

Global Effective Lower Bound and Unconventional Monetary Policy *

Jing Cynthia Wu Ji Zhang
Notre Dame and NBER *Tsinghua PBCSF*

First draft: May 29, 2018
Current draft: August 20, 2018

Abstract

In a standard open-economy New Keynesian model, the effective lower bound causes anomalies: output and terms of trade respond to a supply shock in the opposite direction compared to normal times. We introduce a tractable framework to accommodate for unconventional monetary policy. In our model, these anomalies disappear. We allow unconventional policy to be partially active and asymmetric between countries. Empirically, we find the US, Euro area, UK, and Japan have implemented a considerable amount of unconventional monetary policy: the US and Japan follow the historical Taylor rule, whereas the others have done less compared to normal times.

Keywords: effective lower bound, unconventional monetary policy, shadow rate, New Keynesian model, open economy

*We thank Drew Creal, Charles Engel, Jordi Galí, Peter Karadi, Anna Lipińska, Ken West, as well as participants at the NBER International Seminar on Macroeconomics for helpful comments. Correspondence: cynthia.wu@nd.edu, zhangji@pbcfsf.tsinghua.edu.cn.

1 Introduction

Since the Great Recession, many major central banks of developed economies have faced the effective lower bound (ELB) for their policy interest rates and resorted to unconventional monetary policy to provide further stimulus. In this extraordinary environment, how do we evaluate the role of unconventional monetary policy theoretically and empirically?

In a standard New Keynesian model (e.g., Eggertsson and Woodford (2003) for a closed economy and Cook and Devereux (2013a) for an open economy), the ELB yields to the classic liquidity trap. The central bank cannot further reduce the policy rate, and monetary policy is completely absent. However, emerging empirical studies provide overwhelming evidence to demonstrate the effectiveness of unconventional monetary policy; see, for example, Gagnon et al. (2011), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), Bauer and Rudebusch (2014), and Wu and Xia (2016) for its domestic impact, and Neely (2015), Bauer and Neely (2014), Bowman et al. (2015), and Chen et al. (2016) for its global effects.

We propose a tractable New Keynesian model that incorporates unconventional monetary policy into an otherwise standard model to be consistent with empirical findings. We propose a Taylor (1993)-type policy rule to conveniently summarize both conventional and unconventional monetary policy; see Wu and Zhang (2017) for how to implement a negative policy rate via QE, for example. We extend the framework of Wu and Zhang (2017), where unconventional monetary policy follows the historical Taylor rule by construction. In this paper, we relax this assumption and allow unconventional policy to be potentially less effective, and countries can implement them asymmetrically. Our new model nests the traditional model when monetary policy is absent at the ELB and the model in Wu and Zhang (2017) with fully active unconventional monetary policy. We illustrate our new framework with a two-country setup, similar to Clarida et al. (2002) and Cook and Devereux (2013a), but it can be easily extended to the small-open economy.

During normal times, a negative supply shock from the home country leads to lower home output and terms of trade. In our model, if a sufficient amount of unconventional monetary

policy is implemented, the same results apply for the ELB. On the contrary, the standard model implies an opposite movement of output and terms of trade during a liquidity trap, and we will refer to these movements as anomalies.

The basic mechanism that leads to these anomalies consists of two channels. First, it transmits through inflation and the real interest rate, which works the same way as in a closed-economy macro model. A negative supply shock leads to higher inflation for home goods. At the ELB, the nominal rate does not move, which lowers the real rate. The lower real rate stimulates demand and hence the equilibrium output of the home country. In the open economy with complete financial markets, international trade further amplifies this effect through a depreciation in terms of trade.

When we allow the two countries to implement their respective unconventional monetary policy asymmetrically, we find different results for the home and foreign economies. For the home country, its own policy matters the most, whereas the foreign economy relies on both central banks. More active home or foreign policy is associated with higher welfare, and the most efficient case is obtained when both countries' unconventional policies follow their historical policy rules.

We explore alternative model and parameter specifications for robustness. The anomalies that the ELB makes home inflation and output larger than normal are robust when we exclude international trade, implement a fixed exchange rate or incomplete financial markets, replace the negative supply shock with a positive demand shock, or use a CPI-based Taylor rule instead of a PPI-based rule. We also assess across alternative parameter values the robustness of the anomalies that home output and terms of trade move in the opposite direction at the ELB compared to normal times. We find they are not sensitive to structural parameters, including the Frisch elasticity of labor supply, elasticity of intertemporal substitution, and home bias. Results vary more over parameters governing the preference shock, which creates the ELB environment. We find as long as the ELB lasts for several quarters or longer, the anomalies hold.

Finally, we seek empirical evidence for unconventional monetary policy in the United States, Euro area, United Kingdom, and Japan. First, we test model implications by comparing how output responds to a supply shock in a structural vector autoregression (VAR) between normal times and the ELB. We find that for all four countries and regions, output decreases with a negative shock to the growth rate of total-factor productivity (TFP) regardless of normal times or the ELB. This result is in contrast to the anomaly presented in the standard New Keynesian model, and suggests the four central banks have implemented a considerable amount of unconventional monetary policy.

Next, we quantify unconventional monetary policy using the Taylor rule by comparing what has been done with what should have been done according to the historical rule. Both methods point to the same conclusion, whether it's qualitatively from the VAR or quantitatively from the Taylor rule: the US and Japan operate their unconventional monetary policy similarly to the historical Taylor rule, whereas the Euro area and UK have done less unconventionally compared to what they would normally have done.

The rest of the paper after a brief literature review proceeds as follows. [Section 2](#) describes the theoretical model, and we discuss model implications with and without unconventional monetary policy in [Section 3](#). [Section 4](#) assesses empirical evidence for unconventional monetary policy, and [Section 5](#) concludes.

Literature Our paper is related to several recent papers that investigate policy responses in the global low interest rate environment. Cook and Devereux (2013a) analyze the interaction between monetary and fiscal policy in a global liquidity trap with a two-country New Keynesian model. Cook and Devereux (2013b) compare a currency union with a system with a flexible exchange rate. Fujiwara et al. (2013) focus on the optimal monetary policy. Eggertsson et al. (2016) consider the effectiveness of monetary and fiscal policy during the global secular stagnation, using an open-economy overlapping generations model.

Our empirical analysis of the Taylor rule is related to Hakkio and Kahn (2014). The

main difference is we propose alternative ways to compute the quantity for what should have been done, and our methods are less subject to accumulating and compounding errors and uncertainty over time. Our structural VAR results are consistent with Garín et al. (2016) and Debortoli et al. (2016). The difference is the literature has focused on the US, and our analysis encompasses the US, Euro area, UK, and Japan.

The literature has offered alternative models for unconventional monetary policy. For example, Diba and Loisel (2017) model a liquidity premium as an instrument for unconventional monetary policy. The benefit of our framework is its tractability: it is a straightforward extension of the standard New Keynesian model, and can nest various models proposed in the literature; see Wu and Zhang (2017) for details. The literature has also discussed alternative solutions for the ELB. For example, Boneva et al. (2016) and Gust et al. (2017) propose using nonlinear methods, and Cochrane (2017) recommends exploring alternative equilibrium.

The ELB environment is analogous to a currency union for country-specific shocks; see Eggertsson et al. (2014) and Galí and Monacelli (2016). For a discussion of their slight differences, see Erceg and Lindé (2012). Future work could explore how to carry our framework into that environment.

2 Model

This section describes a two-country open-economy New Keynesian model. Many model ingredients are standard and similar to Clarida et al. (2002) and Cook and Devereux (2013a). The main difference is we do not restrict our attention to the standard setup for the ELB, that is, the nominal interest rate is zero and the monetary authority provides no additional stimulus. Instead, we allow a potential role for unconventional monetary policy, which could be completely inactive, fully active, and anywhere in between. See the setup in [Subsection 2.5](#), and economic implications are discussed in [Section 3](#).

The two countries, home and foreign, are the same size and symmetric. Households consume both home and foreign goods with some preference for the domestically produced products. Firms hire labor to produce differentiated goods, and face Calvo (1983)-type price rigidity. The wage paid to workers is determined in a perfectly competitive labor market without any frictions. Complete markets allow perfect international risk sharing. Monetary policy follows a Taylor (1993) rule.

For the most part, we describe the home economy, and the foreign optimization problems are symmetric, which are denoted by an asterisk superscript *. H stands for home-produced goods, and F is foreign goods.

2.1 Households

2.1.1 Optimization problem

Households maximize their utility over consumption and hours worked:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Xi_t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right], \quad (2.1)$$

where \mathbb{E} is the expectation operator and β is the discount factor. Ξ_t is the preference shifter, and its log $\xi_t = \log(\Xi_t)$ follows $\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi t}$, $\varepsilon_{\xi t} \sim N(0, \sigma_\xi^2)$. C_t is consumption and N_t is labor supply. σ is the elasticity of intertemporal substitution and ϕ is the inverse of the Frisch elasticity of labor supply.

Their budget constraint is

$$P_t C_t + \mathbb{E}_t[Q_{t,t+1} B_{t+1}] = B_t + W_t N_t, \quad (2.2)$$

where P_t is the consumer price index (CPI) and W_t is nominal wage. B_{t+1} is the period $t+1$ random payoff of the asset bought at t for B_t , and $Q_{t,t+1}$ is the stochastic discount factor between t and $t+1$.

Households' Euler equation is

$$\beta \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\Xi_t} \left(\frac{C_t}{C_{t+1}} \right)^\sigma \frac{P_t}{P_{t+1}} \right] = \mathbb{E}_t[Q_{t,t+1}] = \frac{1}{R_t}, \quad (2.3)$$

where $R_t = 1/\mathbb{E}_t[Q_{t,t+1}]$ is the short-term nominal interest rate. Their first-order condition for labor supply satisfies

$$\frac{W_t}{P_t C_t^\sigma} = N_t^\phi. \quad (2.4)$$

2.1.2 Consumption allocation

Households consume both home (H) and foreign (F) goods:

$$C_t = \Phi C_{Ht}^{\nu/2} C_{Ft}^{1-\nu/2}, \quad (2.5)$$

where $\Phi = \left(\frac{\nu}{2}\right)^{\frac{\nu}{2}} \left(1 - \frac{\nu}{2}\right)^{1-\frac{\nu}{2}}$. Households have a preference bias for domestic goods: they allocate $\nu/2$ share of their expenditure to domestic goods and $1 - \nu/2$ to imported goods, where $1 < \nu \leq 2$. The demand curves are

$$C_{Ht} = \frac{\nu}{2} \frac{P_t}{P_{Ht}} C_t \quad (2.6)$$

$$C_{Ft} = \left(1 - \frac{\nu}{2}\right) \frac{P_t}{P_{Ft}} C_t, \quad (2.7)$$

and the CPI aggregates over prices for home goods and foreign goods:

$$P_t = P_{Ht}^{\nu/2} P_{Ft}^{1-\nu/2}. \quad (2.8)$$

C_{Ht} is a constant elasticity of substitution (CES) aggregator over differentiated home

goods:

$$C_{Ht} = \left(\int_0^1 C_{Ht}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad (2.9)$$

where the elasticity of substitution $\theta > 1$. The demand curve for each differentiated good i is

$$\frac{C_{Ht}(i)}{C_{Ht}} = \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\theta},$$

where the producer price index (PPI) is

$$P_{Ht} = \left[\int_0^1 P_{Ht}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}.$$

2.2 Inflation, terms of trade, and exchange rate

Inflation The CPI and PPI inflations are

$$\Pi_t = \frac{P_t}{P_{t-1}} \quad (2.10)$$

$$\Pi_{Ht} = \frac{P_{Ht}}{P_{H,t-1}}. \quad (2.11)$$

Terms of trade The terms of trade are defined as the price of foreign goods relative to domestic goods:

$$\mathcal{T}_t = \frac{P_{Ft}}{P_{Ht}}. \quad (2.12)$$

In the log-linear form, the terms of trade can be expressed in terms of the real interest rate differential:

$$\tau_t = \mathbb{E}_t \left[\sum_{k=0}^{\infty} (rr_{t+k}^* - rr_{t+k}) \right], \quad (2.13)$$

where lowercase letters denote logs: $\tau_t = \log(\mathcal{T}_t)$. $rr_t = r_t - \mathbb{E}_t[\pi_{H,t+1}]$ is the home real interest rate, where $r_t = \log(R_t)$ and $\pi_{Ht} = \log(\Pi_{Ht})$, and rr_t^* is the foreign one. Derivation details can be found in [Appendix A](#).

Exchange rate The law of one price holds

$$\mathcal{E}_t = \frac{P_{Ht}}{P_{Ht}^*} = \frac{P_{Ft}}{P_{Ft}^*}, \quad (2.14)$$

where \mathcal{E}_t is the nominal exchange rate. Our baseline model has a flexible exchange rate. The exchange rate and the terms of trade are related by

$$\mathcal{T}_t = \mathcal{E}_t \frac{P_{Ft}^*}{P_{Ht}}. \quad (2.15)$$

Complete international financial markets with international risk sharing implies

$$\frac{\Xi_t}{C_t^\sigma} = \frac{\Xi_t^*}{(C_t^*)^\sigma} \frac{P_t}{\mathcal{E}_t P_t^*} = \frac{\Xi_t^*}{(C_t^*)^\sigma} \mathcal{T}_t^{1-\nu}. \quad (2.16)$$

2.3 Firms

There is a continuum of firms, indexed by $i \in [0, 1]$. Each firm hires labor and produces differentiated goods with the production function:

$$Y_t(i) = A_t N_t(i), \quad (2.17)$$

where A_t is the exogenous technology process and it follows $\log(A_t) - \log(A) = \rho_A [\log(A_{t-1}) - \log(A)] + \varepsilon_{at}$, where $\log(A)$ is the steady-state value and $\varepsilon_{at} \sim N(0, \sigma_a^2)$. Firms' real marginal cost is

$$MC_t = \frac{(1-g)W_t}{A_t P_{Ht}}, \quad (2.18)$$

where g is the wage subsidy for firms to ensure the efficient output level at the steady state.

