Production Networks, Nominal Rigidities and the Propagation of Shocks*

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Abstract

We develop a multi-sector Calvo model with intermediate inputs to study the quantitative importance of input-output linkages for the real effects of monetary policy, and the propagation of idiosyncratic shocks. We allow for heterogeneity in frequencies of price adjustments across sectors, in sector size, and in sector importance as supplier to other sectors. Intermediate inputs as well as heterogeneous frequencies of price adjustment increase the persistence of the real effects of monetary policy. Heterogeneity in input-output linkages can amplify or dampen these real effects. Fat tails in sector size (Gabaix (2011)) and sectoral heterogeneity in input-output linkages (Acemoglu et al. (2012)) amplify sectoral shocks, but their aggregate impact depends on the interaction of sector size and outdegree with price stickiness. We develop intuition for these results in a series of simplified model economies. Quantitatively, heterogeneity in input-output linkages contributes only marginally to the real effects of monetary policy shocks. Heterogeneity in the frequency of price adjustment creates large real effects of nominal shocks. Heterogeneity in the frequency of price adjustment also amplifies the large effects of heterogeneity in sector size and the effect of heterogeneity in input-output linkages for the propagation of sectoral shocks. We calibrate a 350 sector version of the model to the input-output tables from the Bureau of Economic Analysis and the micro data underlying the producer price index to to reach those conclusions. A less granular calibration with 58 sectors understates the real effects by 25% with a similar impact response of inflation.

JEL classification: E30, E32, E52

Keywords: Input-output linkages, multi-sector Calvo model, monetary policy

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I Introduction

Technology and monetary policy shocks are central to understand business cycle fluctuations (see Ramey (2015)). A high degree of specialization and tightly-linked production networks are a key feature of modern production economies and the network structure is potentially an important propagation mechanism for aggregate fluctuations originating from firm and industry shocks. Gabaix (2011) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) show theoretically idiosyncratic shocks might not wash out and propagate through the network if the input-output structure or the size distribution is sufficiently asymmetric. Idiosyncratic shocks propagate through the network through changes in prices. Real effects of monetary policy are a central question in monetary economics. Most transmission channels of nominal shocks to the real economy rely on some output prices being sticky in the short run.

Rigidities in prices potentially pose an interesting tension between monetary policy and idiosyncratic shocks contributing to aggregate fluctuations. In this paper, we study quantitatively the interaction of heterogeneity in price stickiness, sector size, and input-output structure to answer the question how and to which extent heterogeneity in input-output structure and price stickiness interact to create persistence in the propagation of monetary policy shocks and propagation of idiosyncratic shocks to aggregate fluctuations.

We develop a multi-sector New Keynesian model with intermediate input to tackle these questions. Individual firms in each sector are monopolistically-competitive suppliers of differentiated goods, set prices as in Calvo (1983), and are competitive demanders in the market of homogeneous labor input. Firms also use intermediate inputs in the production of their output. We model the input-output structure as a round-about network as in Basu (1995). Households derive utility from a composite consumption good and leisure. Firms in the model set prices as a markup over a weighted average of expected future marginal costs. Intermediate inputs introduce sluggishness in the process for marginal costs as pricing decisions across firms become strategic complements (see Basu (1995)). After a nominal shock, adjusting firms will change prices less than in a model without intermediate inputs. Many suppliers have not had a chance yet to adjust prices and marginal costs move little. Intermediate inputs mute the price response and result in a larger output response.

We calibrate a 350 sector version of the model to the input-output tables from the Bureau of Economic Analysis (BEA) and the micro data underlying the producer price index from the Bureau of Labor Statistics (BLS) to understand the quantitative implications of the model. Heterogeneity in price stickiness is the main driver of real effects in monetary policy shocks. Input-ouput linkages and heterogeneity in sector size have some effect, but these effects are small compared to the effects of price stickiness. We also find the 350 sector economy has a 25% larger real effects of monetary policy than a less granular 58 sector model. The impact response of inflation, on the contrary, is similar in the two models. This findings cautions against drawing inference for the conduct of monetary policy from the response of inflation to monetary policy shocks.

We identify four distinct channels through which input-output linkages and the heterogeneities can affect the marginal cost process: First, there is a direct effect. Marginal costs of final goods producers are a function of the sector-specific-input price index. Second, sector-specific wages depend indirectly on the input-output linkages since the optimal mix of inputs depends on the relative price of intermediate inputs and labor. Third and fourth, the heterogeneity in participation in total production, consumption, and intermediate inputs create wedges between sectoral participation in total output, production, and total GDP which feed back into marginal costs.

These channels interact in shaping the response to nominal shocks in a very intuitive way: It matters how important a certain input is for a given final goods producer, how sticky or flexible that input is, and then, with respect to the aggregate process, how important the final goods producer is for total consumption. The size and interconnectedness of a sector and the interaction with frequencies of price adjustment matters for the real effects of monetary policy. The convolution of heterogeneities matters because it can generate an economy that for example resembles a perfect flex price economy or an economy with uniformly rigid prices. Based on our model, we predict that a key statistic to capture the effect of this convolution is the dot product of the sectoral outdegrees and the degrees of price flexibility (where the sectoral outdegree is the average of the input-output weights, weighted by each supplier's weight in total production).

We develop precise intuition for the interaction of the three heterogeneities by gradually adding each heterogeneity. We do so analytically where possible, and in a three-sector version of the full model. As a first step, we consider an economy that features input-output linkages which can be homogeneous or heterogeneous. At the same time, Calvo parameters are homogeneous across sectors and sectoral participation in GDP equals sectoral participation in total production. We find that input-output linkages amplify the real effects of monetary policy, like in Nakamura and Steinsson (2008), but heterogeneity in input-output linkages does not matter. The absence of an effect of heterogeneity is intuitive since there are no wedges in sectoral production and consumption shares, and price stickiness is uniform across all firms. The dot product of the outdegrees and price flexibility is

In this first step, we are not only able to show these results analytically, but we also present a new result with respect to the economic environment when monetary policy is "not responsive enough" as in Bhattarai et al. (2014), a special case of which is the zero lower bound (ZLB). We find that input-output linkages can dampen the contraction that follows a large negative demand shock in this highly relevant policy environment. The reason is that input-output linkages create sluggishness in the marginal cost process which mitigates the fall in prices and hence the rise in the real interest rate.

As a second step, we add heterogeneity in Calvo parameters to the basic economy while keeping a mean-preserving spread of price stickiness. Now, heterogeneity in Calvo parameters introduces a hump shape into the previously linear impulse response. The reason is that flex-price firms compete with sticky-price firms and will therefore adjust prices staggered and by less on impact. Relative to a homogeneous network structure, however, we find that heterogeneity in input-output linkages can amplify or dampen the output response. This is due to the interaction between heterogeneous price stickiness and input-output linkages, and depends on which sectors exactly are more or less flexible.

As a third step, we additionally allow for sectors to differ in their weights in GDP from their weights in total production. In this economy, wedges between consumption prices and sectoral production prices open up that influence sectoral marginal costs. Again, we find that heterogeneity in input-output linkages can amplify or dampen the response of GDP to monetary policy shocks. However, now the effect is substantially larger in magnitude than in the previous case because of the additional effect on marginal costs due to the wedges. The economy may now even resemble a flex-price economy depending the interaction of size, importance of suppliers to other sectors, and price stickiness.

Intermediate inputs amplify the real effects of monetary policy but heterogeneity in input-output linkages might either reinforce or dampen the response which is why we study the question quantitatively.

A central finding of our study is that the inflationary response can be very similar across calibrations with different degrees of granularity, whereas the real effects of monetary policy might differ substantially. Specifically, we find a similar impact response of inflation to a monetary policy show in our 350 sector benchmark economy and a less granular, 58 sector model. The 58 sector model understates the real effects of monetary policy, instead, by 25%.

A. Literature Review

Our paper contributes to two strands of the literature: the macroeconomic literature studying the role of production networks on the aggregation of microeconomic shocks to business cycle fluctuations and the monetary economics literature on the propagation role of input-output linkages.

The traditional view in macroeconomics is that shocks at the firm and sector level cannot contribute to aggregate fluctuations as they diversify and wash out in the aggregate. The law of large numbers might not apply if the firm-size distribution is sufficiently fat tailed and idiosyncratic shocks might contribute to business cycle fluctuations (Gabaix (2011)). Acemoglu et al. (2012) build on Long and Plosser (1983) and show that input-output linkages might also mute diversification of sectoral shocks if shocks hit important suppliers to other sectors. Acemoglu et al. (2015) study the importance of networks for aggregate fluctuations empirically and show input-output linkages are important for the propagation of federal spending, trade, technology, and knowledge shocks. Ozdagli and Weber (2016) show empirically that input-output linkages are a key propagation channel of monetary policy shocks to the stock market. Carvalho (2014) provides an overview of this fast growing literature. We contribute to this line of research by adding nominal rigidities into the models and studying the importance of input-output linkages for the propagation of aggregate shocks.

Basu (1995) shows that a round-about production structure can magnify the importance of price rigidities through its effect on marginal costs and results in larger welfare losses of demand-driven business cycles. Huang and Liu (2004) studies in a multi-sector model with round-about production and fixed contract length the persistence of monetary shocks and find theoretically that intermediate inputs amplifies the importance of rigid prices but has no impact on wage stickiness. Nakamura and Steinsson (2009) develop a multi-sector menu cost model and show in a calibration of a six-sector version that heterogeneity in price stickiness together with input-output linkages can explain persistent real effects of nominal shocks with moderate degrees of price stickiness. Carvalho and Lee (2011) show that a multi-sector Calvo model with intermediate inputs can reconcile why firms adjust faster to idiosyncratic shocks than to aggregate shocks (see also Boivin et al. (2009) and Shamloo (2010)). Bouakez, Cardia, and Ruge-Murcia (2014) estimate a multi-sector Calvo model with production networks using aggregate and sectoral data and find evidence for heterogeneity in frequencies of price adjustments across sectors. We extent this literature by allowing for empirically relevant degrees of heterogeneity in input-output linkages and study the importance of networks on the propagation of nominal shocks in a quantitative calibration of a 350 sector model.

Other recent applications of production networks in different areas of macroeconmics are Bigio and Lao (2013) which studies the amplification of financial frictions through production networks and Herskovic (2015) which develops the asset pricing implication of input-output linkages.

II Model

A. Households

Consider a representative household whose utility is

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{k=1}^{K} \int_{\Im_k} g_k \frac{L_{kjt}^{1+\varphi}}{1+\varphi} dj \right) \tag{1}$$

subject to

$$P_t C_t = \sum_{k=1}^{K} W_{kt} \int_{\Im_k} L_{kjt} dj + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t \tag{2}$$

where C_t and P_t respectively are aggregate consumption and aggregate prices to be specified below, L_{kjt} and W_{kt} are labor employed and wages paid by firm j in sector $k = 1, ..., K$, Π_{kt} is transfers from firms in sector k, I_{t-1} is the gross interest rate paid by bonds holding at the beginning of period t, B_{t-1}.

Aggregate consumption is

$$C_t \equiv \left[\sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{3}$$

where C_{kt} is the aggregation of sectoral consumption

$$C_{kt} \equiv \left[n_k^{-1/\theta} \int_{\Im_k} C_{kjt}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \tag{4}$$

where C_{kjt} is the amount of goods produced by firm j in sector k that is demanded for consumption of households.

Note that the elasticity of substitution across sectors η is allowed to be different than the elasticity of substitution within sectors θ. The aggregation weights $\{\omega_{ck}\}$ are also allowed to be different across sectors and satisfy $\sum_{k=1}^{K} \omega_{ck} = 1$. The set of firms in sector k is denoted as $\Im_k$ which has total measure n_k, such that $\sum_{k=1}^{K} n_k = 1$. In equilibrium there is a relation between $\{n_k\}_{r=1}^{K}$, $\{\omega_{ck}\}_{r=1}^{K}$, and the aggregation weights for intermediate inputs to be introduced shortly below in the description of firms in the model.

Households' demand for sectoral composite goods C_{kt} and for each firm' good C_{kjt} respectively are

$$\begin{aligned} C_{kt} &= \omega_{ck} \left(\frac{P_{kt}}{P_t^c} \right)^{-\eta} C_t, \\ C_{kjt} &= \frac{1}{n_k} \left(\frac{P_{kjt}}{P_{kt}} \right)^{-\theta} C_{kt}. \end{aligned}$$

The steady state of the economy is solved in the appendix, but an important result from its solution is that the consumption weights $\{\omega_{ck}\}_{r=1}^{K}$ determine the steady state shares of sectors in total consumption (also interpreted as value-added production). In the following we refer to $\{\omega_{ck}\}_{r=1}^{K}$ as "consumption shares." Outside steady state, the share of sectors in aggregate consumption is distorted by the gap between sectoral prices $\{P_{kt}\}_{r=1}^{K}$ and the consumption aggregate prices P_t^c. In turn, in steady state sectoral consumption is split equally among the measure n_k of firms within the sector but in general the dispersion of firms' demand within sectors is determined by the within-sectors dispersion of prices.

Consumption shares $\{\omega_{ck}\}_{r=1}^{K}$ also enter in the consumption price aggregator P_t^c in (5):

$$P_t^c = \left[\sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{5}$$

while sectoral prices are

$$P_{kt} = \left[\frac{1}{n_k}\int_{\Im_k} P_{kjt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}}. \tag{6}$$

Other key optimality conditions from households are labor supply and the Euler equation:

$$\frac{W_{kt}}{P_t^c} = g_k L_{kjt}^{\varphi} C_t^{\sigma} \text{ for all } k, j, \tag{7}$$

$$\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} I_t \frac{P_t^c}{P_{t+1}^c}\right] = 1 \tag{8}$$

B. Firms

There is a continuum of firms in the economy with total measure one indexed by $j \in [0,1]$. The set of goods is partitioned into a sequence of subsets $\{\Im_k\}_{k=1}^{K}$ respectively with measure $\{n_k\}_{k=1}^{K}$ such that $\sum_{k=1}^{K} n_k = 1$. Each of these subsets is interpreted as a "sector". To avoid confusion, we denote a firm j belonging to sector k as "firm k, j".

The production function of firm k, j is

$$Y_{kjt} = e^{a_{kt}} L_{kjt}^{1-\delta} Z_{kjt}^{\delta}, \tag{9}$$

where a_{kt} is a productivity shock specific to sector k following an AR(1) process with persistence ρ_a, L_{kjt} is labor hired and Z_{kjt} is an aggregator of intermediate inputs demanded by firm k, j which is defined as

$$Z_{kjt} \equiv \left[\sum_{r=1}^{K} \omega_{kr}^{\frac{1}{\eta}} Z_{kjt}(r)^{1-\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{10}$$

where, in turn, $Z_{kjt}(r)$ is the amount of goods from sector r used by firm k, j at period t. The aggregator weights $\{\omega_{kr}\}_{k,r}$ satisfy $\sum_{r=1}^{K} \omega_{kr} = 1$ for all k. A critical ingredient of our analysis is that these weights are allowed to be different across sectors.

$Z_{kjt}(r)$ is also an aggregator according to

$$Z_{kjt}(r) \equiv \left[n_r^{-1/\theta} \int_{\Im_r} Z_{kjt}(r, j')^{1-\frac{1}{\theta}} \, dj' \right]^{\frac{\theta}{\theta-1}} \tag{11}$$

where $Z_{kjt}(r, j')$ is the amount of goods produced by firm j' in sector r that is demanded as input by firm k, j.

$Z_{kjt}(r)$ and $Z_{kjt}(r, j')$ solve

$$\begin{aligned} Z_{kjt}(r) &= \omega_{kr} \left(\frac{P_{rt}}{P_t^k} \right)^{-\eta} Z_{kjt}, \\ Z_{kjt}(r, j') &= \frac{1}{n_r} \left(\frac{P_{rj't}}{P_{rt}} \right)^{-\theta} Z_{kjt}(r) \end{aligned}$$

where $P_{rj't}$ is the price charged by firm j' in sector r. Since, as we derive in the appendix, all prices are identical in steady state, $\{\omega_{kr}\}_{r=1}^{K}$ is the share of costs of firm k, j spent on inputs from sector r, so it is exactly the cell k, r in the Input-Output Tables. In the following, $\{\omega_{kr}\}_{r=1}^{K}$ are referred to as "I/O weights" or "I/O linkages". In turn, the demand of firm k, j for goods produced in sector r are shared equally among all n_r firms in sector r in steady state. Outside the steady state, the share of sector r in the costs of firm k, j is distorted by the gap between sectoral price P_{rt} of sector r and the aggregate price P_t^k relevant for firm k, j. Similarly, dispersion in prices within a given sector determines the dispersion of demand of firm k, j for goods in sector r.

The aggregate price relevant for a firm demand of intermediate inputs depends on which sector the firm belongs to:

$$P_t^k = \left[\sum_{r=1}^{K} \omega_{kr} P_{rt}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{12}$$

because the productive structure of firms varies across sectors depending on their I/O linkages.

In contrast, sectoral prices

$$P_{rt} = \left[\frac{1}{n_r} \int_{\Im_r} P_{rj't}^{1-\theta} dj' \right]^{\frac{1}{1-\theta}} \tag{13}$$

are the same regardless of the demanding firm and are also the same to the one relevant for households' demand.

Another critical ingredient of our model is sectoral heterogeneity in price rigidities. Specifically, we model price rigidities a la Calvo with parameter $\{\alpha_k\}_{k=1}^K$. Thus, the objective of firm j, k is

$$\max_{P_{kjt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s}\alpha_k^s \left[P_{kjt}Y_{kjt+s} - MC_{kjt+s}Y_{kjt+s}\right] \tag{14}$$

where $MC_{kjt} = \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta} A_{kt}^{-1}W_{kt}^{1-\delta}\left(P_t^k\right)^{\delta}$ in reduced form after imposing the efficiency condition in the optimal mix of labor and intermediate inputs:

$$\delta W_{kt}L_{kjt} = (1-\delta)\, P_t^k Z_{kjt}. \tag{15}$$

In principle firms may like to discriminate among the different customers they have, either households or other firms. We assume that the elasticity of substitution across and within sectors, although allowed to be different among them, are the same for all households and all firms. This assumption shuts down the incentives of firms to discriminate among customers, so the optimal pricing problem takes the simple, standard form:

$$\sum_{s=0}^{\infty} Q_{t,t+s}\alpha_k^s Y_{kjt+s}\left[P_{kt}^* - \frac{\theta}{\theta-1}MC_{kjt+s}\right] = 0 \tag{16}$$

where Y_{kjt+s} is the total production of firm k, j at period $t+s$.

Since idiosyncratic shocks $\{a_{kt}\}_{k=1}^K$ are defined at the sectoral level, the optimal adjusting price, P_{kt}^*, is the same for all firms in a given sector. Thus, aggregating among these firms yields the following motion for sectoral prices

$$P_{kt} = \left[(1-\alpha_k)\, P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta}\right]^{\frac{1}{1-\theta}} \text{ for } k = 1, ..., K. \tag{17}$$

C. Monetary policy

Monetary policy controls I_t which is set according to the Taylor rule:

$$I_t = \frac{1}{\beta}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi}\left(\frac{C_t}{\overline{C}}\right)^{\phi_y} e^{\mu_t} \tag{18}$$

where μ_t is a monetary shock which follows an AR(1) process with persistence ρ_μ.

