

The Risky Steady State and the Interest Rate Lower Bound

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¹The views expressed in this presentation are not necessarily those of the Federal Reserve Board of Governors, the Federal Reserve System, or the European Central Bank.

Question

What are the implications of **the effective lower bound (ELB) risk** after liftoff?

- ▶ The federal funds rate is currently above the ELB.
- ▶ In the 2016 July Survey of Primary Dealers, a median respondent sees 25 percent chance that the federal funds rate will return to the ELB in 2016-2018.

Background

One implication of the ELB risk: Deflationary bias

- ▶ The possibility of returning to the ELB leads price-setters to lower prices via expectations.
- ▶ Adam and Billi (2007) and Nakov (2008) made this observation in stylized models.

Is the deflationary bias quantitatively important in empirically realistic models?

What We Do

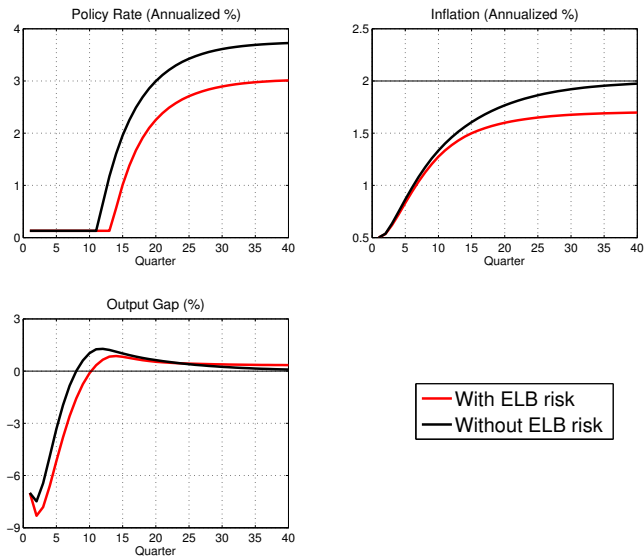
- ▶ We characterize the risky steady state in an empirically rich sticky-price model with an occasionally binding ELB constraint.
 - ▶ The risky steady state is a point where the economy eventually converges to when all exogenous shocks dissipate (Coeurdacier, Rey, and Winant (2011)).
- ▶ By comparing the deterministic and risky steady states, we quantify the effects of the ELB risk on allocations and prices at the steady state.
- ▶ The model is calibrated to match key features of the U.S. economy.

Main Results

- ▶ Inflation and the policy rate are nontrivially lower at the risky steady state than at the deterministic steady state
 - ▶ Inflation falls below the target rate of inflation in the policy rule at the risky steady state.
 - ▶ i.e. the central bank systematically undershoots its inflation objective.
- ▶ The ELB risk reduces inflation by about 20-40 basis points below the target rate.

The “undershooting” result

Figure: The Effect of the ELB Risk on Projections



Why We Care (I)

- ▶ The risky steady state inflation = private sector's long-run inflation expectations.
- ▶ U.S. policymakers project that inflation will eventually return to the target rate of 2 percent (Summary of Economic Projections).
- ▶ This projection is based on the assumption that long-run inflation expectations are anchored at 2 percent (Yellen (2016), various FOMC Minutes).
- ▶ Long-run inflation expectations are unobserved and this assumption could turn out to be wrong.

Why We Care (II)

- ▶ Many measures of long-run inflation expectations have declined last few years and are currently at the low ends of their historical ranges (Michigan Survey, SPF, expectations inferred from financial markets).
- ▶ Our model endogenously generates long-run inflation expectations that are below 2 percent.
- ▶ Our analysis thus provides a cautionary tale for policymakers.

ELB Literature

- ▶ Many papers assume an absorbing state or perfect-foresight (no ELB risk by construction).
- ▶ Most papers examine the implications of the ELB constraint for the economy when the ELB is binding.
- ▶ Adam and Billi (2007) and Nakov (2008) have commented on the implications of the ELB constraint when the ELB is not binding, but only in stylized models.

