The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

E. Pasten\textsuperscript{1}  R. Schoenle\textsuperscript{2}  M. Weber\textsuperscript{3}

\textsuperscript{1}Banco Central de Chile and Toulouse University
\textsuperscript{2}Brandeis University
\textsuperscript{3}Chicago Booth

\textit{Federal Reserve Bank of Cleveland Inflation Conference}
September 29, 2016
**Motivation**

- Fact 1: Sectors are heterogeneous in their input-output linkages with other sectors.
**Motivation**

- **Fact 2:** Sectors are heterogeneous in their frequency of price changes, which is heavy-tailed.
**Motivation**

- Fact 3: Sectors are heterogeneous in their importance for GDP.
Motivation (and literature review)

- How does the interaction of these three forms of heterogeneity affect the propagation of monetary policy shocks?

- Big picture:
  Interaction of heterogeneities with heterogeneous pricing frictions can change identity of sectoral effects on aggregate marginal cost.

- Companion paper: Propagation of idiosyncratic shocks.
Motivation (and literature review)

- Which heterogeneity matters the most for real output effects?
  - Price stickiness >> consumption shares >> input-output linkages.

- Does the choice of granularity matter? N=1? N=350?
  - Most disaggregated calibration yields 34% larger real effects
  - Similar impact response for inflation

- What is the role of heavy tail in frequency of adjustment?

- How does monetary policy rule interact with heterogeneities?
Motivation (and literature review)

- Relevant papers:
  - Many others on markups, real rigidities, trade linkages, etc.
Outline

Two parts:

- Theoretical results
- Empirical analysis
Abridged description of the setup 1/5

Households’ utility:

\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \sum_{k=1}^{K} g_k \frac{L_k^{1+\varphi}}{1 + \varphi} \right) \quad \text{s.t. } BC
\]

where

\[
C_t \equiv \left[ \sum_{k=1}^{K} \omega_{ck} C_{kt}^{\frac{1-\frac{1}{\eta}}{\eta-1}} \right]^{\frac{\eta}{\eta-1}},
\]

\[
C_{kt} \equiv \left[ n_k^{-1/\theta} \int_{\mathcal{S}_k} C_{ktj}^{1-\frac{1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}},
\]

and \( \eta < \theta \).
Abridged description of the setup 2/5

**Firms’ production function:**

\[ Y_{kjt} = L_{kjt}^{1-\delta} Z_{kjt}^\delta, \]

where

\[
Z_{kjt} \equiv \left[ \sum_{r=1}^{K} \omega_{kr} Z_{kjt}(r)^{1-\frac{1}{\eta}} \right]^{\eta/(\eta-1)},
\]

\[
Z_{kjt}(r) \equiv \left[ n_r^{-1/\theta} \int_{\Omega_r} Z_{kjt}(r,j')^{1-\frac{1}{\theta}} dj' \right]^{\theta/(\theta-1)},
\]

and \( \eta < \theta \).
Abridged description of the setup 3/5

Demand for sector $k$, variety producer $j$:

<table>
<thead>
<tr>
<th>from households:</th>
<th>from other firms:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{kt} = \omega_{ck} \left( \frac{P_{kt}}{P^c_t} \right)^{-\eta} C_t,$</td>
<td>$Z_{k'jt}(k) = \omega_{k'k} \left( \frac{P_{kt}}{P^k_t} \right)^{-\eta} Z_{k'jt}$,</td>
</tr>
<tr>
<td>$C_{kjt} = \frac{1}{n_k} \left( \frac{P_{kjt}}{P_{kt}} \right)^{-\theta} C_{kt}.$</td>
<td>$Z_{k'jt}(k,j) = \frac{1}{n_k} \left( \frac{P_{kjt}}{P_{kt}} \right)^{-\theta} Z_{k'jt}(k)$</td>
</tr>
</tbody>
</table>

where price aggregators are:

