i} g_{ji} z_j}_{\text{neighbors' indegree i.e. own outdegree}} \underbrace{- \psi \left(\psi - \frac{2\delta}{\gamma} \right) \sum_{j \neq i} \sum_{m \neq j} g_{ji} g_{jm} z_m}_{\text{volatility of neighbors' accessible network liquidity}}$$

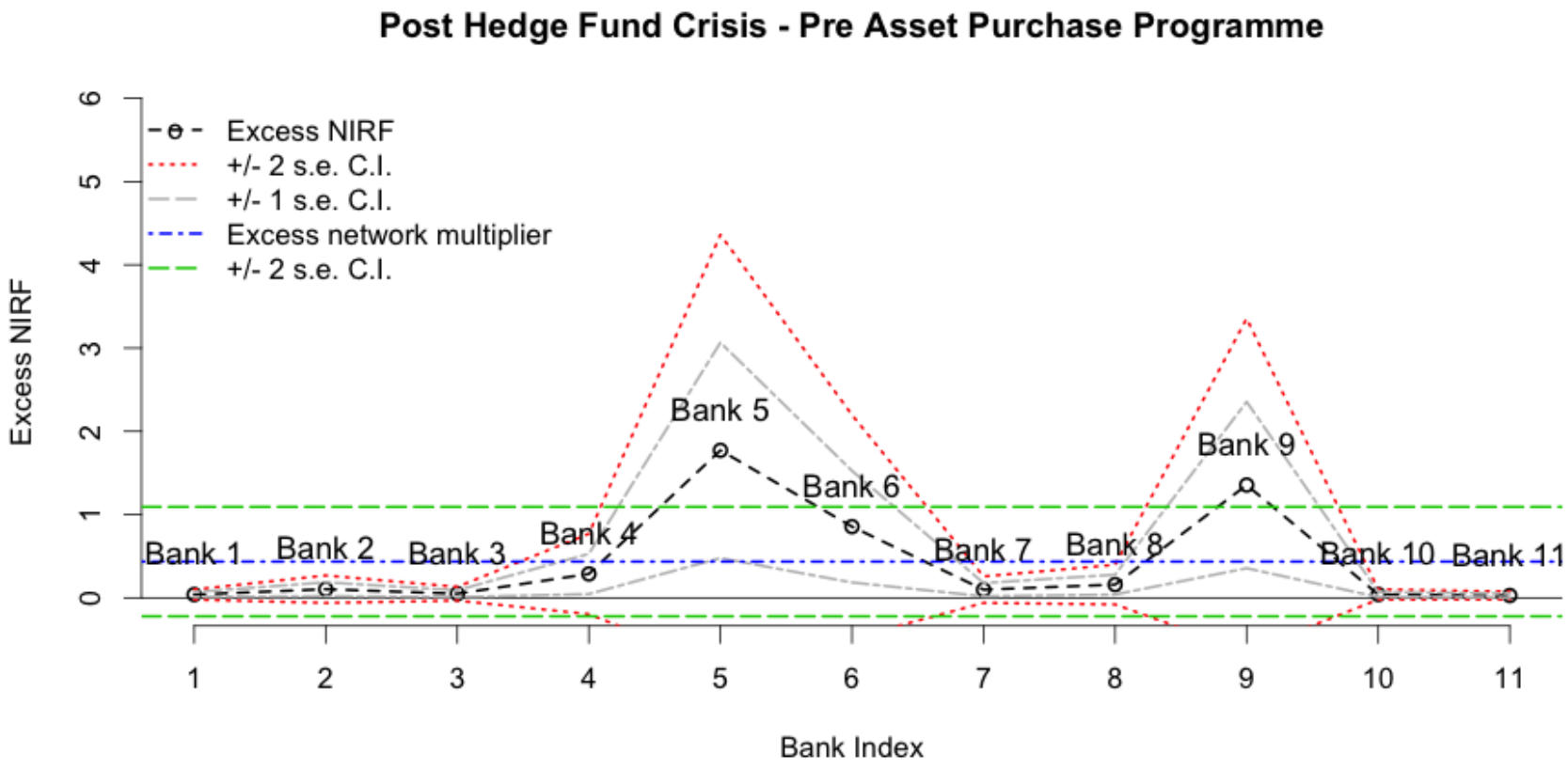
$\Rightarrow$ wedge btw market and CP solutions

Central Planner vs. Market Equilibria

	Period 1	*Period 2*	*Period 3*
Δ% Volatility of Total Liquidity	-90.8%	-64.8%	30.7%
Δ Network Liquidity	-3.47	15.5	-27.5