Note that monetary policy reacts to aggregate consumption inflation and aggregate consumption which here is the best proxy of value-added production (GDP).

D. Equilibrium conditions and definitions

$$\begin{aligned} B_t &= 0, \\ L_{kt} &= \int_{\Im_k} L_{kjt} dj, \\ Y_{kjt} &= C_{kjt} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{k'j't}\left(k, j\right) dj', \\ W_t &\equiv \sum_{k=1}^{K} n_k W_{kt}, \\ L_t &\equiv \sum_{k=1}^{K} L_{kt}. \end{aligned}$$

The first of these equations is the equilibrium in the assets market. The second equation simply aggregates labor within sectors by summing up hours worked in firms belonging to a given sector. The last equation is Walras law for the output produced by firm j in sector k. We use from here on notation j' and k' for firms/sectors from the standing point of a given firm j in sector k. The fourth and fifth equations are definitions of the aggregate wage (which is a weighted average of sectoral wages) and aggregate labor (which linearly sums up hours worked in all sectors).

III Heterogeneity in price stickiness and input-output linkages in the log-linearized system

The full derivation of the solution in steady state as well as of the log-linearized system are relegated to the Appendix. Here we obtain a reduced-form system as we present the channels through which heterogeneity in price stickiness and heterogeneity in input-output linkages across firms affect aggregate outcomes in our model. Small cases denote log-linear deviations from the steady state.

Our reduced-form system has $K+1$ equations and equal number of unknown: value-added production c_t and sectoral prices $\{p_{kt}\}_{k=1}$. The first equation is

$$\sigma \mathbb{E}_t [c_{t+1}] - (\sigma + \phi_c) c_t + \mathbb{E}_t \left[p_{t+1}^c\right] - (1+\phi_\pi) p_t^c + \phi_\pi p_{t-1}^c = \mu_t \tag{19}$$

which is the standard combination of the IS from households' Euler equation in (8) and the monetary authority's Taylor rule in (18). This equation describes the way a monetary shock μ_t is split between variations of value-added production c_t and consumption aggregate prices p_t^c which is

$$p_t^c = \sum_{k=1}^{K} \omega_{ck} p_{kt} \tag{20}$$

The system is completed by the K equations governing the variations in sectoral prices:

$$\beta \mathbb{E}_t [p_{kt+1}] - (1+\beta) p_{kt} + p_{kt-1} = \kappa_k (p_{kt} - mc_{kt}) \tag{21}$$

for $k = 1, ..., K$ where $\kappa_k \equiv (1-\alpha_k)(1-\alpha_k\beta)/\alpha_k$. When the markups of firms in sector k are higher than in steady state, the right-hand side of this equation is positive, so prices in sector k must decrease. The opposite is true when the markup in sector k is smaller than in steady state. Since price stickiness is assumed heterogeneous across sectors, the extent in which prices respond to a given variation in markups depends on κ_k which is heterogeneous across sectors. In the limit, if prices in sector k are fully flexible, $\kappa_k \to \infty$, so $p_{kt} = mc_{kt}$. If prices in sector k are completely rigid, then $\kappa_k \to 0$, so $p_{kt} = 0$.

The effect of sectoral heterogeneity in I/O linkages on the system is captured only in the derivation of log-linearized marginal costs, which comes next.

A. How do I/O linkages affect marginal cost?

We now solve for the log-deviation of marginal costs with special emphasis on the effect of I/O linkages on the system. In particular, we distinguish by the mere existence of these linkages from the need of intermediate inputs for production (i.e., $\delta > 0$), and that these linkages can be heterogeneous across sectors (i.e., $\omega_{kr} \neq \omega_{k'r}$ for all k, all $k' \neq k$, and all r.)

A.1 Preliminary results

Before looking at the effect of I/O linkages on marginal costs, we present their effect on some variables we extensively use later in the analysis.

First, the measure of sectors $\{n_k\}_{k=1}^{K}$ depends on I/O linkages. As derived in the appendix,

$$n_k = \psi\omega_{ck} + (1-\psi)\,\zeta_{kt} \tag{22}$$

where

$$\zeta_k \equiv \sum_{k'=1} n_{k't}\omega_{k'k} \tag{23}$$

In words, the measure n_k of sector k is the weighted sum of the consumption share of sector k, ω_{ck}, and the importance of sector k as supplier in the economy, ζ_k. We refer ζ_k as the "outdegree" of sector k to parallel our analysis with that of Acemoglu et al (2012). The outdegree is obtained summing up the share of sector k as input for all sectors weighted by their respective measure. Since in steady state all firms are identical, n_k is interpreted as the size of sector k.

If intermediate inputs are not used for production, i.e., $\delta = 0$, then $\psi \equiv \delta\,(\theta-1)\,/\theta = 0$, so the sectors' size is determined only by their shares in consumption, $\{\omega_{ck}\}_{k=1}^{K}$. However, when $\delta > 0$, the heterogeneity in I/O linkages introduces heterogeneity in sectoral size. In particular, the outdegree of sector k is higher when sector k is a large supplier to many sectors and/or is a large supplier of large sectors.

It can be shown the vector $\aleph$ of sector sizes $\{n_k\}_{k=1}^{K}$ solves

$$\aleph = (1-\psi)\,[\mathbb{I}_K - \psi\Omega']^{-1}\,\Omega^C \tag{24}$$

where $\mathbb{I}_K$ is the identity matrix of dimension K, Ω is the I/O matrix in steady state with elements $\{\omega_{kk'}\}$ and Ω^C is the vector of $\{\omega_{ck}\}$.

Second, the relevant aggregate prices for demand of intermediate inputs depends on the I/O linkages of the sector which the demanding firm belongs to

$$p_t^k = \sum_{k'=1}^{K} \omega_{kk'}p_{k't}. \tag{25}$$

Thus, the sector-k aggregate price respond more to a variation in prices in a given sector k' when sector k' is a large supplier to sector k.

A.2 Direct effect on sectoral marginal costs

The need of intermediate inputs ($\delta > 0$) has a direct effect on sectoral marginal costs which responds to variations not only in wages, as standard, but also in sectoral prices:

$$mc_{kt} = (1 - \delta)\, w_{kt} + \delta p_t^k - a_{kt} \tag{26}$$

while heterogeneity in I/O linkages enters through sector-k aggregate price, p_t^k. This effect simply captures that, assuming no variation in the production structure of firms in sector k, an increase in the price of a given sector k' implies higher costs of intermediate inputs. This effect is stronger when sector k' is a large supplier of sector k.

A.3 Effect on sectoral wages.

Sectoral wages $\{w_{kt}\}$ are also affected by I/O linkages. This is because the efficient mix of labor and intermediate inputs in equation (15) depends on relative input prices. After impossing this condition, labor demand in sector k for a given production y_{kt} is implictly defined using the production function:

$$y_{kt} = a_{kt} + l_{kt} + \delta \left(w_{kt} - p_t^k \right) \tag{27}$$

In the standard model without I/O linkages ($\delta = 0$), sectoral labor demand is inelastic after conditioning on sectoral production y_{kt} and productivity a_{kt}. Here labor demand depends negatively on wages because higher wages lead firms to substitute labor for intermediate inputs. After combining the log-deviation of the production function and sectoral labor supply

$$w_{kt} - p_t^c = \varphi l_{kt} + \sigma c_t \tag{28}$$

yields

$$w_{kt} = \frac{1}{1 + \delta\varphi} \left[\varphi \left(y_{kt} - a_{kt} \right) + \sigma c_t + \delta\varphi \left(p_t^k - p_t^c \right) \right] + p_t^c \tag{29}$$

The effect of heterogeneity in I/O linkages is captured by the term $\left(p_t^k - p_t^c \right)$ on the right-hand-side of equation (29). In the case of no I/O linkages, wages respond one-to-one to variations in consumption aggregate prices p_t^c through its effect on labor supply. Thus, an increase in prices of sector k' has positive effect on wages in sector k in the extent of the consumption share of sector k', $\omega_{ck'}$. In the economy with I/O linkages, this same effect is present, but a variation in sector k' prices $p_{k't}$ further increases wages in sector k when the I/O linkage of sector k' as supplier of sector k is larger than its consumption share, i.e. $\omega_{kk'} > \omega_{ck'}$. Intuitively, if sector k' is large supplier to sector k, a positive variation in $p_{k't}$ has large effect on increasing the cost of intermediate inputs for firms in sector k, so the demand for labor of these firms increases.

A.4 Effect on sectoral demand

Next we investigate the effect of I/O linkages on the way that variations in total demand y_t is split into sectoral demand, $\{y_{kt}\}_{k=1}^K$. The log-linear total demand for sector k is given by

$$y_{kt} = y_t - \eta \left[p_{kt} - (1-\psi)\, p_t^c - \psi \widetilde{p}_t\right] \tag{30}$$

where

$$\widetilde{p}_t \equiv \sum_{k=1}^{K} n_k p_t^k \tag{31}$$

Sectoral demand depends on its relative price p_{kt} and the weighted average between consumption aggregate prices p_t^c and an "overall sectors" aggregate price $\widetilde{p}_t$ which weights sector-relevant aggregate prices by the size of sectors. After some manipulation, $\widetilde{p}_t$ is

$$\widetilde{p}_t = \sum_{k=1}^{K} \zeta_k p_{kt}, \tag{32}$$

the sum of variations in sectoral prices weighted by their outdegrees $\{\zeta_k\}_{k=1}^K$.

In words, the share of sector k in total demand increases by more after an increase in prices of a given sector k' as sector k' is a larger supplier in the economy, i.e. as $\zeta_{k'}$ is higher. This increase in share is stronger than in an economy with no intermediate inputs $(\delta = 0)$ when $\zeta_{k'} > \omega_{ck'}$.

A.5 Effect on total demand

Finally, we solve for the log-linearized aggregate demand, y_t. Aggregating the log-linearized Walras law across all firms yields

$$y_t = (1-\psi)\, c_t + \psi z_t \tag{33}$$

where z_t is the log-linear deviation of total demand to be used as intermediate inputs. Thus, the need of intermediate inputs create a wedge between total production y_t and value added production, c_t. The log-linear dynamics of z_t around the steady state depends on the heterogeneity of I/O linkages as across sectors. To see this, we solve for z_t by using the log-linearized Walras law, the aggregate production function, the aggregate labor supply and the aggregation of production efficiency:

$$z_t = \frac{\left[(1+\varphi)(1-\psi) + \sigma(1-\delta)\right] c_t - (1+\varphi)\sum_{k'=1}^{K} n_{k'} a_{k't} - (1-\delta)\left(\widetilde{p}_t - p_t^c\right)}{(1-\psi) + \varphi(\delta-\psi)} \tag{34}$$

If there are no I/O linkages, $\delta = 0$, $\psi = 0$ so $y_t = c_t$. But if $\delta > 0$, z_t varies positively with c_t: More value-added production requires more intermediate inputs. Besides, an increase in prices of a given sector k' has negative effect on z_t when $\zeta_{k'} > \omega_{ck'}$. This is because an increase in prices of an important supplier in the economy implies that firms in many and/or the bigger sectors substitute intermediate inputs for labor, so the aggregate demand for intermediate inputs decreases. Note also that heterogeneity in I/O linkages also have an effect on the way that sectoral productivity shocks reduce z_t. In particular, technology shocks of bigger sectors have stronger effect on z_t.

To simplify exposition, we write the relationship between y_t and c_t as

$$y_t = (1+\psi\Gamma_c)\, c_t - \psi\Gamma_a \sum_{k'=1}^{K} n_{k'} a_{k't} - \psi\Gamma_p\left(\widetilde{p}_t - p_t^c\right) \tag{35}$$

where $\Gamma_c \equiv \frac{(1-\delta)(\sigma+\varphi)}{(1-\psi)+\varphi(\delta-\psi)}$, $\Gamma_a \equiv \frac{1+\varphi}{(1-\psi)+\varphi(\delta-\psi)}$, $\Gamma_p \equiv \frac{1-\delta}{(1-\psi)+\varphi(\delta-\psi)}$.

B. Overall solution for log-linearized marginal costs

Using equations obtained above, the expression of log-deviations of marginal costs relative to the steady state depending only on value-added production c_t, sectoral prices $\{n_k\}_{k=1}^{K}$

and exogenous sectoral technology shocks $\{a_k\}_{k=1}^{K}$ is given by

$$
\begin{aligned}
mc_{kt} \;=\; & \left[1+\frac{(1-\delta)\,\varphi\eta}{1+\delta\varphi}\right]p_t^c+\delta\frac{1+\varphi}{1+\delta\varphi}\left(p_t^k-p_t^c\right)+(1-\delta)\,\frac{\varphi\psi\,(\eta-\Gamma_p)}{1+\delta\varphi}\,(\widetilde{p}_t-p_t^c) \qquad (36)\\
& -\frac{(1-\delta)\,\varphi\eta}{1+\delta\varphi}p_{kt}\\
& +\frac{1-\delta}{1+\delta\varphi}\left[\sigma+\varphi\,(1+\psi\Gamma_c)\right]c_t-\frac{1+\varphi}{1+\delta\varphi}a_{kt}-\psi\frac{1-\delta}{1+\delta\varphi}\Gamma_a\sum_{k'=1}^{K}n_{k'}a_{k't}.
\end{aligned}
$$

where p_t^c and $\widetilde{p}_t$ are defined above.

Compare this expression with the one obtained in an otherwise identical economy with no I/O linkages ($\delta=0$):

$$mc_{kt}^{\delta=0}=(1+\varphi\eta)\,p_t^c-\varphi\eta p_{kt}+(\sigma+\varphi)\,c_t-(1+\varphi)\,a_{kt} \qquad (37)$$

The first line of (36) captures the way in which sectoral marginal costs are affected by sectoral prices. In the economy with no I/O linkages, a positive deviation of prices of sector k' has positive effect on marginal costs of other sectors in the extent of the consumption share $\omega_{ck'}$ of sector k' (which are implicit in p_t^c). In the economy with I/O linkages, this "no I/O channel" is mitigated. But I/O linkages create new channels. In particular, $p_{k't}$ has stronger effect on mc_{kt} as sector k' is a bigger supplier to sector k, so $\omega_{kk'}>\omega_{ck'}$ (captured by the second term on the right-hand side of (36)) and as sector k' is a big supplier in the whole economy, so $\zeta_{k'}>\omega_{ck'}$ (captured by the third term on the right hand side of (36)). Overall, the direct effect of variations in $p_{k't}$ on mc_{kt} is

$$\frac{(1-\delta)\left[1+(1-\psi)\,\varphi\eta+\psi\varphi\Gamma_p\right]}{1+\delta\varphi}\omega_{ck'}+\delta\frac{1+\varphi}{1+\delta\varphi}\omega_{kk'}+(1-\delta)\,\frac{\varphi\psi\,(\eta-\Gamma_p)}{1+\delta\varphi}\zeta_{k'}. \qquad (38)$$

Looking now at the forth term on the right-hand side of (36), sectoral marginal costs are decreasing in their own sectoral price. This is because the demand for production of a sector is decreasing in its price, so sectoral wages are decreasing on its sectoral price.

The fifth term on the right-hand side of (36) shows that marginal costs are increasing in the log-deviations of value-added production c_t The sixth term captures the negative effect of a given sector technology shock on its own marginal costs. The seventh term captures the effect of I/O linkages in sectoral marginal costs in the aggregation of idiosyncratic technology shocks.

IV Theoretical results

In this section we study the effect of sectoral heterogeneity in price stickiness and sectoral heterogeneity in I/O linkages on the propagation of shocks. In particular, we look at the response of value-added production c_t to monetary and idiosyncratic technology shocks. We build intuition by studying special cases that gradually add degrees of heterogeneity.

For the study of monetary shocks we shut down sectoral technology shocks, so $a_{kt} = 0$ for all k and t. For the study of the idiosyncratic shocks, we shut down monetary shocks, so $\mu_t = 0$ for all t. Connecting our work to Gabaix (2011) and Acemoglu et al (2012), we further assume that all sectoral technology shocks are iid, so $\rho_a = 0$, and have the same standard deviation ν_a.

A. Homogeneous price stickiness

Assume that the Calvo parameter is homogeneous across sectors, $\alpha_k = \alpha$ in (16) for all k. Then all price equations in (21) may be aggregated by using consumption shares $\{\omega_{ck}\}_{k=1}^K$ to get

$$\beta \mathbb{E}_t \left[\pi_{t+1}^c\right] - \pi_t^c = \kappa \left(p_t^c - \sum_{k=1}^K \omega_{ck} mc_{kt}\right) \equiv \kappa x_t \tag{39}$$

where x_t is the aggregation of sectoral markups and may be interpreted as the inverse of pressure on consumption inflation: positive x_t indicates negative consumption inflation and vice versa. The parameter $\kappa \equiv (1-\alpha)(1-\beta\alpha)/\alpha$ captures the sensitivity of prices to deviations of markups from steady state.

In the following we build intuition only using x_t.

A.1 Monetary shocks

When all sectors have the same degree of price stickiness, the monetary shock affects all sectors equally, so $p_{kt} = p_{k't}$ for all k, k' so $p_t^k = p_t^c = \widetilde{p}_t$ for all k regardless of consumption shares and the heterogeneity in I/O linkages across sectors. Thus, using the equation governing sectoral marginal costs in (36) yields

$$-x_t = \frac{1-\delta}{1+\delta\varphi}\left[\sigma + \varphi\left(1+\psi\Gamma_c\right)\right] c_t \tag{40}$$

which only depends on log-deviations of value-added production c_t. In this case then our model is identical to the standard new Keynesian model, only with the parameter accompaning c_t to be affected by the steady state share δ of intermediate inputs in firms' costs. In this special case, we get closed-form solution for c_t and π_t^c This solution together with some important properties are presented in the next proposition.

Proposition 1 *In an economy where price stickiness is homogeneous across sectors:*

(i) The response of value-added production c_t and consumption inflation π_t^c to a monetary policy shock μ_t is

$$\begin{aligned} c_t &= \Lambda_{\mu c}\mu_t, \\ \pi_t^c &= \Lambda_{\mu\pi}\mu_t. \end{aligned}$$

where $\Lambda_{\mu c} = -\frac{1-\beta\rho}{(1-\beta\rho)\phi_c+\sigma(1-\rho)+(\phi_\pi-\rho)\Psi_c(\delta)\kappa}$, $\Lambda_{\mu\pi} = \frac{\overline{\kappa}_c(\delta)}{1-\beta\rho}\Lambda_{\mu c}$, *and* $\Psi_c(\delta) \equiv \frac{(1-\delta)[\sigma+\varphi(1+\psi\Gamma_c)]}{1+\delta\varphi}$.

(ii) The response of c_t is increasing in δ, the steady state share of intermediate inputs in firms' costs.

(iii) Heterogeneity in I/O linkages is irrelevant for the response of c_t and π_t^c.

Proof. (i) It follows from guess and verify by using equations (19), (39) and (40).