Firms set prices for differentiated goods in the Calvo fashion. A firm can reset its price with probability $1 - \zeta$ in each period. When it does, it chooses \tilde{P}_{Ht} to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} Y_{t+k}(i) (\tilde{P}_{Ht} - P_{H,t+k} MC_{t+k}),$$

subject to the demand curve

$$Y_{t+k}(i) = \left(\frac{\tilde{P}_{Ht}}{P_{H,t+k}} \right)^{-\theta} Y_{t+k},$$

where the stochastic discount factor is $Q_{t,t+k} = Q_{t,t+1} Q_{t+1,t+2} \dots Q_{t+k-1,t+k}$, and the aggregate output is $Y_t = \left[\int_0^1 Y_t(i)^{1-\frac{1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$. The reset price satisfies

$$\tilde{P}_{Ht} = \frac{\theta}{\theta-1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k} \zeta^k (1-g) W_{t+k} P_{H,t+k}^{\theta} Y_{t+k} / A_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k} \zeta^k P_{H,t+k}^{\theta} Y_{t+k}}. \quad (2.19)$$

Firms keep prices constant when they cannot reoptimize. Finally, the PPI evolves according to

$$P_{Ht} = \left[(1-\zeta) \tilde{P}_{Ht}^{1-\theta} + \zeta P_{H,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (2.20)$$

2.4 Market clearing and welfare

The goods market-clearing condition is

$$Y_t = C_{Ht} + C_{Ht}^*. \quad (2.21)$$

The labor market clears when

$$N_t = \int_0^1 N_t(i) di. \quad (2.22)$$

Welfare W is defined as the second-order approximation of households' lifetime utility. Adding two countries together, the world welfare is

$$W^W = W + W^*. \quad (2.23)$$

2.5 Monetary policy and the effective lower bound

The conventional monetary policy follows a Taylor interest-rate rule:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + (1 - \rho_s) [\phi_\pi \hat{\pi}_{Ht} + \phi_y x_t], \quad (2.24)$$

where hatted variables are log deviations from the steady states $\hat{s}_t = s_t - s$ and $\hat{\pi}_{Ht} = \pi_{Ht} - \pi$, $s_t = \log(S_t)$, $s = \log(S)$, $\pi = \log(\Pi)$, and S and Π are the steady-state nominal interest rate and inflation. $x_t = y_t - y_t^n$ is the output gap, $y_t = \log(Y_t)$, and $y_t^n = \log(Y_t^n)$ is the natural level of output, or the equilibrium output under flexible prices when $\zeta = 0$; see more details in [Appendix A.1](#). ρ_s captures the persistence of the interest-rate rule, and ϕ_π and ϕ_y are the sensitivities of the nominal interest rate to inflation and output, respectively. During normal times, when $s_t \geq 0$, $r_t = s_t$.

Effective lower bound and unconventional monetary policy When the ELB binds $s_t < 0$,¹ the nominal interest rate relevant for the economy is

$$r_t = \lambda s_t. \quad (2.25)$$

The case of $\lambda = 0$ and $r_t = 0$ corresponds to the ELB in the standard New Keynesian model, where the nominal interest rate is stuck at zero, and the central bank's unconventional monetary policy fails to intervene with the economy.

¹For simplicity, we take 0 as the lower bound, and hence the ELB becomes the zero lower bound. In practice, the lower bound does not necessarily have to be zero (see Wu and Xia (2016)) or a constant (see Wu and Xia (2017)).

However, a growing literature argues that unconventional monetary policy has a stimulative effect on the economy that is similar to the effect of conventional policy, which implies $\lambda = 1$; for example, see Wu and Xia (2016), Wu and Zhang (2017), Mouabbi and Sahuc (2017), and Debortoli et al. (2016).

Note (2.25) is a tractable framework to summarize all unconventional monetary policy. Wu and Zhang (2017) theorize how unconventional monetary policy such as QE can be conveniently mapped into a negative shadow interest rate that follows the same Taylor rule, making conventional and unconventional monetary policy comparable.

In this paper, we do not impose $\lambda = 0$ or $\lambda = 1$. Rather, we explore all the possibilities along $\lambda \in [0, 1]$. Quantitative analyses of the theoretical model are in Section 3, and empirical results follow in Section 4.

3 Anomalies at the ELB and unconventional monetary policy

This section first discusses analytically and quantitatively the anomalies at the ELB artificially created by the standard New Keynesian model, which does not capture any effort by the central bank's unconventional monetary policy; that is, $\lambda = 0$ in (2.25). By contrast, we demonstrate these anomalies disappear once unconventional monetary policy is introduced in our model. Next, we relax the model assumptions in two steps. First, we allow different degrees of activeness for unconventional monetary policy $\lambda \in [0, 1]$. Second, we further relax $\lambda = \lambda^*$ and allow the two countries to implement unconventional monetary policy differently, and study their interactions. Finally, we consider several alternative model and parameter specifications.

3.1 Anomalies at the ELB

This section presents the anomalies at the ELB: when a negative supply shock hits the economy, output and terms of trade increase, which is the opposite direction from normal times. These anomalies disappear when unconventional monetary policy is implemented. We first derive some analytical results in a simplified setting to gain some intuition, and then relax the simplifying assumptions and present quantitative results.

3.1.1 Analytical results

In this section, we derive analytical properties to provide some intuition with the following simplifying restrictions: $\rho_s = 0$ and $\xi_t = 0 \forall t$, so that for any variable z_t , we can write $\mathbb{E}_t[z_{t+1}] = \rho_a z_t$. The analytical analysis also imposes $\phi_y = 0$ for simplicity. We create the ELB environment with an interest-rate peg at the steady state by modifying (2.25) to $\hat{r}_t = \lambda \hat{s}_t$ and impose $\lambda = 0$. We find the solution that solves for any generic λ first and then impose $\lambda = 0$ for ELB, and ignore other potential equilibria that only arise at the ELB. We will relax all these assumptions in the quantitative Section 3.1.2.

When a supply shock occurs, the inflation differential, output differential, and terms of trade move with technology as follows:

$$\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^* = -2\Theta(1 + \phi)(1 - \rho_a)\sigma_0\Lambda_a\hat{a}_t \quad (3.1)$$

$$\hat{y}_t - \hat{y}_t^* = \Theta(1 + \phi)(\lambda\phi_\pi - \rho_a)(D + 1)\Lambda_a\hat{a}_t \quad (3.2)$$

$$\hat{\tau}_t = \Theta(1 + \phi)(\lambda\phi_\pi - \rho_a)\frac{\sigma(D + 1)}{D}\Lambda_a\hat{a}_t, \quad (3.3)$$

where $\hat{a}_t = \log(A_t) - \log(A)$, $\hat{y}_t = y_t - y$, $\hat{\tau}_t = \tau_t - \tau$, and $y = \log(Y)$ and $\tau = \log(\mathcal{T}_t)$ are the steady-state values. $\Theta = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa} > 0$, $\Lambda_a = \frac{1}{\Theta(\sigma/D+\phi)(\lambda\phi_\pi-\rho_a)(D+1)+2\sigma_0(1-\rho_a)(1-\beta\rho_a)}$, $D = [(\nu - 1)^2 + \sigma\nu(2 - \nu)] > 0$, $\sigma_0 = \sigma - (1 - \nu/2)(\sigma - 1)\nu\sigma/D > 0$. These equations lead to the following proposition:

Proposition 1 *If $\phi_\pi > 1$ and $\Lambda_a > 0$, $\hat{a}_t < 0$ implies $\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^* > 0$, and*

- $\hat{y}_t - \hat{y}_t^* < 0, \hat{\tau}_t < 0$ when $\lambda = 1$.
- $\hat{y}_t - \hat{y}_t^* > 0, \hat{\tau}_t > 0$ when $\lambda = 0$.

Proof: See [Appendix B.1](#).

The contrast between the two cases in [Proposition 1](#) illustrates the anomalies. To demonstrate the intuition, we ignore the home supply shock's foreign effect for now, which we will see is small in the quantitative section. During normal times $\lambda = 1$, a negative home TPF shock lowers the domestic output and terms of trade when the monetary policy obeys the Taylor principal $\phi_\pi > 1$. By contrast, when the ELB binds and the central bank is completely out of the picture, the same shock stimulates its own economy by raising equilibrium output, and increases the terms of trade. In our setting, unconventional monetary policy in [\(2.24\)](#) and [\(2.25\)](#) works the same as the conventional Taylor rule when it is fully active with $\lambda = 1$. Hence, results for unconventional monetary policy are identical to normal times. $\Lambda_a > 0$ is imposed to guarantee inflation moves in the same direction whether $\lambda = 0$ or 1. It is always satisfied for $\lambda = 1$, and when $\lambda = 0$, it is guaranteed by $0 < \rho_a < \bar{\rho}_a$, where the bound is defined in [Appendix B.1](#).

[Proposition 1](#) makes statements about inflation and output differentials, which we then interpret as home quantities by approximating changes to the foreign economy to zero. To see how the home economy moves without approximation, we'll study the special case of $\sigma = 1$ or $\nu = 2$, in which the home shock does not move across the border. The case $\nu = 2$ corresponds to complete home bias and hence no trade, whereas when $\sigma = 1$, wealth and substitution effects in the foreign economy are completely canceled out.

Corollary 1 *If $(\sigma - 1)(\nu - 2) = 0$, $\phi_\pi > 1$, and $\Lambda_a > 0$, $\hat{a}_t < 0$ implies $\hat{\pi}_{Ft}^* = 0$, $\hat{y}_t^* = 0$, $\hat{\pi}_{Ht} > 0$, and*

- $\hat{y}_t < 0, \hat{\tau}_t < 0$ when $\lambda = 1$.
- $\hat{y}_t > 0, \hat{\tau}_t > 0$ when $\lambda = 0$.

Proof: See [Appendix B.2](#).

Corollary 1 illustrates similar anomalies to Proposition 1. The difference is when $\sigma = 1$ or $\nu = 2$, the foreign economy does not move in response to the home TFP shock. Hence, we can make precise statements about the home economy.

Next, we study how international trade contributes to the anomalies at the ELB in Proposition 1. We compare the case with international trade $\nu < 2$ with the no-trade case $\nu = 2$.

Proposition 2 *If $\lambda = 0$, $1 < \sigma < \phi$ and $\Lambda_a > 0$, $\hat{a}_t < 0$ implies $\hat{y}_t - \hat{y}_t^* \geq (\hat{y}_t - \hat{y}_t^*)|_{\nu=2} > 0$, and $\hat{\tau}_t \geq \hat{\tau}_t|_{\nu=2} > 0$.*

Proof: See [Appendix B.3](#).

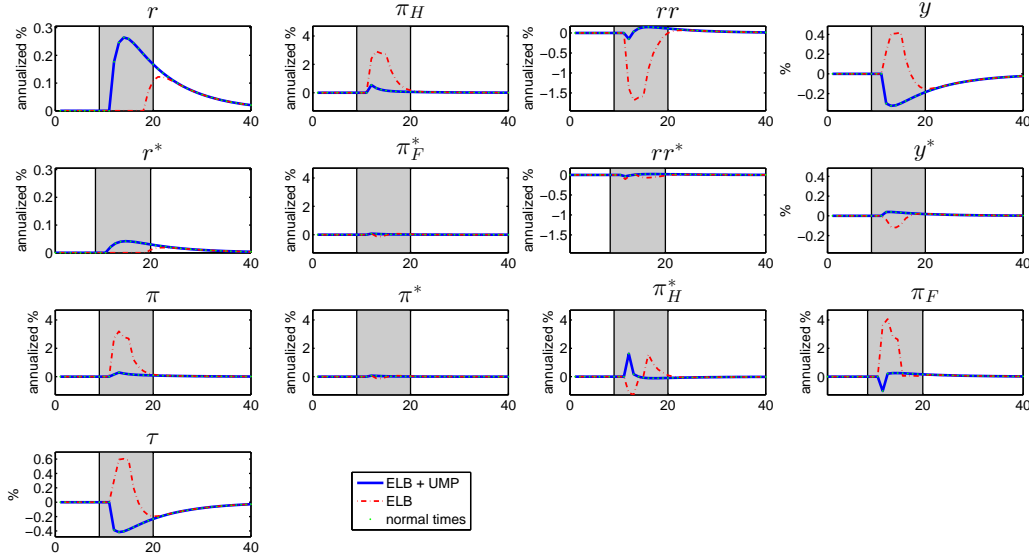
With some mild condition between σ and ϕ , international trade amplifies the impact of the TFP shock on output and terms of trade, which makes the anomalies even more prominent.

3.1.2 Quantitative illustration

Setup for quantitative analysis We set up a quantitative environment here that we will use hereafter, where we relax all the assumptions imposed in Section 3.1.1. We study economies' responses to a negative home TFP shock, which serves as a supply shock. We create the ELB environment with a sequence of preference shocks, and use the occasionally binding method of Guerrieri and Iacoviello (2015) to solve the model. Model details are in [Appendix A](#), and details for calibration and the solution method are in [Appendix C](#).

Results [Figure 1](#) plots impulse response functions of economic quantities to a negative TFP shock in the home country. Green dots are normal times. Red dashed lines represent that the ELB prevails in both countries. The blue lines plot what happens when both countries implement unconventional monetary policy following the historical Taylor rule, that is, $\lambda = \lambda^* = 1$.

Figure 1: **TFP shock in home country**



Notes: A negative technological shock of -0.5% (2 standard deviations) happens in the home country in period 12. To create the ELB environment in the blue and red dashed lines, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23% (about $2/3$ of a standard deviation in each period). We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The blue solid lines represent the case in which fully active unconventional monetary policy is implemented. The red dashed lines represent the case in which, when the policy rate is bounded by zero, no unconventional monetary policy is implemented. The green dotted lines represent normal times with the standard Taylor rule, and the only shock that hits the economy is the TFP shock. The shaded area marks periods 9 - 20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: annualized percentage changes for interest rates and inflation, percentage changes relative to the steady states for output and terms of trade.

Figure 1 illustrates the same anomalies as we discussed in Section 3.1.1. In response to the negative supply shock, output and terms of trade decrease during normal times or with unconventional monetary policy, whereas they increase when the ELB is binding and central banks are absent. Additionally, we also see a contrast for foreign output, albeit in a smaller scale given the origin of the shock.

Next, we calculate welfare, which, unlike variables in Figure 1, are non-linear objects. We compute the total welfare in the presence of all the shocks. We find the case with ELB and no policy intervention suffers from the largest welfare losses for both the home and foreign economies. The losses become much smaller when unconventional monetary policy is

fully active. Note that normal times have slightly higher welfare compared to the case with unconventional monetary policy, because the ELB is created by preference shocks, which introduce some inefficiency.

