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The Demand for Income Tax Progressivity in the Growth Model

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This paper examines the degree of income tax progressivity chosen through a simple majority vote in a model with savings. Households have permanent differences with respect to their labor productivity and their discount factors. The government has limited commitment to future policy so voting is repeated every period. Because the model features mobility within the wealth distribution, the median voter is determined endogenously. In a numerical experiment, the model is initialized to the 1992 U.S. joint distribution of income and wealth as well as several statistics of the federal income tax distribution. Support for a high degree of progressivity is widespread. In the long run, households that vote for lower progressivity have high labor productivity and/or very high wealth. A movement towards greater progressivity decreases aggregate capital and income as well as long-run income and wealth inequality.

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1 Introduction

Nearly all OECD countries have statutory income tax schedules with marginal rates that increase in income. Figure 1 plots the federal statutory marginal tax rate against multiples of the lowest taxable income level in each of twelve OECD countries for the year 2011.[1] While there is considerable variation in tax rates and in tax brackets across countries, one clear feature is that every tax schedule is marginal-rate progressive (i.e., marginal rates are increase monotonically with income)[2] Economists have attempted to explain this feature of the data for many years. These efforts can be divided into two groups: one attempting to derive marginal-rate progressive taxation as a characteristic of an optimal tax code, and the second uncovering it as the outcome of a political process.

The optimal taxation approach generally does not support progressive marginal income tax schedules. In the seminal Mirrlees (1971), for instance, the optimal income tax is very close to linear.[3] In full information settings (i.e., Ramsey style models), the optimal tax code is flat with an exemption for low income[4] The political economy approach bypasses the social planner framework and instead focuses on the process through which fiscal policy is decided (typically majority vote). Within this framework, tax rates should depend, at least in part, on the distributions of income and wealth. [5] In fact, a general finding within political economy models of income tax progressivity is that absent rent-seeking politicians, a democracy with an electorate in which the income-poor outnumber the income-rich will demand a progressive income tax since a majority benefit from income redistribution.[6] ,[7] In these models, the income distribution is exogenously given, and a voter's preference for progressive taxes depends on the voter's resulting net tax bill (i.e., income tax less any tax revenue transfers).[8] This literature has

[1]Data from OECD Tax Database Table I.1.

[2]A few eastern European countries, for example the Slovak Republic and Estonia, have only a single, positive marginal tax rate; however, they all include an exemption for low-income households within which the marginal tax rate is zero, so their codes remain marginal-rate progressive.

[3]See Grochulski (2007) for a notable exception from the Mirrleesian taxation literature

[4]Conesa and Krueger (2006); Conesa et al. (2009); and Saez (2013).

[5]For early examinations of this hypothesis with linear taxation, see Romer (1975), Roberts (1977), and Meltzer and Richard (1981).

[6]Marhuenda and Ortuno-Ortin (1995).

[7]The literature cannot prove that a right-skewed income distribution is sufficient for a marginal-rate progressive taxation equilibrium when the space of admissible income tax functions is defined as *any* nonlinear function. See Hindriks (2001), Klor (2003).

Other papers have shown existence within broad classes of nonlinear tax functions or under more refined equilibrium definitions. See Snyder and Kramer (1988), Roemer (1999), Donder and Hindriks (2004), Carbonell-Nicolau and Klor (2003),Carbonell-Nicolau (2009), and Carbonell-Nicolau and Ok (2007).

[8]A few models have allowed taxes to affect the period income distribution through labor supply decisions, however, the savings channel is not considered. See Snyder and Kramer (1988), Klor (2003), and Donder and

left the dynamic effects of income tax progressivity largely unaddresed. Intertemporal tradeoffs are not considered, and any feedback between current policy and future tax revenues is ignored. Essentially, the literature has concentrated on how politics divides the economic pie today, but has remained largely silent about its affects on the size and distribution of future pies.[9] Central in the argument over the design of income taxes is how marginal tax rates alter incentives to save and through this the amount of income that can be divided in the future.

Including a savings decision is important for three reasons. First, the distribution of wealth affects future production, and thus also future income distributions. A progressive tax not only distorts the optimal savings decisions of households by reducing future marginal returns from capital, but it places the largest marginal tax rates, and thus the strongest disincentives to saving, on households which otherwise tend to save the most.[10]

Second, in the US, wealth is much more concentrated than income and is held primarily by high-income earners.[11] From the perspective of low- and middle-income households (from which the pivotal voter likely arises), this concentration of capital income may be a tempting source for redistribution. Thus, not only could future aggregate income be reduced, but the tax base may erode as high-income households consume their wealth in response. In this way, high transfers in the short run may come at the cost of lower future transfers. On the other hand, increased progressivity reduces marginal tax rates on low-income households, making saving more attractive for them. Generally, the median income level is less than the mean income level, so it is not clear which direction aggregate wealth will move with progressivity without studying a quantitative model[12].

Finally, when both labor and capital are inputs to production, the capital stock influences the prices paid to each factor. Households may not only disagree on tax policy because of differences in income levels, but also because of differences in the composition of their income. Households with a high concentration labor income (relative to capital income) have an incentive to vote for policy which increases aggregate wealth while those with a greater fraction of

Hindriks (2004).

[9]Benabou (2000) is a notable exception. In that paper, the author builds an overlapping generations model in which each family operates its own production technology through accumulated capital and effort. Uninsurable productivity shocks to production lead to income inequality, however, simplifying assumptions about the nature of the shocks imply that the income and wealth distributions remain lognormal over time. When voting power is an increasing function of relative position in the wealth distribution, the author finds that inequality and redistribution are negatively related. In contrast, in this paper the income and wealth distributions is not restricted to a specific functional form.

[10]For an overview of the suboptimality of capital income taxation see Atkeson et al. (1999).

[11]Rodriguez et al. (2002) report a wealth gini of 0.803 and a earnings gini of just 0.553 in the 1998 wave of the SCF.

[12]Carroll and Young (2011) build a model with permanent preference and skill heterogeneity and show that increased progressivity can be associated with higher steady state saving, labor supply, and output.

income from capital have an analogous incentive to see aggregate wealth reduced. In the US data, capital income and labor income are positively, but not perfectly correlated, so there may be considerable disagreement over policy even among households with similar income levels.[13] Again, inferring the direction of factor price movements in response to progressivity changes requires a quantitative model.

This paper builds a general equilibrium neoclassical growth model augmented with a voting mechanism to assess the demand for income tax progressivity. Modeling the political economy of progressive taxation within a dynamic model introduces many complications. First, the wealth and income distributions are endogenous. In this environment, tax policy has more effects than simple income redistribution. Movements in the wealth distribution change factor prices, while changes in the income distribution alter both the size of redistribution and the concentration of the tax burden across households. Moreover, endogenous income and wealth distributions lead to policy being time inconsistent. The government could announce a tax scheme that would satisfy the current pivotal voter, however the identity of the future median voters will be different, and so the government would want to alter its scheme later to satisfy the new pivotal voter.

In order to identify the pivotal voter, previous models have exploited the findings of Chatterjee (1994) that with complete markets and proportional taxation the long-run distribution of wealth is indeterminate and features no mobility along the transition path. When the wealth distribution has these features, the pivotal voter always lies within the same subset of identical households, and equilibrium policy can be uncovered simply by examining these households' preferences. It is not necessary to explicitly calculate the vote nor to solve for the policy preferences of non-pivotal agents. While changes in the wealth and income distribution may alter the indirect preferences over tax policy of the pivotal agent, they never shift political power.[14],[15]

This paper connects the literature on the support for progressive taxation with the literature on dynamic voting. Households have permanent differences in labor productivity and earn income through labor and capital. Income may be consumed or invested toward producing new capital. This is true even if the deviation from linear taxation is very small so results derived from dynamic voting over flat taxation are likely to be quite different from what would arise under progressive taxation. In order to build an accurate approximation to the US data, Carroll and Young (2009) suggest allowing for heterogeneity in discount factors in the spirit of Sarte (1997). This paper adopts that technique, calibrating discount factors and productivities jointly

[13] Carroll and Young (2009) calculates the correlation between labor income and capital income for the 1992, 1995, 1998, 2002, and 2004 SCF waves. These correlations range from 0.14 to 0.43.

[14] See Krusell et al. (1997), Krusell and Rios-Rull (1999); Azzimonti et al. (2006),Azzimonti et al. (2008); Bassetto and Benhabib (2006).

[15] Within stochastic environments, this identification strategy does not hold. For examples, see Corbae et al. (2009) and Bachmann and Bai (2012).

from US household level data on income and wealth.[16] Giving up indeterminacy does come at a cost however: the pivotal voter must be found endogenously.

Heterogeneous abilities and preferences lead to income and wealth heterogeneity which causes disagreement over preferred income tax schedules. In each period, the progressivity of next period's income tax schedule is determined through a simple majority vote. When the model economy is calibrated to the 1992 US joint distribution of income and wealth, the highest degree of progressivity within the policy space is elected by a majority in every period. Preferences for progressivity are strongly decreasing in income and in wealth. Examination of individual household value functions shows that most households have nearly "bang-bang" preferences for progressivity. At low wealth levels, the household prefers high progressivity, and at high levels of wealth it prefers low progressivity. Somewhere between low and high wealth, there exists a narrow interval over which intermediate degrees of progressivity are favored. Finally, as long as its labor productivity isn't too high, a household prefers more progressivity as the ratio of its labor income to its total income increases. This is because in this model higher progressivity makes effective labor more scarce relative to capital, inducing a rise in the wage rate.

Comparing the long-run effects from increased progressivity with an alternative case under which the parameter governing progressivity is exogenously fixed at its initial level, increased progressivity leads to lower long-run aggregate wealth and income. The elasticity of the capital stock, and therefore the elasticity of factor prices, to progressivity is very small. A 12.0% increase in income tax progressivity leads to only a 1.3% decrease in long-run aggregate wealth.[17] In addition, the equilibrium path with high tax progressivity leads to more equal income and wealth distributions. Interestingly, transfers are higher on the more progressive path for only the very early periods of transition. Wealthy households quickly adjust their savings in response to higher progressivity, which leads to a sharp decline in their income and thus in tax revenue as well.

Finally, a household's demand for progressivity is not tightly linked to its resulting net tax bill (i.e., its taxes paid less transfers) in the period in which policy changes. While higher income levels, and by consequence higher net tax bills, do affect households' preferred policies, households that would pay more under a more progessive tax code may still favor it if their income is highly concentrated in labor income. In the long run, households with negative net taxes compose only a minority of the group voting for highly progressive policy. This finding suggests that the simple static story of income redistribution fails to entirely capture the

[16] For a further discussion of the long-run distribution of income and wealth in a similar environment see Carroll and Young (2011).

[17] Wenli and Sarte (2004) find that the long run effects of reducing progressivity by 7% on GDP growth are small, ranging from -0.12% to -0.34%.

motivation for progressive income taxation.

The remainder of the paper is organized into five sections. Section 2 details the model and explains the equilibrium concept. Section 3 explains the steady state analytics. Section 4 describes the calibration strategy and the quantative experiment. Section 5 discusses the results. The final section concludes.

2 Model

The model economy consists of three sectors: households, firms, and a government. In this outline of the model, capital letters denote aggregate variables and lower case letters denote individual-specific variables.

2.1 Households

This sector is comprised of a unit continuum of infinitely-lived households which differ with respect to their subjective discount factor, β, and permanent labor productivity, ε. Each household belongs to one of a finite number, I, of types. A type i is a pair (β_i, ε_i), and ψ_i is the fraction of the total population comprised by type i. Each household has the same period utility function $u(c)$ which is assumed to be strictly increasing and concave and to obey the Inada conditions. Lifetime utility for a household of type i is given by the time-separable function

$$\max_{\{c_{it}, k_{i,+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_i^t u(c_{it}) \tag{1}$$

subject to

$$\begin{aligned} s.t.\ c_{it} + k_{i,t+1} &\leq y_{it} - \tau(y_{it}) + T_t + k_{it} &(2)\\ y_{it} &= w_t \varepsilon_i \bar{h} + r_t k_{it} &(3)\\ k_t &\geq 0 &(4)\end{aligned}$$

where c_{it} and k_{it} are household i's consumption and wealth, respectively, in period t. Households supply a fixed number of hours, $\bar{h}$, and therefore a fixed number of effective hours as well.[18]

[18] Allowing for elastic labor supply would introduce a significantly greater computational challenge while not changing the resulting path of votes. The households that would adjust their hours the most have high labor productivity and zero wealth, and they would reduce their hours. This would drive up the wage which, as the analysis below indicates, increases the support for progressivity. In this way, the model is biased against progressivity.

w_t and r_t are the payments to effective labor and to capital. Total income, y_{it}, is the sum of income from labor and from capital. $\tau(y_{it})$ is the total tax bill paid on income of y_i at time t before lump transfers, T_t.

2.1.1 The tax bill

Both $\tau(y)$, the tax bill function, and $\tau_y(y)$, the marginal tax rate, are assumed to be nonnegative for all income and strictly monotonic increasing $\forall t$ with $\tau(y) = 0$ at $y = 0$. In words, the tax bill is progressive according to the definition from Musgrave and Thin (1948) since the average tax bill $\frac{\tau(y)}{y}$ is increasing in income. It is also marginal rate progressive; that is, at the margin, an additional unit of income increases the tax bill by more than the previous unit did.[19]

The tax bill takes the following function form:

$$\tau(y) = y\xi\left(\frac{y}{z}\right)^{\phi} \tag{5}$$

for some z. This progressive tax function is used in Wenli and Sarte (2004).[20] It has the convenient property that ϕ captures the degree of progressivity in the tax schedule, measured as the marginal tax rate, $\tau_y(y)$, divided by the average tax rate, $\frac{\tau(y)}{y}$[21], such that higher ϕ implies more progressivity.

As ϕ changes, marginal tax rates may not remain well-ordered across all income levels. As a result, there is a built-in potential for the single-peakedness over ϕ' to fail. To illustrate, figure 2 plots the marginal tax function for three values of ϕ. For income level below y_{low} (above y_{high}), marginal tax rates fall (rise) with progressivity; however, for incomes between y_{low} and y_{high}, the highest marginal tax rate occurs when ϕ takes an intermediate value.[22] Because savings responses are sensitive to the marginal tax rate, it is possible for a household to prefer either high progressivity or low progressivity over an intermediate value which violates single-peakedness.

To eliminate this potential problem, the marginal tax function above is altered slightly to allow z to depend upon ϕ. Ideally, the function should pivot about a central tax rate as ϕ changes. If ϕ increases (i.e., more progressivity), then marginal tax rates above the pivot rate will rise while those below will decline. Given any two marginal tax functions, one described

[19] Under the restriction that a household's tax burden cannot exceed its total income, a tax function being marginal-rate progressivity is equivalent to it being strictly convex.

[20] In Wenli and Sarte (2004), z equals mean income.

[21] $\tau_y(y) = \frac{(1+\phi)}{y}\tau(y)$ so $\frac{\tau_y(y)}{\frac{\tau(y)}{y}} = 1 + \phi$.