(ii) It follows from comparative statics.

(iii) It follows from observing that $\Lambda_{\mu c}$ and $\Lambda_{\mu\pi}$ do not depend on I/O linkages, $\{\omega_{kk'}\}_{k,k'=1}^{K}$. ■

This proposition is useful for benchmarking. Both value-added production c_t and consumption inflation π_t^c respond negatively to a positive monetary policy shock. The negative response of c_t gets amplified by the need of intermediate inputs for production. Intuitively, since prices are sticky but wages are fully flexible, when the share δ of intermediate inputs in firms' costs increases, the response of marginal costs become more sluggish to the monetary shock. This effect feeds back into the sluggishness of prices, amplifying the overall effect. This result is equivalent to the one found by Basu (1995) and is why in the literature the introduction of intermediate inputs is interpreted as a source of strategic complementarity (for instance, in Nakamura and Steinsson, 2010).

The response of consuption inflation π_t^c is more involving. On the one hand, as explained for the response of c_t, higher share of intermediate inputs δ makes π_t^c to have more sluggish response to a monetary shock. On the other hand, the amplified response of c_t pushes down the response of π_t^c. As a result, the response of c_t is enhanced by more than the response of π_t^c when δ is increased. This is also true in the different cases we study below that gradually add degrees of heterogeneity in price stickiness and heterogeneity in I/O linkages. This is the rationale why, in the following, we build intuition on the mechanisms that we gradually introduce affecting monetary non-neutrality by taking the response of π_t^c as given.

Another important result in Proposition 1 is that the I/O structure of the economy is irrelevant for the propagation of monetary shocks. As a matter of fact, the response of c_t is identical to one in which there is only one sector. We conclude then that, regarding monetary non-neutrality, sectors are defined only as the subset of firms with the same degree of price stickiness while the I/O linkages within sectors is irrelevant.

A.2 Idiosyncratic shocks

Since idiosyncratic shocks are sector-specific, there is dispersion of sectoral prices, so using (36) yields

$$\begin{aligned} -x_t &= \delta \frac{1+\varphi}{1+\delta\varphi} \sum_{k=1}^{K} \omega_{ck} \left(p_t^k - p_t^c\right) + (1-\delta) \frac{\varphi\psi\left(\eta - \Gamma_p\right)}{1+\delta\varphi} \left(\widetilde{p}_t - p_t^c\right) \\ &\quad + \frac{1-\delta}{1+\delta\varphi} \left[\sigma + \varphi\left(1+\psi\Gamma_c\right)\right] c_t - \frac{1+\varphi}{1+\delta\varphi} \sum_{k=1}^{K} \omega_{ck} a_{kt} - \psi \frac{1-\delta}{1+\delta\varphi} \Gamma_a \sum_{k=1}^{K} n_k a_{kt}. \end{aligned} \tag{41}$$

We are interested in the way that idiosyncratic shocks may create volatility in value-added output c_t. The standard diversification argument (Lucas, 1977) claims, by the law of large numbers, that the rate of decay of aggregate volatility is quick. Gabaix (2011) challenges this argument in an economy without I/O linkages where some "firms" are disproportionally larger than others. If this is the case, there are conditions in which the rate of decay is much slower than predicted by the standard law of large numbers, so idiosyncratic volatility can generate sizable aggregate fluctuations.

In our model, all aggregate effects of idiosyncratic shocks are captured by x_t: If the volatility of x_t is zero, then the volatility of c_t (and π_t^c) is also zero. The next proposition revisits Gabaix's argument in our model.

Proposition 2 *In an economy where price stickiness is homogeneous across sectors, I/O linkages across sectors equal consumption shares, $\omega_{kk'} = \omega_{ck'}$ for all k, k', and sectoral technology shocks are such that $\mathbb{E}(a_{kt}) = 0$ and $\mathbb{V}(a_{kt}) = v_a$ for all k, t:*

(i) Heterogeneity of I/O linkages is irrelevant for aggregate volatility.

(ii) When the distribution of consumption shares $\{\omega_{ck}\}_{k=1}^{K}$ follows a power-law distribution $P(S > \varpi) = a\varpi^{-\xi}$ with $\xi \geq 1$, the volatility of value-added production v_c decays according to

$$v_c \sim \begin{cases} \frac{v_a}{\log K}\Lambda_{ac} & \text{for } \xi = 1, \\ \frac{v_a}{K^{1-1/\xi}}\Lambda_{ac} & \text{for } \xi \in [1, 2], \\ \frac{v_a}{K^{1/2}}\Lambda_{ac} & \text{for } \xi \geq 2. \end{cases} \tag{42}$$

where

$$\Lambda_{ac} = \frac{\phi_\pi \Psi_a(\delta)\kappa}{\phi_\pi \Psi_c(\delta)\kappa - (\sigma + \phi_c)} \tag{43}$$

with $\Psi_a(\delta) \equiv \frac{1+\varphi+\psi(1-\delta)\Gamma_a}{1+\delta\varphi}$ is the multiplier of an aggregate technology shock on volatility of value-added output.

Proof. (i) When $\omega_{kk'} = \omega_{ck'}$ for all k, k', then $p_t^k = p_t^c$, so $\widetilde{p}_t = p_t^c$. Further, from (22), $n_k = \zeta_k = \omega_{ck}$ for all k. Therefore

$$-x_t = \frac{1-\delta}{1+\delta\varphi}\left[\sigma + \varphi(1+\psi\Gamma_c)\right]c_t - \frac{1+\varphi+\psi(1-\delta)\Gamma_a}{1+\delta\varphi}\sum_{k=1}^{K}\omega_{ck}a_{kt}, \tag{44}$$

which does not depend on I/O linkages $\{\omega_{kk'}\}_{k,k'=1}^{K}$.

(ii) For the rate of decay, see See Gabaix (2011). For the closed-form solution of Λ_{ac}, verify by using equations (19), (39) and the expression for x_t in (i). ■

This proposition states three important results. The first is that heterogeneity in I/O linkages only matter when it induces sector-relevant aggregate prices to respond differently to the shock. In this case, sectors differ in their importance as suppliers of intermediate inputs, but all sectors have the same productive structure such that outdegrees are equal to consumption shares. The second result is the revisiting of the Gabaix's argument in our model with sectors. In our model consumption shares also are the sectoral shares in value-added output. Thus, if the distribution of consumption shares is fat-tailed enough, meaning that there is enough number of sectors disproportionally large, the rate of decay of aggregate volatility is slower than assumed by the standard diversification argument, $K^{1/2}$. In the limit, when the distribution of consumption shares is linear, $\xi = 1$, the rate of convergence is $\log K$. The third result is that, the parameter Λ_{ac} is interpreted as the scale of the volatility in c_t induced by volatility of an aggregate technology shock. Proposition 2 solves for Λ_{ac} which scale depends on the degree of price rigidity. As prices approach to be fully flexible, $\kappa \to \infty$ so $\Lambda_{ac} \to \frac{\Psi_a(\delta)}{\Psi_c(\delta)}$. In contrast, as prices approach to be fully rigid, $\kappa \to 0$ so $\Lambda_{ac} \to 0$. This result highlights the importance of price rigidity for the propagation of idiosyncratic technology shocks: If prices are fully rigid, there is no effect at all on value-added output.

As the next step, we focus on revisiting the key result of Acemoglu et al (2012) in our model. They build on Gabaix (2011) to show that the fat-tailed distribution of sectors (or firms) is not a necessary condition to violate the standard diversification argument. The same effect can be obtained when production is organized as a network when there is enough number of sectors that are central in the network.

Since we do not have closed-form solution, for our analysis regarding idiosyncratic shocks we use from here on the following definition.

Definition 1 *Let $\{\chi_k\}_{k=1}^K$ be the multipliers of sectoral technology shocks on volatility v_c of value-added output c_t such that*

$$v_c = \Lambda_{ac}\sqrt{\sum_{k=1}^{K} \chi_k^2 v_a} \tag{45}$$

where Λ_{ac} is a normalization that ensures $\sum_{k=1}^K \chi_k = 1$, and v_a is the volatility of sectoral technology shocks (which is homogeneous for all sectors).

We define $\{\chi_k\}$ as the multipliers of sectoral technology shocks on volatility of value-added output. We normalize these multipliers by Λ_{ac}, so $\sum_{k=1}^k \chi_k = 1$. These multipliers capture the net effect of all mechanisms in our model that contribute to generate aggregate volatility from each sectoral shock. This definition exploits a general representation of aggregate volatility in linear rational expectation models when the source of fluctuations are idiosyncratic shocks. For instance, when $\zeta_k = \omega_{ck}$ for all k Proposition 2 applies, so it can be shown that $\chi_k = \omega_{ck}$ for all k and Λ_{ac} solves exactly as in Proposition 2.

The next proposition uses these sectoral multipliers to revisit the main result in Acemoglu et al (2012) and makes it interact with the main result in Gabaix (2011) in the context of our new-Keynesian model.

Proposition 3 *(i) If $\{\chi_k\}_{k=1}^K$ follows a power-law distribution $P(S > \varpi) = a\varpi^{-\xi}$ with $\xi \geq 1$, the volatility of value-added production v_c decays according to*

$$v_c \sim \begin{cases} \frac{v_a}{\log K}\Lambda_{ac} & \text{for } \xi = 1, \\ \frac{v_a}{K^{1-1/\xi}}\Lambda_{ac} & \text{for } \xi \in [1,2], \\ \frac{v_a}{K^{1/2}}\Lambda_{ac} & \text{for } \xi \geq 2. \end{cases} \tag{46}$$

(ii) χ_k is large when sector k has large consumption share, it is large supplier in the economy, i.e. ζ_k is large relative to ω_{ck}, it is large supplier to large suppliers in the economy, it is large supplier to sectors with large consumption share, and/or it is a large supplier to sectors which are large suppliers to sectors with large consumption share.

Proof. (i) See Gabaix (2011). ■

This proposition introduces network effects in our economy. For this, it is necessary that heterogeneity in I/O linkages makes outdegrees different from consumption shares at least for some sectors. This means that we need some sectors to be more important as suppliers than other sectors. In part (i), we simply reinterpret the condition for the violation of the difersification argumen by using $\{\chi_k\}_{k=1}^K$.

To build intuition on the mechanisms involved, assume that that prices are fully flexible, so $x_t = 0$. Further assume that $a_{kt} = 1$ and $a_{-kt} = 0$ for all $-k \neq k$. Using (41), c_t solves

$$c_t = \Psi_c(\delta)^{-1}\left[\begin{array}{c} \frac{1+\varphi}{1+\delta\varphi}\omega_{ck}a_{kt} + \psi\frac{1-\delta}{1+\delta\varphi}\Gamma_a n_k a_{kt} - (1-\delta)\frac{\varphi\psi(\eta-\Gamma_p)}{1+\delta\varphi}\sum_{k'=1}^K(\zeta_{k'} - \omega_{ck'})p_{k't} \\ -\delta\frac{1+\varphi}{1+\delta\varphi}\sum_{k'=1}^K\left(\sum_{r=1}^K\omega_{cr}\omega_{rk'} - \omega_{ck'}\right)p_{k't} \end{array}\right] \tag{47}$$

Inspecting this equation allows to see the way a technology shock in sector k affects value-added output c_t. By the first term in the RHS, the multiplier of a sectoral shock on value-added output is larger when its consumption share is large. This is the mechanism in Gabaix (2011) presented in Proposition 2. The second term in the RHS captures that the same is true when a sector has large outdegree since n_k is increasing in ζ_k. This effect is present in Acemoglu et al (2012).

The third and forth terms are more involved. Prices of the shocked sector k react negatively to the shock. Thus, by the third term when $k' = k$, the multiplier of sector k is increasing in its outdegree, i.e. its importance as supplier in the economy. Besides, the price of other sectors react negatively to the shock in sector k depending on how much their marginal costs decrease responding to the shock and by how large their demand decrease due to substitution of their goods by goods from the sector hit by the positive technology shock. Focusing on the third term on the RHS when $k' \neq k$, the multiplier of sector k is increasing as sector k' is a larger supplier to sectors which have high outdegree. Thus, this is the effect captured in Acemoglu et al (2012) by their second-order outdegree, which measures the importance of sectors as suppliers of large sectors.

The forth term in the RHS captures the interaction between the effects highlighted by Gabaix (2011) and Acemoglu et al (2012): For $k' = k$, this term implies that multiplier of a technology shock to a given sector k is larger when the shocked sector is large supplier to sectors with large consumption share, so $\sum_{r=1}^{K} \omega_{cr}\omega_{rk} > \omega_{ck}$. For $k' \neq k$, this term implies that the multiplier is also increasing as the shocked sector k is large supplier of sectors which in turn are large suppliers of sectors with large consumption shares (when $k' \neq k$).

When prices are rigid, the same four terms exists, but similarly to Proposition 2, the scale Λ_{ac} of aggregate volatility that can be generated is decreasing in the degree of price rigidity.

B. Heterogeneous price stickiness when I/O linkages are irrelevant

We now look at the case when there is heterogeneity of price stickiness across sectors while the effect of heterogeneity of I/O linkages is shut down. Similarly to Proposition 2, heterogeneity in I/O linkages are irrelevant for the effect of monetary shocks and idiosyncratic shocks on value-added output when all sectors require the same combination of intermediate inputs such that outdegrees equal consumption shares, $\zeta_k = \omega_{ck}$ for all k. Note that this condition does not imply that the I/O matrix must be homogeneous – it only does when consumption shares are homogeneous across sectors.

When price stickiness is heterogeneous across sectors, aggregating equations for sectoral prices using consumption shares yields

$$\beta \mathbb{E}_t \left[\pi_{t+1}^c\right] - \pi_t^c = \sum_{k=1}^{K} \kappa_k \omega_{ck} \left(p_{kt} - mc_{kt}\right) \equiv \kappa x_t. \tag{48}$$

Comparing this equation to (39), the heterogeneous degree of price stickiness enters now in the aggregation of sectoral log-deviations of sectoral markups in the RHS of (48). Intuitively, the dynamics of consumption inflation must now consider the fact that sectoral prices absorb at different speeds the deviations of their markups from steady state. Due to this effect, the system cannot be reduced to two equations and two unknowns, c_t and π_t^c. Now we must solve for c_t and $\{p_{kt}\}_{k=1}^{K}$. We do not have closed-form solutions either. However, we still use x_t as a device to organize the analysis. As above, we study independently monetary and idiosyncratic shocks with the latter assumed iid.

B.1 Monetary shocks

The heterogeneity in price stickiness implies that sectoral prices do not have the same response to a monetary shock. However, under the conditions assumed in this section, the sectoral weights of all aggregate prices are the same, so $p_t^k = p_t^c = \widetilde{p}_t$ for all k. Thus, using (36), x_t solves

$$-x_t = \frac{\overline{\kappa}}{\kappa} \left[\left(1 + \frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right) \sum_{k=1}^{K} \left(1 - \frac{\kappa_k}{\overline{\kappa}}\right) \omega_{ck} p_{kt} + \frac{1-\delta}{1+\delta\varphi} \left[\sigma + \varphi\left(1 + \psi\Gamma_c\right)\right] c_t \right]. \tag{49}$$

where $\kappa \equiv (1-\overline{\alpha})(1-\beta\overline{\alpha})/\overline{\alpha}$ with $\overline{\alpha} = \sum_{k=1}^{K} \omega_{ck}\alpha_k$ and $\overline{\kappa} = \sum_{k=1}^{K} \omega_{ck}\kappa_k$ with $\kappa_k \equiv (1-\alpha_k)(1-\beta\alpha_k)/\alpha_k$. In words, we make a distinction between κ – the ability of prices to absorb log-deviations of markups if price stickiness is homogeneous across sectors and equals the average degree of price stickiness – and $\overline{\kappa}$ – the average capacity of sectoral prices to absorb log-deviations in markups. Since κ_k is a highly non-linear function of α_k, κ and $\overline{\kappa}$ are not the same. We introduce this distinction to highlight that sectoral dispersion of price stickiness has a scale effect relative to the case when price stickiness is homogeneous. In particular, κ_k is a decreasing, convex function of α_k with $\kappa_k \to \infty$ as

$\alpha_k \to 0$ and $\kappa_k \to 0$ as $\alpha_k \to 1$. Thus, by Jensen's inequality,

$$\kappa < \overline{\kappa} \tag{50}$$

which increases the parameter accompaning c_t in (49). Thus, by this sole effect, heterogeneity in price stickiness dampens the response of c_t to a monetary shock.

A force in opposite direction introduced by heterogeneity in price stickiness is captured by the first term in (49). As highlighted by Carvalho and Schwartzman (2015), prices of more flexible sectors change more often but the first of these changes after the monetary shock captures most of the response of the shock. In other words, only the first price change of a firm counts for the flexibility of aggregate prices to the monetary shock. Thus, relative to the case when $\kappa_k = \overline{\kappa}$ for all k (i.e., abstracting from the scale effect between κ and $\overline{\kappa}$ explained above), aggregate consumption prices are more sticky when there is heterogeneity of price stickiness. Thus, by this sole effect, the response of c_t to a monetary shock gets amplified. This force is captured in the first term in the RHS of (49) which becomes positive on impact after a monetary shock. This is because the most flexible prices (those with $\kappa_k > \overline{\kappa}$) decrease by more on impact due to the shock than the most sticky sectors (those with $\kappa_k > \overline{\kappa}$). Thus, if the negative response of consumption inflation is the same when κ_k are heterogeneous than when $\kappa_k = \overline{\kappa}$ for all k, then x_t also has the same response in both cases. Then, since the first term in the RHS of (49) responds positively, the negative response of c_t must stronger.

The next proposition summarizes these two forces, so it needs no proof.

Proposition 4 *When heterogeneity in I/O linkages are irrelevant for the propagation of monetary shocks, higher dispersion of price stickiness involves two opposite forces of a monetary shock on the response of value-added output* c_t*, so the overall effect is ambiguous.*

Therefore, whether heterogeneity of price stickiness strengthens or dampens monetary non-neutrality turns out to be an empirical question which we answer in a subsequent section when we calibrate our model to US data.

B.2 Idiosyncratic shocks

After we introduce sectoral technology shocks, x_t becomes

$$-x_t = \frac{\overline{\kappa}}{\kappa}\left[\begin{array}{c}\left(1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right)\sum_{k=1}^{K}\left(1-\frac{\kappa_k}{\overline{\kappa}}\right)\omega_{ck}p_{kt}+\frac{1-\delta}{1+\delta\varphi}\left[\sigma+\varphi\left(1+\psi\Gamma_c\right)\right]c_t \\ -\frac{1+\varphi+\psi(1-\delta)\Gamma_a}{1+\delta\varphi}\sum_{k=1}^{K}\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}a_{kt}\end{array}\right]. \tag{51}$$

This expression must be compared to the one derived in Proposition 2. The first observation is on the last term in the RHS of (51). In Proposition 2, this term captures the Gabaix's effect – if the distribution of the share of sectors in value-added output is fat-tailed (in our model, consumption shares $\{\omega_{ck}\}_{k=1}^{K}$), the diversification argument is violated. Here, what is crucial for the violation of the diversification argument is that the convoluted distribution of consumption shares and the capacity of prices to absorb log-deviations in markups, i.e. $\left\{\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}\right\}_{k=1}^{K}$, is fat-tailed. Intuitively, the aggregate propagation of idiosyncratic technology shocks crucially depends on the responsiveness of prices to the shock. If the larger sectors are also the most flexible sectors, then the distribution of $\left\{\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}\right\}_{k=1}^{K}$ has even fatter tails than $\{\omega_{ck}\}_{k=1}^{K}$. Conversely, the diversitication argument may apply even when consumption shares are fat-tailed if some of the larger sectors are the most sticky sectors in the economy. In particular, since $\kappa_k \to \infty$ as $\alpha_k \to 0$, the introduction of price stickiness has the potential to generate large aggregate volatility from idiosyncratic shocks even if consumption shares (and oudegrees, which effect here is shut down) do not have the properties to violate the diversification argument.