Our paper examines the implications of the ELB constraint for the economy **when the ELB is not binding in an empirically serious model.**

Outline of the Talk

- ▶ Stylized Model

- ▶ Empirical Model

A Stylized Model

A standard New Keynesian model:

- ▶ Discrete Time, Infinite Horizon.
- ▶ Discount factor shocks: $\{\delta_t\}_{t=1}^{\infty}$.
- ▶ $\delta_t - 1 = \rho(\delta_{t-1} - 1) + \epsilon_t$, $\epsilon_t \sim N(0, \sigma_{\epsilon}^2)$.

- ▶ A representative household.
- ▶ A final good producer.
- ▶ A continuum of intermediate-good producers, indexed by $i \in [0, 1]$.
 - ▶ subject to quadratic price-adjustment costs.
- ▶ The government.

An Equilibrium

Given any P_0 and a stochastic process for $\{\delta_t\}_{t=1}^{\infty}$, an equilibrium can be characterized by $\{C_t, N_t, Y_t, w_t, \Pi_t, R_t\}_{t=1}^{\infty} \equiv \{d_t\}_{t=1}^{\infty}$ satisfying

$$C_t^{-\chi_c} = \beta \delta_t R_t E_t C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1} \quad (1)$$

$$w_t = N_t^{\chi_n} C_t^{\chi_c} \quad (2)$$

$$\begin{aligned} \frac{Y_t}{C_t^{\chi_c}} \left[\varphi \left(\frac{\Pi_t}{\bar{\Pi}} - 1 \right) \frac{\Pi_t}{\bar{\Pi}} - (1 - \theta) - \theta w_t \right] \\ = \beta \delta_t E_t \frac{Y_{t+1}}{C_{t+1}^{\chi_c}} \varphi \left(\frac{\Pi_{t+1}}{\bar{\Pi}} - 1 \right) \frac{\Pi_{t+1}}{\bar{\Pi}} \end{aligned} \quad (3)$$

$$Y_t = C_t + \frac{\varphi}{2} \left[\frac{\Pi_t}{\bar{\Pi}} - 1 \right]^2 Y_t \quad (4)$$

$$Y_t = N_t \quad (5)$$

$$R_t = \max \left[1, \frac{1}{\beta} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_\pi} \right] \quad (6)$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ and $w_t = \frac{W_t}{P_t}$.

The Risky Steady State Generically Defined

- ▶ Γ_t : a vector of endogenous variables in the model under investigation.
- ▶ X_t : a vector of exogenous variables. Let X_{SS} denote the steady state of X_t .
- ▶ $f(\cdot, \cdot)$: a vector of policy functions mapping (i) endogenous variables in the previous period and (ii) today's realizations of exogenous variables into (iii) endogenous variables today.

The risky steady state, Γ_{RSS} , is a fixed point of the economy:

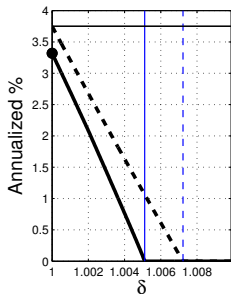
$$\Gamma_{RSS} = f(\Gamma_{RSS}, X_{SS})$$

Table: Parameterization for the Stylized Model

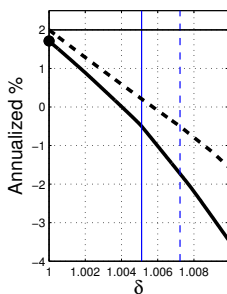
Parameter	Description	Value
β	Discount rate	$\frac{1}{1+0.004365}$ *
χ_c	Inverse intertemporal elasticity of substitution for C_t	1.0
χ_n	Inverse labor supply elasticity	1
θ	Elasticity of substitution among intermediate goods	11
φ	Price adjustment cost	200
$400(\bar{\pi} - 1)$	(Annualized) target rate of inflation	2.0
ϕ_π	Coefficient on inflation in the Taylor rule	1.5
ϕ_y	Coefficient on the output gap in the Taylor rule	0
ρ	AR(1) coefficient for the discount factor shock	0.8
σ_ϵ	The standard deviation of shocks to the discount factor	$\frac{0.24}{100}$
	*The implied prob. that the policy rate is at the lower bound	10%

*The implied deterministic steady-state policy rate is 3.75 percent.