[22] While the size of this income interval may seem small, it contains approximately 16% of households in the final income distribution, and a even larger fraction of househould pass through this interval at some point during the evolution of the distribution to its terminal steady state.

by ϕ_0 and the other by ϕ_1, $z(\phi)$ must satisfy the condition

$$\tau_y(\tilde{y};\phi_0) = \tau_y(\tilde{y};\phi_1),$$

where $\tilde{y}$ is the income level associated with the pivot rate, implying

$$z(\phi_1) = \left\{ \left[\frac{(1+\phi_1)}{(1+\phi_0)} \right] \tilde{y}^{\phi_1-\phi_0} z(\phi_0)^{\phi_0} \right\}^{\frac{1}{\phi_1}}.$$[23], [24]

Figures 3 plot the marginal tax rate for several values of ϕ given $\tilde{y}=1$ and $z_0=1$. Notice that as ϕ increases the marginal tax remains the same for $\tilde{y}$, rises for all $y>\tilde{y}$, and falls for all $y<\tilde{y}$. It should be pointed out that this normalization procedure is not sufficient to guarentee single-peakedness since there other general equilibrium factors which influence a household's preference over ϕ'. Nevertheless, it does address one potential pitfall.

2.2 Firms and Government

Each period, households rent their effective labor, N, and capital, K, to a stand-in firm in return for wages and rent. With labor and capital as inputs, the firm produces a good which may be consumed or invested for future production. Let the production technology be Cobb-Douglas with capital's share denoted by α. Under the assumption that markets are competitive, factors of production are paid their marginal product so that

$$w_t = (1-\alpha) K_t^{\alpha} N_t^{-\alpha} \tag{6}$$

$$r_t = \alpha K_t^{\alpha-1} N_t^{1-\alpha} - \delta \tag{7}$$

, where δ is depreciation of capital. The government raises tax revenue to finance wasteful government spending, $\bar{G}$. Any surplus revenue is returned to the households as a lump-sum transfer,

$$T_t = \sum_i^I \psi_i \tau(y_{it}) - \bar{G}. \tag{8}$$

T_t is restricted to be non-negative so lump-sum taxation is not a policy instrument available to the government. Furthermore, the government does not have access to a commitment technology.

[23] Notice that since z does not depend upon income y, this augmented tax function preserves the identity from (5), $1+\phi = \frac{\tau_y(y)}{\frac{\tau(y)}{y}}$.

ϕ_0 will be uncovered from calibration, and $z(\phi_0)$ will be defined to equal 1.

[24] Because τ_y is not bounded above by 1, an upper bound of 0.999 is imposed in the quantitative experiment.

2.3 Voting

In each period t, the degree of progressivity in $t+1$ is determined through simple majority rule in pairwise competition. This is the only policy choice considered. The median voter theorem does not hold generally for multidimensional policy spaces, and voting equilibria are not guaranteed to exist. The existence of equilibria in political economy models rarely comes without conceding to some restrictive assumptions. If a richer policy space were permitted, very strong assumptions on voter preferences would have to be made in order to guarantee the existence of a Condorcet winner (i.e., a policy that would receive majority support against any other policy in a one-on-one competition) .[25]

2.4 Recursive Formulation

Let Γ be the distribution of wealth and assume that it follows the law of motion $\Gamma^{'} = H(\Gamma, \phi)$. For simplicity, denote by Γ_i the wealth holdings of an i-th type household. I assume that the progressivity of taxation evolves over time according to $\Psi(\Gamma, \phi)$. It should be stressed here that the assumptions about H and Ψ imply that this analysis is restricted to Markov equilibria. That these functions depend only upon Γ and ϕ is the concept of a "minimum state variable" as discussed in Krusell and Rios-Rull (1999). Together, the distribution of wealth and the degree of progressivity provide sufficient information to calculate current prices and transfers. It is assumed that the markets for investment, consumption, and labor clear every period. Mathematically,

$$K = \sum_i \psi_i k_i \tag{9}$$

$$N = \bar{h} \sum_i \psi_i \varepsilon_i \tag{10}$$

$$\sum_i \psi_i c_i + K' + G = F(K, N) + (1-\delta)K \tag{11}$$

The household problem may be expressed recursively as the following dynamic programming problem:

$$v_i(k, \Gamma, \phi) = \max_{c,k'} u(c) + \beta_i v_i(k', \Gamma', \phi')$$

subject to

[25] Common solutions to the difficulties arising from multidimensional conflict are not feasible in this model primarily because of the multidimensionality of heterogeneity. Generally, these solutions amount to projecting the multidimensional conflict down into a unidimensional characteristic space over which policy preferences are easily ordered. In this model, the relationship between β and ε and preferred policy is not easily reduced to a single dimension.

$$
\begin{aligned}
c + k' &\leq y - \tau(y;\phi) + k + T & (12)\\
y &= w\bar{h}\varepsilon_i + rk & (13)\\
k' &\geq 0 & (14)\\
\Gamma' &= H(\Gamma, \phi) & (15)\\
\phi' &= \Psi(\Gamma, \phi). & (16)
\end{aligned}
$$

Solving this problem yields decision rules $c = g_i(k, \Gamma, \phi)$ and $k' = h_i(k, \Gamma, \phi)$ for consumption and savings, respectively. Following Krusell and Rios-Rull (1999), this next section distinguishes a *competitive economic equilibrium* and a *politico-economic equilibrium*.

2.5 Equilibrium Concept

2.5.1 Competitive Economic Equilibrium

A competitive economic equilibrium (CEE) takes the evolution of tax policy, $\Psi(\Gamma, \phi)$, as given. As will be seen in the next section, to find its preferred value of ϕ', each household must evaluate the outcome associated with any candidate ϕ'. When the economy evolves according to Ψ and H, any ϕ will lead to a sequence of future tax progressivities and wealth distributions. From this sequence, a household can determine its welfare associated with a given ϕ' and rank all ϕ' in the policy space accordingly. The definition of a recursive competitive economic equilibrium is now formally stated.

Definition 1 *Given Ψ, a CEE is a set of functions $\left\{\{v_i, g_i, h_i\}_{i\in I}, H, r, w, T\right\}$ such that:*

1. *Given $\{H, r, w, T\}$, v_i, g_i, and h_i solve the recursive problem for type i households for all $i \in I$.*
2. *Factor markets clear.*
3. *T clears the government budget constraint.*
4. *The economy-wide resource constraint is satisfied.*
5. *$\Gamma_i = H_i(\Gamma, \phi) = h_i(\Gamma_i, \Gamma, \phi)$. In words, the i-th element of the wealth distribution implied by the law of motion H is consistent with the optimal saving decision of the i-th type for all i.*

2.5.2 Politico-Economic Equilibrium

In a politico-economic equilibrium (PEE), $\Psi(\Gamma, \phi)$ is determined endogenously. To uncover the equilibrium Ψ, households solve a *one-period deviation* problem. Every household knows that ϕ' will follow Ψ from tomorrow onward, but today ϕ' is permitted to deviate from the rule $\Psi(\Gamma, \phi)$. Because of this, the distribution of wealth may no longer evolves according to H. Instead, in the period in which the vote occurs, the Γ' will follow a new rule $\tilde{H}(\Gamma, \phi, \phi')$, after which it resumes following H in all future periods. Formally, the problem is stated as follows:

$$\tilde{v}_i(k, \Gamma, \phi; \phi') = \max_{c,k'} \left\{ u(c) + \beta_i v_i(k', \Gamma', \phi') \right\} \tag{17}$$

subject to

$$\begin{aligned} c + k' &\leq y - \tau(y; \phi) + k + T && (18) \\ y &= w\bar{h}\varepsilon_i + rk && (19) \\ k' &\geq 0 && (20) \\ \Gamma' &= \tilde{H}(\Gamma, \phi, \phi'). && (21) \end{aligned}$$

$\tilde{v}_i$ here differs from v_i in that it depends directly upon the ϕ' chosen in the current period. Clearly, different values of ϕ' induce different savings decisions and thus differents paths of aggregate wealth and income, as well as transfers and factor prices.

To arrive at its preferred policy, a household must also account not only for how its choice of ϕ' will affect future wealth distributions, but also how it will affect future elections. The pivotal voter today will be the pivotal voter in any future vote with zero probability. There are two reasons for this. First, a household's position in the wealth and income distribution is not fixed over time so the ordering of households' preferences for policy will not remain constant over time either. Second, voting is competitive in the sense that even if the same household type contained the median voter every period, the median-voter household would be selected randomly from among the infinite households of that type. Each household, then, considers how its vote today, should it be decisive, would influence the entire sequence of future wealth distributions and policy decisions, though the laws of motion

$$\begin{aligned} \phi'' &= \Psi\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right) \\ \Gamma'' &= H\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right) \\ \phi''' &= \Psi\left(H\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right), \Psi\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right)\right) \\ \Gamma''' &= H\left(H\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right), \Psi\left(\tilde{H}(\Gamma, \phi, \phi'), \phi'\right)\right) \\ &\ldots \end{aligned}$$

Solving this problem returns decision rules $\tilde{g}_i\left(k;\Gamma,\phi,\phi'\right)$ and $\tilde{h}_i\left(k;\Gamma,\phi,\phi'\right)$ for consumption and savings, respectively.

Given knowledge of the future effects of each ϕ', the household selects it most preferred progressivity value

$$\phi^i = \arg\max_{\phi'} \tilde{v}_i. \tag{22}$$

Definition 2 *A PEE then can be formally stated as a set of functions* $\left\{\{v_i, g_i, h_i\}_{i\in I}, H, r, w, T\right\}$ *and* $\left\{\tilde{v}_i, \tilde{g}_i, \tilde{h}_i, \tilde{H}\right\}$ *such that:*

1. $\left\{\{H, r, w, T\}, \{v_i, g_i, h_i\}_{i\in I}\right\}$ is a CEE.
2. $\left\{\tilde{v}_i, \tilde{g}_i, \tilde{h}_i\right\}_{i\in I}$ solve (17)-(21) and $\tilde{H}$ implies $\Gamma_i' = \tilde{H}_i = \tilde{h}_i$ for all i.
3. For all i, ϕ^i satisfies (22).
4. ϕ^i_{med} is such that $\sum_{i:\phi^i\leq\phi^i_{med}} \psi_i = \sum_{i:\phi^i\geq\phi^i_{med}} \psi_i = 0.5$.
5. $\Psi(\Gamma,\phi) = \phi^i_{med}(\Gamma_{med},\Gamma,\phi)$

Condition (1.) says that politico-economic equilibrium must satisfy the definition of competitive equilibrium. (2.) says that it must be the solution to the one-period deviation problem. (3.)restricts each agent to vote for the policy that is in its best interest, while (4.) declares the median-voter to be the decisive vote. Finally, (5.) is a rational expectations condition which says that the policy rule agents believe operates in the economy must be consistent with the preferred tax policy of the median voter.

3 Steady State

Although a full solution to this model can only be uncovered with numerical solution methods, some insights can be gained from a brief analytical study of its steady state. In the long-run, the marginal rate of substitution of consumption goes to unity so the optimal savings decision for a type-i household is described by the following equation:

$$1 \geq \beta_i\left[\left(1-\tau_y\left(y_i;\phi\right)\right)r+1\right] \tag{23}$$

where

$$y_i = w\varepsilon_i\bar{h} + rk_i \tag{24}$$

and (23) holds with equality if and only if $k_i > 0$.

Given that τ_y is monotonically increasing, two facts are immediately apparent:

1. There is only one value of income which can satisfy (23) with equality for β_i.
2. Among households with wealth above the borrowing limit in the long-run, a higher discount factor is associated with higher income.

I now formally define a β-*group.*

Definition 3 *A β-group is a collection of all households in I with the same value of β.*

All households in a β-group have the same discount factor, but they may differ in their labor productivity. The long-run savings behavior of households within a β-group follow the general results in Carroll and Young (2009). Within a β-group, all households with positive assets have the same long run level of income. Among these households, those with greater labor productivity earn a greater fraction of their income from labor than do less productive households in the same β-group. Households that are especially productive in labor may hit the lower bound on savings. These households will have higher income than the positive wealth households in the group.

3.1 The effect of changes in tax policy on the long-run income and savings of households

When ϕ changes, there are several effects on a household's long-run wealth and income. Because the tax structure is nonlinear, the net result of these effects will differ from household to household depending upon each one's characteristics. [26] To understand the long-run effects of changes in progressivity on each household, it is helpful to express (23) as

$$\frac{\beta_i^{-1} - 1}{r} \geq 1 - \tau_y \left(y_i; \phi\right). \tag{25}$$

The tax rate effect An increase in ϕ, raises τ_y for types with income above $\tilde{y}$ and discourages saving. High-income households that are not already at the borrowing limit gradually reduce their wealth, decreasing their income and their marginal tax rates over time. Eventually, either τ_y declines enough that saving once again becomes optimal or the borrowing limit will be reached. The tax rate effect works in the reverse for low income types. They will increase their savings and their income. In aggregate, the tax rate effect reduces long run income and wealth inequality.

[26] Here an increase in ϕ will be examined. The results from a decrease in progressivity are the opposite.

The factor price effect. A higher long-run interest rate implies a higher marginal benefit from saving. For all households with wealth above the borrowing limit this effect decreases long-run wealth and income. A higher value of r decreases the LHS of (25). For equality to be restored, τ_y, and therefore also y_i, must rise as well. Because wages and interest rates move in opposite directions, an increase in r implies less labor income for all households. Since in the long-run income rises, capital income must increase to offset the change in labor income. The magnitude of the change in labor income will be largest for households with high ε so these households will also have large changes in wealth. Households at the lower limit after tax policy changes will experience an unambiguous decline in long-run income, due entirely to the decrease in wages.

Figures 4 and 5 compare the two effects when ϕ increases and r increases.[27] Both types hold positive wealth before and after the policy change. To find the tax rate effect, calculate long-run income when r is held fixed, that is keeping the LHS of (25) constant. Since (25) holds with equality, while $\tau_y(y)$ decreases, y must rise so that the marginal tax rate is the same before and after the increase in ϕ. Turning to the figures, this can be seen as the distance from point A to point B. This change in income keeps the marginal tax rate the same before and after reform. In equilibrium, however, the household ends up at point C. This additional increase in income is due to the factor price effect through r. Comparing the low-income and high-income cases, the factor price effect moves income in the same direction for both types, while the tax rate effect is positive for a low-income household and negative for a high-income household. The change in the marginal tax function is particularly severe for high income households, and clearly the tax rate effect strongly dominates causing income to decline in the long run. For low income households however the marginal tax function does not decline as much as progressivity rises. As a consequence most of the increase in income can be attributed to the change in r.

3.2 The Steady-State Distributions of Income and Wealth

As stated above, all else equal, increasing the progressivity of the tax schedule reduces income inequality. The relationship between the long-run wealth distribution and household heterogeneity is best seen in the definition of income. Rearranging (24) yields

$$k_i = \frac{y_i(\beta_i, \tau_y, r)}{r} - \frac{w\bar{h}}{r}\varepsilon_i.$$

For households with the same labor productivity, those with larger discount factors will have more wealth. Within a β-group long-run wealth declines with ε at a rate of $\frac{w\bar{h}}{r}$. Notice that

[27] The values of income in these plots are taken from the the initial steady state $(\phi, r) = (0.71, 0.138)$ and the final steady state. $(\phi, r) = (0.8, 0.139)$.

this implies wealth inequality within every β-group is negatively related to r. Looking across β-groups an increase in r causes the LHS of (25) to decrease. The factor price effect increases the long-run income of all households with positive wealth, however because the marginal tax rate function is concave high-income types must increase income more than low-income types for equality in (25) to be restored. This leads to an increase in wealth inequality between β-groups. Whether overall wealth inequality increases or decreases in response to a change in factor prices is ambiguous The question is further complicated by the existence of a lower bound on assets. Once assets have been depleted to zero, a household's income is composed entirely of labor income which it cannot reduce to lower its tax burden. This suggests that, to the extent that some households in the model have zero wealth, removing the non-negativity constraint would lead to lower income inequality and greater wealth inequality.

