Rational Sunspots

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[1]The views expressed are solely the responsibility of the authors and should not to be interpreted as reflecting the views of Sveriges Riksbank.

Introduction

In this paper:
We propose a generalization of the Rational Expectations framework to estimate temporary unstable paths

Premise
RE generally implies multiple solutions

- Explosive
- Stable

How can we get uniqueness? (Sargent and Wallace ,1973; Phelps and Taylor, 1977; Taylor, 1977; Blanchard, 1979)

Stability Criterion: Transversality conditions
In saddle paths dynamics only one solution is stable

This became the standard in Macroeconomics (Blanchard and Kahn, 1980)

Example: U.S. Great Inflation period

Is it appropriate to rule out unstable paths from the empirical analysis?

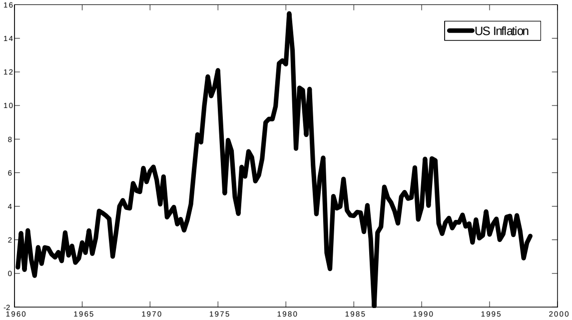

Figure: CPI inflation, quarterly data. Sample: 1960Q1 - 1997Q4

Is there any evidence that inflation is described (at least for a while) by unstable equilibria?

Outline Paper / Talk

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    - A novel way to introduce sunspots in a RE model to take into account the possibility of unstable paths

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   - U.S. inflation dynamics in the 70's are better described by unstable rational equilibrium paths

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   - Example of U.S. Great inflation (LS model and data)
   - U.S. inflation dynamics in the 70's are better described by unstable rational equilibrium paths
   - Unstable paths can be empirically relevant, also within the context of RE

A simple example: multiple RE solutions

Consider the following model inspired by Cochrane (2011), including the Fisher equation (1) and the Taylor rule (2):

$$i_t = r + E_t \pi_{t+1} \tag{1}$$

$$i_t = r + \phi \pi_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \tag{2}$$

$$\pi_t = \frac{1}{\phi} E_t \pi_{t+1} + e_t, \qquad e_t \sim i.i.d.N(0, \sigma_e^2) \tag{3}$$

Equation (3) has an infinite number of solutions:

$$E_t \pi_{t+1} = \phi \pi_t - \phi e_t => \pi_{t+1} = \phi \pi_t - \phi e_t + \eta_{t+1}$$

where $E_t \eta_{t+1} = 0$.

Multiple RE solutions

Muth (1961) and Blanchard (1979): UCM => All the solutions for π_t are described by

$$\begin{aligned}\pi_t &= \sum_{j=1}^{\infty} \phi^j (b-1) e_{t-j} + b e_t + \sum_{j=1}^{\infty} \frac{b}{\phi^j} E_t e_{t+j} \qquad (4)\\ \pi_t &= \phi \pi_{t-1} - \phi e_{t-1} + b e_t\end{aligned}$$

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- Easy to recognize two particular cases:
    - "pure" forward-looking $(b = 1)$

$$\pi_t^F = \sum_{j=0}^{\infty} \left(\frac{1}{\phi}\right)^j E_t e_{t+j} = e_t$$

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  - "pure" backward-looking solution $(b = 0)$

$$\pi_t^B = -\sum_{j=1}^{\infty} \phi^j e_{t-j} = \phi \pi_{t-1}^B - \phi e_{t-1}$$

Bubbles

The interpretation for b

- All the solutions can be written as a linear combination of the forward and the backward one (Blanchard, 1979):

$$\pi_t = (1 - b)\pi_t^B + b\pi_t^F$$

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- For $b \neq 0$, the expected value = an exponentially weighted average of the past observations (Muth, 1961)

$$E_t\pi_{t+1} = (b-1)\sum_{i=1}^{\infty}\left(\frac{\phi}{b}\right)^i \pi_{t+1-i}$$

Natural interpretation for b: the way people form expectations

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Natural interpretation for b: the way people form expectations

- b defines the importance the agents give to the past data, both in *absolute* terms (b vs 1), and in *relative* terms.
  Infinite solutions = infinite way we can set that weights => how to choose?