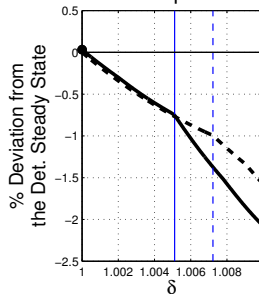
Nominal Interest Rate



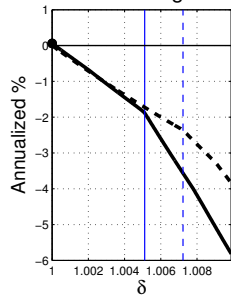
Inflation



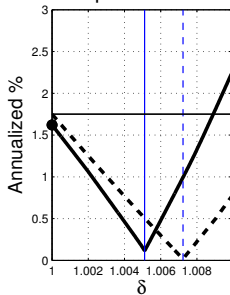
Output



Real Wage



Exp. Real Rate



— With uncertainty
 - - - Without uncertainty

— Deterministic St.St.
 • Risky St.St.

Mechanism

- ▶ $\pi_t = \kappa w_t + \beta E_t \pi_{t+1} \Rightarrow \pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t w_{t+k}$
- ▶ The distribution of real wage (w_{t+1}) when $\delta_t = 1.004$ and $400(R_t - 1) = 0.75$.

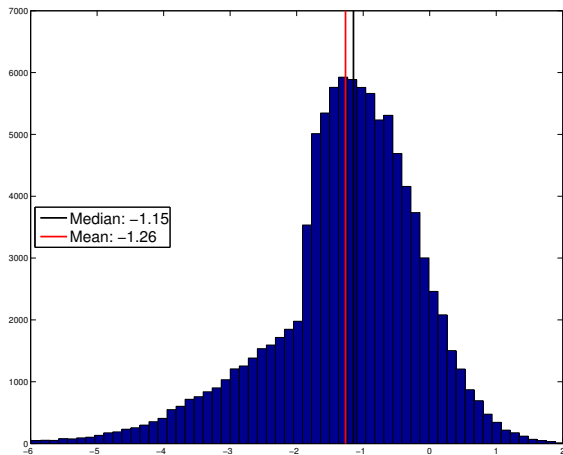


Table: The Risky Steady State in the Stylized Model

	Inflation	Output gap	Policy rate
Deterministic steady state	2	0	3.75
Risky steady state	1.71	0.03	3.32
(Wedge)	(-0.29)	(0.03)	(-0.43)
Risky steady state w/o the ELB	1.99	-0.02	3.72
(Wedge)	(-0.01)	(-0.02)	(-0.03)

Outline of the Talk

- ▶ Stylized Model
- ▶ Empirical Model

Our empirical model adds four features common in the DSGE literature to the stylized model:

- ▶ Consumption Habits
- ▶ Sticky Wages
- ▶ Interest-Rate Smoothing
- ▶ Non-stationary TFP

Table: Parameter Values for the Empirical Model

Parameter	Description	Parameter Value
β	Discount rate	0.99875
a	Trend growth rate of productivity	$\frac{1.25}{400}$
χ_c	Inverse intertemporal elasticity of substitution for C_t	1.0
ζ	Degree of consumption habits	0.5
χ_n	Inverse labor supply elasticity	0.5
θ_p	Elasticity of substitution among intermediate goods	11
θ_w	Elasticity of substitution among intermediate labor	4
φ_p	Price adjustment cost	1000*
φ_w	Wage adjustment cost	300*
Interest-rate feedback rule		
$400(\bar{\pi} - 1)$	(Annualized) target rate of inflation	2.0
ρ_R	Interest-rate smoothing parameter in the Taylor rule	0.8
ϕ_π	Coefficient for inflation in the Taylor rule	3
ϕ_y	Coefficient for the output gap in the Taylor rule	0.25
$400(R_{ELB} - 1)$	(Annualized) effective lower bound	0.13
Shock		
ρ_d	AR(1) coefficient for the discount factor shock	0.85
$\sigma_{\epsilon,d}$	The standard deviation of shocks to the discount factor	$\frac{0.69}{100}$

*The corresponding Calvo parameters are 0.85 and 0.85.