We now turn attention to the first term in the RHS of (51). The same two forces present for monetary shocks are also present here. On the one hand, heterogeneity of price stickiness makes $\overline{\kappa} > \kappa$ which here increase the scale of aggregate volatility that idiosyncratic shocks can generate but it is irrelevant regarding the diversification argument. On the other hand, the speed of response of some sectors versus others make aggregate prices to be more sticky, which decreases the scale of aggregate volatility.

The next proposition summarizes this result.

Proposition 5 *When price stickiness is heterogeneous across sectors and I/O linkages across sectors equal consumption shares, $\omega_{k'k} = \omega_{ck}$ for all k, k':*

(i) The multiplier χ_k of a technology shock on sector k on volatility v_c of value-added

output is increasing in $\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}$, the product of the capacity of its prices to respond to a deviation of markups from the steadt state and the consumption share of this sector.

(ii) The scale Λ_{ac} of the response of value-added output to idiosyncratic technology shocks is reduced relative to an economy without heterogeneity of price stickiness.

Again, the effect of heterogeneity of price stickiness for the aggregate propagation of idiosyncratic shocks is an empirical question that we answer in Section 5.

C. The general case

We now study an economy where price stickiness is heterogeneous across sectors and there is no restriction on the I/O linkages across sectors. The equation governing π_t^c is still (48) but now we use the whole expression in (36) for sectoral marginal costs.

C.1 Monetary shocks

The relevant expression for x_t regarding monetary shocks now is

$$-x_t = \frac{\overline{\kappa}}{\kappa}\left[\begin{array}{c} \left(1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right)\sum_{k=1}^{K}\left(1-\frac{\kappa_k}{\overline{\kappa}}\right)\omega_{ck}p_{kt}+\frac{\delta(1+\varphi)}{1+\delta\varphi}\sum_{k=1}^{K}\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}\left(p_t^k-p_t^c\right) \\ +(1-\delta)\frac{\varphi(\psi\eta+\delta\Gamma_p)}{1+\delta\varphi}\sum_{k=1}^{K}\left(\zeta_k-\omega_{ck}\right)p_{kt}+\frac{1-\delta}{1+\delta\varphi}\left[\sigma+\varphi\left(1+\psi\Gamma_c\right)\right]c_t \end{array}\right] \tag{52}$$

The new terms relative to previous sections are the second and third terms on the right hand side of (52). The second term is positive after a monetary shock when the most sticky sectors (those which prices decrease by less due to the shock) are big suppliers of the sectors with higher consumption share and/or most flexible sectors (those with the higher κ_k). If this term is positive, a given variation in x_t implies stronger variation of c_t.Intuitively, the real effect of monetary shocks is stronger when the marginal costs of the most flexible sectors are more sticky. This happens when the most sticky sectors are big suppliers of the most flexible sectors.

Besides, the third term on the RHS of (52) reinforces this effect. In particular, this term also reacts positively to the shock if the sectors with outdegrees higher than consumption shares are the most sticky sectors. This effect captures the fact that marginal costs of firms are more sticky when the most sticky sectors are the most sticky.

The next proposition puts these mechanisms together with those described in sections 4.1 and 4.2.

Proposition 6 *Overall, the response of value-added output to a monetary shock is amplified*

(i) The share of intermediate inputs in firms' costs is high.

(ii) There is high dispersion of price stickiness across sectors

(iii) Sectors with large outdegree are also the most sticky sectors.

(iv) The most sticky sectors are big suppliers of sectors with high consumption shares and/or the most flexible sectors.

C.2 Idiosyncratic shocks

The relevant expression for x_t regarding idiosyncratic shocks is

$$-x_t = \frac{\overline{\kappa}}{\kappa}\left[\begin{array}{c} \left(1+\frac{(1-\delta)\varphi\eta}{1+\delta\varphi}\right)\sum_{k=1}^{K}\left(1-\frac{\kappa_k}{\overline{\kappa}}\right)\omega_{ck}p_{kt}+\frac{\delta(1+\varphi)}{1+\delta\varphi}\sum_{k=1}^{K}\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}\left(p_t^k-p_t^c\right) \\ +(1-\delta)\frac{\varphi(\psi\eta+\delta\Gamma_p)}{1+\delta\varphi}\sum_{k=1}^{K}\left(\zeta_k-\omega_{ck}\right)p_{kt}+\frac{1-\delta}{1+\delta\varphi}\left[\sigma+\varphi\left(1+\psi\Gamma_c\right)\right]c_t \\ -\frac{1+\varphi}{1+\delta\varphi}\sum_{k=1}^{K}\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}a_{kt}-\frac{\psi(1-\delta)\Gamma_a}{1+\delta\varphi}\sum_{k=1}^{K}\zeta_k a_{kt}\end{array}\right] \tag{53}$$

We must compare this expression with (41) and (51). As in the case with homogeneous price rigidity in (41), the terms in the last row aggregating across sectoral technology shocks gets split in two. The "Gabaix effect" (the first term in the lasr row) is affected by the heterogeneity in price stickiness in one important way: It is not the distribution of consumption shares $\{\omega_{ck}\}_{k=1}^{K}$ what matters; it is the convoluted distribution of the consumption shares and the decree of price rigidity of sectors, $\left\{\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}\right\}_{k=1}^{K}$. The "Acemoglu effect" (the last term in the last row) does not really get affected. However, the interaction of the two effects is affected by the heterogeneity of price stickiness, which is captured in the term

$$\sum_{k=1}^{K}\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}\left(p_t^k-p_t^c\right) \tag{54}$$

Now, a sector has higher multiplier in aggregate volatility when it is a big suppliers of sectors with high $\frac{\kappa_k}{\overline{\kappa}}\omega_{ck}$, i.e., sectors with large consumption share.or very flexible.

All other effects remain the same as in the previous section. The next proposition summarizes all effects together.

Proposition 7 *The multiplier χ_k of sector k on volatility of value added ouput is larger when* *(i) Sector k has large share in value-added output*

(ii) Sector k is very flexible relative to other sectors

(iii) Sector k is a large supplier in the economy.

(iv) Sector k is a large supplier of large suppliers in the economy

(v) Sector k is a large supplier of sectors with large share in value-added-output

(vi) Sector k is a large supplier of the most flexible sectors.

V Data

A. Input-Output Data

A.1 Bureau of Economic Analysis Input and Output Tables

This section discusses the benchmark input-output (IO) tables published by the Bureau of Economic Analysis (BEA) at the U.S. Department of Commerce and how we employ these tables to create an industry to industry matrix of dollar trade flows.

The Bureau of Economic Analysis produces benchmark Input-Output tables which detail the dollar flows between all producers and purchasers in the US. Producers include all industrial and service sectors as well as household production. Purchasers include industrial sectors, households, and government entities. The BEA constructs the IO tables using Census data that is collected every five years. The BEA has published IO tables every five years beginning in 1982 and ending with the most recent tables in 2012.

The IO tables are based on both NAICS and SIC industry codes. Prior to 1997, the IO tables are based on SIC codes.

The IO tables consist of two basic national-accounting tables – a "make" table and a "use" table. The make table shows the production of commodities by industries. Rows present industries, and columns present commodities that each industry produces. Looking across columns for a given row, we see all commodities produced by a given industry are identified. The sum of the entries adds up to industry's output. Looking across rows for a given column, we see all industries producing a given commodities. The sum of the entries adds up the output of that commodity. The use table contains the uses of commodities by intermediate and final users. The rows in the use table contain the commodities and the columns show the industries and final users that utilize them.

The sum of the entries in a row is the output of that commodity. The columns document the products each industry uses as inputs and the three components of "value added" – compensation of employees, taxes on production and imports less subsidies, and gross operating surplus. The sum of the entries in a column adds up to industry output.

We utilize the IO tables for 2002, to create an industry network of trade flows. The BEA defines industries at two levels of aggregation, detailed and summary accounts. We use the detailed accounts to create industry-by-industry trade flows at the four digit IO industry aggregations

A.2 Industry Aggregations

The 2002 IO tables are based on the 2002 NAICS codes. The BEA provides concordance tables between NAICS codes and IO industry codes. We follow the BEA's IO classifications with minor modifications to create our industry classifications for the subsequent calibration. We account for duplicates when six-digit NAICS codes are not as detailed as the IO codes. In some cases, different IO industry codes are defined by an identical set of NAICS codes. For example, for the 2002 IO tables, SIC codes that map to both Dairy farm products(010100) and Cotton (020100). We aggregate industries with overlapping NAICS codes to remove duplicates.

A.3 Identifying Supplier to Customer Relationships

We use the make and use tables to create a table of trade flows between industries. The make table is an industry-by-commodity matrix which contains the amounts of commodities produced by each industry in producer prices. Every industry is designated as a primary producer for a certain commodity, and is often a secondary producer for other commodities. The use table is a commodity-by-industry matrix which contains the amount of commodities consumed by each industry in producers prices. We combine the make and use tables to construct an industry-by-industry matrix which details how much of an industry inputs are produced by other industries. The direct requirements table would be an alternative. These tables specify the amount of gross output of an industry that is required for the production of a given level of final uses.

We use the make table ($MAKE$) to determine the share of each commodity c that each industry i produces. We call this matrix share which is an industry-by-commodity matrix. We define the market share of industry i's production of commodity c as

$$SHARE = MAKE \odot (\mathbb{I} \times MAKE)^{-1}_{i,j}. \tag{55}$$

where $\mathbb{I}$ is a matrix of ones with suitable dimensions.

We multiply the share and use table (USE) to calculate the dollar amount that industry i sells to industry j. We label this matrix revenue share ($REVSHARE$), which is a supplier industry-by-consumer industry matrix:

$$REVSHARE = (SHARE \times USE). \tag{56}$$

We use revenue share matrix to calculate the percentage of industry j's inputs purchased from industry i and label the resulting matrix $SUPPSHARE$:

$$SUPPSHARE = REVSHARE \odot ((MAKE \times \mathbb{I})^{-1}_{i,j})^{\top}. \tag{57}$$

The input-share matrix in equation (57) is an industry-by-industry matrix and therefore consistently maps into our model. The direct requirements table is an commodity-by-industry matrix and the mapping to our theoretical model is therefore less straightforwards. A commodity-by-commodity direct requirements table would be an alternative to our approach of modeling input-output relations but is not readily available. We report calibration results using direct requirements in the appendix for comparison with the literature (see, e.g., Acemoglu et al. (2012)).

B. Price Stickiness Data

We use the confidential microdata underlying the PPI at the BLS to calculate the frequency of price adjustment at the industry level.[1] The PPI measures changes in selling prices from the perspective of producers, and tracks prices of all goods-producing

[1]The data has been used before in Nakamura and Steinsson (2008), Goldberg and Hellerstein (2011), Bhattarai and Schoenle (2014), Gilchrist, Schoenle, Sim, and Zakrajšek (2015), Gorodnichenko and Weber (2016), Weber (2015), and D'Acunto, Liu, Pflueger, and Weber (2016).

industries such as mining, manufacturing, and gas and electricity, as well as the service sector.[2]

The BLS applies a three-stage procedure to determine the individual sample goods. In the first stage, the BLS compiles a list of all firms filing with the Unemployment Insurance system to construct the universe of all establishments in the United States. In the second and third stages, the BLS probabilistically selects sample establishments and goods based on either the total value of shipments or on the number of employees. The BLS collects prices from about 25,000 establishments for approximately 100,000 individual items on a monthly basis. The BLS defines PPI prices as "net revenue accruing to a specified producing establishment from a specified kind of buyer for a specified product shipped under specified transaction terms on a specified day of the month." Prices are collected via a survey that is emailed or faxed to participating establishments. Individual establishments remain in the sample for an average of seven years until a new sample is selected to account for changes in the industry structure.

We calculate the frequency of price adjustment at the good level, FPA, as the ratio of price changes to the number of sample months. For example, if an observed price path is \$4 for two months and then \$5 for another three months, one price change occurs during five months and the frequency is 1/5. We aggregate goods-based frequencies to BEA industry level.

The overall mean monthly frequency of price adjustment is 22.15%, which implies an average duration, $-1/ln(1-SAU)$, of 3.99 months. Substantial heterogeneity is present in the frequency across sectors, ranging from as low as 4.01% for the semiconductor manufacturing sector (duration of 24.43 months) to 93.75% for dairy production (duration of 0.36 months).

VI Empirical Results

A. Calibration

We calibrate a 350 sector versions of the model of section II. We use the make and use tables from the BEA to construct input shares across sectors (see section V). We measure

[2]The BLS started sampling prices for the service sector in 2005. The PPI covers about 75% of the service sector output. Our sample ranges from 2005 till 2011.

sector size as a sectors' share of value added in total value added. We construct sectoral frequencies of price adjustment using the micro data underlying the PPI at the BLS. The granularity of the input-output data determines the definition of sectors for the PPI data. In total, we have three sources of heterogeneity: different combinations of intermediate inputs for production, different sector sizes, and heterogeneous Calvo rates across sectors. We calibrate our model at different levels of detail to analyze how the different degrees of heterogeneity interact. Carvalho (2006) shows that a more granular definition of sectors results in larger real effects of monetary policy. Lucas (1977) instead argues that finer definitions of sectors lowers the aggregate effects of idiosyncratic shocks. We discuss the most granular case with 350 sectors in detail below and delegate a six- and 58-sector model to the appendix.

Figure 1 plots the unweighted sectoral frequencies of price adjustment for a 350 sector model. We see a large fraction of sectors having a mean monthly frequency of price adjustment of 0.1. There is a large right tail in the frequency which is a novel feature of our paper. Previous papers only studies heterogeneity at more aggregate levels and did not have a large, pronounced right tail in the frequency distribution. We study the impact of heavy tails in the frequency distribution below.

We calibrate the model at monthly frequency using standard parameter values in the literature (see Table 2). The coefficient of relative risk aversion, σ is 1 and $\beta = 0.99.75$ implying an annual risk-free interest rate of 3%. We set $\phi = 2$ implying a Frisch elasticity of labor supply of 0.5. We set θ, the average share of inputs in the production function to 0.5 in line with Basu (1995) and empirical estimates. We set the within sector elasticity of substitution θ to 6 implying a steady-state markup of 20% and the across sector elasticity of substitution η to 2 in line with Carvalho and Lee (2011). We set the parameters in the Taylor rule to standard values of $\phi_{pi} = 1.24$ and $\phi_c = 0.33/12$ (see Rudebusch (2002)). The persistence of monetary and (idiosyncratic) technology shocks are $\rho = 0.9$. We investigate the robustness of our findings to permutations in parameter values below.

B. Monetary Policy Shocks

In this section, we study the response of consumption, inflation, and real marginal costs to a one-percent monetary policy shock. A calibration of the model economy with homogeneous Calvo rates equal to the average Calvo rate, equal sector sizes, and input-output structure serves as benchmark to understand the importance of different degrees of heterogeneity and their interactions. We develop intuition analogous to the order in Section IV. Our main empirical result is that heterogeneity in price stickiness is the main driver behind real effects of monetary policy. At the same time, the interaction of heterogeneous price stickiness, sector size and input-output linkages can lower or amplify real effects but only by small amounts. This result depends on the level of granularity, as well as the specification of the monetary policy rule. The response of inflation is also mainly driven by heterogeneity in the frequency of price changes across sectors, but little by heterogeneity in sector size or input-output linkages.

We calibrate six different cases to arrive at these results. We start with an economy with perfectly flexible prices in which consumption and input-output linkages are homogeneous, and add one kind of heterogeneity at a time. Table 3 lists the different combinations of frequencies of price adjustments across sectors, sector sizes, and input-output linkages we study. Table 4 and Figure 2 show our results.

In our first case, prices are fully flexible and adjust fully on impact. We allow for the existence of input-output linkages, but we constrain them to be homogeneous and uniform. When all sectors have the same degree of price stickiness, the monetary shock affects all sectors equally and consumption prices, p_t^c, and sector-relevant prices, p_t^k, are identical (see discussion around Proposition 1). Our model boils down to a textbook New Keynesian model in which the state-share of intermediate inputs, δ, also affects the response of consumption. In a New Keynesian model with fully flexible prices, prices fully absorb the monetary policy shock and we do not see any effect on real consumption or marginal costs (blue lines in Figure 2).

We add homogeneous price stickiness across sectors that is equal to a consumption-share weighted average in the economy of 18.35% in case 2. We know from the discussion of case 1 that our model behaves like a standard New Keynesian model. Price stickiness reduces the impact response of inflation by more than 40% (-1.64 vs. -2.94, see Table 4), and leads to a large impact drop in consumption of 3.46%. Both the inflation and consumption responses are very persistent. Figure 2 shows the response in the blue lines with diamonds.

We do not study the interaction of homogeneous price stickiness, heterogeneous sector size, and heterogeneous input-output structure. We know from proposition 1) that the response of consumption and inflation is independent from heterogeneity in sector size and input-output structure when price stickiness is homogeneous across sectors.

Case 3 studies and economy with heterogeneous price stickiness but homogeneous sector size, and input-output structure. Sectoral prices react differently to a monetary shock due to the heterogeneity in price stickiness. Under the assumptions of case 3, however, sectoral weights of all aggregate prices are the same, so $p_t^k = p_t^c = \widetilde{p}_t$ for all sectors k. We know from Proposition 4 heterogeneity in price stickiness introduces 2 countervailing forces on the response to monetary shocks and the consumption and inflation response is ambiguous.[3] We see in Figure 2 (red-dashed line) the negative selection effect introduced in Carvalho and Schwartzman (2015) dominates and real effects of monetary policy are substantially larger and the price effect is muted.[4] The real effects of monetary policy increase by more than 60%, and the cumulative consumption response more than doubles compared to an economy with homogeneous but equal average price stickiness (see Table 4). The inflation response, instead is substantially muted (see red-dashed line in Figure 2). Our results confirms the intuition for economies with strategic complementary in price setting of Carvalho and Schwartzman (2015) and Nakamura and Steinsson (2009).

