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# The Information Effect, Policy Rule Ambiguity, and the Decomposition of Monetary Policy Surprises*

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**Abstract**

I study a New Keynesian economy in which the private sector has imperfect information about both the underlying shock and the monetary policy rule. The information effect of monetary policy makes the two sources of imperfect information interact with each other. On the normative side, I show that the conventional certainty equivalence result regarding optimal monetary policy breaks down. The optimal policy rule becomes less responsive to the underlying shock, and depends on the degree of rule ambiguity. On the positive side, in a dynamic New Keynesian model calibrated to the US economy, I decompose monetary policy surprises into shock uncertainty, rule ambiguity, and the interaction effect. I find that the interaction effect accounts for more than 70 percent of the variance of monetary policy surprises.

*Keywords:* monetary policy; information frictions; central bank; inflation expectations
*JEL classification:* E52, E58, E31, D83, D84

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# 1 Introduction

High-frequency monetary policy surprises—which are changes in short-term interest rates in a narrow window around FOMC announcements—have become a commonly used measure of monetary policy shocks (Kuttner, 2001; Gürkaynak, Sack, and Swanson, 2005). However, a large body of recent work, argues that these surprises contain more than "pure" policy shocks. Two theories have emerged to explain the empirical patterns of high-frequency monetary policy surprises: the *"Fed information effect"* (Nakamura and Steinsson, 2018) and the *"Fed response to news"* channel (Bauer and Swanson, 2023a,b). For the *"Fed information effect"* to appear, the private sector has to have information frictions on the underlying shock to which the policy rate responds. It therefore uses the policy rate to infer information about the realization of the underlying shock. For the *"Fed response to news"* channel to work, the private sector has to have information frictions on the reaction function of the policy rate. Even if the private sector and the central bank observe the same public information on the underlying economic fundamentals, the private sector cannot perfectly predict the interest rate due to its underestimation of the policy reaction function.

These two theories are usually interpreted as being in competition with each other. In this paper, I argue that these two sources of information frictions presented in the theories interact with each other, and that their interaction has first-order implications both for optimal monetary policy and for the interpretation of high-frequency monetary policy surprises. Intuitively, for the private sector to learn from the policy rate about the central bank's private information of the underlying shock, the private sector must first understand how the interest rate responds to economic shocks. If the monetary policy rule is ambiguous to the private sector, then a higher degree of policy rule ambiguity means that less information can be learned from interest rate decisions.

I formalize this idea in a New Keynesian model with two sources of imperfect information. I assume that the cost-push shock is the only underlying fundamental shock and the central bank observes it perfectly. Agents in the private sector, including households and firms, have imperfect information about it, and use priors and public signals to update expectations in each period. The private sector knows the functional form of the central bank's interest-rate rule but does not exactly know the coefficient on the public signal. The private-sector's perceived coefficient differs from the true one by a random error. I refer to this information friction as *policy rule ambiguity*,[1] and interpret it as how well the private sector understands

[1]The term echoes Cukierman and Meltzer (1986), in which noise in monetary control makes it harder for the public to infer the policymaker's objectives from its actions. It should not be confused with ambiguity in the decision-theoretic sense: the private sector in this paper is a Bayesian who knows the distribution of the error in its perceived rule coefficient but not its realization.

the monetary policy reaction function.

Under rational expectations, the private sector uses the interest rate to update expectations about the cost-push shock. To do so, it inverts the interest rate to an unbiased signal of the cost-push shock with a noise component. The noise component in this interest rate-implied signal becomes the product of the error in the perceived rule coefficient and the noise in the public signal. I refer to this product structure as the *interaction effect*.

To analytically study the normative implications of this interaction effect, I start with a static version of the model, where I assume that the cost-push shock is i.i.d., and that monetary policy cannot be path-dependent. I compare this baseline model with two benchmarks, one having perfect information and the other having no information effect. In the perfect-information benchmark case, the private sector knows both the cost-push shock and the rule coefficients perfectly. In the no-information-effect benchmark case, the private sector has imperfect information about both the shock and the rule coefficient. It forms expectations about the cost-push shock using only its prior and the noisy public signal, not the interest rate. In this case, the private-sector's expectations are exogenous and monetary policy affects the economy only through its direct effect.

Comparing the two benchmark cases, I show that the certainty equivalence principle regarding optimal monetary policy applies. That is, the optimal monetary policy rule under the perfect-information benchmark and the no-information-effect benchmark are the same, except that in the latter case the actual cost-push shock is replaced by its expected value. The optimal policy rule does not depend on the degree of rule ambiguity. This echoes the classic result of Svensson and Woodford (2003) for optimal policy under partial information.[2] The key assumption for this certainty equivalence result to hold is that rule ambiguity enters the central bank's loss additively and independently of the policy instrument.

Turning to the case in which monetary policy has an information effect, the first main result is that the information effect overturns certainty equivalence. Once the private sector learns from the interest rate, the precision of the interest rate as a signal depends on the central bank's own response to the shock. A stronger response makes the interest rate more informative about the actual cost-push shock, and at the same time, it also increases the weight the private sector places on the interest rate and therefore amplifies the pass-through of rule-ambiguity noise into the private-sector's expectations. Due to this learning mechanism, rule ambiguity enters the central bank's loss function non-additively. The certainty equivalence principle breaks down and the optimal policy rule depends on the degree of rule ambiguity. This mechanism is related to Swanson (2004), in which certainty equivalence

[2]Certainty equivalence was first established by Simon (1956) and Theil (1957) for linear-quadratic control problems with additive uncertainty.

fails because the central bank's policy interacts with its own signal-extraction problem; here, it fails because the central bank's policy decision interacts with the private sector's signal-extraction problem.

The second main result is that optimal policy responds less aggressively to the fundamental shock. Internalizing that a strong response to the cost-push shock would amplify the fluctuations in private-sector beliefs to either the cost-push shock or to the noise shock, the central bank restrains its response to the cost-push shock. When extending the model to a dynamic setting, with path-dependent monetary policy, this attenuation becomes even stronger. In response to a positive cost-push shock, instead of loading all policy tightening in the first period, the central bank responds with a delay and continues to tighten monetary policy in the subsequent periods. A delayed response is optimal in this case because it lowers expected future inflation and reduces the stabilization bias in the current period (Woodford, 1999; Clarida, Galí, and Gertler, 1999). With the information effect, this inertial response has an additional benefit: it allows the central bank to stabilize the shock while revealing little about it.

The third result links optimal policy to the decomposition of monetary policy surprises. The monetary policy surprise is calculated as the central bank's response to the part of the shock unanticipated by the private sector, minus the error in the perceived monetary policy conditional on the noisy public signal. Its variance decomposes into three components: *shock uncertainty*, which scales with the square of the central bank's response to the shock; *pure rule ambiguity*, which would remain even if the public signal was perfectly precise; and the *interaction effect*, which arises only because both frictions are present. An attenuated policy response to the underlying shock lowers the share of shock uncertainty and raises the share of the rule ambiguity and the interaction effect.

To quantify the decomposition of monetary policy surprises, I extend the model to a dynamic setting and calibrate the degree of information frictions to the US economy. The key set of parameters that govern the degree of information frictions includes: the volatility of cost-push innovations, the degree of information frictions about the cost-push shock, and the degree of rule ambiguity. I simultaneously calibrate these three parameters to match three moments of US data: the volatility of headline consumer price index (CPI) inflation, the variance of CPI inflation surprises, and the volatility of high-frequency monetary policy surprises around scheduled FOMC announcements.[3]

[3]The monetary policy surprises are the series of Bauer and Swanson (2023b), and are updated by the Federal Reserve Bank of San Francisco. The CPI surprise is the difference between the actual month-over-month headline CPI inflation rate and the market consensus estimate a few days before the data release. The market consensus estimate is obtained from Money Market Services. (Source: Informa Global Markets, accessed from Haver Analytics)

Under the calibration, the interaction effect accounts for 70.7 percent of the variance in monetary policy surprises, for 22.2 percent of pure rule ambiguity, and for only 7.1 percent of shock uncertainty. The small share of shock uncertainty does not mean that the private sector is well informed about the shock, or that the information effect is unimportant. Instead, the public signal is estimated to be very noisy, as implied by the large standard deviation of headline CPI surprises relative to the standard deviation of actual CPI month-over-month changes. Since the central bank internalizes the information effect of its policy decisions, it optimally chooses an inertial policy response.

Sensitivity analysis shows that the interaction share is governed by the noise fraction of the public signal, $\sigma_s^2/(\sigma_u^2+\sigma_s^2)$, and is nearly invariant to the degree of rule ambiguity itself: rule ambiguity determines the *scale* of monetary policy surprises, while the degree of information frictions about the underlying shock determines their *composition*. The interaction share remains above one half unless the public signal becomes considerably more precise relative to the volatility of the underlying shock.

The rest of the paper is organized as follows. After a review of the related literature, Section 2 presents the static model and the two benchmarks. Section 3 introduces the information effect and explains how the information effect leads to the interaction between the two sources of information frictions. It also characterizes optimal policy under the interaction effect and the decomposition of monetary policy surprises. Section 4 extends the model to a dynamic setting, and Section 5 calibrates the dynamic model and quantifies the decomposition. Section 6 concludes the paper.

## Related Literature

This paper contributes to two strands of the literature. It first contributes to the literature on the interpretations of high-frequency monetary policy surprises. The view of the "Fed information effect" argues that monetary policy surprises reflect the information gap between the central bank and the private sector regarding underlying economic fundamentals. Nakamura and Steinsson (2010) show evidence supporting this view: professional forecasters positively revise their output forecasts after a positive monetary policy surprise, contrary to the predictions of a conventional New Keynesian model. Additional supportive evidence is given by Campbell et al. (2012) and Cieslak and Schrimpf (2019). The empirical findings of the "Fed information" also motivate the decompositions of monetary policy surprises to investigate the quantitative importance of the "Fed information effect" (Jarociński and Karadi (2020) and Miranda-Agrippino and Ricco (2021), among others). Hoesch, Rossi, and Sekhposyan (2023), however, find that the information channel has weakened in more recent

periods.

The other prominent theory is the "Fed response to news" channel, proposed by Bauer and Swanson (2023a,b), who argue that high-frequency monetary policy surprises reflect that the private sector underestimates the actual responsiveness of monetary policy to publicly observable economic news. Consistent with this view, the policy rule perceived by market participants varies considerably over time and can be revised in response to monetary policy actions (Hamilton, Pruitt, and Borger, 2011; Bauer, Pflueger, and Sunderam, 2024). My paper nests both interpretations in a single model and shows that they are not separable. Rather, their interaction has important normative and positive implications.

My paper is most closely related to Ricco and Savini (2025), who decompose monetary policy surprises into three components: an exogenous monetary policy shock, a shock to the policy rule parameters, and the central bank's private information about the underlying shock. While both Ricco and Savini (2025) and my paper share the same goal of decomposing monetary policy surprises, Ricco and Savini (2025) take an empirical approach and provide a time-series answer: for each individual FOMC meeting they estimate how much of the monetary policy surprise came from each component using data on the yield curve. Different from their approach, the decomposition in this paper is structural and grounded in a New Keynesian model with optimal monetary policy.[4] This structural approach allows for the interaction between the two sources of information frictions, which is shown to account for most of the variance in high-frequency monetary policy surprises.

The second strand of the literature is modeling the central bank's information effect in a New Keynesian model. Ellingsen and Söderström (2001), Baeriswyl and Cornand (2010), Berkelmans (2011), and Melosi (2017) model the information effect (or signaling effect) of monetary policy when the private sector includes the interest rate in its information set to update expectations about the underlying state of the economy. Tang (2015) and Jia (2023) characterize optimal policy under the information effect of monetary policy. Han and Jia (2025) extend the analysis to a two-way learning framework, in which the central bank learns from inflation and output and the private sector learns from the interest rate. My paper also relates to the work on central bank transparency and communication Cukierman and Meltzer (1986); Faust and Svensson (2001); Blinder et al. (2008); Eusepi and Preston (2010)). My results point to a complementary role for communication: the clarity of the reaction function determines the effect of the interest rate, and thereby shapes the optimal policy rule.

[4]Another difference is that in Ricco and Savini (2025), "shock to the rule" captures a real change in the policy rule parameter that is empirically measured, whereas "rule ambiguity" in this paper models the difference between the private-sector's expected and the actual policy rule parameter.

# 2 The Static Model and the Two Benchmarks

In this section, I first describe a dynamic New Keynesian model in which the private sector is forward-looking but its expectations are subject to imperfect information. I then shut down the forward-looking components and turn the model to a static case to analytically study the optimal policy under different information sets.

