Swing Pricing for Mutual Funds: Breaking the Feedback Loop Between Fire Sales and Fund Runs

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May 15, 2018

Abstract

We develop a model of the feedback between mutual fund outflows and asset illiquidity. Alert investors anticipate the impact on a fund's net asset value (NAV) of other investors' redemptions and exit first at favorable prices. This first-mover advantage may lead to fund failure through a cycle of falling prices and increasing redemptions. Our analysis shows that (i) the first-mover advantage introduces a nonlinear dependence between a market shock and the aggregate impact of redemptions on the fund's NAV; (ii) as a consequence, there is a critical magnitude of the shock beyond which a run brings down the fund; (iii) properly designed *swing pricing* transfers liquidation costs from the fund to redeeming investors and, by removing the nonlinearity stemming from the first-mover advantage, it reduces these costs and prevents fund failure. Achieving these objectives requires a larger swing factor at larger levels of outflows. The swing factor for one fund may also depend on policies followed by other funds.

Key words: mutual funds, first-mover advantage, swing price, fire sales, financial stability
JEL Classification: G01, G23, G28

1 Introduction

The size of the open-end mutual fund industry has increased substantially in recent years. In the United States, the total assets managed by open-end mutual funds grew by \$6.8 trillion over the last decade.[1] In particular, fixed income mutual funds posted significant net inflows: 16.3% of

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[1]See the Flow of Funds Accounts, Z.1 Financial Accounts of the United States, published by the Federal Reserve Board. Compare the table L.122 for March 2006, reporting that the total value of financial assets held by mutual funds in 2005 is \$6.05 trillion, with the table for March 2016, which indicates that the total value of assets held by mutual funds in 2015 is \$12.9 trillion.

outstanding corporate bonds held in the U.S. are owned by mutual funds as of 2017, up from 3.5% in 1990.[2]

Liquidity management by funds has attracted regulators' attention, because of the structural *liquidity mismatch* in open-end mutual funds: funds offer same-day liquidity to their investors, but the assets they hold may not be as easy to sell on short notice, such as in the case of corporate and emerging market bond funds. To meet investor redemptions, a fund may be forced to sell assets at reduced prices, but investors' redeemed shares are paid at the end-of-day net asset value (NAV), which may not account for the total liquidation costs incurred in subsequent days. The liquidity mismatch creates an incentive for investors to redeem their shares early, as they anticipate that the cost of other investors' redemptions will be reflected in the future NAV of the fund.

In extreme stress scenarios, this *first-mover advantage* can trigger a fund run. A prominent example is the junk-bond fund Third Avenue Focused Credit. Impacted by heavy redemptions, from July to December 2015, the fund lost more than half of its market value, falling below $1 billion from an initial value of $2.1 billion. In December, Third Avenue suspended redemptions and began liquidating the fund because it could not meet withdrawal requests by selling shares of its assets at "rational" prices. In its application to the SEC for the approval of the redemption block, Third Avenue wrote:

> If the relief is not granted, and the Fund is unable to suspend redemptions, the institutional investors would likely be best positioned to take advantage of any redemption opportunity, to the detriment of those investors – most likely, retail investors – who remain in the Fund. These remaining investors would suffer a rapidly declining net asset value and an even further diminished liquidity of the Fund's securities portfolio. The relief would help avoid such an outcome.

In October 2016, the Securities and Exchange Commission announced the adoption of amendments to Rule 22c-1 to promote liquidity risk management in the open-end investment company industry. The rule, effective on November 19, 2018, allows open-end funds to use "swing pricing" under certain circumstances. Swing pricing allows a fund to adjust ("swing") its net asset value per share to effectively pass on the costs stemming from shareholder purchase or redemption activity to the shareholders associated with that activity; see Securities and Exchange Commission (2016).

We develop a theoretical framework for the analysis of this rule and its implications for financial stability. Our analysis shows the following. (i) The first-mover advantage magnifies fire sale effects and introduces a crucial nonlinear dependence between the aggregate price impact due to redemptions and an initial market shock. (ii) There is a critical threshold for the market shock beyond which the fire-sale driven amplification leads to the failure of the fund, in the sense that the fund is unable to repay shares of redeeming investors at the promised NAV. (iii) Swing pricing, under an ideal implementation, transfers the cost of liquidation from the fund to the redeeming investors, and – importantly – reduces this cost by removing the nonlinear amplification stemming

[2] See Table L.213, respectively Table L.212, in the Flow of Funds Accounts, Z.1 Financial Accounts of the United States, published by the Federal Reserve Board in September 2017, respectively in September 1996.

from the first-mover advantage. (iv) Swing pricing as currently applied in practice may not achieve these objectives, because funds apply a fixed adjustment instead of an adjustment that increases with the number of investors' redemptions. (v) In an economy with multiple funds which all adopt swing pricing, the NAV adjustment required to remove all cross-fund externalities would be lower than in the case that some funds do not apply swing pricing while others charge a swing price that removes only their own fund's externalities.

Our analysis builds on empirical work exploring the connection between market liquidity, mutual fund performance, and investor flows. Several studies, including Chordia (1996), Ferson and Schadt (1996), Sirri and Tufano (1998), and Warther (1995) have documented relationships between investor flows and fund performance; Edelen (1999) in particular finds that negative abnormal returns in open-end mutual funds can be explained by the liquidation costs induced by investor flows. Other important contributions include Chen et al. (2010) and Goldstein et al. (2017), which study the sensitivity of outflows to underperformance in the context of equity and fixed income funds, respectively. Goldstein et al. (2017) compare the flow-to-performance relation of funds holding liquid assets with that of funds holding illiquid assets. They show that the funds holding illiquid assets are more sensitive to bad performance, because the liquidity mismatch and the externalities imposed by the first redeeming investors on those who remain in the fund create an incentive to exit the fund.

Few other works have explored the theoretical underpinnings of the interactions between asset illiquidity, market stress, and redemption flows. In Chen et al. (2010), the authors present a model to explain why only some investors redeem in response to a fund's bad performance. They attribute this behavior to informational asymmetries: investors receive different signals about the fund's future performance; some investors believe that improved future performance can compensate for the costs of liquidation in the face of an immediate redemption, while others believe the opposite. Chordia (1996) studies the use of load fees to discourage redemptions in a model with redemptions driven by investor liquidity shocks, rather than by fund performance; load fees are fixed and, unlike swing pricing, do not respond to the level of redemptions. Lewrick and Schanz (2017b) develop an equilibrium model which yields the welfare-optimal swing price, and discuss its dependence on trading costs and investors' liquidity needs. In contrast to these models, all building on the foundational work on bank runs by Diamond and Dybvig (1983), in our study a run arises from the withdrawal of forward-looking investors in response to an initial market shock. Zeng (2017) develops a dynamic model of an open-end mutual fund that holds illiquid assets and manages its cash buffer over time. He argues that even if redeeming investors were internalizing the liquidation costs they create, there would still be a negative externality imposed on the fund, which needs to rebuild its cash position at a later date by selling illiquid assets, a costly operation. While the focus of Zeng (2017) is on the cash management policy and its dynamic relation with shareholder redemptions, our focus is on how the feedback between market and liquidity shocks is reinforced through the first-mover advantage and stopped by an appropriate swing pricing rule. Different from Zeng (2017), the redemption mechanism in our study is triggered by an exogenous market shock which not only reduces the value of a fund share, but also exerts downward pressure on the price of the asset, and in extreme scenarios brings the fund down. Morris et al. (2017) study, both theoretically and empirically, how asset managers manage liquidity when they interact with redeeming investors. They analyze the trade-off between cash hoarding and pecking order liquidity management. They find that if the costs of future fire sales are high relative to the liquidity discount that applies to instantaneous liquidation, funds hoard cash and liquidate more assets than necessary to meet current redemptions.

Our paper is also related to the literature studying the asset pricing implications of forced sales by leveraged financial institutions (e.g., banks), which need to comply with prescribed balance sheet requirements (e.g., Adrian and Shin (2010)). The typical mechanism works as follows: After an initial market shock, leverage ratios may deviate from their targets, prompting the institutions to sell illiquid assets to return to their targets. The aggregate impact of asset liquidation on prices is linear in the size of the exogenous market shock (see Capponi and Larsson (2015), Duarte and Eisenbach (2015) and Greenwood et al. (2015)). The mechanism of fire sales triggered by redemptions of mutual funds, however, is different due to their unique institutional structure: because of the first-mover advantage, the value of a fund share and the price of an asset share depend *nonlinearly* on the initial market shock. A larger shock creates a stronger incentive to redeem early, forcing the fund to liquidate superlinearly with respect to the size of the shock. Our model shows that only in an idealized setting without a first-mover advantage (or, equivalently, with an appropriate swing price) is the impact of redemptions on prices linear. These findings imply that treating the mutual fund structure like that of a bank, and ignoring institutional features of the first-mover advantage, would underestimate the effects.[3] The asset pricing implications of investor redemptions may be significant, especially in periods of market stress or if the fund is managing illiquid assets, such as high-yield or emerging market corporate debt.

We build an analytically tractable model that mimics the redemption mechanism identified by the empirical literature on mutual fund flows and use it to explain the effects of the liquidity mismatch in open-end mutual funds. Our model features a continuum of investors with heterogeneous levels of tolerance to the fund's performance: a decrease in the fund's NAV leads to an increasing fraction of investors exiting the fund, consistent with the empirical studies of Chen et al. (2010) and Goldstein et al. (2017).[4] We capture the first-mover advantage by assuming that some investors are sophisticated and anticipate the impact on the fund's NAV of other investors' redemptions. We refer to those investors as first movers. In response to a negative shock to the fund's NAV, investors redeem their shares: the fund may be forced to sell shares of its assets at unfavorable prices, leading to a further drop in the fund's NAV. The first movers anticipate this drop in the NAV and sell before it materializes, thus imposing an even larger externality on the fund (see Figure 2).

[3]Cetorelli et al. (2016a), Cetorelli et al. (2016b) and Fricke and Fricke (2017) quantify the impact of mutual fund fire sales on asset prices, and conclude that the funds' aggregate vulnerability of U.S. open-end mutual funds is small (compared to banks). Their analysis, however, does not account for the first-mover advantage.

[4]This heterogeneity may be caused by different levels of risk aversion, investment horizons, liquidity needs or, as in the theoretical model of Chen et al. (2010), different beliefs on the long-term ability of the fund to recover from an instantaneous shock.

We show that if the initial market shock exceeds a certain critical threshold, the front-running incentive and the number of early redemptions become so large that the fund is unable to repay investors at the nominal NAV, because of the significant price drop of its asset shares. For brevity, we refer to this outcome as a fund *failure*.[5] Our model can thus be adopted as a reverse stress testing tool: after calibrating it to fund flow data, via econometric specifications proposed in the empirical literature (e.g., Goldstein et al. (2017) and Ellul et al. (2011)), we can find the critical shock size that triggers the fund failure for each given level of asset illiquidity. Notably, our model can be used to design stress testing scenarios that explicitly incorporate the risk of a financial run. This in turn enables the positive analysis of regulatory measures targeting fund stability and prevention of fund runs, such as minimum cash requirements and adoption of swing pricing.

We propose a formal definition of swing pricing that captures the adjustment to the end-of-day NAV required to remove the first-mover advantage. While stylized, our definition embodies the salient features of the amended SEC 22c-1 Rule. We show that, to eliminate the first-mover advantage, the swing price should be linear in the size of redemptions, with a slope determined by the illiquidity of the asset. This linear specification makes swing pricing effective even under scenarios of extreme market stress. In fact, swing pricing turns the one-sided first-mover advantage into a trade-off: by redeeming early, investors avoid the costs imposed by their redemptions on the fund's future NAV; on the other hand, a crowding of redemptions results in a larger swing price for redeemers. The major benefit of swing pricing stems from the reduction in the magnitude of early redemptions: by removing the first-mover advantage, a smaller number of investors exit the fund, and the fund is required to sell less of its assets at a discount. Swing pricing results not only in a transfer of the liquidation cost, but also – and more importantly – in a reduction of this cost. In particular, our model shows that swing pricing removes incentives to run that could lead to a fund failure.

Many European mutual funds adopt a flat swing price when redemptions hit a certain threshold (Lewrick and Schanz (2017a)). The empirical results in Lewrick and Schanz (2017a) show that such a swing price is effective in normal times. However, in periods of heavy outflows, like during the 2013 U.S. "taper tantrum", funds appear not to have benefited from the adoption of the swing price rule. These empirical observations are consistent with our theoretical predictions: to be effective in periods of intense market stress, the swing price should be strictly increasing in the amount of redemptions. The empirical studies by Chernenko and Sunderam (2016), Chernenko and Sunderam (2017), and Jiang et al. (2017) discuss more traditional liquidity management policies followed by mutual funds such as cash buffering and cost-effective liquidation strategies.

Our study sheds some light on how open-end mutual funds may pose a threat to financial stability. As argued by Feroli et al. (2014), the absence of leverage is not enough to dismiss potential financial risks: in a downturn scenario, intermediaries that exhibit a procyclical behavior exert an additional adverse pressure on the market. Empirical evidence (Chen et al. (2010)) indicates that

[5] In explaining its liquidity rules, the SEC frequently refers to "reducing the risk that funds would be unable to meet redemption and other legal obligations." See, for example, *Federal Register*, November 18, 2016, vol. 81, no. 223, p.82158 and p.82235.)

when returns are negative, mutual funds tend to liquidate assets, thus magnifying market shocks as opposed to absorbing them. Portfolio commonality exposes funds to similar market risks, and hence large capital outflows often occur simultaneously at several funds. This exacerbates the impact of redemptions on the fund and asset performance (Coval and Stafford (2007), Koch et al. (2016)). In illiquid markets where the presence of the mutual fund industry is prominent (for instance, U.S. corporate bonds), financial distress can escalate and lead to market tantrums, with negative consequences on the real economy. Feroli et al. (2014) discuss a model where funds' fire sales are triggered by relative performance concerns. Our study instead analyzes the fire-sales amplifications driven by the first-mover advantage. In an extension of our baseline model to multiple funds, we show how portfolio commonality and simultaneous redemptions generate cross-fund externalities and exacerbate the price pressure from mutual funds' asset sales. A fund's swing pricing rule should therefore not only account for the externalities imposed on the fund by its own redeeming investors, but also for those imposed by redeeming investors of other funds. Interestingly, we show that if a fund charges a swing price that accounts for the externalities imposed by all funds' first movers and all other funds do the same, then such a price would be lower than in the case where other funds do not adopt swing pricing, even if the swing price charged by the fund were to account only for the externalities imposed by its own first movers. The intuition underlying this phenomenon is that no amplification due to first-mover redemptions occurs when all funds apply swing pricing. If some of these funds were not to apply swing pricing, then their first movers' redemptions would amplify the pressure on prices imposed by first movers of other funds which did apply swing pricing, hence requiring a larger adjustment to the end-of-day NAV.

The rest of the paper is organized as follows. We present the model in Section 2. Section 3 introduces swing pricing and analyzes its preventive role against fund failure. We study how first-mover advantage gets amplified in the presence of multiple funds in Section 4. Section 5 concludes the paper. Additional discussions and proofs of technical results are delegated to the Appendix.

2 The Model

An open-end mutual fund holds and manages Q_0 units of an illiquid asset (each unit of the asset can be thought of as a unit of the portfolio managed by the fund), and it holds an amount C_0 of cash. The market price at time 0 of an asset share is P_0. Investors hold N_0 mutual fund shares, which they may redeem (sell back to the fund) at any time. The value of one fund share at time 0 is $S_0 = \frac{Q_0 \cdot P_0 + C_0}{N_0}$, i.e., the total wealth of the fund, including its total asset value and cash, divided by the number of shares issued by the fund. For example, if Vanguard holds 100 IBM shares and issues 200 fund shares then $Q_0 = 100$ and $N_0 = 200$.

We assume that selling shares of the asset impacts the price by an amount

$$\Delta P = \gamma \Delta Q,$$

where ΔQ is the number of traded shares of the asset, ΔP is the resulting price change, and $\gamma > 0$

is a measure of the asset's illiquidity (when $\gamma = 0$, the asset is perfectly liquid). The parameter γ can also be viewed as the slope of the inverse demand curve.[6] Investors redeem fund shares in response to bad short-term performance of the fund: the number ΔR of redeemed fund shares is assumed to be proportional to the (negative) change in value of a fund share ΔS:

$$\Delta R = -\beta \Delta S,$$

where β represents the sensitivity of alert investors to bad performance.[7] We focus on negative market shocks and take $\beta > 0$.[8]

There are two types of redeeming investors: first movers and second movers. First movers are fast and forward-looking: they observe a negative shock to the market, anticipate other investors' redemptions, compute the overall impact on the value of a fund share, and immediately react by redeeming their shares. The fund pays redeeming investors the cash value of their shares at the end-of-day NAV.[9] Because costly asset liquidations have not yet occurred, the first movers' position is valued at an NAV that does not account for these costs. The forward-looking behavior of some investors provides an explanation for the redemption patterns observed empirically in Chen et al. (2010): first movers anticipate that asset liquidation worsens the fund performance especially if the fund manages illiquid assets, therefore the amount of shares they redeem grows with the illiquidity of the underlying asset (see Section 3.1 and Figure 4). Second movers are slow and react mechanically to observed bad performance of the fund: they redeem gradually, hence the fund can anticipate their actions and liquidate assets simultaneously with their redemptions. First movers would typically be institutional investors, while the behavior of second movers mimics that of most retail investors.[10] We illustrate the timeline of the model in Figure 1.