The stability criterium (e.g., Blanchard, 79)

$$\pi_t = \phi \pi_{t-1} - \phi e_{t-1} + b e_t$$

Is the stability criterium sufficient to identify a unique path?

1. If $\phi > 1$ *YES* determinacy, by imposing $b = 1 =$ f.l. solution
2. If $\phi < 1$ *NO* indeterminacy

$=>$ *"Sunspot equilibria can often be constructed by randomizing over multiple equilibria of a general equilibrium model, and models with indeterminacy are excellent candidates for the existence of sunspot equilibria since there are many equilibria over which to randomize."* Benhabib and Farmer (1999, p.390)

Introducing sunspot equilibria: any RE path

We have infinite equilibria because:

- there is an infinite number of ways of forming expectations
- all of them coherent with the Muth's REH
- parametrized by b

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hence we introduce sunspots randomizing over b:

$$b_t = b_t(\zeta_t) \tag{5}$$

where ζ_t i.i.d., orthogonal to the fundamental shocks e_s ($s = 1, 2, ...$), and $E_t\zeta_t = 0 \; \forall t$.

Sources of multiplicity

Solution UCM:

$$\pi_{t+1} = \phi\pi_t - \phi e_t + be_{t+1}$$

There are two sources of multiplicity => expectation error:

$$\eta_{t+1}(e_{t+1}, \zeta_{t+1}) = be_{t+1} + \zeta_{t+1}$$

where ζ_{t+1} = sunspot or non-fundamental error.

This paper considers the FIRST term: intrinsic multiplicity of RE solutions

Introducing sunspot equilibria: drifting parameters and unstable paths

If $b_t = b_{t-1} + \zeta_t$, and $\zeta_t \sim i.i.d.N(0, \sigma^2_\zeta)$, then

$$\pi_t = \theta_t \pi_{t-1} - \theta_t e_{t-1} + b_t e_t$$

with $\theta_t = \phi \frac{(1 - b_t)}{(1 - b_{t-1})}$ (with $b_{t-1} \neq 1$ otherwise FL solution).

- Same form as $\pi_t = \phi \pi_{t-1} - \phi e_{t-1} + b e_t$

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- Drifting parameters and stochastic volatility within the rational expectations framework. Cogley and Sargent (2005), Primiceri (2005).
- Intuition:agents can modify in every period the expectation formation process
- Reconsidering unstable paths: $|\phi| > 1$ *and* b_t *temporarily different from one* $=>$ estimate the process for b_t and check

Example: U.S. Great Inflation period

Is it appropriate to rule out unstable paths from the empirical analysis?

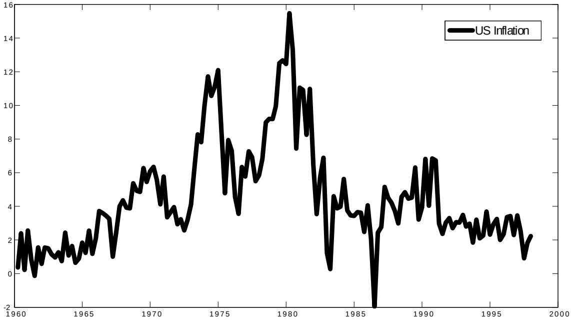


Figure: CPI inflation, quarterly data. Sample: 1960Q1 - 1997Q4

Example: Lubik and Schorfheide (2004) model

$$\begin{aligned} x_t &= E_t(x_{t+1}) - \tau(R_t - E_t(\pi_{t+1})) + g_t \\ \pi_t &= \beta E_t(\pi_{t+1}) + \kappa(x_t - z_t) \\ R_t &= \rho_R R_{t-1} + (1-\rho_R)(\psi_1 \pi_t + \psi_2(x_t - z_t)) + \varepsilon_{R,t} \end{aligned}$$

and

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}; \qquad z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$$

allow for non-zero correlation between the two shocks: ρ_{gz}

Compare two "models": M_S (stable solutions) and M_U (unstable solutions).

Eig

The estimation strategy

We use an econometric strategy to deal with the following issues:

i) the model has stochastic volatility, then the likelihood distribution is not Gaussian;

ii) we are interested in tracking the behavior of b_t, that can be considered as a stochastic latent process;

iii) we would like to study the fit of different models, and eventually compare them, during different periods.