## 2.1 Private-Sector Equilibrium

Building on Tang (2015), Jia (2023), and Han and Jia (2025). the equilibrium of aggregate output and inflation can be described by the IS curve and the Phillips curve, given by[5]

$$y_t = \mathbb{E}_t^p y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t^p \pi_{t+1}\right) \tag{1}$$

$$\pi_t = \beta\theta\, \mathbb{E}_t^p \pi_{t+1} + (1-\theta)\, \mathbb{E}_t^p \pi_t + \kappa\theta y_t + u_t \tag{2}$$

where $\sigma$ is the inter-termporal elasticity of substitution, $\beta$ is the discount factor on future consumption relative to today, $\theta$ is the Calvo parameter (the probability that the current price remains effective in the next period), and $\kappa$ is the slope of the Phillips curve.[6]

The cost-push shock, denoted by $u_t$, is the only fundamental shock in the private sector. A common assumption in the literature on information frictions is that cost-push shocks are firm specific, and are a composite of an aggregate component and an idiosyncratic component. This assumption would imply that each firm's own cost-push shock serves as a private signal of the aggregate cost-push shock, and thereby individual firms' expectations on the aggregate economy are heterogeneous, leading to the higher-order-beliefs problem. The focus of this paper is the information asymmetry between the central bank and the private sector, and I therefore abstract from the higher-order-beliefs problem by assuming that all agents in the private sector have homogeneous expectations, denoted by $\mathbb{E}_t^p$.

The IS curve in Equation (1) implies that current demand increases with expected future output and expected future inflation. The household is assumed to have perfect information on the interest rate, and the interest rate has a direct effect on output.

The Phillips curve in Equation (2) shows that current inflation rises with expected future inflation, as firms are forward-looking under a Calvo pricing strategy. It also shows that prices

[5] See Appendix A for derivations of the IS and the Phillips curves from the private-sector's optimization problems under imperfect information.

[6] Letters in lower cases represent log deviations from their steady-state values (i.e., $x_t \equiv ln(X_t/X)$. Equation (1) is derived from the inter-temporal consumption decision of the household, combined with the market clearing condition. Equation (2) is derived from the optimal pricing decision of individual firms, combined with individual prices aggregated to inflation, under the assumption of homogeneous expectations. See Jia (2023) for details.

react to the current output level, as the current output level is reflected in the equilibrium wage, which individual firms can observe perfectly. I assume firms observe their individual marginal costs perfectly, and adjust prices optimally in response to changes in marginal costs, conditional on getting the Calvo lottery to reset prices. Therefore the actual cost-push shock affects inflation. However, I assume that firms do not use their firm-specific marginal costs as a signal to infer the changes in the aggregate price level. Each firm relies on public information to form $\mathbb{E}_t^p u_t$.[7]

**The Static Case** I now impose two assumptions that turn the model into a static case. First, I assume that the auto-correlation in the cost-push shock is zero. Second, I assume that the central bank cannot commit across periods. In other words, it optimizes its interest-rate decisions period-by-period. In this case, expected future inflation and output stay at the zero steady-state level, so output is only determined by the interest rate, given by $y_t = -\frac{1}{\sigma}$. Inflation becomes $\pi_t = (1-\theta)\,\mathbb{E}_t^p \pi_t + \kappa\theta y_t + u_t$. To express expected inflation as the expected cost-push shock, apply $\mathbb{E}_t^p$ to both sides of Equation (2) and get

$$\mathbb{E}_t^p \pi_t = (1-\theta)\,\mathbb{E}_t^p \pi_t + \kappa\theta\,\mathbb{E}_t^p y_t + \mathbb{E}_t^p u_t = \kappa y_t + \frac{1}{\theta}\,\mathbb{E}_t^p u_t \tag{3}$$

where the second equation makes the use of $\mathbb{E}_t^p y_t = y_t$ since the interest rate is observable. Substituting $\mathbb{E}_t^p \pi_t$ in Equation (2) with $\mathbb{E}_t^p u_t$ and re-arranging, the static case can be characterized by

$$y_t = -\frac{1}{\sigma}\, i_t \tag{4}$$

$$\pi_t = \kappa\, \hat{y}_t + \frac{1-\theta}{\theta}\, \mathbb{E}_t^p u_t + u_t \tag{5}$$

## 2.2 Imperfect Information and Monetary Policy Rule Ambiguity

I study two sources of imperfect information: imperfect information about the underlying shock and imperfect information about the policy rule.

**Imperfect Information about the Underlying Shock** I assume that the private sector does not observe $u_t$ directly. In each period, the private sector makes Bayesian updates about $u_t$ using a prior belief and a noisy public signal, given by

[7] This assumption allows both the actual shock and the expected shock to play a role in inflation. This assumption can be thought of as equivalent to the assumption that firms use all available signals, including their firm-specific shocks, but the idiosyncratic components of their firm-specific shocks (in logs) have infinite variance. Or, equivalently, aggregate components of the shocks are arbitrarily small relative to the idiosyncratic components.

- the prior, $u_t^0 \neq 0$ with precision $\tau_\mu = 1/\sigma_\mu^2$.
- the noisy signal, $s_t = u_t + \varepsilon_t^s$ with precision $\tau_s = 1/\sigma_s^2$.

**The Monetary Policy Rule** The central bank is assumed to have perfect information. It observes the actual cost-push shock ($u_t$), as well as the prior ($u_t^0$) and the signal ($s_t$) in the private sector.

In this static case, the central bank's loss function reduces to the loss in the current period, i.e.,

$$\mathcal{L} = \pi_t^2 + \lambda \hat{y}_t^2, \tag{6}$$

subject to the Phillips curve (Equation (5). Given the Bayesian updating process in the private sector, I conjecture (and prove later) that the optimal interest rate takes the following form:

$$i_t = F_u u_t + F_p u_t^0 + F_s s_t \tag{7}$$

**The Policy Rule Ambiguity** I assume that the private sector knows the functional form of the interest rate as specified in Equation (7), but has imperfect information about the values of the parameters. Specifically, I assume that the private sector knows the coefficients $F_u$ and $F_p$ exactly, but not $F_s$. I use the phrase "ambiguity" to refer to the imperfect information about $F_s$ to distinguish it from the fact that the private sector updates beliefs about the cost-push shock.

Instead, the private sector perceives the coefficient on $s_t$ to be $\hat{F}_s(t) \neq F_s$, and does not update its perceived policy rule. I assume that the difference between the perceived and the actual coefficient is a random noise, $\zeta_t$

$$\hat{F}_s(t) = F_s + \zeta_s(t), \qquad \zeta_s(t) \sim (0, \sigma_{F_s}^2), \quad \zeta_s(t) \perp (u_t, \varepsilon_t^s) \tag{8}$$

$\sigma_{F_s}^2$ measures the degree of policy rule ambiguity. The private sector knows the degree of policy rule ambiguity, $\sigma_{F_s}^2$, but does not know the realization of $\zeta_s$. $\sigma_{F_s} > 0$ can be regarded as a measure of the precision of central bank communication, or the degree to which the private sector understands the policy decision process of the central bank. Improving communication can be modeled as reducing $\sigma_{F_s}^2$, but in this paper, $\sigma_{F_s}^2$ is exogenous and constant over time.

## 2.3 The Perfect Information Benchmark

The *perfect-information (PI)* benchmark is the case when the private sector has perfect information about both the realization of the cost-push shock and the policy rule. In this

case, prior beliefs or public signals do not play a role in determining equilibrium inflation and output. The central bank's optimization problem leads to the standard first-order condition:

$$\kappa\pi_t + \lambda\hat{y}_t = 0. \tag{9}$$

Substituting $\hat{y}_t$ with $i_t$ using the IS curve, the optimal interest rate under perfect information is given by

$$i_t^{PI} = \frac{\sigma\kappa}{(\kappa^2+\lambda)\,\theta}\,u_t \tag{10}$$

Writing the equilibrium interest rate in the general form of Equation (7), the coefficients of the optimal interest rate rule under perfect information are given by

$$F_u^{PI} = \frac{\sigma\kappa}{(\kappa^2+\lambda)\,\theta} \qquad F_p^{PI} = 0 \qquad F_s^{PI} = 0 \tag{11}$$

The central bank optimally responds to $u_t$ only, not to noise or priors.

## 2.4 The No-Information-Effect Benchmark

When the private sector does not use the interest rate as a signal to infer information about $u_t$, the degree of information frictions about $u_t$ and the degree of policy ambiguity ($\sigma_{F_s}$) are independent of each other. I now study this *no-information-effect (NI)* case as the other benchmark.

The expected cost-push shock is formed as the private sector weighs the prior ($u_t^0$) and the noisy signal ($s_t$) by their precision:

$$E_t^{p,NI}[u_t] = \alpha_0^{NI} u_t^0 + \alpha_s^{NI} s_t \tag{12}$$

where $\alpha_0^0 = \tau_\mu/(\tau_\mu+\tau_s)$ and $\alpha_s^0 = \tau_s/(\tau_\mu+\tau_s)$.

In this case, the IS curve is the same as in Equation (4). The interest rate has a direct effect on output and affects inflation only through its effect on output. The Phillips curve can be written as

$$\pi_t = -\frac{\kappa}{\sigma} i_t + \frac{1-\theta}{\theta}\,\mathbb{E}_t^p\, u_t + u_t \equiv -\frac{\kappa}{\sigma} i_t + \Delta_t \tag{13}$$

where $\Delta_t \equiv \frac{1-\theta}{\theta}\,\mathbb{E}_t^p\, u_t + u_t = \Gamma\,\mathbb{E}_t^p\, u_t + u_t$ collects all terms in the Phillips curve that are exogenous to $i_t$, including both the expected shock $\Gamma\,\mathbb{E}_t^p\, u_t$ and the actual shock, $u_t$.

Substituting the Phillips curve with the first-order-condition of the interest rate yields

$$i_t = \frac{\kappa\sigma}{\kappa^2+\lambda}\,\Delta_t \tag{14}$$

Substituting the expectation term with the prior and the noisy signal, the optimal interest rate rule is $i_t = F_u^{NI} u_t + F_p^{NI} u_t^0 + F_s^{NI} s_t$, where the coefficients are

$$F_u^{NI} = \frac{\kappa\sigma}{\kappa^2+\lambda}, \qquad F_s^{NI} = \frac{\kappa\sigma\Gamma\alpha_s^0}{\kappa^2+\lambda}, \qquad F_p^{NI} = \frac{\kappa\sigma\Gamma\alpha_0^0}{\kappa^2+\lambda} \tag{15}$$

**Lemma 1.** *Certainty equivalence regarding optimal monetary policy holds when policy rule ambiguity does not interact with imperfect information about the underlying shock. Specifically, the optimal monetary policy depends only on the actual and the expected cost-push shock, not $\sigma_s^2$ or $\sigma_{Fs}^2$.*

*Proof:* This result is a direct implication from the comparison between the optimal interest rate in the perfect information case (Equation (10)) and in the no-information-effect case (Equation (14)). $\Delta_t$ is the effect of the cost-push shock on inflation under imperfect information and is exogenous to the central bank. In addition, the coefficients of the optimal interest rate in the no-interaction benchmark (Equation (15)) also show that the optimal monetary policy rule does not depend on $\sigma_{Fs}$. The difference between the perceived and the actual policy rule does not affect the optimal monetary policy.

# 3 The Information Effect of Monetary Policy

The two sources of information frictions can potentially interact with each other if the private sector uses the interest rate as a signal and infers information about $u_t$. In this section, I first show that this information effect leads to an interaction between the degree of information frictions about the underlying shock and the degree of policy rule ambiguity (Section 3.1). I then solve for the optimal policy in Section 3.2 and analyze how the optimal policy rule affects the size of the interaction effect in Section 3.3.

## 3.1 The Interaction between Two Sources of Information Frictions

When the private sector extracts information about $u_t$ from $i_t$, it first subtracts the part of $i_t$ that is a response to $u_t^0$ and $s_t$, and by doing so it converts $i_t$ to an unbiased signal about $u_t$. Specifically,

$$\tilde{i}_t \equiv \frac{i_t - F_p u_t^0 - \hat{F}_s s_t}{F_u} = \frac{(F_u u_t + F_p u_t^0 + F_s s_t) - F_p u_t^0 - (F_s+\zeta_s) s_t}{F_u} = u_t - \frac{\zeta_s s_t}{F_u} \tag{16}$$

Due to policy rule ambiguity, i.e., $\zeta_s(t)$, the private sector cannot perfectly back out the central bank's private information on $u_t$. The constructed signal $\tilde{i}_t$ would perfectly reveal $u_t$

if there is no ambiguity of $F_s$ ($\zeta_s(t) = 0, \forall t$).