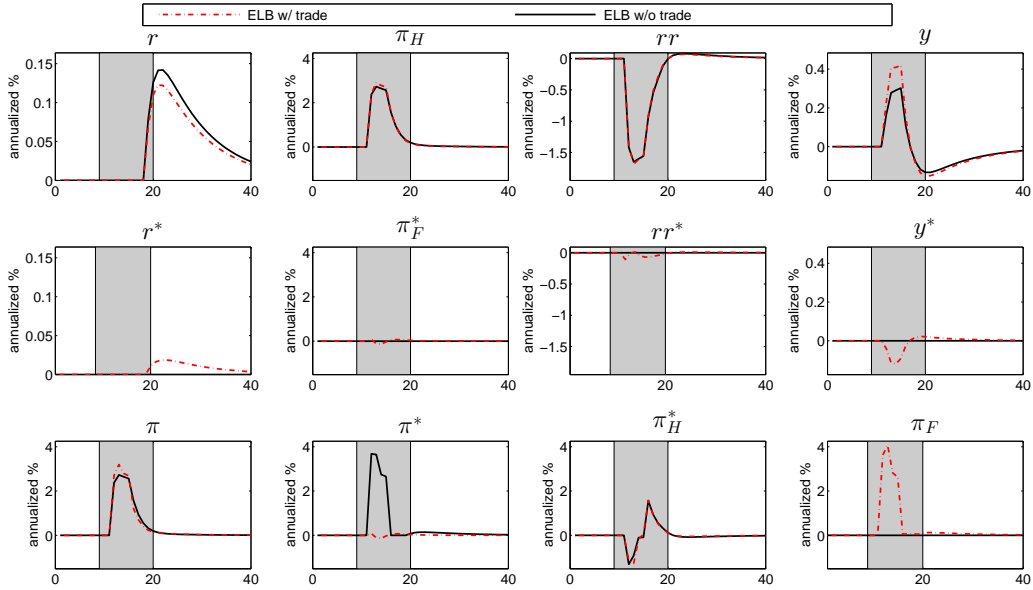
3.1.3 Mechanism

The basic mechanism that leads to these results consists of two channels. First, it transmits through inflation and the real interest rate, which works the same as in a closed-economy macro model; see Wu and Zhang (2017), for example. A negative supply shock leads to a higher inflation for home goods. During normal times or with unconventional monetary policy, the nominal interest rate increases as a response, which leads to a higher real interest rate. However, at the ELB, the nominal rate does not move, which lowers the real rate. The lower real rate stimulates the demand and hence equilibrium output in the home country.

The open-economy model contains an additional international channel. To illustrate this channel, we plot in [Figure 2](#) the ELB cases with and without international trade. The red lines are identical to the red lines in [Figure 1](#). The case without trade is in black, where $\nu = 2$.

Without trade, the foreign economy does not react to the home shock, which is consistent with [Corollary 1](#). International trade brings this shock across the border into the foreign economy, which in turn further amplifies its effect on the home output: the home country is expected to have a lower real interest rate compared to the foreign country, which increases the terms of trade through [\(2.13\)](#). Higher terms of trade imply a decrease in the relative price of home goods, and therefore households shift their demand toward home goods. Hence, international trade reduces foreign output and further increases home production. This result is consistent with [Proposition 2](#).

Figure 2: Trade vs. no trade at the ELB



Notes: A negative technological shock of -0.5% happens in the home country in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The red dashed lines represent the case with trade, and the black solid lines represent the case without trade $\nu = 2$. In both cases, the policy rate is bounded by zero and no unconventional monetary policy is implemented. The shaded area marks periods 9 - 20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade.

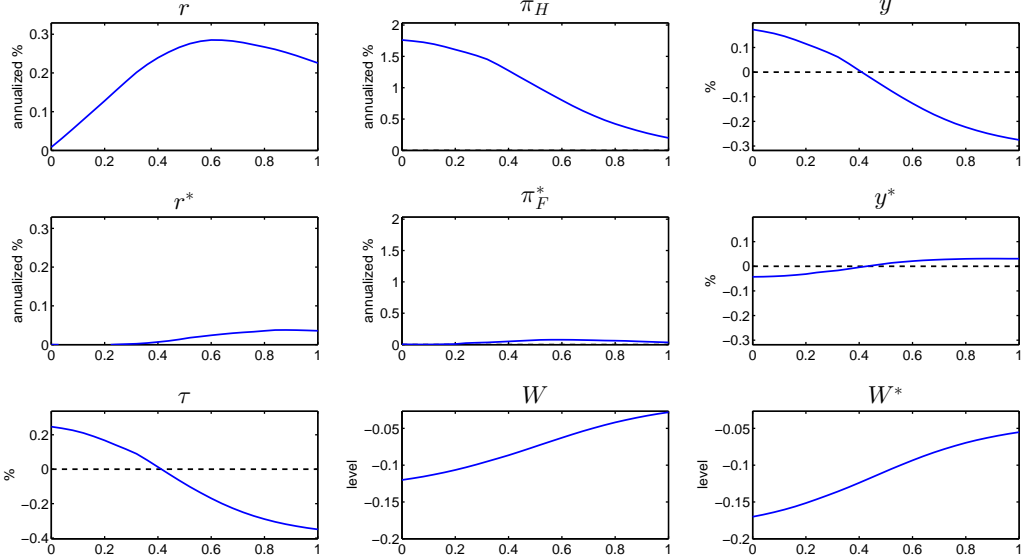
3.2 Partially active unconventional monetary policy

We have studied the limiting cases in which unconventional monetary policy is either completely absent, $\lambda = 0$, or fully active, $\lambda = 1$. In this section, we explore all possibilities along $\lambda \in [0, 1]$ and allow unconventional monetary policy to be partially active. Section 3.2.1 imposes $\lambda = \lambda^*$ that both central banks' interventions are equally active, and we turn to the asymmetric case in Section 3.2.2.

3.2.1 Symmetric case

Figure 3 summarizes the impulse response of each economic variable to the TFP shock to a one-dimensional object and plots it as a function of λ , which is the same as λ^* . For the first

Figure 3: Varying degrees of activeness of unconventional monetary policy

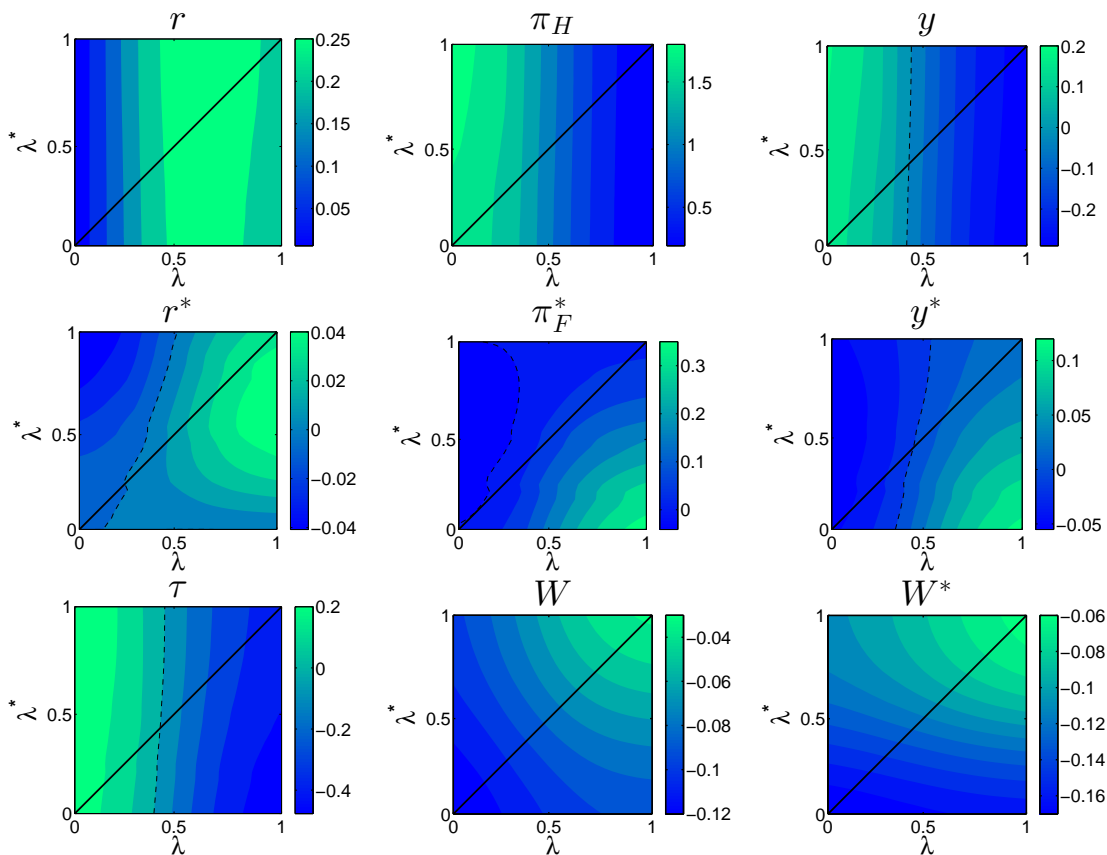


Notes: For all the variables but W and W^* , we plot the average impulse responses from period 12 to the end of the ELB to the home country’s negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: $\lambda = \lambda^*$. Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

seven variables, we plot the average response during the ELB. For $\lambda = 0$ ($\lambda = 1$), they are equal to the average values of the red (blue) lines from period 12 to 19 (20) in Figure 1. Both the home output and terms of trade decrease from positive to negative as unconventional monetary policy becomes more active, whereas the foreign output increases from negative to positive. Lifetime welfare, W and W^* , increases when unconventional monetary policy becomes more active, and fully active unconventional monetary policy is the closest to being efficient.

Interestingly, the home country’s nominal interest rate does not increase monotonically with λ . Combining (2.24) and (2.25), the nominal rate depends on the products $\lambda\pi_{Ht}$ and λy_t . While λ increases, both inflation and output decrease. For small λ , when λ increases, r_t increases. At some point, the rate of decrease in π_{Ht} and y_t outweighs the increase of λ , and hence r_t decreases.

Figure 4: **Asymmetric unconventional monetary policy**



Notes: For all the variables but W and W^* , we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: λ ; Y-axis: λ^* . The color from light green to dark blue corresponds to high to low values. The units are annualized percentage for interest rates and inflation, percentage for output and terms of trade, and level for welfare. The 45-degree lines represent the symmetric case $\lambda = \lambda^*$. The dashed lines are the 0 contours.

3.2.2 Asymmetric case

Next, we further relax the assumption $\lambda = \lambda^*$ and allow two countries to operate their unconventional monetary policy differently, and study their interactions. Figure 4 plots summary responses, which are defined in Section 3.2.1, to the TFP shock as functions of λ and λ^* . Different colors represent different values for economic quantities, where light green (dark blue) represents higher (lower) values. The 45-degree lines correspond to the symmetric case $\lambda = \lambda^*$ in Figure 3.

For the home country, mainly its own policy matters: the more active its central bank

is in implementing unconventional monetary policy, the lower its output and inflation. It is similar for the terms of trade: a higher λ is associated with smaller terms of trade.

The foreign economy, as well as welfare, relies on both central banks. A more active home unconventional policy or a less active foreign policy is associated with higher foreign inflation and output. For welfare, more active home or foreign policy is associated with higher welfare. The most efficient case happens when both countries' policies are fully active.

3.3 International dimensions

This and the next subsection explore alternative specifications and serve as a robustness check, with this section focusing on the international dimensions. Section 3.3.1 excludes trade, Section 3.3.2 studies fixed exchange rate, and Section 3.3.3 investigates incomplete financial markets.

3.3.1 No-trade case

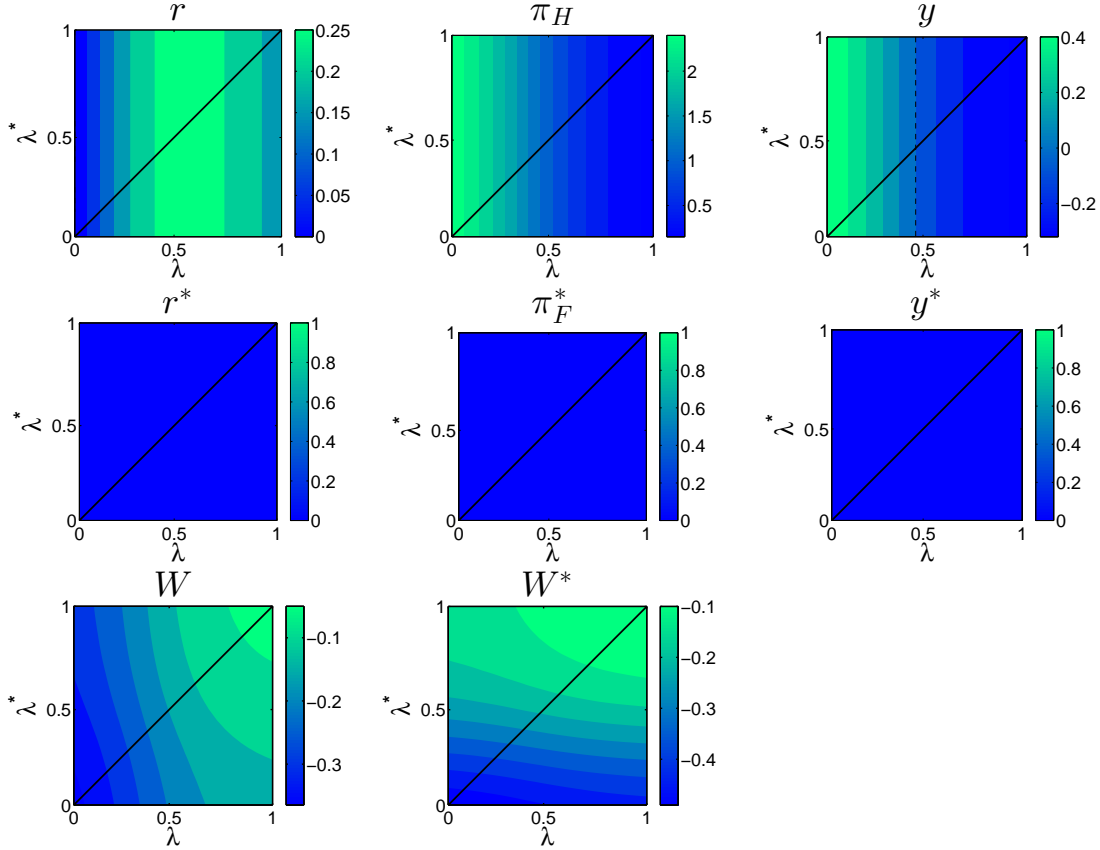
Figure 5 plots the summary responses to the TFP shock as functions of both λ and λ^* for the case with no international trade, which is instrumented by $\nu = 2$. Unlike in Figure 4, the home economic indicators only move with the home policy indicator λ . The foreign economy in the second row does not move regardless of monetary policy. Welfare, on the other hand, still depends on monetary policies of both countries.