We assume a continuum of investors, and use $\pi \in (0, 1)$ to denote the proportion of first movers and $1 - \pi$ for the proportion of second movers. At time 0 there is a negative market shock ΔZ, which translates into a shock $\Delta S_0 := \frac{C_0+Q_0(P_0+\Delta Z)}{N_0} - S_0 = \frac{Q_0}{N_0}\Delta Z$ to the value of a fund share. In the remainder of the section, we discuss the mechanism through which an exogenous market shock

[6] The linear dependence of the price impact on the traded volume emerges in equilibrium in the seminal paper of Kyle (1985), and is the predominantly used assumption in the empirical literature on fire sales. Greenwood et al. (2015) and Duarte and Eisenbach (2015) calibrate a linear price impact model of fire-sales spillovers resulting from banks deleveraging; Coval and Stafford (2007) estimate the price impact coefficient using forced purchases and sales of stocks by mutual funds; Ellul et al. (2011) use similar methodologies in other asset markets.

[7] Throughout the paper, we work under the natural conditions $P_0 \geq 0$, $-\Delta Q \leq Q_0$, $\Delta R \leq N_0$ and $-\Delta S \leq S_0$. Violation of these conditions imply the failure of the fund, as defined in Section 3.1.

[8] Our model can also be used to study the effect of positive market shocks and capital inflow. Sensitivity of flows to past performance is not symmetric: it tends to be convex (see, for instance, Ippolito (1992)) for funds specialized in more liquid assets, and concave (see, for instance, Goldstein et al. (2017)) for funds specializing in more illiquid assets. Hence, depending on the sign of ΔS, different sensitivity parameters β can be used.

[9] SEC rule 22c-1 requires an open-end mutual fund to redeem shares based on the next NAV calculated after a redemption request is received, and it requires that the NAV be calculated daily. This calculation is typically done at the end of the day.

[10] In revising rules for money market funds, the SEC wrote that "the first investors to redeem from a stable value money market fund that is experiencing a decline in its NAV benefit from a 'first-mover advantage'," and also comments that "We further believe history shows that, to date, institutional investors have been significantly more likely than retail investors to act on this incentive." See *Federal Register*, August 14, 2014, vol. 79, no. 157, p.47774.

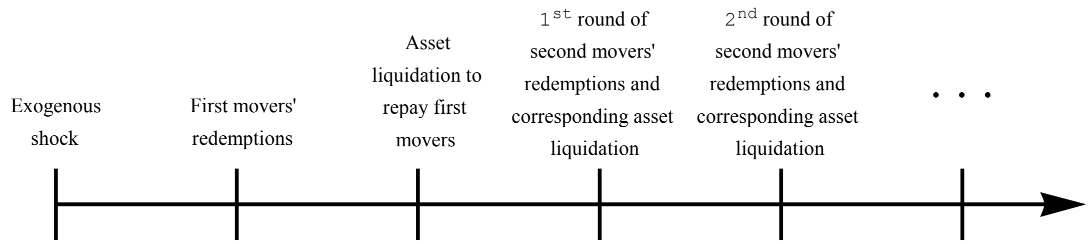

Figure 1: Timeline of the model.

triggers a fund run. We also discuss how the fire-sale amplification driven by redeeming investors of the mutual fund is fundamentally different from that of leverage constrained financial institutions, due to the unique institutional structure of a mutual fund.

Any model of fire sales relies on some friction that constrains or deters arbitrageurs from stepping in to buy when an asset price falls below fundamental value. In our setting, the potential buyers include fund investors who choose not to sell. Our model abstracts from the underlying source of market illiquidity and captures these effects in reduced form through the parameter γ and the actions of the first movers. In other words, γ measures price impact net of any buying by bargain shoppers, and the liquidation by first movers anticipates the extent to which second movers will sell as the share price falls.

2.1 Second Movers

To describe the behavior of second movers, we begin with the case $\pi = 0$, a fund without first movers. While first movers would redeem their shares immediately after the shock, before the fund starts liquidating the asset, redemptions by second movers are slower and happen simultaneously with the liquidation executed by the fund. This captures the behavior of investors who do not exploit the fund's liquidity mismatch. Their redemptions proceed through multiple rounds: each round of redemptions drives down the price and, in turn, triggers a new round. Concretely, in the n-th round, second movers observe a change ΔS_n^{sm} in the value of a fund share and redeem accordingly. To pay back the redeemed shares, the fund liquidates shares of the asset. Costly liquidation affects the value of the fund, causing an additional change ΔS_{n+1}^{sm} in the value of a fund share, and hence further redemptions by second movers.

Throughout the paper, we study the baseline model of a mutual fund which holds a zero cash buffer, i.e., $C_0 = 0$. The inclusion of a cash buffer does not alter the main findings, and is studied in Section 3.5 and Appendix C. We also assume that, initially, the number of shares issued by the fund equals the number of asset shares, i.e., $N_0 = Q_0$. This assumption does not qualitatively impact our conclusions, but leads to more interpretable expressions.

In response to the change in value of a fund share at the n-th round, ΔS_n^{sm}, second movers redeem

$$\Delta R_{n+1}^{sm} = -\beta \Delta S_n^{sm}$$

shares of the fund. Second movers incur the liquidation costs of their redemptions (they do not enjoy the first-mover advantage) and receive the cash amount $\Delta R^{sm}_{n+1} \times (S^{sm}_n + \Delta S^{sm}_n + \Delta S^{sm}_{n+1})$; in practice, this means that each round of redemptions may unfold over a few days, so the liquidation costs are incurred as these investors sell. The number of shares the fund needs to liquidate to repay second movers is

$$\Delta Q^{sm}_{n+1} = -\Delta R^{sm}_{n+1} \frac{S^{sm}_n + \Delta S^{sm}_{n+1}}{P^{sm}_n + \Delta P^{sm}_{n+1}}, \tag{2.1}$$

where $\Delta P^{sm}_{n+1} := \gamma \Delta Q^{sm}_{n+1}$ is the price impact generated by the liquidation of shares needed to repay second movers, and

$$\Delta S^{sm}_{n+1} = \frac{(Q^{sm}_n + \Delta Q^{sm}_{n+1})(P^{sm}_n + \Delta P^{sm}_{n+1})}{N^{sm}_n - \Delta R^{sm}_{n+1}} - S^{sm}_n. \tag{2.2}$$

The change in value ΔS^{sm}_{n+1} of a fund share will trigger a new round of redemptions, yielding $Q^{sm}_{n+1} = Q^{sm}_n + \Delta Q^{sm}_{n+1}$, $P^{sm}_{n+1} = P^{sm}_n + \Delta P^{sm}_{n+1}$, $S^{sm}_{n+1} = S_n + \Delta S^{sm}_{n+1}$, $N^{sm}_{n+1} = N^{sm}_n - \Delta R^{sm}_{n+1}$, where N^{sm}_n is the number of fund shares before the n-th round of second movers' redemptions.

As the number of liquidation rounds increase, the change in price of the asset and of a fund share converges to a fixed point $(\Delta P^{sm}_{tot}, \Delta S^{sm}_{tot})$ which can be explicitly computed.

Proposition 2.1. *Assume $\pi = 0$ and $\gamma\beta < 1$. Given an initial market shock $\Delta Z < 0$, the aggregate impact of the redemptions by second movers on the price of the asset and of a fund share is*

$$\Delta P^{sm}_{tot} = \Delta S^{sm}_{tot} = \frac{\Delta Z}{1 - \gamma\beta}.$$

Notice that there are two levels of ownership: an investor can own the asset either directly or through the fund. Because $N_0 = Q_0$, the two modes of ownership are initially equivalent: a share of the fund has the same value as the price of a share of the asset in the market. Proposition 2.1 states that in the absence of first movers, the market price of the asset and the value of a fund share also coincide at the end of the liquidation process. The execution costs of second movers simultaneously drive the asset price and the value of a fund share, and there is no additional externality imposed on the fund.

The liquidation costs due to second movers' redemptions grow linearly with the exogenous market shock ΔZ, and increase both with the illiquidity of the asset γ and with the sensitivity to the fund's performance β. The liquidation of asset shares in response to a negative market shock is not caused by the institutional structure of the mutual fund, because investors would have sold the asset anyway even if they were holding it directly without the fund's intermediation. For small values of $\gamma > 0$, the change in value of a fund share ΔS^{sm}_{tot} caused by all second movers' redemptions admits the representation

$$\Delta S^{sm}_{tot} \approx \Delta S^{sm}_0 + \gamma\beta \Delta S^{sm}_0 + \gamma^2\beta^2 \Delta S^{sm}_0 + \cdots . \tag{2.3}$$

Each term of the sum reflects a new round of redemptions. Each round has an impact on the value of a fund share, and the final value is the aggregate outcome of the redemption and liquidation process.

2.2 First Movers

Prior to investors' redemptions, the value of a fund share changes to reflect the exogenous shock. First movers observe this change, and additionally foresee the overall impact of other investors' redemptions on the fund performance. Hence, they react not to the initial change ΔS_0, but to the final change in value of a fund share, ΔS_{tot}, that takes into account all liquidation costs due to other investors' redemptions. In fact, first-mover investors who would exit the fund if the change in its NAV were ΔS_{tot} but would remain in the fund if it were ΔS_0, know that they would only receive the cash equivalent of the diluted NAV if they redeemed after other investors' redemptions. Hence, they redeem before the aggregate impact of redemptions on the NAV, ΔS_{tot}, is realized. By doing so, they receive the cash amount $S_0 + \Delta S_0$ per redeemed fund share.

Assume that there are only first movers, i.e. $\pi = 1$. The exogenous market shock ΔZ induces an immediate change $\Delta S_0^{fm} := \Delta Z$ in the value of a fund share. Although all first movers redeem jointly prior to the fund's asset liquidation, some of them react to the initial observed shock, while others redeem anticipating the impact of other investors' redemptions on the fund's NAV. We compute the total number of first movers recursively: at each iteration we include the first movers who redeem anticipating the impact on the fund's NAV of the first movers identified in the previous iteration. First movers reacting to the initial shock redeem $\Delta R_0^{fm} = -\beta \Delta S_0^{fm}$ shares of the fund. The fund has not yet started to liquidate asset shares to repay investors, but is legally obliged to repay the cash equivalent of the NAV of a fund share. Thus, the fund would need to liquidate ΔQ_0^{fm} units of the asset to raise the level of cash needed to repay these investors:

$$-\Delta Q_0^{fm} \times (P_0 + \Delta Z + \gamma \Delta Q_0^{fm}) = \Delta R_0^{fm} \times (S_0 + \Delta S_0^{fm}).$$

The left-hand side is the revenue for the fund after selling the asset and taking into account the execution costs. The right-hand side is the amount of cash that the fund owes to these redeeming investors, and thus it does not account for the execution costs of the fund. Notice that because liquidation is costly, the fund needs to sell more units of the asset to account for these deadweight losses. Since first movers exit at an NAV which has not yet internalized the liquidation costs, these costs need to be absorbed by the fund. The drop in the fund's NAV is

$$\Delta S_1^{fm} = \frac{(Q_0 + \Delta Q_0^{fm}) \times (P_0 + \Delta Z + \gamma \Delta Q_0^{fm})}{N_0 - \Delta R_0^{fm}} - S_0.$$

Because of the price impact and the liquidity mismatch, the change in value of a fund share after these first movers' redemptions ΔS_1^{fm} is larger than the initial change ΔS_0^{fm}. Because first movers are forward-looking, they redeem not only in response to the initial shock, but also anticipating the impact of those redemptions: these two groups of first movers together redeem $\Delta R_1^{fm} = -\beta \Delta S_1^{fm}$ shares. This larger number of redemptions causes an even larger reduction ΔS_2^{fm} in the value

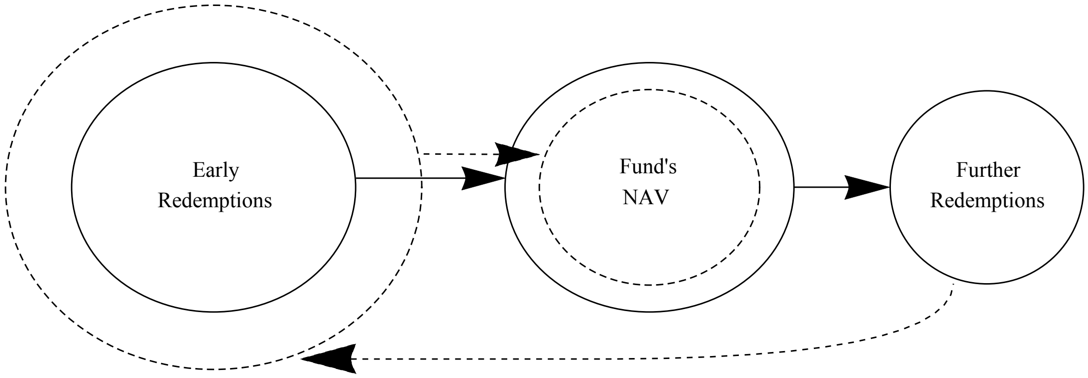

Figure 2: Description of the mechanism that gives rise to the first-mover advantage. The number ΔR_n^{fm} of early redemptions leads to a further drop in the fund's NAV. In response to this drop, other investors would redeem fund shares. These investors are aware that if they were not to redeem immediately, their shares would be valued at a lower NAV. Hence, they join early redeemers and withdraw their investment simultaneously with them, imposing an even larger externality on the fund and the investors that remain in the fund.

of a fund share. Hence, a higher number of investors redeem fund shares. Taken to the limit, this iterative procedure ends at a fixed point (see Figure 2), which is attained precisely when the aggregate change in value of a fund share coincides with the change first movers anticipate. Formally, in the n-th iteration,

$$\begin{aligned}
\Delta R_n^{fm} &= -\beta \Delta S_n^{fm}, \quad \text{[investors redeem]} \\
-\Delta Q_n^{fm} \times (P_0 + \Delta Z + \gamma \Delta Q_n^{fm}) &= \Delta R_n^{fm} \times (S_0 + \Delta S_0^{fm}), \quad \text{[fund sells assets]} \\
\Delta S_{n+1}^{fm} &= \frac{(Q_0 + \Delta Q_n^{fm}) \times (P_0 + \Delta Z + \gamma \Delta Q_n^{fm})}{N_0 - \Delta R_n^{fm}} - S_0. \quad \text{[share value falls]}
\end{aligned}$$

If the sequence ΔS_n^{fm} converges, the limit ΔS_{tot}^{fm} is a fixed point of the system

$$\begin{aligned}
\Delta R_{tot}^{fm} &= -\beta \Delta S_{tot}^{fm}, \\
-\Delta Q_{tot}^{fm} \times (P_0 + \Delta Z + \gamma \Delta Q_{tot}^{fm}) &= \Delta R_{tot}^{fm} \times (S_0 + \Delta S_0^{fm}), \\
\Delta S_{tot}^{fm} &= \frac{(Q_0 + \Delta Q_{tot}^{fm}) \times (P_0 + \Delta Z + \gamma \Delta Q_{tot}^{fm})}{N_0 - \Delta R_{tot}^{fm}} - S_0.
\end{aligned}$$

2.3 With First and Second Movers

We consider now a fund with both first and second movers ($0 < \pi < 1$). First and second movers react to a change in value of a fund share ΔS by redeeming, respectively, $\Delta R^{fm} = -\pi\beta\Delta S$ and $\Delta R^{sm} = -(1-\pi)\beta\Delta S$ fund shares. The actions of first and second movers are intertwined:

(i) second movers are slower and redeem their shares after first movers' withdrawals. Hence, the initial change in value of a fund share they observe depends on the amount redeemed by first movers;

(ii) first movers are forward-looking: they anticipate the liquidation costs due not only to other first movers, but also to second movers.

We illustrate the model timeline in Figure 1. We present the exact mathematical formulation of this mechanism in Appendix A. The aggregate outcome of the liquidation procedure which accounts for both first and second movers' redemptions – if it converges – results in the final asset price change ΔP_{tot} and final fund share value change ΔS_{tot}. We characterize these changes for small γ in Proposition 2.2.

Proposition 2.2. *For small γ, the changes in asset price and value of a fund share after redemptions by first and second movers are*

$$\Delta P_{tot} = \Delta Z + \gamma\beta\Delta Z + \gamma^2 \left(\beta^2 \Delta Z - \beta \frac{\pi^2 \beta^2 \Delta Z^2}{N_0 + \pi\beta\Delta Z} - \frac{\pi^2\beta^2\Delta Z^2}{P_0 + \Delta Z} \frac{N_0 + \beta\Delta Z}{N_0 + \pi\beta\Delta Z} \right) + o(\gamma^2), \tag{2.4}$$

$$\Delta S_{tot} = \Delta Z + \gamma \left(\beta\Delta Z - \frac{\pi^2\beta^2\Delta Z^2}{N_0 + \pi\beta\Delta Z} \right) + o(\gamma). \tag{2.5}$$

To quantify the externalities imposed by the first movers on the fund, compare the expression (2.5) to the asymptotic expansion (2.3), recalling that $\Delta S_0^{sm} = \Delta Z$ in the absence of first movers. As expected, the impact of the liquidation process on the value of a fund share is higher when some investors are first movers, because first movers do not internalize the costs imposed on the fund by their redemptions. As a consequence, a share of the fund will be worth less than a share of the asset after first movers' redemptions. The term $\gamma\frac{\pi^2\beta^2\Delta Z^2}{N_0+\pi\beta\Delta Z}$ is, at the first order, the fraction of the liquidation cost due to first movers' redemptions that needs to be absorbed by each remaining investor in the fund. This may be understood as follows. The numerator $\pi^2\beta^2\Delta Z^2$ captures the cost incurred by the fund when it liquidates shares to repay first movers. In fact, at the first order, first movers redeem $\Delta R_{tot}^{fm} \approx \pi\beta\Delta Z$ fund shares and the fund trades $\Delta Q^{fm} \approx \pi\beta\Delta Z$ shares of the underlying asset to repay first movers. The price per share of the asset is $P^{fm} = P_0 + \Delta Z + \gamma\Delta Q^{fm}$, hence the liquidation cost due to first movers is $\gamma\Delta Q^{fm} \times \Delta Q^{fm} \approx \gamma\pi^2\beta^2\Delta Z^2$. The cost is quadratic in quantities, because price impact per share is linear in quantities. The denominator $N_0 + \pi\beta\Delta Z$ represents the amount of outstanding shares after redemptions by first movers.