Case 4 introduces heterogeneity in consumption weights. Input-output linkages are also heterogeneous but equal to consumption weights, $\omega_{kk'} = \omega_{ck}$. This assumption implies outdegrees equal the weight of sector k in consumption, $\zeta_k = \omega_{ck}$ and hence the aggregate consumption price index, p_t^c, and sector-k aggregate price index, p_t^k, are equal for all sectors k. As a results, there are no effects from input-output linkages on the total markup in the economy, x_t, hence consumption and inflation. Allowing for heterogeneity in consumption weights increases the real effects of monetary policy by 10% and reduces the impact response of inflation by 3% relative to case 3. Heterogeneity in sector size makes little difference for the impact response of consumption and inflation to monetary policy shock but increase the cumulative real effects by 20% and lower the total response of inflation by more than 60% compared to heterogeneity in price stickiness (compare blue-diamond line to red-dashed line and red-dashed line to black-dotted line in Figure

[3]Carvalho and Schwartzman (2015) derive close-form results only for an economy with strategic neutrality in price setting.

[4]Absent strategic interactions in price setting, selection effects characterize the real effects of monetary policy. Real effects are larger if older prices are less likely to change.

2).

Case 5 examines an economy in which input-output linkages are homogeneous but different from consumption weights. It is now no longer the case $p_t^k = p_t^c$ and $\zeta_k = \omega_{ck}$ which opens up two additional wedges (see equation (52)). Real effects of monetary policy increase if sticky-price sectors are large suppliers to flexible-price sectors (making their marginal costs sticky) or to sector important for aggregate consumption. Empirically, we do not find an economically significant effect of these two wedges on either output or inflation relative to the previous cases (see green-dashed line in Figure 2).

Case 6 studies the interaction of all three hetereogeneities: heterogeneity in price stickiness, sector size, and input-output linkages. The real effects of monetary policy and the inflation response is not qualitatively or quantitatively substantially different from the previous cases once we allow for heterogeneity in the frequency of price adjustment (see magenta asterisks in Figure 2). The reason the interaction of all three heterogeneities is not important comes from the empirical fact that sectoral price stickiness, sector size, and outdegrees are almost uncorrelated.

To summarize, heterogeneity in price stickiness is the main driver of real effects in monetary policy shocks in our calibration of a 350 sector economy to the empirical distribution of price stickiness from the BLS and the input-output structure from the BEA. Input-ouput linkages and heterogeneity in sector size have some effect, but these effects are small compared to the effects of price stickiness. These findings suggest there is no strong systematic relationship between price flexibility and the importance of sectors as suppliers of flexible sectors, or the economy as a whole.

The real effects of monetary policy are due to the following three components:

$$\begin{aligned} -x_t &= \Lambda_0(\delta) \sum_{k=1}^{K} (\overline{\kappa} - \kappa_k)\, \omega_{ck} p_{kt} + \delta \Lambda_1(\delta) \sum_{k=1}^{K} \kappa_k \omega_{ck} \left(p_t^k - p_t^c\right) \\ &\quad + \psi \Lambda_2(\delta)\, \overline{\kappa} \sum_{k=1}^{K} (\zeta_k - \omega_{ck})\, p_{kt} + \Lambda_3(\delta)\, c_t \end{aligned}$$

Figure 3 decomposes the overall response of consumption into the three components. Heterogeneity in price stickiness is responsible for most of the overall response of consumption (black line). Heterogeneity in consumption shares and input-output linkages add (red-diamond line and blue-dashed line), the second and third terms, lower the response of consumption to a monetary policy shock in our baseline calibration (negative contribution) but are smaller in absolute terms. The figure confirms the results of the calibration and documents why heterogeneity in price stickiness drives most of the real effects of monetary policy.[5]

Table 5 reports the cumulative real effects of monetary policy shocks for the ten least (Panel A) and most (Panel B) responsive sectors for our different cases. We know from the discussion above and in Section IV all sectors are equally responsive in cases 1 and 2. Once we allow for heterogeneity in price stickiness in case 3 we a somewhat heterogeneous response for the most flexible price sectors in Panel A. The heterogeneity in real effects across sectors is substantially amplified for the most responsive sectors. The heterogeneity in case 3 is actually largest across cases which confirms our findings above that heterogeneity in consumption shares or input-output structure might not amplify real effects of monetary policy shocks. Panel C and D report the BEA industry classification codes of the most and least responsive sectors. We see the identity of the most and least responsive sectors varies substantially across cases indicating the convolution of different heterogeneities is important.

B.1 **Robustness and Alternative Specifications**

In this subsection we study the robustness of our baseline findings to variation in parameters of the Taylor rule, changes in preference parameters, and a specification with exogenous nominal demand rather than closing the model with a Taylor rule. We report the results in Figure 4 and Table 6 for the same six cases we studied in the previous section.

First, we increase ϕ_π, the systematic response of monetary policy to inflation in the Taylor rule, from a baseline value of 1.24 to 2.5.

We see in Figure 4 a similar response of inflation independent of whether we study heterogeneous or homogeneous price stickiness, sector size, and input-output structure. The impact response of inflation is, however, roughly cut in half which comes from less strong demand effects. The inflation response tends to be more persistent with all three forms of heterogeneity leading to larger cumulative inflation responses in an economy with

[5]Equation (58) decomposes the overall effect in the inverse inflation pressure into three components. To develop intuition we set it equal to the real consumption response.

$\phi_\pi = 2.5$ than in an economy with $\phi_\pi = 1.24$ (compare Panel B of Table 4 and Table 6).

The higher systematic response to inflation in the Taylor rule reduces the impact response of consumption by a factor of three across different cases (compare Panel A of Table 4 and Table 6). A model with heterogeneous price stickiness but homogeneous sector size and input-output structure has a similar impact response to an economy in which all three forms of heterogeneities interact (case 3 versus case 6). A higher weight on inflation stabilization in the Taylor rule for a given demand shock results in a larger stabilization of output in the standard New Keynesian model. We see a similar results for the cumulative real effects of a demand shock in case 2, an economy with homogeneous price stickiness across sector and no heterogeneity in sector size or input-output structure. One we allow for heterogeneous price stickiness, the cumulative real effects of a monetary shock only contract by 40% with a more stringent response to inflationary pressure in the Taylor rule. Once we add heterogeneity in sector size and input-output structure, we even see larger cumulative real effects or a less stark drop in an economy with a more systematic response to inflation despite smaller real effects on impact. We see in Panels A and C of Table 4 and Table 6 the different forms of heterogeneity introduce a more sluggish and persistent response in consumption and real marginal costs which explains the large real effects of demand shocks despite the smaller effects on impact. This findings is reminiscent to the responder-nonresponder framework discussant in Carvalho (2006) and the selection effect of Carvalho and Schwartzman (2015).

Changes in the systematic response to output growth has little impact on the response of real consumption, inflation, or real marginal costs (not tabulated).

Second, we study the effect of changes in risk aversion, σ, the Frisch elasticity, φ, the average input share in production, δ, the elasticity of substitution within and across sectors, η, θ and the persistence of shocks, ρ, in our full-blown model (case 6 in Table 3). Specifically, we set (baseline parameters in parentheses) $\sigma = 2(1)$, $\varphi = 1(2)$, $\delta = 0.7(0.5)$, $\eta = 6(2)$, $\theta = 10(6)$, and $\rho = 0.95(0.90)$. Figure 5 and Table 7 report our findings.

Overall, we see our results in the baseline calibration of Table 4 and Figure 2 are robust to variations in parameter values. The only exception is the increase of the coefficient of relative risk aversion, σ, from a baseline value of 1 to 2. The intratemporal rate of substitution from leisure to consumption determines the real wage. The drop in consumption results in a drop in the real wage which increases in σ. Lower real wages lower the response of real marginal costs and the overall demand pressure.

Third, we study a calibration of our baseline model with baseline parameter values but close the model positing exogenous nominal demand. Figure 6 and Table 8 report our findings. The red-dashed line represents our baseline response with Taylor rule and the green dash-dotted line represents the response for a model with exogenous nominal demand. Real marginal costs barely move in the model with exogenous demand resulting in small and transient impact response of inflation and a one-percentage-point response of consumption on impact. The impact response of consumption is smaller by a factor of 6 compared to the impact response with a Taylor rule.

Last, we compare the calibration of the 350 sector economy to a less granular 58 sector model. Figure 7 and Table 8 report our finding.[6] Real effects of monetary policy are 32% larger in the more granular 350 sector economy compared to the 58 sector calibration. The impact response of inflation, on the contrary, is only 13% smaller in the 350 sector economy compared to the 58 sector economy. This finding cautions against drawing inference for policy from the response of inflation to shocks as small-scale models might substantially underestimate the real effects. The differential response, however, is only true in a model with Taylor rule and vanishes once we close the model with exogenous nominal demand (see appendix).

C. Idiosyncratic Productivity Shocks

In this section we use our data to construct a measure of how idiosyncratic shocks may generate fluctuations in value-added output in our economy within our calibrated model. Once again, we construct our empirical results in stages by obtaining results using calibrations of our model that gradually add degrees of heterogeneity.

Our general methodology is to compute the multipliers $\{\chi_k\}_{k=1}^{K}$ and scale Λ_{ac} in Definition 1 such that

$$v_c = \Lambda_{ac}\sqrt{\sum_{k=1}^{K}\chi_k^2 v_a} \tag{58}$$

and then estimate the parameter ξ of the power-law in the distribution of multipliers to apply Proposition 3.

We compute χ_k using the MA representation of value-added output in our model

[6] We report robustness results for the 58 sector calibration in the online appendix.

when there are only technology shocks to sector k

$$c_t = \sum_{\tau=0}^{\infty} \gamma_{k\tau} a_{kt-\tau} \tag{59}$$

so

$$\begin{aligned} \widetilde{\chi}_k &= \sqrt{\sum_{\tau=0}^{T} \gamma_{k\tau}^2}, \\ \Lambda_{ac} &= \sum_{k=1}^{K} \widetilde{\chi}_k, \\ \chi_k &= \frac{\widetilde{\chi}_k}{\Lambda_{ac}}. \end{aligned}$$

Note that the parameter Λ_{ac} is not directly interpretable as the multiplier of iid aggregate technology shocks to value-added volatility, which is

$$\Lambda_{ac}^{agg} = \sqrt{\sum_{k=1}^{K} \left(\sum_{\tau=0}^{T} \gamma_{k\tau} \right)^2}, \tag{60}$$

when price stickiness is heterogeneous. This is because in this case the response of c_t does not die on impact in spite of sectoral technoloigy shocks are iid. However, quantitative Λ_{ac} and Λ_{ac}^{agg} are very close, so we only report Λ_{ac}.

To obtain the parameter ξ in the power law distribution, we follow Gabaix (2011) and Acemoglu et al. (2012) by running an OLS regression of the counter-cumulative density function to the sequence of multipliers $\{\chi_k\}_{k=1}^{k}$ using the correction suggested by Gabaix and Ibragimov (2011) for small samples. The tail of the counter-cumulative distribution corresponds to the 20% largest multipliers. Table 9 summarizes the different cases we study and Table 10 reports the results.

We start with a case where there is no price stickiness and consumption shares as well as I/O linkages are homogeneous across sectors. In this case our model has a closed-form solution. Following Proposition

$$\Lambda_{ac}^{case\ 1} = \frac{\Psi_a(\delta)}{\Psi_c(\delta)}, \chi_k^{case\ 1} = \frac{1}{K}. \tag{61}$$

Therefore

$$v_c = \frac{\Lambda_{ac} v_a}{\sqrt{K}} \tag{62}$$

which is considered a quick rate of convergence although Λ_{ac} is large. For 350 sectors, this means that $\sqrt{\sum_{k=1}^{K} \chi_k^2} = \sqrt{350} = .05$.

The second case studied is flexible prices when consumption shares matches those in the data. I/O linkages are set equal to consumption shares, so they do not have any active role in aggregate volatility, as assumed in section 4.2. We find that, in line with Gabaix (2011), the share of sectors in value-added output is very fat-tailed, with parameter $\xi = 1.0029$ and statistically not different than one (standard deviation .17). This regression fits well the data on the 20% of the distribution of mutipliers, with $R^2 = .95$. The assymptopic standard deviation of value added converges then at a rate K close to $\log(K)$. For 350 sectors, $\sqrt{\sum_{k=1}^{K} \chi_k^2} = .17$, more than three times higher than when consumption shares are assumed homogeneous (case 1).

Case 3 assumes flexible prices, homogeneous consumption shares, and I/O linkages in steady state replicate our data. We find that network effects by themselves have sizable but much weaker effect that the heterogeneity in consumption shares on generating aggregate volatility from idiosyncratic technology shocks. The parameter $\xi = 1.6123$, with standard deviation .27 and $R^2 = .97$ and $\sqrt{\sum_{k=1}^{K} \chi_k^2}$ for our 350 sectors economy equals .1, the double of that in the homogeneous economy with flexible prices. One possible reason why our network effects is weaker than suggested by Acemoglu et al. (2012) is because we leave out some sectors which prices are not available in teh PPI sample. Many of these sectors are goverment sectors, like Federal General Government Defense, Federal General Government Non-Defense, Federal Government Enterprises, State and Local General Government and State and Local Government Enterprises. Besides data availability, the reason of lack of pricing data for these sectors is that these sectors do not behave as standard produtctive sectors in the economY. Thus, the mechanism in which shocks to these sectors propagate to the overall economy are arguably different to those in standard productive sectors.

Case 4 puts together heterogeneity of consumption shares and I/O linkages that in steady state replicate the data in an economy with flexible prices. Naturally the result is that the parameter of the power-law is between those computed when only on of these two dimensions of heterogeneity were active. The tail of the distribution of multipliers in this case inherit much of the property of the multiplier of consumption shares, with $\xi = 1.1$, standard deviation .19 and $R^2 = .93$. Overall, for 350 sectors, this means $\sqrt{\sum_{k=1}^{K} \chi_k^2}$ that of the flexible economy.

In cases 5 to 10 prices are assumed sticky, either homogeneous in its weighted mean level using consumption shares(cases 5-8) or heterogeneous across sectors (cases 9-10). In all these cases the scale parameter Λ_{ac} is much smaller than that in our flex-price calibrations capturing the augmented effect of price stickiness on aggregate fluctuations originated in aggregate shocks.

Regarding the violation argument, the parameter ξ in the power-law regression is exactly the same only when consumption shares are assumed heterogeneous to match in steady state those in the data (case 6). In case 7, the form of the distribution of multipliers does not violate the diversification argument, with $\xi = 2.0967$. In case 8, which puts together the heterogeneity in consumption shares and I/O linkages when price stickiness is homogeneous. The parameter $\xi = 1.0585$, its standard deviation is .18, the fit of the regression on the 20% tail is $R^2 = .94$, and $\sqrt{\sum_{k=1}^{K} \chi_k^2}$ for 350 sectors is .15.

Cases 9 and 10 calibrate price stickiness to match the sectoral frequency of price changes in the PPI data. Case 9 shuts down any other form of heterogeneity. We find that $\xi = 1.17388$ with standard deviation .29 and $R^2 = .84$, which is smaller than for other cases but still high. This case shows that heterogeneity of price stickiness as in the data is enough to violate the diversification argument in our model, with $\sqrt{\sum_{k=1}^{K} \chi_k^2} = .09$, almost twice higher than in the homogeneous economy. Finally, case 3 puts together all three types of heterogeneity. The parameter $\xi = .9895$, pointwise smaller than one but statistically not different than one at 95% since its standard deviation is .17. The pointwise estimation of ξ implies that aggregate volatility converges at a even slower rate than $\log(K)$. For our 350 sectors economy, $\sqrt{\sum_{k=1}^{K} \chi_k^2} = .18$.

VII Concluding Remarks

Asymmetries in sector size and input-output structure across sectors can contribute to aggregate fluctuations arising from idiosyncratic shocks which are propagated through prices. Heterogeneity in prices stickiness is a key ingredient in many macro models to generate persistent real effects of monetary policy shocks. In this paper we quantitatively study the tension between price stickiness, real effects of monetary policy, and aggregate fluctuations arising from sectoral shocks.

We find heterogeneity in price stickiness is the central mechanism for large and persistent real effects of nominal shocks. Consistent, we also document that small-scale models might substantially underestimate real effects even though the impact response of inflation is almost identical.

We develop a multi-sector New Keynesian model with intermediate inputs and calibrate a 350 sector version of the model to the input-output tables from the Bureau of Economic Analysis and the micro data underlying the producer price index from the Bureau of Labor Statistics to reach these conclusions.

References

Acemoglu, D., U. Akcigit, and W. Kerr (2015). Networks and the macroeconomy: An empirical exploration. *In preparation for: NBER Macroannual*.

Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The network origins of aggregate fluctuations. *Econometrica 80*(5), 1977–2016.

Basu, S. (1995). Intermediate goods and business cycles: Implications for productivity and welfare. *The American Economic Review 85*(3), pp. 512–531.

Bhattarai, S., G. Eggertsson, and R. Schoenle (2014, February). Is Increased Price Flexibility Stabilizing? Redux. NBER Working Papers 19886, National Bureau of Economic Research, Inc.

Bhattarai, S. and R. Schoenle (2014). Multiproduct firms and price-setting: Theory and evidence from u.s. producer prices. *Journal of Monetary Economics 66*(0), 178–192.

Bigio, S. and J. Lao (2013). Financial frictions in production networks. *Unpublished Manuscript, Columbia University*.

Boivin, J., M. P. Giannoni, and I. Mihov (2009). Sticky prices and monetary policy: Evidence from disaggregated us data. *The American Economic Review 99*(1), 350–384.

Bouakez, H., E. Cardia, and F. Ruge-Murcia (2014). Sectoral price rigidity and aggregate dynamics. *European Economic Review 65*, 1–22.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398.

Carvalho, C. (2006). Heterogeneity in price stickiness and the real effects of monetary shocks. *The B.E. Journal of Macroeconomics 2*(1), 1.

Carvalho, C. and J. W. Lee (2011). Sectoral price facts in a sticky-price model. *Unpublished Manuscript, PUC-Rio*.

Carvalho, C. and F. Schwartzman (2015). Selection and monetary non-neutrality in time-dependent pricing models. *Journal of Monetary Economics 76*, 141 – 156.

Carvalho, V. M. (2014). From micro to macro via production networks. *The Journal of Economic Perspectives*, 23–47.

D'Acunto, F., R. Liu, C. Pflueger, and M. Weber (2016). Flexible prices and leverage. *Unpublished manuscript, University of Chicago*.

Gabaix, X. (2011). The granular origins of aggregate fluctuations. *Econometrica 79*(3), 733–772.

Gabaix, X. and R. Ibragimov (2011). Rank- 1/2: a simple way to improve the ols estimation of tail exponents. *Journal of Business & Economic Statistics 29*(1), 24–39.

Gilchrist, S., R. Schoenle, J. Sim, and E. Zakrajšek (2015). Inflation dynamics during the financial crisis. *Unpublished Manuscript, Brandeis University*.

Goldberg, P. P. and R. Hellerstein (2011). How rigid are producer prices? *FRB of New York Staff Report*, 1–55.

Gorodnichenko, Y. and M. Weber (2016). Are sticky prices costly? Evidence from the stock market. *American Economic Review 106*(1), 165–199.

Herskovic, B. (2015). Networks in production: Asset pricing implications. *Unpublished Manuscript, UCLA*.

Huang, K. X. and Z. Liu (2004). Input–output structure and nominal rigidity: the persistence problem revisited. *Macroeconomic Dynamics 8*(02), 188–206.