3.3.2 Fixed exchange rate

Our baseline model implements a flexible exchange rate. An alternative is a fixed exchange rate regime, and we implement it by letting the foreign country follow the home country's interest rate and react to the exchange rate:

$$\hat{r}_t^* = \hat{r}_t - \phi_e \hat{e}_t, \tag{3.4}$$

Figure 5: No-trade case

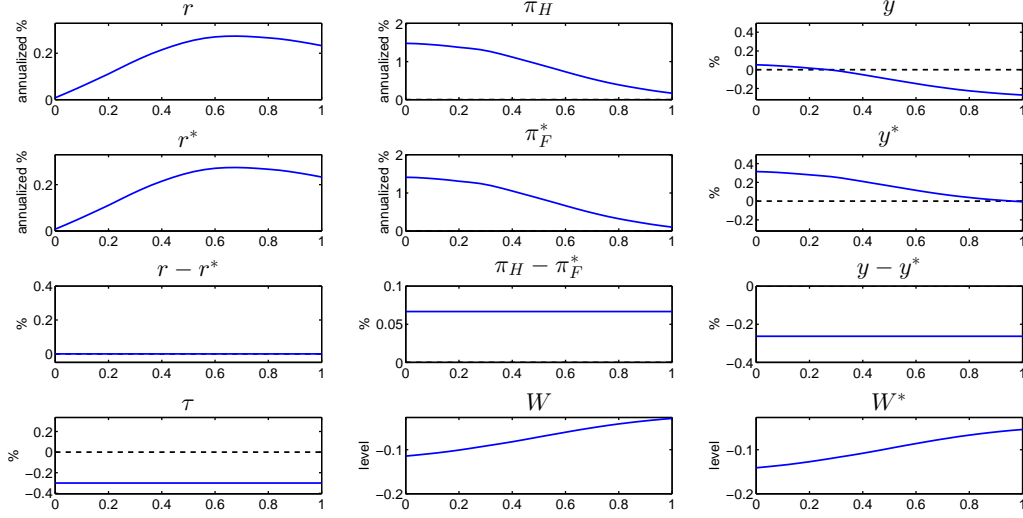


Notes: For all the variables but W and W^* , we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: λ ; Y-axis: λ^* . The color from light green to dark blue corresponds to high to low values. The units are annualized percentage for interest rates and inflation, percentage for output, and level for welfare. The 45-degree lines represent the symmetric case $\lambda = \lambda^*$. The dashed lines are the 0 contours. The no-trade case is created by $\nu = 2$.

where $\hat{e}_t = \log(\mathcal{E}_t) - \log(\mathcal{E})$, and $\mathcal{E} = 1$ is the steady-state exchange rate. ϕ_e is the sensitivity of the foreign nominal interest rate to the exchange rate, and $\phi_e > 0$ ensures the equilibrium $r_t^* = r_t$, and $\hat{e}_t = 0$. See [Appendix B.5](#) for details.

Imposing the same restrictions as in [Section 3.1.1](#), we can solve the economy as functions

Figure 6: Fixed exchange rate



Notes: For all the variables but W and W^* , we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: $\lambda = \lambda^*$. Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

of TFP:

$$\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^* = 2\sigma_0\Theta(1 + \phi)(1 - \rho_a)\Lambda_a^F \hat{a}_t \quad (3.5)$$

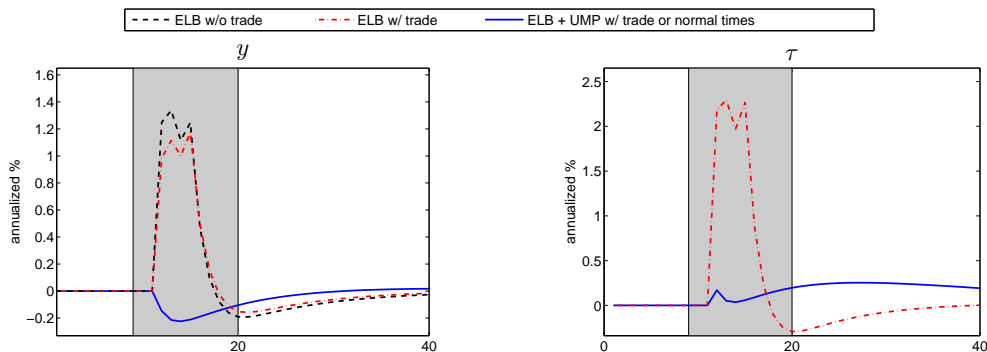
$$\hat{y}_t - \hat{y}_t^* = 2\rho_a(1 + \phi)(D + 1)\Lambda_a^F \hat{a}_t \quad (3.6)$$

$$\hat{\tau}_t = \frac{2\rho_a\sigma}{D}(1 + \phi)(D + 1)\Lambda_a^F \hat{a}_t, \quad (3.7)$$

where $\Lambda_a^F = \frac{1}{2\sigma_0(1-\rho_a)(1-\beta\rho_a)-\Theta\rho_a(\sigma/D+\phi)(D+1)}$. Equations (3.5) - (3.7) show that with the fixed exchange-rate rule, the terms of trade, inflation differential, and output differential behave the same regardless of how active unconventional monetary policy is. See [Appendix B.5](#) for derivation. [Figure 6](#) illustrates the same pattern numerically when we relax the assumptions on the parameters; see the third row and τ .

Compared to [Figure 3](#), [Figure 6](#) shows that for smaller λ , while the home economy

Figure 7: **Incomplete markets**



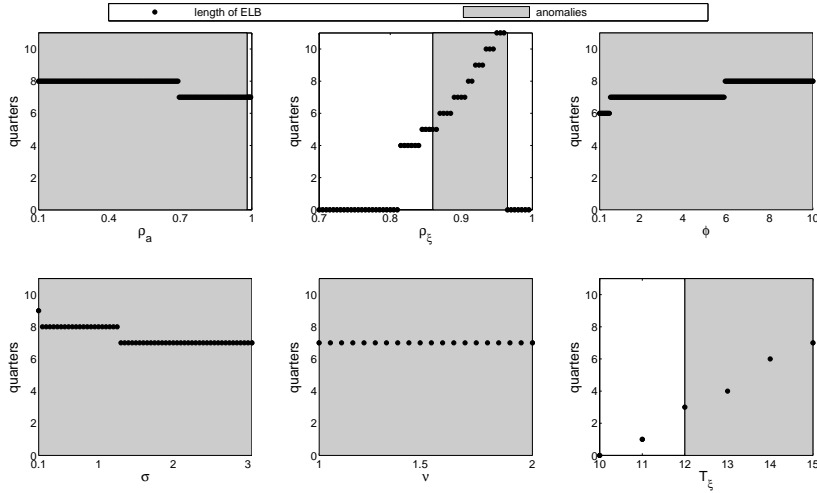
Notes: The international financial markets are incomplete. A negative technological shock of -0.5% happens in the home country in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks, and only plot the additional effect of the technological shock. The black dashed line represents the ELB case without trade, the red dotted-dashed lines represent the case with trade, and the blue solid lines represent the case with trade and UMP, which is the same as in normal times. The shaded area marks periods 9 - 20, for which both countries stay at the ELB with only the preference shocks and without unconventional monetary policy. X-axis: time in quarters; Y-axis: percentage changes relative to the steady states.

seems to be less volatile under the fixed exchange rate rule, the foreign economy is much more susceptible to the home shock. Here is the basic mechanism. As a result of the negative TFP shock, the inflation rate for home goods increases. With a fixed exchange rate, the complementarity between the home and foreign goods puts upward pressure on the price of foreign goods and downward pressure on that of home goods. In equilibrium, the two inflation rates increase by similar amounts. When monetary policy intervention is insufficient, real interest rates decrease both domestically and internationally. Consequently, this shock stimulates the home and foreign economies. The foreign output increases more than the domestic counterpart, because the TFP shock has a direct negative impact on the home output. Similar to [Figure 3](#), welfare in [Figure 6](#) also increases when unconventional monetary policy becomes more active.

3.3.3 Incomplete financial markets

The benchmark model in [Section 2](#) and our analyses thus far assume international financial markets are complete. This section examines incomplete markets. The contrast between

Figure 8: **Anomalies with alternative parameter values**



Notes: X-axis: ρ_a in the top left panel, ρ_ξ in the top middle panel, ϕ in the top right panel, σ in the bottom left panel, ν in the bottom middle panel, and T_ξ in the bottom right panel. Y-axis: time in quarters. Black dots: the number of ELB periods after the TFP shock. Gray shades: anomalies exist; white areas: anomalies do not exist.

the red dashed line and blue line in the left panel of [Figure 7](#) demonstrates the anomaly discussed in [Subsection 3.1](#) still exists for output. See details of the model in [Appendix D](#).

However, whether the financial markets are complete or not does affect trade-related quantities. When the market is complete, terms of trade decrease normally in response to a negative home TFP shock. However, they increase in the setting of incomplete financial markets, which is consistent with what [Enders and Müller \(2009\)](#) find. Moreover, international trade lowers how much output increases at the ELB, mitigating the anomaly.

3.4 Alternative specifications

This section continues exploring alternative specifications: [Section 3.4.1](#) explores alternative parameter spaces. [Section 3.4.2](#) focuses on a demand shock, and [Section 3.4.3](#) assesses an alternative monetary policy rule.

3.4.1 Alternative parameter space

This section assesses the robustness of anomalies discussed in [Subsection 3.1](#) across alternative parameter values, where we define anomalies when the maximum response of y and τ are positive at the ELB.²

[Figure 8](#) illustrates the existence of anomalies when we vary the persistence of the TFP dynamics ρ_a , the persistence of the preference shifter ρ_ξ , Frisch elasticity of labor supply ϕ , elasticity of intertemporal substitution σ , home bias ν , and the length of preference shocks T_ξ , one at a time, and set other parameters as in the baseline calibration. Gray areas mark that anomalies exist, whereas white areas correspond to the parameter space where anomalies do not appear.

The anomalies are not sensitive to structural parameters $\rho_a, \phi, \sigma, \nu$: they exist as long as $\rho_a < 0.98$. This finding is consistent with the condition $0 < \rho_a < \bar{\rho}_a$ in [Proposition 1](#) that guarantees $\Lambda_a > 0$. They always exist for all $\phi \in [0.1, 5]$, $\sigma \in [0.1, 3]$, and $\nu \in [1, 2]$.

Results vary more over parameters related to the preference shock. The gray shades correspond to $0.86 \leq \rho_\xi \leq 0.9650$ or $T_\xi \geq 12$. Fundamentally, whether anomalies exist depends on how long the ELB lasts, which varies substantially over ρ_ξ and T_ξ . When ρ_ξ is too small or too large or when T_ξ is too small, the number of ELB periods is not large enough to generate anomalies. In the case of $\rho_\xi (T_\xi)$, anomalies are supported if ELB lasts six (three) quarters or longer.

3.4.2 Demand shock

We have presented our main results with a supply shock. In this section, we illustrate what happens when a demand shock occurs. To derive analytical results, we impose the same simplifying restrictions as in [Section 3.1.1](#). To obtain implications for the home economy,

²Results for an alternative definition, the average response of y or τ being positive, are very similar.

we further impose $\nu = 2$ and express the economies as follows:

$$\hat{y}_t = (1 - \rho_\xi)(1 - \beta\rho_\xi)\Lambda_\xi\hat{\xi}_t \quad (3.8)$$

$$\hat{\pi}_{Ht} = \Theta(\sigma + \phi)(1 - \rho_\xi)\Lambda_\xi\hat{\xi}_t \quad (3.9)$$

$$\hat{\tau}_t = -\Theta(\sigma + \phi)(\lambda\phi_\pi - \rho_\xi)\Lambda_\xi\hat{\xi}_t \quad (3.10)$$

$$\hat{y}_t^* = \hat{\pi}_{Ft}^* = 0, \quad (3.11)$$

where $\hat{\xi}_t = \xi_t - \xi$ and $\Lambda_\xi = \frac{1}{\Theta(\sigma+\phi)(\lambda\phi_\pi-\rho_\xi)+\sigma(1-\rho_\xi)(1-\beta\rho_\xi)}$. Hence,

Proposition 3 *If $\nu = 2$, $\phi_\pi > 1$, and $\Lambda_\xi > 0$, $\hat{\xi}_t > 0$ implies $\hat{\pi}_{Ft}^* = 0$, $\hat{y}_t^* = 0$, and*

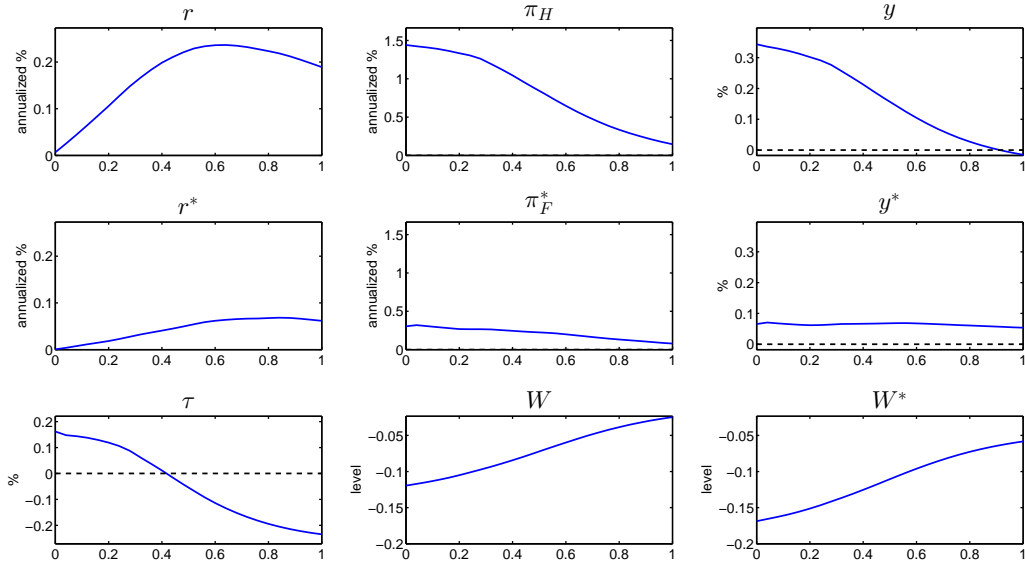
- $\hat{\pi}_{Ht}|_{\lambda=0} > \hat{\pi}_{Ht}|_{\lambda=1} > 0$
- $\hat{y}_t|_{\lambda=0} > \hat{y}_t|_{\lambda=1} > 0$
- $\hat{\tau}_t|_{\lambda=1} < 0$, $\hat{\tau}_t|_{\lambda=0} > 0$.

Proof: See [Appendix B.4](#).

When a positive demand shock occurs, inflation and output always increase regardless of whether monetary policy is implemented. The anomalies exist in the sense that the ELB amplifies the movements. Moreover, the terms of trade still move in opposite directions for $\lambda = 0$ and $\lambda = 1$. Similar to Λ_a in the case of a supply shock, we impose $\Lambda_\xi > 0$ to guarantee inflation moves in the same direction for $\lambda = 0$ and $\lambda = 1$. $\Lambda_\xi > 0$ is guaranteed by $0 < \rho_\xi < \bar{\rho}_\xi$, where the bound is defined in [Appendix B.4](#).