The last term in Equation (16) is the noise component of the interest rate, and it is a composite of the error in the perceived policy rule, $\zeta_s(t)$, and the public signal, $s_t$. We further define the composite noise and its variance under linear approximation:

$$\xi_t \equiv \zeta_s(t) s_t = \zeta_s(t)(u_t + \varepsilon_t^s), \qquad \text{Var}(\xi_t) = \sigma_{F_s}^2(\sigma_u^2 + \sigma_s^2) \equiv D_{F_s} \tag{17}$$

The precision of the interest rate as a signal is:

$$\tau_i \equiv \frac{F_u^2}{D_{F_s}} = \frac{F_u^2}{\sigma_{F_s}^2(\sigma_u^2 + \sigma_s^2)} \tag{18}$$

Equation (18) shows that $\bar{\tau}_i$ decreases in both $\sigma_{F_s}$ and $\sigma_s$: both a higher level of rule ambiguity and a less precise signal of the fundamental shock make the interest rate a noisier signal. A larger $F_u$ (larger response to the cost-push shock) makes the interest rate a more precise signal.

By Bayes' rule, the private sector weighs the prior, the public signal, and the interest rate by their precision to update expectations about the cost-push shock. The posterior expectation is given by

$$\mathbb{E}_t^p u_t = \alpha_0 u_t^0 + \alpha_s s_t + \alpha_i \tilde{i}_t = \alpha_0 u_t^0 + (\alpha_s + \alpha_i) u_t + \alpha_s \varepsilon_t^s - \frac{\alpha_i}{F_u} \xi_t \tag{19}$$

where

$$\alpha_0 \equiv \frac{\tau_u}{\bar{\tau}}, \quad \alpha_s \equiv \frac{\tau_s}{\bar{\tau}}, \quad \alpha_i \equiv \frac{\bar{\tau}_i}{\bar{\tau}}, \qquad \bar{\tau} \equiv \tau_u + \tau_s + \bar{\tau}_i$$

The last term in Equation (19), $\frac{\alpha_i}{F_u}\xi_t$ captures the **interaction** between the two sources of information frictions. Suppose $u_t > 0$ and both the prior and the noise in the public signal are zero. In this case, $i_t = (F_u + F_s)u_t$. If $\zeta_s(t) > 0$, the private sector mistakenly believes the interest rate is more sensitive to $s_t$ than the actual interest rate is, and therefore infers a smaller $u_t$ from $i_t$.

Notice that $\xi_t = \zeta_s s_t$, and thus a non-zero $\xi_t$ requires not only rule ambiguity ($\zeta_s \neq 0$) but also a non-zero $s_t$. If, on the contrary, the private sector receives $s_t = 0$, then the private sector understands that the change in the interest rate is entirely attributed to the known response, $F_u u_t + F_p u_t^0$. In this case, the private sector can get perfect information about $u_t$ from the interest rate, and the rule ambiguity does not play a role.

Equation(18) suggests that the value of $F_u$ affects the precision of the information effect, which I explain in the following lemma.

**Lemma 2.** *A higher $F_u$ makes the interest rate a more precise signal of the underlying cost-*

*push shock and, simultaneously, makes the private-sector's inference more sensitive to the noise from policy rule ambiguity.*

*Intuition:* The central bank can make the interest rate a stronger signal of the cost-push shock - increasing the sensitivity of $\mathbb{E}_t^p u_t$ to $u_t$ - by raising $F_u$. With a larger $F_u$, a larger fraction of the change in the interest rate is due to a response to the cost-push shock, making $\hat{i}_t$ more informative about the actual shock. The private sector places more weight on $\hat{i}_t$, and less on the prior and the noisy signal. However, as the private sector places more weigh on $\hat{i}_t$, the effect of the combined noise ($\xi_t = \zeta_s(t)s_t$) on $\mathbb{E}_t^p u_t$ is simultaneously amplified.

*Proof:* From Equation (19) it is easy to see that the sensitivity of $\mathbb{E}_t^p u_t$ to $u_t$ is $S(F_u) = \alpha_s + \alpha_i$, and the sensitivity of $\mathbb{E}_t^p u_t$ to $\xi_t$ is $N(F_u) = \frac{\alpha_i}{F_u}$. Both $S(F_u)$ and $N(F_u)$ are strictly increasing in $F_u$:

$$\frac{\partial \alpha}{\partial F_u} = \frac{2\tau_u \bar{\tau}_i}{\bar{\tau}^2 F_u} > 0, \qquad \frac{\partial(\alpha_i/F_u)}{\partial F_u} = \frac{\tau_u + \tau_s}{D_{F_s}\bar{\tau}^2} > 0 \tag{20}$$

## 3.2 Optimal Policy Rule

I study a central bank that conducts Ramsey optimization ex-ante, before the realization of shocks and noises. The central bank chooses $(F_u, F_s, F_p)$ to minimize the weighted sum of inflation volatility and output volatility,[8] subject to the IS curve (Equation (4)) and the Phillips curve (Equation (5)), where $\mathbb{E}_t^p u_t$ in the Phillips curve is given by Equation (19).

To solve for the optimal values of $(F_u, F_s, F_p)$, first write output and inflation in terms of policy rule coefficients. Substituting the interest rate rule into the IS curve leads to the following equation of output:

$$\hat{y}_t = -\frac{1}{\sigma}\left[(F_u + F_s)u_t + F_s\varepsilon_t^s + F_p u_t^0\right] \tag{21}$$

The output gap depends on $u_t$, $\varepsilon_t^s$, and $u_t^0$ — but not on $\xi_t$. This is because since households expect future equilibrium to stay at their zero steady state, expectations do not affect output. Therefore, only the direct effect of the interest rate matters in determining output.

Substituting the interest-rate rule into the Phillips curve and the expected shocks, inflation becomes

$$\pi_t = \underbrace{\left(1 - \frac{\kappa(F_u + F_s)}{\sigma} + \Gamma\alpha\right)}_{P_u} u_t + \underbrace{\left(\Gamma\alpha_s - \frac{\kappa F_s}{\sigma}\right)}_{P_s}\varepsilon_t^s + \underbrace{\left(\Gamma\alpha_0 - \frac{\kappa F_p}{\sigma}\right)}_{P_p} u_t^0 - \underbrace{\frac{\Gamma\alpha_i}{F_u}}_{P_\xi}\xi_t \tag{22}$$

where $\Gamma \equiv (1-\theta)/\theta$.

[8] Since the only underlying shock is the inefficient cost-push shock, natural output stays the same. Therefore, the output gap is the same as output throughout the paper.

Since all shocks are mutually independent with mean zero, the central bank's optimization problem in the explicit form is written as

$$\min_{F_u,F_s,F_p} \mathcal{L}(F_u,F_s,F_p) = \mathrm{Var}(\pi_t) + \lambda \mathrm{Var}(\hat{y}_t) \tag{23}$$

where

$$\mathrm{Var}(\pi_t) = P_u^2\sigma_u^2 + P_s^2\sigma_s^2 + P_p^2\sigma_\mu^2 + P_\xi^2 D_{F_s} \tag{24}$$

$$\mathrm{Var}(\hat{y}_t) = \frac{1}{\sigma^2}\left[(F_u+F_s)^2\sigma_u^2 + F_s^2\sigma_s^2 + F_p^2\sigma_\mu^2\right] \tag{25}$$

Before solving for the optimal values of $(F_u, F_s, F_p)$ I first explain why the optimal policy rule is different from the ones in the two benchmark cases.

**Lemma 3.** *Certainty equivalence regarding optimal monetary policy fails when there is an information effect of monetary policy. Specifically, optimal monetary policy depends on the degree of rule ambiguity, $\sigma_{Fs}^2$.*

*Intuition:* The reason that certainty equivalence breaks down is that $F_u$ enters the central bank's loss function in a non-linear way. In the two benchmark cases, the private-sector's expectations are independent of $F_u$. In contrast, with the information effect, varying $F_u$ changes the Kalman gain coefficient, $\tau_i$ and $\tau_s$, leading to a first-order effect on inflation and output.

*Proof:* See Appendix D.1.

The solution to $(F_u, F_s, F_p)$ comes from the first-order conditions. The first-order condition with respect to $F_u$ does not take a linear form, due to how $F_u$ enters the Kalman gains coefficients. Although the first order conditions with respect to $F_s$ and $F_p$ yield explicit solutions, the solutions depend on $\tau_i(F_u)$, which is non-linear in $F_u$. Therefore, the solution to the central bank's optimization problem relies on numerical methods. See Appendix B for the first-order conditions and the outline of the numerical method.

Figure 1 plots the optimal policy rule coefficients with an increasing degree of policy rule ambiguity ($\sigma_{Fs}$). The perfect information benchmark and the no-information-effect benchmark are shown by the red dashed line and the blue dotted line, respectively. $F_u$ is smaller in the no-information-effect benchmark than in the perfect information benchmark, because the effect of $u_t$ on $\pi_t$ is smaller due to imperfect information. However, loss is higher under the no-information-effect benchmark, because noise shocks and priors also move inflation, increasing the unconditional variance of inflation. The optimal monetary policy needs to respond to noise shocks and priors, causing output fluctuations as well.

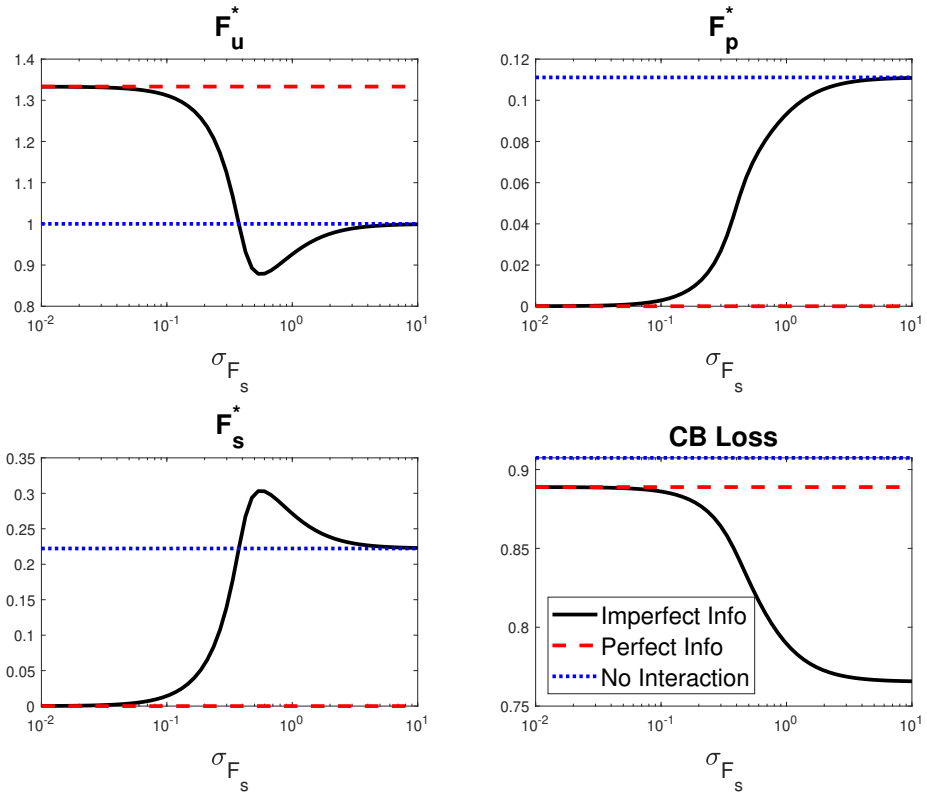



Figure 1: Optimal Policy Rule Coefficients at Varying Degree of Rule Ambiguity

When $\sigma_{Fs}$ becomes infinitely large, the private sector knows exactly how the interest rate responds to shocks and can perfectly back out the central bank's information about $u_t$. Therefore, the economy approaches the perfect information benchmark, and the optimal monetary policy coefficients approaches that under the perfect information benchmark. When $\sigma_{Fs}$ becomes infinitely large, the private sector cannot back out any information about $u_t$ from the interest rate, which is equivalent to the no-information-effect benchmark. The optimal monetary policy in the baseline model approaches that in the no-information-effect benchmark.