Then, the econometric strategy is based on Bayesian methods, in particular on *Particle filtering*, and on *Sequential model monitoring*, based on Carvalho, Johannes, Lopes and Polson (2010)

Particle filter

Particle learning by *Carvalho, Johannes, Lopes and Polson (2010)*

1. Marginalization. θ: all latent states different from b; y: data

$$p(\theta, b|y) = \underbrace{p(\theta|y, b)}_{\text{Kalman Filter}} \quad \underbrace{p(b|y)}_{\text{Particle Filter}}$$

2. Parameter learning (Particle learning by CJLP 2010)

▸ PL

Priors and Distributions

Table 1

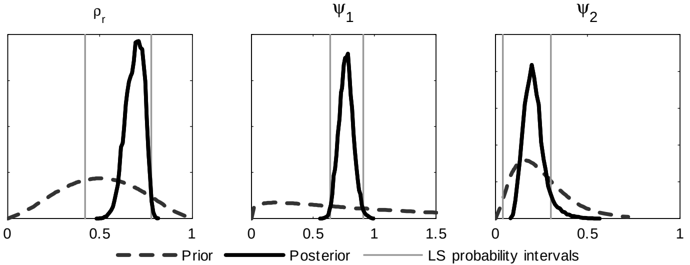
Prior Distributions			
Parameter	Density	Mean	Variance
ψ_1	Gamma	1.1	1
ψ_2	Gamma	0.25	0.15^2
ρ_R	Beta	0.5	0.2^2
π^*	Gamma	4	4
r^*	Gamma	2	1
κ	Gamma	0.5	0.2
τ^{-1}	Gamma	2	0.5^2
ρ_g	Beta	0.7	0.1^2
ρ_z	Beta	0.7	0.1^2
σ_R^2	Inverse Gamma	0.31^2	0.16^2
σ_b^2	Inverse Gamma	0.005	0.005
Variance Covariance	Density	Scale	Degrees of freedom
Σ_{gz}	Inverse Wishart	$3\begin{bmatrix} 0.4^2 & 0 \\ 0 & 1.2^2 \end{bmatrix}$	5

Estimates Great Inflation sample

	Pre- Volcker 1960:I - 1979:II		
Parameter	M_S	M_U	LS
ψ_1	0.77 [0.68 0.87]	0.31 [0.19 0.49]	0.77 [0.64 0.91]
ψ_2	0.2 [0.13 0.31]	0.22 [0.16 0.34]	0.17 [0.04 0.30]
ρ_R	0.69 [0.61 0.76]	0.54 [0.47 0.66]	0.60 [0.42 0.78]
π^*	1.83 [1.34 2.38]	4.03 [2.52 5.94]	4.28 [2.21 6.21]
r^*	1.41 [1.16 1.86]	1.42 [1.07 2.11]	1.13 [0.63 1.62]
κ	0.12 [0.09 0.17]	0.09 [0.07 0.12]	0.77 [0.39 1.12]
τ^{-1}	3.38 [2.54 4.21]	3.07 [2.49 3.59]	1.45 [0.85 2.05]
ρ_g	0.74 [0.70 0.77]	0.76 [0.73 0.79]	0.68 [0.54 0.81]
ρ_z	0.82 [0.78 0.85]	0.84 [0.79 0.87]	0.82 [0.72 0.82]
ρ_{gz}	0.12 [0.09 0.17]	0.06 [0.04 0.09]	0.14 [−0.4 0.71]
σ_R	0.21 [0.19 0.25]	0.16 [0.14 0.19]	0.23 [0.19 0.27]
σ_g	0.20 [0.18 0.24]	0.16 [0.14 0.19]	0.27 [0.17 0.36]
σ_z	0.82 [0.69 1.00]	0.62 [0.54 0.74]	1.13 [0.95 1.30]
σ_ς	0.05 [0.04 0.06]	0.07 [0.04 0.09]	0.20 [0.12 0.27]

90% credibility interval in brackets

Stable Model, Great Inflation

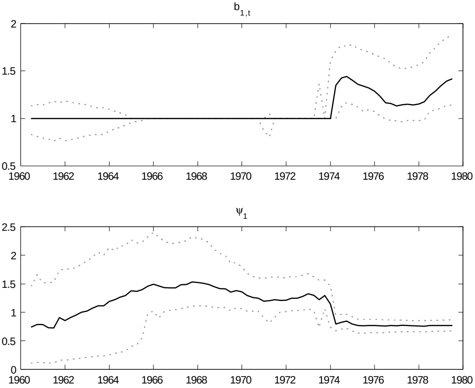


M_S : Comparison between the posterior distributions of the policy parameters and the probability intervals of LS

$=>$ Very similar results to LS

IRFs

Stable Model, Great Inflation



Estimated path for $b_{1,t}$ - stable model M_s - Great Inflation subsample (1^{st} panel); sequential inference on the parameter ψ_1 (2^{nd} panel).