Table: Key Moments

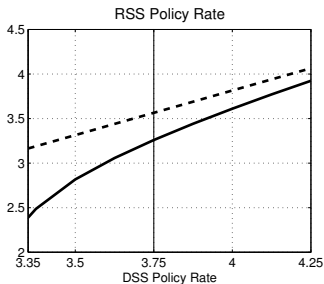
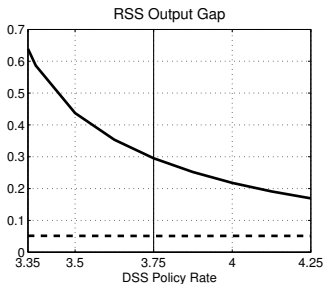
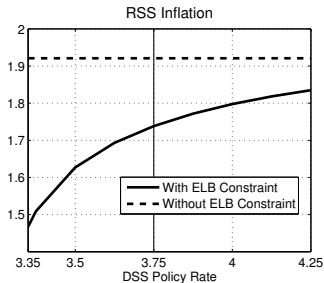
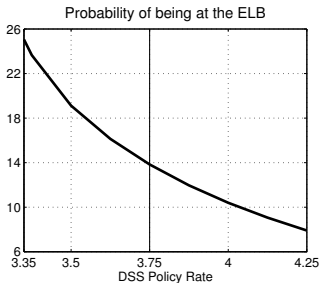
Moment	Variable	Model	Data [†] (1995Q3–2015Q2)
St.Dev.(·)	Output gap	3.0	2.9
	Inflation	0.31	0.52
	Policy rate	2.34	2.34
$E[X R_t = R_{ELB}]$	Output gap	-3.7	-5.2
	Inflation	1.21	1.48
	Policy rate	0.13	0.13
ELB	Frequency	13.8%	32.5%
	Expected/Actual Duration	8.6 quarters	26 quarters

Table: The Risky Steady State in the Empirical Model

	Inflation	Output gap	Policy rate
Deterministic steady state	2	0	3.75
Risky steady state	1.74	0.30	3.26
(Wedge)	(-0.26)	(0.30)	(-0.49)
Risky steady state w/o the ELB	1.92	0.05	3.56
(Wedge)	(-0.08)	(0.05)	(-0.19)

Long-Run Interest Rates

- ▶ Substantial uncertainty regarding the level of long-run equilibrium interest rates (Hamilton et al. (2015)).
- ▶ We vary the deterministic steady-state policy rate by varying the trend productivity growth parameter.
 - ▶ Motivated by the recent attention given to uncertainty regarding the long-run growth (Fernald (2014) and Gordon (2014)).



† DSS stands for “Deterministic Steady State” and RSS stands for “Risky Steady State”

Other Sensitivity Analyses

- ▶ Parameters in the policy rule.
- ▶ Adding other shocks (MP and TFP shocks).
- ▶ Price and wage indexation.

Summary

- ▶ The ELB constraint can adversely affect the economy even after liftoff via expectations.

- ▶ The ELB risk reduces the steady state inflation by 20-40 basis points.

Thoughts

- ▶ In linear models, the following three objects coincide:
 - ▶ The inflation-target parameter in the policy rule.
 - ▶ RSS inflation.
 - ▶ Unconditional average of inflation.
- ▶ As a result, modelers can simply assign the central bank's inflation objective to the inflation-target parameter in the policy rule and no complication arises.
- ▶ In stochastic nonlinear models, they do not. It becomes important for modelers to understand the differences between the three objects and how they relate to the central bank's inflation objective.
- ▶ “The Risk-Adjusted Monetary Policy Rule” (Nakata and Schmidt (2016)) for more details.

Extra slides

Households

$$\max_{\{C_t, N_t, B_t\}_{t=1}^{\infty}} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s \right] \left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} \right]$$

subject to

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} + P_t \Phi_t$$

where $B_0 = 0$ and δ_1 is given.