Long, J. B. and C. Plosser (1983). Real business cycles. *The Journal of Political Economy*, 39–69.

Lucas, R. E. (1977). Understanding business cycles. In *Carnegie-Rochester conference series on public policy*, Volume 5, pp. 7–29. Elsevier.

Nakamura, E. and J. Steinsson (2008). Five facts about prices: A reevaluation of menu cost models. *Quarterly Journal of Economics 123*(4), 1415–1464.

Nakamura, E. and J. Steinsson (2009). Monetary non-neutrality in a multi-sector menu cost model. *Quarterly Journal of Economics 125*(3), 961–1013.

Ozdagli, A. and M. Weber (2016). Monetary policy through production networks: Evidence from the stock market. *Unpublished Manuscript, University of Chicago*.

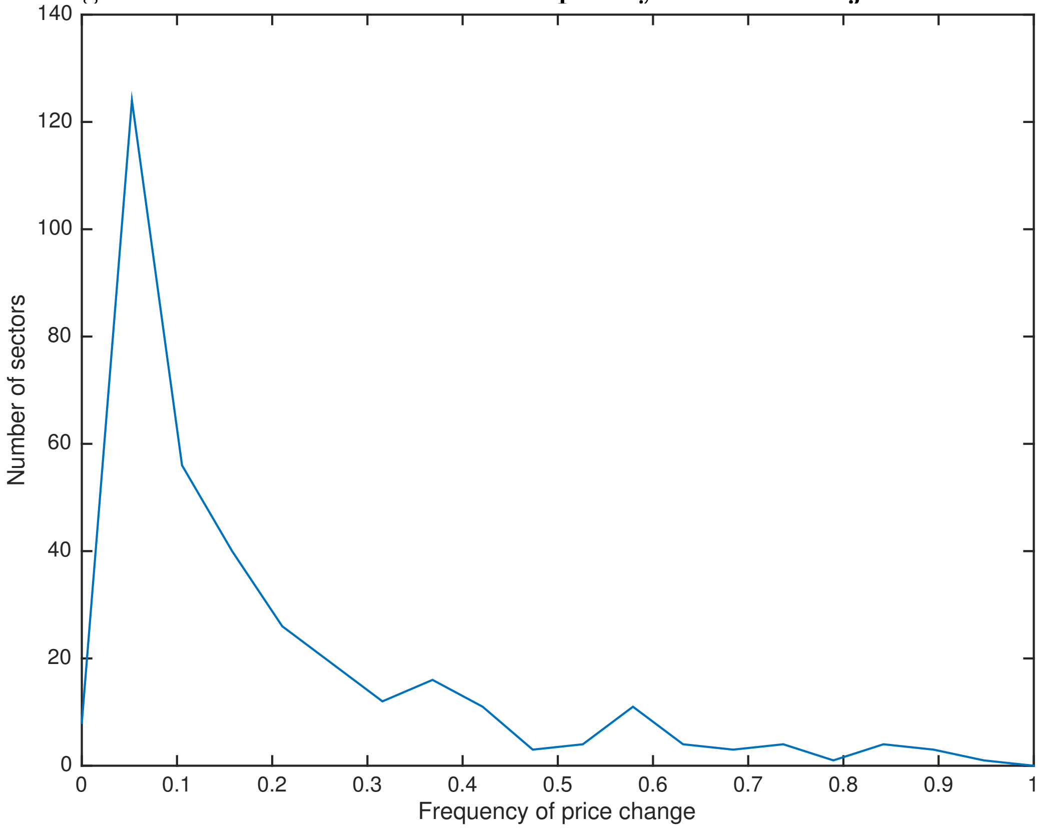
Ramey, V. A. (2015). Macroeconomic shocks and their propagation. *Handbook of Macroeconomics, forthcoming*.

Rudebusch, G. D. (2002). Term structure evidence on interest rate smoothing and monetary policy inertia. *Journal of Monetary Economics 49*(6), 1161–1187.

Shamloo, M. (2010). Price setting in a model with production chains: Evidence from sectoral data. *IMF Working Papers*, 1–50.

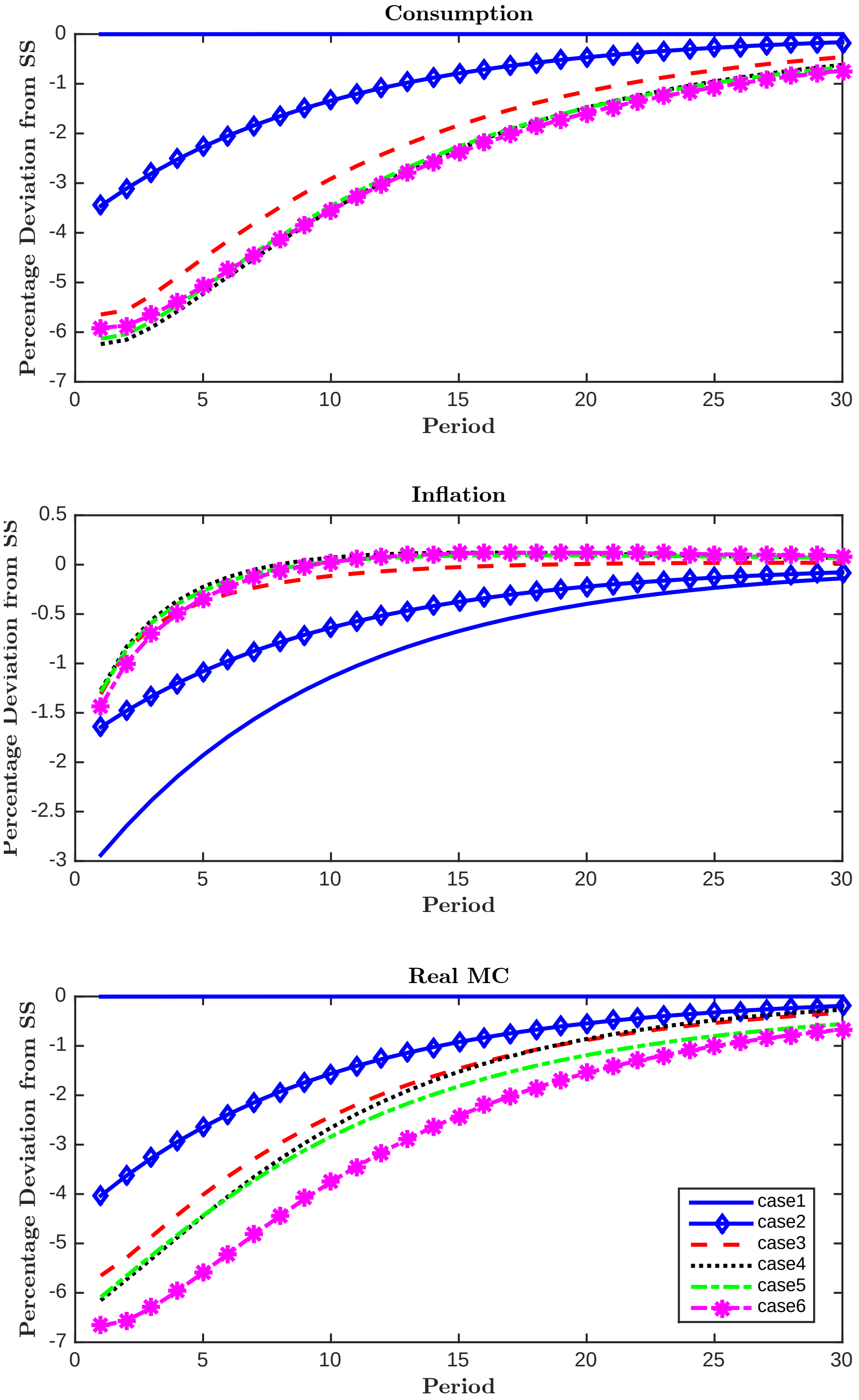
Weber, M. (2015). Nominal rigidities and asset pricing. *Unpublished manuscript, University of Chicago Booth School of Business*.

Figure 1: **Distribution of the Frequency of Price Adjustment**



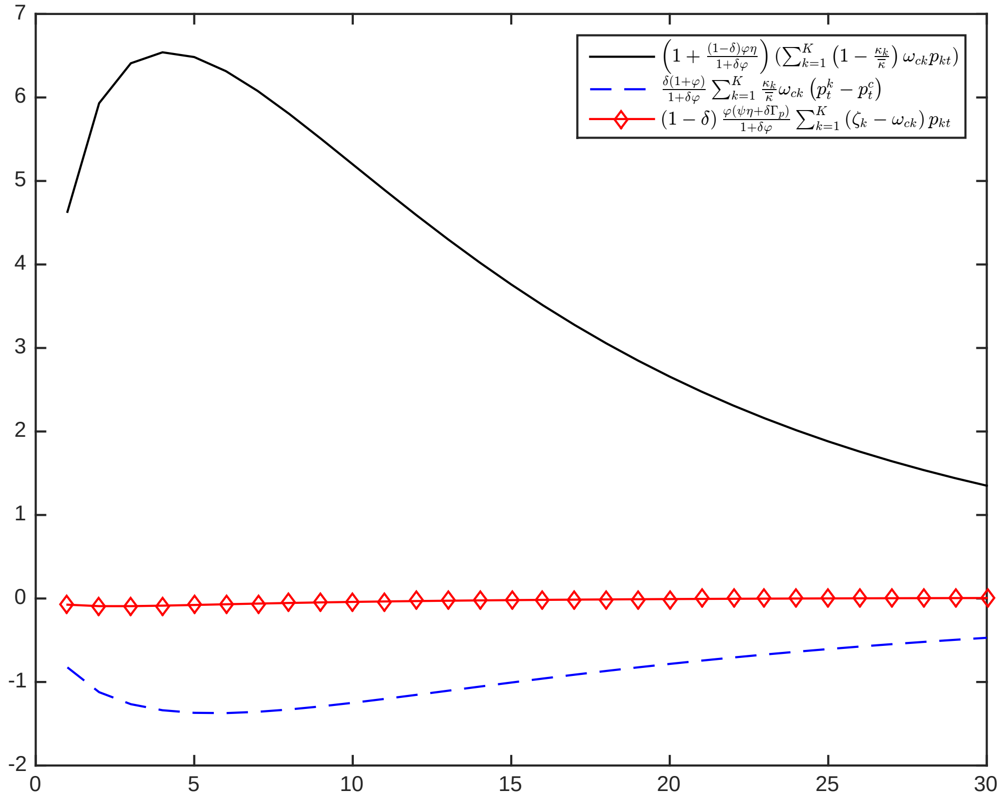
This figure plots the distribution of the frequency of price adjustment for a 350 sector model.

Figure 2: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock**



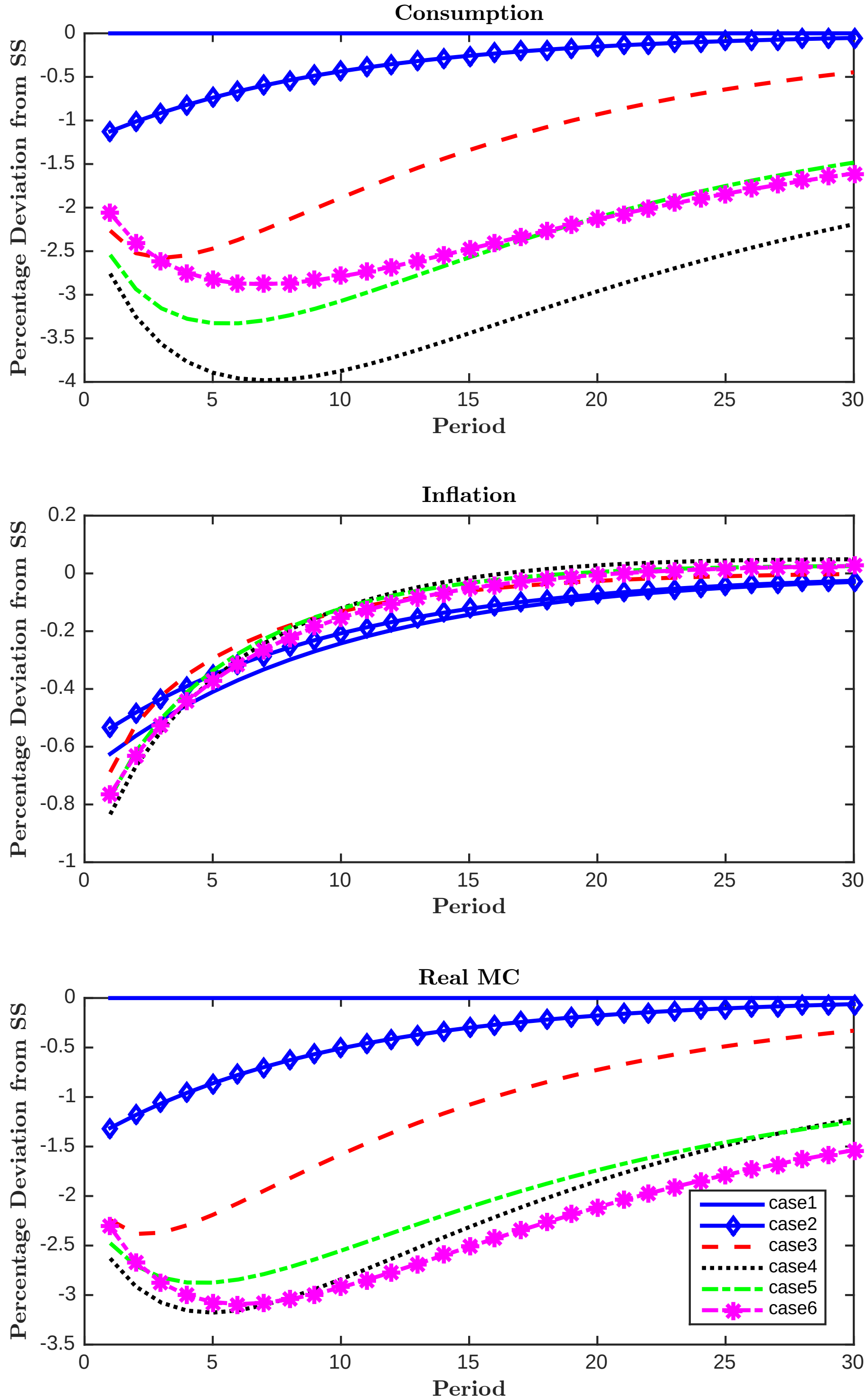
This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard deviation monetary policy shock for a 350 sector model for different cases (see Table 3).

Figure 3: **Decomposition of the Overall Response of Markups into Components**



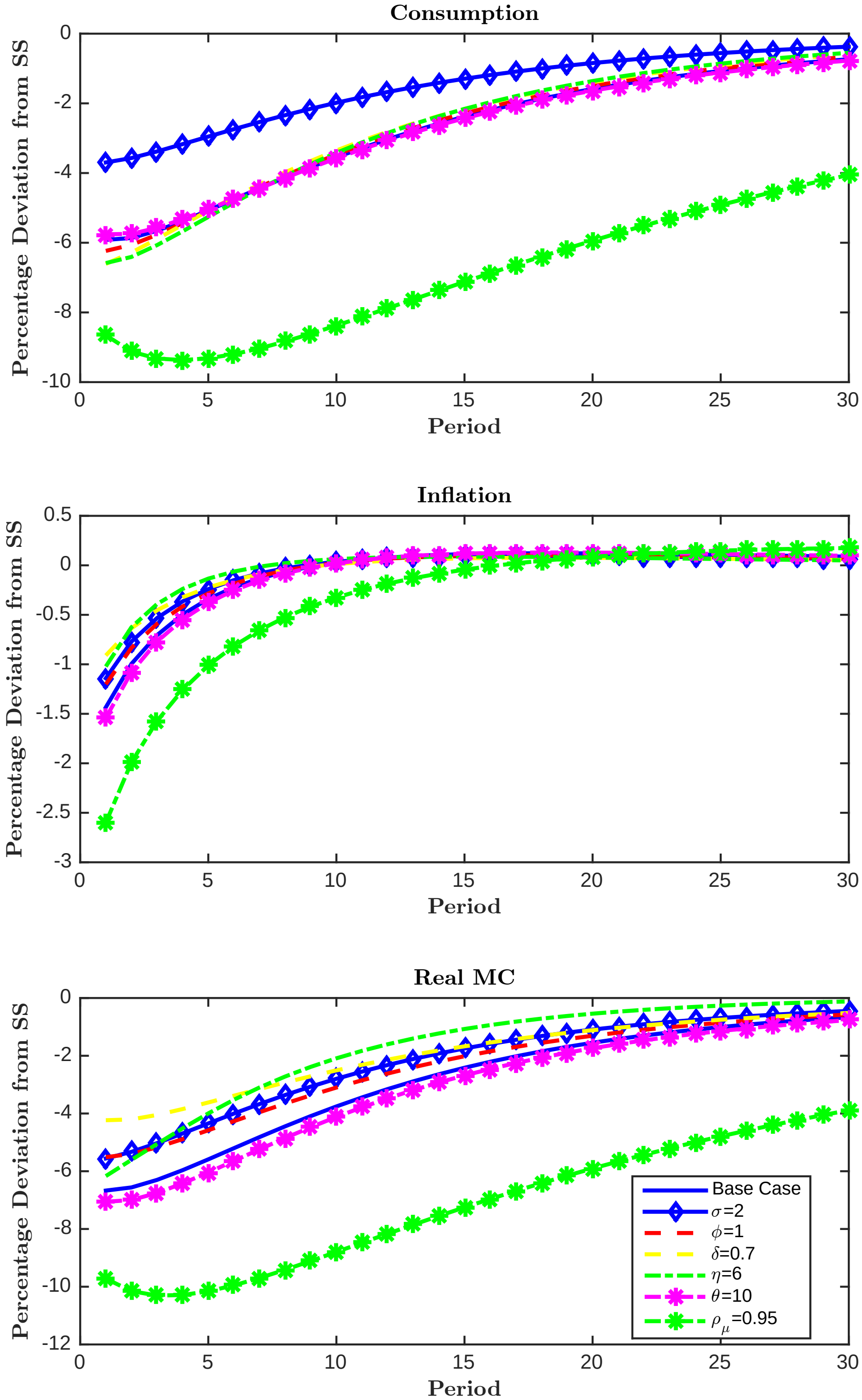
This figure decomposes the overall response of markups to a one-standard deviation monetary policy shock for a 350 sector model into the three different components discussed in Section IV.