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Unstable Model, Great Inflation

The behavior of $b_{1,t}$

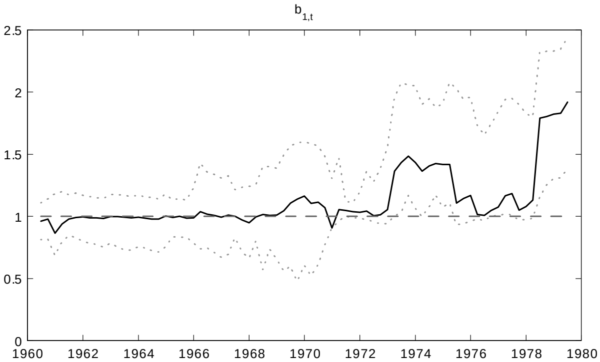
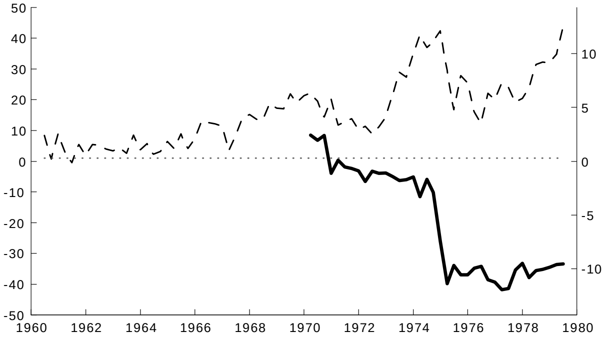


Figure: Estimated path of $b_{1,t}$ for the unstable model M_U in the Great Inflation subsample

AES

Comparing the relative fit of Ms/Mu

Sequential Bayes Factor West (1986): $2\ln(W_t)$ and the inflation rate



The Bayes Factor strongly favours the unstable model

▸ AES ▸ BMM

Estimates Great Moderation sample

	Post-82 1982:IV - 1997:IV		
Parameter	M_S	M_U	LS
ψ_1	2.18 [1.33 3.41]	0.42 [0.12 1.08]	2.19 [1.38 2.99]
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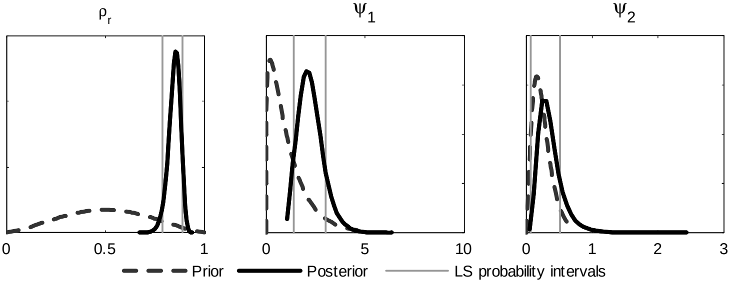
Under M_S => determinacy => $b = 1$

Stable Model, Great Moderation



M_S : Comparison between the posterior distributions of the policy parameters and the probability intervals of LS.

Unstable model: Great Moderation

The behavior of $b_{1,t}$

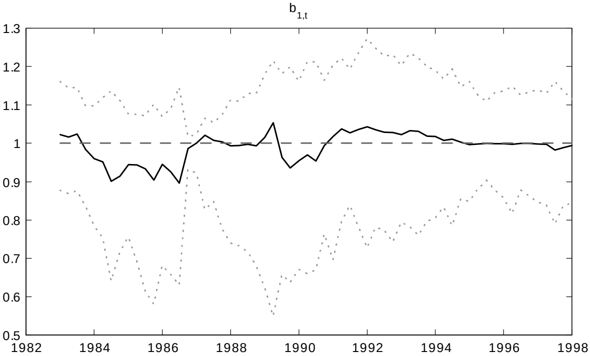


Figure: Estimated path of $b_{1,t}$ for the unstable model M_U in the Post-82 subsample