Interestingly, the first-mover advantage not only reduces the value of a fund share, but also negatively affects the market price of the asset. However, the asset price in the presence of first movers differs from that in the absence of first movers only at the second order in γ (see equation (2.4)). This is because the first-mover advantage affects the asset price only indirectly, while it directly impacts the value of a fund share: as more investors exit the fund in response to the NAV drop caused by the first-mover advantage, the fund needs to further liquidate asset shares hence exacerbating price impact.

More precisely, there are two forces contributing to price impact. The first is the higher flow of investors' redemptions, including both first and second movers, triggered by the first-mover advantage: because the return of a fund share ΔS_{tot} is lower in the presence of first movers, the number of shares redeemed by first and second mover investors is higher, triggering more fire sales by the fund, and leading to a worse market price for the asset, as captured by the term $\beta \frac{\pi^2 \beta^2 \Delta Z^2}{N_0 + \pi \beta \Delta Z}$. The second force is the increased amount of asset sales required to meet investors' redemptions: to repay first movers, the fund needs to liquidate an additional number $\gamma \frac{\pi^2 \beta^2 \Delta Z^2}{P_0 + \Delta Z}$ of asset shares, on top of the number of redeemed shares ΔR^{fm}_{tot}, to cover the liquidation costs ($\Delta Q^{fm} \approx \Delta R^{fm}_{tot} + \gamma \frac{\pi^2 \beta^2 \Delta Z^2}{P_0 + \Delta Z}$). This yields a second order effect on market prices. A proportion of this cost is borne by second movers, so it is normalized by $\frac{N_0 + \beta \Delta Z}{N_0 + \pi \beta \Delta Z}$, which is the number of shares held by the remaining investors in the fund over the number of shares held by remaining investors and second-mover redeemers.

2.4 Redemption Outflows Versus Bank Deleveraging

Existing literature has analyzed price linkages arising when financial institutions manage their balance sheets to comply with prescribed leverage requirements. Greenwood et al. (2015) show that the amplification effects on prices arising when banks liquidate assets to target their leverage are linear in the exogenous shock, if one takes into account only the first round of deleveraging. Capponi and Larsson (2015) confirm this linear dependence even if one accounts for higher order effects caused by repeated rounds of deleveraging needed to restore banks' leverages to their targets. The banks' deleveraging mechanism is essentially equivalent to the redemption mechanism of the fund in the absence of first movers: each round of deleveraging has an impact on the price, and leads to a successive round of asset liquidation because it depresses prices. In the absence of first movers ($\pi = 0$), the aggregate impact of redemptions on prices is still linear (see Proposition 2.1). The iterative redemption procedure executed by second movers converges to a fixed point if $\gamma\beta < 1$.[11]

The presence of first movers introduces an important structural difference between the fire sale mechanism imposed by leverage targeting and that triggered by mutual fund redemptions. After accounting for the first-mover advantage, Proposition 2.2 shows that the aggregate impact of liquidation on asset prices is no longer linear in the exogenous shock. This point is illustrated in Figure 3, which compares the total price drop $-\Delta P_{tot}$ resulting from a shock ΔZ with (solid) and without (dashed) first movers, for two different liquidity regimes. Additionally, in the presence of first movers, the condition required for the convergence of this procedure takes a more complex form and depends crucially on the size of the initial shock.

[11] Such a condition is equivalent to assuming that the matrix in equation (4) in Greenwood et al. (2015) (or the systemicness matrix defined in Equation (2.2) of Capponi and Larsson (2015)) has spectral radius smaller than 1. In economic terms, this means that a round of deleveraging causes another round of deleveraging that is smaller than the previous one. In particular, the condition for the convergence of this liquidation procedure is independent of the initial market shock ΔZ.

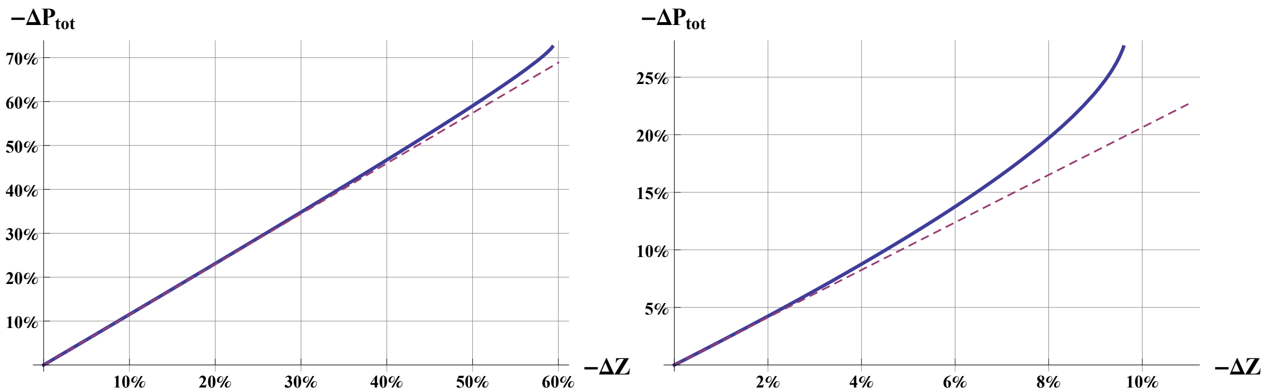

Figure 3: The graphs show the aggregate impact on market prices of an initial market shock, for $\pi = 75\%$ (solid line) and $\pi = 0$ (dashed line). The asset illiquidity parameter γ is 0.5×10^{-8} (left panel) and 2×10^{-8} (right panel). These values of γ correspond to typical liquidity levels for corporate bonds, as reported in the empirical study by Ellul et al. (2011). The flow-performance relation $\frac{\beta}{N_0}$ is chosen to be 0.859, consistently with the estimates in Goldstein et al. (2017). In the presence of first movers, the impact on prices grows superlinearly with the size of the shock, if the asset is illiquid.

3 Fund Failure, Swing Pricing, and Stress Testing

The incentive to redeem early increases with the illiquidity of the asset managed by the fund. If the fund's asset is too illiquid, the first-mover advantage may induce enough early redemptions to bring down the fund. Swing pricing is intended to stop the transfer of liquidation costs from first movers to investors remaining in the fund. In this section, we provide a formal definition of the swing price which achieves this objective. We also construct a stress testing scenario to demonstrate how swing pricing can prevent the fund failure.

3.1 Redemption Flow and Fund Failure

Investors redeem shares in response to bad performance by the fund. If the asset managed by the fund is illiquid, then the feedback between fund performance, redemption flow, and asset liquidation increases the incentive to redeem early. Hence, for a given initial shock, a higher illiquidity of the asset triggers a larger redemption outflow (see Figure 4). This prediction of our model is consistent with the flow-to-performance relation in the mutual funds industry identified by empirical literature (Chen et al. (2010), Goldstein et al. (2017)).

In the absence of first movers, the redemption procedure converges if $\gamma\beta < 1$, which ensures that each subsequent round of redemption is smaller than the previous one. If first movers are present, convergence is not guaranteed even if $\gamma\beta < 1$, and it strongly depends on the size ΔZ of the initial shock. The procedure fails to converge if a shock of large size forces the failure of the fund.[12]

[12] The fund may decide to suspend redemptions if it foresees that they would be insufficient to repay exiting investors. This happened in the case of Third Avenue Focused Credit, a junk-bond fund which experienced heavy

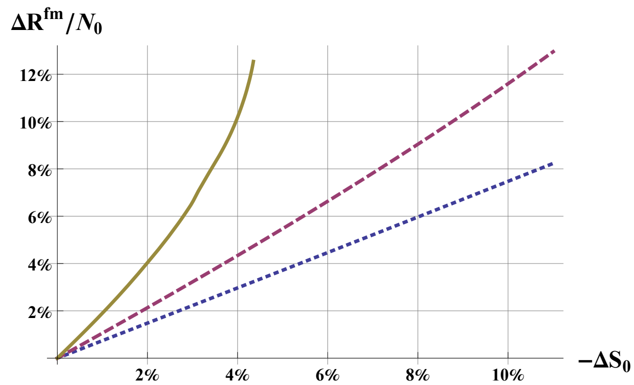

Figure 4: The graph shows the outflow due to first movers in response to an exogenous shock on the fund's NAV. The flow-to-performance relation depends on the liquidity of the asset held by the fund: the asset illiquidity parameter γ is 0.5×10^{-8} (dotted line), 1.5×10^{-8} (dashed line), and 2.5×10^{-8} (solid line).

Proposition 3.1. *Assume $\pi > 0$ and a negative shock $\Delta Z < 0$. There exists a critical value $\Delta Z^* < 0$ such that the iterative redemption procedure converges if and only if $|\Delta Z| \leq |\Delta Z^*|$. Furthermore, $|\Delta Z^*|$ decreases with the illiquidity parameter γ of the asset.*

If $\pi > 0$ and the exogenous shock ΔZ is sufficiently large, the number of investors that redeem early is so high that the fund becomes unable to repay them. This can be intuitively understood as follows. For each additional fund share redeemed by first movers, the marginal cost of liquidation is increasing while the marginal proceeds of first movers stay constant. The fund may eventually run short of asset shares or obtain negligible marginal revenue from asset sales.

Figure 5 plots the relation between the critical value ΔZ^* and the asset illiquidity parameter γ. We set $\frac{\beta}{N_0} = 0.859$ in all numerical examples, consistently with the estimates provided in the empirical study by Goldstein et al. (2017). If the asset is perfectly liquid ($\gamma = 0$), there is no first-mover advantage. As γ increases, so does the risk of a fund run. In particular, if the illiquidity of the asset is larger, the critical threshold on the shock size that leads to a fund run, and consequently to the fund's failure, is smaller (in absolute value).

Figure 6 illustrates how the iterative liquidation procedure that yields the total change in value of a fund share fails to converge if the asset is not sufficiently liquid.[13]

Despite the temporary nature of price impact, the fund may still be unable to meet investors' redemptions before prices revert to fundamentals. Even if the fund survives the run, its NAV never recovers completely. We refer to Appendix B for a detailed discussion on temporary and permanent NAV losses.

redemptions in the period from July to December, 2015.

[13] We have assumed a fixed γ for tractability. If a fund were to sell its more liquid assets first, γ would increase as the fund sold more assets, further amplifying the effects in the figures.

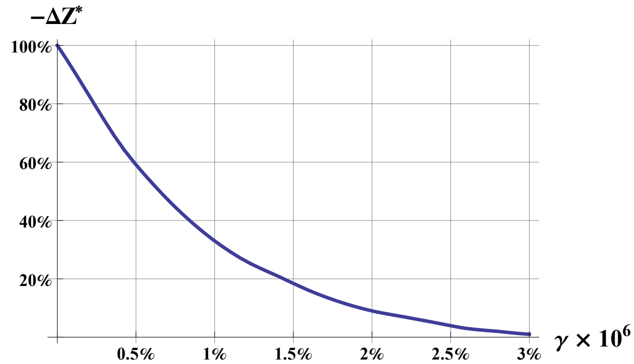

Figure 5: The graph shows the critical level ΔZ^* as a function of the illiquidity parameter γ. The horizontal axis reports the price impact per \$1 million. The proportion π of first movers is 75%.

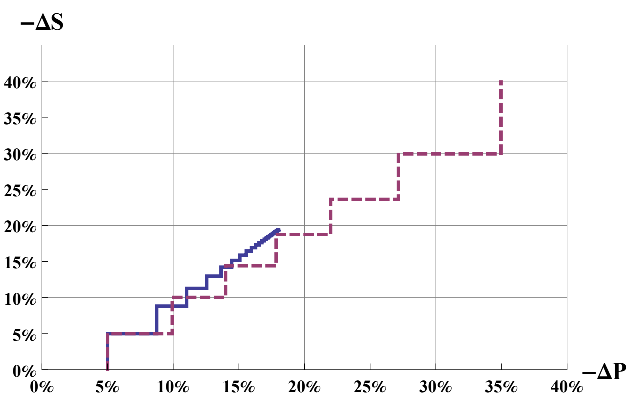

Figure 6: The graph shows the iterative liquidation procedure that yields the aggregate changes in fund share value and asset price for $\gamma = 2.4 \times 10^{-8}$ (solid line), and for $\gamma = 3 \times 10^{-8}$ (dashed line). The liquidation procedure ends at a fixed point if $\gamma = 2.4 \times 10^{-8}$, while the fund becomes unable to repay its redeeming investors if $\gamma = 3 \times 10^{-8}$. The proportion π of first movers is 75%.

3.2 Swing Pricing

Let ΔS^{sw} be the adjustment applied to the value of a fund share when first movers are paid: the fund makes a (negative) adjustment to the fundamental value of a fund share $S_0 + \Delta Z$ and it pays back $S_0 + \Delta Z + \Delta S^{sw}$ for each share redeemed by first movers.

Definition 3.2. *Let $\Delta S_{tot}^{\pi=0}$ be the aggregate change in value of a fund share in the absence of first movers (that is, with $\pi = 0$). For $\pi > 0$, assume that the fund pays first movers a cash amount equal to $S_0 + \Delta Z + \Delta S^{sw}$ for each redeemed share. The adjustment ΔS^{sw} is a swing price if the resulting aggregate change in value of a fund share ΔS_{tot} is equal to $\Delta S_{tot}^{\pi=0}$.*

Swing pricing is thus the adjustment to the value of a fund share that makes the first movers internalize all externalities imposed on the fund. Recall that in the absence of first movers, the value change of a fund share is

$$\Delta S_{tot} = \frac{\Delta Z}{1 - \beta\gamma}.$$

Proposition 3.3. *The swing price, as specified in Definition 3.2, is uniquely given by*

$$\Delta S^{sw} = \gamma \frac{\pi\beta\Delta Z}{1 - \beta\gamma}.$$

Viewed as a function of the number of redemptions from first movers, the swing price takes the form

$$\Delta S^{sw} = -\gamma \Delta R_{tot}^{fm}. \tag{3.1}$$

Notice that the use of swing pricing not only removes the first mover's advantage but also removes the adverse effect that first movers have on the price of an asset share: when swing prices are used, ΔP_{tot} coincides with the price change in the absence of first movers. Hence, not only does swing pricing benefit the fund, it also mitigates the negative impact on the price of an asset share caused by first movers' redemptions.[14]

The swing price is high if there is a large redemption outflow or a strong amplification of the exogenous shock. To see this, notice that the quantity $\pi\beta\Delta Z$ is the number of fund shares withdrawn by first movers during their first round of redemptions. Hence, $\gamma\pi\beta\Delta Z$ is the price impact from the first round of these redemptions. The term $\frac{1}{1-\beta\gamma}$ quantifies how the exogenous shock is amplified: a change ΔS in the value of a fund share triggers investors' redemptions, which in turn causes a further drop in value and leads to additional redemptions. The aggregate outcome of this process is a change in value $\frac{\Delta S}{1-\beta\gamma}$, which coincides with the change in value observed in a fund consisting only of second movers (see the result in Proposition 2.1).

Eq. (3.1) indicates that the swing price can be expressed in terms of the number of first-mover redemptions.[15] This is an endogenous, but approximately observable, quantity. To compute the

[14]Stale prices for assets held by a mutual fund distort the fund's NAV to the benefit of certain investors, as in Zitzewitz (2006). Swing pricing can be seen as correcting soon-to-be-stale prices.

[15]The swing price is proportional to the number of redeemed shares because of the linearity of the inverse demand function. In general, the swing pricing formula would depend on the specification of the inverse demand function.

swing price, the fund does not need to know the sensitivity β to bad fund performance. The fund only needs to estimate the level of illiquidity γ of its asset and observe the quantity of redemptions from first movers – the redemptions that occur before the fund starts liquidating shares of the asset, as indicated in the timeline in Figure 1. In practice, it may be difficult to distinguish first-mover redemptions from other transactions, but the fund would know the quantity of redemptions from institutional and retail accounts and could compare an account's transactions with its past activity. Using the aggregate flow of redemptions, both from first and second movers, our swing pricing rule would yield a more conservative adjustment.

If the fund applies swing pricing, the externalities imposed by first movers are no longer borne by the remaining investors, but instead internalized by first movers. Crucially, first movers who would have redeemed in anticipation of the diluted NAV caused by other redemptions no longer do so in the presence of swing pricing. Thus, by removing the benefits of front-running, swing pricing reduces the total liquidation costs.

If γ is large, the initial amount ΔR_0^{fm} of shares redeemed in response to the market shock may account only for a small fraction of the total first movers' redemptions. In other words, the liquidation cost eliminated by swing pricing may be much larger than the cost that is merely transferred from one set of investors to another.

3.3 Swing Pricing Practices

Starting in November 2018, the amendments to Rule 22c-1 by the Securities and Exchange Commission allow U.S.-based mutual funds to adopt swing pricing. Swing pricing is already used in other jurisdictions, particularly Luxembourg. The vast majority of funds adopt a swing pricing rule defined by a redemption threshold and a pre-determined swing factor: when net redemptions exceed the threshold, the fund applies a fixed percentage adjustment to its NAV. Such an adjustment differs from the swing pricing formula in Proposition 3.3. Therefore, it does not remove the first-mover advantage and cannot guarantee prevention of a fund run and failure; see Figure 7. According to the survey by Association of the Luxembourg Fund Industry (2015), some asset managers already apply or are considering applying multiple swing factors, depending on the level of redemptions. Our study supports such an implementation of swing pricing. Our analysis identifies two important features that yield an effective swing price:

1) The adjustment should take into account the dependence of the asset price on traded quantities. As the liquidation cost per traded share increases with the number of liquidated shares, the swing price should also increase with the flow of redemptions. A fixed swing price may have limited efficacy during periods of heavy outflows.