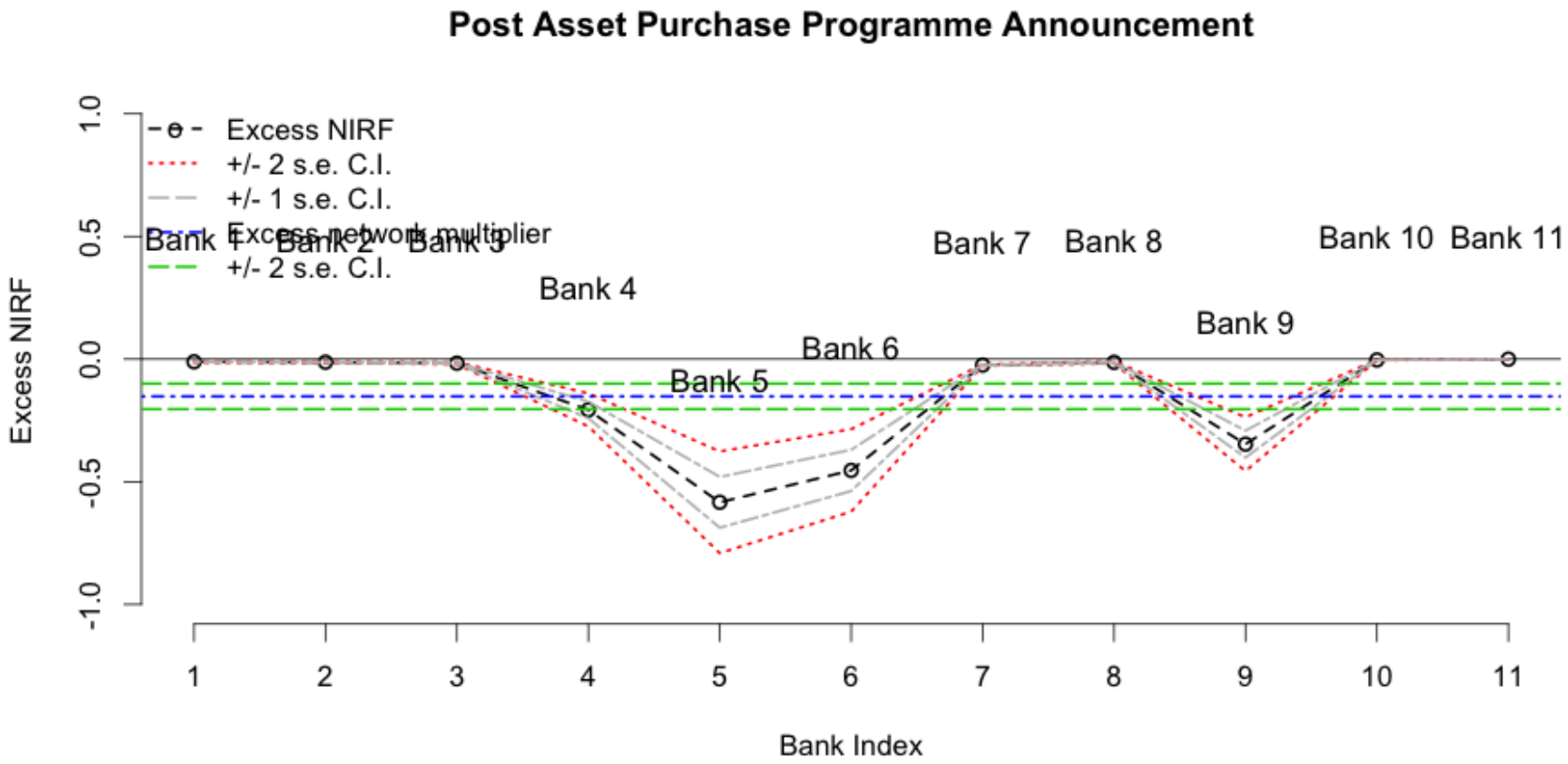
The three subsamples are indexed by $j = 1, 2, 3$, and $\bar{\mathbf{G}}_j$ is the average $\mathbf{G}_t$ in subsample j. The table reports: in first row, $100 \times \left[\left(\frac{Var(Z^p(\hat{\phi}_j, \bar{\mathbf{G}}_j))}{Var(Z^*(\hat{\phi}_j, \bar{\mathbf{G}}_j))} \right)^{\frac{1}{2}} - 1 \right]$; in second row, $\mathbf{1}' \left[\mathbf{M}^p \left(\hat{\phi}_j, \bar{\mathbf{G}}_j \right) - \mathbf{M} \left(\hat{\phi}_j, \bar{\mathbf{G}}_j \right) \right] \hat{\bar{\mu}}_j$.