**Proposition 1.** *For all* $\sigma^2_{F_s} \in (0, \infty)$*:*

$$A^*(\sigma^2_{F_s}) = F^*_u + F^*_s < F^{*,NI}_u + F^{*,NI}_s = A^{*,NI} \tag{26}$$

*The total response to the cost-push shock in the optimal monetary policy rule is strictly smaller when the central bank internalizes the information effect, compared to that in the no-information-effect benchmark.*

*Intuition:* $A = F_u + F_s$ is the total response to the $u_t$ shock. In the *NI* benchmark and

in the *PI* benchmark, $A$ is chosen as the central bank makes the tradeoff between output gaps and inflation deviations. This tradeoff is the same whether or not the information effect exists. In both cases, $i_t|_{u_t} = Au_t$. The difference between our baseline case and the *NI* benchmark arises only through the Kalman weights: in our baseline case $\alpha_s < 1$ and the expectation channel is less responsive to $s_t$ than in the *NI* benchmark. This weakens the effectiveness of $A$ in stabilizing inflation through the expectations channel, making the marginal stabilization benefit of raising $A$ lower in the baseline than in the *NI* benchmark.

In addition, conditional on the optimal allocation between $F_u$ and $F_s$, reducing $A$ reduces the volatility of inflation through the effect on $\mathbb{E}_t^p u_t$. This result is implied from Lemma 2. When the central bank reduces $F_u$, it reduces the sensitivity of $\mathbb{E}_t^p u_t$ to both the underlying shock and the noise shock, which is welfare improving.

*Proof:* See Appendix D.2

Figure 2 plots the equilibrium inflation and output after four shocks at varying degrees of rule ambiguity. Panel (a) shows our baseline model. After a positive cost-push shock, inflation is positive and output is negative, as the central bank tightens monetary policy to partially offset the inflation pressure induced by the positive cost-push shock. After either a positive noise shock or a positive prior, inflation is positive, as both raise inflation expectations, which has an effect similar to that of an actual cost-push shock. A positive $\xi_t$ shock captures the case that there is a positive signal and the private-sector's perceived policy rule coefficient is larger than the actual one. In this case, the inflation rate is $P_s + P_\xi < 0$. This is because while the interest rate responds to the noise shock by $i_t = F_s$, the private sector infers a negative cost-push shock (see Equation (16)), as it believes the higher interest rate is due to a tighter policy rule rather than a positive cost-push shock. In comparison, in both the perfect-information benchmark and the no-interaction benchmark, a $\xi_t$ shock does not create any fluctuations in inflation or output.

The bottom right panel shows the equilibrium after a positive combined shock of rule ambiguity and a noise shock. For example, in the absence of an actual cost-push shock, the private sector receives positive noise about the cost-push shock and mistakenly believes the interest rate would be more sensitive to the noise shock than it actual is. In this case, the central bank raises the interest rate, and when the private sector observes the interest rate, it regards it as an additional signal of a higher cost-push shock.

## 3.3 Monetary Policy Surprises

The monetary policy surprise ($MPS_t$) is defined as the difference between the actual and the pre-FOMC expected interest rate. The expected interest rate is conditional on the private

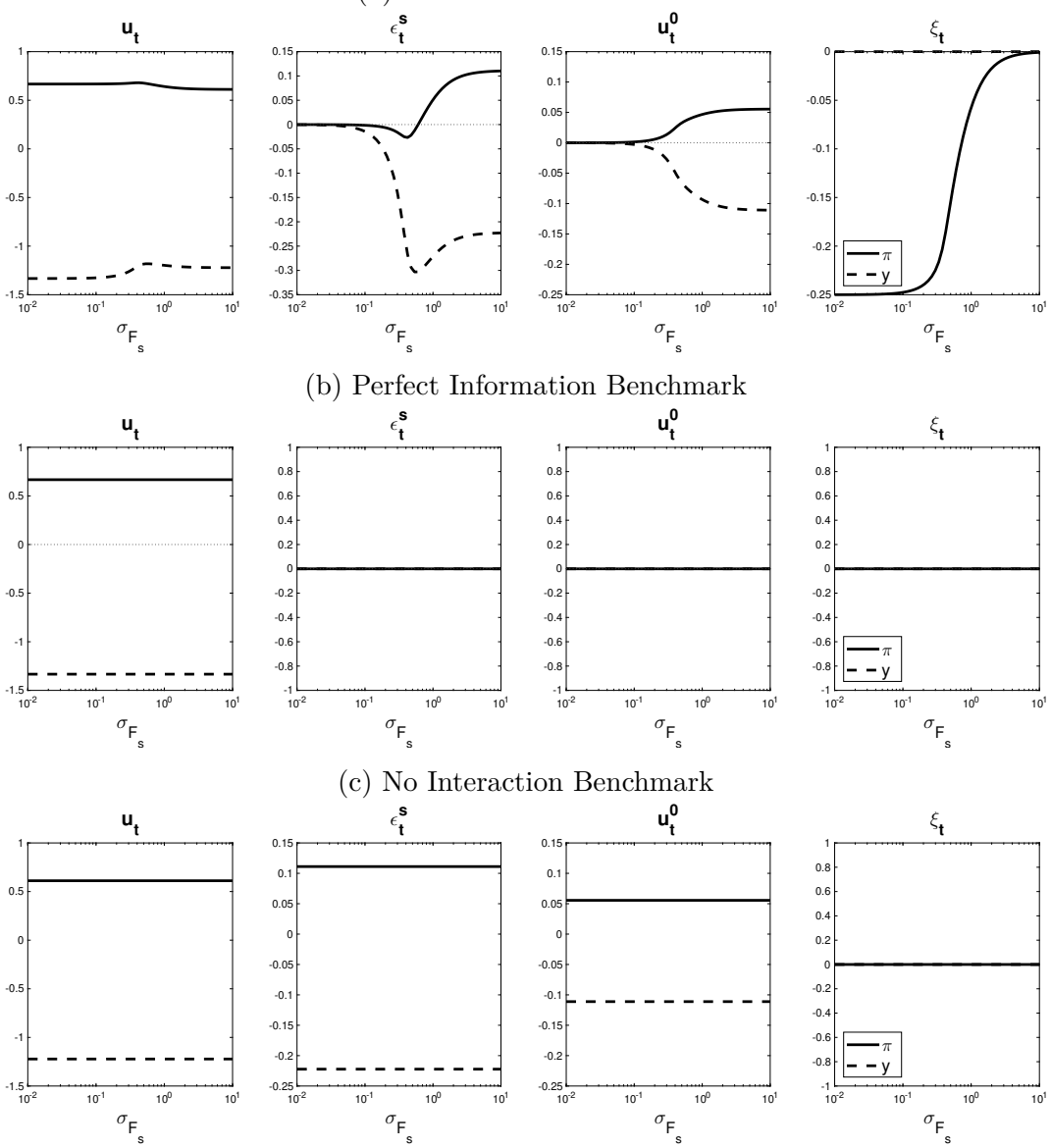



Figure 2: Equilibrium Inflation and Output under Optimal Policy

sector's imperfect information about both the cost-push shock and the policy rule coefficient, given by

$$\mathbb{E}_t^{pre}[i_t] = F_u \, \mathbb{E}^{pre}[u_t] + \hat{F}_s s_t \tag{27}$$

The monetary policy surprise is

$$MPS_t \equiv i_t - \mathbb{E}_t^{pre}[i_t] = F_u(u_t - \mathbb{E}^{pre}[u_t]) - \zeta_s s_t \tag{28}$$

Notice that the term $F_s s_t$ cancels out because the private sector uses $\hat{F}_s s_t$ in its expectation

of $i_t$, and $F_s s_t - \hat{F}_s s_t = -\zeta_s s_t$. Taking the variance of $MPS_t$ and expanding $\sigma^2_{F_s}\text{Var}(s_t)$ yields the variance decomposition of monetary policy surprises:

$$\text{Var}(\text{MPS}_t) = \underbrace{F_u^2 Var(u_t - \mathbb{E}_t^{pre}[u_t]}_{\text{shock uncertainty}} + \underbrace{\sigma^2_{F_s}\sigma^2_u}_{\text{policy rule ambiguity}} + \underbrace{\sigma^2_{F_s}\sigma^2_s}_{\text{interaction effect}} \tag{29}$$

Equation (29) decomposes the variance of monetary policy surprises into three components:

- shock uncertainty: the private sector understands that the interest rate reacts to the cost-push shock with the sensitivity of $F_u$, but does not know the realization of $u_t$. The size of this component scales with $F_u$.
- policy rule ambiguity: the private sector is uncertain about $F_u$. Even if the public signal of the cost-push shock is perfect, i.e., $\sigma_s^2 = 0$ so $s_t = u_t$, the private sector's expected interest rate is still different from the actual interest rate because the perceived policy rule coefficient is different from the actual rule coefficient.
- interaction effect: the joint effect of shock uncertainty and rule ambiguity. The private sector has imperfect information about the shock to which the interest rate responds and has imperfect information about how much the interest rate responds to the shock. The interaction effect amplifies the $MPS$ variance and this interaction effect can be reduced by either a smaller $\sigma_s$ or a smaller $\sigma_{Fs}$

In the perfect-information benchmark, since the private sector knows the underlying shock and the policy rule perfectly, it can perfectly predict the interest rate, and therefore there is no monetary policy surprise. In both the *no-information-effect* benchmark and our baseline model, both sources of information frictions exist. In both cases, $MPS_t$ can be decomposed into three components. The difference is the size of each component in total $MPS_t$.

**Proposition 2.** *The share of the interaction effect, calculated as*

$$\text{share}_{interact} = \frac{\sigma^2_{F_s}\sigma^2_s}{F_u^2\text{Var}(u_t - \mathbb{E}_t^{pre}u_t) + \sigma^2_{F_s}(\sigma^2_u + \sigma^2_s)}, \tag{30}$$

*is strictly decreasing in $F_u$ for $F_u > 0$.*

*Intuition:* When $F_u$ decreases, Lemma 2 implies that the expected cost-push shock changes less with the actual cost-push shock. Under rational expectations, both the actual and the expected interest rate respond less to the actual cost-push shock, and the share

of shock uncertainty reduces. The size of the policy rule ambiguity and the interaction effect does not change with $F_u$. As the total variance of monetary policy surprises reduces and the size of the interaction effect does not change, the share of the interaction effect becomes larger.

In the next section, I quantify the share of the interaction effect in a calibrated dynamic model.

# 4 The Dynamic Model

To quantify the size of the interaction effect that matches the empirically estimated monetary policy surprises, I extend my model to a dynamic setting.

## 4.1 Dynamic Setup

I extend the simple model in two respects in the dynamic setting. First, I allow the underlying fundamental shock to be persistent. Specifically, the cost-push shock follows an AR(1) process, given by

$$u_t = \rho u_{t-1} + \eta_t, \qquad \eta_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_\eta^2), \qquad \sigma_\eta^2 = \sigma_u^2(1-\rho^2) \tag{31}$$

where $\rho \in (0,1)$ is persistence and $\sigma_u^2$ is the unconditional variance. The innovation $\eta_t$ is independent of all other shocks.

Second, I allow the interest rate to be path-dependent. To maintain a similar information structure, I assume the optimal interest rate rule takes the following form:

$$i_t = F_u u_t + F_l \, \mathbb{E}_{t-1}^p \, u_{t-1} + F_s s_t. \tag{32}$$

where $F_u$, $F_s$, $F_l$ are policy coefficients optimally chosen by the central bank, and $\mathbb{E}_{t-1}^p \, u_{t-1}$ is the private sector's expected cost-push shock in the last period, known to both the CB and the private sector at the beginning of period $t$.

**Shock Uncertainty** As assumed in the static case, the private sector has imperfect information about the cost-push shock as well as the policy rule coefficient. Specifically, the private sector receives a noisy signal of the cost-push shock in each period, given by

$$s_t = u_t + \varepsilon_t^s, \qquad \varepsilon_t^s \overset{iid}{\sim} \mathcal{N}(0, \sigma_s^2)$$

**Policy Rule Ambiguity** The private sector does not know exactly how the interest rate responds to $s_t$. It only observes a noisy signal of the coefficient, denoted by $\hat{F}_s$. The private sector's perceived policy rule coefficient on $s_t$ may be different from the true $F_s$. While the actual coefficient, $F_s$ is constant, the difference between the perceived and the actual coefficient is time varying and is normally distributed around zero, given by

$$\hat{F}_s(t) = F_s + \zeta_s(t), \qquad \zeta_s \overset{iid}{\sim} \mathcal{N}(0, \sigma^2_{F_s}).$$

where $\zeta_s$ is i.i.d. across periods and independent of all other shocks.

The assumed functional form of the interest rate allows the private sector to back out the fraction of the change in the interest rate that is a signal to the current cost-push shock. Since $F_l \, \mathbb{E}^p_{t-1} \, u_{t-1}$ is in the private sector's information set, the private sector subtracts it and $\hat{F}_s s_t$ from $i_t$:

$$\tilde{u}_t = \frac{i_t - F_l \, \mathbb{E}^p_{t-1} \, u_{t-1} - \hat{F}_s s_t}{F_u} = u_t - \frac{\zeta_s s_t}{F_u} \tag{33}$$

$\tilde{u}_t$ is the information content in the interest rate. It is an imperfect signal of $u_t$ and its noise comes from the interaction between shock uncertainty and rule ambiguity, i.e., $-\zeta_s s_t / F_u$, with variance $\sigma^2_{F_s}(\sigma^2_u + \sigma^2_s)/F^2_u$.