Figure 4: **Inflation Response to Monetary Policy Shock** ($\phi_\pi = 2.5$)



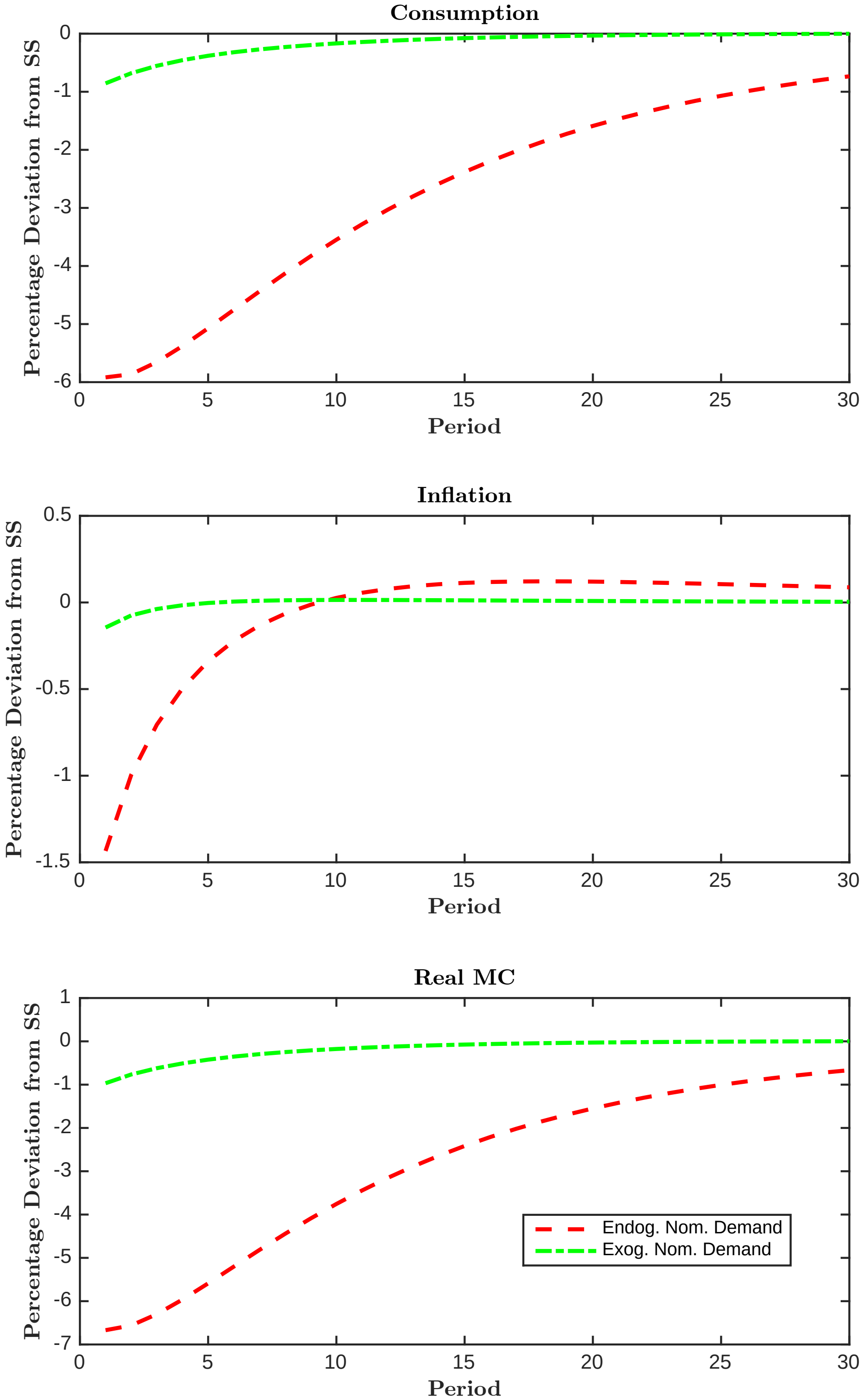
This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard deviation monetary policy shock for a 350 sector model for different cases (see Table 3) with a coefficient on inflation in the Taylor rule of $\phi_\pi = 2.5$.

Figure 5: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock (variations in parameters)**



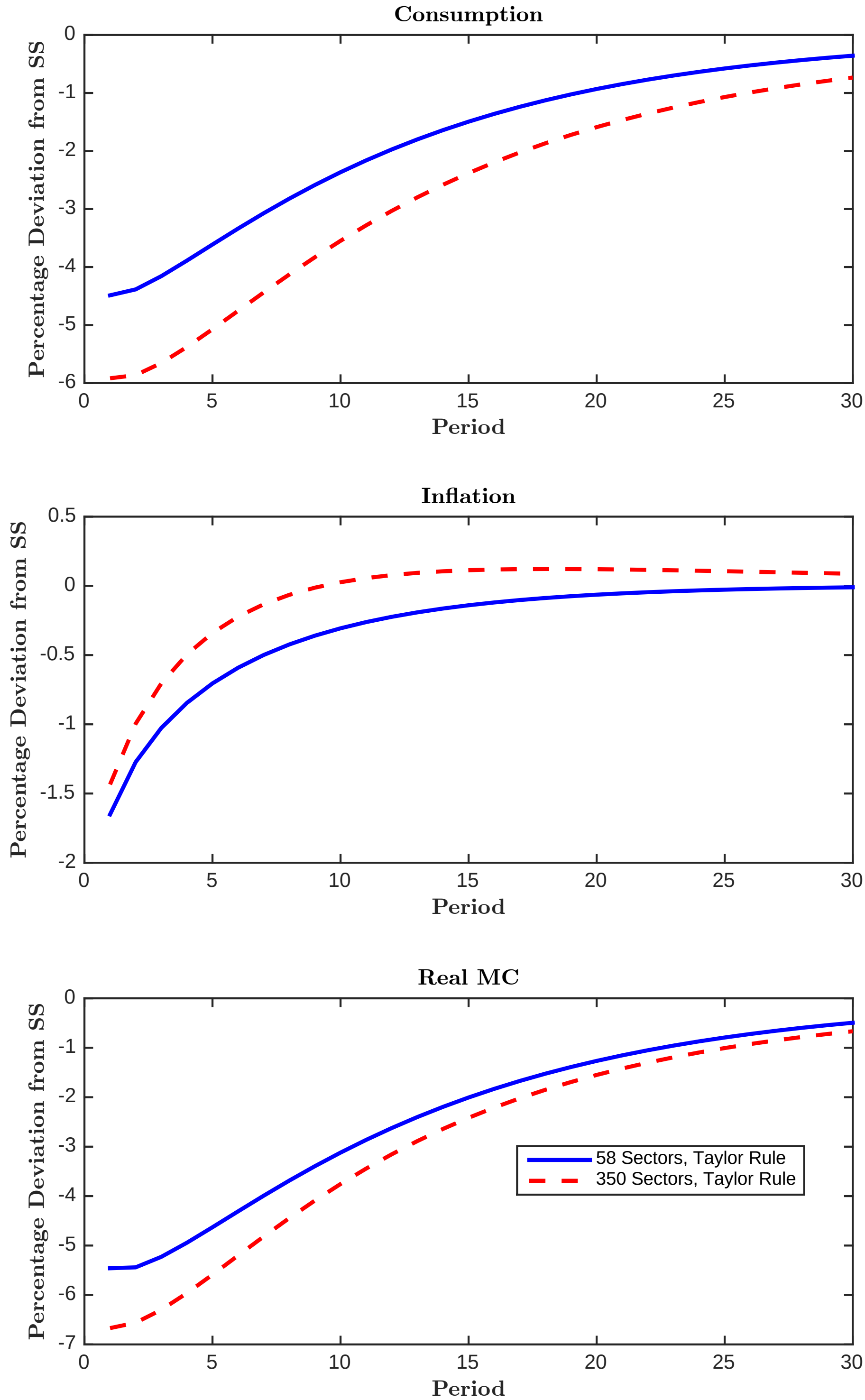
This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard deviation monetary policy shock for a 350 sector model for different cases (see Table 3) for different values of structural parameters.

Figure 6: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock (exogenous nominal demand)**



This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard deviation monetary policy shock for a 350 sector model for case 6 (see Table 3) closing the model with positing exogenous nominal demand.

Figure 7: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock (58 vs 350 sector economy)**



This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard deviation monetary policy shock for a 58 and 350 sector model for case 6 (see Table 3).

Table 1: Descriptive Statistics

The table reports the moments of the frequency of price adjustment, FPA, distribution for a 58 sector model in Panel A. and a 350 sector model in Panel B.

	Mean	Median	Std	25^{th} Pct	75^{th} Pct
	Panel A. 58 Sector Economy				
FPA	0.19	0.14	0.16	0.08	0.25
	Panel b. 350 Sector Economy				
FPA	0.19	0.12	0.20	0.06	0.26

Table 2: Calibration Parameters

This table reports the parameter values of the calibration of the model developed in Section IV.

β	0.9975	Monthly discount factor
σ	1	Relative risk aversion
φ	2	Inverse of Frisch elasticity
δ	0.5	Average inputs share in production function
η	2	Elasticity of substitution across sectors
θ	6	Elasticity of substitution within sectors
ϕ_π	1.24	Responsiveness of monetary policy to consumption inflation
ϕ_c	0.33/12	Responsiveness of monetary policy to output variations
ρ	0.9	Persistence of shocks (equal across shocks)

Table 3: **Overview of Calibration Cases**

This table details the frequencies, consumption weights, and input-output linkages for the different cases employed in the calibration.

	Frequencies	Consumption Weights	Input-Output Linkages
Case 1	flexible	homogeneous	homogeneous
Case 2	sticky, homogeneous	homogeneous	homogeneous
Case 3	sticky, heterogeneous	homogeneous	homogeneous
Case 4	sticky, heterogeneous	heterogeneous	heterogeneous (size weights)
Case 5	sticky, heterogeneous	heterogeneous	homogeneous
Case 6	sticky, heterogeneous	heterogeneous	heterogeneous

Table 4: **Response to Monetary Policy Shock**

This table reports the impact response, the cumulative impulse response as well as the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption, inflation, and real marginal costs for a 350 sector economy for different cases (see Table 3).

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
			Panel A. Consumption			
Impact	0.00	−3.46	−5.64	−6.24	−6.13	−5.92
Cumulative IRF	−0.03	−33.11	−68.75	−82.19	−81.09	−82.64
Persistence	0.87	0.87	0.85	0.85	0.84	0.88
			Panel B. Inflation			
Impact	−2.94	−1.64	−1.31	−1.27	−1.29	−1.43
Cumulative IRF	−28.16	−15.75	−4.74	−1.29	−2.00	−2.29
Persistence	0.87	0.87	0.87	0.87	0.87	0.89
			Panel C. Real Marginal Costs			
Impact	0.00	−4.03	−6.58	−7.16	−7.08	−7.35
Cumulative IRF	−0.03	−38.62	−80.21	−81.64	−89.62	−105.80
Persistence	0.87	0.87	0.90	0.91	0.90	0.92

Table 5: **Response to Monetary Policy Shock: Sorted by Cumulative Response**

This table reports the cumulative real consumption response to a one-percent monetary policy shock for a 350 sector economy for different cases (see Table 3). Panel A reports the response of the least responsive sectors and Panel B reports the response of the most responsive sectors. Panel C and D list the sector numbers following the BEA classification.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
	Panel A. Cumulative Consumption Response: Least Responsive					
Least responsive	−0.05	−60.69	−46.25	−49.31	−44.06	−9.73
2	−0.05	−60.69	−46.50	−49.58	−44.33	−9.74
3	−0.05	−60.69	−46.51	−49.59	−44.33	−10.37
4	−0.05	−60.69	−46.53	−49.61	−44.36	−12.88
5	−0.05	−60.69	−46.69	−49.78	−44.53	−13.68
6	−0.05	−60.69	−46.73	−49.82	−44.57	−14.14
7	−0.05	−60.69	−46.75	−49.84	−44.59	−14.56
8	−0.05	−60.69	−46.78	−49.88	−44.62	−14.74
9	−0.05	−60.69	−47.41	−50.55	−45.29	−15.68
10	−0.05	−60.69	−47.52	−50.66	−45.40	−15.74
	Panel B. Cumulative Consumption Response: Most Responsive					
Most responsive	−0.05	−60.69	−375.63	−282.25	−314.51	−353.53
2	−0.05	−60.69	−324.40	−265.21	−290.26	−328.79
3	−0.05	−60.69	−297.53	−251.78	−272.49	−305.32
4	−0.05	−60.69	−296.68	−251.31	−271.89	−304.91
5	−0.05	−60.69	−291.68	−248.54	−268.32	−301.30
6	−0.05	−60.69	−287.35	−246.09	−265.18	−296.78
7	−0.05	−60.69	−278.02	−240.61	−258.24	−288.13
8	−0.05	−60.69	−262.75	−231.17	−246.45	−274.13
9	−0.05	−60.69	−260.51	−229.74	−244.68	−274.10
10	−0.05	−60.69	−255.53	−226.51	−240.70	−270.58

continued on next page

Table 5: **Continued from Previous Page**

This table reports the cumulative real consumption response to a one-percent monetary policy shock for a 350 sector economy for different cases (see Table 3). Panel A reports the response of the least responsive sectors and Panel B reports the response of the most responsive sectors. Panel C and D list the sector numbers following the BEA classification.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
	Panel C. BEA Sector Code: Least Responsive					
Least responsive	324191	315290	311920	311920	311920	311920
2	321992	311330	315290	315290	315290	315290
3	315290	713B00	48A000	48A000	48A000	315210
4	315210	332710	332710	332710	332710	33441A
5	311920	1119B0	33441A	33441A	33441A	335120
6	311330	335120	336212	336212	336212	48A000
7	2122A0	324191	326130	326130	326130	336212
8	112120	321992	335314	335314	335314	326130
9	1119B0	311920	339950	339950	339950	339950
10	111910	315210	335120	335120	335120	333295
	Panel D. BEA Sector Code: Most Responsive					
Most responsive	326130	112120	713B00	713B00	713B00	332710
2	332710	2122A0	112120	112120	112120	713B00
3	333295	33441A	333295	333295	333295	112120
4	33441A	333295	1119B0	1119B0	1119B0	335314
5	335120	326130	324191	324191	324191	1119B0
6	335314	336212	321992	321992	321992	324191
7	336212	111910	2122A0	2122A0	2122A0	321992
8	339950	48A000	111910	111910	111910	2122A0
9	48A000	339950	311330	311330	311330	111910
10	713B00	335314	315210	315210	315210	311330

Table 6: **Response to Monetary Policy Shock ($\phi_\pi = 2.5$)**

This table reports the impact response, the cumulative impulse response as well as the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption and inflation for a 350 sector economy for different cases (see Table 3) with a coefficient on inflation in the Taylor rule of $\phi_\pi = 2.5$.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
			Panel A. Consumption			
Impact	0.00	−1.13	−2.26	−2.76	−2.54	−2.07
Cumulative IRF	−0.01	−10.78	−42.54	−95.97	−74.00	−69.54
Persistence	0.87	0.87	0.90	0.93	0.92	0.94
			Panel B. Inflation			
Impact	−0.62	−0.54	−0.69	−0.83	−0.78	−0.76
Cumulative IRF	−5.98	−5.13	−3.91	−3.62	−3.76	−4.25
Persistence	0.87	0.87	0.92	0.94	0.94	0.94
			Panel C. Real Marginal Costs			
Impact	0.00	−1.31	−2.64	−3.11	−2.92	−2.59
Cumulative IRF	−0.01	−12.57	−49.63	−87.16	−80.32	−86.38
Persistence	0.87	0.87	0.94	0.94	0.94	0.93

Table 7: **Response to Monetary Policy Shock (variations in parameters)**

This table reports the impact response, the cumulative impulse response as well as the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption and inflation for a 350 sector economy for different cases (see Table 3) for different values of structural parameters.

	Base	$\sigma = 2$	$\phi = 1$	$\delta = 0.7$	$\eta = 6$	$\theta = 10$	$\rho = 0.95$
				Panel A. Consumption			
Impact	−5.92	−3.70	−6.23	−6.61	−6.58	−5.76	−8.65
Cumulative IRF	−82.64	−46.81	−81.42	−79.66	−80.41	−83.13	−208.48
Persistence	0.88	0.87	0.89	0.90	0.78	0.88	0.91
				Panel B. Inflation			
Impact	−1.43	−1.14	−1.20	−0.91	−1.02	−1.54	−2.60
Cumulative IRF	−2.29	−1.61	−1.88	−1.53	−0.89	−2.59	−10.18
Persistence	0.89	0.88	0.89	0.89	0.83	0.89	0.92
				Panel C. Real Marginal Costs			
Impact	−7.35	−6.12	−5.81	−4.21	−7.98	−7.79	−10.78
Cumulative IRF	−105.80	−79.99	−81.66	−60.92	−80.91	−115.74	−262.60
Persistence	0.92	0.91	0.92	0.91	0.90	0.92	0.94

Table 8: **Response to Monetary Policy Shock (exogenous nominal demand and 58 sector economy)**

This table reports the impact response, the cumulative impulse response as well as the persistence of the response defined as AR(1) coefficient due to a one-percent monetary policy shock for consumption, inflation, and real marginal costs for a 350 sector economy with Taylor Rule, exogenous nominal demand, and a 58 sector economy with Taylor Rule for case 6 (see Table 3).

	350 Sectors Taylor Rule	350 Sectors Exogenous Demand	58 Sectors Taylor Rule
		Panel A. Consumption	
Impact	−5.92	−0.86	−4.49
Cumulative IRF	−82.64	−5.01	−55.18
Persistence	0.88	0.76	0.90
		Panel B. Inflation	
Impact	−1.43	−0.14	−1.65
Cumulative IRF	−2.29	−0.05	−9.39
Persistence	0.89	0.77	0.78
		Panel C. Real Marginal Costs	
Impact	−7.35	−1.05	−5.46
Cumulative IRF	−105.80	−6.46	−71.80
Persistence	0.92	0.82	0.91

Table 9: **Overview of Calibration Cases (idiosyncratic shocks)**

This table details the frequencies, consumption weights, and input-output linkages for the different cases employed in the calibration for idiosyncratic shocks.