Period 2: $NIRF^e\left(\phi, \bar{\mathbf{G}}, 1\right)$ – Risk Key Players



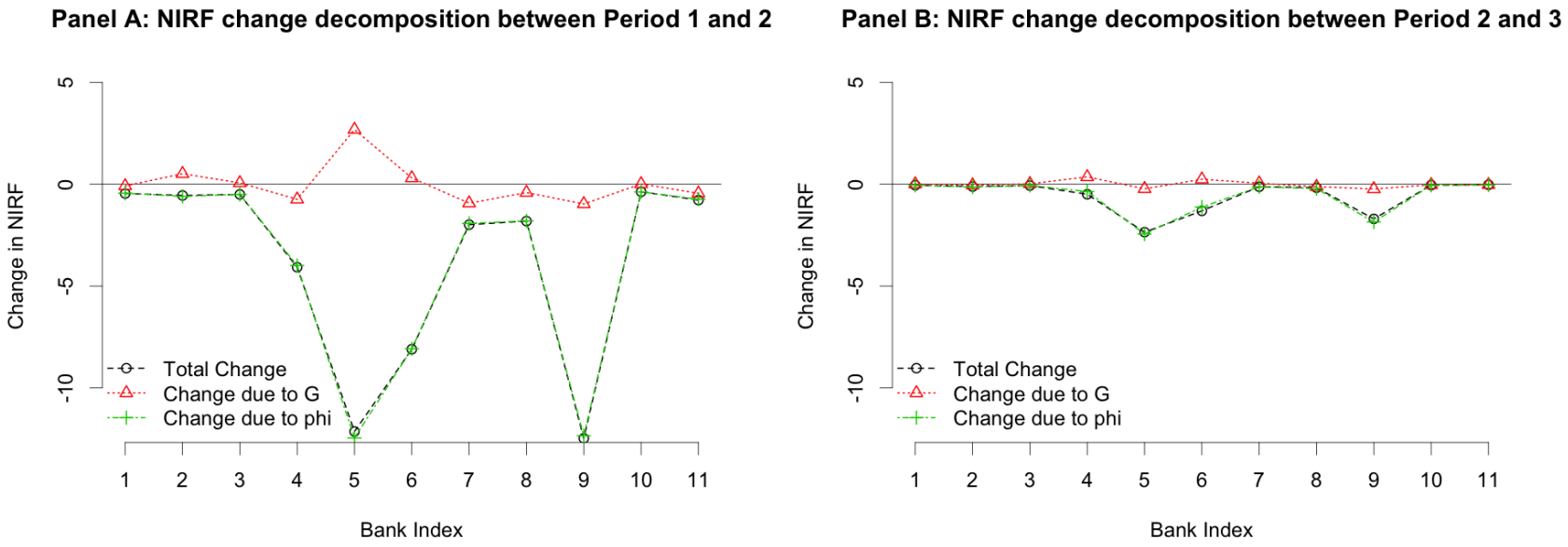
Note: network risk reduction despite increased borrowing & lending