For numerical analysis, we relax parameter restrictions and introduce a preference shock in period 12 to serve as a positive demand shock. [Figure 9](#) illustrates how impulse responses vary over the unconventional monetary policy indicator λ . Many of the patterns look very similar to those in [Figure 3](#). Inflation and output in the home country and terms of trade decrease when λ increases. While home output stays mostly positive, terms of trade move from positive to negative. On the other hand, welfare increases when unconventional monetary

Figure 9: Demand shock



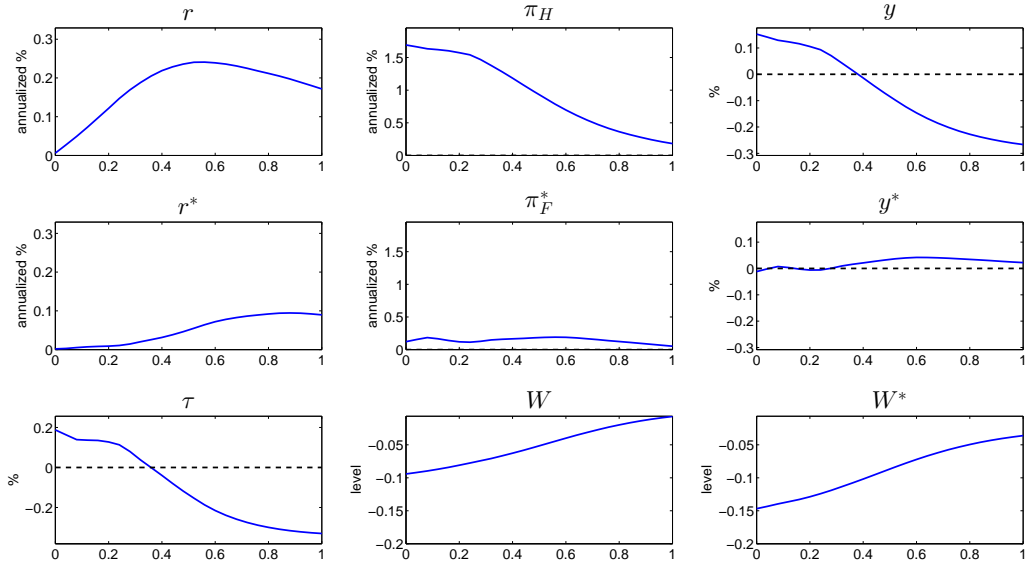
Notes: For all the variables but W and W^* , we plot the average impulse responses from period 12 to the end of the ELB to the home country's preference shock of 0.77% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23%. We difference out the effect of preference shocks and only plot the additional effect of the negative preference shock. W and W^* are the discounted lifetime welfare. X-axis: $\lambda = \lambda^*$. Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

policy becomes more active. The home country's nominal interest rate displays a humped shape.

Whether it is a negative supply shock or a positive demand shock, the basic mechanism works similarly through the inflation and real interest-rate channel. A positive home demand shock increases its CPI inflation, which involves both home and foreign goods. At the ELB, the nominal rates do not move, which lowers the real rates. Lower real rates further stimulate demand. When unconventional monetary policy is active, the nominal interest rates in both countries increase in response, which increases the real interest rates and lowers demand. Due to the origin of the shock, the movement of the domestic real rate is larger than its foreign counterpart. Therefore, at the ELB, real rates decrease and terms of trade increase, whereas with fully active unconventional monetary policy, terms of trade decrease.

The difference between the demand and supply shocks is their direct impact on output.

Figure 10: **CPI vs. PPI - based Taylor rule**



Notes: For all the variables but W and W^* , we plot the average impulse responses from period 12 to the end of the ELB to the home country's negative TFP shock of -0.5% in period 12. To create the ELB environment, a series of negative preference shocks occurs in both countries in periods 1 - 15, and the total shock size in each country is 23% . We difference out the effect of preference shocks and only plot the additional effect of the technological shock. W and W^* are the discounted lifetime welfare. X-axis: $\lambda = \lambda^*$. Y-axis: annualized percentage changes for interest rates and inflations, percentage changes relative to the steady states for output and terms of trade, and level for welfare.

Whereas a positive demand shock increases consumption, a negative supply shock decreases output. Therefore, the equilibrium output is higher in the case of a demand shock.

3.4.3 CPI - based Taylor rule

Our baseline specification of the Taylor rule in (2.24) relies on the PPI inflation. A viable alternative is to have the central bank respond to the CPI inflation instead:

$$\hat{s}_t = \rho_s \hat{s}_{t-1} + (1 - \rho_s) [\phi_\pi \hat{\pi}_t + \phi_y x_t]. \quad (3.12)$$

Figure 10 shows how economic quantities vary with $\lambda = \lambda^*$ when the central bank adopts the alternative Taylor rule. The economies behave similarly to those with the PPI-based rule in Figure 3. The impulse responses for the domestic economy and the terms of trade are lower

if the monetary policy is implemented based on the CPI inflation for most λ , whereas the foreign quantities are higher in this case.

4 Empirical evidence on unconventional monetary policy

This section empirically investigates unconventional monetary policies at the ELB in the US, Euro area, UK, and Japan, and compares them with their corresponding conventional policies. First, we test model implications by comparing impulse responses in a VAR between normal times and the ELB. This exercise allows us to draw some qualitative conclusions for the four countries and regions. Next, to further draw some quantitative conclusions, we rely on the Taylor rule to compare what has been done with what should have been done.

4.1 Vector autoregression

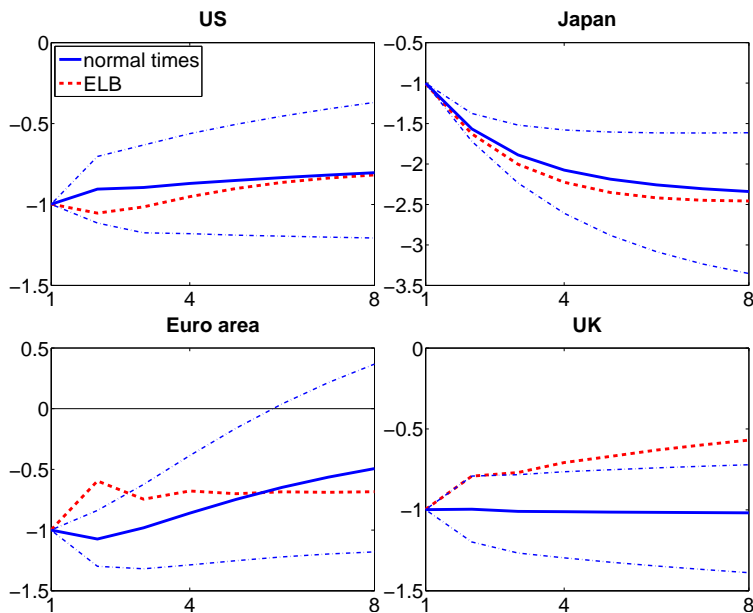
This section analyzes unconventional monetary policy in a VAR framework. We quantify empirically how output responds to a TFP shock in the US, Euro area, UK, and Japan. Then we compare our empirical results with implications from our theoretical model in [Section 3](#) to draw conclusions.

Following Galí and Gambetti (2009), we measure TFP with labor productivity. Our VAR has two variables: the growth rate of labor productivity³, $\Delta(y_t - n_t)$, and the log of per-capita hours, n_t . We use a first-order VAR due to the short sample in the quarterly frequency. We identify TFP shocks through the Cholesky decomposition by ordering labor productivity first, which assumes a shock to hours has no contemporaneous impact on labor productivity growth.

We estimate the VAR for the pre-ELB and ELB samples separately. The two samples

³We use the growth rate for stationarity.

Figure 11: **Impulse response of output to a productivity shock**



Notes: Impulse responses of output to a -1% shock to labor productivity growth. The blue lines are normal times, with solid being medians and dashed lines representing 90% confidence intervals. The red dashed lines are the median impulse responses at the ELB. X-axis: time in quarters; Y-axis: percentage changes in output.

span from 1985Q2 - 2007Q4⁴ and 2009Q1 - 2015Q4 for the US, 1999Q1 - 2011Q3 and 2011Q4 - 2017Q4 for the Euro area, 1993Q1 - 2009Q1 and 2009Q2 - 2017Q4 for the UK, and 1981Q1 - 1999Q2 and 1999Q3 - 2017Q4 for Japan. The detailed data sources for the four countries and regions are in [Appendix E](#).

[Figure 11](#) plots impulse responses of output to a -1% shock to labor productivity growth for the four countries and regions.⁵ Blue represents normal times with medians in the solid lines, and 90% confidence intervals in the dashed lines. Red represents the central tendencies at the ELB. We find that for all four countries and regions, output decreases with a negative TFP shock regardless of normal times or the ELB. This similarity result is in contrast to the anomaly presented by the standard New Keynesian model in [Subsection 3.1](#), and is evidence for unconventional monetary policy.

The top left panel is for the US. We find the impulse response at the ELB is initially

⁴We end the pre-ELB sample prior to the Great Recession.

⁵Output is calculated as $y_{t+j} = \sum_{\tau=0}^j \Delta(y_{t+\tau} - n_{t+\tau}) + n_{t+j}$, where $y_{t-1} - n_{t-1} = 0$.

slightly lower than normal times, and then the red and blue lines track each other closely after five quarters. Moreover, the red line is within the confidence interval in blue. This result suggests $\lambda \approx 1$ and is potentially slightly larger than 1, which is consistent with Garín et al. (2016) and Debortoli et al. (2016). We find a similar comparison in the top right panel for Japan, which is consistent with Wieland’s (2017) findings.

The bottom left panel is for the Euro area, and the bottom right panel is for the UK. Both of them show that output decreases less at the ELB than in normal times. The differences between the red and blue lines are statistically significant in both cases, with the UK being more pronounced.

These findings suggest unconventional monetary policy is as active as usual in the US and Japan, and it is less active for the Euro area and UK. Overall, we find $\lambda_{US} \approx \lambda_{Japan} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$.

4.2 Taylor rule

In [Subsection 4.1](#), the VAR qualitatively sorts the effectiveness of unconventional monetary policy among the four regions and countries. However, the key parameter λ enters the VAR non-linearly, and consequently, the VAR does not provide a direct numerical measure for it. For this purpose, we direct attention to the Taylor rule in this section.

We calculate how active unconventional monetary policy is by comparing what has been done at the ELB, measured by the shadow rates of Wu and Xia (2016) and Wu and Xia (2017),⁶ with the interest rates implied by the historical Taylor rule.

We estimate the historical Taylor rule,

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 \pi_{Ht} + \beta_3 x_t + \varepsilon_t, \tag{4.1}$$

which is the empirical version of [\(2.24\)](#), via ordinary least squares, using the pre-ELB sample,

⁶Shadow rates are available for the US, the Euro area, and UK, and they can be downloaded from Cynthia Wu’s website: <https://sites.google.com/view/jingcynthiawu/shadow-rates>.

and label the estimates $\tilde{\beta}_0, \tilde{\beta}_1, \tilde{\beta}_2, \tilde{\beta}_3$. When the ELB is binding, the desired interest rate implied by the historical Taylor rule can be calculated as follows:

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 s_{t-1} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \quad (4.2)$$

Next, we calculate the activeness of unconventional monetary policy by comparing the implemented monetary policy at ELB and the desired interest rate. Specifically, we regress observed shadow rate r_t on the imputed \tilde{s}_t per (2.25).

Now we turn our attention to s_{t-1} in (4.2). We propose two methods below to proxy it.

Simple method The simple method uses the observed shadow rate r_{t-1} as a proxy for s_{t-1} . Hence, (4.2) becomes

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 r_{t-1} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \quad (4.3)$$

The benefit of the simple method is that the shadow rate is observable to us. Hence, the calculation is simple and robust.

Iterative method To measure s_{t-1} more accurately in the case of small λ , we leverage the relationship in (2.25), and replace (4.2) with

$$\tilde{s}_t = \tilde{\beta}_0 + \tilde{\beta}_1 r_{t-1} / \tilde{\lambda} + \tilde{\beta}_2 \pi_{Ht} + \tilde{\beta}_3 x_t. \quad (4.4)$$

Now we face a fixed-point problem: (4.4) relies on $\tilde{\lambda}$ to compute \tilde{s}_t , whereas to obtain $\tilde{\lambda}$, we regress r_t on \tilde{s}_t . We propose an iterative procedure to solve this fixed-point problem. First, we give an initial guess for λ : $\tilde{\lambda}^{(0)}$. Then we iterate over the following two steps until convergence:

1. Based on $\tilde{\lambda}^{(i)}$, compute $\{\tilde{s}_t^{(i)}\}_{t=1}^T$ using (4.4).

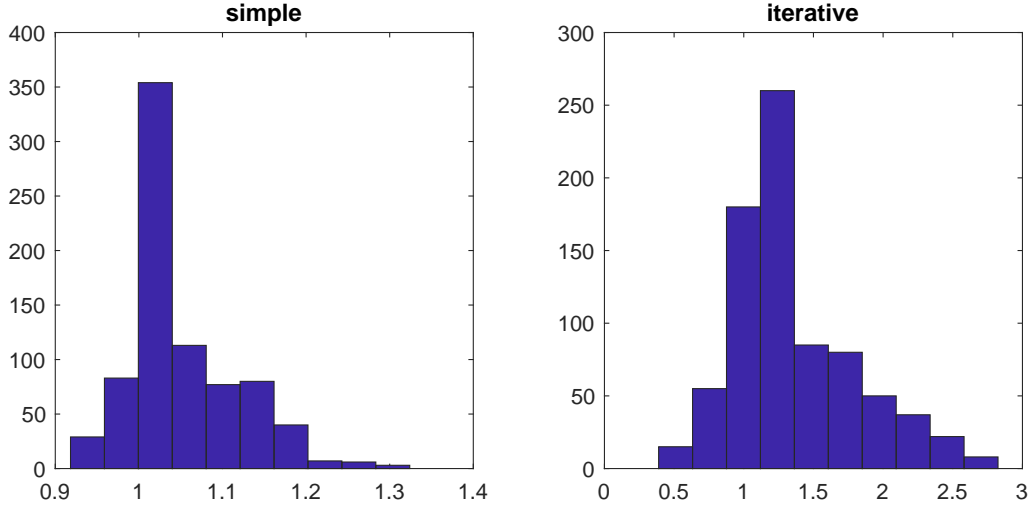
2. Regress r_t on $\tilde{s}_t^{(i)}$ and compute $\tilde{\lambda}^{(i+1)}$.

Empirical results We begin with the US. We measure π_{Ht} with the inflation based on the GDP price deflator, x_t is the real GDP minus potential GDP, and before the ELB, r_t corresponds to the effective fed funds rate. The pre-ELB and ELB samples are the same as in [Subsection 4.1](#). The details of the data are in [Appendix E](#). The estimate of the simple method is 1.02, and is 1.12 from the iterative method. We conclude that the US unconventional monetary policy is as active as, if not more active than, the historical Taylor rule.

The Taylor rule is known to vary over different sample periods, and researchers' choices of sample periods in the literature are far from unanimous. We quantify the variation of our estimates by varying the pre-ELB estimation sample: the beginning of the sample ranges from $t_0 \in \{1982Q1 : 1990Q1\}$ and the end of the sample varies from $t_1 \in \{2003Q1 : 2008Q4\}$, which covers the majority of popular choices. We compute a λ for each combination of t_0 and t_1 and plot its distribution across all possible combinations in [Figure 12](#). The left panel plots the distribution for the simple method, and the right panel uses the iterative method. They both center around 1: the median for the simple method is 1.03, and is 1.19 for the iterative method. The standard error for the simple method is 0.065, and is 0.45 for the iterative method. The iterative method displays a larger variation across different sample periods than the simple method. On the other hand, the results from the simple method might be biased if λ is far from 1. This is the classic bias-variance tradeoff. The high persistence of the interest rate r_t contributes to both issues.