4 Quantitative Experiment

4.1 Initialization

Period utility is assumed to be a CRRA function of consumption

$$u\left(c_t\right) = \frac{c_t^{1-\gamma}}{1-\gamma},$$

with $\gamma = 2$.

A representative sample of the US economy is constructed using observations of the US income and wealth distribution from the 1992 wave of the Survey of Consumer Finances data set. Each observation i in the SCF contains of an income value, $\tilde{y}_i$, a wealth value, $\tilde{k}_i$, and a population weight, $\tilde{\psi}_i$, which is assigned by the survey. The 1992 wave contains 3906 households. Unfortunately, it is not computationally feasible to use so many types. Instead, the sample joint distribution of income and wealth must be approximated by a coarser distribution. The coarse distribution has 51 types of households. This means that 153 parameters describing the preferences, productivities, and population weights of the household types are inferred from the data. In addition, ξ and ϕ, the tax function parameters, are set to match the average tax rate and the average marginal tax rate from the NBER TAXSIM data for 1992. Finally the transfer, $\bar{T}$ clears the government budget constraint.

As shown in Carroll and Young (2009), in deterministic models with heterogeneous productivities a non-degenerate long run distribution of wealth implies that the distribution of household discount factors have a one-to-one relation with the marginal tax function. To initialize, the model assumes a steady state in 1992 and that no households were restricted from borrowing. Given a market clearing interest rate, β_i can be backed out as a function of household i's marginal tax rate,

$$\beta_i = \left[\left(1 - \tau_y\left(y_i\right)\right) r + 1\right]^{-1}.$$

If $\tau_y(y)$ is strictly increasing then every y is assigned to a unique $\tau_y(y)$ and therefore also to a unique β.[28]

α is set to 0.36 to match labor's share of income in the data. G_t is assumed to be $\bar{G} = 0.08$. δ is 0.05 so that in the initial steady state investment is 15% of aggregate income. $\bar{Y}$, the aggregate level of income, is set equal to 1. $\bar{K}$, the initial aggregate capital stock, is set to 3.0. $\bar{h}$ is assumed to be 0.33. z_0 and $\tilde{y}$, the initial base value of z and the income for which marginal taxes remain constant, are set equal to initial mean income.[29]

One advantage of this initialization strategy is its ability to well approximate the US distributions of income and wealth. Table 1 compares moments from the data with those from the coarse distribution. In general, the coarse distribution does a good job of characterizing the inequality in income and wealth: considerable variance in income and extreme variance in wealth, a strong positive covariance, and significant right-skewedness.[30] Although the inequality in the data is still larger than in the coarse distribution, this is primarily due to observations in the data of very wealthy individuals. With sufficient grid points, the coarse approximation could do a better job, but for these gains one must increase the number of types and thus tradeoff large amounts of computational time. Preliminary work with 267 types showed insignificant changes to the results.

Looking at each marginal distribution in closer detail, while the coarse approximation underestimates wealth in the middle to upper part of the distribution, it does a very good job of characterizing the income distribution along income percentiles. Figures 6 and 7 compare the cumulative distribution functions of income and wealth, respectively, from the model approximation to those from the SCF. Given the greatly reduced number of household types, the coarse distributions follows the SCF distributions relatively well. This is especially true for the wealth distribution which is encouraging given this study's particular emphasis on savings.

Each period, households vote on a value for ϕ' between 0.2 and 0.8. Single-peaked preferences cannot be guaranteed for every household in every possible state. When households consider off-

[28]If $\tau_y(y)$ is flat however any y is associated with the same marginal tax rate and therefore mapped to the same β. In order to get a wealth distribution with a significant upper tail, I place a limit on the marginal tax rate of 0.396. This is the highest tax bracket in the US statutory income tax code for 1992. By capping the marginal tax rate at 0.396, the calibrated distribution can capture more of the right-skewness in income and asset holdings found in the US data without resorting to extremely high discount factors.

This paper uses value function iteration to solve the model. High discount factors are known to make convergence of the value function very slow. Moreover, with high discount factors small approximation errors can disrupt convergence making the process potentially unstable. Since the effects of progressivity on high-income households are clear in the results, assigning extremely high discount factors to rich households would not provide any additional insight.

In all voting periods however, the marginal tax function associated with a given ϕ will be strictly increasing.

[29]For more detail on the calibration method see Appendix B.

[30]Because variance is not a unit-independent measure, Table 1 reports the coefficient of variation.

equilibrium paths, they do so presuming that other households vote sincerely. Single-peakedness is tested numerically for every possible state. Given that each individual household has zero population mass, the assumption of sincere voting may be thought of as arising from very large coordination costs[31].

Because the initial distribution is calibrated assuming a marginal tax function with a flat region, while the policy space over which households vote contains only marginal tax functions that are strictly increasing, the economy would not remain in the initial steady state even if ϕ remained unchanged from its initial calibrated level in every period. Figure 8 compares the marginal tax function with the ceiling to the one with the same value of ϕ and no ceiling. High-income households face considerably greater marginal tax rates once the ceiling on this function is removed. All else equal, the adjustment to strictly increasing functions decreases these households' desire to save which will have a sizeable effect on the capital stock in the economy. For this reason, two experiments are run: one where tax progressivity is determined by a vote and another where $\phi^{'}$ remains fixed at its initial value but the cap on high income marginal tax rates is lifted. All results arising from the voting case will be referred to as *on the political equilibrium path*. The economy with voting does approach a steady state in the limit. This steady state is called the *political equilibrium steady state.* In the alternative scenario, where just the ceiling is removed and no voting takes place, the transition path is called the *fixed policy path*, and the steady state is called the *fixed policy steady state.*

Appendix A details the computational algorithm for solving the model.

4.2 Steady State Comparison

4.2.1 Inequality

Table 2 shows the long-run effects on the distribution of wealth in both the political equilibrium and fixed policy steady states. Comparing the political equilibrium and initial steady states, wealth is more spread out with more progressive taxes. Wealth among the highest income quintile declines by 36.5% while wealth among the remaining 80% of households increases substantially. The starkest example of this behavior comes from the model's bottom quinitile which increases savings by an astounding 5,410%! There is a denominator effect in operation here. This quintile holds only a 0.2% share of the capital stock initially. Table 3 shows the share of steady state wealth held by each quintile. While wealth holdings are still significantly skewed over the income distribution, capital in the economy is much more evenly distributed in the political equilibrium steady state.

[31] In the baseline case described, only 9040 of a possible 2409750 (or 0.38%) state variables and household combinations display non-single-peaked preferences. In these cases, the value function is essentially flat, only minute "wiggles" cause single-peakedness to be violated.

Long-run income inequality is reduced only slightly with more progressivity. The variances, skewnesses, and Gini coefficients of the long-run income and wealth distributions are reported in Table 4. Under both the voting and fixed policy cases, there are large reductions in the variance and skewness of income and wealth, as well as in their associated Gini coefficients, from the initial steady state. Within each β-group households trade off labor income and capital income one-for-one, and wealth holdings are very sensitive to changes in the marginal tax function and in factor prices. For many types after-tax returns are too low to induce them to save. When the full equilibrium is compared to the case where only the cap on high income tax rates is removed, however, the effect of income tax progressivity on inequality is modest. A 12.0% increase in ϕ produces a 2% greater reduction in the income gini and of 2.4% greater reduction in the wealth gini. The median capital holding in the political equilibrium steady state is 2.1 while in the fixed policy steady state, it is 1.93.

4.2.2 Aggregates

Because roughly two-thirds of the initial capital stock is owned by the top income quintile, this decline in wealth at the upper end has a significant effect on aggregate wealth. Table 5 compares the percentage change in economy-wide variables between the initial and political equilibrium steady states to those between the initial and fixed policy steady states. Under the fixed policy case the aggregate capital stock declines more than when voting occurs.

This may seem surprising since allowing voting only further increases marginal tax rates on the households that (initially) hold most of the wealth. The after-tax return to saving declines dramatically for these households, leading them to dissave rapidly. In the long run, however, that dissaving is partially offset by increased saving from low- and middle-income households who, as a result of voting for high progressivity, either pay lower or only slightly higher marginal tax rates. For this second set of households, the rise in the rental rate of capital is sufficient to induce increased saving. In addition, whether voting is or is not permitted, long run average income and consumption fall. Income decreases by only 0.6 percent when policy is fixed. When policy is decided by voting, it falls by 0.3 percent. The reason is analogous to that for the capital stock. Average long run consumption falls substantially in either case, primarily because of the dissaving behavior of initially rich, high-income households.

4.3 Transitional Dynamics

While steady state analysis is helpful for understanding households' decisions, given the forward-looking behavior of voters in this model, the transition path induced by policy change plays a critical role. This is especially true for economies which converge slowly to their steady states since long run consumption levels may not be close approximations to consumption levels in early transitional periods, and because due to discounting, early consumption levels have many times more weight in a voter's decision than consumption in distant periods. For some initial distributions, early transitional dynamics may make some policies politically unpopular even if those policies lead to steady states in which a majority of households enjoy more consumption than under the fixed policy path.

Figure 9 plots the paths of aggregate wealth, mean income, the government transfer, and mean consumption. The transition path can be divided into two stages. The first stage is characterized by declines in aggregate wealth, income, and consumption, and for most periods, transfers as well. The second stage is longer than the first and marked by a more gradual rise in aggregate behavior. The path with fixed policy follows a similar pattern implying that this "dip" behavior arises from the removal of the ceiling on the marginal tax rate function. The dip can be understood by considering that households' responses to the fiscal policy differ both in direction (some households increase wealth, others decrease wealth) as well as in rate. Under either the political equilibrium path or the fixed policy path, high income households immediately face a dramatic rise in their marginal tax rates. In the political equilibrium, this is primarily due to the removal of the marginal tax rate ceiling while in the fixed policy path, it is due *exclusively* to the ceiling's removal. High marginal tax rates discourage savings. This effect, along with significantly greater tax bills after the policy change, causes wealth to decrease rapidly.

Meanwhile, low-income households increase their wealth for two reasons. First, along the political equilibrium, these households' marginal tax rates decline, making consumption more expensive at the margin. Second, in both the voting and non-voting cases, the rapid depletion of aggregate capital increases the interest rate. The factor price effect discussed earlier showed that in isolation r rising causes households to increase their income. The tax rate effect dominates the factor price effect for high-income households while the two effects work in the same direction for low-income households.

Figure 10 plots the wealth paths of two households, one with low income and the other with high income. In the first stage of transition, the high-income household reduces its wealth very quickly while the low-income household increases its wealth slowly. During this time, the interest rate rises, incentivizing the low-income household to increase its wealth accumulation and the high-income household to slow its dissaving. As the transition enters stage two, the dissaving by high-income households has slowed sufficiently that saving by the other households causes aggregate wealth to begin rising. Notice that in the political equilibrium (increased progressivity combined with the removal of the tax ceiling), the tax effect is even more powerful than along the fixed-policy equilibrium. Aggregate wealth (and income) declines more quickly, and the second stage arrives sooner on the political equilibrium transition.

With or without voting, the transfer increases initially because of the increased tax bill on high income households. Not surprisingly, the transfer increase is larger for higher values of ϕ' (i.e., more progressivity).[32] The additional amount of transfer quickly diminishes as aggregate income declines through the first stage of transition. In fact, despite being initially higher, the transfer under the political equilibrium falls below the transfer level for the fixed-policy equilibrium after the sixth period. This highlights the importance of modeling redistribution with an endogenous income distribution. In this case, a large transfer can only be supported temporarily since tax policy distorts the savings decisions of the high-income households. Interestingly, unlike aggregate income, the transfer does not rise in the second stage of transition. This is because a progressive tax schedule attempts to collect the largest share of taxes from the portion of the income distribution that is shrinking while lowering the tax burden on the portion that is growing. For this reason, the total tax collected does not rise when average income does.

Finally, there are several other facts about the two transition paths to note. First, it takes a considerable amount of time for the voting equilbrium to overtake the non-voting equilibrium in terms of aggregate wealth and income. It takes 13 periods for the political equilibrium path to overtake the fixed-policy path. Second, aggregate variables converge to their steady state values more rapidly than individual household variables, suggesting that the effects of progressive tax policy changes on inequality take much longer to evolve than its effects on economy-wide measures.

4.4 Voting Decision

4.4.1 On the equilibrium path

The most important factors determining a household's preference over tax progressivity are its current income and its labor productivity. Not surprisingly, as a household's income increases, all else equal, it prefers less progressivity. Figure 11 plots households' preferred tax policy against their income in the long run (i.e., 5000 periods after the initial vote) and demonstrates a clear negative relationship between income and progressivity. A similar pattern appears in every voting period. In fact, the bottom 81% of households by income always vote for the most progressive tax policy. The economic logic for why income and preferred progressivity are negatively related is a recurrence of the lesson from previous static models: low-income households prefer a more progressive tax because it imposes a lower net tax to them; high-income households oppose progressivity because it raises their net tax. This logic however only captures part of the story. From the logic above, one would expect that the distribution of preferred tax policy across income would be degenerate. There would be an income level below which all households would want the maximum progressivity in the policy space and above which

[32] The average tax bill in the economy equals the transfer plus a fixed level of government spending. Therefore, this account of the transfer holds true for the average tax bill as well.

all households would want the least progressive choice available. Figure 11 however, shows that there is a wide range of income over which households want intermediate levels of progressivity. In some cases, even households with the same level of income vote for considerably different policies.

The reason that some households vote for intermediate progressivity levels is that they can differ dramatically in the way that their income is composed. Households with low labor productivity will, in the long-run, have income derived almost entirely from capital. Meanwhile, those with high labor productivity will have very little (or zero) capital income. Because tax policy affects savings incentives, and therefore the equilibrium ratio of capital to labor input, it alters the prices paid to capital and labor. Since a more progressive policy ultimately leads to a higher capital stock, it also increases the aggregate wage and decreases the rental rate for capital because labor is supplied inelastically. Households that earn most of their income from labor gain from the change in factor prices. Figure 12 plots preferred tax policy across ε for several income levels. All the lowest income households vote for very high progressivity regardless of productivity level. The direct effect of tax policy on net transfers dominates for them. Among higher income levels, preferred progressivity is increasing in ε. At these income levels, the net tax effect still dominants for the low productivity households, however at sufficiently high levels of ε, the positive effect of progressivity on wages leads them to vote for more progressive policy.

4.4.2 Off the equilibrium path

One advantage of using the approach in this paper is that the voting rule $\phi^i(k, \Gamma, \phi)$ is uncovered in the computation. This rule reveals how a household would vote in states of the world which do not appear along the equilibrium path. Figure 13 shows preferred ϕ' as a function of k for three households from the same β-group but with different productivities. Because ϕ^i has a high-dimension, the values of the other states are chosen from those corresponding to the nearest gridpoints to their long run equilibrium values. Preferences are nearly bang-bang. For each type, there is a narrow interval $[k_{low}, k_{high}]$ such that it prefers maximum progressivity for all wealth levels below k_{low} and minimum progressivity for wealth above k_{high}. Within $[k_{low}, k_{high}]$, the preferred ϕ' decreases sharply in wealth. As ε increases, k_{low} and k_{high} decrease so that higher productivity types stop supporting maximum progressivity at lower levels of wealth.