Define the total noise variance in the signal:

$$D_{total} \equiv \sigma^2_{F_s}(\sigma^2_u + \sigma^2_s) \tag{34}$$

The effective precision of $i_t$ as a signal about $u_t$ is:

$$\bar{\tau}_i = \frac{F^2_u}{D_{total}} = \frac{F^2_u}{\sigma^2_{F_s}(\sigma^2_u + \sigma^2_s)}, \tag{35}$$

which is the same as in the static case (Equation (18)).

## 4.2 Belief-Updating Process

The private sector does Bayesian updating on the current cost-push shock by optimally combining last-period beliefs ($\mathbb{E}_{t-1} \, u_{t-1}$) and current-period signals ($s_t$ and $\tilde{u}_t$):

$$\mathbb{E}^p_t \, u_t = \rho(1 - \alpha^p) \, \mathbb{E}^p_{t-1} \, u_{t-1} + \alpha^p(\alpha_s s_t + \alpha_i \tilde{u}_t) \tag{36}$$

where $\alpha^p$ is the steady-state Kalman gain that solves the Riccati equation. $\alpha_s$ and $\alpha_i$ are the Kalman weights for the public signal and the signal of the interest rate, given by $\alpha_s = \tau_s / \bar{\tau}_{si}$,

$\alpha_i = \bar{\tau}_i/\bar{\tau}_{si}$, and $\bar{\tau}_{si} = \tau_s + \bar{\tau}_i$. Substituting $\tilde{u}_t$ from (33):

$$\mathbb{E}_t^p u_t = \rho(1-\alpha^p)\,\mathbb{E}_{t-1}^p u_{t-1} + \alpha^p \alpha u_t + \alpha^p \alpha_s \varepsilon_t^s - \frac{\alpha^p \alpha_i}{F_u}\zeta_s s_t \tag{37}$$

## 4.3 Equilibrium Solution

The equilibrium is solved by undetermined coefficients. To conjecture the equilibrium of inflation and output, notice that the IS and the Phillips curves imply that the equilibrium variables are functions of the actual and the expected cost-push shock, and that the beliefs-formation process implies that the expected cost-push shock is a linear function of $\left[u_t, \mathbb{E}_{t-1}^p u_{t-1}, \varepsilon_t^s, \xi_t\right]$. We therefore conjecture that equilibrium inflation and output take the following form:

$$y_t = \phi_u u_t + \phi_o \mathbb{E}_{t-1}^p u_{t-1} + \phi_s \varepsilon_t^s + \phi_\xi \xi_t \tag{38}$$

$$\pi_t = \psi_u u_t + \psi_o \mathbb{E}_{t-1}^p u_{t-1} + \psi_s \varepsilon_t^s + \psi_\xi \xi_t \tag{39}$$

With the conjectured equilibrium variables, the private sector forms expected future variables as

$$\mathbb{E}_t^p y_{t+1} = (\phi_u \rho + \phi_o)\,\mathbb{E}_t^p u_t \tag{40}$$

$$\mathbb{E}_t^p \pi_{t+1} = (\psi_u \rho + \psi_o)\,\mathbb{E}_t^p u_t, \tag{41}$$

where we make use of $\mathbb{E}_t^p[u_{t+1}] = \rho \mathbb{E}_t^p u_t$, and the fact that $\varepsilon_{t+1}^s$ and $\xi_{t+1}$ are mean zero and not predictable at time $t$.

The equilibrium is solved by matching the coefficients on $\left[u_t, \mathbb{E}_{t-1}^p u_{t-1}, \varepsilon_t^s, \xi_t\right]$ for both $y_t$ and $\pi_t$ between the conjectured reduced-form in Equations (38) and (39) and the structural form of the IS and the Phillips curves in Equations (1) and (2). See Appendix C for details.

## 4.4 Optimal Monetary Policy

The central bank chooses the coefficients of the interest rate rule $(F_u, F_s, F_l)$ ex-ante by minimizing the unconditional period-loss:

$$\min_{F_u, F_s, F_l} \mathcal{L}(F_u, F_s, F_l) = \mathrm{Var}(\pi_t) + \lambda\,\mathrm{Var}(\hat{y}_t) \tag{42}$$

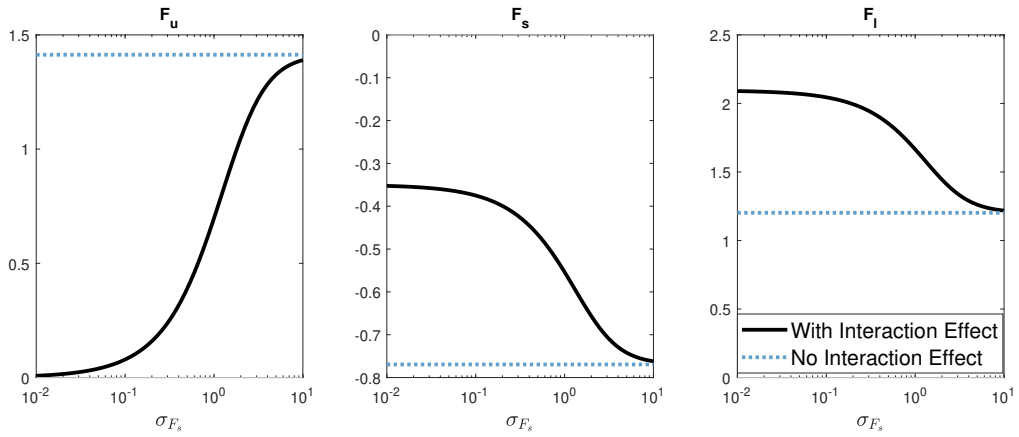



Figure 3: Optimal Policy Rule Coefficients in the Dynamic Model

Substituting equilibrium inflation and output with the cost-push shock and the belief-updating process, the loss can be written in terms of

$$\mathcal{L} = (\psi_u^2 + \lambda\phi_u^2)\sigma_u^2 + (\psi_o^2 + \lambda\phi_o^2)\text{Var}(\mathbb{E}_{t-1}^p u_{t-1}) + (\psi_s^2 + \lambda\phi_s^2)\sigma_s^2 + (\psi_\xi^2 + \lambda\phi_\xi^2)\sigma_{F_s}^2\left(\sigma_u^2 + \sigma_s^2\right) \quad (43)$$

where $\text{Var}(\mathbb{E}_{t-1}^p u_{t-1}) = (\alpha^p)^2[(\alpha_s + \alpha_i)^2\sigma_u^2 + \alpha_s^2\sigma_s^2]/(1 - \rho^2(1-\alpha^p)^2)$.

Solving the coefficients of the optimal policy rule relies on numerical methods. I numerically search for the set of $\{F_u, F_s, F_l\}$ such that the period-loss is minimized. Figure 3 plots the values of $\{F_u, F_s, F_l\}$ at varying degrees of rule ambiguity. Other parameters are calibrated to match macroeconomic moments in the US economy (Section 5). The solid black line and the dashed blue line plot the baseline model under the interaction effect and the no-interaction-effect benchmark, respectively.

The left panel of Figure 3 shows that compared to the no-interaction-effect benchmark, when the central bank internalizes the information effect of its policy decisions, it optimally reduces the interest-rate's response to the cost-push shock. The less ambiguous the policy rule is (smaller $\sigma_{Fs}$), the more precise is the information revealed from the interest rate, and in which case the central bank reduces $F_u^*$ to a larger extent. When $\sigma_{Fs} \to \infty$, the interest rate contains essentially no information about the cost-push shock. As the information effect is not present, $F_u$ approaches the level under the no-interaction-effect benchmark. This mechanism is the same as in the static case.

What is different from the static case is that optimal $F_s$ is negative in both the no-interaction-effect benchmark and the baseline model. This is because in the dynamic model, the central bank can compensate the attenuated on-impact response to $u_t$ by a delayed overshooting in the interest rate, captured by a larger response to $\mathbb{E}_{t-1}^p u_{t-1}$ (right panel). The benefits of a delayed response to the cost-push shock is well known in the New Keynesian

Table 1: Externally Calibrated Parameters

| Description | Parameter | Value | Source |
|---|---|---|---|
| Household discount factor | $\beta$ | 0.99 | Standard calibration |
| Inverse intertemporal elasticity | $\sigma$ | 1.00 | Log utility of consumption |
| Frisch elasticity of labor | $\varphi$ | 1.00 | Unitary |
| Calvo price-stickiness | $\theta$ | 0.875 | Average price duration |
| CB weight on output gap | $\lambda$ | 0.25 | Equal weight |
| persistence of $u_t$ | $\rho$ | 0.85 | Smets and Wouters (2007) |

literature, as it reduces the stabilization bias. (See Woodford (1999) and Galí (2015), for example.) After a positive cost-push shock, it is optimal for the central bank to commit to tightening policy in the future to reduce expected future inflation, as lowering future inflation puts downward pressure on current inflation. In my model, a delayed response yields another benefit: as $F_l$ increases, the central bank can further reduce $F_u$, which reduces the information effect.

After a positive noise shock ($\varepsilon_t^s > 0$), the private sector mistakenly regards it as a positive cost-push shock, and expects a higher interest rate in the next period due to the delayed overshooting. In this case, expected future inflation and output are negative, putting downward pressure on inflation in the current period. The central bank partially offsets this downward pressure by a negative $F_s$.

# 5 Quantitative Analysis

This section studies the positive implications of the interaction between the information effect of monetary policy and the policy rule ambiguity. I first calibrate the model parameters (Section 5.1) and decompose high-frequency monetary policy surprises under model implications (Section 5.2).

## 5.1 Calibration

The model is calibrated to the US economy at FOMC frequency. The first set of parameters is externally calibrated. I directly assign values to this set of parameters using standard New Keynesian structural parameters. Table 1 reports this set of parameters, together with the values and sources.

Specifically, the discount factor, $\beta$, is set to 0.99, standard for quarterly or monthly calibration. I set $\varphi = 1$ and $\sigma = 1$, assuming a unitary Frisch elasticity of labor supply and log utility of consumption. I choose $\theta = 0.875$, which is mapped from a $\theta = 0.75$ under

Table 2: Internally Calibrated Parameters

| Description | Parameter | Value | Target | Empirical | Model |
|---|---|---|---|---|---|
| Std. Dev. of Noise Shock | $\sigma_s$ | 0.316 | $SD(\pi_t - \mathbb{E}_t^p \pi_t)$ | 0.127 | 0.126 |
| Std. Dev. of Ambiguity Shock | $\sigma_{Fs}$ | 0.129 | $SD(MPS_t)$ | 0.050 | 0.051 |
| Std. Dev. of Cost-Push Shock | $\sigma_\eta$ | 0.097 | $SD(\pi_t)$ | 0.220 | 0.219 |

quarterly calibration, targeting the average price duration of approximately 4 quarters. $\lambda$ the weight on output gap stabilization in the central bank's objective function, is set to 0.25, which gives the central bank a larger weight on price stabilization than maximum employment. I assign $\rho = 0.85$, slightly lower than the persistence of the price markup and wage markup shocks estimated in Smets and Wouters (2007). This is because cost-push shocks stem from various sources, including both persistence shocks and temporary shocks such as oil price shocks. This is also consistent with using headline CPI as the empirical target in the internal calibration process.

The remaining three parameters are the key parameters that govern the degree of information frictions and rule ambiguity, which are the standard deviation of the fundamental shock ($\sigma_\eta$), the noise of the public signal ($\sigma_s$), and the noise in the perceived policy rule coefficient ($\sigma_{Fs}$). I calibrate this set of parameters internally to match three empirical targets in the US economy: the standard deviation of the actual headline CPI inflation rate, the standard deviation of the difference in the market-expected and the actual CPI inflation rate, and the standard deviation of monetary policy surprises under high-frequency identification. I summarize their calibrated values and their targeted moments in Table 2, together with the empirical and model-implied moments.

The internally calibrated parameters are estimated by generalized method of moments (GMM), matching three empirical moments to their model-implied counterparts.[9] Even though every targeted moment is determined simultaneously by all parameters, in what follows, I discuss each moment in relation to the parameter for which the moment yields the most identification power.