2) Investors should be informed about a fund's swing pricing mechanism. Liquidation costs are reduced, and not just transferred, only when investors understand that the first-mover advantage has been eliminated. In practice, the disclosed information should not allow any investor to use the implemented swing pricing mechanism to their advantage: funds rarely

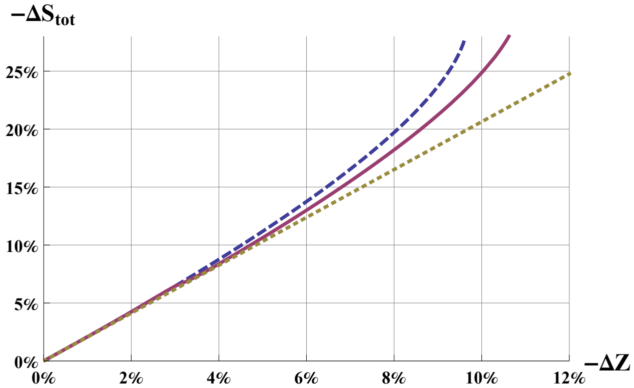

Figure 7: The graph shows the change in value of a fund share with the swing price specified in Proposition 3.3 (dotted line), without swing price (dashed line), and with a fixed NAV adjustment applied when more than 5% of investors exit the fund (solid line). The proportion π of first movers is 75%.

disclose their swing thresholds, and they do not report the specific days on which the NAV was swung.[16]

A few asset managers have expressed concerns that swing pricing may increase the volatility of a fund's NAV; see Securities and Exchange Commission (2016), Section III-C. Our analysis shows that an effective swing price alleviates fire sales and prevents NAV dilution, hence mitigating large fluctuations in the fund's NAV, particularly in periods of market stress. For additional information on swing pricing practices, we refer to Malik and Lindner (2017), Investment Company Institute (2016), and Association of the Luxembourg Fund Industry (2015).

3.4 A Stress Testing Example

We illustrate how a calibrated version of our model can be used for stress testing. We quantify the first-mover advantage for both high and low liquidity regimes, and we compute the threshold on the shock size beyond which redemptions would lead to fund failure.

We calibrate the model parameters using empirical estimates from the literature on fund flows and abnormal returns due to fire sales for corporate bond funds. We normalize the initial price of the asset and the value of a fund share to \$1, so $P_0 = S_0 = \$1$. Goldstein et al. (2017) estimate the flow-performance relation for corporate bond mutual funds: in the case of negative fund performance, the value of $\frac{\beta}{N_0}$ is approximately 0.859. This relation is asymmetric in the fund's performance: if the fund performance is positive, the corresponding value is 0.238.

To estimate the illiquidity parameter γ we follow Ellul et al. (2011), who analyze the impact of fire sales in the corporate bond market. To estimate deviations of prices from (unobservable) fundamentals, the authors analyze the temporary drop of bond prices after a downgrade and their

[16]In 2013, the Autorité des marchés financiers allowed the use of swing pricing in France, explicitly requiring the fund to not disclose details that would let investors place strategic orders, taking advantage of the swing pricing mechanism.

rebound to the fundamental value. The price impact per \$1 million is on the order of 1% (ranging from 0.4% to 1.9% in different years and with different sets of controls). We consider two illiquidity regimes for the asset: a regime of typical liquidity with price impact of 1% per \$1 million, and a regime of high illiquidity with price impact of 2.5% per \$1 million. We assume that the fund holds \$30 million in the asset, and apply a market shock that reduces the current asset price by 5%, so $\frac{\Delta Z}{P_0} = -5\%$.

By the *endogenous shock*, we mean $\Delta S^{\pi}_{tot} - \Delta Z$, which is the difference between the total change in value of a fund share after all redemptions and the initial shock ΔZ. As before, π refers to the fraction of first movers. Table 1 decomposes the endogenous shock into contributions from the first, second, and third round of second mover redemptions, in the absence of first movers ($\pi = 0$). Recall that the second movers respond to and then contribute to a sequence of price declines. Their cumulative impact generates endogenous shocks of 1.74% and 9.05%, for price impact parameters of 1×10^{-8} and 2.5×10^{-8}, respectively. Without first movers, the change in value of a fund share and of the asset are identical.

3.4.1 Impact of first-mover advantage and fund run

Figure 8 highlights the additional impact on the value of a fund share triggered by first movers' redemptions. When the price impact parameter γ and the proportion of first movers π are both large, the recursive procedure that determines the number of shares redeemed by first movers does not converge, and the value of a fund share collapses. Recall that the cash obtained from selling shares of the asset is a quadratic function of the number of shares sold. When the fund sells a large quantity of the asset and the price impact is high, the marginal revenue from the sale might become negative: by selling an additional share, the fund may experience a lower revenue compared to not doing so, because of the significant drop in the share price caused by this additional sale. Figure 8 illustrates a situation in which the fund fails to retrieve the cash amount required to repay the first movers.

3.4.2 Swing pricing prevents fund runs

The adoption of swing pricing prevents a fund run. To see this, compare Figures 8 and 9. When the price impact parameter is 1×10^{-8}, the impact of first movers (when all redeeming investors are first movers) is roughly 0.08% and the swing price is below 1.80%. Intuitively, if 5% of the investors are redeeming their shares and the fund decides to apply swing pricing, the externalities generated by first movers – previously imposed on the whole fund – are now internalized by the first movers. This means that the swing price should roughly be $0.08\% \times \frac{1}{5\%} = 1.60\%$, which is not too far from the exact swing price displayed in Figure 9. This argument does not apply if the price impact is higher and equal to 2.5×10^{-8}. Under these circumstances, the externalities imposed by first movers on the fund are 50 times larger. The swing price, however, is only five times higher.

When price impact is small, the recursive procedure that determines the number of shares redeemed by first movers converges very quickly. If price impact is large, the convergence is slow

Price Impact	Endogenous Shock	First Round	Second Round	Third Round
$1{\times}10^{-8}$	-1.74%	-1.29%	-0.33%	-0.09%
$2.5{\times}10^{-8}$	-9.05%	-3.22%	-2.08%	-1.34%

Table 1: Endogenous shock $\Delta S_{tot} - \Delta Z$ caused by fire sales when there are no first movers, and contributions of successive rounds of redemptions to this shock.

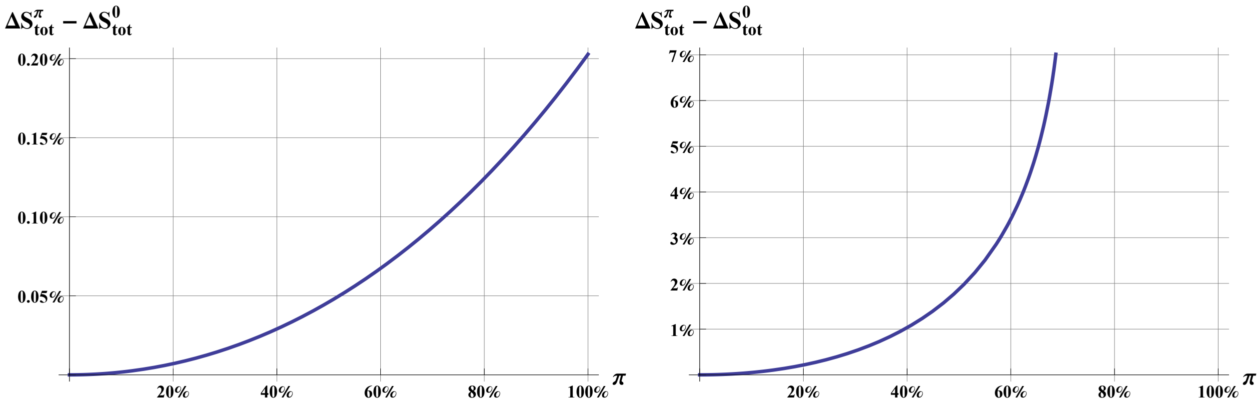

Figure 8: The graphs show the impact of first movers' redemptions on the value of a fund share, i.e. $\Delta S^{\pi}_{tot} - \Delta S^{0}_{tot}$. We set the price impact parameter $\gamma = 1 \times 10^{-8}$ (left panel), and $\gamma = 2.5 \times 10^{-8}$ (right panel). For the larger value of γ, the impact diverges if $\pi \geq 70\%$.

(or the procedure may not even converge). The swing price removes the first-mover advantage in that it stops the recursive procedure after the first round: if the first movers that react to the shock $\Delta S_0^{fm} := \Delta Z$ have to pay the swing price, there is no liquidation cost transferred to the fund, hence no reason why first movers who would react to subsequent drops in NAV in the absence of swing pricing should redeem their shares early. While swing pricing may not have a strong effect when the fund holds liquid assets, it acts as a stabilizing force if the fund holds illiquid assets. The swing price adjustment is relatively small compared to the enormous costs of a fund run triggered by first movers.

3.5 Swing Pricing in the Presence of a Cash Buffer

Holding a cash buffer allows open-end mutual funds to meet redemptions without the immediate need of costly asset liquidation. However, the fund does not necessarily avoid asset sales completely. This means that the presence of a cash buffer does not eliminate the first-mover advantage. The fund may still be susceptible to a run under stressed market conditions, and asset liquidation is still necessary if the value of redeemed fund shares exceeds the amount of cash held by the fund. Even if cash buffers and swing pricing are both tools to mitigate the downward pressure on prices, they have different economic roles. If the fund manages liquidity through a cash buffer, the externalities from redemptions are not internalized by the redeeming investors, but imposed on the remaining investors in the fund.

We discuss how the model from Section 2 generalizes to the case that the fund holds an amount

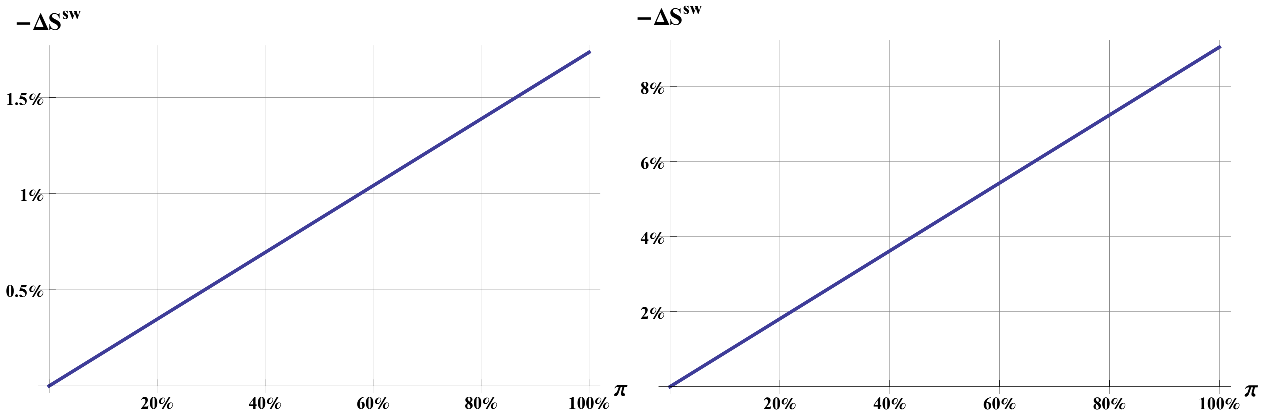

Figure 9: The swing price as a function of π. We set the price impact parameter $\gamma = 1 \times 10^{-8}$ (left panel), and $\gamma = 2.5 \times 10^{-8}$ (right panel).

C_0 of cash resources, in addition to shares of the illiquid asset. Assume that the fund uses cash first to pay redeeming investors. Once the cash resources are exhausted, the fund sells shares of the illiquid asset to raise the level of cash needed to meet the remaining redemptions. Depending on the amount of redemptions, the cash buffer C_0 can be used to cover both first and second mover redemptions, only first mover redemptions, or neither of those. For a given initial market shock ΔZ, there exist levels of cash C^* and C_* such that one of the following situations happens.

(i) $C_0 > C^*$. The fund holds enough cash to repay all redeeming investors.

(ii) $C^* > C_0 > C_*$. The fund holds enough cash to repay first movers, but shares of the asset need to be sold to repay second movers.

(iii) $C_0 < C_*$. The fund needs to liquidate asset shares to repay first movers.

In case (i), no asset liquidation occurs and hence the price change reflects fundamentals: $\Delta P_{tot} = \Delta Z$. In case (ii), the fund sells shares of the asset only to repay second movers, hence there is no liquidation cost passed from first movers to other investors. In case (iii), the first-mover advantage arises. Its impact on the price of the asset and of a fund share remain qualitatively the same (see Appendix C for details). Next, we describe how the swing price changes in the presence of a cash buffer. In the following, x^+ denotes the positive part of x.

Proposition 3.4. *Assume $Q_0 = N_0$ and define $L := \frac{S_0+\Delta Z}{P_0+\Delta Z}$, a conversion factor between the value of a fund share and the price of the asset. In the presence of a cash buffer C_0, the swing price is*

$$\Delta S^{sw} = -\gamma L^2 \left(\Delta R^{fm}_{tot} - \frac{C_0}{S_0 + \Delta Z} \right)^+,$$

where $\Delta R^{fm}_{tot} = -\pi\beta\Delta S_{tot}$, and ΔS_{tot} is the change in value of a fund share after accounting for all redemptions.

The presence of a cash buffer introduces a threshold on redemptions beyond which the swing price is charged to first movers. Additionally, the total quantity of redemptions is lower because the presence of a cash buffer mitigates the self-reinforcing feedback mechanism between share redemptions and asset liquidation (compare the expression of ΔS_{tot} in Proposition C.1 with that in Proposition 2.1).

4 Systemic Amplification of the First-Mover Advantage

A fund's liquidity mismatch not only negatively affects its own non-redeeming investors, but also other funds holding the same asset. Early redemptions by first movers of a fund increase the incentive of other funds' investors to redeem early, hence driving down the price of the asset. The resulting cross-fund negative externalities magnify the negative pressure imposed on the price of an asset share. Section 4.1 studies swing pricing in an economy with multiple funds. Section 4.2 analyzes the benefits resulting from the simultaneous application of swing pricing by all funds.

4.1 First-Mover Advantage with Common Asset Ownership

We consider two funds that hold the same illiquid asset; the asset may be thought of as representative of their entire portfolios. Let β_1 and β_2 denote the sensitivity to bad performance of investors in fund 1 and 2, respectively. We use π_1 and π_2 to denote the fractions of first movers in fund 1 and 2, respectively. Consistent with previous sections, we make the assumption that the initial number of asset shares equals the initial number of fund shares for each fund: $Q_{0,i} = N_{0,i}$ for $i = 1, 2$. The simplified setting of two funds with common asset ownership allows us to highlight the amplification channel of fire-sale externalities across funds.

For $i = 1, 2$, let $\Delta S_{tot,i}$ be the aggregate change in NAV of fund i caused by all redemptions, both of fund 1 and 2. The redemptions by first movers of a fund exacerbate liquidation losses of the other fund, which simultaneously experiences redemptions of its own first movers in response to the same negative market shock of the asset. The total impact of these redemptions on the value of a share of fund 1 is (omitting terms of higher order in γ)

$$\Delta S_{tot,1} \approx \Delta Z + \underbrace{\gamma \left(\beta_1 \Delta Z - \frac{(\beta_1 \pi_1 \Delta Z)^2}{N_{0,1} + \beta_1 \pi_1 \Delta Z} \right)}_{\text{Own Impact}} + \underbrace{\gamma \left(\beta_2 \Delta Z - \frac{(\beta_2 \pi_2 \Delta Z)(\beta_1 \pi_1 \Delta Z)}{N_{0,1} + \beta_1 \pi_1 \Delta Z} \right)}_{\text{Other Fund's Impact}}.$$

In addition to the price impact due to an individual fund, as in equation (2.4), there is a cross-fund price impact which imposes additional negative pressure on the asset price:

$$\Delta P_{tot} \approx \Delta Z + (\text{Impact from Fund 1}) + (\text{Impact from Fund 2}) + (\text{Cross-impact}),$$

where the analytical expressions of the above price impact terms are given in Remark D.4.

If investors of multiple funds holding overlapping asset portfolios redeem fund shares simultaneously, the feedback between fund performance, outflow and asset liquidation is reinforced. Additionally, the first movers of each fund anticipate the other fund's outflow and redeem a higher number of shares compared to the case when each fund liquidates in isolation. This cross-fund liquidity effect increases the downward pressure imposed on the price. To eliminate the first-mover advantage, each fund needs to consider the impact of the other fund. If both funds implement swing pricing, the adjustment is[17]

$$\Delta S^{sw}_{both} = -\gamma(\Delta R^{fm}_{tot,1} + \Delta R^{fm}_{tot,2}). \tag{4.1}$$

Hence, the swing price charged by two funds with common asset ownership is higher than the swing price in the case of a single fund; compare equations (3.1) and (4.1).

4.2 The Benefits of Cooperative Swing Pricing

Swing pricing may not be adopted uniformly across the mutual fund industry. A fund implementing swing pricing may apply an adjustment that only neutralizes the execution costs imposed on the fund by the redemptions of its own first movers, or it may apply an adjustment that also anticipates the effect of other funds' first movers. In the former case, the fund is still impacted by a run at another mutual fund.