The impact of the discount factor, β, on preferences for ϕ' as a function of wealth is small. Despite nearly identical preferences for ϕ', two households with the same ε may vote for different ϕ' because they differ in wealth. Given two households with the same ε, the less patient household will generally have lower income and therefore less wealth.[33] In some cases, this wealth difference may be large enough so that the low-β household always votes for the highest

[33] It is possible for two households to have the same income level in the long run despite having different discount factors. This would happen if both households have zero wealth.

level of progressivity while the high-β household always votes for the lowest value.

As for other state variables, current progressivity, ϕ, has an quantitatively negligible effect on the preferences of households. For greater values of R, if a household supports high progressivity at some levels of individual wealth, the wealth level for which that household begins to favor lower levels of progressivity increases.[34] K has a small effect on households' policy preferences, and the direction of effect depends upon how concentrated total income is in a single factor. For households who earn income almost exclusively from labor (capital) an increase in K tends to increase (decrease) the preference for progressivity. In a capital abundant environment wages are relatively high meaning that households with high labor productivity will be relatively income-rich, inducing them to vote against more progressive income taxes. By the same logic, households with low labor productivity (and thus highly concentrated in capital income) will be relatively income-poor and thus value progressivity.

4.4.3 Net taxes as a predictor of votes

Most of the literature on the demand for income tax progressivity has focused on the distribution of net tax burden (taxes less transfers) to predict policy. The idea is that a winning policy must induce a reduction in tax burden for a majority of voters. In this model, that intuition does not hold up for every household. Although it is true that all households with negative net tax bills support the highest degree of progressivity, these households do not form a decisive majority on their own. Other households join them in voting for high progressivity despite facing a positive tax bill. These other households have low, but positive, net taxes and a high concentration of total income from labor.

The best analog to the prediction from the static literature, however, is between the net tax bill under the equilibrium policy and the net tax bill from a one-time deviation in progressivity. Viewed this way, the net tax burden does a good job of predicting the winning vote and a decent job of predicting individual votes. To see this, the following counterfactual is run. For each household, compare the net tax bills resulting from two different one-period deviations The first deviation is to $\phi' = \phi^*$, the household's preferred policy, and the second is to $\phi' = 0.2$, the lowest degree of progressivity in the policy space. To do this, the tax bill under both policies is calculated assuming that income does not change in one period[35]. The counterfactual transfer under each policy is derived using the equilibrium law of motion for government revenue. Given the tax bill and transfer for each policy, the net tax bills can be calculated.

[34] Since R does not clear the government budget constraint at the grid points, contemplating alternative R amounts to uncoupling tax progressivity and tax revenue. Therefore, households can reduce their tax bills by imposing higher progressivity without affecting their redistribution.

[35] While it would be best to allow income to be endogenous for these counterfactuals, it is not possible to do so. Fixing these income values for a one-period deviation should induce only a small error since the counterfactual policy under consideration is not too drastic a change and one period is not long for income to evolve.

If every household votes for the policy which yields the lowest net tax bill to them, then for any household, subtracting the net tax bill under its preferred policy from the smallest net tax bill under either $\phi' = 0.2$ or $\phi' = 0.8$, should yield a non-negative number. In other words, if the intution from the static literature holds up, then the net tax bill under the preferred policy can be no larger than that under any other policy. Figure 14 displays the absolute difference between the net tax bill with $\phi' = \phi^*$ and the net tax bill under the best alternative policy (i.e., whichever ϕ' from the set [0.2 0.8] leads to a smaller net tax bill) plotted by labor productivity. The blue circles mark households whose votes are consistent with the static literature's prediction. These household's vote for a progressivity level that yields a net tax that is no greater than that under the best alternative policy. For many of these households the difference is zero, meaning that their preferred policy and the best alternative are identical. The red diamonds are households who would have a lower net tax if $\phi' = 0.2$ but vote for a $\phi' > 0.2$. These households have labor productivity above the mean (2.03) though they are not the most productive. Nevertheless, they have a high concentration of total income from labor, and so the wage benefit of higher progressivity alters their votes. Finally, the green squares are the counterparts to the red diamonds. These households would enjoy a lower net tax if $\phi' = 0.8$, but vote for less progressivity instead. They have lower than average labor productivity and a high concentration of income from capital. Because the rental rate declines with progressivity, they vote for $\phi' < 0.8$. Despite failing to correctly predict the votes of these households, the net tax bill does a good job of predicting the winning policy. The blue circles account for 95% of the households in the model.

4.5 Extensions

4.5.1 Fixed Factor Prices

While the impact of factor prices on household decisions was discussed in the baseline model, it is illustrative to point out the quantitative differences between the baseline (i.e., with voting) and an alternative case in which factor prices are fixed. Under this scenario the rental rate and wage do not respond to changes in the capital stock and instead remain at their initialized values. Because the capital stock under the baseline declines from its initial value, the fixed rental rate will be low relative to its market clearing value, and the wage will be too high. The value of this experiment is that it allows changes in the economy due to movements in factor prices to be disentangled from changes arising from the change in marginal tax rates and in disposable income. The first difference between the baseline and fixed price economy is that with fixed factor prices progressivity receives even more support than in the baseline. As shown in Carroll and Young (2011), in this model environment, a higher rental rate implies higher long run income for every household with positive wealth, given the same marginal tax function.

The reason for this rests in the steady state Euler equation (25). A higher rental rate reduces the LHS of (25). Equality can only be restored if the long run marginal tax rate is higher, or equivalently, long run income is higher. In the fixed price economy, all households with positive wealth are necessarily poorer in the long run under fixed prices because the rental rate is low. Additionally, because tax policy cannot alter future factor prices, only the direct impact of progressivity on the net tax bill matters, and so these poorer households show more support for progressivity.

Aggregate activity is lower overall in the fixed price economy. Table 6 reports these values for the experiment along with those from the baseline The long run capital stock and aggregate income are only 83% and 95%, respectively, of their baseline values. The transfer is also reduced in the fixed price case as well. Both wealth and income inequality are a bit higher. Figure 15 compares the transition path under fixed factor prices to that under the baseline. Under fixed prices, the first stage of the transition path is only noticeable after a very long period time. Without an endogenous rise in the rental rate, there is no channel offset the negative effect on the saving decisions of high-income households or to further encourage the positive savings incentives for other households resulting from a more progressive tax reform. As a result, it takes a very long time for savings from low- and middle-income households to have a discernable influence on aggregate wealth.

4.5.2 Alternative γ

As a robustness check, the voting experiment is run for other values of intertemporal elasticity of substitution. Results for long run levels of income or capital are very similar. More importantly, changing its value does not change the voting outcome in any period; the most progressive policy is preferred in every vote. In addition, it does not change the basic path of transition. There is a "dip" period followed by a smooth rise to the new steady state capital stock level. The rate of transition to the new steady state, however, is different. Generally, as γ increases the capital stock adjusts more slowly, reaching the bottom of the dip earlier and emerging sooner as well.

5 Conclusion

This paper has examined the popular choice for the progressivity of income taxation in a neo-classical growth model. Within a model that is calibrated to well-estimate the distributions of wealth and income in the United States, there is strong support for progressive income taxation even accounting for its effects on future income distributions and tax revenues. In the long run, support for a high level of progressivity comes from a coalition of low-income households and middle-income households whose income is primarily derived from labor. Although the distribution of net tax bills predicts the winning policy, for some households the consequences of progressivity for factor prices causes them to vote for policies which increase their net tax bills.

In this model, increasing income tax progressivity leads to a lower long run capital stock due to the decrease in the savings of high-income households. Large reductions in income and wealth inequality occur from more progressivity, however the primary reason for this is a sharp increase in the tax rates on high income households induced by the removal of the initial upper bound on the marginal tax function. Once this change is accounted for, higher progressivity leads to only a small reduction in both income and wealth inequality.

The model presented here only begins to scratch the surface of the political economy of progressive income taxation within a model with savings. One potentially interesting research topic would be to include some income uncertainty to the environment. Carroll and Young (2011) shows that the long-run consequences of progressive tax reforms can be qualitatively different depending upon whether heterogeneity arises from uninsurable income shocks as in Aiyagari (1994) or from permanent differences in preferences, labor productivity, and the disutility from labor. Adding uninsurable idiosyncratic labor productivity shocks to the model here would reduce the sensitivity of savings to taxation since households would have a precautionary saving motive as in Aiyagari (1994) so the impact of policy on factor prices could be quite different. Uncertainty would also create another reason for a below-average income household to vote for progressive taxation: social insurance. If such a household faced an earnings shock process with a reasonable degree of persistence, then its income is likely to be low in the future as well. Without a redistribution mechanism, this household would self-insure to prevent very low future consumption. If this household were the pivotal voter, it would be tempted to exchange self-insurance for social insurance by redistributing income. It is likely that the structure of a progressive tax would make social insurance even more attractive to this household since unlike a proportional tax, an increase in progressivity yields larger transfers (though perhaps only in the short run) without increasing the pivotal voter's tax rate.

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Appendices

A Algorithm for Solving Recursive Politico-Economic Equilibrium

Due to the large size of the model, a hybridization of the computational algorithms of Krusell et al. (1998), Krusell and Rios-Rull (1999), and Corbae et al. (2009) is used. In order to solve the household's savings decision, tomorrow's prices, r' and w', the next period transfer, T', and the tax policy two periods in the future, ϕ'' must be known. These values depend directly upon the current distribution of wealth, Γ, however the high dimensionality of Γ makes using this state variable computationally unfeasible. Instead this paper develops a deterministic hybrid of Krusell et al. (1998).[36] As in Krusell and Smith, the key idea is to approximate Γ with a finite set of moments. The moments used in this paper are the mean level of capital K and the current period tax revenue, R. Along with the current tax policy ϕ, the laws of motion for these state variables are approximated by a set of log-linear equations.

$$\log\left(K'\right) = a_0 + a_1 \log\left(K\right) + a_2 \log\left(\phi\right) + a_3 \log\left(R\right) + a_4 \log\left(\phi'\right) \tag{26}$$

$$\log\left(\phi'\right) = b_0 + b_1 \log\left(K\right) + b_2 \log\left(\phi\right) + b_3 \log\left(R\right) + b_4 \log\left(\phi'\right) \tag{27}$$

$$\log\left(R'\right) = c_0 + c_1 \log\left(K\right) + c_2 \log\left(\phi\right) + c_3 \log\left(R\right) + c_4 \log\left(\phi'\right) \tag{28}$$

The problem to be solved is the following:

$$\tilde{v}\left(i,k,K,\phi,R;\phi'\right) = \max_{k'} u\left(c\left(k'\right)\right) + \beta_i v\left(i,k',K',\phi',R'\right)$$

The algorithm proceeds in the following manner:

1. Let $\left\{a_j^n, b_j^n, c_j^n\right\}_{j=0}^{4}$ and $v^n\left(i,k',K',\phi',R'\right)$ be the current guess for the continuation function v and the coefficients to the laws of motion on the n^{th} iteration.

2. Construct grids for k, K, ϕ, R, and ϕ'.

3. For each type i, loop over every combination of K, ϕ, R, and ϕ'.

   (a) To find the value of the continuation function, forecast K' and R' using equations above (26)-(28).

   (b) Linearly interpolate v^n in the K' and R' directions. Fit cubic splines to v^n in k' direction to approximate both the value function and its first derivative. Let these approximations be $\varpi\left(i,k',\phi'\right)$ and $\varpi_{k'}\left(i,k',\phi'\right)$ respectively.

[36] Corbae et al. (2009) also use a similar method.

(c) Find k'^* such that either

$$u'\left(c\left(k'^*\right)\right) = \beta_i \varpi_{k'}\left(i, k'^*, \phi'\right)$$

is satisfied or $k'^* = 0$.

(d) Step 3(c) returns an array $q\left(i, k, K, \phi, R; \phi'\right)$ and $h\left(i, k, K, \phi, R; \phi'\right)$, the value function and savings decision, respectively, under policy ϕ' for a household of type i given k, K, ϕ, and R.

4. Fit cubic splines to q in the ϕ' direction.[37]

5. Find the value of ϕ' which maximizes the cubic approximation to q. This yields $\theta^*(i, k, K, \phi, R)$ the preferred tax policy choice of a type i household given the states k, K, ϕ, and R.

6. Using the rules, $h\left(i, k, K, \phi, R; \phi'\right)$ and $\theta^*(i, k, K, \phi, R)$ simulate the economy for N periods. Choose N such that the difference between k_{N-1} and k_N is small.

   (a) The initial transfer, T_0, is given from the initial distribution, however future transfers must be found along the equilibrium path. Next period's government budget clearing transfer can be found from the current periods h and θ^*.

   (b) At each vote, order $\theta^*(i, .)$ from lowest to highest. The equilibrium ϕ' is the value of $\theta^*(i, .)$ which solves

$$\sum_{\{i:\theta^*(i,.)\leq\phi'\}} \psi_i = \sum_{\{i:\theta^*(i,.)\geq\phi'\}} \psi_i = 0.5. \tag{29}$$

   Because the number of types is finite, these sums will never equal 0.5. To deal with this, I take a weighted average of the value of $\theta^*(i, .)$ which is closest to 0.5 from below and the one that is closest from above.

7. The simulation returns a sequence $\{K_s, \phi_s, R_s\}_{s=0}^{N+1}$. Run OLS on this data to get new values for the coefficients to the laws of motion, a^{new}, b^{new}, and c^{new}.

8. For some λ_1,$\lambda_2 \in (0, 1]$, update the value function and the laws of motion according to $v^{n+1} = (1 - \lambda_1) q + \lambda_1 v^n$ and $x^{n+1} = (1 - \lambda_2) x^{new} + \lambda_2 x^n$ where $x = [a; b; c]$.

9. Iterate on 3-8 until the $\left\|v^{n+1} - v^n\right\|^\infty$and $\left\|x^{n+1} - x^n\right\|^\infty$ are less than some tolerance.

[37] Here is where I check for the single-peakedness of the indirect utility function in the policy direction. For each ϕ'_s on the ϕ'-grid, evaluate $q_{diff} = q\left(\blacklozenge, \phi_{s+1}\right) - q\left(\blacklozenge, \phi_s\right)$. If the sign of q_{diff} changes more than once, then single-peakedness is violated. Note that although single peakedness is never violated in any iteration on the value function for the experiments reported in this paper, it is only necessary that single-peakedness be satisfied for the converged value function, saving rules, and laws of motion.

B Initialization Method

The goal is to back out preferences β_i and labor productivity ε_i from household level data on income and wealth by using the steady state Euler equations and the definition of income. In this way, one may calibrate β_i and ε_i so that the long-run distribution from the model closely approximates the data. Since there are 3906 households in the 1992 SCF, it is not computationally feasible to assign a type in the model to every household in the data. The distribution in the data is then "coarsened" by reducing the number of types to 51. In addition to Table 1, Tables 7 and 8 show that key features of the SCF distribution can still be captured by this coarse approximation. The initialization steps used for the numerical exercise are presented below.