<table>
<thead>
<tr>
<th>for households:</th>
<th>for other firms:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^c_t = \left[ \sum_{r=1}^{K} \omega_{cr} P^{1-\eta}_{rt} \right]^{\frac{1}{1-\eta}}$,</td>
<td>$P^{k'}<em>t = \left[ \sum</em>{r=1}^{K} \omega_{k'r} P^{1-\eta}_{rt} \right]^{\frac{1}{1-\eta}}$,</td>
</tr>
<tr>
<td>$P_{rt} = \left[ \frac{1}{n_r} \int_{\Omega_r} P^{1-\theta}_{rjt} dj \right]^{\frac{1}{1-\theta}}$.</td>
<td>$P_{rt} = \left[ \frac{1}{n_r} \int_{\Omega_r} P^{1-\theta}_{rjt} dj \right]^{\frac{1}{1-\theta}}$.</td>
</tr>
</tbody>
</table>
Abridged description of the setup 4/5

**Sectoral heterogeneity of price stickiness:**

\[
\sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k s Y_{kjt+s} \left[ P_{kjt}^* - \frac{\theta}{\theta - 1} MC_{kjt+s} \right] = 0
\]

where

\[
MC_{kjt} = \frac{1}{1 - \delta} \left( \frac{\delta}{1 - \delta} \right)^{-\delta} W_{kt}^{1-\delta} (P_t^k)^\delta
\]

and sectoral prices follow

\[
P_{kt} = \left[ (1 - \alpha_k) P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]
Abridged description of the setup 5/5

Equilibrium conditions:

\[ Y_{kjt} = C_{kjt} + \sum_{k' = 1}^{K} \int_{\mathcal{S}_{k'}} Z_{k'jt} (k, j) \, dj', \]

\[ Y_t = C_t + Z_t \]

+ the usual equations:

- sectoral labor supply,
- Euler equation,
- production efficiency condition,
- Taylor rule, and
- standard equilibrium conditions.
Mechanism: Intuition

The effect of monetary policy shocks depends on the properties of marginal cost:

- Highlight individual feedback mechanisms
- Analytical expressions from simplified model
Mechanism: Intuition

Defining aggregate price indices:

\[ CPI : \quad p^c_t = \sum_{k=1}^{K} \omega_{ck} p_{kt}, \]

sectoral input price index:

\[ p^k_t = \sum_{k' = 1}^{K} \omega_{kk'} p_{k't} \]

Sectoral participation in total production (note: \( \psi = Z/Y \)):

\[ n_k = (1 - \psi) \omega_{ck} + \psi \left( \sum_{k' = 1}^{K} n_{k'} \omega_{k'k} \right) \text{ for all } k. \]

\[ \rightarrow \hat{\Omega} = (1 - \psi) \left[ I - \psi \Omega' \right]^{-1} \Omega^c \]
Mechanism: Intuition

Feedback at the sectoral level:

\[ mc_{kt} = (1 - \delta) w_{kt} + \delta p^k_t \]

and \( w_{kt} \) solves

\begin{align*}
\quad \text{(labor sup)} & w_{kt} = \varphi l_{kt} + \sigma c_t + p^c_t, \\
\quad \text{(sect prod. fn)} & y_{kt} = l_{kt} + \delta \left( w_{kt} - p^k_t \right), \\
\quad \text{(sect demand)} & y_{kt} = y_t - \eta \left[ p_{kt} - (1 - \psi) p^c_t - \psi \tilde{p}_t \right]
\end{align*}

where \( \tilde{p}_t = \sum_{k=1}^K \left( \sum_{k'=1}^K n_{k'k} \omega_{k'k} \right) p_{kt} \).
Mechanism: Intuition

Overall, sectoral prices follow

\[ \beta E_t [p_{kt+1}] - (1 + \beta)p_{kt} + p_{kt-1} = -\kappa_k (mc_{kt} - p_{kt}) \]
\[ \beta E_t [\pi_{kt+1}] - \pi_{kt} = \kappa_k x_{kt} \]

where \( \kappa_k \equiv (1 - \alpha_k)(1 - \alpha_k \beta)/\alpha_k \) and

\[ x_{kt} = (1 + \varphi \eta) \gamma_1 (\delta) (p^c_t - p_{kt}) + \gamma_2 (\delta) (p^k_t - p^c_t) + \gamma_3 (\delta) (\tilde{p}_t - p^c_t) + (\sigma + \varphi) \gamma_4 (\delta) c_t \]