If only one fund were to adopt swing pricing, the NAV adjustment it would need to offset the impact of first movers at all funds would be larger than the adjustment required if the fund operated in isolation. If both funds adopt swing pricing, the NAV adjustment required to remove all first-mover externalities would be smaller than in the case that one fund does not apply swing pricing while the other does; in fact, it is even smaller than the adjustment required for one fund to remove its own first movers' externalities (see Figure 10).

To make these statements precise, suppose only fund 2 adopts swing pricing. Let ΔS^{sw}_{loc} be the NAV adjustment that makes fund 2's first movers internalize their liquidation costs. This is the swing price leading to the same change in NAV as if $\pi_1 > 0$ and $\pi_2 = 0$. Let ΔS^{sw}_{glob} be the swing price for fund 2 that offsets the effect of first movers at both funds, leading to the same change in NAV as if $\pi_1 = 0, \pi_2 = 0$. (See Appendix D for mathematical details.) We now have the following result.

Proposition 4.1. *Suppose $\pi_1, \pi_2 > 0$, and suppose that only fund 2 applies swing pricing. For small γ,*

$$|\Delta S^{sw}_{both}| \leq |\Delta S^{sw}_{loc}| \leq |\Delta S^{sw}_{glob}|.$$

The intuition underlying this result is as follows: The externalities imposed on a fund by its first movers are amplified by other funds' first movers. Hence, if only a single fund applies swing pricing, the NAV adjustment required to eliminate these externalities needs to account for the fire-sale amplification driven by other funds' first movers. On the other hand, if each fund were to adopt swing pricing, cross-fund amplification due to first movers' redemptions would be eliminated.

[17] see Proposition D.5

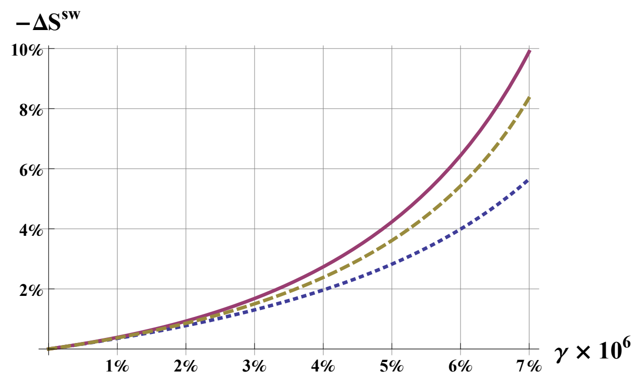

Figure 10: Comparison of the swing prices ΔS^{sw}_{both} (dotted line), ΔS^{sw}_{loc} (dashed line) and ΔS^{sw}_{glob} (solid line) for different levels of illiquidity γ. The horizontal axis reports the price impact per \$1 million. The proportion of first movers in each fund is $\pi_1 = \pi_2 = 75\%$.

A mutual fund that does not adopt swing pricing still benefits from the implementation of swing pricing by other funds, because of the reduced selling pressure imposed on it by the other funds' first movers. The presence of mutual funds that do not implement swing pricing imposes a cost on the first movers of funds that do adopt swing pricing, because their exit NAV is smaller than in the case that all funds cooperate in the adoption of swing pricing.

5 Concluding Remarks

Our study models and quantifies the externalities stemming from the liquidity mismatch in open-end mutual funds. By analyzing the interactions between fund performance, net outflows, and asset liquidity, we provide a unified framework that delivers several predictions:

- The first-mover advantage amplifies the effects of fire sales and introduces a nonlinear relation between the aggregate price impact and the magnitude of the exogenous shock.
- The first-mover advantage may trigger a cascade of redemptions following bad fund performance, leading to asset sales that further drive down prices and generate further redemptions, potentially to a point where the fund may be unable to repay redeeming investors, and thus fails.
- Our definition of swing pricing neutralizes the first-mover advantage. It does so by transferring the costs of liquidation to redeeming investors. Importantly, it also reduces these costs by eliminating the incentive for investors to redeem earlier. At large levels of redemptions, the required swing adjustment is larger than the fixed adjustment seen in practice.

The major policy implication of our study is the provision of an ideal yet simple swing pricing rule, which is based on roughly observable quantities. Funds only need to account for the net outflows of first movers and estimate the illiquidity of the asset to decide how much to adjust their

NAV. We have assumed a single level of liquidity for all of a fund's holdings. Extrapolating to more general cases, our proposed adjustment suggests a need to partition a fund's portfolio into liquidity buckets. The current SEC 22e-4 Rule requires funds to divide their assets into buckets based on time for liquidation, but our analysis points to the importance of distinguishing by liquidation costs as well, because a fund may be forced to sell assets quickly to meet redemptions.

The amendments to the SEC 22c-1 Rule on swing pricing impose a 2% cap, relative to the fund's NAV, on the swing factor. Such a constraint may limit the efficacy of swing pricing in periods of severe market illiquidity. To prevent an overly aggressive use of swing pricing, other forms of regulatory oversight such as an appropriate disclosure clause on the adopted swing pricing mechanism should be considered.

Our analysis shows that greater benefits are attained if swing pricing is consistently applied by all mutual funds. Under these circumstances, the externalities imposed on the funds are internalized by their first movers at a lower cost, compared to the case when some funds apply swing pricing but others do not. The discretionary adoption of swing pricing is likely to affect the distribution of investor flows. For example, funds that do not implement swing pricing may appeal to alert investors that can exit the fund at zero cost, but be less attractive for inattentive investors who would prefer to be safeguarded against a fund run and therefore lean towards funds with swing pricing. Modeling this behavior may lead to a separation between institutional investors (often first movers) concentrated at funds that do not adopt swing pricing, and retail investors (typically second movers) participating in funds that adopt swing pricing.

Because of portfolio commonality, mutual funds act as a channel of contagion across assets, and conversely common assets are a channel of contagion across funds. After a shock to an asset's price, a fund that is required to repay its redeeming investors may also liquidate other assets in its portfolio, thus creating an endogenous selling pressure on assets that are not directly impacted by the initial market shock. Hence, even mutual funds that do not hold assets affected by the initial market shock may be impacted, if their portfolio is composed of other assets that were liquidated in response to that shock. This indirect mechanism of contagion due to overlapping portfolios is not specific to mutual funds, but common across intermediaries constrained by regulatory or contractual obligations. We leave the construction of such a richer framework for future research.

A The Mechanics with First and Second Movers' Redemptions

If the recursive redemption procedure followed by first movers converges, the total amount of fund shares they redeem is

$$\Delta R_{tot}^{fm} = -\pi\beta\Delta S_{tot}, \tag{A.1}$$

where ΔS_{tot} is the aggregate change in fund share value, which includes the shock ΔZ and accounts for the externalities imposed on the fund both by first and second movers' redemptions. The number of shares ΔQ_{tot}^{fm} the fund needs to trade to repay first movers, and the change ΔS_{tot}^{fm} in the value of a fund share after first movers' redemptions are given by the following equations:

$$
\begin{aligned}
-\Delta Q_{tot}^{fm} \times (P_0 + \Delta Z + \gamma \Delta Q_{tot}^{fm}) &= \Delta R_{tot}^{fm} \times (S_0 + \Delta S_0^{fm}), \\
\Delta S_{tot}^{fm} &= \frac{(Q_0 + \Delta Q_{tot}^{fm}) \times (P_0 + \Delta Z + \gamma \Delta Q_{tot}^{fm})}{N_0 - \Delta R_{tot}^{fm}} - S_0.
\end{aligned} \tag{A.2}
$$

Second movers start their recursive withdrawal procedure after the fund has met first movers' redemptions. Hence, $\Delta S_0^{sm} = \Delta S_{tot}^{fm}$, i.e., the initial change in value of a fund share observed by second movers is equal to ΔS_{tot}^{fm}, which includes the shock ΔZ and accounts for the externalities imposed by the first movers on the second movers. These externalities reflect the aggregate impact on prices generated by the liquidation process by first movers. In the limit (if it exists), the recursive procedure followed by second movers converges to the solution to the system of equations

$$
\begin{aligned}
\Delta R_{tot}^{sm} &= -(1-\pi)\beta \Delta S_{tot}, \\
\Delta Q_{tot}^{sm} &= -\Delta R_{tot}^{sm} \frac{S_0 + \Delta S_{tot}}{P_0 + \Delta P_{tot}}, \\
\Delta P_{tot} &= \Delta Z + \gamma(\Delta Q_{tot}^{fm} + \Delta Q_{tot}^{sm}), \\
\Delta S_{tot} &= \frac{(Q_0 + \Delta Q_{tot}^{fm} + \Delta Q_{tot}^{sm})(P_0 + \Delta P_{tot})}{N_0 - \Delta R_{tot}^{fm} - \Delta R_{tot}^{sm}} - S_0.
\end{aligned} \tag{A.3}
$$

B Swing Pricing Removes Permanent NAV Losses From Temporary Asset Losses

We think of the fundamental drop in asset price from P_0 to $P_0 + \Delta Z$ as permanent. Further drops in the asset price due to forced selling are temporary. We consider a loss in the fund's share price temporary if it is recovered once the asset price returns to $P_0 + \Delta Z$. Otherwise, the loss in the fund's NAV is permanent: the fund does not recover the loss when the fire-sale effect is undone.

The following proposition decomposes ΔS_{tot} into a temporary and a permanent component. We show that the adoption of swing pricing reduces the permanent component so that prices only reflect changes due to fundamentals, i.e. it is only driven by the initial shock ΔZ. The permanent change in value of a fund share is

$$
\Delta S^p := \frac{(Q_0 + \Delta Q_{tot}^{fm} + \Delta Q_{tot}^{sm})(P_0 + \Delta Z)}{N_0 - \Delta R_{tot}^{fm} - \Delta R_{tot}^{sm}} - S_0,
$$

where the asset shares held by the fund are valued at the fundamental price $P_0 + \Delta Z$, instead of the fire sale price $P_0 + \Delta P_{tot}$.

Proposition B.1. *The following statements hold:*

- *If $\pi = 0$ or the fund adopts swing pricing, then $\Delta S^p = \Delta Z$.*

- *If $\pi > 0$, then at first order in γ*[18]

$$\Delta S^p = \Delta Z - \gamma \frac{\pi^2 \beta^2 \Delta Z^2}{N_0 + \pi\beta\Delta Z} + o(\gamma).$$

The temporary price impact can be devastating if the fund does not survive the run. If the initial market shock equals the critical threshold ΔZ^*, the fund survives the run and the temporary component of the price impact may dominate over the permanent component. However, a *phase transition* would occur if $|\Delta Z| > |\Delta Z^*|$. Under this condition, the fund is unable to repay its first movers.

It is worth noticing that second movers only indirectly, i.e., through the forward looking behavior of first movers, affect the permanent component of the NAV loss. If the asset price has already (partly) recovered before second movers redeem, the first movers would redeem less. As a result, the temporary price impact would be lower. Moreover, the fund would need to sell a smaller quantity of asset shares to repay first movers at par, and thus the permanent NAV loss would also be lower. Hence, early price recovery partly mitigates the feedback effect in our model.

C First-Mover Advantage in the Presence of a Cash Buffer

To quantify the impact of the first-mover advantage, we consider first the case $\pi = 0$ without first movers. We introduce some notation to make the final expressions more readable: $P^{\Delta Z} := P_0 + \Delta Z$ is the price of the asset after the shock, $S^{\Delta Z} := \frac{C_0 + P^{\Delta Z} Q_0}{N_0}$ is the value of a fund share after the shock, $K := \frac{C_0}{S^{\Delta Z}}$ is the maximum amount of shares that can be redeemed without triggering liquidation of asset shares, $E := -(-\beta \Delta Z \frac{Q_0}{N_0} - K)$ is (at order 0 in γ) the amount of shares that need to be liquidated after the cash buffer is depleted, and $L := \frac{S^{\Delta Z}}{P^{\Delta Z}}$ is a conversion factor between the value of a fund share and the price of the asset.

Proposition C.1. *Assume $\pi = 0$ and $-\beta \frac{Q_0}{N_0} \Delta Z > K$. The aggregate change in asset price ΔP_{tot} and fund share value ΔS_{tot} are*

$$\Delta P_{tot} = \Delta Z + \gamma L \frac{E}{1 - \beta\gamma L^2},$$
$$\Delta S_{tot} = \frac{Q_0}{N_0} \Delta Z + \gamma L^2 \frac{E}{1 - \beta\gamma L^2}.$$

For small γ, the asymptotic expansions for ΔS_{tot} and ΔP_{tot} are

$$\Delta P_{tot} = \Delta Z + \gamma L E + \gamma^2 \beta L^3 E + o(\gamma^2),$$
$$\Delta S_{tot} = \frac{Q_0}{N_0} \Delta Z + \gamma L^2 E + \gamma^2 \beta L^4 E + o(\gamma^2).$$

[18]The second order term is $\gamma^2 \beta (\pi\beta\Delta Z)^2 \left(\frac{2\pi\Delta Z}{(N_0 + \pi\beta\Delta Z)(P_0 + \Delta Z)} - \frac{1-\pi}{N_0 + \pi\beta\Delta Z} - \frac{\pi N_0^2}{(N_0 + \pi\beta\Delta Z)^3} - \frac{N_0}{(N_0 + \pi\beta\Delta Z)^2} \right)$.

We now consider the case when first movers' redemptions exceed the cash level, $\Delta R_{tot}^{fm} > K$, and study the impact on the value of fund shares and on the asset price arising in the presence of the first mover advantage.

Proposition C.2. *Assume that $\Delta R_{tot}^{fm} > K$. The aggregate change in asset price ΔP_{tot} and fund share value ΔS_{tot} are*

$$\Delta P_{tot} = \Delta Z + \gamma LE + \gamma^2 \left(\beta L^3 E - \beta L^3 \frac{E_\pi^2}{N_0 + \beta \Delta Z \frac{Q_0}{N_0} \pi} - L^2 \frac{N_0 + \beta \Delta Z \frac{Q_0}{N_0}}{N_0 + \beta \Delta Z \pi \frac{Q_0}{N_0}} \frac{E_\pi^2}{P^{\Delta Z}} \right) + o(\gamma^2),$$

$$\Delta S_{tot} = \frac{Q_0}{N_0} \Delta Z + \gamma L^2 \left(E - \frac{E_\pi^2}{N_0 + \beta \Delta Z \frac{Q_0}{N_0} \pi} \right) + o(\gamma),$$

where $E_\pi := -(-\beta \pi \Delta Z \frac{Q_0}{N_0} - K)$ is (at order 0 in γ) the amount of shares that needs to be liquidated to repay first movers.

The impact of the first-mover advantage when the fund holds both risky assets and cash is similar to the case that the fund does not hold any cash. The first-mover advantage affects the price of the asset only at second order in γ. The term $\beta \frac{Q_0}{N_0} \Delta Z \pi + K$ represents the amount of shares redeemed by first movers that cannot be paid back with cash and that, therefore, cause liquidation of asset shares. The impact of these sales on the value of a fund share is quadratic and has to be normalised by the amount of remaining shares of the fund.

C.1 The Cost of Cash Replenishment

If the fund desires to maintain a target level of cash and has used its available cash to repay redeeming investors, it may eventually need to sell assets to restore its original cash position. While the fund is time constrained by contractual agreements to repay redeeming investors, it is arguably not in immediate need of reinstating its target cash level. Even funds that invest in illiquid assets and are not subject to time constraints may reduce the cost of raising cash; for example, they can decide not to reinvest maturing bonds, wait for an opportunity to sell assets at favorable prices, or keep the flow of cash from entering investors uninvested.

The findings of our analysis would remain qualitatively the same if the cost for replenishing the fund's cash buffer were to be modeled. We present an extension of the baseline model which provides evidence that this cost is small compared to the overall change in the fund's NAV triggered by the initial market shock ΔZ. We model the longer time frame at disposal of the fund to revert to the target cash-to-asset ratio by assuming that the fund sells asset shares with a market price impact equal to $\gamma_{CR} = \epsilon \times \gamma$, where $\epsilon \in [0, 1]$. This reflects the fact that the liquidity depends not only on the asset itself, but also on the time window available to the fund to liquidate the asset. Without stringent time constraints, the fund incurs a lower cost to liquidate asset shares, and hence the asset illiquidity parameter is lower.

Let $c_{prop} := \frac{C_0}{C_0 + P_0 Q_0}$ be the initial proportion in cash of the fund's assets. The fund aims at this target in the long run. We assume that after all rounds of redemptions from first and second movers

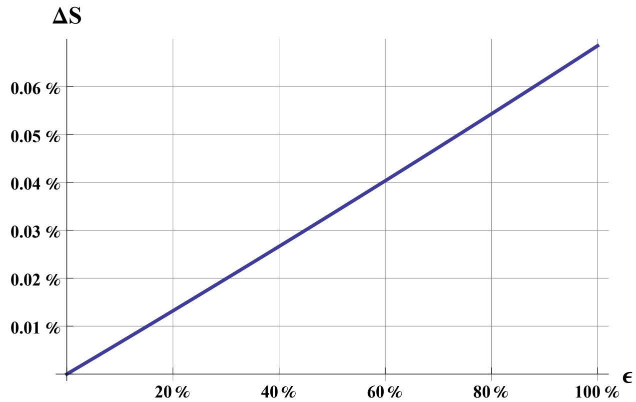

Figure 11: Impact on the fund's NAV of asset sales to replenish cash position for various values of asset illiquidity (ϵ is long-term asset illiquidity over short-term asset illiquidity). The initial shock size is 5% and $\gamma = 2.5 \times 10^{-8}$.

have concluded, the asset price slowly recovers from its sell-off value $P_0 + \Delta P_{tot}$ to its fundamental value $P_0 + \Delta Z$. Hence, after all cash has been depleted due to redemptions and the asset price has rebounded, the fund's NAV is

$$S_f := \frac{(Q_0 + \Delta Q_{tot}^{fm} + \Delta Q_{tot}^{sm})(P_0 + \Delta Z)}{N_0 - \Delta R_{tot}^{fm} - \Delta R_{tot}^{sm}}.$$

The number of asset shares ΔQ_{CR} the fund needs to sell to reinstate the cash allocation c_{prop} is given by the solution of the system:

$$\frac{C_f}{C_f + (Q_0 + \Delta Q_{tot}^{fm} + \Delta Q_{tot}^{sm} + \Delta Q_{CR})(P_0 + \Delta Z)} = c_{prop},$$
$$-\Delta Q_{CR}(P_0 + \Delta Z + \gamma_{CR}\Delta Q_{CR}) = C_f.$$

The cost of cash replenishment on the fund's NAV is

$$\Delta S_{CR} := \frac{C_f + (Q_0 + \Delta Q_{tot}^{fm} + \Delta Q_{tot}^{sm} + \Delta Q_{CR})(P_0 + \Delta Z)}{N_0 - \Delta R_{tot}^{fm} - \Delta R_{tot}^{sm}} - S_f$$

Figure 11 illustrates the cost of cash replenishment on the fund's NAV for ϵ ranging from 0 to 1. The cost is small compared to the size of the initial asset market shock.