	Frequencies	Consumption Weights	Input-Output Linkages
Case 1	flexible	homogeneous	homogeneous
Case 2	flexible	heterogeneous	consumption shares
Case 3	flexible	homogeneous	heterogeneous
Case 4	flexible	heterogeneous	heterogeneous
Case 5	sticky, homogeneous	homogeneous	homogeneous
Case 6	sticky, homogeneous	heterogeneous	consumption shares
Case 7	sticky, homogeneous	homogeneous	heterogeneous
Case 8	sticky, homogeneous	heterogeneous	heterogeneous
Case 9	sticky, heterogeneous	homogeneous	homogeneous
Case 10	sticky, heterogeneous	heterogeneous	heterogeneous

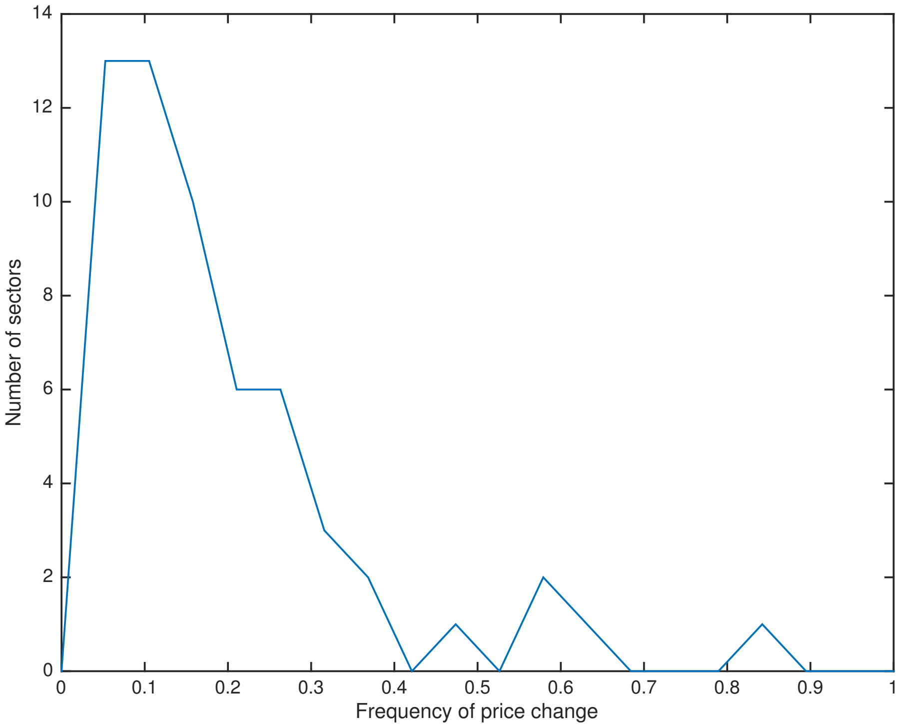
Table 10: **Idiosyncratic Shocks**

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9	Case 10
Λ_{ac}	3.29	3.29	3.29	3.29	0.17	0.17	0.17	0.17	0.18	0.24
ξ	-	1.0029	1.6123	1.1000	-	1.0029	2.0967	1.0585	1.7388	0.9895
σ_ξ	-	0.17	0.27	0.19	-	0.17	0.35	0.18	0.29	0.17
R^2	-	0.96	0.97	0.93	-	0.95	0.99	0.94	0.84	0.92
$\sqrt{\sum_{k=1}^{K} \chi_k^2}$	0.05	0.17	0.1	0.15	0.05	0.17	0.07	0.15	0.09	0.18

Online Appendix: Production Networks, Nominal Rigidities, and the Propagation of Shocks

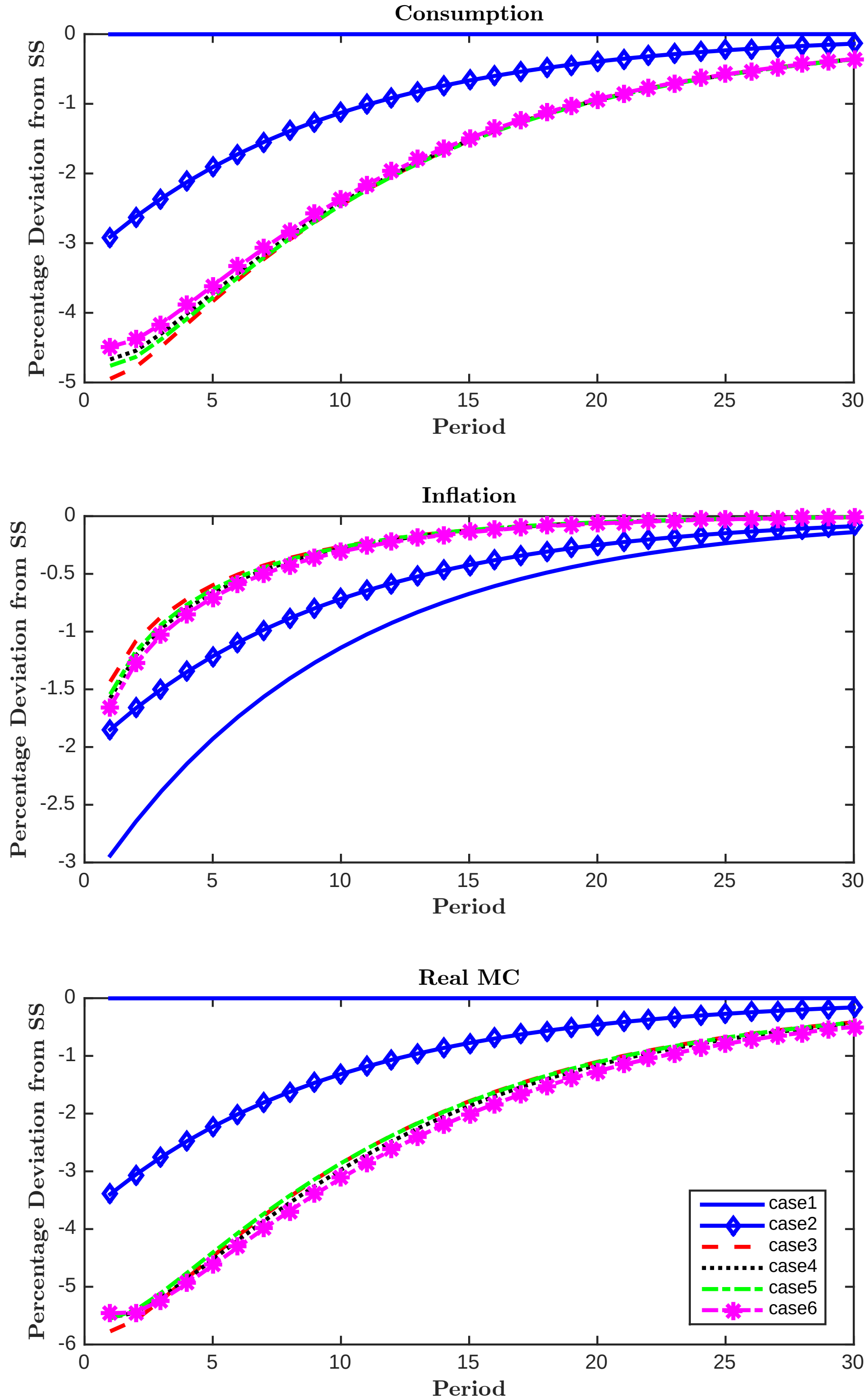
Ernesto Pasten, Raphael Schoenle, and Michael Weber

Not for Publication

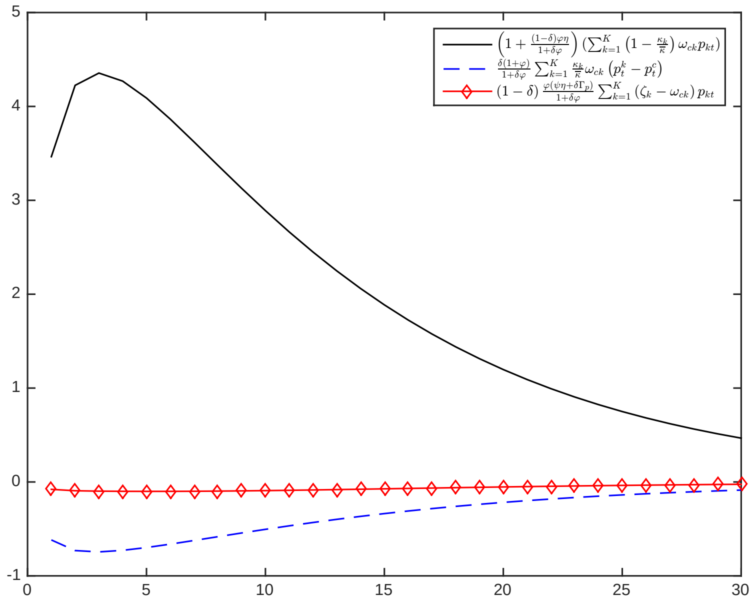


This figure plots the distribution of the frequency of price adjustment for a 58 sector model.

Figure A.1: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock**

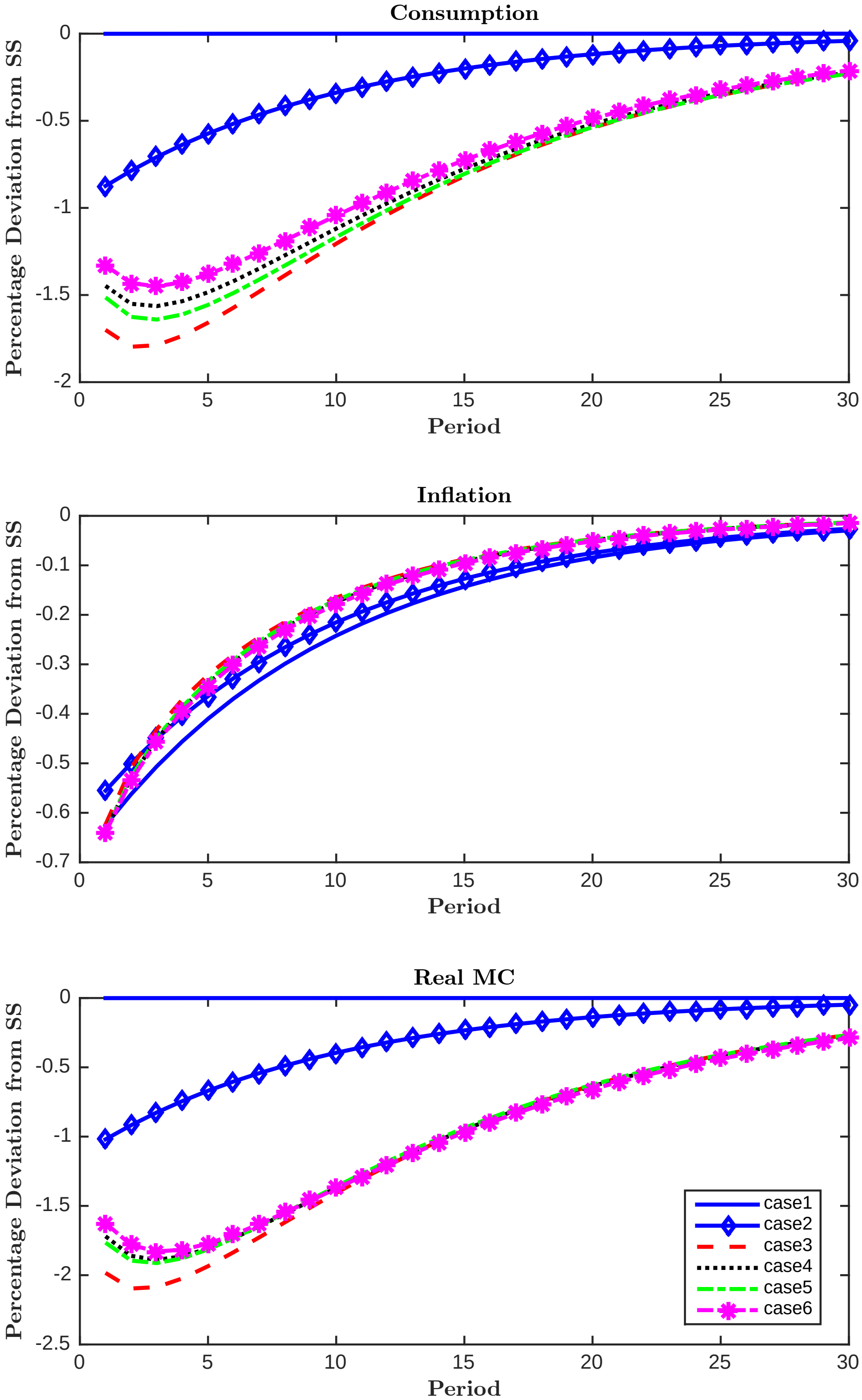


This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard deviation monetary policy shock for a 58 sector model for different cases (see Table 3).



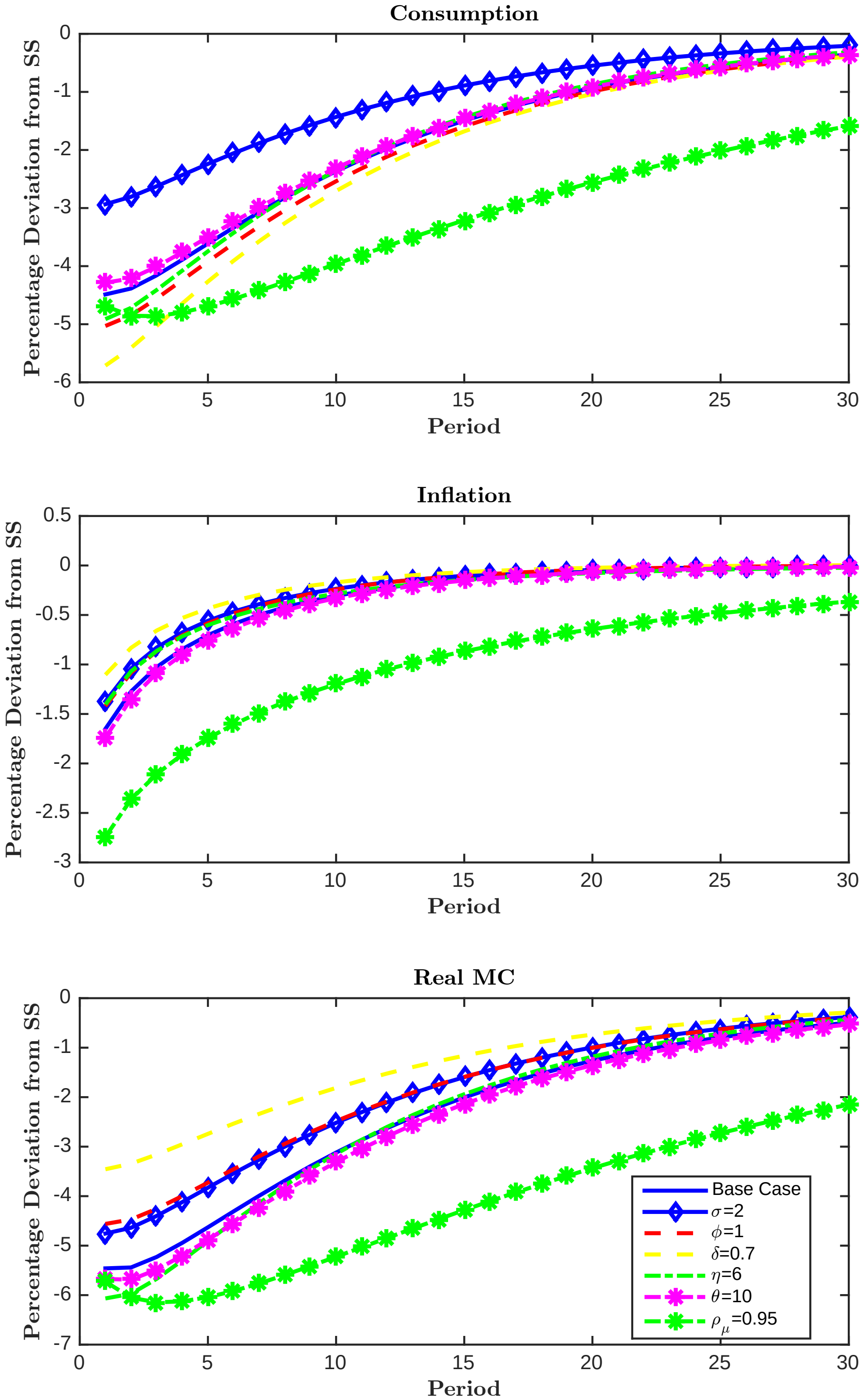
This figure decomposes the overall response of consumption to a one-standard deviation monetary policy shock for a 58 sector model into the three different component discussed in Section IV.

Figure A.2: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock ($\phi_\pi = 2.5$)**



This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard deviation monetary policy shock for a 58 sector model for different cases (see Table 3) with a coefficient on inflation in the Taylor rule of $\phi_\pi = 2.5$.

Figure A.3: **Response of Real Consumption, Inflation, and Real Marginal Costs to Monetary Policy Shock (Variations in parameters)**



This figure plots the impulse response function of real consumption, inflation, and real marginal costs to a one-standard deviation monetary policy shock for a 58 sector model for different cases (see Table 3) for different values of structural parameters.

Table A.1: **Response to Monetary Policy Shock: Sorted by Cumulative Response**

This table reports the cumulative real consumption response to a one-percent monetary policy shock for a 58 sector economy for different cases (see Table 3). Panel A reports the response of the least responsive sectors and Panel B reports the response of the most responsive sectors. Panel C and D list the sector numbers following the BEA classification.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
	Panel A. Cumulative Consumption Response: Least Responsive					
Least responsive	−0.05	−51.10	−39.57	−38.71	−40.51	−22.79
2	−0.05	−51.10	−41.92	−41.08	−42.85	−24.46
3	−0.05	−51.10	−42.85	−42.02	−43.79	−24.51
4	−0.05	−51.10	−42.93	−42.11	−43.87	−25.89
5	−0.05	−51.10	−44.91	−44.14	−45.87	−29.48
6	−0.05	−51.10	−48.68	−48.03	−49.68	−34.87
7	−0.05	−51.10	−50.38	−49.80	−51.42	−36.63
8	−0.05	−51.10	−52.94	−52.47	−54.02	−37.47
9	−0.05	−51.10	−55.45	−55.10	−56.59	−39.59
10	−0.05	−51.10	−56.03	−55.71	−57.19	−50.30
	Panel B. Cumulative Consumption Response: Most Responsive					
Most responsive	−0.05	−51.10	−245.19	−259.22	−253.78	−269.79
2	−0.05	−51.10	−239.51	−253.08	−247.86	−261.32
3	−0.05	−51.10	−227.84	−240.45	−235.68	−249.38
4	−0.05	−51.10	−210.89	−222.12	−218.01	−227.49
5	−0.05	−51.10	−189.72	−199.24	−195.95	−212.71
6	−0.05	−51.10	−184.21	−193.28	−190.20	−207.28
7	−0.05	−51.10	−183.44	−192.44	−189.40	−200.50
8	−0.05	−51.10	−180.88	−189.68	−186.73	−196.79
9	−0.05	−51.10	−173.49	−181.70	−179.03	−190.32
10	−0.05	−51.10	−165.74	−173.33	−170.96	−179.01

continued on next page

Table A.1: **Continued from Previous Page**

This table reports the cumulative real consumption response to a one-percent monetary policy shock for a 58 sector economy for different cases (see Table 3). Panel A reports the response of the least responsive sectors and Panel B reports the response of the most responsive sectors. Panel C and D list the sector numbers following the BEA classification.

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
	Panel C. BEA Sector Code: Least Responsive					
Least responsive	324	321	452	452	452	323
2	323	339	5411	5411	5411	521CI
3	339	493	493	493	493	452
4	3364OT	713	3364OT	3364OT	3364OT	22
5	333	486	713	713	713	493
6	321	211	22	22	22	3364OT
7	23	42	339	339	339	486
8	22	212	486	486	486	333
9	212	621	333	333	333	5411
10	211	324	321	321	321	321
	Panel D. BEA Sector Code: Most Responsive					
Most responsive	42	61	323	323	323	212
2	452	22	621	621	621	339
3	486	23	521CI	521CI	521CI	621
4	493	3364OT	42	42	42	211
5	521CI	21CI	211	211	211	81
6	5411	323	81	81	81	61
7	61	452	212	212	212	713
8	621	81	324	324	324	324
9	713	333	23	23	23	23
10	81	5411	61	61	61	42