Period 3: $NIRF^e\left(\phi, \bar{\mathbf{G}}, 1\right)$ – Risk Key Players



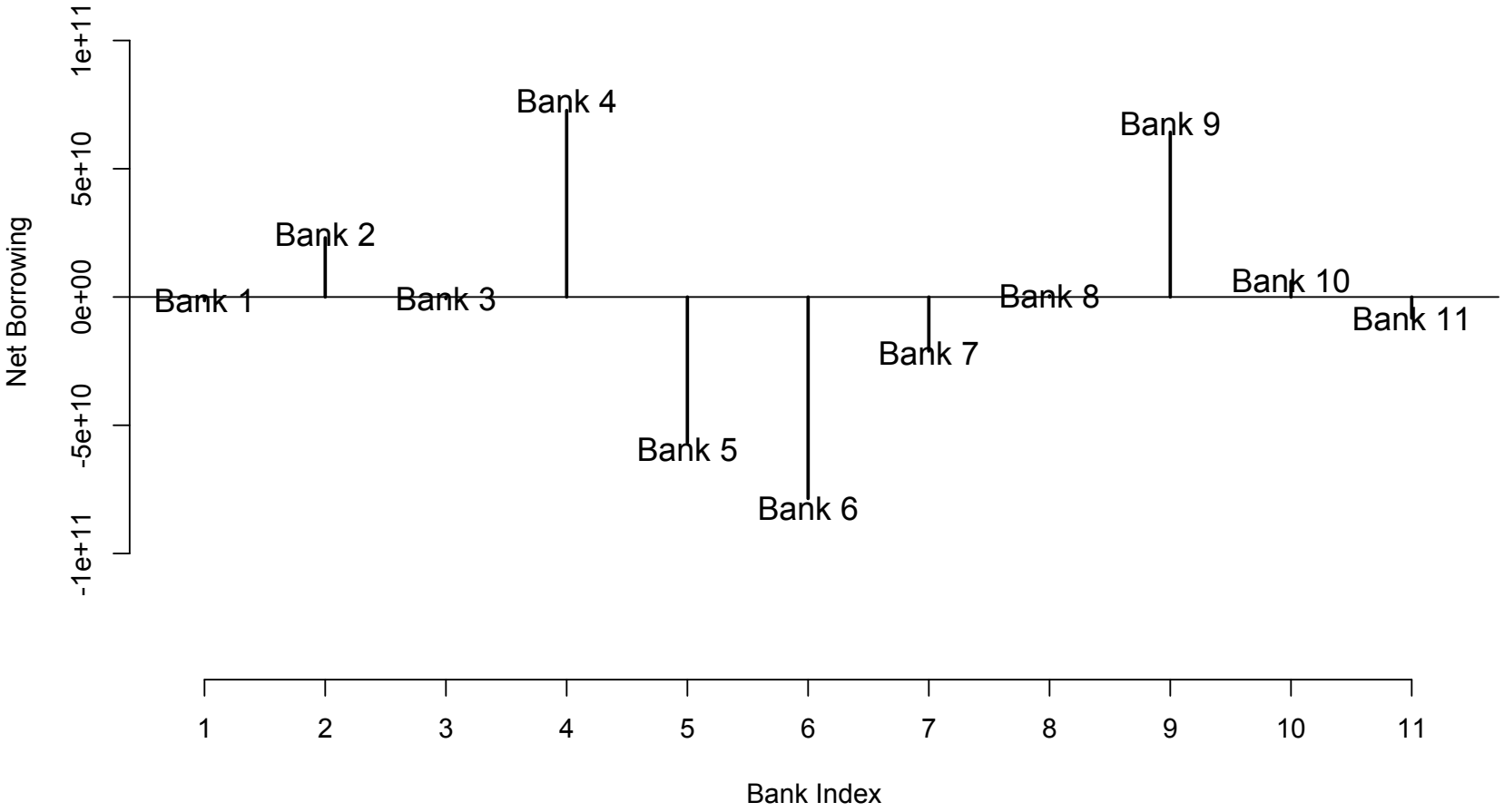
NIRF P1