D Multiple Funds and Swing Pricing

Consider two funds that hold shares of the same asset. First movers of each fund $i = 1, 2$ redeem $\Delta R_{tot,i}^{fm}$ fund shares, and fund i liquidates $\Delta Q_{tot,i}^{fm}$ asset shares to meet these redemption requests, where

$$\Delta R^{fm}_{tot,i} = -\beta_i \pi_i \Delta S_{tot,i} \tag{D.1}$$
$$-\Delta Q^{fm}_{tot,i} \times (P_0 + \Delta Z + \gamma(\Delta Q^{fm}_{tot,1} + \Delta Q^{fm}_{tot,2})) = \Delta R^{fm}_{tot,i}(S_{0,i} + \Delta Z).$$

Second movers of each fund i redeem $\Delta R^{sm}_{tot,i}$ fund shares and, consequently, fund i liquidates $\Delta Q^{sm}_{tot,i}$ asset shares, where

$$\Delta R^{sm}_{tot,i} = -\beta_i(1 - \pi_i)\Delta S_{tot,i} \tag{D.2}$$
$$\Delta Q^{sm}_{tot,i} = -\Delta R^{sm}_{tot,i} \frac{S_0 + \Delta S_{tot,i}}{P_0 + \Delta P_{tot}}.$$

The change in value $\Delta S_{tot,i}$ of a fund i's share, for $i = 1, 2$, and the change in price of an asset share ΔP_{tot}, are given by the solution to the following system of equations:

$$\begin{aligned} \Delta P_{tot} &= \Delta Z + \gamma(\Delta Q^{fm}_{tot,1} + \Delta Q^{fm}_{tot,2} + \Delta Q^{sm}_{tot,1} + \Delta Q^{sm}_{tot,2}), \\ \Delta S_{tot,1} &= \frac{(Q_0 + \Delta Q^{fm}_{tot,1} + \Delta Q^{sm}_{tot,1})(P_0 + \Delta P_{tot})}{N_0 - \Delta R^{fm}_{tot,1} - \Delta R^{sm}_{tot,1}} - S_{0,1}, \\ \Delta S_{tot,2} &= \frac{(Q_0 + \Delta Q^{fm}_{tot,2} + \Delta Q^{sm}_{tot,2})(P_0 + \Delta P_{tot})}{N_0 - \Delta R^{fm}_{tot,2} - \Delta R^{sm}_{tot,2}} - S_{0,2}. \end{aligned} \tag{D.3}$$

Proposition D.1. *Let $\Delta R_i := -\beta_i \Delta S_{tot,i}$ be the amount of redeemed shares. Assume that $\pi_1 = \pi_2 = 0$, and that the number of asset shares equals the number of fund shares for each fund: $Q_i = N_i$ for $i = 1, 2$.*
The change in value of fund i's share $\Delta S_{tot,i}$ (for $i = 1, 2$) and the change in the asset price ΔP_{tot} are

$$\Delta S_{tot,i} = \Delta Z + \gamma \frac{E_1 + E_2}{1 - (\beta_1 + \beta_2)\gamma}, \tag{D.4}$$
$$\Delta P_{tot} = \Delta Z + \gamma \frac{E_1 + E_2}{1 - (\beta_1 + \beta_2)\gamma},$$

where $E_i = \beta_i \Delta Z$.

Remark D.2. *Cross-price impact effects are important. The impact on the funds' share value and the asset price imposed by the simultaneous liquidation procedure of multiple funds is larger that the sum of the impacts of each individual fund without accounting for spillover effects:*

$$\Delta P_{tot} \approx \Delta Z + \underbrace{\gamma E_1 + \gamma^2 \beta_1 E_1}_{\textit{Fund 1 Impact}} + \underbrace{\gamma E_2 + \gamma^2 \beta_2 E_2}_{\textit{Fund 2 Impact}} + \underbrace{\gamma^2(\beta_1 E_2 + \beta_2 E_1)}_{\textit{Cross-impact}}.$$

Proposition D.3. *Assume that $\pi_1, \pi_2 > 0$. Define $E_i = \beta_i \Delta Z$, $E^{\pi}_i = \beta_i \pi_i \Delta Z$, $Rem^{\pi}_i = N_i + \beta_i \pi_i \Delta Z$ the number of remaining shares after first mover redemptions at order 0 in γ and $Rem_i =$*

$N_i + \beta_i \Delta Z$ the number of remaining shares after first and second mover redemptions at order 0 in γ.
For small γ, the change in value of fund i's share is

$$\Delta S_{tot,i} = \Delta Z + \gamma \left((E_1 + E_2) - \frac{E_i^\pi (E_1^\pi + E_2^\pi)}{Rem_i^\pi} \right) + o(\gamma).$$

For small γ, the change in the asset price is

$$\begin{aligned}\Delta P_{tot} = \Delta Z + \gamma(E_1 + E_2) + \gamma^2 \Bigg(& (\beta_1 + \beta_2)(E_1 + E_2) \\ & - \beta_1 E_1^\pi \frac{E_1^\pi + E_2^\pi}{Rem_1^\pi} - \beta_2 E_2^\pi \frac{E_1^\pi + E_2^\pi}{Rem_2^\pi} \\ & - E_1^\pi \frac{E_1^\pi + E_2^\pi}{P_0 + \Delta Z} \frac{Rem_1}{Rem_1^\pi} - E_2^\pi \frac{E_1^\pi + E_2^\pi}{P_0 + \Delta Z} \frac{Rem_2}{Rem_2^\pi} \Bigg) + o(\gamma^2).\end{aligned}$$

Remark D.4. *The expressions in Proposition D.3 can be restated as*

$$\Delta S_{tot,1} \approx \Delta Z + \underbrace{\gamma \left(E_1 - \frac{(E_1^\pi)^2}{Rem_1^\pi} \right)}_{\text{Own Impact}} + \underbrace{\gamma \left(E_2 - \frac{E_2^\pi E_1^\pi}{Rem_1^\pi} \right)}_{\text{Other Fund's Impact}},$$

$$\begin{aligned}\Delta P_{tot} \approx \Delta Z & + \underbrace{\gamma E_1 + \gamma^2 \left(\beta_1 E_1 - \beta_1 \frac{(E_1^\pi)^2}{Rem_1^\pi} - \frac{(E_1^\pi)^2}{P^{\Delta Z}} \frac{Rem_1}{Rem_1^\pi} \right)}_{\text{Impact from Fund 1}} \\ & + \underbrace{\gamma E_2 + \gamma^2 \left(\beta_2 E_2 - \beta_2 \frac{(E_2^\pi)^2}{Rem_2^\pi} - \frac{(E_2^\pi)^2}{P^{\Delta Z}} \frac{Rem_2}{Rem_2^\pi} \right)}_{\text{Impact from Fund 2}} \\ & + \underbrace{\gamma^2 \left(\beta_1 E_2 + \beta_2 E_1 - \beta_1 \frac{E_1^\pi E_2^\pi}{Rem_1^\pi} - \beta_2 \frac{E_1^\pi E_2^\pi}{Rem_2^\pi} - \frac{E_1^\pi E_2^\pi}{P^{\Delta Z}} \frac{Rem_1}{Rem_1^\pi} - \frac{E_1^\pi E_2^\pi}{P^{\Delta Z}} \frac{Rem_2}{Rem_2^\pi} \right)}_{\text{Cross-impact}}.\end{aligned}$$

Proposition D.5. *Assume that $\pi_1, \pi_2 > 0$, and that the number of asset shares equals the number of fund shares for each fund: $Q_i = N_i$ for $i = 1, 2$. Assume both fund 1 and fund 2 apply swing pricing. The swing price of fund $i = 1, 2$ is*

$$\Delta S_{both}^{sw} = \gamma \frac{E_1^\pi + E_2^\pi}{1 - (\beta_1 + \beta_2)\gamma}.$$

Viewed as a function of the number of redemptions from first movers, the swing price takes the form

$$\Delta S_{both}^{sw} = -\gamma(\Delta R_{tot,1}^{fm} + \Delta R_{tot,2}^{fm}), \tag{D.5}$$

where $\Delta R_{tot,i}^{fm}$ is the number of shares redeemed by first movers of fund $i = 1, 2$.

Proposition D.6. *Assume that $\pi_1 > 0$ and $\pi_2 = 0$, and that the number of asset shares equals* *the number of fund shares for each fund: $Q_i = N_i$ for $i = 1, 2$. For small γ, the change in value of fund 2's share $\Delta S_{tot,2}$ is*

$$\Delta S_{tot,2} = \Delta Z + \gamma(\beta_1+\beta_2)\Delta Z + \gamma^2\left((\beta_1+\beta_2)^2\Delta Z - (E_1^\pi)^2\frac{Rem_1 + \beta_1(P_0+\Delta Z)}{(P_0+\Delta Z)Rem_1^\pi}\right) + o(\gamma^2). \tag{D.6}$$

If only one fund applies swing pricing, the fund may decide to implement an adjustment that removes either the impact of first movers of both funds or only the impact of its own first movers. Swing price ΔS^{sw}_{loc} is computed such that the fund attains the change in NAV (D.6), while swing price ΔS^{sw}_{glob} is computed such that the fund's NAV change is (D.4).

Proposition D.7. *Assume that $\pi_1, \pi_2 > 0$, and that the number of asset shares equals the number of fund shares for each fund: $Q_i = N_i$ for $i = 1, 2$. Assume that only fund 2 applies swing pricing. For small γ,*

$$\Delta S^{sw}_{loc} = \Delta S^{sw}_{both} + \gamma^2\frac{E_1^\pi(\beta_1(P_0+2\Delta Z)(Rem_2^\pi - \Delta Z\pi_1(\beta_1\pi_1+\beta_2\pi_2)) + N_1(N_2 - \beta_1\Delta Z\pi_1))}{(P_0+\Delta Z)Rem_1^\pi} + o(\gamma^2),$$

$$\Delta S^{sw}_{glob} = \Delta S^{sw}_{loc} + \gamma^2\frac{\beta_1\pi_1}{\beta_2\pi_2}\frac{Rem_2^\pi}{Rem_1^\pi}\frac{E_1^\pi(\beta_1(P_0+\Delta Z)+Rem_1)}{P_0+\Delta Z} + o(\gamma^2).$$

E Technical Proofs

Lemma E.1. *Assume that first movers redeem ΔR^{fm}_{tot} fund shares and the fund trades ΔQ^{fm}_{tot} asset shares to repay first movers. After first movers' redemptions, the fund holds $Q^{fm} := Q_0 + \Delta Q^{fm}_{tot}$ asset shares and there are $N^{fm} := N_0 - \Delta R^{fm}_{tot}$ outstanding fund shares, the change in asset price is $\Delta P^{fm} := \Delta Z + \gamma\Delta Q^{fm}_{tot}$ and the change in value of a fund share is $\Delta S^{fm} := \frac{Q^{fm}P^{fm}}{N^{fm}} - S_0$. Assume that $\beta\gamma(1-\pi)\left(\frac{Q^{fm}}{N^{fm}}\right)^2 < 1$. The changes in asset price and fund share value after second movers' redemptions are given by*

$$\Delta P_{tot} = \Delta P^{fm} + \beta\gamma(1-\pi)\frac{Q^{fm}}{N^{fm}}\frac{\Delta S^{fm}}{1-\beta\gamma(1-\pi)\left(\frac{Q^{fm}}{N^{fm}}\right)^2},$$

$$\Delta S_{tot} = \frac{\Delta S^{fm}}{1-\beta\gamma(1-\pi)\left(\frac{Q^{fm}}{N^{fm}}\right)^2}.$$

Proof. After all first movers have redeemed their shares, second movers observe the change in value of a fund share $\Delta S^{sm}_0 := \frac{Q^{fm}P^{fm}}{N^{fm}} - S_0$ and the change in asset price $\Delta P^{sm}_0 := \Delta P^{fm}$. At each round of redemptions, second movers redeem $\Delta R^{sm}_{n+1} = -\beta(1-\pi)\Delta S^{sm}_n$ shares and the fund sells $\Delta Q^{sm}_{n+1} = -\Delta R^{sm}_{n+1}\frac{S^{sm}_n + \Delta S^{sm}_{n+1}}{P^{sm}_n+\Delta P^{sm}_{n+1}}$, where $\Delta P^{sm}_n := \gamma\Delta Q^{sm}_n$ and S^{sm}_n (resp. P^{sm}_n) is defined recursively as $S^{sm}_n := S^{sm}_{n-1} + \Delta S^{sm}_n$ with $S^{sm}_0 := S_0 + \Delta S^{sm}_0$ (resp. $P^{sm}_n := P^{sm}_{n-1} + \Delta P^{sm}_n$ with

$P_0^{sm} := P_0 + \Delta P_0^{sm}$). The change in value of a fund share after the n-th round of redemptions is $\Delta S_{n+1}^{sm} = \frac{(Q_n^{sm}+\Delta Q_{n+1}^{sm})(P_n^{sm}+\Delta P_{n+1}^{sm})}{N_n^{sm}-\Delta R_{n+1}^{sm}} - S_n^{sm}$, where Q_n^{sm} (resp. N_n^{sm}) is defined recursively as $Q_n^{sm} := Q_{n-1}^{sm} + \Delta Q_n^{sm}$ with $Q_0^{sm} := Q^{fm}$ (resp. $N_n^{sm} := N_{n-1}^{sm} - \Delta R_n^{sm}$ with $N_0^{sm} := N^{fm}$). It can be immediately verified that at each iteration, we obtain

$$\Delta S_{n+1}^{sm} = \gamma\beta(1-\pi)\left(\frac{Q^{fm}}{N^{fm}}\right)^2 \Delta S_n^{sm},$$
$$\Delta P_{n+1}^{sm} = \gamma\beta(1-\pi)\frac{Q^{fm}}{N^{fm}}\Delta S_n^{sm},$$

with $\Delta P_0^{sm} = \Delta P^{fm}$ and $\Delta S_0^{sm} = \Delta S^{fm}$. The result follows from the equalities $\Delta P_{tot} = \sum_{n=0}^{\infty} \Delta P_n^{sm}$ and $\Delta S_{tot} = \sum_{n=0}^{\infty} \Delta S_n^{sm}$. □

Proof of Proposition 2.1. It follows directly from Lemma E.1 after setting $\pi = 0$, $Q_0 = N_0$, $\Delta Q^{fm} = 0$, $\Delta R^{fm} = 0$, $\Delta P^{fm} = \Delta Z$, and $\Delta S^{fm} = \Delta Z$. □

Proof of Proposition 2.2. The change in value of a fund share following all redemptions is computed iteratively. Because we assume $Q_0 = N_0$, the initial negative shock to the value of a fund share is $\Delta S_0 = \Delta Z$. If first movers anticipate a negative change ΔS_n in the value of a fund share after the n-th round of redemptions, then they redeem $\Delta R_{n+1}^{fm} = -\beta\pi\Delta S_n$ fund shares. To repay first movers, the fund has to trade ΔQ_{n+1}^{fm} asset shares, where $-\Delta Q_{n+1}^{fm} \times (P_0 + \Delta Z + \gamma\Delta Q_{n+1}^{fm}) = \Delta R_{n+1}^{fm}(S_0 + \Delta S_0)$. Notice that the number of redemptions ΔR_{n+1}^{fm} (resp. the negative change in holdings ΔQ_{n+1}^{fm}) decreases (resp. increases) with ΔS_n. By rewriting the equation as $\frac{Q_0+\Delta Q_{n+1}^{fm}}{N_0-\Delta R_{n+1}^{fm}} = 1 - \frac{\gamma}{P_0+\Delta Z}\frac{(\Delta Q_{n+1}^{fm})^2}{N_0-\Delta R_{n+1}^{fm}}$, we can immediately see that the fraction $\frac{Q_0+\Delta Q_{n+1}^{fm}}{N_0-\Delta R_{n+1}^{fm}}$ is strictly increasing in ΔS_n. The smallest number of asset shares the fund has to trade to repay first movers is

$$\Delta Q_{n+1}^{fm} = -\frac{P_0 + \Delta Z - \sqrt{(P_0+\Delta Z)^2 - 4\gamma\Delta R_{n+1}^{fm}(P_0+\Delta Z)(Q_0/N_0)}}{2\gamma}.$$

The change in value of a fund share due to first movers' redemptions is

$$\Delta S_{n+1}^{fm} = \frac{(Q_0 + \Delta Q_{n+1}^{fm})(P_0 + \Delta Z + \gamma\Delta Q_{n+1}^{fm})}{N_0 - \Delta R_{n+1}^{fm}} - S_0.$$

The redemptions by second movers further amplify the downward pressure on the value of a fund share: using Lemma E.1 we obtain that, after second mover redemptions, the change in value of a fund share is

$$\Delta S_{n+1} = \frac{\Delta S_{n+1}^{fm}}{1 - \beta\gamma(1-\pi)\left(\frac{Q_0+\Delta Q_{n+1}^{fm}}{N_0-\Delta R_{n+1}^{fm}}\right)^2}. \tag{E.1}$$

Since $\frac{Q_0+\Delta Q_{n+1}^{fm}}{N_0-\Delta R_{n+1}^{fm}}$ increases with ΔS_n, we obtain that ΔS_{n+1}^{fm} and ΔS_{n+1} also increase with ΔS_n.