Model Comparison

Conclusions

- **Generalization of RE** approach / novel way of introducing sunspots

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- Great Inflation caused by drifting unstable expectations => **different policy implications**
- We should explore the possibility of (temporarily) **unstable paths**
  - Gourieroux et al. (1982): Asymptotically equal stationary path => bubble
  - Other possibilities: myopic agents, measurement errors

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EXTRA

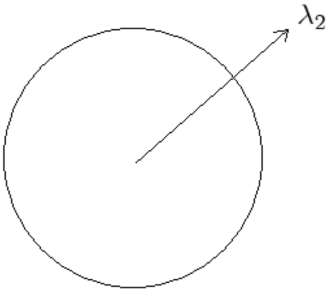
Multiple RE solutions: Bubble literature

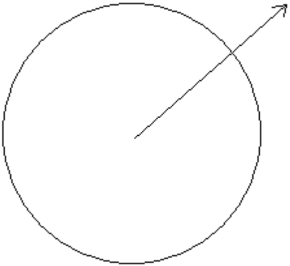
The solution can be rewritten as

$$\pi_t = \sum_{j=1}^{\infty} \phi^j (b-1) e_{t-j} + b e_t + \sum_{j=1}^{\infty} \frac{b}{\phi^j} E_t e_{t+j} =$$

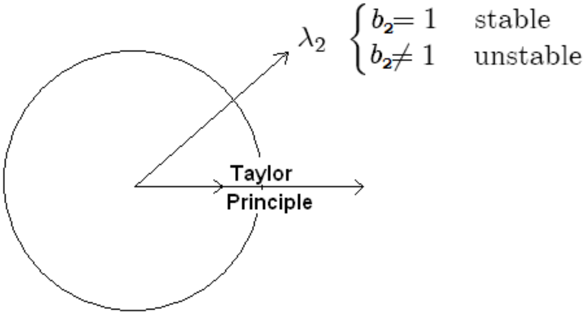
$$= \sum_{j=0}^{\infty} \frac{1}{\phi^j} E_t e_{t+j} + \underbrace{(b-1)\left[\sum_{j=1}^{\infty} \phi^j e_{t-j} + e_t + \sum_{j=1}^{\infty} \frac{1}{\phi^j} E_t e_{t+j}\right]}_{B}$$

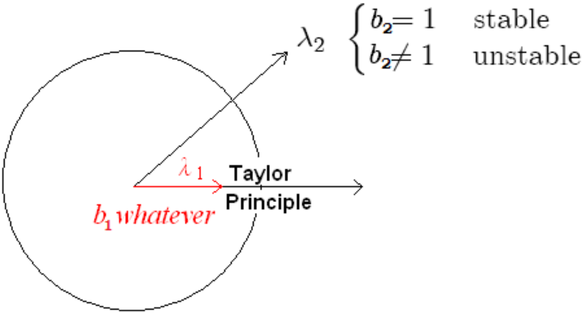
see Burmeister, Flood and Gaber (1983) ▸ Solution


λ_2



$$\lambda_2 \quad \begin{cases} b_2 = 1 & \text{stable} \\ b_2 \neq 1 & \text{unstable} \end{cases}$$


λ_2
$b_2 = 1$ stable
$b_2 \neq 1$ unstable
Taylor Principle



λ_2
$b_2 = 1$ stable
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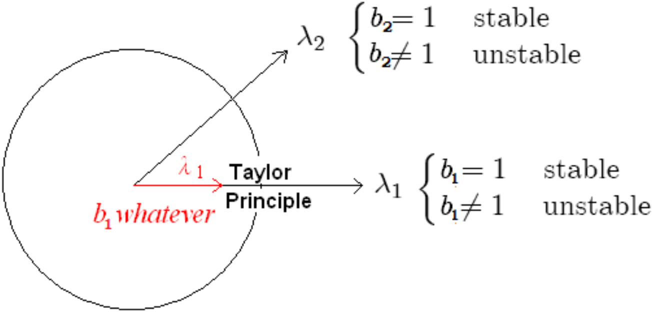
λ_1
Taylor
Principle
b_1 whatever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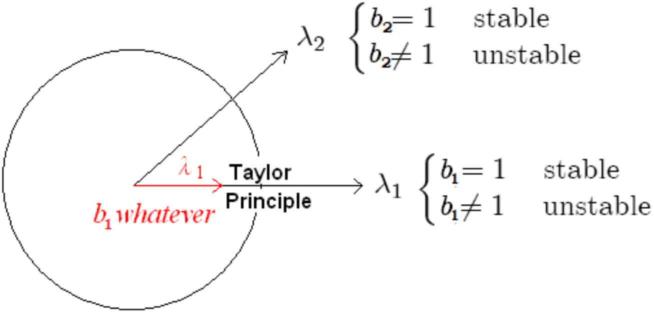


Compare two "models": M_S (stable solutions) and M_U (unstable solutions).