1. Let $Y = 1$, $K = 3$, $\delta = 0.05$, and $\alpha = 0.36$.

2. $$N = \frac{(Y - rK)}{w}$$

   and

   $$w = (1 - \alpha)\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha - 1}},$$

   imply that

   $$r = \alpha\left(\frac{K}{N}\right)^{\alpha - 1} - \delta = 0.138.$$

3. Guess the tax function parameters (ξ, ϕ) and the long run transfer, T.

4. Fix a range of income and wealth values over which to place grid points and partition the income interval and the wealth interval into n_y and n_k segments, respectively. While it is permissible to make these grid points evenly spaced, because of the skewness of the data a better approximation can be achieved by bunching more grid points at the lower ends of the intervals. This paper uses the function

   $$z_{i+1} = z_i + \exp\left(c + \frac{d * i}{n}\right)$$

   where c and d are constants and n is the number of grid points. For income set $c = -1.2$ and $d = 9.5$, and for wealth set c is -0.8 and d is 7.2. 30 grid points in each direction are used. For every combination of income and wealth on the grid, define a rectangular box such that the vertices of the box lie at the midpoints between the current grid point and its four neighbors (2 neighbors in the income direction and 2 neighbors in the wealth direction). For example, let $(x_{y,j}, x_{k,m})$ be the combination of the j^{th} income grid point and the m^{th} wealth grid point. The box assigned to this point would have vertices

$$\left(\frac{x_{y,j}-x_{y,j-1}}{2},\frac{x_{k,m}-x_{k,m-1}}{2}\right),\ \left(\frac{x_{y,j}-x_{y,j-1}}{2},\frac{x_{k,m+1}-x_{k,m}}{2}\right)$$
$$,\ \left(\frac{x_{y,j+1}-x_{y,j}}{2},\frac{x_{k,m}-x_{k,m-1}}{2}\right),\ and\ \left(\frac{x_{y,j+1}-x_{y,j}}{2},\frac{x_{k,m+1}-x_{k,m}}{2}\right).$$

Add the population weights from the SCF data of each household whose income and wealth fall inside the box and assign that weight to the grid point.

5. Normalize the type weights so that $\sum_{i=1}^{n_y * n_k} \psi_i = 1$. To reduce computational load for the model, if $\psi_i < 1.0 * 10^{-8}$ reset $\psi_i = 0$. Let the number of types with non-zero weight be $n_t \leq n_y * n_k$. Normalize the grid points for income and wealth such that $\sum_{i=1}^{n_t} \psi_i x_{y,i} = 1$. $\sum_{i=1}^{n_t} \psi_i x_{k,i} = 3$.

6. Because wealth in the data may be composed of many types of assets each yielding a different return, while the model has only one asset, it is possible for some wealth levels in the data to imply negative income at r. To avoid this, these observations are removed. 62 households are eliminated by this condition.

7. Check that $\sum_{i=1}^{n_t} \psi_i \frac{\tau(y_i)}{y_i} = 0.132$, $\sum_{i=1}^{n_t} \psi_i \tau_y(y_i) = 0.227$, and that $\bar{T} + G = \sum_{i=1}^{n_t} \psi_i \tau(y_i)$. If so then go to step 8, else update $[\xi, \phi, T]]$, and return to step 4.

8. Iterating on steps $(5-7)$ returns vectors $\{k_i\}_{i=1}^{n_t}$ and $\{y_i\}_{i=1}^{n_t}$. For each i, solve for β_i and ε_i which solve

$$\beta_i = [(1-\tau_y(y_i))\, r + 1]^{-1} \tag{30}$$
$$\varepsilon_i = \frac{y_i - r k_i}{w\bar{h}} \tag{31}$$

C Accuracy of Approximations to the Laws of Motion

This appendix details a test for the accuracy of the approximations to the laws of motion, (26)-(28). Because the coefficients to these approximations come from running OLS on a deterministic path and because 0.80 is the only realization of progressivity along this path, it seems reasonable to ask how well households evaluate the future effects on capital and government revenue from choosing alternative ϕ' values. To address this question, divide the ϕ'-grid into 20 evenly-spaced points. For each of these 20 alternative ϕ' points, impose that ϕ' value as the first realization of ϕ' and simulate the economy using the decision rules solved from the model.

Then estimate the coefficients for the laws of motion implied by that path. If the approximation used in the model is close then these coefficients should not change much.

The coefficients changed the most when ϕ' is initially fixed to be 0.2. Table 9 compares the coefficient values from the laws of motion in the model equilibrium to those implied by this counterfactual case. The final columns give the maximum absolute percentage error in any period and the average percentage error from using the model laws of motion to forecast the K', ϕ', and R' on the alternative path. By either measure, the errors are very small. These results are interpreted to mean that (26)-(28) do a sufficiently good job of approximating the paths of K', ϕ', and R' in response to a one-period deviation in policy.