[9]The GMM estimator solves: $\hat{\Theta} = \arg\min_\Theta \big(\hat{m} - m(\Theta)\big)' W \big(\hat{m} - m(\Theta)\big)$, where $\Theta = (\sigma_\eta, \sigma_s, \sigma_{Fs}, \sigma_e)$. $\hat{m}$ is the vector of empirical moments, $m(\Theta)$ denotes the model-implied moments evaluated at the optimal policy $(F_u^*(\Theta), F_s^*(\Theta), F_l^*(\Theta))$, and $W$ is a diagonal weighting matrix with entries proportional to the inverse squared values of the empirical moments. Since the system is exactly identified (three moments, three parameters), the choice of $W$ does not affect the point estimates. At each evaluation of $m(\Theta)$, I solve for the equilibrium inflation and output coefficients under the optimal policy rule and calculate the model-implied targeted moments, where the optimal policy rule coefficients $(F_u^*, F_s^*, F_l^*)$ are the ones that minimizes the central bank's loss function.

$\sigma_s$ **from** $SD(\pi_t - \mathbb{E}_t^{pre}\pi_t)$**.** Most of the commonly used surveys of inflation expectations contain forecasts for the current quarter, next quarter, next year, or longer terms. To be consistent with the empirical measurement of monetary policy surprises under high frequency identification, I use nowcast expectations, the market consensus estimates of the economic indicators a few days before the official release. I use nowcast expectations because they will not be contaminated by forecast errors. The difference between the actual and the market-expected values in the nowcasts is only due to imperfect information in the current period. I obtain market consensus estimates of month-over-month headline CPI inflation rates from Money Market Services, which gathers predictions from professional forecasters for a variety of economic indicators leading up to the indicators' official release. The CPI surprise is constructed by taking the difference between the actual and the median estimate of the month-to-month headline CPI rate, and the data sample covers 1988 to 2023.

In the model, the private sector's *pre-FOMC* inflation expectation is formed after observing $s_t$ and before observing the interest rate $i_t$. Their pre-FOMC expected cost-push shock is:

$$\mathbb{E}_t^{pre}[u_t] = \rho(1-\alpha^p)\,\mathbb{E}_{t-1}^{p}\,u_{t-1} + \alpha^p\alpha_s s_t \tag{44}$$

Applying the expectation operator to Equation (22) conditional on the private sector's pre-FOMC information set yields:

$$\mathbb{E}^{pre}[\pi_t] = \psi_u\,\mathbb{E}^{pre}[u_t] + \psi_o\,\mathbb{E}_{t-1}^{p}\,u_{t-1} + \psi_s\,\mathbb{E}^{pre}[\varepsilon_t^s] + \psi_\xi\,\mathbb{E}^{pre}[\xi_t] \tag{45}$$

The private sector's pre-FOMC expected noise shocks and policy rule ambiguity are not the unconditional mean of zero. Instead, $\mathbb{E}^{pre}[\varepsilon_t^s] = s_t - \mathbb{E}^{pre}[u_t]$, since the private sector observes $s_t$ and forms $\mathbb{E}^{pre}[u_t]$, so $E^{pre}[\varepsilon_t^s] = \mathbb{E}^{pre}[s_t - u_t] = s_t - \mathbb{E}^{pre}[u_t]$. In addition, $\mathbb{E}^{pre}[\xi_t] = \mathbb{E}^{pre}[\zeta_s s_t] = s_t \cdot E[\zeta_s] = 0$, since the private sector knows $s_t$ but not the realization of $\zeta_s$, and uses its zero unconditional mean.

Substitute $\mathbb{E}^{pre}[\varepsilon_t^s]$ and $\mathbb{E}^{pre}[\xi_t]$ and get

$$\mathbb{E}^{pre}[\pi_t] = (\psi_u - \psi_s)\,\mathbb{E}^{pre}[u_t] + \psi_o\,\mathbb{E}_{t-1}^{pre}\,u_{t-1} + \psi_s s_t \tag{46}$$

Substituting Equation (44) into Equation (46):

$$E^{pre}[\pi_t] = \Big[(\psi_u - \psi_s)\alpha^p\alpha_s + \psi_s\Big]s_t + \Big[(\psi_u - \psi_s)\rho(1-\alpha^p) + \psi_o\Big]\,\mathbb{E}_{t-1}^{pre}\,u_{t-1} \tag{47}$$

Take the difference between the actual inflation and the pre-FOMC expected inflation:

$$\pi_t - \mathbb{E}^{pre}[\pi_t] = (\psi_u - \psi_s)\,(u_t - \mathbb{E}^{pre}[u_t]) + \psi_\xi\,\xi_t \tag{48}$$

Equation (46) shows that the difference between the actual and the expected inflation is explained by two terms: imperfect information about the fundamental shock, $u_t - E^{pre}[u_t]$, and policy rule ambiguity, $\xi_t$, and the two terms are orthogonal. Therefore, the standard deviation is

$$SD(\pi_t - \mathbb{E}^{pre}[\pi_t]) = \sqrt{(\psi_u - \psi_s)^2 P + \psi_\xi^2 \sigma_{F_s}^2 (\sigma_u^2 + \sigma_s^2)} \tag{49}$$

where $P = \text{Var}(u_t - E_t^{pre}[u_t])$.

$\sigma_s$ has the highest identification power from $SD(\pi_t - E^{pre}[\pi_t])$ because $\sigma_s$ affects it through two channels. First, a larger $\sigma_s$ reduces the precision of the public information about the cost-push shock, raising the variance of the difference between the actual and the expected cost-push shock. Second, rule ambiguity scales with $\sigma_s^2$. The variance of rule ambiguity mechanically increases with $\sigma_s^2$ as $\xi_t = \zeta_s s_t$.

**$\sigma_{Fs}$ from SD(MPS$_t$).** The empirical measure of monetary policy surprises is the high-frequency monetary policy surprises as constructed in Bauer and Swanson (2023b).[10] The standard deviation of the monetary policy surprises in all scheduled FOMC meetings from 1988 to 2023 is 0.050.

In the model, as in the static case, the monetary policy surprises are the difference between the actual and the pre-FOMC expected interest rate. Appling the pre-FOMC expectation operator to the interest rate in Equation (32) and then taking the difference, $MPS_t$ in the dynamic case is

$$\text{MPS}_t = \underbrace{F_u(u_t - \mathbb{E}^{pre}[u_t])}_{\text{shock uncertainty}} - \underbrace{\zeta_s s_t}_{\text{rule uncertainty}} \tag{50}$$

which is the same as the $MPS_t$ in the static case (Equation (28)). This is because although the interest rate has additional terms responding to lagged varaibles in the dynamic case, this endogenous response is perfectly known by the private sector, and therefore does not show up in the surprise of the interest rate. The variance decomposition is also the same as in the static case, given by Equation (29).

Given that $F_u^*$ is small, the total variance of monetary policy surprises is primarily informative about $\sigma_{Fs}$, the degree of rule ambiguity. This is because a larger $\sigma_{Fs}$ increases both the variance of pure rule ambiguity and the variance of the interaction term.

**$\sigma_\eta$ from SD($\pi_t$).** While all shocks affect equilibrium inflation, Var($\pi_t$) is primarily driven by $\sigma_\eta$, because the persistence in $u_t$ amplifies the cost-push innovation by $\sigma_u^2 = \sigma_\eta^2/(1-\rho^2)$,

[10] Data used in this paper are the updated series, accessed from https://www.frbsf.org/research-and-insights/data-and-indicators/monetary-policy-surprises/

whereas noise shocks are temporary shocks. In addition, as $\sigma_s$ is estimated to be small, the private sector puts a small weight on this noisy signal. As $F_u^*$ is small, the interest rate provides imprecise information about the realization of $u_t$, and therefore, the weight on the interest rate is also small. As a result, actual inflation does not change much with noisy signals, and the variance of actual inflation is mostly informative about the variance of the cost-push shock. I use month-over-month headline CPI from 1980 to 2020 as the empirical measure of the actual inflation process.

## 5.2 Decomposing Monetary Policy Surprises

Re-writing the variance decomposition of monetary policy surprises is similar to the static case, which is

$$\text{Var}(\text{MPS}_t) = \underbrace{F_u^{*2}(\rho, \sigma_\eta, \sigma_s, \sigma_{F_s}) \cdot \text{Var}(u_t - \mathbb{E}_t^{pre}[u_t])}_{\text{shock uncertainty}} + \underbrace{\sigma_{F_s}^2 \cdot \sigma_\eta^2/(1-\rho^2)}_{\text{rule ambiguity}} + \underbrace{\sigma_{F_s}^2 \sigma_s^2}_{\text{interaction}} \tag{51}$$

Using the calibrated parameters, the share of each component in the total variance of monetary policy surprises is shown in Table 3.

Table 3: Variance Decomposition of Monetary Policy Surprises

| total $Var(MPS_t)$ | shock uncertainty | rule ambiguity | interaction effect |
|---|---|---|---|
| 100% | 7.1% | 22.2% | 70.7% |

The shock uncertainty accounts for only 7 percent of the total variance of monetary policy surprises, primarily because $F_u^*$ in the dynamic case is very small. This is because the central bank wants to minimize information about the cost-push shock, and the dynamic setting allows it to do so by delaying the response to the next period (responding to $\mathbb{E}_{t-1}^p u_{t-1}$).

Rule ambiguity accounts for about 20 percent of the total variance of monetary policy surprises. The major fraction of the total variance of monetary policy surprises is attributed to the interaction effect (70.7%). This is because under the calibration, both $\sigma_{Fs}$ and $\sigma_s$ are large, implying that the private sector has relatively imperfect information about both the fundamental shock and the policy rule coefficient, and the combination of these two sources of imperfect information leads the interaction effect to have a larger share in the variance of monetary policy surprises. Notice that imperfect information about the cost-push shock (a large $\sigma_s$) does not imply that a large fraction of $mps_t$ should be attributed to shock uncertainty, because the interest rate may respond little to the shock on impact, as it does under optimal monetary policy.

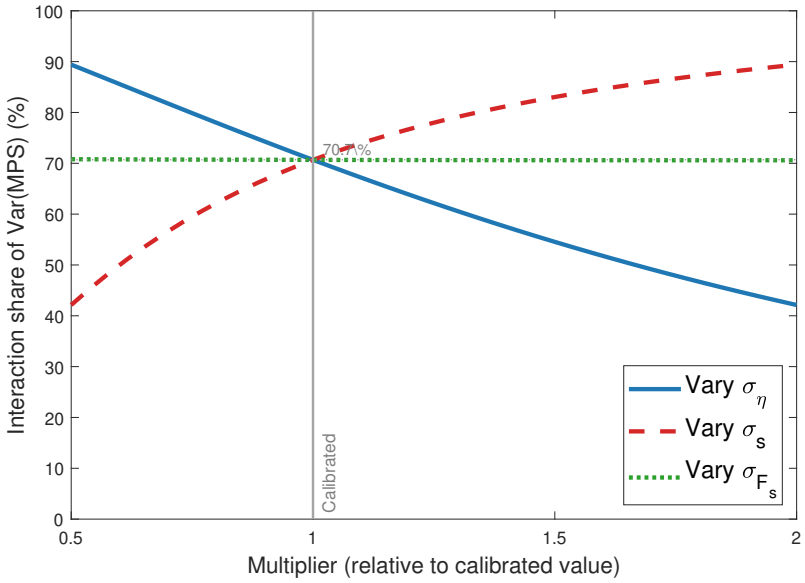



Figure 4: Share of the Interaction Effect in Var($MPS_t$)

I now discuss how varying each of the internally calibrated parameters may change the variance decomposition of monetary policy surprises. Figure 4 shows the share of the interaction term in the total variance of monetary policy surprises, and each line plots the case when changing one of the $\{\sigma_\eta,\ \sigma_s,\ \sigma_{Fs}\}$ while holding other parameters fixed at the calibrated values.

Increasing $\sigma_\eta$ can substantially decrease the interaction share, because as $\sigma_\eta$ increases, agents place less weight on their prior and more weight on the signal, $s_t$, when updating expected cost-push shock. The rule ambiguity share becomes larger, as it scales up with $\sigma_\eta$, and the share of the interaction term decreases.

Increasing $\sigma_s$ can substantially increase the interaction share. Intuitively, the interaction term in the $MPS_t$ comes from the private sector not knowing the size of the noise shock, and how much the interest rate responds to the noise shock. The larger the variance of the noise shock, the larger the share of the interaction term.

Interestingly, the share of the interaction term is nearly invariant to $\sigma_{Fs}$. This is because $\sigma_{Fs}$ changes all three components. First, both the rule ambiguity term and the interaction term variances are proportional to $\sigma^2_{Fs}$. Second, within the range of parameter values considered here, $F^*_u$ rises linearly with $\sigma_{Fs}$, making the shock uncertainty term also proportional to $\sigma^2_{Fs}$.