Model

Parameter learning

Particle learning by *Carvalho, Johannes, Lopes and Polson (2010)*

Assume that the posterior for some parameters ψ is function of a set of sufficient statistics s_t recursively updated

$$
\begin{aligned}
p(\psi|\theta_{0:t}, y_{0:t}) &= p(\psi|s_t) \\
s_t &= S(s_{t-1}, \theta_{0:t}, y_{0:t})
\end{aligned}
$$

and consider s_t as a latent state with deterministic evolution.

- When it is not possible use Liu and West approach. Approximating the posterior distribution with mixtures of Normals.

▸ PF

Bayesian model monitoring (West 1986)

Compare two models: M_S and M_U.

For $t = 1 ... T$

- Compute the predictive likelihood: $p(y_t | y_{0:t-1}, M_i) \quad i = S, U$
- Compute the likelihood ratio

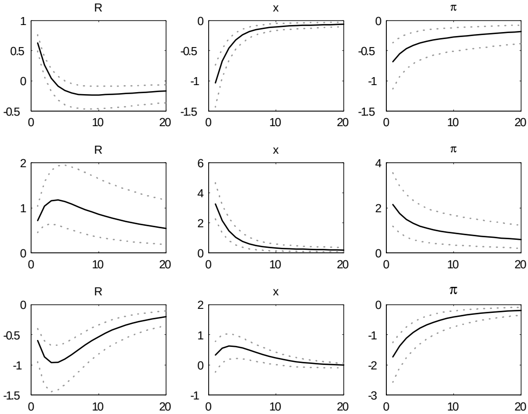
$$H_t = \frac{p(y_t | y_{0:t-1}, M_S)}{p(y_t | y_{0:t-1}, M_U)}$$

- Compute $W_t(k) = H_t H_{t-1} ... H_{t-k+1}$ (Kass and Raftery, 1995)

$W_t(k)$ is called the sequential Bayes factor and it assesses the fit of the most recent k observations.

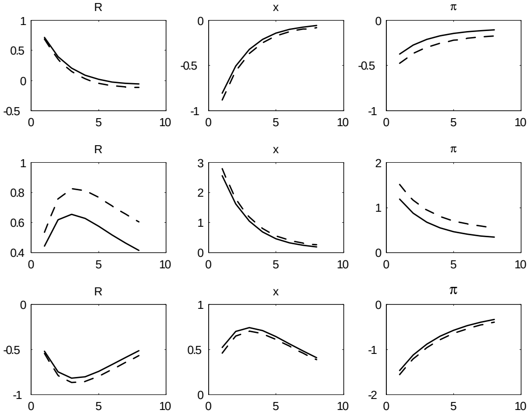
BF

Stable Model, Great Inflation



Transmission mechanism of structural shocks: GIRF in the M_S model.

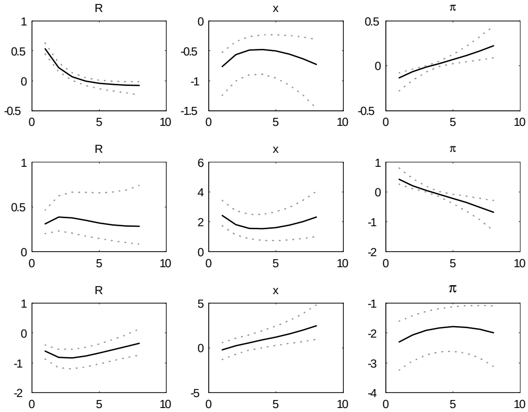
Stable Model, Great Inflation



Transmission mechanism of sunspot shock: GIRF in the M_S model: solid line: $b_1 = 1.3$, dashed line: $b_1 = 1.5$.