Figure 1: Marginal tax schedules across countries

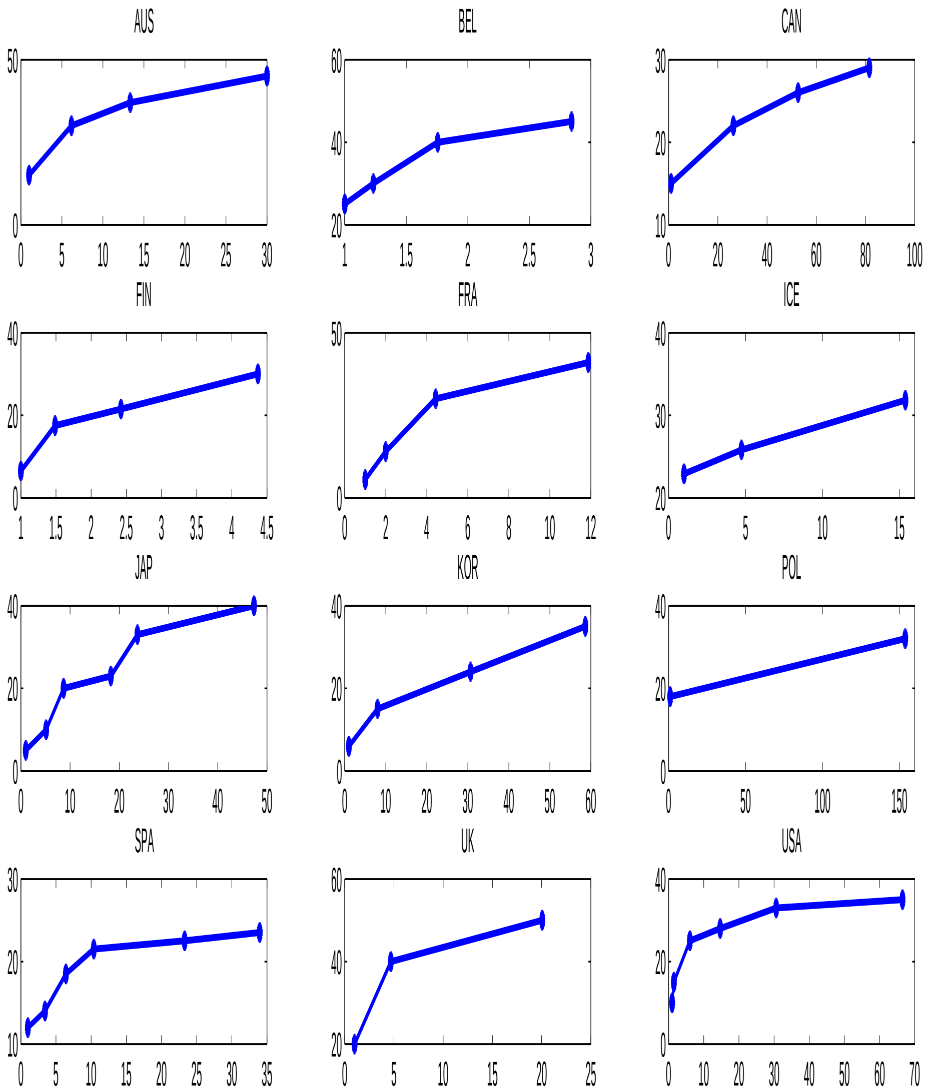

Figure 2: Failure of single-peakedness

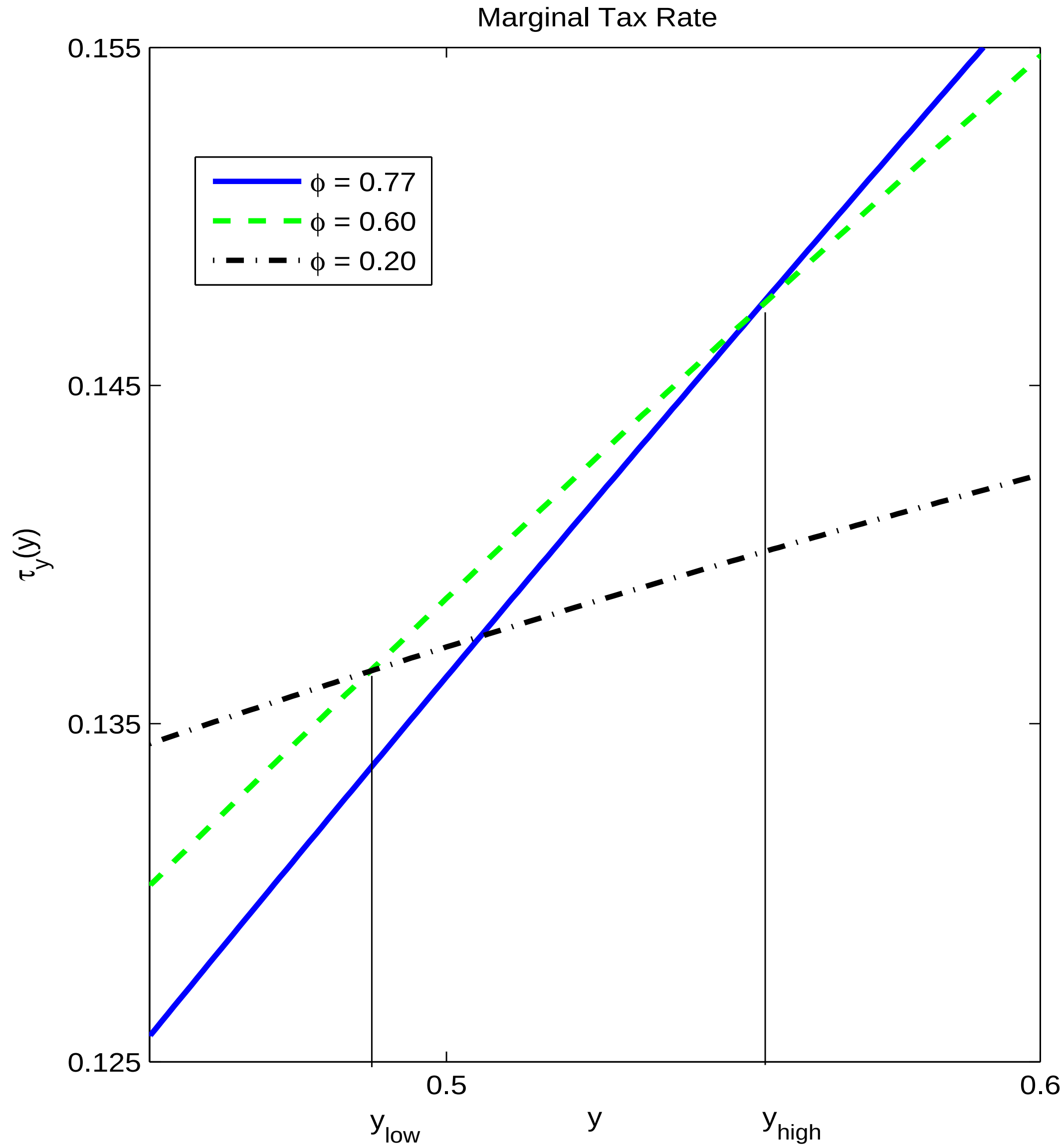

Figure 3: Marginal tax rate

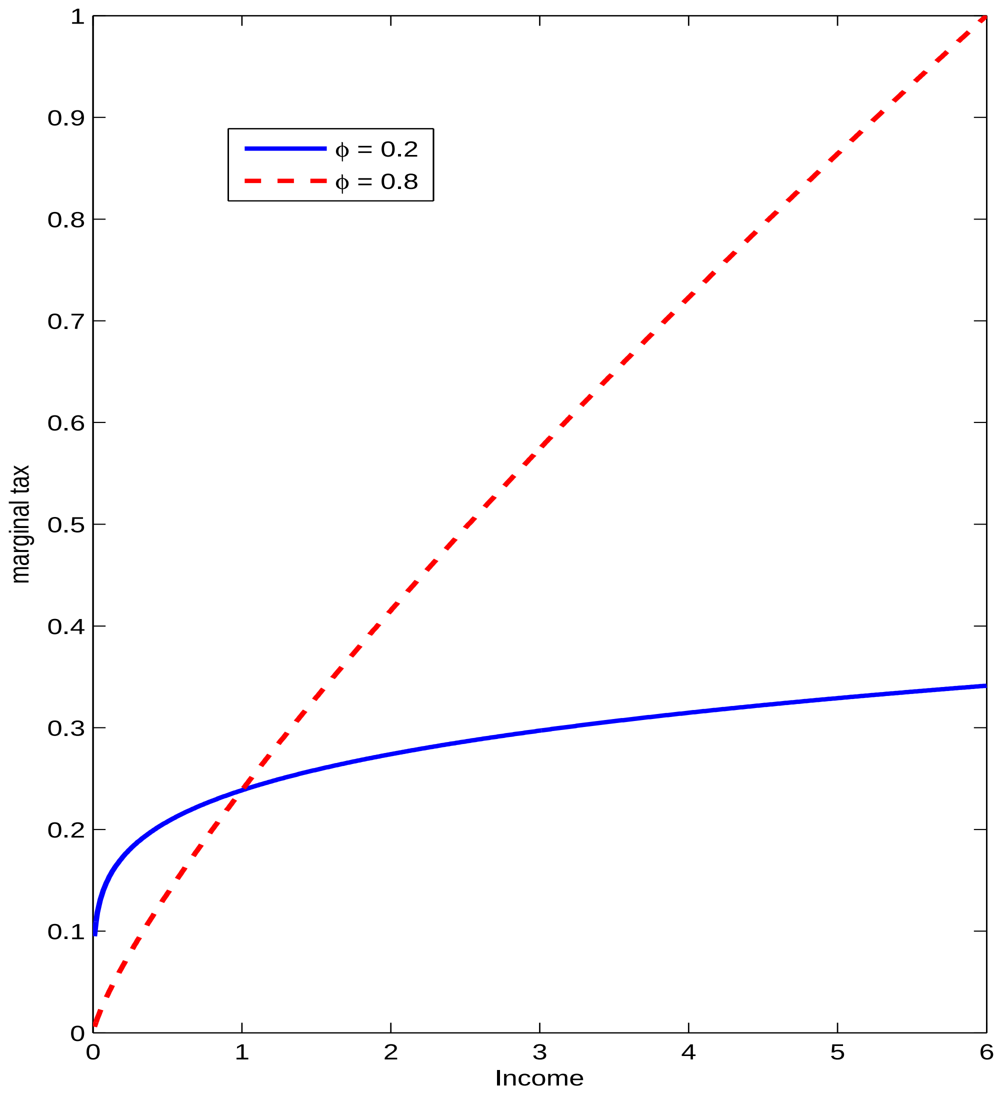

Figure 4: Tax rate and factor price effects for low income household

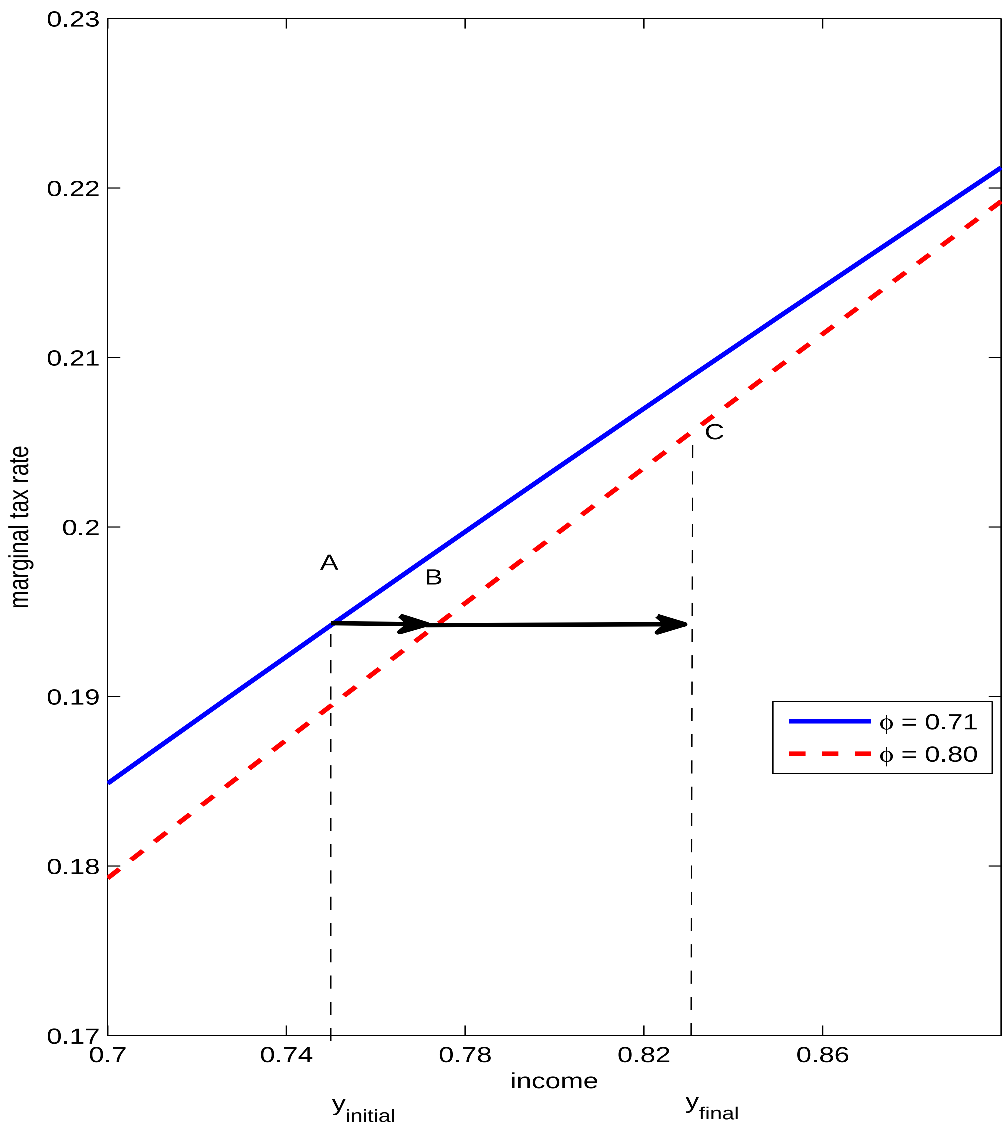

Figure 5: Tax rate and factor price effects for high income household

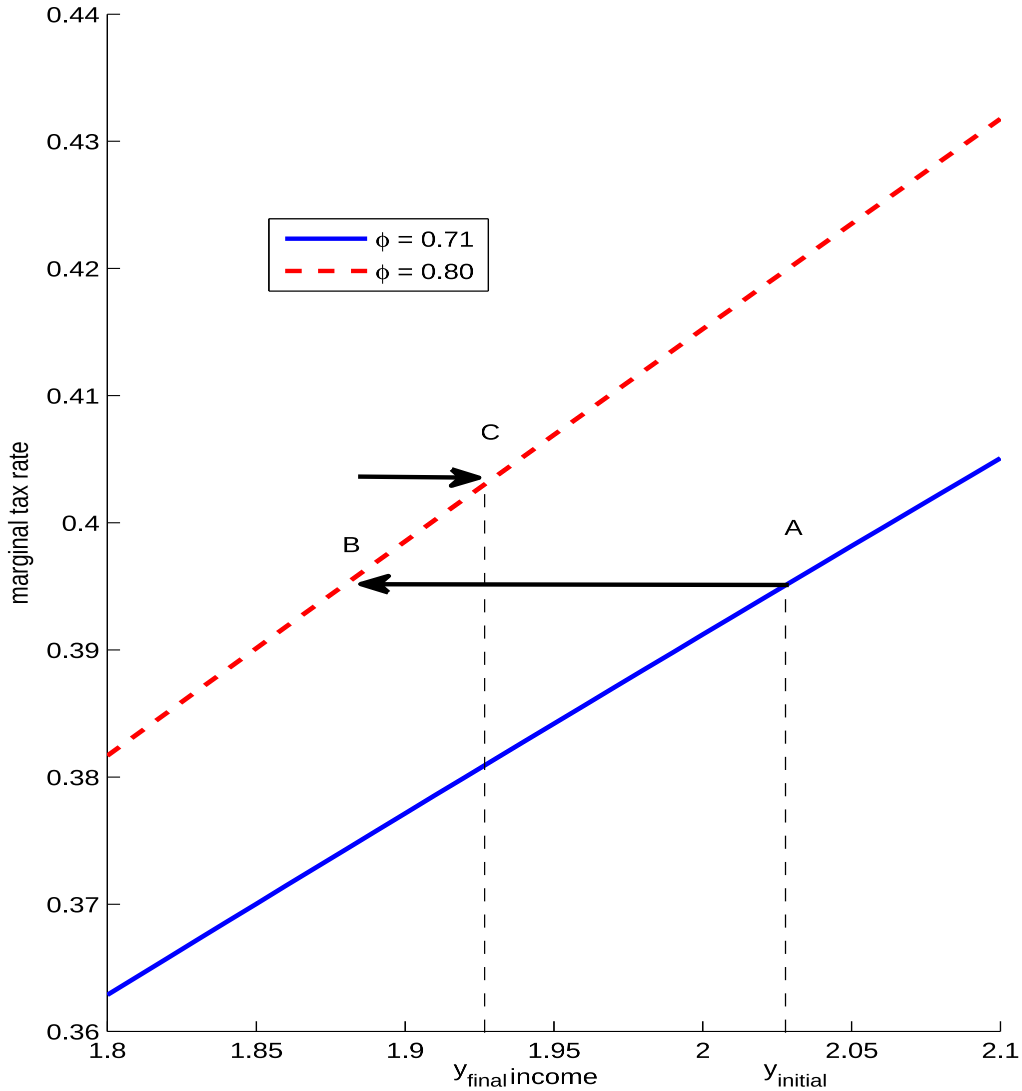

Figure 6: CDF of income

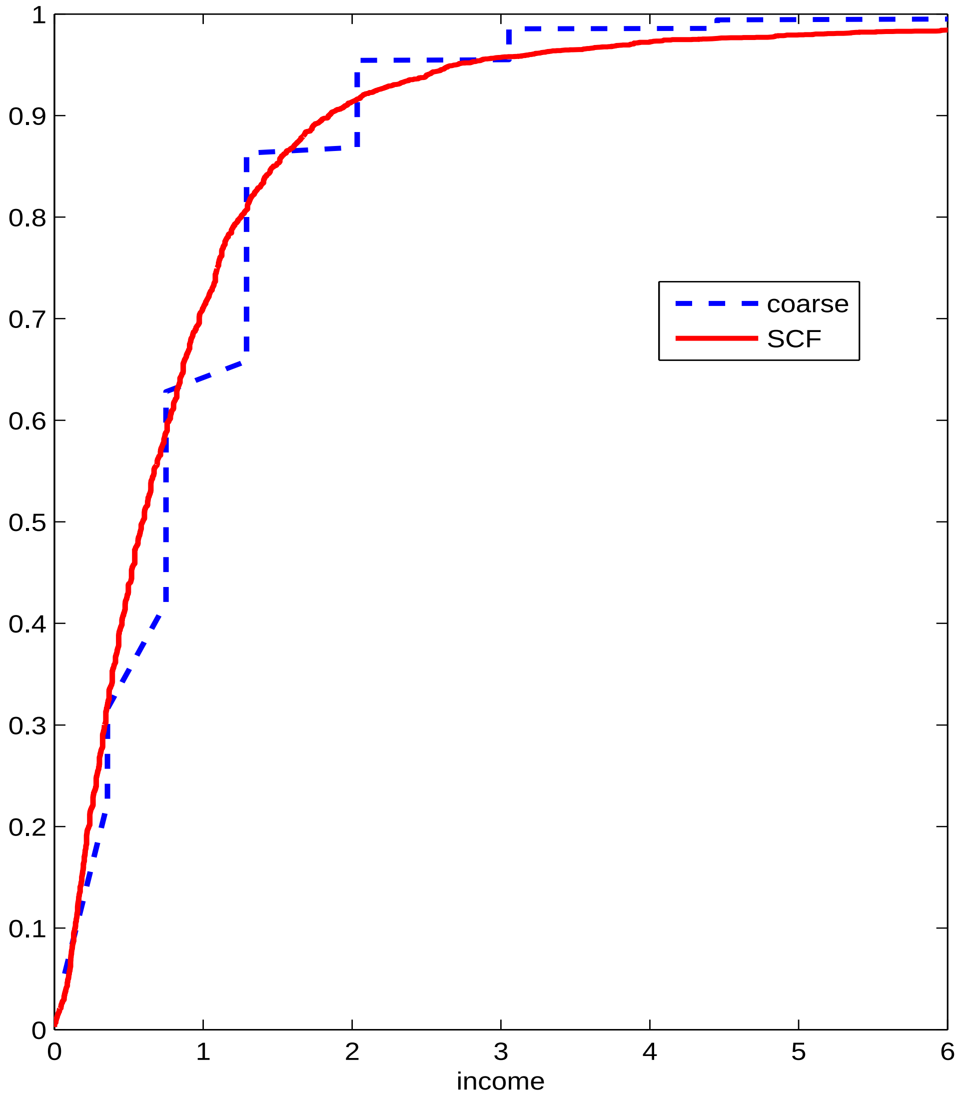

Figure 7: CDF of wealth

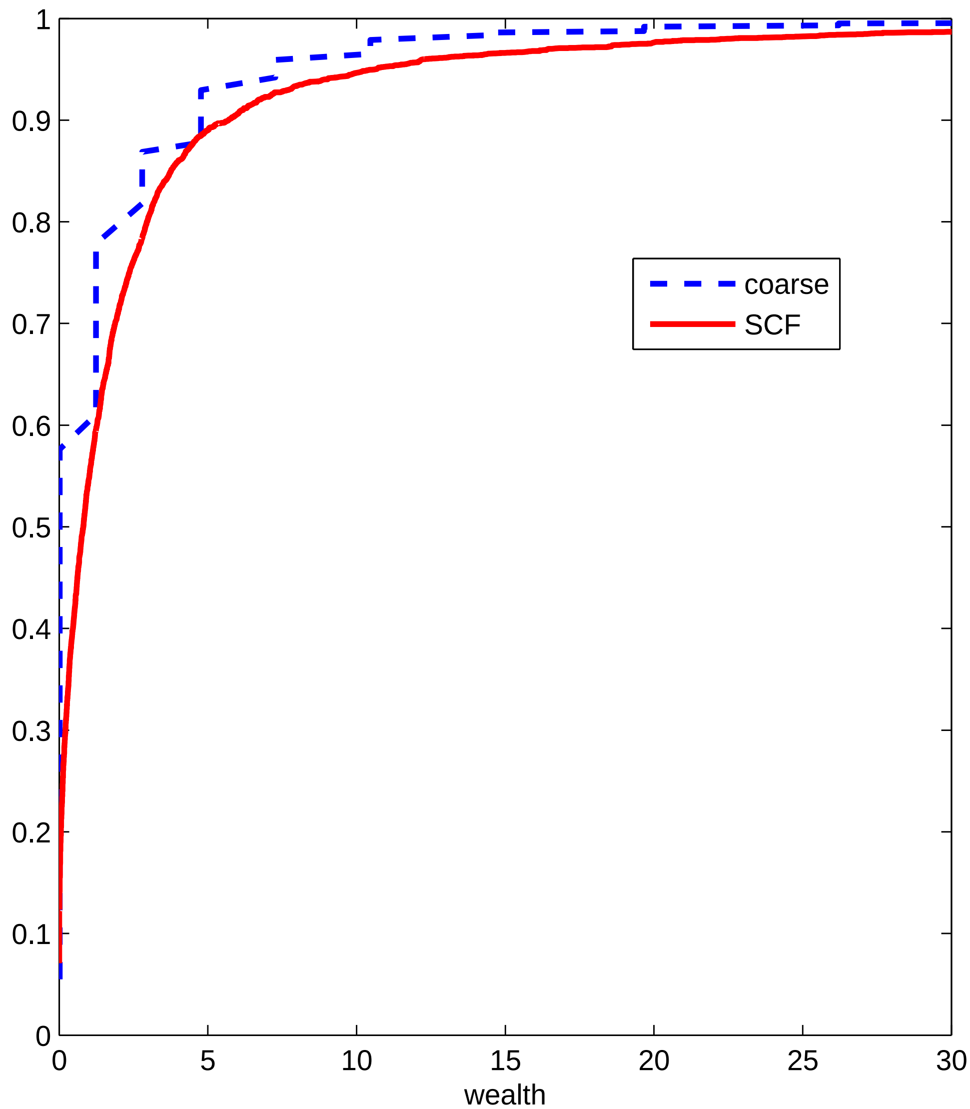

Figure 8: Marginal tax function in calibration

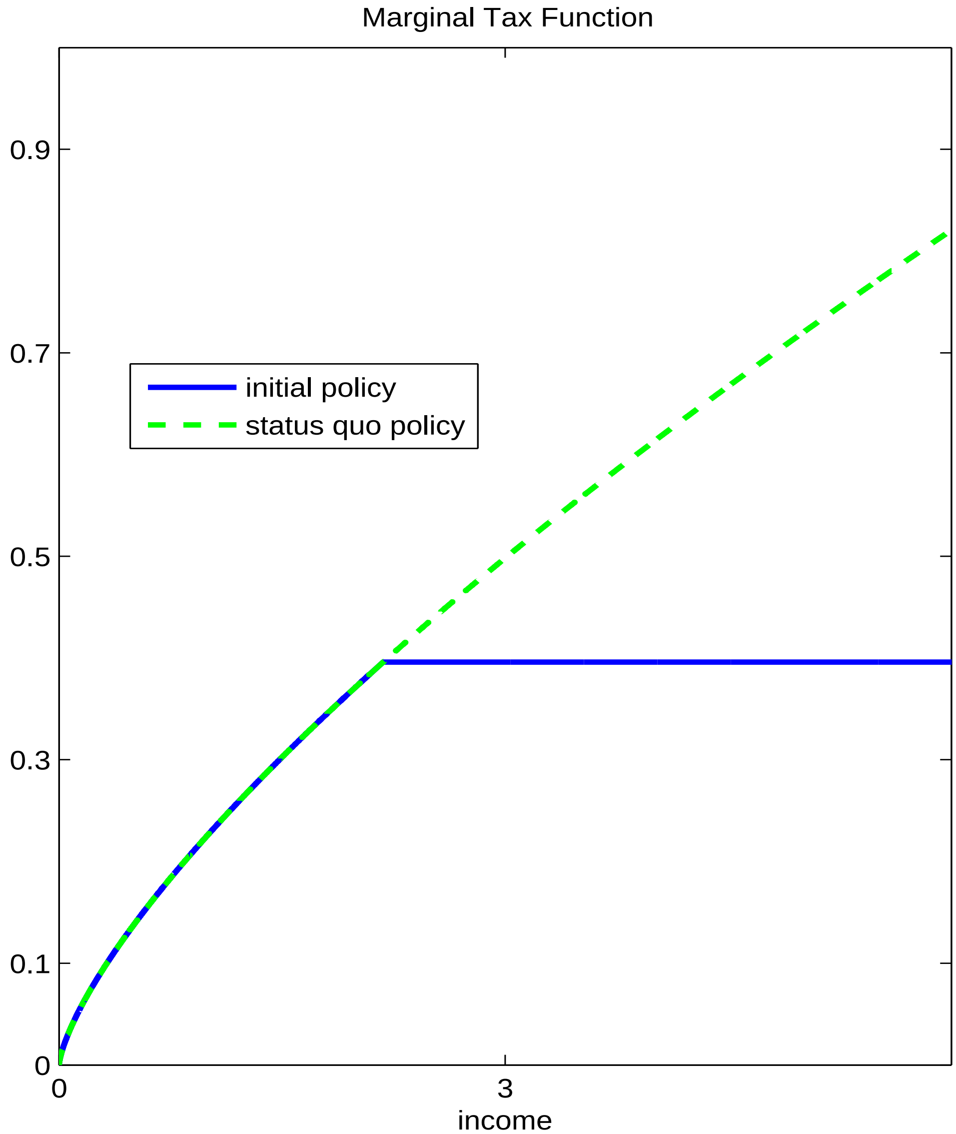

Figure 9: Transition path

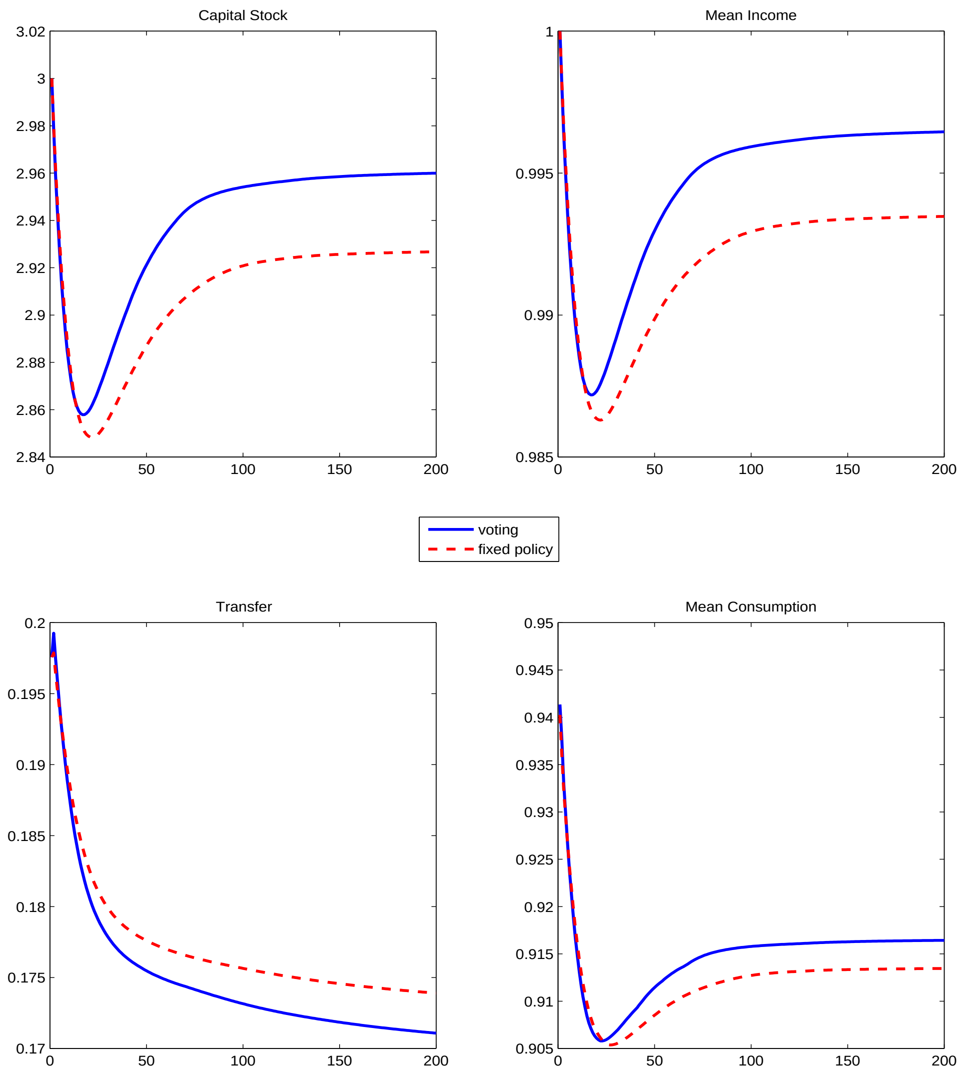

Figure 10: Wealth over transition for two household types

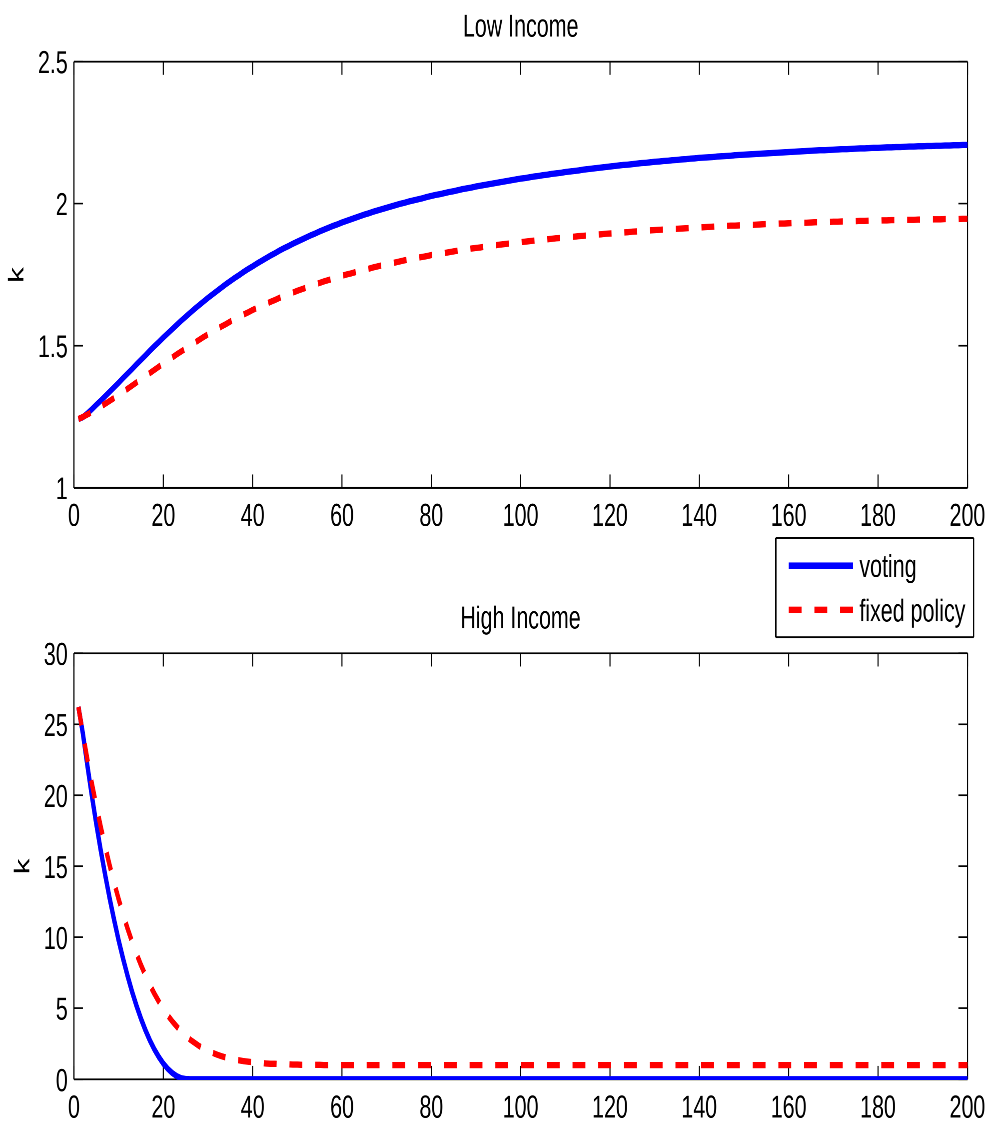

Figure 11: Preferred policy by income

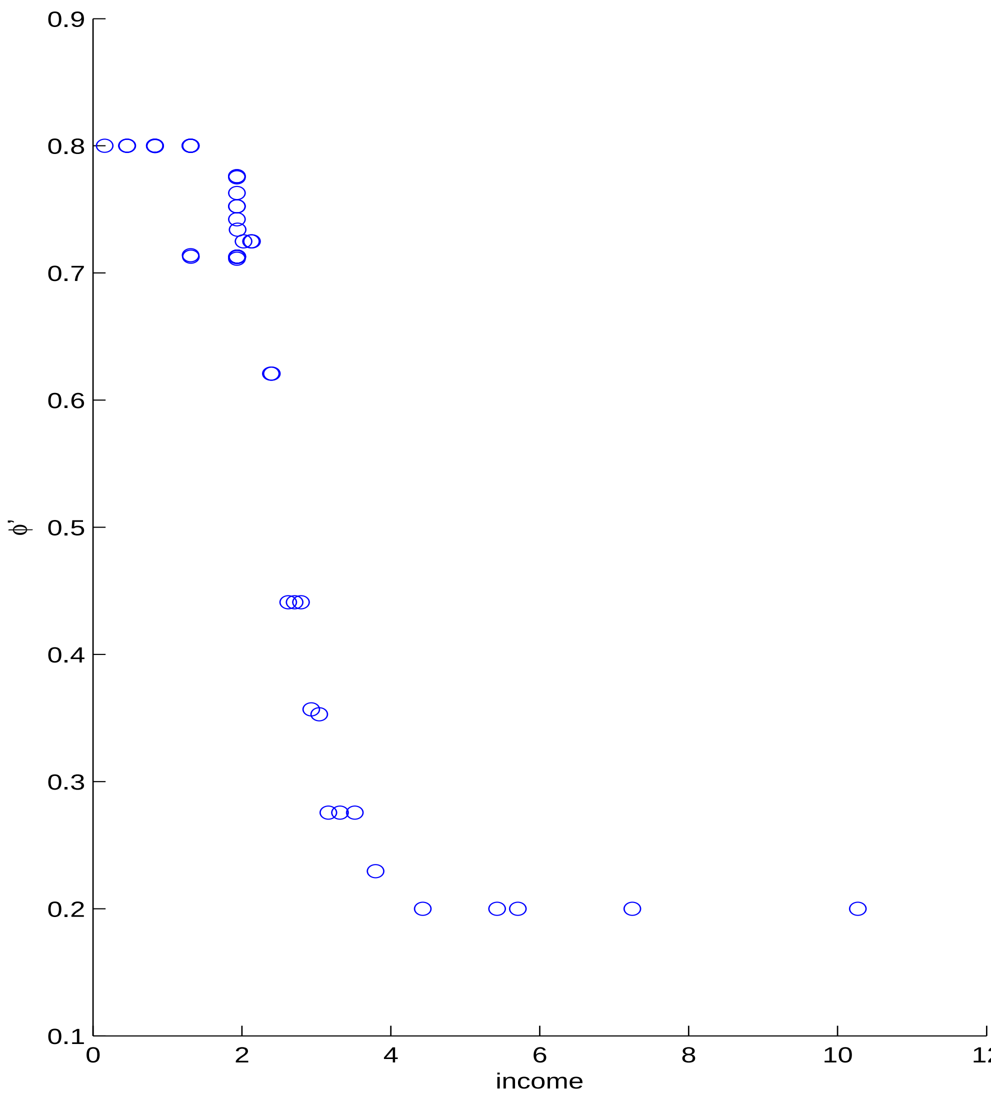

Figure 12: Preferred policy by labor productivity for several income levels

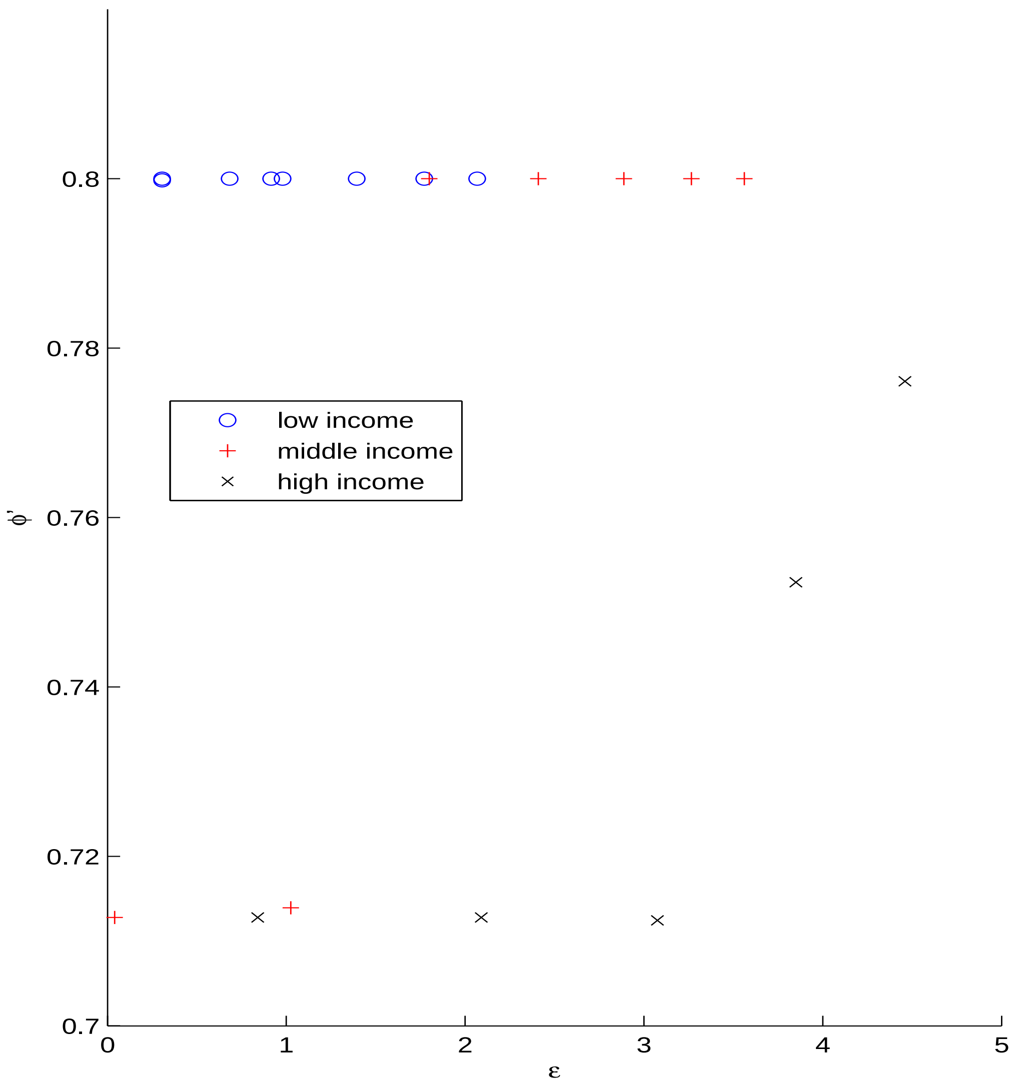

Figure 13: Preferred progressivity across wealth by ε

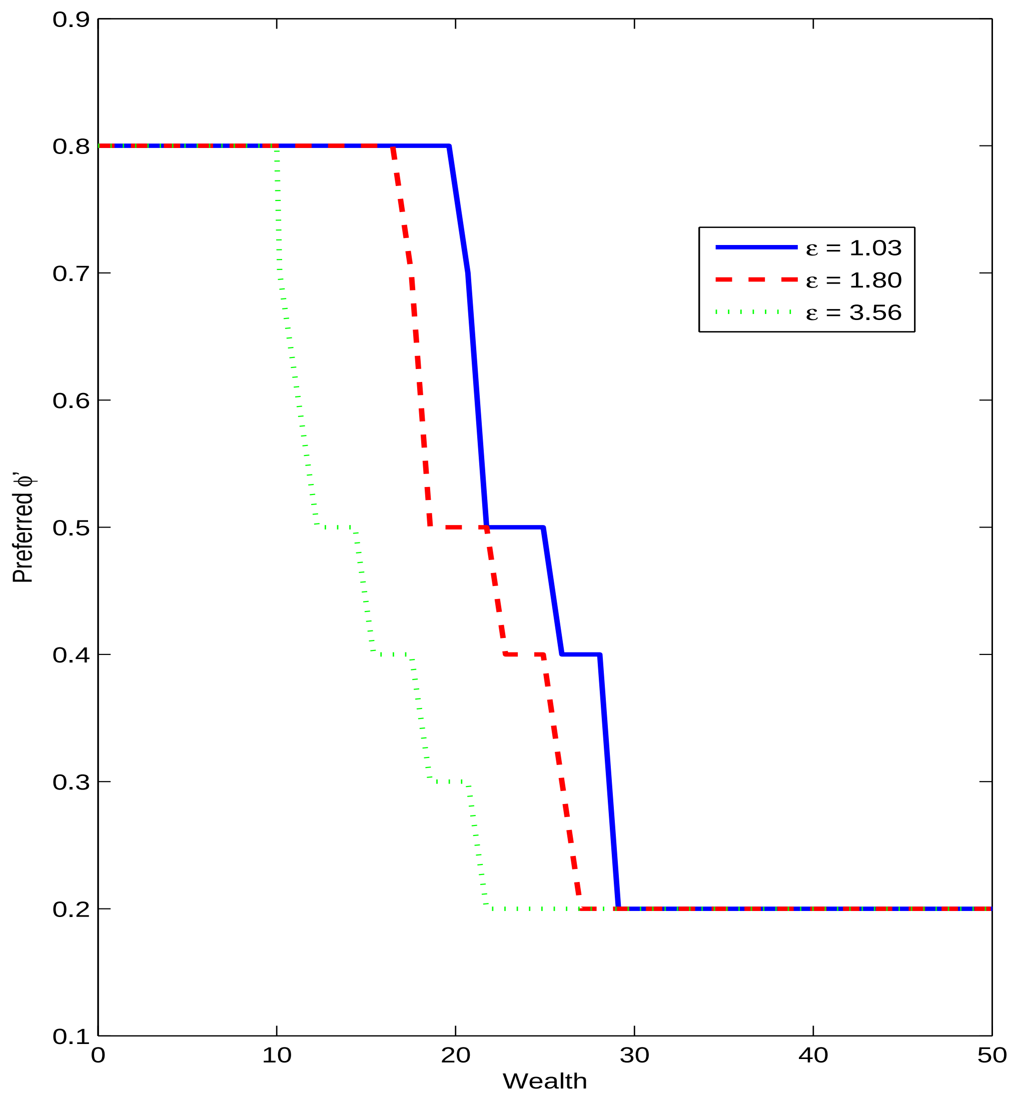

Figure 14: Absolute difference in net tax bill under preferred policy and best alternative policy