After plugging the expressions for ΔS^{fm}_{n+1}, ΔQ^{fm}_{n+1} and ΔR^{fm}_{n+1} into ΔS_{n+1}, we may rewrite the expression for ΔS_{n+1} as

$$\Delta S_{n+1} = f_\gamma(\Delta S_n), \tag{E.2}$$

where

$$f_\gamma(x) = \frac{2\beta\pi\Delta Z x + N_0(\Delta Z - P_0 + \sqrt{P_0+\Delta Z}\sqrt{P_0+\Delta Z+4\beta\pi\gamma x})}{2(N_0+\beta\pi x)(1 - \frac{\beta(1-\pi)(P_0+\Delta Z - 2\gamma N_0 - \sqrt{P_0+\Delta Z}\sqrt{P_0+\Delta Z+4\beta\pi\gamma x})^2}{4\gamma(N_0+\beta\pi x)^2})}. \tag{E.3}$$

Because ΔS_{n+1} is strictly increasing in ΔS_n, the function $f_\gamma(x)$ is strictly increasing when $x<0$. Furthermore, it can be immediately verified that $f_\gamma(\Delta Z) \le f_\gamma(0) = \frac{\Delta Z}{1-\beta\gamma(1-\pi)} < \Delta Z$. By iterating the relation (E.2), it follows that the sequence $\{\Delta S_n\}_{n\ge 0}$ is strictly decreasing: if $\Delta S_n < \Delta S_{n-1}$, then $\Delta S_{n+1} = f_\gamma(\Delta S_n) < f_\gamma(\Delta S_{n-1}) = \Delta S_n$. The limit of this sequence, if it exists, must be a fixed point ΔS_{tot} of the function $f_\gamma(\cdot)$, i.e. $\Delta S_{tot} = f_\gamma(\Delta S_{tot})$. Notice that $f_0(x) := \lim_{\gamma\to 0^+} f_\gamma(x) = \Delta Z$. Hence, the initial shock $\Delta S_0 = \Delta Z$ is a fixed point of $f_\gamma(\cdot)$ when $\gamma = 0$. Because the dependence of $f_\gamma(\cdot)$ on γ is continuous, there exists $\gamma^* > 0$ such that for $0<\gamma<\gamma^*$ there exists a solution to the fixed point equation $x = f_\gamma(x)$.

We can Taylor expand the fixed point $\Delta S_{tot}(\gamma)$ around $\gamma = 0$ to obtain $\Delta S_{tot}(\gamma) = \Delta S_{tot}(0) + \gamma\frac{\partial \Delta S_{tot}(\gamma)}{\partial\gamma}|_{\gamma=0} + o(\gamma)$. Since $\lim_{\gamma\to 0^+} f_\gamma(\Delta S_{tot}(\gamma)) = \Delta Z$, we get that $\Delta S_{tot}(0) = \Delta Z$. By differentiating both sides of the fixed point equation $\Delta S_{tot}(\gamma) = f_\gamma(\Delta S_{tot}(\gamma))$ with respect to γ, we obtain $\frac{\partial \Delta S_{tot}}{\partial\gamma} = \frac{\partial f_\gamma(\Delta S_{tot})}{\partial\gamma} + \frac{\partial f_\gamma(\Delta S_{tot})}{\partial \Delta S_{tot}}\frac{\partial \Delta S_{tot}}{\partial\gamma}$. It can be verified that $\frac{\partial f_\gamma(\Delta S_{tot})}{\partial \Delta S_{tot}}|_{\gamma=0} = 0$ and $\frac{\partial f_\gamma(\Delta S_{tot})}{\partial\gamma}|_{\gamma=0} = \beta\Delta Z - \frac{\pi^2\beta^2\Delta Z^2}{N_0+\pi\beta\Delta Z}$. Hence, $\Delta S_{tot} = \Delta Z + \gamma\left(\beta\Delta Z - \frac{\pi^2\beta^2\Delta Z^2}{N_0+\pi\beta\Delta Z}\right) + o(\gamma)$.

From Lemma E.1, we get that $\Delta P_{tot} = \Delta Z + \gamma\Delta Q^{fm} + \beta\gamma(1-\pi)\frac{Q_0+\Delta Q^{fm}}{N_0-\Delta R^{fm}}\Delta S_{tot}$, where both ΔQ^{fm} and ΔR^{fm} are functions of ΔS_{tot}. Given the asymptotic expansion in γ for ΔS_{tot}, we can compute the expansion for ΔP_{tot}: $\lim_{\gamma\to 0^+}\Delta P_{tot} = \Delta Z$, $\lim_{\gamma\to 0^+}\frac{\Delta P_{tot}-\Delta Z}{\gamma} = \beta\Delta Z$ and $\lim_{\gamma\to 0^+}\frac{\Delta P_{tot}-\Delta Z-\gamma\beta\Delta Z}{\gamma^2} = \beta^2\Delta Z - \beta\frac{\pi^2\beta^2\Delta Z^2}{N_0+\pi\beta\Delta Z} - \frac{\pi^2\beta^2\Delta Z^2}{P_0+\Delta Z}\frac{N_0+\beta\Delta Z}{N_0+\pi\beta\Delta Z}$. □

Proof of Proposition 3.1. In the proof of Proposition 2.2, we have shown that ΔS_{tot} is the limit, if it exists, of the sequence $\{\Delta S_n\}_{n\ge 0}$, defined as $\Delta S_{n+1} = f_\gamma(\Delta S_n)$, with f_γ given in (E.3) and $\Delta S_0 = \Delta Z$. It can be verified immediately that if $\Delta Z = 0$, then $\Delta S_{tot} = 0$ is the unique fixed point of $f_\gamma(\cdot)$.

Notice that the maximum amount of cash the fund can retrieve from asset sales is $\max_{\Delta Q}\Delta Q(P_0+\Delta Z+\gamma\Delta Q) = \frac{(P_0+\Delta Z)^2}{4\gamma}$. Hence, the fund becomes unable to repay first movers when $\Delta R^{fm}(S_0+\Delta S_0) > \frac{(P_0+\Delta Z)^2}{4\gamma}$, where the left-hand side is the amount of cash the fund owes to first movers. In other words, if first movers redeem $\Delta R^{fm} = -\beta\pi\Delta S$ in response to an anticipated final change in value of a fund share ΔS, this solvency-type condition reads as $\Delta S < -\frac{P_0+\Delta Z}{4\gamma\pi\beta}$ (recall that $Q_0 = N_0$). Hence, if $\Delta S_0 = \Delta Z < -\frac{P_0}{1+4\gamma\beta\pi}$, the fund becomes unable to meet its first movers' redemption requests.

Throughout the proof, we will write $f_\gamma(x,\Delta Z)$ to highlight the dependence of $f_\gamma(x)$ on ΔZ. Define the solvency set $\mathcal{S} := \{\Delta Z : \forall n\ \Delta S_n \in [-\frac{P_0+\Delta Z}{4\beta\pi\gamma}, 0],$ where $\Delta S_{n+1} = f_\gamma(\Delta S_n,\Delta Z)$ and $\Delta S_0 = \Delta Z\}$. This is the set of initial shocks ΔZ such that the fund remains solvent after each iteration of the procedure yielding the aggregate change in the value of a fund share ΔS_{tot}. We have already shown that if $\Delta Z < -\frac{P_0}{1+4\gamma\beta\pi}$, then ΔZ does not belong to $\mathcal{S}$.

Define $\Delta Z^* := \inf \mathcal{S}$. We have already argued that $-\frac{P_0}{1+4\gamma\beta\pi} \leq \Delta Z^*$. In order to prove that the fund remains solvent for any initial shock $\Delta Z \geq \Delta Z^*$, it is sufficient to show that $f_\gamma(x, \Delta Z)$ is an increasing function of ΔZ for any $x \in [-\frac{P_0+\Delta Z}{4\gamma\pi\beta}, 0]$. For a given quantity of first movers' redemptions ΔR^{fm}, it can be seen immediately that the amount of asset shares ΔQ^{fm} the fund trades to repay first movers is an increasing function of ΔZ. Hence, $\Delta S^{fm} := \frac{(Q_0+\Delta Q^{fm})(P_0+\Delta Z+\gamma\Delta Q^{fm})}{N_0-\Delta R^{fm}} - S_0$ is also an increasing function of ΔZ, and so, combining equations (E.1) and (E.2), we obtain $f_\gamma\left(-\frac{\Delta R^{fm}}{\beta\pi}, \Delta Z\right) = \frac{\Delta S^{fm}(\Delta Z)}{1-\beta\gamma(1-\pi)\left(\frac{Q_0+\Delta Q^{fm}(\Delta Z)}{N_0-\Delta R^{fm}}\right)^2}$. In other words, for any $x \in [-\frac{P_0+\Delta Z}{4\gamma\pi\beta}, 0]$, $f_\gamma(x, \Delta Z)$ is increasing in ΔZ. It follows that if $\Delta Z_2 < \Delta Z_1$ and $\Delta Z_1 \notin \mathcal{S}$, then $\Delta Z_2 \notin \mathcal{S}$. This shows that the fund remains solvent for any $\Delta Z \geq \Delta Z^*$.

To highlight the dependence of the solvency set $\mathcal{S}$ on γ, we write $\mathcal{S}_\gamma$. It can easily be seen that $f_\gamma(x, \Delta Z)$ is decreasing in γ for any $\Delta Z \in [-\frac{P_0}{1+\gamma\beta\pi}, 0]$ and any $x \in [-\frac{P_0+\Delta Z}{4\gamma\pi\beta}, 0]$. Let $\gamma_1 < \gamma_2$. Because $f_\gamma(x, \Delta Z)$ is decreasing in γ and $-\frac{P_0+\Delta Z}{4\beta\pi\gamma}$ is increasing in γ, we obtain that $\mathcal{S}_{\gamma_2} \subset \mathcal{S}_{\gamma_1}$. This implies that $\Delta Z^*(\gamma_2) \geq \Delta Z^*(\gamma_1)$ and concludes the proof. □

Proof of Proposition 3.3. Assume the fund adjusts its NAV by ΔS^{adj} when first movers redeem. The final asset price change ΔP_{tot} and final fund share value change ΔS_{tot} are the solution to the system of equations (A.1–A.2–A.3), where the first line of (A.2) gets replaced by $-\Delta Q^{fm}_{tot} \times (P_0 + \Delta Z + \gamma\Delta Q^{fm}_{tot}) = \Delta R^{fm}_{tot} \times (S_0 + \Delta S^{fm}_0 + \Delta S^{adj})$.

It can be immediately verified that $(\Delta S^{adj}, \Delta P_{tot}, \Delta S_{tot}) = \left(\gamma\frac{\pi\beta\Delta Z}{1-\beta\gamma}, \frac{\Delta Z}{1-\beta\gamma}, \frac{\Delta Z}{1-\beta\gamma}\right)$ is a solution to this system of equations. This means that $\gamma\frac{\pi\beta\Delta Z}{1-\beta\gamma}$ is a swing price. Since $\Delta R^{fm}_{tot} = -\pi\beta\Delta S_{tot} = -\pi\beta\frac{\Delta Z}{1-\beta\gamma}$, we get that $\Delta S^{sw} = -\gamma\Delta R^{fm}_{tot}$.

Notice that ΔQ^{fm}_{tot} is a strictly decreasing function of ΔS^{adj}. Since ΔS_{tot} increases with ΔQ^{fm}_{tot}, also ΔS_{tot} is a strictly decreasing function of ΔS^{adj}. This implies that the swing price is unique. □

Proof of Proposition 3.4. Notice that if $\Delta R^{fm}_{tot} \leq \frac{C_0}{S_0+\Delta Z}$, the fund has enough cash to repay first movers, therefore in this case the swing price is 0. Assume that ΔZ and C_0 are such that $\Delta R^{fm}_{tot} \geq \frac{C_0}{S_0+\Delta Z}$. Since the fund first uses cash to repay first movers, it needs to liquidate assets to raise only the cash equivalent of $\Delta R^{fm}_{tot} - \frac{C_0}{S_0+\Delta Z}$ fund shares. Hence, the final change ΔP_{tot} in asset price and the final change ΔS_{tot} in the fund share value are given by the solution to the system of equations (A.1–A.2–A.3), where the first line of (A.2) gets replaced by $-\Delta Q^{fm}_{tot} \times (P_0 + \Delta Z + \gamma\Delta Q^{fm}_{tot}) = (\Delta R^{fm}_{tot} - \frac{C_0}{S_0+\Delta Z})(S_0 + \Delta S^{fm}_0 + \Delta S^{adj})$. It can be verified that $(\Delta S^{adj}, \Delta P_{tot}, \Delta S_{tot}) = \left(-\gamma L^2(\Delta R^{fm} - \frac{C_0}{S_0+\Delta Z}), \Delta Z + \gamma L\frac{E}{1-\beta\gamma L^2}, \Delta Z + \gamma L^2\frac{E}{1-\beta\gamma L^2}\right)$ is a solution to this system of equations. Since ΔS_{tot} is also the change in value of a fund share in the absence of first movers (see Proposition C.1), the adjustment corresponds to the swing price. □

Proof of Proposition 4.1. The second order terms in the expansion formulas given in Proposition D.7 are strictly negative. The result follows immediately. □

Proof of Proposition B.1. Notice that $\Delta S^p = (S_0 + \Delta S_{tot}) \times \frac{P_0+\Delta Z}{P_0+\Delta P_{tot}} - S_0$. Assume first $\pi = 0$.

Because $Q_0 = N_0$, we have $P_0 = S_0$. From Proposition 2.1, we get $\Delta P_{tot} = \Delta S_{tot}$. It follows immediately that $\Delta S^p = \Delta Z$.

Assume now $\pi > 0$. From Proposition 2.2, we get the asymptotic expansions in γ of ΔP_{tot} and ΔS_{tot}. Plugging these expressions into ΔS^p yields the result. $\square$

Proof of Proposition C.1. Notice that if $-\beta \frac{Q_0}{N_0} \Delta Z \leq K$, the fund is not required to liquidate asset shares to repay investors who react to the initial market shock. Hence, there is no pressure imposed on the asset price, and $\Delta P_{tot} = \Delta S_{tot} = \Delta Z$. This implies that $\Delta R^{sm}_{tot} = -\beta \frac{Q_0}{N_0} \Delta Z$.