back

Unstable Model, Great Inflation



Transmission mechanism of structural shocks: GIRF in the M_U model

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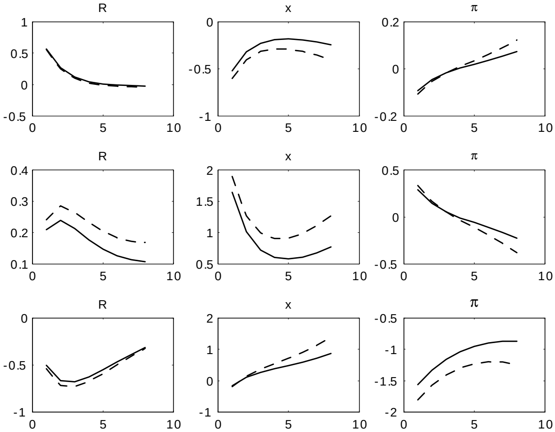


Transmission mechanism of sunspot shock: GIRF in the M_U model: solid line: $b_1 = 1.3$, dashed line: $b_1 = 1.5$.

back

Asymptotically equal stationary path (AES)

Process for b

$$b_t = \begin{cases} \theta b_{t-1} + 1 - \theta + u_t & \text{with probability } 1/\theta \\ 1 & \text{with probability 1-}1/\theta \end{cases}$$

where u_t is white noise, so that $E(u_t) = 0$, $\forall t$. $=> E(b_t) = b_{t-1}$

This process converge to 1 with probability 1 , but it is perturbed by u_t.

back

AES Model, Great Inflation

The behavior of $b_{1,t}$

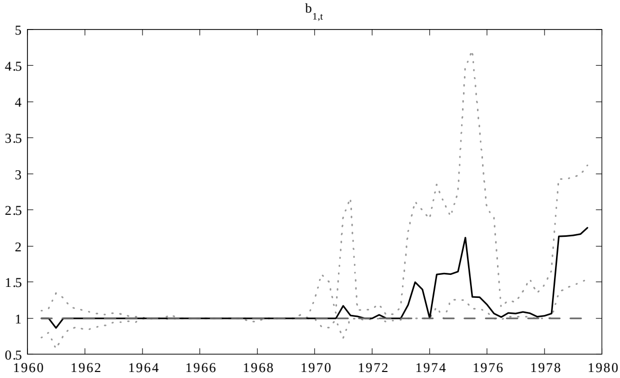
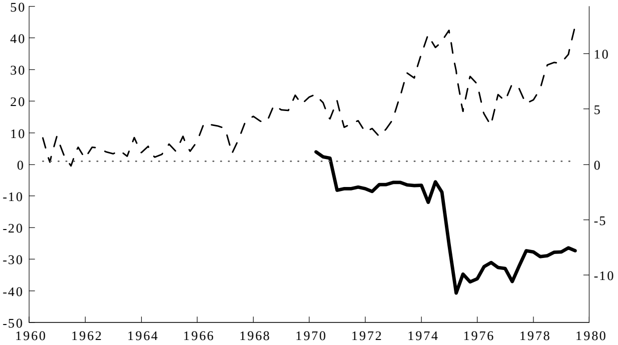


Figure: Estimated path of AES $b_{1,t}$ for the unstable model M_U in the Great Inflation subsample

▸ Mu

Comparing the relative fit of Ms/AESMu

Sequential Bayes Factor West (1986): $2\ln(W_t)$ and the inflation rate



The Bayes Factor strongly favours the **AES** model

▸ Mu

Comparing the relative fit of Ms/Mu: Great Moderation

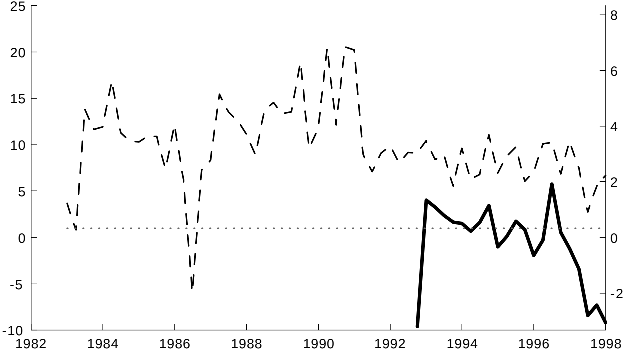


Figure: Sequential Bayes Factor West (1986): $2\ln(W_t)$ and the inflation rate

b1