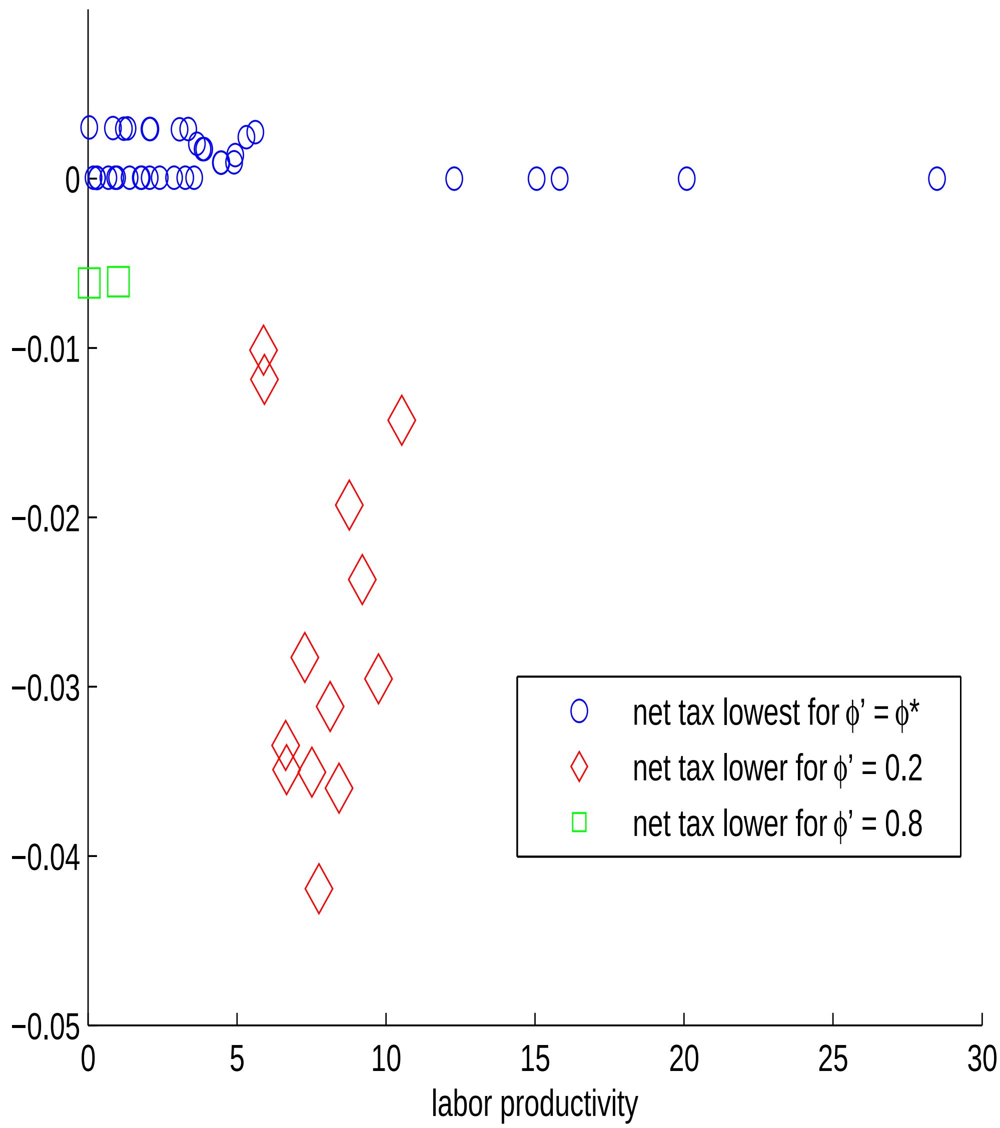

Figure 15: Transition paths of capital under flexible and fixed factor prices

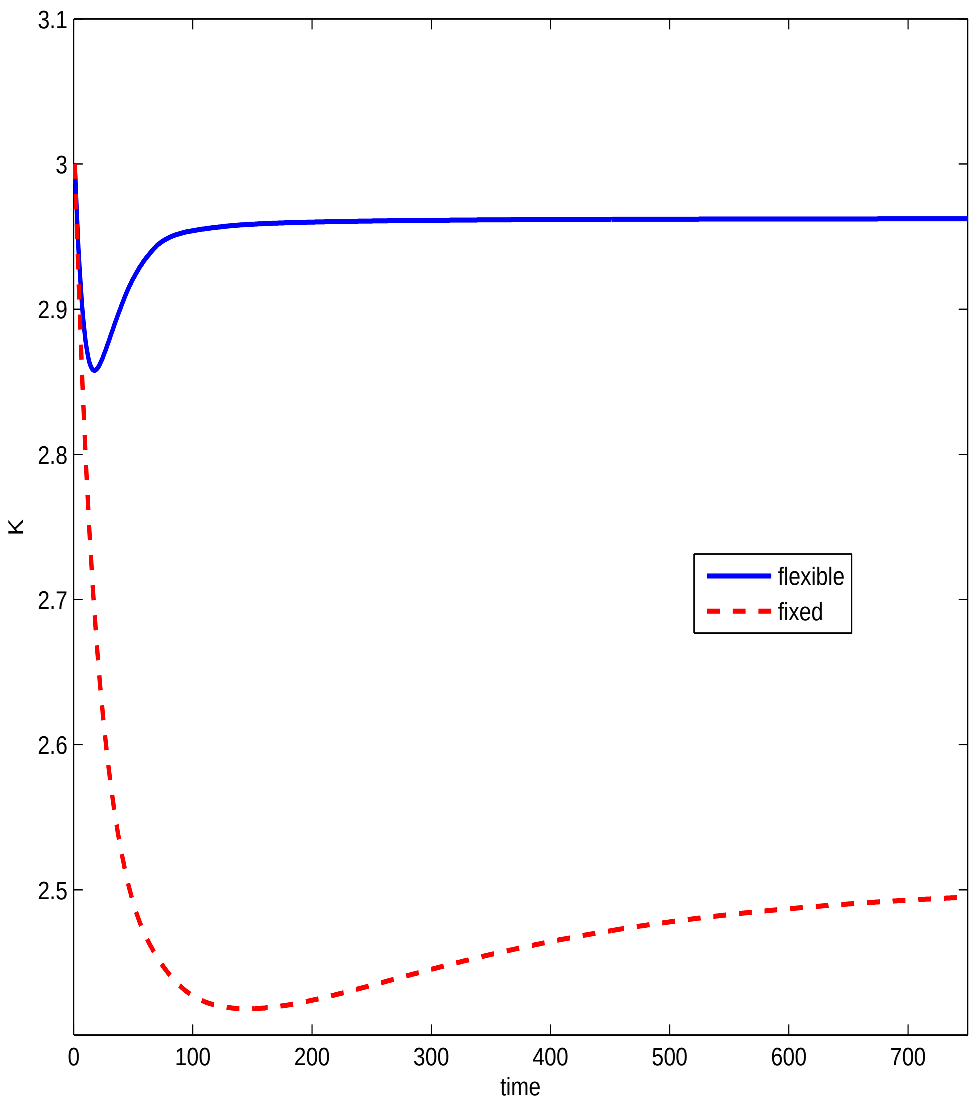

Table 1							
	$\frac{std(y)}{mean(y)}$	$\frac{std(k)}{mean(k)}$	Cov(y,k)	Skew(y)	Skew(k)	Gini(y)	Gini(k)
SCF	1.83	5.96	7.91	47.00	142.03	0.50	0.78
Coarse	0.91	1.97	3.85	4.11	9.52	0.40	0.68

SCF income gini from Weicher (1996). SCF wealth gini from Kennickell and Woodburn (1999).

Table 2										
Percentage changes across initial income distribution by quintile										
	k					y				
Floating r	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
PE SS	$5,410.1$	136.4	32.5	36.5	-36.5	35.6	18.8	8.6	5.8	-13.6
FP SS	$5,001.8$	113.4	20.4	35.6	-32.8	32.9	15.6	5.7	5.5	-12.2
Fixed r										
PE SS	$3,603.8$	43.1	13.1	-3.7	-78.4	31.8	10.5	4.5	-0.9	-34.1
FP SS	$2,242.0$	3.8	-14.0	-18.7	-77.8	19.8	0.9	-4.8	-4.5	-33.8
	c					$\tau(y) - T$				
Floating r	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
PE SS	22.9	14.2	5.2	4.4	-15.3	-4.2	-5.9	-5.4	33.7	3.6
FP SS	21.4	11.6	2.6	4.1	-13.5	-4.1	-5.9	-5.3	35.3	6.2
Fixed r										
PE SS	10.4	2.5	-0.5	-3.8	-46.9	-38.1	-53.8	-115.1	38.0	-34.9
FP SS	1.6	-6.1	-8.9	-7.5	-45.4	-42.0	-57.7	-107.7	38.3	-37.6