If $-\beta \frac{Q_0}{N_0} \Delta Z > K$, the fund sells asset shares after all available cash has been used to repay redeeming investors: $\Delta Q^{sm}_{tot} = -(\Delta R^{sm}_{tot} - K) \frac{S_0 + \Delta S_{tot}}{P_0 + \Delta P_{tot}}$, where $\Delta R^{sm}_{tot} = -\beta \Delta S_{tot} > K$. The change in value of a fund share solves $\Delta S_{tot} = \frac{(Q_0 + \Delta Q^{sm}_{tot})(P_0 + \Delta P_{tot})}{N_0 - \Delta R^{sm}_{tot}} - S_0$, while the change in price of an asset share solves $\Delta P_{tot} = \Delta Z + \gamma \Delta Q^{sm}_{tot}$. It can be verified that the pair $(\Delta P_{tot}, \Delta S_{tot})$, where

$$\Delta P_{tot} = \Delta Z + \gamma L \frac{E}{1 - \beta \gamma L^2},$$
$$\Delta S_{tot} = \frac{Q_0}{N_0} \Delta Z + \gamma L^2 \frac{E}{1 - \beta \gamma L^2},$$

is a solution to these equations. $\square$

Proof of Proposition C.2. Since $\Delta R^{fm}_{tot} > K$, the fund needs to sell asset shares to repay first movers. The change in value of a fund share ΔS_{tot} and the change in price of an asset share ΔP_{tot} are given by the solution of the system of equations (A.1–A.2–A.3), where the first line of (A.2) gets replaced by $-\Delta Q^{fm}_{tot} \times (P_0 + \Delta Z + \gamma \Delta Q^{fm}_{tot}) = (\Delta R^{fm}_{tot} - K)(S_0 + \frac{Q_0}{N_0} \Delta Z)$, or equivalently by

$$\Delta Q^{fm}_{tot} = -\frac{P_0 + \Delta Z - \sqrt{(P_0 + \Delta Z)^2 - 4\gamma(\Delta R^{fm}_{tot} - K)(S_0 + \Delta Z Q_0/N_0)}}{2\gamma}.$$

As in the proof of Proposition 2.2, we find an approximate solution $(\Delta P_{tot}, \Delta S_{tot})$ for small γ: $\Delta P_{tot}(\gamma) = \Delta P_{tot}(0) + \gamma \frac{\partial \Delta P_{tot}(\gamma)}{\partial \gamma}|_{\gamma=0} + \frac{\gamma^2}{2} \frac{\partial^2 \Delta P_{tot}(\gamma)}{\partial \gamma^2}|_{\gamma=0} + o(\gamma^2)$ and $\Delta S_{tot}(\gamma) = \Delta S_{tot}(0) + \gamma \frac{\partial \Delta S_{tot}(\gamma)}{\partial \gamma}|_{\gamma=0} + o(\gamma)$. The last two equations in the system (A.3) may be rewritten as $\Delta P_{tot} = g_P(\gamma, \Delta P_{tot}, \Delta S_{tot})$ and $\Delta S_{tot} = g_S(\gamma, \Delta P_{tot}, \Delta S_{tot})$ for appropriately defined functions g_P and g_S. By letting γ to 0^+ in these equations and solving for $\Delta P_{tot}(0)$ and $\Delta S_{tot}(0)$, we obtain the solutions $\Delta P_{tot}(0) = \Delta Z$ and $\Delta S_{tot}(0) = \frac{Q_0}{N_0} \Delta Z$. Differentiating both sides of each equation with respect to γ and evaluating the derivatives at $\gamma = 0$, we obtain $\frac{\partial \Delta P_{tot}(\gamma)}{\partial \gamma}|_{\gamma=0} = LE$, $\frac{\partial \Delta S_{tot}(\gamma)}{\partial \gamma}|_{\gamma=0} = L^2(E - \frac{E^2_\pi}{N_0 + \beta \Delta Z \frac{Q_0}{N_0} \pi})$ and $\frac{\partial^2 \Delta P_{tot}(\gamma)}{\partial \gamma^2}|_{\gamma=0} = 2 \left(\beta L^3 E - \beta L^3 \frac{E^2_\pi}{N_0 + \beta \Delta Z \frac{Q_0}{N_0} \pi} - L^2 \frac{N_0 + \beta \Delta Z \frac{Q_0}{N_0}}{N_0 + \beta \Delta Z \pi \frac{Q_0}{N_0}} \frac{E^2_\pi}{P^{\Delta Z}} \right)$. $\square$

Proof of Proposition D.1. The proof proceeds along similar lines as the proofs of Lemma E.1 and Proposition C.1. The change in asset price ΔP_{tot} and the change in fund i's NAV $\Delta S_{tot,i}$, for $i = 1, 2$, are given by the solution of the system of equations (D.1–D.2–D.3) with $\pi_i = 0$, and therefore $\Delta R^{fm}_{tot,i} = 0$ and $\Delta Q^{fm}_{tot,i} = 0$, for $i = 1, 2$. It can be verified that a solution to these equations is given by the triplet $(\Delta S_{tot,1}, \Delta S_{tot,2}, \Delta P_{tot})$ defined as $\Delta S_{tot,1} = \Delta S_{tot,2} = \Delta P_{tot} = \Delta Z + \gamma \frac{E_1 + E_2}{1 - (\beta_1 + \beta_2)\gamma}$. $\square$

Proof of Proposition D.3. The proof follows the same lines as the proofs of Proposition 2.2 and Proposition C.2. The change in asset price ΔP_{tot} and the change in fund i's NAV $\Delta S_{tot,i}$, for $i = 1, 2$, are given by the solution of the system of equations (D.1–D.2–D.3). We rewrite the equations in the system (D.3) as $\Delta P_{tot} = g_P(\gamma, \Delta P_{tot}, \Delta S_{tot,1}, \Delta S_{tot,2})$, $\Delta S_{tot,1} = g_{S,1}(\gamma, \Delta P_{tot}, \Delta S_{tot,1}, \Delta S_{tot,2})$ and $\Delta S_{tot,2} = g_{S,2}(\gamma, \Delta P_{tot}, \Delta S_{tot,1}, \Delta S_{tot,2})$ for appropriately defined functions g_P, $g_{S,1}$ and $g_{S,2}$. Differentiating both sides of these equations with respect to γ and evaluating them at $\gamma = 0$, yields the coefficients of the asymptotic expansions $\Delta P_{tot}(\gamma) = \Delta P_{tot}(0) + \gamma \frac{\partial \Delta P_{tot}(\gamma)}{\partial \gamma}|_{\gamma=0} + \frac{\gamma^2}{2} \frac{\partial^2 \Delta P_{tot}(\gamma)}{\partial \gamma^2}|_{\gamma=0} + o(\gamma^2)$ and $\Delta S_{tot,i}(\gamma) = \Delta S_{tot,i}(0) + \gamma \frac{\partial \Delta S_{tot,i}(\gamma)}{\partial \gamma}|_{\gamma=0} + o(\gamma)$, for $i = 1, 2$. □

Proof of Proposition D.5. The proof follows the same lines as the proofs of Proposition 3.3 and Proposition 3.4. The NAV that fund i's first movers receive is adjusted by an amount ΔS_i^{adj}, for $i = 1, 2$. Hence, the change in asset price ΔP_{tot} and the change in fund i's NAV $\Delta S_{tot,i}$, for $i = 1, 2$, are given by the solution of the system of equations (D.1–D.2–D.3), where the second line of the system (D.1) gets replaced by $-\Delta Q^{fm}_{tot,i} \times (P_0 + \Delta Z + \gamma(\Delta Q^{fm}_{tot,1} + \Delta Q^{fm}_{tot,2})) = \Delta R^{fm}_{tot,i}(S_{0,i} + \Delta Z + \Delta S_i^{adj})$, for $i = 1, 2$. It can be verified that $\Delta S_i^{adj} = \gamma \frac{E_1^\pi + E_2^\pi}{1-(\beta_1+\beta_2)\gamma}$, $\Delta Q^{fm}_{tot,i} = \frac{E_i^\pi}{1-(\beta_1+\beta_2)\gamma}$, for $i = 1, 2$, and $\Delta S_{tot,1} = \Delta S_{tot,2} = \Delta P_{tot} = \Delta Z + \gamma \frac{E_1+E_2}{1-(\beta_1+\beta_2)\gamma}$ solve the system of equations. It follows that $\Delta S^{sw}_{both} := \gamma \frac{E_1^\pi + E_2^\pi}{1-(\beta_1+\beta_2)\gamma}$ is the swing price. The swing price also satisfies the relation $\Delta S^{sw}_{both} = -\gamma(\Delta R^{fm}_{tot,1} + \Delta R^{fm}_{tot,2})$. □

Proof of Proposition D.6. The proof follows the same lines as the proofs of Proposition 2.2, Proposition C.2 and Proposition D.3. By assumption, it is only fund 1 to have first-mover investors. Hence, the change in asset price ΔP_{tot} and the change in fund i's NAV $\Delta S_{tot,i}$, for $i = 1, 2$, are given by the solution to the system of equations (D.1–D.2–D.3) with $\pi_2 = 0$. Therefore, $\Delta R^{fm}_{tot,2} = 0$ and $\Delta Q^{fm}_{tot,2} = 0$. The asymptotic expansions for ΔP_{tot}, $\Delta S_{tot,1}$ and $\Delta S_{tot,2}$ can be computed as in the proof of Proposition D.3. □

Proof of Proposition D.7. The NAV that fund 2's first movers receive is adjusted by an amount ΔS^{adj}. Fund 1's NAV does not get adjusted. Hence, the change in asset price ΔP_{tot} and the change in fund i's NAV $\Delta S_{tot,i}$, for $i = 1, 2$, are given by the solution of the system of equations (D.1–D.2–D.3), where the second line of the system (D.1) gets replaced by the equations

$$\begin{aligned} &-\Delta Q^{fm}_{tot,1} \times (P_0 + \Delta Z + \gamma(\Delta Q^{fm}_{tot,1} + \Delta Q^{fm}_{tot,2})) = \Delta R^{fm}_{tot,1}(S_{0,1} + \Delta Z), \\ &-\Delta Q^{fm}_{tot,2} \times (P_0 + \Delta Z + \gamma(\Delta Q^{fm}_{tot,1} + \Delta Q^{fm}_{tot,2})) = \Delta R^{fm}_{tot,2}(S_{0,2} + \Delta Z + \Delta S^{adj}). \end{aligned}$$

We rewrite the equations in the system (D.3) as

$$\begin{aligned} \Delta P_{tot} &= g_P(\gamma, \Delta S^{adj}, \Delta P_{tot}, \Delta S_{tot,1}, \Delta S_{tot,2}), \\ \Delta S_{tot,1} &= g_{S,1}(\gamma, \Delta S^{adj}, \Delta P_{tot}, \Delta S_{tot,1}, \Delta S_{tot,2}), \\ \Delta S_{tot,2} &= g_{S,2}(\gamma, \Delta S^{adj}, \Delta P_{tot}, \Delta S_{tot,1}, \Delta S_{tot,2}). \end{aligned} \tag{E.4}$$

Next, we compute the asymptotic expansion for small γ of $\Delta S^{sw}_{glob} = \Delta S^{sw}_{glob}(0) + \gamma \frac{\partial \Delta S^{sw}_{glob}(\gamma)}{\partial \gamma}|_{\gamma=0} + \frac{\gamma}{2}\frac{\partial^2 \Delta S^{sw}_{glob}(\gamma)}{\partial \gamma^2}|_{\gamma=0} + o(\gamma^2)$. Proposition D.1 states that, for $\pi_1 = \pi_2 = 0$, the change in NAV of fund 2 is

$$\Delta S_{tot,2} = \Delta Z + \gamma(\beta_1 + \beta_2)\Delta Z + \gamma^2(\beta_1 + \beta_2)^2 \Delta Z + \cdots . \tag{E.5}$$

By definition, the adjustment ΔS^{sw}_{glob} is the one that fund 2 needs to apply to guarantee that $\Delta S_{tot,2}$ admits the asymptotic expansion E.5. Hence, if $\Delta S^{adj} = \Delta S^{sw}_{glob}$, then $\Delta S_{tot,2}(0) = \Delta Z$, $\frac{\partial \Delta S_{tot,2}(\gamma)}{\partial \gamma}|_{\gamma=0} = (\beta_1+\beta_2)\Delta Z$ and $\frac{\partial^2 \Delta S_{tot,2}(\gamma)}{\partial \gamma^2}|_{\gamma=0} = 2(\beta_1+\beta_2)^2\Delta Z$. By letting γ to 0^+ on both sides of each equation in E.4 and using that $\Delta S_{tot,2}(0) = \Delta Z$, we get that $\Delta P_{tot}(0) = \Delta S_{tot,1}(0) = \Delta Z$ and $\Delta S^{adj} = 0$. By differentiating both sides of each equation in E.4 with respect to γ, evaluating the derivatives at $\gamma = 0$ and using that $\frac{\partial \Delta S_{tot,2}(\gamma)}{\partial \gamma}|_{\gamma=0} = (\beta_1 + \beta_2)\Delta Z$, we obtain the values for $\frac{\partial \Delta P_{tot}(\gamma)}{\partial \gamma}|_{\gamma=0}$, $\frac{\partial \Delta S_{tot,1}(\gamma)}{\partial \gamma}|_{\gamma=0}$ and $\frac{\partial \Delta S^{adj}(\gamma)}{\partial \gamma}|_{\gamma=0}$. By differentiating the same equations again, we can compute $\frac{\partial^2 \Delta S^{adj}(\gamma)}{\partial \gamma^2}|_{\gamma=0}$. The resulting values for $\Delta S_{tot,2}(0)$, $\frac{\partial \Delta S_{tot,2}(\gamma)}{\partial \gamma}|_{\gamma=0}$ and $\frac{\partial^2 \Delta S_{tot,2}(\gamma)}{\partial \gamma^2}|_{\gamma=0}$ are the coefficients in the asymptotic expansion of ΔS^{sw}_{glob}.

Proposition D.6 states that if $\pi_1 > 0$ and $\pi_2 = 0$, then fund 2's change in NAV is

$$\Delta S_{tot,2} = \Delta Z + \gamma(\beta_1 + \beta_2)\Delta Z + \gamma^2 \left((\beta_1 + \beta_2)^2 \Delta Z - (E^{\pi}_1)^2 \frac{\text{Rem}_1 + \beta_1(P_0 + \Delta Z)}{(P_0 + \Delta Z)\text{Rem}^{\pi}_1} \right) + \cdots . \tag{E.6}$$

The same methodology used to compute the asymptotic expansion for ΔS^{sw}_{glob}, can be applied to determine the expansion for ΔS^{sw}_{loc}. Because the adjustment ΔS^{sw}_{loc} is such that $\Delta S_{tot,2}$ admits the asymptotic expansion E.6, in this case $\Delta S_{tot,2}(0) = \Delta Z$, $\frac{\partial \Delta S_{tot,2}(\gamma)}{\partial \gamma}|_{\gamma=0} = (\beta_1 + \beta_2)\Delta Z$ and $\frac{\partial^2 \Delta S_{tot,2}(\gamma)}{\partial \gamma^2}|_{\gamma=0} = 2\left((\beta_1 + \beta_2)^2 \Delta Z - (E^{\pi}_1)^2 \frac{\text{Rem}_1 + \beta_1(P_0+\Delta Z)}{(P_0+\Delta Z)\text{Rem}^{\pi}_1} \right)$. The coefficients for the asymptotic expansion of ΔS^{sw}_{loc} can now be found repeating the same procedure used for ΔS^{sw}_{glob}. □

References

Adrian, T. and Shin, H. (2010). Liquidity and leverage. *Journal of Financial Intermediation*, 19(3):418–437.

Association of the Luxembourg Fund Industry (2015). Swing Pricing. *Survey.*

Capponi, A. and Larsson, M. (2015). Price contagion through balance sheet linkages. *Review of Asset Pricing Studies*, 5(2):227–253.

Cetorelli, N., Duarte, F., and Eisenbach, T. (2016a). Are asset managers vulnerable to fire sales? *Liberty Street Economics Blog.*

Cetorelli, N., Duarte, F., Eisenbach, T., and Eisner, E. (2016b). Quantifying potential spillovers from runs on high-yield funds. *Liberty Street Economics Blog.*

Chen, Q., Goldstein, I., and Jiang, W. (2010). Payoff complementarities and financial fragility: Evidence from mutual fund outflows. *Journal of Financial Economics*, 97(2):239–262.

Chernenko, S. and Sunderam, A. (2016). Liquidity transformation in asset management: Evidence from the cash holdings of mutual funds. *NBER Working Paper*, (22391).

Chernenko, S. and Sunderam, A. (2017). Do fire sales create externalities? *Working Paper, Purdue University and Harvard Business School*.

Chordia, T. (1996). The structure of mutual fund charges. *Journal of Financial Economics*, 41(1):3–39.

Coval, J. and Stafford, E. (2007). Asset fire sales (and purchases) in equity markets. *Journal of Financial Economics*, 86(2):479–512.

Diamond, D. W. and Dybvig, P. H. (1983). Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):401–419.

Duarte, F. and Eisenbach, T. (2015). Fire-sale spillovers and systemic risk. *Federal Reserve Bank of New York Staff Reports*, (645).

Edelen, R. M. (1999). Investor flows and the assessed performance of open-end mutual funds. *Journal of Financial Economics*, 53(3):439–466.

Ellul, A., Jotikasthira, C., and Lundblad, C. T. (2011). Regulatory pressure and fire sales in the corporate bond market. *Journal of Financial Economics*, 101(3):596–620.

Feroli, M., Kashyap, A. K., Schoenholtz, K. L., and Shin, H. S. (2014). Market tantrums and monetary policy. *Chicago Booth Paper*.

Ferson, W. E. and Schadt, R. W. (1996). Measuring fund strategy and performance in changing economic conditions. *The Journal of Finance*, 51(2):425–461.

Fricke, C. and Fricke, D. (2017). Vulnerable asset management? The case of mutual funds. *Bundesbank Discussion Paper*, (32).

Goldstein, I., Jiang, H., and Ng, D. T. (2017). Investor flows and fragility in corporate bond funds. *Journal of Financial Economics*, 126(3):592–613.

Greenwood, R., Landier, A., and Thesmar, D. (2015). Vulnerable banks. *Journal of Financial Economics*, 115(3):471–485.

Investment Company Institute (2016). ICI Comment Letter on SEC Liquidity Proposal.

Ippolito, R. A. (1992). Consumer reaction to measures of poor quality: Evidence from the mutual fund industry. *The Journal of Law and Economics*, 35(1):45–70.

Jiang, H., Li, D., and Wang, A. (2017). Dynamic liquidity management by corporate bond mutual funds. *Working Paper, Michigan State University and Federal Reserve Board.*

Koch, A., Ruenzi, S., and Starks, L. (2016). Commonality in liquidity: a demand-side explanation. *The Review of Financial Studies*, 29(8):1943–1974.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society*, pages 1315–1335.

Lewrick, U. and Schanz, J. (2017a). Is the price right? Swing pricing and investor redemptions. *BIS Working Papers*, (664).

Lewrick, U. and Schanz, J. F. (2017b). Liquidity risk in markets with trading frictions: What can swing pricing achieve. *BIS Working Papers*, (663).

Malik, S. and Lindner, P. (2017). On swing pricing and systemic risk mitigation. *IMF Working Paper: On Swing Pricing and Systemic Risk Mitigation*, 17(159).

Morris, S., Shim, I., and Shin, H. S. (2017). Redemption risk and cash hoarding by asset managers. *Journal of Monetary Economics*, 89:71–87.

Securities and Exchange Commission (2016). Investment company swing pricing. *RIN 3235-AL61.*

Sirri, E. R. and Tufano, P. (1998). Costly search and mutual fund flows. *The Journal of Finance*, 53(5):1589–1622.

Warther, V. A. (1995). Aggregate mutual fund flows and security returns. *Journal of Financial Economics*, 39(2-3):209–235.

Zeng, Y. (2017). A dynamic theory of mutual fund runs and liquidity management. *Job Market Paper, Harvard University.*

Zitzewitz, E. (2006). How widespread was late trading in mutual funds? *American Economic Review*, 96(2):284–289.