Compare to \( \delta = 0 \) :

\[ x_{kt}^{\delta=0} = (1 + \varphi \eta) (p^c_t - p_{kt}) + (\sigma + \varphi) c_t \]
Analytical Results

Simplified model:
- utility log in consumption, linear in leisure
- exogenous nominal demand, one-time permanent shock
- future fully discounted: $p_{kt}^* = mc_{kt}$

Intuition carries through in full model, and calibration.
Analytical Results

Three propositions on effect of monetary policy:
- Homogeneous price stickiness
- Heterogeneous price stickiness but irrelevance of heterogeneity in I/O linkages
- Heterogeneous price stickiness and no restriction on I/O linkages

Two propositions on effect of granularity
Analytical Results: Monetary Policy Shock

- Assume that $\alpha_k = \alpha$ for all $k$, so $\kappa_k = \kappa$ for all $k$ (homogeneous stickiness)
- Unrestricted consumption shares $\{\omega_{ck}\}$ and I/O linkages $\{\omega_{kk'}\}$

**Proposition**

Following a permanent monetary shock $m$ at time $t^*$, heterogeneity of $\{\omega_{ck}\}$ and $\{\omega_{kk'}\}$ is irrelevant for the response of output. The existence of I/O linkages amplifies the response of output.

**Proof.**

Follows directly from

$$\tilde{p}_t^C (\alpha) = \left[ 1 - \left( \frac{\alpha}{1 - \delta (1 - \alpha)} \right)^{t-t^*+1} \right] m \text{ for } t \geq t^* \quad (1)$$
Analytical Results: Monetary Policy Shock

- Assume that $\omega_{ck} = \omega_{kk}'$ (no price wedge), otherwise unrestricted.
- Assume that $\{\alpha_k\}$ are heterogeneous, $E[\alpha_k] = \alpha$

Proposition

Following a permanent monetary shock,

1. The sectoral heterogeneity of price stickiness and consumption shares are irrelevant for the response of output to the monetary shock on impact.

2. $p^c_t \leq p^c_t(\alpha)$ for $t > t^*$. The response of the aggregate consumption prices for $t \geq t^*$ is weakly decreasing in the dispersion of price stickiness.
Analytical Results: Monetary Policy Shock

- No restrictions, allowing for $\omega_{ck} \neq \omega_{kk'}$ (price wedge)

Proposition

1. The response of $p^c_t$ is now weaker on impact when
   \[ u_k \equiv \sum_{k'=1}^{K} \omega_{ck'} (1 - \alpha_{k'}) \omega_{k' k} > (1 - \bar{\alpha}) \omega_{ck} \text{ for the sectors with the most sticky prices.} \]

2. The response of $p^c_t$ for $t > t^*$ is now more persistent when for sectors with the most sticky prices either of the following conditions hold: (i) $\bar{\omega}_k \equiv \frac{1}{K} \sum_{k'=1}^{K} \omega_{k' k} > \omega_{ck}$, (ii) $u_k > (1 - \bar{\alpha}) \omega_{ck}$, (iii) $\text{COV}(\omega_{ck'} \alpha_{k'}^T (1 - \alpha_{k'}) , \omega_{k' k}) > 0$.

where first-order and second-order outdegrees are

\[ u_k \equiv \sum_{k'=1}^{K} \omega_{ck'} (1 - \alpha_{k'}) \omega_{k' k}, \quad v_k \equiv \sum_{k'=1}^{K} u_{k'} (1 - \alpha_{k'}) \omega_{k' k}. \]
Summary of Results: Monetary Policy Shock

**The real effects of monetary policy shocks are bigger if**

- share of intermediate inputs is high,
- sticky-price sectors are important suppliers to the rest of the economy,
- sticky-price sectors are important suppliers to flexible-price sectors,
- sticky-price sectors are important suppliers to large sectors,
- sticky-price sectors have large covariance of connectedness with function of customer size and price stickiness.
Analytical Results: Effect of Aggregation

What is the effect of sectoral aggregation?