Network Amplification Mechanism: ϕ and **G**

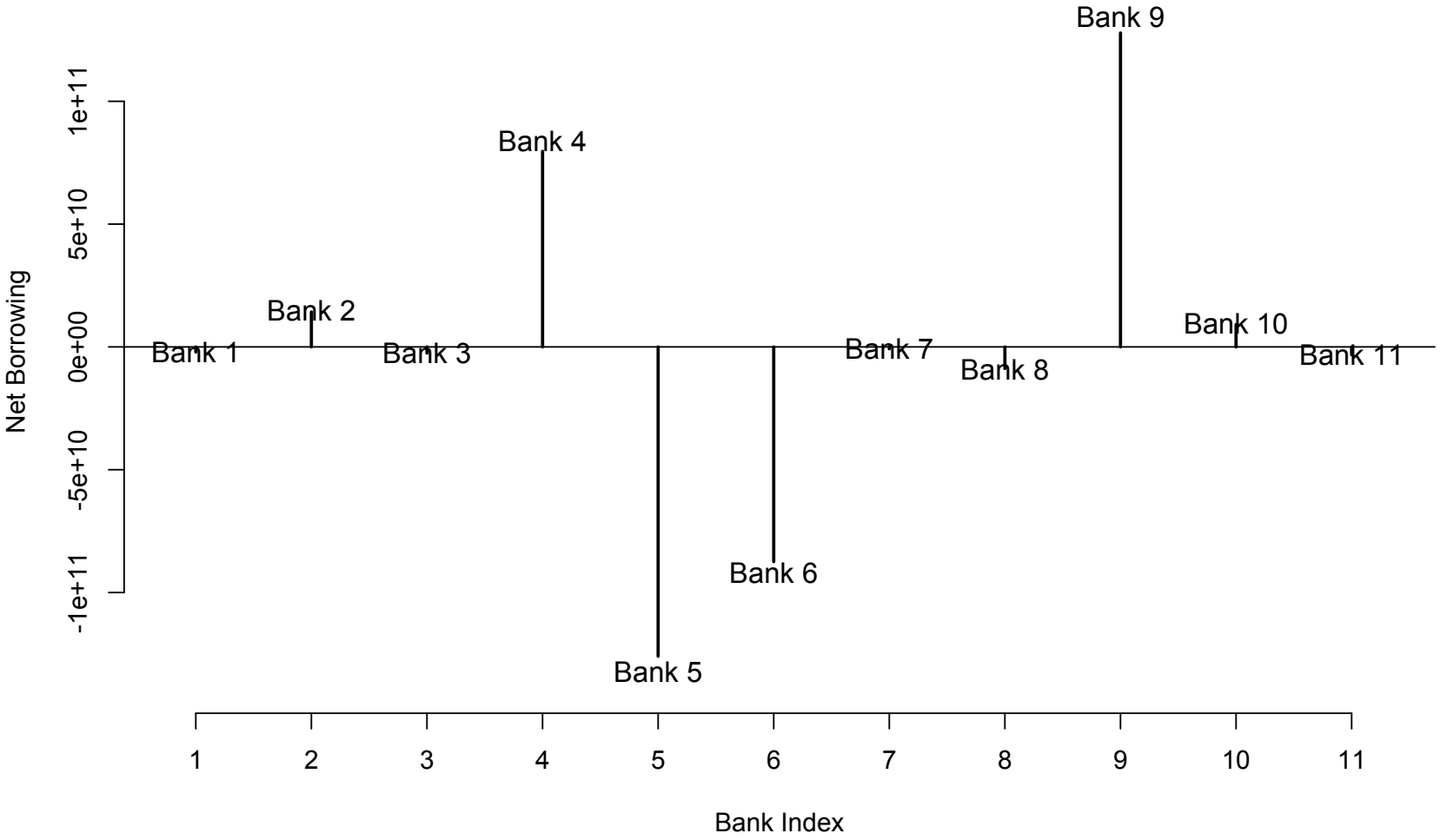


Decomposition of total change in the NIRFs between periods.