Figure 4 implies that the size of the interaction share of the variance of monetary policy surprises is determined by the fraction of noise in the signal, $\frac{\sigma^2_s}{\sigma^2_u+\sigma^2_s}$. A noisier signal, either

due to a larger $\sigma_s$ or due to a smaller $\sigma_u$ ($\sigma_u^2 = \sigma_\eta^2/(1-\rho^2)$), results in a larger share of the interaction term in the total variance of monetary policy surprises.

# 6 Conclusion

This paper studies a New Keynesian economy in which the private sector has imperfect information about both the fundamental shock and the central bank's policy rule. When monetary policy has an information effect, these two sources of information frictions interact with each other: ambiguity about how the *Fed's Response to News* reduces the *Fed's Information Effect* channel.

This interaction effect breaks the certainty equivalence principle regarding optimal monetary policy. The optimal rule depends on the degree of rule ambiguity, and it leads the central bank to restrain its response to the fundamental shock. The restrained response, in turn, shifts the composition of monetary policy surprises toward the components driven by rule ambiguity. In a dynamic model calibrated to US data, the interaction effect accounts for about 70 percent of the variance of high-frequency monetary policy surprises.

Several extensions are left for future work. Rule ambiguity is exogenous and constant in this paper; endogenizing it through the central bank optimizing its communication strategy, or allowing the private sector to learn about the rule over time as documented by Bauer, Pflueger, and Sunderam (2024), would shed light on the dynamics of perceived rules and the value of communication.

# References


Baeriswyl, Romain and Camille Cornand. 2010. "The Signaling Role of Policy Actions." *Journal of Monetary Economics* 57 (6):682–695. URL https://doi.org/10.1016/j.jmoneco.2010.06.001.

Bauer, Michael D., Carolin E. Pflueger, and Adi Sunderam. 2024. "Perceptions about Monetary Policy." *Quarterly Journal of Economics* 139 (4):2227–2278. URL https://doi.org/10.1093/qje/qjae021.

Bauer, Michael D. and Eric T. Swanson. 2023a. "An Alternative Explanation for the 'Fed Information Effect'." *American Economic Review* 113 (3):664–700. URL https://doi.org/10.1257/aer.20201220.

———. 2023b. "A Reassessment of Monetary Policy Surprises and High-Frequency Identification." *NBER Macroeconomics Annual* 37:87–155. URL https://doi.org/10.1086/723574.

Berkelmans, Leon. 2011. "Imperfect Information, Multiple Shocks, and Policy's Signaling Role." *Journal of Monetary Economics* 58 (4):373–386. URL https://doi.org/10.1016/j.jmoneco.2011.07.002.

Blinder, Alan S., Michael Ehrmann, Marcel Fratzscher, Jakob De Haan, and David-Jan Jansen. 2008. "Central Bank Communication and Monetary Policy: A Survey of Theory and Evidence." *Journal of Economic Literature* 46 (4):910–945. URL https://doi.org/10.1257/jel.46.4.910.

Campbell, Jeffrey R., Charles L. Evans, Jonas D. M. Fisher, and Alejandro Justiniano. 2012. "Macroeconomic Effects of Federal Reserve Forward Guidance." *Brookings Papers on Economic Activity* Spring. URL https://doi.org/10.1353/eca.2012.0004.

Cieslak, Anna and Andreas Schrimpf. 2019. "Non-Monetary News in Central Bank Communication." *Journal of International Economics* 118:293–315. URL https://doi.org/10.1016/j.jinteco.2019.01.012.

Clarida, Richard, Jordi Galí, and Mark Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (4):1661–1707. URL https://doi.org/10.1257/jel.37.4.1661.

Cukierman, Alex and Allan H. Meltzer. 1986. "A Theory of Ambiguity, Credibility, and Inflation under Discretion and Asymmetric Information." *Econometrica* 54 (5):1099–1128. URL https://doi.org/10.2307/1912324.

Ellingsen, Tore and Ulf Söderström. 2001. "Monetary Policy and Market Interest Rates." *American Economic Review* 91 (5):1594–1607. URL https://doi.org/10.1257/aer.91.5.1594.

Eusepi, Stefano and Bruce Preston. 2010. "Central Bank Communication and Expectations Stabilization." *American Economic Journal: Macroeconomics* 2 (3):235–271. URL https://doi.org/10.1257/mac.2.3.235.

Faust, Jon and Lars E. O. Svensson. 2001. "Transparency and Credibility: Monetary Policy with Unobservable Goals." *International Economic Review* 42 (2):369–397. URL https://doi.org/10.1111/1468-2354.00114.

Galí, Jordi. 2015. *Monetary policy, inflation, and the business cycle: An introduction to the New Keynesian framework and its applications*. Princeton University Press.

Gürkaynak, Refet S., Brian Sack, and Eric T. Swanson. 2005. "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements." *International Journal of Central Banking* 1 (1):55–93. URL https://www.ijcb.org/journal/v1n1/do-actions-speak-louder-words-response-asset-prices-monetary-policy-actions-and.

Hamilton, James D., Seth Pruitt, and Scott Borger. 2011. "Estimating the Market-Perceived Monetary Policy Rule." *American Economic Journal: Macroeconomics* 3 (3):1–28. URL https://doi.org/10.1257/mac.3.3.1.

Han, Zhao and Chengcheng Jia. 2025. "Asymmetric Information, Two-Way Learning, and the Fed Information Effect." Working Paper 23-32R, Federal Reserve Bank of Cleveland. URL https://doi.org/10.26509/frbc-wp-202332r.

Hoesch, Lukas, Barbara Rossi, and Tatevik Sekhposyan. 2023. "Has the Information Channel of Monetary Policy Disappeared? Revisiting the Empirical Evidence." *American Economic Journal: Macroeconomics* 15 (3):355–387. URL https://doi.org/10.1257/mac.20200068.

Jarociński, Marek and Peter Karadi. 2020. "Deconstructing Monetary Policy Surprises—The Role of Information Shocks." *American Economic Journal: Macroeconomics* 12 (2):1–43. URL https://doi.org/10.1257/mac.20180090.

Jia, Chengcheng. 2023. "The informational effect of monetary policy and the case for policy commitment." *European Economic Review* 156:104468. URL https://doi.org/10.1016/j.euroecorev.2023.104468.

Kuttner, Kenneth N. 2001. "Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market." *Journal of Monetary Economics* 47 (3):523–544. URL https://doi.org/10.1016/S0304-3932(01)00055-1.

Melosi, Leonardo. 2017. "Signalling Effects of Monetary Policy." *Review of Economic Studies* 84 (2):853–884. URL https://doi.org/10.1093/restud/rdw050.

Miranda-Agrippino, Silvia and Giovanni Ricco. 2021. "The Transmission of Monetary Policy Shocks." *American Economic Journal: Macroeconomics* 13 (3):74–107. URL https://doi.org/10.1257/mac.20180124.

Nakamura, Emi and Jón Steinsson. 2010. "Monetary Non-Neutrality in a Multisector Menu Cost Model." *Quarterly Journal of Economics* 125 (3):961–1013. URL https://doi.org/10.1162/qjec.2010.125.3.961.

Nakamura, Emi and Jón Steinsson. 2018. "High-Frequency Identification of Monetary Non-Neutrality: The Information Effect." *Quarterly Journal of Economics* 133 (3):1283–1330. URL https://doi.org/10.1093/qje/qjy004.

Ricco, Giovanni and Emanuele Savini. 2025. "Decomposing Monetary Policy Surprises: Shock, Information, and Policy Rule Revision." Tech. rep., Centre for Economic Policy Research. URL https://cepr.org/publications/dp20166.

Simon, Herbert A. 1956. "Dynamic programming under uncertainty with a quadratic criterion function." *Econometrica* :74–81URL https://doi.org/10.2307/1905261.

Smets, Frank and Rafael Wouters. 2007. "Shocks and frictions in US business cycles: A Bayesian DSGE approach." *American Economic Review* 97 (3):586–606. URL https://doi.org/10.1257/aer.97.3.586.

Svensson, Lars E. O. and Michael Woodford. 2003. "Indicator Variables for Optimal Policy." *Journal of Monetary Economics* 50 (3):691–720. URL https://doi.org/10.1016/S0304-3932(03)00030-8.

Swanson, Eric T. 2004. "Signal Extraction and Non-Certainty-Equivalence in Optimal Monetary Policy Rules." *Macroeconomic Dynamics* 8 (1):27–50. URL https://doi.org/10.1017/S1365100504020279.

Tang, Jenny. 2015. "Uncertainty and the signaling channel of monetary policy." Working Paper 15-8, Federal Reserve Bank of Boston. URL https://ideas.repec.org/p/fip/fedbwp/15-8.html.

Theil, Henri. 1957. "A Note on Certainty Equivalence in Dynamic Planning." *Econometrica* 25 (2):346–349. URL https://doi.org/10.2307/1910260.

Woodford, Michael. 1999. "Optimal Monetary Policy Inertia." Working Paper 7261, National Bureau of Economic Research. URL https://doi.org/10.3386/w7261.

# Appendices

## A Micro-Foundation of the IS and the Phillips Curve under Imperfect Information

This section provides details of the derivation of the equilibrium aggregate output and inflation from households' and firms' optimization problems.

### A.1 The Household

The preferences of the representative household are defined over the aggregate consumption good, $C_t$, and the labor supplied to each firm, $N_t(j)$, as

$$\mathbb{E}_t^p \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t) - \int V(N_t(j)) dj \right\}, \tag{A.1}$$

The aggregate good $C_t$ consists of a continuum of intermediate goods:

$$C_t = \left( \int_0^1 C_t(j)^{1-\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{A.2}$$

where $C_t(j)$ is the consumption of intermediate good $j$ in period $t$.

The household maximizes expected utility subject to the intertemporal budget constraint:

$$\int_0^1 P_t(j) C_t(j) dj + B_{t+1} \leq \int_0^1 W_t(j) N_t(j) dj + (1+i_t) B_t + \Pi_t, \tag{A.3}$$

where $B_t$ is the risk-free bond with nominal interest rate $i_t$, which is determined by the central bank. $\Pi_t$ is the lump-sum component of household income, which includes dividends from ownership of all firms. $W_t(j)$ and $N_t(j)$ are the labor wage and labor supply for firm $j$, respectively.

The household's optimization problem can be solved in two stages. In the second stage, the household allocates intermediate goods consumption to minimize the cost of expenditure, given the level of aggregate goods consumption, which yields the demand for the intermediate good:

$$C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon} C_t \qquad \forall j, \tag{A.4}$$

where $P_t = \left[ \int_0^1 P_t(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$. Households have perfect information regarding individual prices when making demand decisions for individual goods (Equation (A.4)).

In the first stage, the household chooses its aggregate consumption, $C_t$, and its savings

in the risk-free bond, $B_{t+1}$. I assume that the utility of consuming the aggregate good takes the following forms: $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$, where $\sigma$ is the inverse of the intertemporal elasticity of substitution and the parameter.

The intertemporal consumption decision leads to the following Euler equation:

$$C_t^{-\sigma} = \beta(1+i_t)\,\mathbb{E}_t^p\left(C_{t+1}^{-\sigma}\frac{P_t}{P_{t+1}}\right). \tag{A.5}$$

The resource constraint implies that $c_t^j = y_t^j \ \forall j$, which further implies $c_t = y_t$. So, the log-linear approximation to the Euler equation can be written as:

$$y_t = \mathbb{E}_t^p\, y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t^H\, \pi_{t+1}\right) \tag{A.6}$$

In addition to the consumption-saving decision, the household also makes the intratemporal consumption-leisure decision, which sets the marginal rate of substitution between leisure and consumption equal to the real wage. Assuming that the dis-utility of labor takes the form of $V(N_{jt}) = \frac{N_{jt}^{1+\varphi}}{1-\varphi}$, the real wage rate in equilibrium is given by

$$\frac{N_t^{\varphi}(j)}{C_t^{-\sigma}} = \frac{W_t(j)}{P_t}. \tag{A.7}$$

The log-linear approximation of Equation (A.7) yields

$$\varphi n_t(j) + \sigma y_t = w_t(j), \tag{A.8}$$

where $w_t$ denotes the log approximated real wage, $log(W_t/P_t)$. Integrate Equation (A.8) and get:

$$\int w_t(j) = \varphi \int n_t(j)dj + \sigma y_t. \tag{A.9}$$

The labor supply decision is made under perfect information, as we follow the literature to assume that the equilibrium wage clears the labor market. In other words, firms and workers sign contracts that specify higher wage rates for aggregate overtime.