For the Euro area and UK, quarterly real potential GDP is not available. Hence, we replace x_t in [\(4.1\)](#) with output growth Δy_t , measured by the growth rate of real GDP. The pre-ELB r_t for the Euro area is the 3-month Euro Interbank Offered Rate (Euribor), and it is the Bank of England policy rate for the UK. The details of the data are in [Appendix E](#). For the Euro area, $t_0 \in \{1998Q2 : 1999Q1\}$ and $t_1 \in \{2009Q1 : 2011Q3\}$. The ELB

Figure 12: **Distribution of λ for the US**



Notes: $t_0 \in \{1982Q1 : 1990Q1\}$, and $t_1 \in \{2003Q1 : 2008Q4\}$. For each combination of t_0 and t_1 , estimate a λ from t_0 to t_1 . Then plot the distribution across all possible combinations of t_0 and t_1 . Left panel: simple method; right panel: iterative method.

period is from $t_1 + 1$ to 2017Q4. The median estimate for the simple (iterative) method is 0.998 (0.63) with a standard error of 0.031 (1.07). For the UK, $t_0 \in \{1993Q1 : 2003Q1\}$, $t_1 = 2009Q1$, and the ELB period is from 2009Q2 to 2017Q4. The median from the simple (iterative) method is 0.98 (0.39), with a standard error of 0.10 (4.10). Note the variations across samples are larger for the Euro area - especially for the UK - than for the US, partly due to a shorter sample.

In summary, all three central banks have implemented a considerable amount of unconventional monetary policy: the US operates following the historical Taylor rule, the Euro area and UK are less active, or $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$, which is consistent with what we find in [Subsection 4.1](#).

Both the VAR and Taylor-rule based methods are used to measure how active unconventional monetary policy is. There are some subtle differences between them. The VAR measures the effectiveness of unconventional policy on the real economy, whereas the shadow rate measures how much it changes the yield curve. The fact that we get coherent results from both methods implies the transmission from the yield curve to the real economy is

similar across these countries and regions, and the key difference among them is how much monetary policy changes the yield curve and hence the shadow rate.

5 Conclusion

We have introduced a new open-economy New Keynesian model. Our model provides a tractable framework that allows for unconventional monetary policy when the ELB is binding. We find when unconventional monetary policy operates following the historical Taylor rule, the anomalies in a standard model, namely, that output and terms of trade increase in response to a negative supply shock, disappear. The anomalies in the standard model are robust to alternative model and parameter specifications. Our model allows unconventional policy to be partially active and potentially asymmetric between the two countries. Empirically, we assess unconventional monetary policy across the US, Euro area, UK, and Japan. Both the VAR analysis and the Taylor rule point to the same conclusion: the US and Japan have operated their unconventional monetary policy following the historical Taylor rule. Although both the Euro area and UK have also implemented a considerable amount of unconventional policies, they have done less than what they normally would have.

References

- Bauer, Michael D. and Christopher J. Neely**, “International channels of the Fed’s unconventional monetary policy,” *Journal of International Money and Finance*, 2014, 44, 24 – 46.
- **and Glenn D. Rudebusch**, “The Signaling Channel for Federal Reserve Bond Purchases,” *International Journal of Central Banking*, 2014, 10 (3), 233–289.
- Benigno, Gianluca and Pierpaolo Benigno**, “Exchange rate determination under interest rate rules,” *Journal of International Money and Finance*, 2008, 27 (6), 971–993.
- Benigno, Pierpaolo**, “Price Stability with Imperfect Financial Intergration,” *Journal of Money, Credit, and Banking*, 2009, 41 (1), 26–57.
- Boneva, Lena Maren, R. Anton Braun, and Yuichiro Waki**, “Some unpleasant properties of loglinearized solutions when the nominal rate is zero,” *Journal of Monetary Economics*, 2016, 84, 216–232.
- Bowman, David, Juan M. Londono, and Horacio Sapriza**, “U.S. unconventional monetary policy and transmission to emerging market economies,” *Journal of International Money and Finance*, 2015, 55, 27 – 59. Macroeconomic and financial challenges facing Latin America and the Caribbean after the crisis.
- Calvo, Guillermo A.**, “Staggered prices in a utility-maximizing framework,” *Journal of Monetary Economics*, 1983, 12 (3), 383–398.
- Chen, Qianying, Andrew Filardo, Dong He, and Feng Zhu**, “Financial crisis, US unconventional monetary policy and international spillovers,” *Journal of International Money and Finance*, 2016, 67, 62 – 81.
- Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno**, “Risk Shocks,” *American Economic Review*, 2014, 104 (1), 27–65.

- Clarida, Richard, Jordi Galí, and Mark Gertler**, “A simple framework for international monetary policy analysis,” *Journal of Monetary Economics*, 2002, 49 (5), 879 – 904.
- Cochrane, John H.**, “The new-Keynesian liquidity trap,” *Journal of Monetary Economics*, 2017, 92, 47–63.
- Cook, David and Michael B. Devereux**, “Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap,” *American Economic Journal: Macroeconomics*, July 2013, 5 (3), 190–228.
- **and Michael B Devereux**, “Exchange Rate Flexibility under the Zero Lower Bound: the Need for Forward Guidance,” November 2013.
- Debortoli, Davide, Jordi Galí, and Luca Gambetti**, “On the Empirical (Ir)Relevance of the Zero Lower Bound Constraint,” 2016. Universitat Pompeu Fabra working paper.
- Diba, Behzad and Olivier Loisel**, “Pegging the interest rate on bank reserves: a resolution of New Keynesian puzzles and paradoxes,” 2017. Working paper.
- Eggertsson, Gauti and Michael Woodford**, “The Zero Interest-Rate Bound and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 2003, (1), 139–211.
- , **Andrea Ferrero, and Andrea Raffo**, “Can structural reforms help Europe?,” *Journal of Monetary Economics*, 2014, 61, 2–22.
- Eggertsson, Gauti B., Neil R. Mehrotra, Sanjay R. Singh, and Lawrence H. Summers**, “A Contagious Malady? Open Economy Dimensions of Secular Stagnation,” *IMF Economic Review*, 2016, 64 (4), 581–634.
- Enders, Zeno and Gernot J. Müller**, “On the international transmission of technology shocks,” *Journal of International Economics*, 2009, 78, 45–59.
- Erceg, Christopher J. and Jesper Lindé**, “Fiscal consolidation in an open economy,” *American Economic Review: Papers and Proceedings*, 2012, 102 (3), 186–191.

- Fernández-Villaverde, Jesus, Grey Gordon, Guerrón-Quintana, and Juan F. Rubio-Ramírez**, “Nonlinear Adventures at the Zero Lower Bound,” *Journal of Economic Dynamics and Control*, 2015, 57, 182–204.
- Fujiwara, Ippei, Tomoyuki Nakajima, ao Sudo, and Yuki Teranishi**, “Global liquidity trap,” *Journal of Monetary Economics*, 2013, 60, 936–949.
- Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack**, “The financial market effects of the Federal Reserve’s large-scale asset purchases,” *International Journal of Central Banking*, 2011, 7, 3–43.
- Galí, Jordi and Luca Gambetti**, “On the Sources of the Great Moderation,” *American Economic Journal: Macroeconomics*, 2009, 1 (1), 26–57.
- **and Tommaso Monacelli**, “Understanding the gains from wage flexibility: the exchange rate connection,” *American Economic Review*, 2016, 106 (12), 3829–3868.
- Garín, Julio, Robert Lester, and Eric Sims**, “Are supply shocks contractionary at the ZLB? Evidence from utilization-adjusted TFP data.,” 2016. NBER working paper No. 22311.
- Guerrieri, Luca and Matteo Iacoviello**, “OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily.,” *Journal of Monetary Economics*, 2015, 70, 22–38.
- Gust, Christopher, Edward Herbst, David López-Salido, and Matthew E. Smith**, “The empirical implications of the interest-rate lower bound,” *American Economic Review*, 2017, 107 (7), 1971–2006.
- Hakkio, Craig S. and George A. Kahn**, “Evaluating monetary policy at the zero lower bound,” *The Macro Bulletin, Federal Reserve Bank of Kansas City*, 7 2014.

- Hamilton, James D. and Jing Cynthia Wu**, “The effectiveness of alternative monetary policy tools in a zero lower bound environment,” *Journal of Money, Credit, and Banking*, 2012, 44 (s1), 3–46.
- Krishnamurthy, Arvind and Annette Vissing-Jorgensen**, “The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy,” *Brookings Papers on Economic Activity*, 2011, 2, 215–265.
- Mouabbi, Sarah and Jean-Guillaume Sahuc**, “Evaluating the macroeconomic effects of the ECB’s unconventional monetary policies,” 2017. Working paper.
- Neely, Christopher J.**, “Unconventional monetary policy had large international effects,” *Journal of Banking & Finance*, 2015, 52, 101 – 111.
- Taylor, John B.**, “Discretion versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 1993, 39, 195–214.
- Wieland, Johannes F.**, “Are Negative Supply Shocks Expansionary at the Zero Lower Bound?,” 2017. Working paper, University of California, San Diego.
- Wu, Jing Cynthia and Fan Dora Xia**, “Measuring the macroeconomic impact of monetary policy at the zero lower bound.,” *Journal of Money, Credit, and Banking*, 2016, 48 (2-3), 253–291.
- **and** –, “Negative Interest Rate Policy and Yield Curve,” 2017. Chicago Booth Working paper.
- **and Ji Zhang**, “A Shadow Rate New Keynesian Model,” 2017. NBER Working Paper No. 22856.

Appendix A Model appendix

Appendix A.1 Flexible-price equilibrium

For any variable Z_t , Z_t^n represents its flexible-price counterpart. In the flexible-price economy, real marginal cost is a constant

$$\frac{\theta - 1}{\theta} \equiv MC_t^n = \frac{(1 - g)(W_t^n/P_{Ht}^n)}{A_t} = \frac{(1 - g)(W_t^n/P_t^n)(\mathcal{T}_t^n)^{1-\nu/2}}{A_t}, \quad (\text{A.1})$$

and the optimal wage subsidy satisfies $\frac{\theta}{\theta-1} = 1 - g$. Combining the labor-supply condition, $(C_t^n)^\sigma (N_t^n)^\phi = W_t^n/P_t^n$, the production function, $Y_t^n = A_t N_t^n$, and the real marginal cost (A.1), we have

$$(C_t^n)^\sigma (N_t^n)^\phi = A_t (\mathcal{T}_t^n)^{\nu/2-1}. \quad (\text{A.2})$$

The risk-sharing condition holds as follows:

$$\frac{\Xi_t}{(C_t^n)^\sigma} = \frac{\Xi_t^*}{(C_t^{n*})^\sigma} \frac{P_t^n}{\mathcal{E}_t P_t^{n*}} = \frac{\Xi_t^*}{(C_t^{n*})^\sigma} (\mathcal{T}_t^n)^{1-\nu}. \quad (\text{A.3})$$

The market-clearing conditions are

$$Y_t^n = \frac{\nu}{2} (\mathcal{T}_t^n)^{1-\nu/2} C_t^n + (1 - \frac{\nu}{2}) (\mathcal{T}_t^n)^{\nu/2} C_t^{n*} \quad (\text{A.4})$$

$$Y_t^{n*} = \frac{\nu}{2} (\mathcal{T}_t^n)^{-1+\nu/2} C_t^{n*} + (1 - \frac{\nu}{2}) (\mathcal{T}_t^n)^{-\nu/2} C_t^n. \quad (\text{A.5})$$

Appendix A.2 Log-linearized equations

Log-linearizing the consumption-savings decision in (2.3) and its foreign counterpart yields⁷

$$\hat{r}_t - \mathbb{E}_t \left[-(\hat{\xi}_{t+1} - \hat{\xi}_t) + \sigma(\hat{c}_{t+1} - \hat{c}_t) + \hat{\pi}_{t+1} \right] = 0 \quad (\text{A.6})$$

$$\hat{r}_t^* - \mathbb{E}_t \left[-(\hat{\xi}_{t+1}^* - \hat{\xi}_t^*) + \sigma(\hat{c}_{t+1}^* - \hat{c}_t^*) + \hat{\pi}_{t+1}^* \right] = 0. \quad (\text{A.7})$$

The labor-supply decision in (2.4) becomes

$$\hat{w}_t = \hat{c}_t + \phi \hat{n}_t \quad (\text{A.8})$$

$$\hat{w}_t^* = \hat{c}_t^* + \phi \hat{n}_t^*. \quad (\text{A.9})$$

The market-clearing condition in (2.21) becomes

$$\hat{y}_{Ht} = \left[\frac{\nu}{2} \hat{c}_t + \left(1 - \frac{\nu}{2}\right) \hat{c}_t^* \right] + \nu \left(1 - \frac{\nu}{2}\right) \hat{\tau}_t \quad (\text{A.10})$$

$$\hat{y}_{Ft}^* = \left[\frac{\nu}{2} \hat{c}_t^* + \left(1 - \frac{\nu}{2}\right) \hat{c}_t \right] - \nu \left(1 - \frac{\nu}{2}\right) \hat{\tau}_t. \quad (\text{A.11})$$

The international risk-sharing condition (2.16) is

$$\hat{\xi}_t - \sigma \hat{c}_t = \hat{\xi}_t^* - \sigma \hat{c}_t^* + p_t - e_t - p_t^* = \hat{\xi}_t^* - \sigma \hat{c}_t^* + (1 - \nu) \hat{\tau}_t. \quad (\text{A.12})$$

The production function in (2.17) becomes

$$\hat{y}_t = \hat{a}_t + \hat{n}_t \quad (\text{A.13})$$

$$\hat{y}_t = \hat{a}_t^* + \hat{n}_t^*. \quad (\text{A.14})$$

⁷We will omit “log-linearize” and “foreign counterpart” hereafter for brevity.