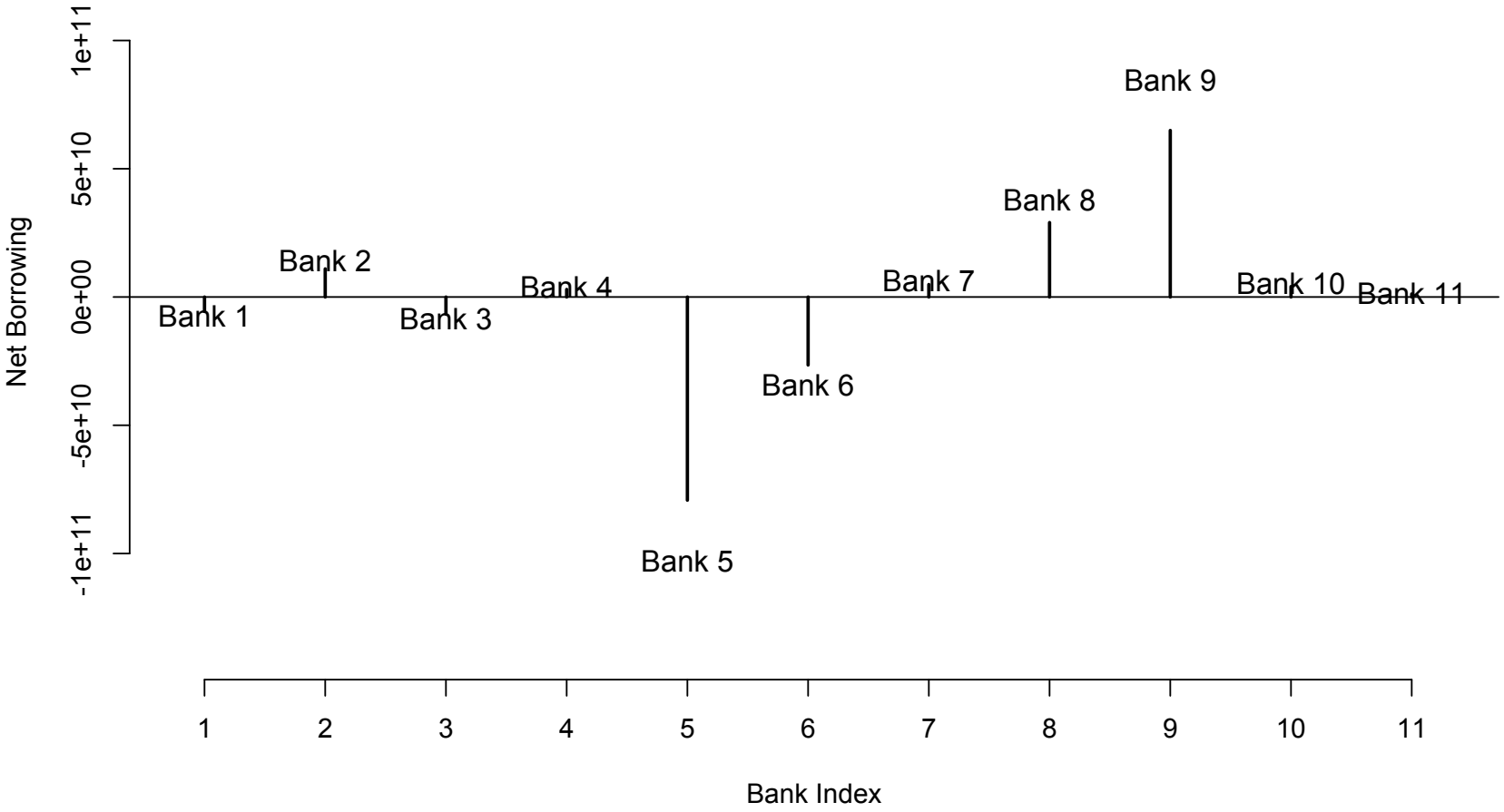
Period 1: Net Borrowing



Period 2: Net Borrowing

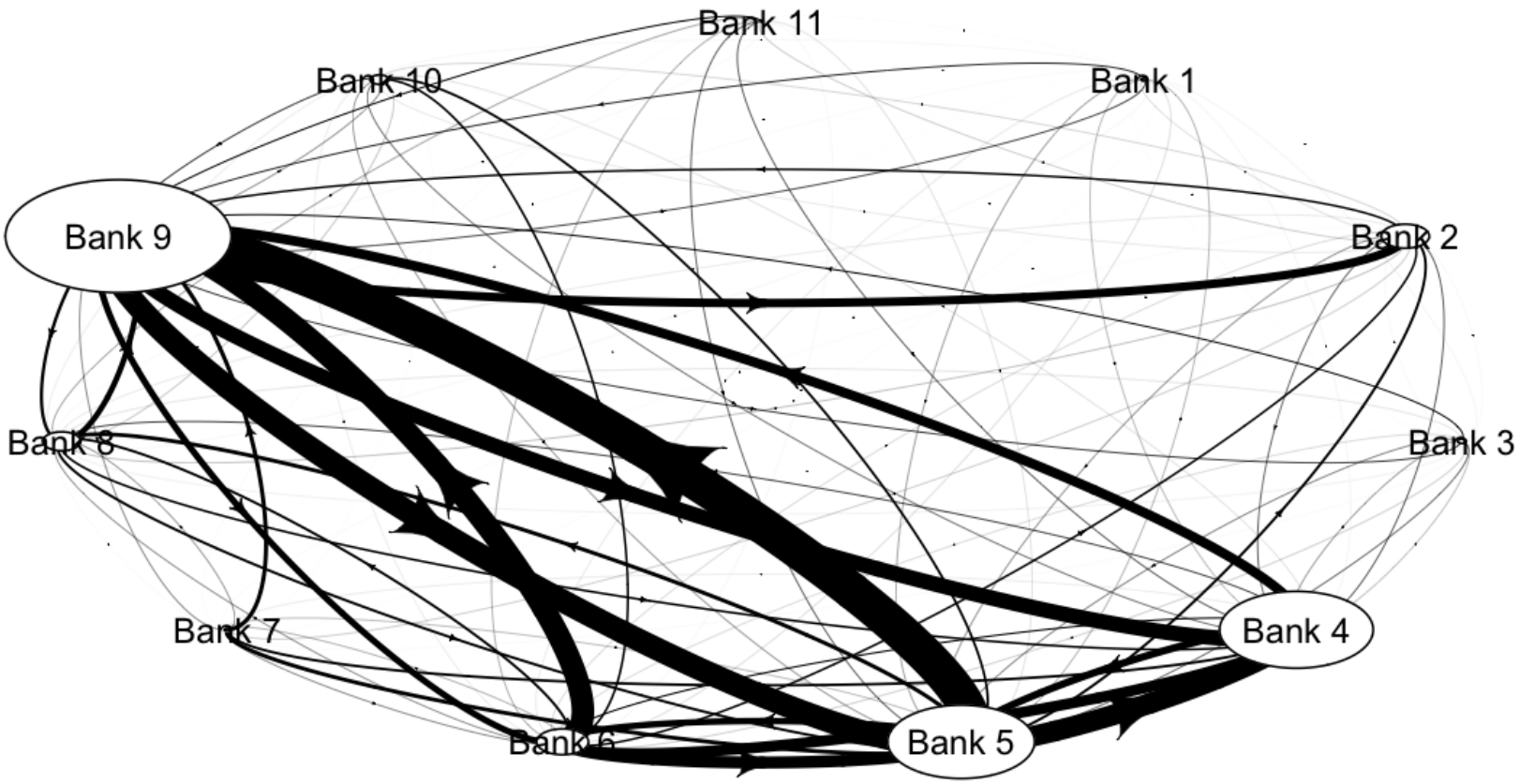