Table 3					
Share of Steady State Wealth by Quintile					
	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Initial	0.2	5.8	15.1	12.2	66.8
PE SS	8.6	13.6	19.5	16.4	41.9
FP SS	8.1	12.4	18.1	16.5	44.8

Table 4						
	$\frac{std_y}{mean_y}$	$skew_y$	$gini_y$	$\frac{std_k}{mean_k}$	$skew_k$	$gini_k$
Initial	0.91	4.1	0.399	1.97	9.5	0.680
PE SS	0.46	2.9	0.327	23.4	5.2	0.417
FP SS	0.48	2.8	0.335	25.8	5.1	0.433

Table 5		
	% Change	
	equil.	fixed ϕ'
ϕ'	12.0	0
K	-1.3	-2.4
Y	-0.3	-0.6
C	-2.6	-2.8
T	-14.2	-12.7
avg. tax	-14.2	-12.7
rent	0.8	1.6
wage	-0.5	-0.9

Table 6: Steady state with fixed prices

	Percentage of Baseline Value
K	84.5
Y	96.0
C	95.6
T	90.2

Table 7

Percentiles of Income Distribution

	5%	25%	50%	75%	95%	99%
SCF	0.09	0.29	0.59	1.09	2.70	8.00
Coarse	0.07	0.36	0.75	1.29	2.03	4.45

Table 8						
Percentiles of Wealth Distribution						
	5%	25%	50%	75%	95%	99%
SCF	-0.2	0.12	0.81	2.36	10.60	38.10
Coarse	0.02	0.02	0.02	1.24	7.27	19.66

Table 9							
	constant	K	ϕ	R	ϕ'	max % error	avg. % error
K'	0.00	0.948	-0.030	-0.029	0.037	1.2	0.004
	0.00	0.948	-0.030	-0.029	0.037		
ϕ'	-0.261	0.00	0.00	0.00	0.00	0.0	0.000
	-0.261	0.00	0.00	0.00	0.00		
R'	0.0	0.00	0.00	0.997	0.021	2.2	0.005
	0.0	0.0	0.0	0.998	0.016		