$\beta \delta_t$ is the relative weight the agent puts on the future utility flows at time t :

$$\beta \delta_0 U(C_1, N_1, G_1) + \beta^2 \delta_0 \delta_1 U(C_2, N_2, G_2) + \beta^3 \delta_0 \delta_1 \delta_2 U(C_3, N_3, G_3) + \dots$$

Firms

A final-good firm aggregates intermediate goods by CES technology.

Intermediate-good firms:

$$\max_{\{P_{i,t}\}_{t=1}^{\infty}} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s \right] \lambda_t \left[P_{i,t} Y_{i,t} - W_t N_{i,t} - P_t \frac{\varphi}{2} \left[\frac{P_{i,t}}{\bar{\pi} P_{i,t-1}} - 1 \right]^2 Y_t \right]$$

subject to

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t} \right]^{-\theta} Y_t \quad \& \quad Y_{i,t} = N_{i,t}$$

$$P_{i,0} = P_0 \text{ for some given constant } P_0 > 0$$

Government's Policy

- ▶ The supply of the government bond is zero.
- ▶ The nominal interest rate is set according to a truncated Taylor rule.

$$R_t = \max \left[1, \frac{1}{\beta} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \right]$$

Risk-Adjusted Fisher Relation

Standard Fisher Relation:

$$R_{DSS} = \frac{\Pi_{DSS}}{\beta}$$

Risk-Adjusted Fisher Relation

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$$R_{DSS} = \frac{\Pi_{DSS}}{\beta}$$

Risk-Adjusted Fisher Relation:

$$R_{RSS} = \frac{\Pi_{RSS}}{\beta} \cdot \frac{1}{E_{RSS} \left[\left(\frac{C_{RSS}}{C_{t+1}} \right)^{\chi_c} \frac{\Pi_{RSS}}{\Pi_{t+1}} \right]}$$

where $E_{RSS}[\cdot] := E[\cdot | \delta_t = 1]$

Risk-Adjusted Fisher Relation

Standard Fisher Relation:

$$R_{DSS} = \frac{\Pi_{DSS}}{\beta}$$

Risk-Adjusted Fisher Relation:

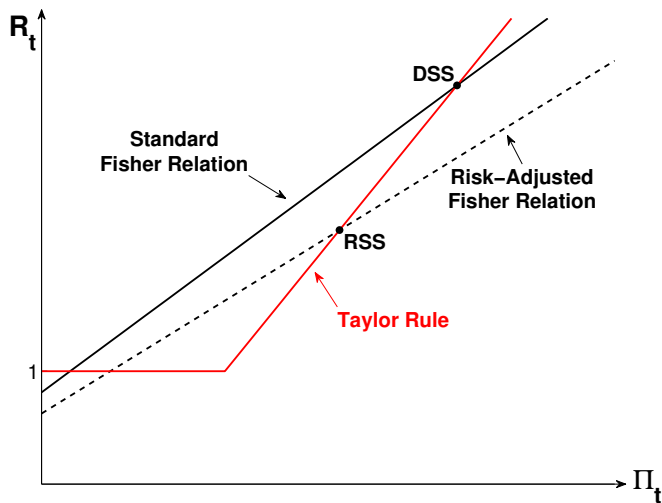
$$R_{RSS} = \frac{\Pi_{RSS}}{\beta} \cdot \frac{1}{E_{RSS} \left[\left(\frac{C_{RSS}}{C_{t+1}} \right)^{\chi_c} \frac{\Pi_{RSS}}{\Pi_{t+1}} \right]}$$