- Granularity matters for the real effects of monetary policy
  - Convexification of price stickiness
  - Size heterogeneity and I/O linkages can amplify effect
Analytical Results: Effect of Aggregation

Theoretical results from comparison of two economies

- Finely disaggregated economy:
  - \( K \) sectors, \( \{\alpha_k, \omega_{ck}, \omega_{ks}\} \)
  - \( p_t^c = \sum_{k=1}^{K} \omega_{ck}p_{kt} \)

- Coarse, aggregated economy:
  - \( K/2 \) sectors, \( \{\bar{\alpha}'_k, \bar{\omega}_{ck}', \bar{\omega}_k's'\} \)
  - \( \bar{p}_t^c = \sum_{k'=1}^{K/2} \bar{\omega}_{ck}'\bar{p}_{k't} \)

- Sectors randomly combined, with appropriate weights.

⇒ Two propositions.
Analytical Results: Effect of Aggregation

Proposition

Given no I/O linkages ($\delta = 0$), the difference of consumption prices

$$\bar{p}_t^c - p_t^c = \sum_{k'=1}^{K/2} \bar{\omega}_{ck'} \left[ \lambda_{k'} \alpha_{2k'}^{t-t^*+1} + (1 - \lambda_{k'}) \alpha_{2k'}^{t-t^*+1} - \bar{\alpha}_{k'}^{t-t^*+1} \right] m$$

is such that

1. $p_t^c = \bar{p}_t^c$ for $t = t^*$
2. $p_t^c < \bar{p}_t^c$ for $t > t^*$
3. $\bar{p}_t^c - p_t^c$ is increasing in the dispersion of Calvo parameters among merged sectors
4. $\bar{p}_t^c - p_t^c$ is increasing in $\bar{\omega}_{ck'}$ for merged sectors with the highest dispersion of Calvo parameters
Analytical Results: Effect of Aggregation

Proposition

When $\delta \in (0, 1)$ and $\omega_{sk} = \omega_{ck}$ $\forall s, k$, the amplification effect of monetary non-neutrality introduced by intermediate inputs is stronger in the finely-disaggregated economy relative to the coarsely-disaggregated economy when the dispersion of frequency of price changes is higher among the merged sectors.

$$p_t^c - \bar{p}_t^c = -\frac{1 - \delta}{1 - \delta (1 - \alpha)} \sum_{k' = 1}^{K/2} \bar{\omega}_{ck'} \left( \lambda_{k'} \alpha_{2k'}^{t-t^*} + 1 \right) \left( 1 - \lambda_{k'} \right) \left( \alpha_{2k'}^{t-t^*} - 1 \right) + \frac{\delta}{1 - \delta (1 - \alpha)} \sum_{\tau = 1}^{t-t^*} \left( \sum_{k' = 1}^{K/2} \bar{\omega}_{ck'} \left[ \lambda_{k'} \left( 1 - \alpha_{2k'}^{t-t^*} \right) \alpha_{2k'}^{\tau} + 1 \right] - \left( 1 - \lambda_{k'} \right) \left( 1 - \alpha_{2k'}^{t-t^*} \right) \alpha_{2k'}^{\tau} \right) \bar{p}_{t-\tau}$$ (2)
Allowing for full, unrestricted heterogeneity such that $\omega_{sk} \neq \omega_{ck}$ can lead to even larger amplification.

How large are these effects? Empirical question.
Calibration

Calibration goal:

- Which type of heterogeneity matters most?
  - 6 cases with incremental degrees of heterogeneity
- Role of monetary policy?
- Does the level of aggregation matter?
  - Decrease aggregation from 350 to 8 sectors
  - Value-added sectoral output shares from BEA
  - 2002 I-O tables from BEA to get industry-to-industry linkages
  - Stickiness from BLS PPI micro data
- What is the role of heavy-tailed price stickiness?
Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>$8, 350$</td>
<td>Number of sectors</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0.9975$</td>
<td>Monthly discount factor to get 3% annual risk-free rate</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$1$</td>
<td>Relative risk aversion</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>$2$</td>
<td>Inverse of Frisch elasticity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0.7$</td>
<td>Intermediate inputs share in production function</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$2$</td>
<td>Elasticity of substitution across sectors</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$6$</td>
<td>Elasticity of substitution across firms within sectors</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>$1.24$</td>
<td>Responsiveness of monetary policy to inflation</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>$0.33/12$</td>
<td>Responsiveness of monetary policy to GDP variations</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$0.9$</td>
<td>Persistence of shocks</td>
</tr>
</tbody>
</table>