Combining (2.18) and the labor-supply decision (2.4) results in the real marginal costs:

$$\hat{m}c_t = \phi \hat{n}_t + \sigma \hat{c}_t - \hat{a}_t + (1 - \nu/2) \hat{\tau}_t \quad (\text{A.15})$$

$$\hat{m}c_t^* = \phi \hat{n}_t^* + \sigma \hat{c}_t^* - \hat{a}_t^* - (1 - \nu/2) \hat{\tau}_t. \quad (\text{A.16})$$

The CPI price in (2.8) yields to

$$p_t = \frac{\nu}{2} p_{Ht} + \left(1 - \frac{\nu}{2}\right) p_{Ft} \quad (\text{A.17})$$

$$p_t^* = \frac{\nu}{2} p_{Ft}^* + \left(1 - \frac{\nu}{2}\right) p_{Ht}^*. \quad (\text{A.18})$$

The definitions of CPI (2.10) and PPI inflation (2.11) are

$$\hat{\pi}_t = p_t - p_{t-1} \quad (\text{A.19})$$

$$\hat{\pi}_t^* = p_t^* - p_{t-1}^* \quad (\text{A.20})$$

$$\hat{\pi}_{Ht} = p_{Ht} - p_{H,t-1} \quad (\text{A.21})$$

$$\hat{\pi}_{Ft} = p_{Ft} - p_{F,t-1}. \quad (\text{A.22})$$

Combining (2.18), (2.19), and (2.20), the dynamics for the PPI inflation can be written as

$$\hat{\pi}_{Ht} = \beta \mathbb{E}_t \hat{\pi}_{H,t+1} + \Theta \hat{m}c_t \quad (\text{A.23})$$

$$\hat{\pi}_{Ft}^* = \beta \mathbb{E}_t \hat{\pi}_{F,t+1}^* + \Theta \hat{m}c_t^*, \quad (\text{A.24})$$

where $\Theta = \frac{(1-\beta\zeta)(1-\zeta)}{\zeta} > 0$. The definitions for terms of trade (2.12) and nominal exchange rate (2.14) are

$$\hat{\tau}_t = p_{Ft} - p_{Ht} \quad (\text{A.25})$$

$$p_{Ht} = e_t + p_{Ht}^* \quad (\text{A.26})$$

$$p_{Ft} = e_t + p_{Ft}^*. \quad (\text{A.27})$$

Combining (A.19) - (A.22) and (A.25) - (A.27), the CPI inflation can be expressed as a function of PPI inflation and terms of trade:

$$\hat{\pi}_t = \hat{\pi}_{Ht} + \left(1 - \frac{\nu}{2}\right) \Delta \hat{\tau}_t \quad (\text{A.28})$$

$$\hat{\pi}_t^* = \hat{\pi}_{Ft}^* - \left(1 - \frac{\nu}{2}\right) \Delta \hat{\tau}_t. \quad (\text{A.29})$$

The labor-supply decision (A.2) in the flexible-price economy becomes

$$\sigma \hat{c}_t^n + \phi \hat{n}_t^n = a_t + (\nu/2 - 1) \hat{\tau}_t^n \quad (\text{A.30})$$

$$\sigma \hat{c}_t^{n*} + \phi \hat{n}_t^{n*} = a_t^* - (\nu/2 - 1) \hat{\tau}_t^n. \quad (\text{A.31})$$

The international risk-sharing condition (A.3) in the flexible-price economy is

$$\sigma(\hat{c}_t^n - \hat{c}_t^{n*}) - (\hat{\xi}_t - \hat{\xi}_t^*) - (\nu - 1) \hat{\tau}_t^n = 0. \quad (\text{A.32})$$

The market-clearing conditions (A.4) and (A.5) in the flexible-price economy are

$$\hat{y}_{Ht}^n = \left[\frac{\nu}{2} \hat{c}_t^n + \left(1 - \frac{\nu}{2}\right) \hat{c}_t^{n*} \right] + \nu \left(1 - \frac{\nu}{2}\right) \hat{\tau}_t^n \quad (\text{A.33})$$

$$\hat{y}_{Ft}^{n*} = \left[\frac{\nu}{2} \hat{c}_t^{n*} + \left(1 - \frac{\nu}{2}\right) \hat{c}_t^n \right] - \nu \left(1 - \frac{\nu}{2}\right) \hat{\tau}_t^n. \quad (\text{A.34})$$

The output gaps are defined as

$$x_t = y_t - y_t^n \quad (\text{A.35})$$

$$x_t^* = y_t^* - y_t^{n*}. \quad (\text{A.36})$$

Equations (A.6) to (A.36) and the monetary policy rules (2.24) and (2.25) and their foreign counterparts summarize all equilibrium conditions.

Appendix A.3 Exchange rate, terms of trade, and interest rates

Combining the Euler equations (A.6) and (A.7) with the international risk-sharing condition (A.12), we obtain

$$\mathbb{E}_t [\Delta e_{t+1}] = r_t - r_t^*. \quad (\text{A.37})$$

Combining (A.37), (A.25), and (A.27), we get

$$\hat{r}_t = (\hat{r}_t^* - \mathbb{E}_t[\hat{\pi}_{F,t+1}^*]) - (\hat{r}_t - \mathbb{E}_t[\hat{\pi}_{H,t+1}]) + \mathbb{E}_t[\hat{r}_{t+1}] = \hat{r}_t^* - \hat{r}_t + \mathbb{E}_t[\hat{r}_{t+1}]. \quad (\text{A.38})$$

Solving (A.38) forward under the stationarity condition $\lim_{k \rightarrow \infty} \mathbb{E}_t \hat{r}_{t+k} = 0$, we obtain (2.13).

Appendix B Analytical results and proofs

Appendix B.1 Proof of Proposition 1

In Appendix B.1 to Appendix B.3, there is only a home country's TFP shock, that is, $\hat{a}_t^* = \hat{\xi}_t = \hat{\xi}_t^* = 0$. Combining the market-clearing conditions (A.10) and (A.11) and the international risk-sharing condition (A.12), terms of trade can be expressed as a function of relative output:

$$\hat{r}_t = \sigma/D(\hat{y}_t - \hat{y}_t^*). \quad (\text{B.1})$$

Combining the Euler equations (A.6) and (A.7) with the market-clearing conditions (A.10) and (A.11), international risk sharing (A.12), and the definition of terms of trade (A.25), we get the IS curves for the home and foreign countries:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma_0}(\hat{r}_t - \mathbb{E}_t \hat{\pi}_{H,t+1}) + K_2(\mathbb{E}_t \hat{y}_{t+1}^* - \hat{y}_t^*) \quad (\text{B.2})$$

$$\hat{y}_t^* = \mathbb{E}_t \hat{y}_{t+1}^* - \frac{1}{\sigma_0}(\hat{r}_t^* - \mathbb{E}_t \hat{\pi}_{F,t+1}^*) + K_2(\mathbb{E}_t \hat{y}_{t+1} - \hat{y}_t), \quad (\text{B.3})$$

where $\sigma_0 = \sigma - K_1$, $K_1 = (1 - \nu/2)(\sigma - 1)\nu\sigma/D = \frac{\sigma}{2} \frac{D-1}{D}$, $D = [(\nu - 1)^2 + \sigma\nu(2 - \nu)]$, and $K_2 = K_1/\sigma_0$. Take the difference between the home and foreign IS curves,

$$(\hat{r}_t - \hat{r}_t^*) - \mathbb{E}_t(\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*) = \sigma_0(1 - K_2)\mathbb{E}_t[(\hat{y}_{t+1} - \hat{y}_{t+1}^*) - (\hat{y}_t - \hat{y}_t^*)]. \quad (\text{B.4})$$

The monetary policy rules are

$$\hat{r}_t = \lambda\phi_\pi \hat{\pi}_{Ht} \quad (\text{B.5})$$

$$\hat{r}_t^* = \lambda\phi_\pi \hat{\pi}_{Ft}^*. \quad (\text{B.6})$$

Substitute them into (B.2) - (B.4):

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \frac{1}{\sigma_0} (\lambda \phi_\pi \hat{\pi}_{Ht} - \mathbb{E}_t \hat{\pi}_{H,t+1}) + K_2 (\mathbb{E}_t \hat{y}_{t+1}^* - \hat{y}_t^*) \quad (\text{B.7})$$

$$\hat{y}_t^* = \mathbb{E}_t \hat{y}_{t+1}^* - \frac{1}{\sigma_0} (\lambda \phi_\pi \hat{\pi}_{Ft}^* - \mathbb{E}_t \hat{\pi}_{F,t+1}^*) + K_2 (\mathbb{E}_t \hat{y}_{t+1} - \hat{y}_t) \quad (\text{B.8})$$

$$\lambda \phi_\pi (\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^*) - \mathbb{E}_t (\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*) = \sigma_0 (1 - K_2) \mathbb{E}_t [(\hat{y}_{t+1} - \hat{y}_{t+1}^*) - (\hat{y}_t - \hat{y}_t^*)]. \quad (\text{B.9})$$

Combining the labor-supply conditions (A.8) and (A.9), production functions (A.13) and (A.14), the risk-sharing condition (A.12), and the market-clearing conditions (A.10) and (A.11), the real marginal costs can be derived as

$$\begin{aligned} \hat{m}c_t &= \phi \hat{n}_t + \sigma \hat{c}_t - \hat{a}_t + (1 - \nu/2) \hat{\tau}_t \\ &= \phi \hat{y}_t - (1 + \phi) \hat{a}_t + \sigma \hat{c}_t + (1 - \nu/2) \hat{\tau}_t \\ &= \phi \hat{y}_t - (1 + \phi) \hat{a}_t + \sigma \hat{y}_t - \sigma (1 - \nu/2) \left(\nu - \frac{\nu - 1}{\sigma} \right) \hat{\tau}_t + (1 - \nu/2) \hat{\tau}_t \\ &= K \hat{y}_t - (1 + \phi) \hat{a}_t + K_1 \hat{y}_t^*, \end{aligned} \quad (\text{B.10})$$

where $K = \sigma + \phi - K_1$. The foreign country's counterpart is

$$\hat{m}c_t^* = K \hat{y}_t^* + K_1 \hat{y}_t. \quad (\text{B.11})$$

Combining (B.10) and (B.11) with (A.23) and (A.24), the New Keynesian Phillips curves (NKPCs) are

$$\hat{\pi}_{Ht} = \beta \mathbb{E}_t \hat{\pi}_{H,t+1} + \Theta K \hat{y}_t - \Theta (1 + \phi) \hat{a}_t + \Theta K_1 \hat{y}_t^* \quad (\text{B.12})$$

$$\hat{\pi}_{Ft}^* = \beta \mathbb{E}_t \hat{\pi}_{F,t+1}^* + \Theta K \hat{y}_t^* + \Theta K_1 \hat{y}_t. \quad (\text{B.13})$$

The difference is

$$\hat{\pi}_{Ht} - \hat{\pi}_{Ft}^* = \beta \mathbb{E}_t (\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*) + \Theta (K - K_1) (\hat{y}_t - \hat{y}_t^*) - \Theta (1 + \phi) \hat{a}_t. \quad (\text{B.14})$$

Next, we solve the system of equations in (B.9) and (B.14). When $\lambda \phi_\pi > 1$, the Blanchard-Kahn condition is satisfied, and the system has a unique solution, which is (3.1), (3.2). Next, (B.1) implies (3.3).

In our model, $\Theta > 0$, $1 + \phi > 0$, $1 - \rho_a > 0$, $D > 0$, $D + 1 > 0$, $\sigma > 0$, $\sigma_0 > 0$.

- When $\lambda = 1$ and $\phi_\pi > 1$, $\Lambda_a > 0$ and $\lambda \phi_\pi - \rho_a > 0$.
- When $\lambda = 0$, the denominator of Λ_a is a convex quadratic function of ρ_a with one root between 0 and 1 and another root larger than 1. We solve the root within the unit circle $\bar{\rho}_a = \frac{2\sigma_0(1+\beta) + \Theta(\sigma/D + \phi)(D+1) - \sqrt{[2\sigma_0(1+\beta) + \Theta(\sigma/D + \phi)(D+1)]^2 - 16\sigma_0^2\beta}}{4\sigma_0\beta}$, and $0 < \rho_a < \bar{\rho}_a$ guarantees $\Lambda_a > 0$. Moreover, $\lambda \phi_\pi - \rho_a < 0$.

■

Appendix B.2 Proof of Corollary 1

When $\sigma = 1$ or $\nu = 2$, $K_1 = K_2 = 0$, so that $\sigma_0 = \sigma$, $K = \sigma + \phi$, and $D = 1$. For the foreign economy, (B.8) and (B.13) yield to

$$\hat{\pi}_{Ft}^* = \hat{y}_t^* = 0. \quad (\text{B.15})$$

The solution to (B.7) and (B.12) for the home economy is

$$\hat{y}_t = \Theta (\lambda \phi_\pi - \rho_a) (1 + \phi) \Lambda_a \hat{a}_t \quad (\text{B.16})$$

$$\hat{\pi}_{Ht} = -\Theta (1 - \rho_a) (1 + \phi) \Lambda_a \hat{a}_t, \quad (\text{B.17})$$

and (B.1) implies $\hat{\tau}_t = \sigma \hat{y}_t$. ■

Appendix B.3 Proof of Proposition 2

When $\lambda = 0$, (3.2) and (3.3) become

$$\hat{y}_t - \hat{y}_t^* = -\rho_a \Theta(1 + \phi)(D + 1)\Lambda_a \hat{a}_t \quad (\text{B.18})$$

$$\hat{\tau}_t = -\rho_a \Theta(1 + \phi) \frac{\sigma(D + 1)}{D} \Lambda_a \hat{a}_t. \quad (\text{B.19})$$

First,

$$\frac{\partial D}{\partial \nu} = 2(1 - \sigma)(\nu - 1) < 0,$$

given $\sigma > 1$. Next, take the derivative of the coefficient in (B.18) with respect to D :

$$\begin{aligned} & \frac{\partial [-\rho_a \Theta(1 + \phi)(D + 1)\Lambda_a]}{\partial D} \\ &= -\rho_a \Theta(1 + \phi)\Lambda_a \left\{ 1 + (D + 1)\Lambda_a \left[\rho_a \Theta(\phi - \sigma/D^2) + \frac{\sigma(1 - \beta\rho_a)(1 - \rho_a)}{D^2} \right] \right\}. \end{aligned} \quad (\text{B.20})$$

Note $\phi - \sigma/D^2$ is an increasing function of D and hence a decreasing function of ν . Therefore, $\phi - \sigma/D^2 \geq \phi - \sigma/D^2|_{\nu=2} = \phi - \sigma > 0$, and $\frac{\partial [-\rho_a \Theta(1 + \phi)(D + 1)\Lambda_a]}{\partial D} < 0$. That is, $-\rho_a \Theta(1 + \phi)(D + 1)\Lambda_a$ is increasing in ν and negative. When $\hat{a}_t < 0$, $\hat{y}_t - \hat{y}_t^* \geq \hat{y}_t - \hat{y}_t^*|_{\nu=2} > 0$.