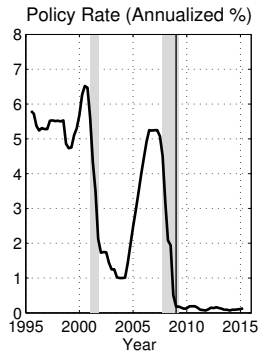
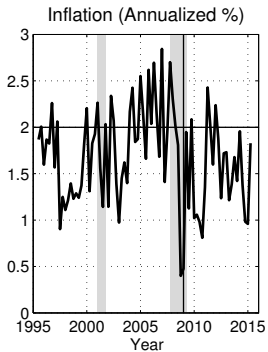
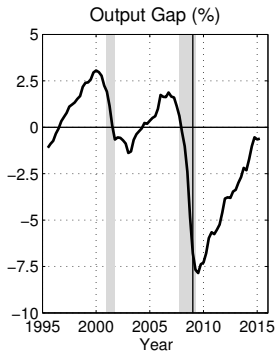
where $E_{RSS}[\cdot] := E[\cdot | \delta_t = 1]$ and

$$\frac{1}{E_{RSS} \left[\left(\frac{C_{RSS}}{C_{t+1}} \right)^{\chi_c} \frac{\Pi_{RSS}}{\Pi_{t+1}} \right]} < 1,$$

if the distributions of C and Π are negatively skewed.

Risk-Adjusted Fisher Relation





† The output gap measure is from FRB/US model. Inflation rate is computed as the annualized quarterly percentage change (log difference) in core PCE price index. The annualized federal funds rate is used as the measure for the policy rate.

Policy Implications

Can the central bank do something to mitigate the deflationary bias?

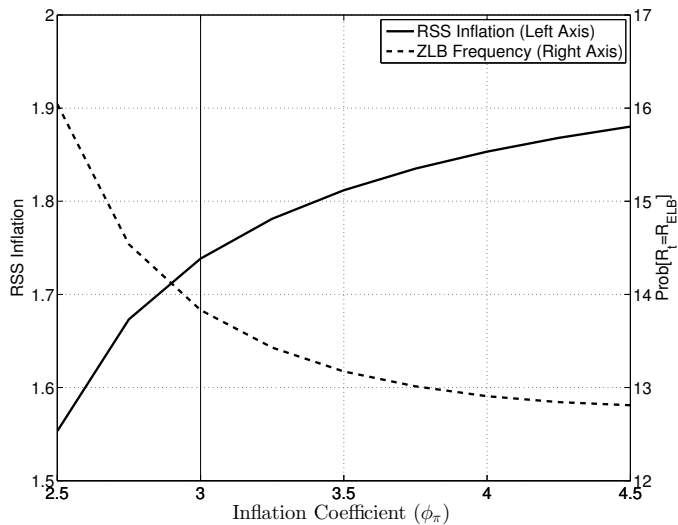
Recall that the policy rule is given by

$$R_t = \max[R_{ELB}, R_t^*]$$

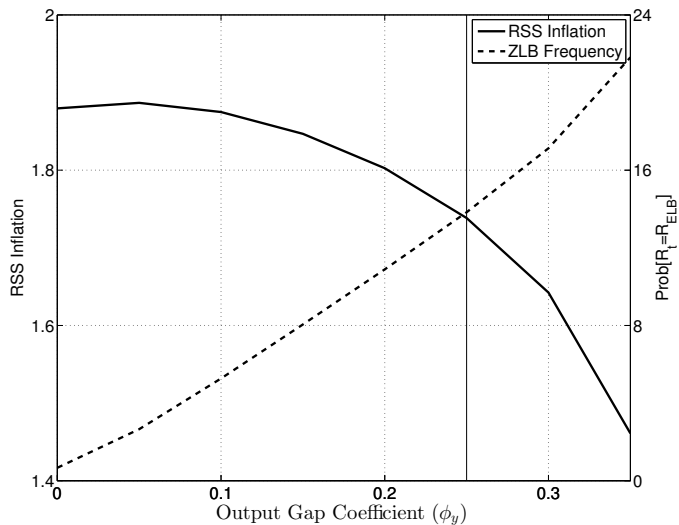
where

$$\frac{R_t^*}{\bar{R}} = \left(\frac{R_{t-1}^*}{\bar{R}} \right)^{\rho_r} \left(\frac{\pi_t^p}{\bar{\pi}^p} \right)^{(1-\rho_r)\phi_\pi} \left(\frac{\tilde{Y}_t}{\bar{Y}} \right)^{(1-\rho_r)\phi_y}$$