Monetary shock: $\mu_t = 1$
### Calibration

#### Overview of Calibration Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Prices</th>
<th>Consumption Weights</th>
<th>Input-Output Linkages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>flexible</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 2</td>
<td>sticky, homogeneous</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 3</td>
<td>sticky, heterogeneous</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 4</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>heterogeneous (size weights)</td>
</tr>
<tr>
<td>Case 5</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 6</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>heterogeneous</td>
</tr>
</tbody>
</table>
Calibration

<table>
<thead>
<tr>
<th>Case</th>
<th>Prices</th>
<th>Consumption Weights</th>
<th>Input-Output Linkages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>flexible</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 2</td>
<td>sticky, homogeneous</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 3</td>
<td>sticky, heterogeneous</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 4</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>heterogeneous (size weights)</td>
</tr>
<tr>
<td>Case 5</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 6</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>heterogeneous</td>
</tr>
</tbody>
</table>
## Overview of Calibration Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Prices</th>
<th>Consumption Weights</th>
<th>Input-Output Linkages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>flexible</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 2</td>
<td>sticky, homogeneous</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 3</td>
<td>sticky, heterogeneous</td>
<td>homogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 4</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>heterogeneous (size weights)</td>
</tr>
<tr>
<td>Case 5</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>homogeneous</td>
</tr>
<tr>
<td>Case 6</td>
<td>sticky, heterogeneous</td>
<td>heterogeneous</td>
<td>heterogeneous</td>
</tr>
</tbody>
</table>
Calibration

Which heterogeneity matters most?

- Heterogeneity of price stickiness most important driver of real effects.
- Lesser role for heterogeneous consumption shares and even less for input-output linkages.
The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

Calibration Results

![Consumption Graph](image)

![Inflation Graph](image)

![Real MC Graph](image)
The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

Calibration Results

58 Sectors

Sector Size

Sector Frequency

0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16

0

0.1

0.2

0.3

0.4

0.5

0.6

0.7

0.8

0.9
The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

Calibration Results

Role of monetary policy:

- Large effect of endogenizing demand through monetary policy.
- Large effect of more systematic response to inflation.
The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

Calibration Results

![Graphs showing consumption, inflation, and real MC deviations from SS over periods.](image)
Increase systematic response of monetary policy \((\phi_\pi = 2.5)\):
The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

Calibration Results

[Graphs showing consumption, inflation, and real MC over periods with different cases]
Finer granularity of calibration increases real effects:

- 25% (34%) larger cumulative consumption response for 350 vs. 58 (8) sectors
- Steepest gradient between 148/60, and 60/20 sector aggregations

Similar impact response for inflation.
The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

Calibration Results

Consumption

Inflation

Real MC
The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

Calibration Results

- Consumption
- Inflation
- Real MC
Role of fat tails in frequency of price changes:

- Experiment 1: cut right tail
  - Censor 20% of upper tail
  - Assign mean to the right censored sectors
  - Shift entire distribution to the right to preserve mean

- Experiment 2: heavier left tail
  - Right-censor at f=18%
  - Mirror at 0.18
  - Put uniform mass to the left to preserve mean
The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy

Calibration Results

![Graphs showing consumption, inflation, and real MC deviations over periods for different scenarios.]

- Consumption
- Inflation
- Real MC

- 350 Sectors, Baseline
- 350 Sectors, 20% Right-Trimmed
- 350 Sectors, Left Tail
Conclusion:

- Convolution of heterogeneity of price stickiness with sector size and input-output linkages creates rich set of theoretical predictions.

- Empirically:
  - Heterogeneity of price stickiness most important driver of real effects; lesser role for heterogeneous consumption shares and even less for input-output linkages.
  - More granular calibration yields up to 34% larger real effects but similar impact response for inflation, steep gradient.
  - Results highly sensitive to monetary policy specification.
  - Heavy tails of frequency of price changes affect real effects.

- Ongoing work: idiosyncratic shocks, oil price shocks, optimal monetary policy.