Next, for the coefficient in (B.19),

$$\begin{aligned} & \frac{\partial [-\rho_a \sigma \Theta(1 + \phi) \frac{D+1}{D} \Lambda_a]}{\partial D} \\ &= -\rho_a \sigma \Theta(1 + \phi)\Lambda_a \left\{ -\frac{1}{D^2} + (D + 1)\Lambda_a \left[\rho_a \Theta(\phi - \sigma/D^2) + \frac{\sigma(1 - \beta\rho_a)(1 - \rho_a)}{D^2} \right] \right\} \\ &= -\rho_a \sigma \Theta(1 + \phi)\Lambda_a \left\{ \frac{\sigma(1 - \beta\rho_a)(1 - \rho_a)(D + 1)\Lambda_a - 1}{D^2} + (D + 1)\Lambda_a \rho_a \Theta(\phi - \sigma/D^2) \right\} \\ &= -\rho_a \sigma \Theta(1 + \phi)\Lambda_a \left\{ \frac{1}{D^2} \frac{\sigma(1 - \beta\rho_a)(1 - \rho_a)(D + 1) - 1/\Lambda_a}{1/\Lambda_a} + (D + 1)\Lambda_a \rho_a \Theta(\phi - \sigma/D^2) \right\} \\ &= -\rho_a \sigma \Theta(1 + \phi)\Lambda_a \left\{ \frac{1}{D^2} \frac{\sigma/D(1 - \beta\rho_a)(1 - \rho_a)(D + 1)(D - 1) + \rho_a \Theta(\sigma/D + \phi)(D + 1)}{1/\Lambda_a} \right. \\ & \quad \left. + (D + 1)\Lambda_a \rho_a \Theta(\phi - \sigma/D^2) \right\}. \end{aligned} \quad (\text{B.21})$$

D is decreasing in ν : $D \geq D|_{\nu=2} = 1$. Therefore, (B.21) is negative, and $-\rho_a \Theta(1 + \phi) \frac{\sigma(D+1)}{D} \Lambda_a$ is negative and increasing in ν , or when $\hat{a}_t < 0$, $\hat{\tau}_t \geq \hat{\tau}_t|_{\nu=2} > 0$. ■

Appendix B.4 Proof of Proposition 3

In this section, there is only a home country's demand shock, and $\hat{a}_t = \hat{a}_t^* = \hat{\xi}_t^* = 0$. The terms of trade can be expressed as a function of the relative output:

$$\hat{\tau}_t = \sigma/D(\hat{y}_t - \hat{y}_t^*) - \frac{1}{\nu - 1} \hat{\xi}_t. \quad (\text{B.22})$$

The IS curves are

$$\hat{y}_t - \mathbb{E}_t \hat{y}_{t+1} = -\frac{1}{\sigma_0} (\lambda \phi_\pi \hat{\pi}_{Ht} - \mathbb{E}_t \hat{\pi}_{H,t+1}) + K_2 \mathbb{E}_t (\hat{y}_{t+1}^* - \hat{y}_t^*) - \frac{D + \nu - 1}{2D\sigma_0} \mathbb{E}_t \Delta \hat{\xi}_{t+1} \quad (\text{B.23})$$

$$\hat{y}_t^* - \mathbb{E}_t \hat{y}_{t+1}^* = -\frac{1}{\sigma_0} (\lambda \phi_\pi \hat{\pi}_{Ft}^* - \mathbb{E}_t \hat{\pi}_{F,t+1}^*) + K_2 \mathbb{E}_t (\hat{y}_{t+1} - \hat{y}_t) + \frac{\nu - 1 - D}{2D\sigma_0} \mathbb{E}_t \Delta \hat{\xi}_{t+1}. \quad (\text{B.24})$$

The NKPCs are

$$\hat{\pi}_{Ht} = \beta \mathbb{E}_t \hat{\pi}_{H,t+1} + \Theta K \hat{y}_t + \Theta K_1 \hat{y}_t^* + \frac{K_1(\nu - 1)}{\sigma} \hat{\xi}_t \quad (\text{B.25})$$

$$\hat{\pi}_{Ft}^* = \beta \mathbb{E}_t \hat{\pi}_{F,t+1}^* + \Theta K \hat{y}_t^* + \Theta K_1 \hat{y}_t - \frac{K_1(\nu - 1)}{\sigma} \hat{\xi}_t. \quad (\text{B.26})$$

When $\nu = 2$, we have $D = 1$, $K_1 = K_2 = 0$, $\sigma_0 = \sigma$, and $K = \sigma + \phi$, so that (B.23) to (B.26) can be simplified to

$$\hat{y}_t - \mathbb{E}_t \hat{y}_{t+1} = -\frac{1}{\sigma} (\lambda \phi_\pi \hat{\pi}_{Ht} - \mathbb{E}_t \hat{\pi}_{H,t+1}) - \frac{1}{\sigma} \mathbb{E}_t \Delta \hat{\xi}_{t+1}, \quad (\text{B.27})$$

$$\hat{y}_t^* - \mathbb{E}_t \hat{y}_{t+1}^* = -\frac{1}{\sigma} (\lambda \phi_\pi \hat{\pi}_{Ft}^* - \mathbb{E}_t \hat{\pi}_{F,t+1}^*), \quad (\text{B.28})$$

$$\hat{\pi}_{Ht} = \beta \mathbb{E}_t \hat{\pi}_{H,t+1} + \Theta K \hat{y}_t, \quad (\text{B.29})$$

$$\hat{\pi}_{Ft}^* = \beta \mathbb{E}_t \hat{\pi}_{F,t+1}^* + \Theta K \hat{y}_t^*. \quad (\text{B.30})$$

Solving the system of equations (B.22) and (B.27) to (B.30), we obtain (3.8) - (3.11).

When $\lambda = 0$, the denominator of Λ_ξ is a quadratic function of ρ_ξ , with one root between 0 and 1 and the other root larger than 1. $\Lambda_\xi > 0$ requires $0 < \rho_\xi < \bar{\rho}_\xi$, where

$$\bar{\rho}_\xi = \frac{[1 + \beta + \Theta(1 + \phi)] - \sqrt{[1 + \beta + \Theta(1 + \phi)]^2 - 4\beta}}{2\beta} < 1.$$

Given $\Lambda_\xi > 0$, we have $\Lambda_\xi|_{\lambda=0} > \Lambda_\xi|_{\lambda=1} > 0$ because

$$\Theta(\sigma + \phi)(\phi_\pi - \rho_\xi) + \sigma(1 - \rho_\xi)(1 - \beta\rho_\xi) > -\rho_\xi\Theta(\sigma + \phi) + \sigma(1 - \rho_\xi)(1 - \beta\rho_\xi).$$

Therefore, the coefficients in (3.8) and (3.9) are larger when $\lambda = 0$ than when $\lambda = 1$. For the terms of trade, the term $\lambda\phi_\pi - \rho_\xi$ in (3.10) is positive when $\lambda = 1$ and negative when $\lambda = 0$. ■

Appendix B.5 Fixed exchange rate

The IS curve (B.4), NKPC (B.14), and the home country's monetary policy rule (B.5) are the same as in the flexible exchange rate economy, whereas the foreign country's monetary policy becomes (3.4).

Substituting (3.4) into (A.37), we have $\mathbb{E}_t \hat{e}_{t+1} = (1 + \phi_e) \hat{e}_t$. $\phi_e > 0$ ensures a unique and non-explosive equilibrium, which is $e_t = e$, where e is the steady-state value of the exchange rate. Therefore,

$$\hat{e}_t = 0. \quad (\text{B.31})$$

A similar argument can be found in Benigno and Benigno (2008).

Substitute (B.5), (3.4), and (B.31) into (B.4) and (B.14), and we obtain

$$\begin{aligned} -\mathbb{E}_t(\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*) &= \sigma_0(1 - K_2)\mathbb{E}_t[(\hat{y}_{t+1} - \hat{y}_{t+1}^*) - (\hat{y}_t - \hat{y}_t^*)] \\ \hat{\pi}_{Ht} - \hat{\pi}_{Ft}^* &= \beta \mathbb{E}_t(\hat{\pi}_{H,t+1} - \hat{\pi}_{F,t+1}^*) + \Theta(K - K_1)(\hat{y}_t - \hat{y}_t^*) - \Theta(1 + \phi)\hat{a}_t. \end{aligned}$$

The solution of this system is given by (3.5) - (3.6). And (3.6) and (B.1) yield to (3.7).

Appendix C Setup for quantitative analysis

Calibration We calibrate structural parameters according to the standard macro and international literature. The discount factor is $\beta = 0.99$, so the steady-state quarterly risk-free nominal interest rate is 1%. The intertemporal elasticity of substitution is $\sigma = 2$, and the Frisch labor-supply elasticity is $\phi = 3$. The elasticity of substitution among differentiated goods is $\theta = 6$, implying the steady-state price markup is 1.2. The price stickiness parameter is $\zeta = 0.75$, meaning the average time between two price adjustments is

one year. The persistence of the Taylor rule is $\rho_r = 0.8$, and the sensitivities of the policy rate to inflation and output are $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. $\nu = 1.5$ implies a significant home bias. The persistence and standard deviation of the TFP shock are $\rho_a = 0.9$ and $\sigma_a = 0.0025$, according to Fernández-Villaverde et al. (2015). The persistence and standard deviation of the preference shock are $\rho_\xi = 0.9$ and $\sigma_\xi = 0.023$, according to Christiano et al. (2014).

ELB environment To create an ELB environment, we impose a series of negative preference shocks on both countries. The shocks occur in periods 1-15, and the total shock size in each country is 23%. These shocks push down the nominal interest rate to zero at period 9 and keep it there until period 20 when there is no unconventional monetary policy.

Negative TFP shock In addition to the preference shocks, we hit the home country with a one-time negative TFP shock with a size of -0.5% at period 12.

Solution method When $\lambda = 1$, the model is linear, so we use the standard method for solving the rational expectations models. When $\lambda < 1$, we use the occasional binding method of Guerrieri and Iacoviello (2015).

Appendix D Incomplete asset markets

Following Benigno (2009), there is no longer complete international risk sharing in the model with incomplete asset markets, where only two nominal non-contingent bonds are traded. Then the international risk sharing condition (2.16) no longer holds, and the household's budget constraint (2.2) in the baseline model changes to:

$$P_t C_t - \frac{B_{Ht+1}}{R_t} + \frac{\mathcal{E}_t B_{Ft+1}}{R_t^*} + \frac{\ell P_t}{2R_t^*} \left(\frac{\mathcal{E}_t B_{Ft+1}}{P_t} - \bar{a} \right)^2 = \mathcal{E}_t B_{Ft} - B_{Ht} + W_t N_t + TR_t + \mathcal{D}_t, \quad (\text{D.1})$$

where B_{Ht+1} is the debt issued in units of risk-free nominal bond denominated in H currency, and the nominal interest rate on this bond is R_t . B_{Ft+1} is the holding of risk-free nominal bond denominated in units of foreign currency, and the nominal interest rate on this bond is R_t^* . The assumption that households of the home country hold assets denominated in foreign currency and issue debt in the home currency, reflects the net international position of the US economy. We assume a quadratic transition cost when deviating the real foreign bond position from a constant real value, denoted by \bar{a} ; ℓ is nonnegative, measuring this cost in terms of units of the consumption index, and is rescaled by the factor $1/R_t^*$ for analytical convenience. The quadratic cost serves for the purpose of determining the steady state and getting rid of the indeterminacy problem. TR_t includes government transfer and the revenues obtained from the transaction costs paid by households in the foreign country when trading home country bonds, and \mathcal{D}_t is the profits from firms.

The first-order conditions of home country households with respect to domestic and foreign bonds are:

$$\beta \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\Xi_t} \frac{C_t^\sigma}{C_{t+1}^\sigma} \frac{R_t}{\Pi_{t+1}} \right] = 1 \quad (\text{D.2})$$

$$\beta \mathbb{E}_t \left[\frac{\Xi_{t+1}}{\Xi_t} \frac{C_t^\sigma}{C_{t+1}^\sigma} \frac{R_t^*}{\Pi_{t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right] = 1 + \ell \left(\frac{\mathcal{E}_t B_{Ft+1}}{P_t} - \bar{a} \right). \quad (\text{D.3})$$

Utility maximization of households in the foreign country yields the counterparts of equation (D.2) and (D.3). The equilibrium in the asset markets requires that

$$B_{Ht} - B_{Ht}^* = 0 \quad (\text{D.4})$$

$$B_{Ft} - B_{Ft}^* = 0. \quad (\text{D.5})$$

Combining the household's and government budget constraints, we obtain the aggregate budget constraint of the home country:

$$P_t C_t - \frac{B_{Ht+1}}{R_t} + \frac{\mathcal{E}_t B_{Ft+1}}{R_t^*} + \frac{\ell P_t}{2R_t^*} \left(\frac{\mathcal{E}_t B_{Ft+1}}{P_t} - \bar{a} \right)^2 = \mathcal{E}_t B_{Ft} - B_{Ht} + P_{Ht} Y_t + \frac{\ell^* P_t}{2R_t} \left(\frac{B_{Ht+1}^*}{\mathcal{E}_t P_t^*} - \bar{a}^* \right)^2. \quad (\text{D.6})$$

Appendix E Data

- Shadow rates are downloaded from Cynthia Wu's website:
<https://sites.google.com/site/jingcynthiawu/home/wu-xia-shadow-rates>.
- The U.S. macroeconomic variables are downloaded from the Database of the Federal Reserve Bank of St. Louis (FRED) at <http://research.stlouisfed.org/fred2/>.
 - Real GDP (GDPC): billions of chained 2009 dollars, seasonally adjusted.
 - Real potential GDP (GDPPOT): billions of chained 2009 dollars, not seasonally adjusted.
 - GDP deflator (GDPDEF): index 2009=100, seasonally adjusted.
 - Effective federal funds rate (FEDFUNDS): percent.
 - Real output per hour of all persons (nonfarm business sector) (OPHNFB): index 2009=100, seasonally adjusted.
 - Hours of all persons (nonfarm business sector) (HOANBS): index 2009=100, seasonally adjusted.
 - Civilian noninstitutional population (CNP16OV): thousands of persons.
- The Euro area macroeconomic variables are from the ECB Statistical Data Warehouse at <http://sdw.ecb.europa.eu/home.do>.
 - Real GDP: reference year 1995, calendar and seasonally adjusted.
 - GDP deflator: index 1995=1, calendar and seasonally adjusted.
 - Policy rate: 3-month Euribor.
 - Euro area 19 (fixed composition) total economy labor productivity (per hours worked): index, chain linked volume (rebased), calendar and seasonally adjusted.
 - Euro area 19 (fixed composition) total economy hours worked: hours, calendar and seasonally adjusted.
 - Euro area 19 (fixed composition) employment: thousands of persons, calendar and seasonally adjusted.
- The UK macroeconomic variables are downloaded from the Office for National Statistics at <https://www.ons.gov.uk/> and the FRED.
 - Real GDP: seasonally adjusted.
 - GDP deflator: index 1995=1, seasonally adjusted.
 - Bank of England policy rate: percent per annum.
 - Output per hour: index 2015=100, seasonally adjusted.
 - Average actual weekly hours of work for all workers: millions, seasonally adjusted.
 - Population aged 16 and over: thousands of persons.
- The Japan macroeconomic variables are downloaded from the FRED.
 - Total labor productivity: growth rate same period previous year, seasonally adjusted.
 - Hours worked (manufacturing): index 2010=100, seasonally adjusted.
 - Working age population: aged 15-64, persons, seasonally adjusted.