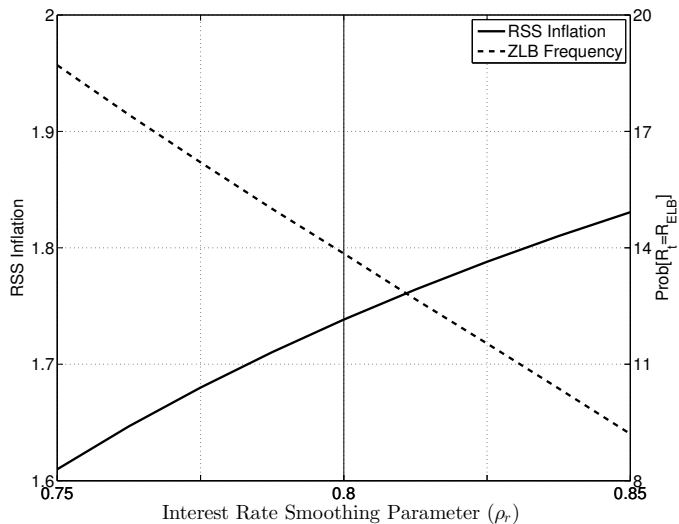
Policy Implications (I)



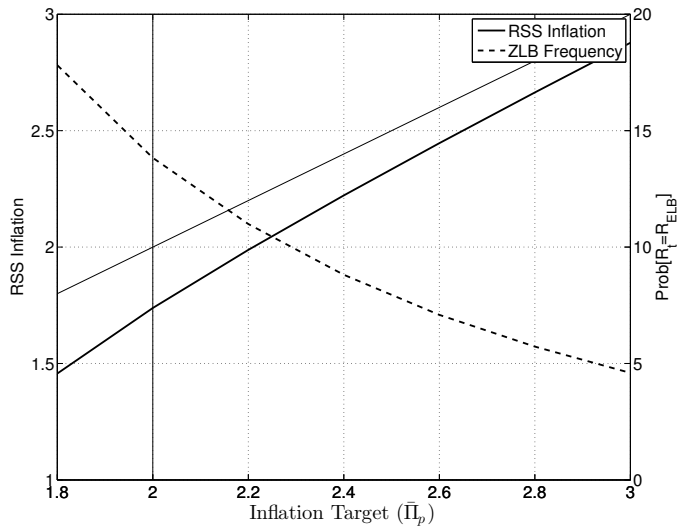
Policy Implications (II)



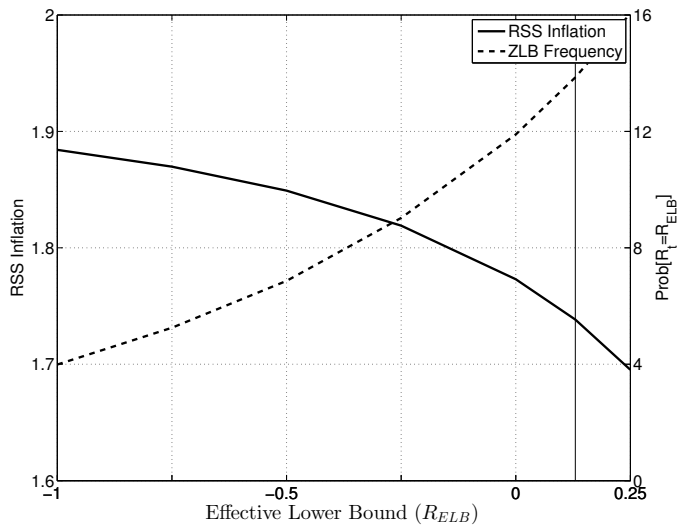
Policy Implications (III)



Policy Implications (IV)



Policy Implications (V)



Related Literature

Risky Steady State:

- ▶ Coeurdacier et al. (2011), Devereux and Sutherland (2011), and de Groot (2013).
- ▶ These papers study, and develop computational techniques to solve, differentiable economies.

We analyze a non-differentiable economy.