Period 3: Net Borrowing

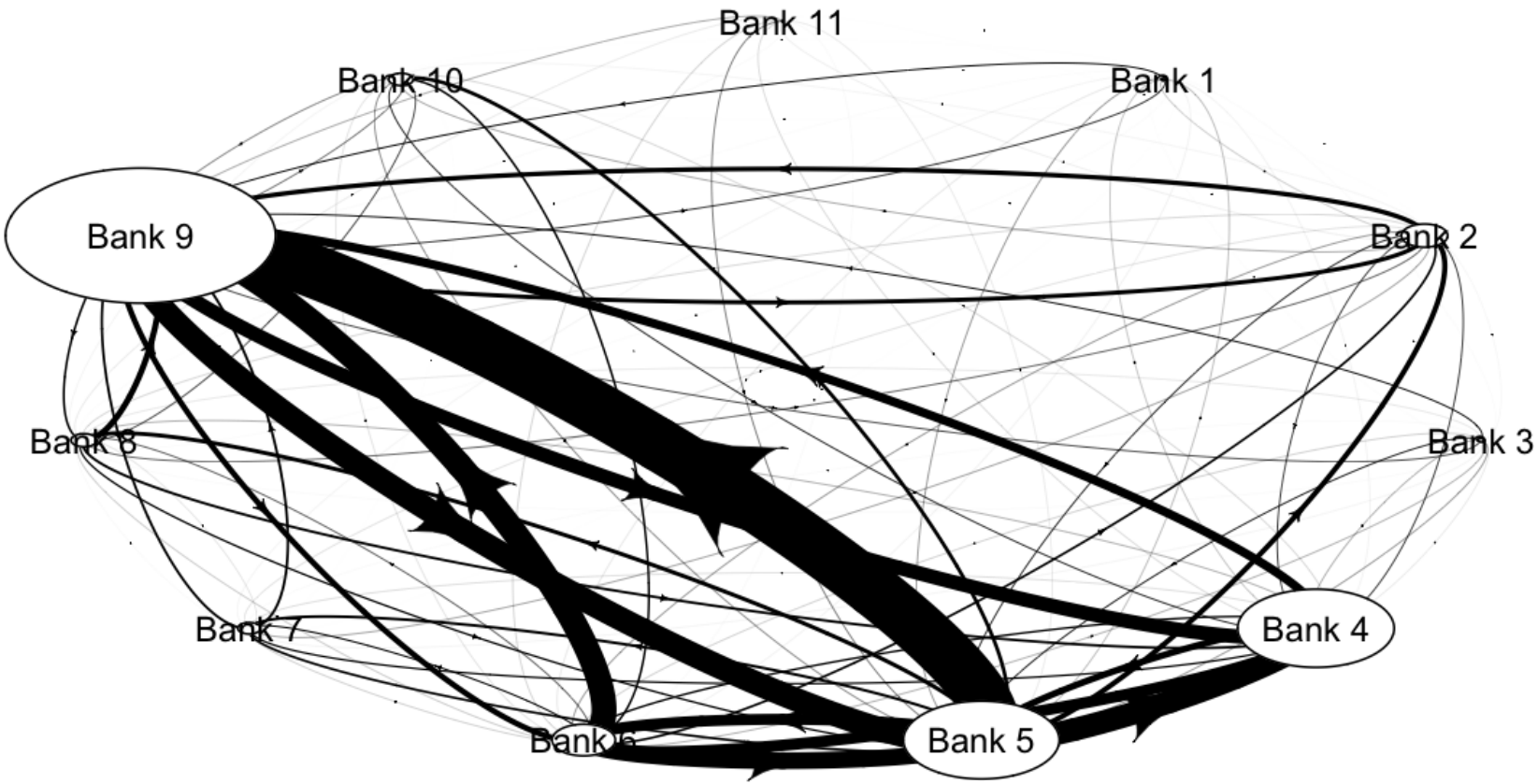


Borrowing P1
Rolling $\hat{\phi}$

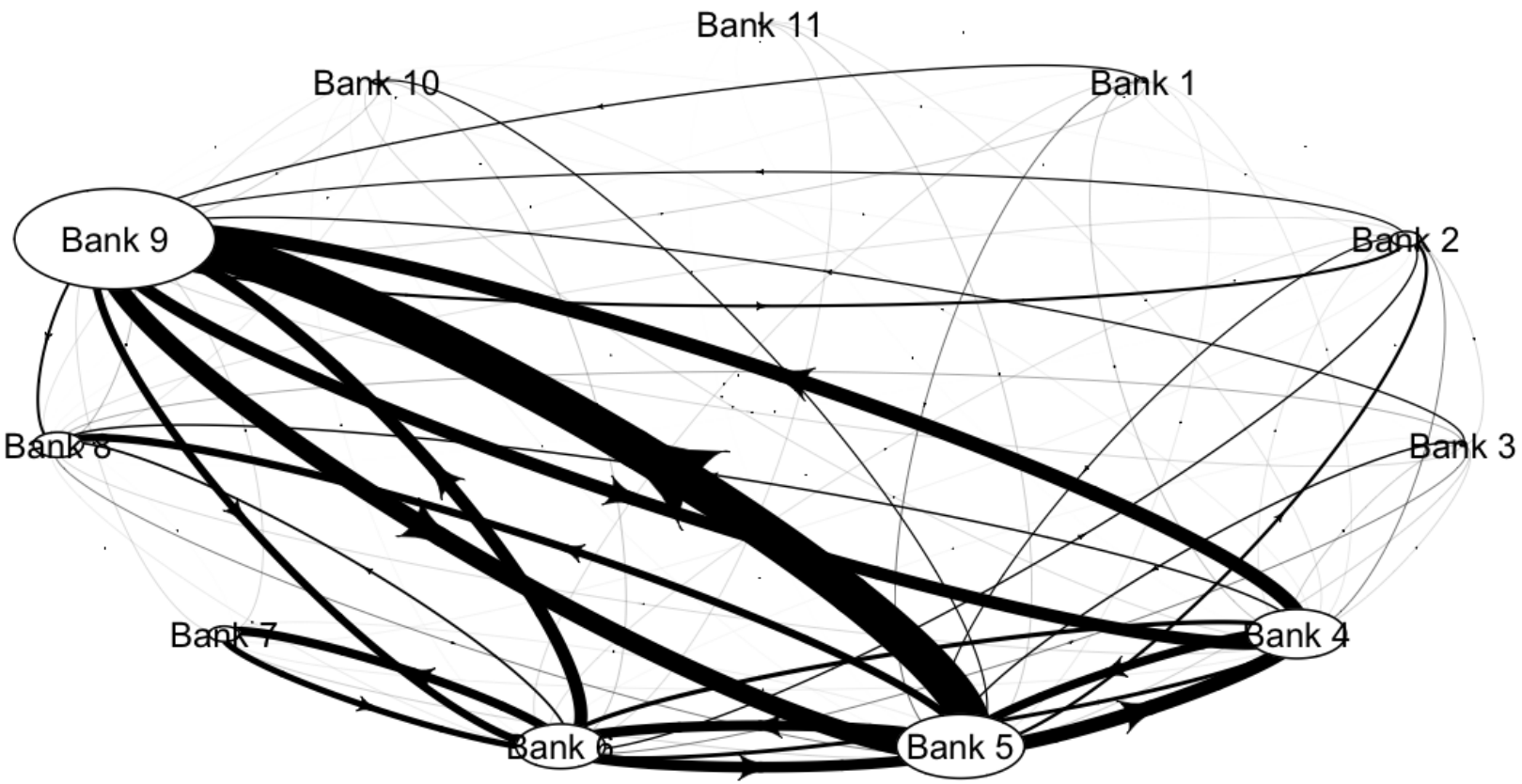
Period 1: Network Borrowing/Lending Flows



Period 2: Network Borrowing/Lending Flows



Period 3: Network Borrowing/Lending Flows



Flows P1