The production function is linear in labor and has constant technology that is normalized to one, given by $Y_t^j = N_t^j$. Combining the production function and the demand function of of $y_t(j)$ leads to

$$\int n_t(j)dj = y_t + \int(-\epsilon)(p_t(j) - p_t) = y_t \tag{A.10}$$

Substituting $\int n_t(j)dj$ in (A.9) with (A.10) results in

$$\int w_t(j) = (\varphi + \sigma) y_t \tag{A.11}$$

### A.2 Firms

When choosing the optimal resetting price, firm $j$ expects the demand function of its product as specified in Equation (A.4), and hires labor according to the production function to meet the demand. The log-linear approximation to the firm's optimal resetting price is:

$$p_t^*(j) = (1-\beta\theta)\,\mathbb{E}_t^j \left\{ \Sigma(\beta\theta)^k \left[ p_{t+k} + u_{t+k}^w(j) + w_{t+k}(j) \right] \right\} \tag{A.12}$$

The Calvo assumption implies that the aggregate price index is an average of the price charged by the fraction $1-\theta$ of firms that reset their prices at $t$, and the fraction $\theta$ of firms whose prices remain the same as in the last period. Thus, the aggregate price under log-linear approximation is:

$$p_t = \theta p_{t-1} + (1-\theta) \int p_t^*(j) dj \tag{A.13}$$

Subtract $p_{t-1}$ from both sides and get:

$$\pi_t = (1-\theta) \left( \int p_t^*(j) - p_{t-1} \right) \tag{A.14}$$

Under the assumption that beliefs about aggregate variables are homogeneous, we can simply integrate individual prices without dealing with the higher-order beliefs problem. The integral of individual prices is given by

$$\int p_t^*(j) dj = (1-\beta\theta) \left( \mathbb{E}_t^s p_t + u_t^w + w_t \right) + (1-\beta\theta) \Sigma_{k=1}^{\infty} (\beta\theta)^k \, \mathbb{E}_t^s \left( p_{t+k} + u_{t+k}^w + w_{t+k} \right) \tag{A.15}$$

To write (A.15) in difference equations, we first express

$$\beta\theta \int \mathbb{E}_t^s p_{t+1}^*(j) dj = (1-\beta\theta) \Sigma_{k=1}^{\infty} \mathbb{E}_t^s \left( p_{t+k} + u_{t+k}^w + w_{t+k} \right) = \beta\theta \, \mathbb{E}_t^s p_{t+k}^* \tag{A.16}$$

Subtract (A.16) from (A.15) and substitute $\int w_t(j)$ with $y_t$ according to Equation (A.11):

$$\int p_t^*(j) dj - \beta\theta \, \mathbb{E}_t^s p_{t+1} = (1-\beta\theta) \, \mathbb{E}_t^s p_t + (1-\beta\theta) u_t^w + (1-\beta\theta)(\varphi+\sigma) y_t \tag{A.17}$$

Subtract $p_{t-1}$ from both sides and get

$$\int p_t^*(j) dj - p_{t-1} = \beta\theta \left( \mathbb{E}_t^s p_{t+1}^* - \mathbb{E}_t^s p_t \right) + \mathbb{E}_t^s p_t - p_{t-1} + (1-\beta\theta) u_t^w + (1-\beta\theta)(\varphi+\sigma) y_t \tag{A.18}$$

Apply (A.14) and get

$$\pi_t = \beta\theta\,\mathbb{E}_t^s\,\pi_{t+1} + (1-\theta)\,\mathbb{E}_t^s\,\pi_t + (1-\theta)(1-\beta\theta)u_t^w + (1-\beta\theta)(1-\theta)(\varphi+\sigma)y_t \tag{A.19}$$

Now, inflation can be written as

$$\pi_t = \beta\theta\,\mathbb{E}_t^p\,\pi_{t+1} + (1-\theta)\,\mathbb{E}_t^p\,\pi_t + \kappa\theta\hat{y}_t + u_t \tag{A.20}$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)(\varphi+\sigma)}{\theta}$, and $u_t = (1-\theta)(1-\beta\theta)u_t^w$

# B Solution Method for the Static Model

The optimal $(F_u^*, F_s^*, F_p^*)$ minimize $\mathcal{L}$. We write the loss as:

$$\mathcal{L} = (P_u^2 + \lambda Q_u^2)\sigma_u^2 + (P_s^2 + \lambda Q_s^2)\sigma_s^2 + (P_p^2 + \lambda Q_p^2)\sigma_\mu^2 + (P_\xi^2 + \lambda Q_\xi^2)\,D_{F_s} \tag{B.1}$$

**FOC for $F_u$** The FOC is:

$$\frac{d\mathcal{L}}{dF_u} = \left.\frac{\partial\mathcal{L}}{\partial F_u}\right|_{\alpha_i \text{ fixed}} + \frac{\partial\mathcal{L}}{\partial\alpha_i}\cdot\frac{d\alpha_i}{dF_u} = 0 \tag{B.2}$$

where $d\alpha_i/dF_u = 2F_u\tau_s/[D_{F_s}(\bar{\tau})^2] > 0$.

Because (B.2) is nonlinear in $F_u$ (through $\alpha_i(F_u)$), no closed-form solution exists. A fixed-point algorithm is required. However, the numerical method can be facilitated if one writes $F^*$ as functions of $\alpha$, which can be derived from FOCs on $F_p$, $F_s$, and $A = F_u + F_s$.

**FOC for $F_p$** $F_p$ enters the loss only through the prior channel $(P_p^2 + \lambda Q_p^2)\sigma_\mu^2$. Substituting $P_p = \Gamma\alpha_0 + \kappa Q_p$ and $Q_p = -F_p/\sigma$:

$$P_p = \Gamma\alpha_0 - \frac{\kappa F_p}{\sigma} \tag{B.3}$$

The FOC is:

$$\frac{\partial\mathcal{L}}{\partial F_p} = 2\left(P_p\cdot\frac{\partial P_p}{\partial F_p} + \lambda Q_p\cdot\frac{\partial Q_p}{\partial F_p}\right)\sigma_\mu^2 = 0 \tag{B.4}$$

Since $\partial P_p/\partial F_p = -\kappa/\sigma$ and $\partial Q_p/\partial F_p = -1/\sigma$:

$$\left(\Gamma\alpha_0 - \frac{\kappa F_p}{\sigma}\right)\left(-\frac{\kappa}{\sigma}\right) + \lambda\left(-\frac{F_p}{\sigma}\right)\left(-\frac{1}{\sigma}\right) = 0 \tag{B.5}$$

Rearranging:

$$-\frac{\kappa\Gamma\alpha_0}{\sigma}+\frac{\kappa^2 F_p}{\sigma^2}+\frac{\lambda F_p}{\sigma^2}=0 \tag{B.6}$$

$$F_p^*=\frac{\sigma\kappa\Gamma\alpha_0}{\kappa^2+\lambda} \tag{B.7}$$

which is a closed-form solution, given $\alpha_0$.

**FOC for $F_s$ and for $A=F_u+F_s$** $F_s$ enters through the $u_t$ channel (via $F_u+F_s$ in $Q_u$), the $\varepsilon_t^s$ channel (via $Q_s=-F_s/\sigma$), and the rule ambiguity only indirectly (since $\bar{\tau}_i=F_u^2/D_{F_s}$ does not depend on $F_s$). Differentiating:

$$\frac{\partial\mathcal{L}}{\partial F_s}=2\sigma_u^2\Big[P_u\Big(-\frac{\kappa}{\sigma}\Big)+\lambda Q_u\Big(-\frac{1}{\sigma}\Big)\Big]+2\sigma_s^2\Big[P_s\Big(-\frac{\kappa}{\sigma}\Big)+\lambda Q_s\Big(-\frac{1}{\sigma}\Big)\Big]=0 \tag{B.8}$$

Substituting $P_u$, $Q_u$, $P_s$, $Q_s$:

$$\sigma_u^2\left[\frac{\kappa(P_u+\lambda Q_u/\kappa)}{\sigma}\right]+\sigma_s^2\left[\frac{\kappa(P_s+\lambda Q_s/\kappa)}{\sigma}\right]=0 \tag{B.9}$$

This is a linear equation in $F_u+F_s$ and $F_s$. Define $A\equiv F_u+F_s$ (total response to $u_t$). Then $F_u=A-F_s$ and the FOC separates:

$$\text{From } u_t \text{ channel:}\quad \frac{\partial}{\partial A}\big[(P_u^2+\lambda Q_u^2)\sigma_u^2\big]=0\ \Rightarrow\ A^*=\frac{\sigma\kappa(\alpha_s+\alpha_i+1)}{\kappa^2+\lambda}\cdot\frac{1}{1+\Gamma} \tag{B.10}$$

$$\text{From } \varepsilon_t^s \text{ channel:}\quad \frac{\partial}{\partial F_s}\big[(P_s^2+\lambda Q_s^2)\sigma_s^2\big]=0\ \Rightarrow\ F_s^*=\frac{\sigma\kappa\Gamma\alpha_s}{\kappa^2+\lambda} \tag{B.11}$$

Substituting $1/(1+\Gamma)=\theta$ and $1+\Gamma(\alpha_s+\alpha_i)=1+\Gamma(1-\alpha_0)$:

$$A^*=\frac{\sigma\kappa(1+\Gamma(1-\alpha_0))}{(\kappa^2+\lambda)(1+\Gamma)}\cdot\frac{1+\Gamma}{1} \tag{B.12}$$

Working directly from the FOC for $A$ at optimal $F_s^*$, substituting $P_u=1+\Gamma(\alpha_s+\alpha_i)+\kappa Q_u$ and $Q_u=-A/\sigma$:

$$\Big(1+\Gamma(1-\alpha_0)-\frac{\kappa A}{\sigma}\Big)\Big(-\frac{\kappa}{\sigma}\Big)+\lambda\Big(-\frac{A}{\sigma}\Big)\Big(-\frac{1}{\sigma}\Big)=0 \tag{B.13}$$

which gives:

$$A^*=F_u^*+F_s^*=\frac{\sigma\kappa(1+\Gamma(1-\alpha_0))}{\kappa^2+\lambda} \tag{B.14}$$

$$F_s^*=\frac{\sigma\kappa\Gamma\alpha_s}{\kappa^2+\lambda} \tag{B.15}$$

$$F_u^* = A^* - F_s^* = \frac{\sigma\kappa}{\kappa^2+\lambda}\left[1+\Gamma(1-\alpha_0)-\Gamma\alpha_s\right] = \frac{\sigma\kappa(1+\Gamma\alpha_i)}{\kappa^2+\lambda} \tag{B.16}$$

Both $F_s^*$ and $A^*$ are linear in the Kalman weights $(\alpha_0, \alpha_s, \alpha_i)$ and are solved in closed form once those weights are known.

**Fixed-Point Algorithm** The key observation is that at fixed Kalman weights $(\alpha_0, \alpha_s, \alpha_i)$, the FOCs for $F_p$, $F_s$, and $F_u$ all have closed-form solutions:

$$F_p^*(\alpha_0) = \frac{\sigma\kappa\Gamma\alpha_0}{\kappa^2+\lambda} \tag{B.17}$$

$$F_s^*(\alpha_s) = \frac{\sigma\kappa\Gamma\alpha_s}{\kappa^2+\lambda} \tag{B.18}$$

$$F_u^*(\alpha_i) = \frac{\sigma\kappa(1+\Gamma\alpha_i)}{\kappa^2+\lambda} \tag{B.19}$$

where $F_u^*(\alpha_i)$ follows from (B.14) and (B.15) combined: $F_u^* = A^* - F_s^*$.

Crucially, these solutions require only $\alpha_i$ (which determines $\bar{\tau}_i$, $\bar{\tau}$, and hence $\alpha_0$, $\alpha_s$). But $\alpha_i$ itself depends on $F_u$ through $\bar{\tau}_i = F_u^2/D_{F_s}$. This creates a fixed-point equation in $F_u$.

Substituting (B.19) into $\bar{\tau}_i = F_u^2/D_{F_s}$:

$$\bar{\tau}_i = \frac{1}{D_{F_s}}\left(\frac{\sigma\kappa(1+\Gamma\alpha_i)}{\kappa^2+\lambda}\right)^2 \tag{B.20}$$

where $\alpha_i = \bar{\tau}_i/(\tau_u+\tau_s+\bar{\tau}_i)$. This is a scalar fixed-point equation in $\bar{\tau}_i > 0$ (equivalently in $\alpha_i \in (0,1)$).

The numerical method to find the fixed-point solution takes the following steps:

1. Set $\alpha_i^0 = 0$ (the *NI* benchmark starting point). This gives $F_u^0 = \sigma\kappa/(\kappa^2+\lambda) = F_u^{*,NI}$.

2. Compute Kalman weights given $F_u^k$.

$$\bar{\tau}_i^k = \frac{(F_u^k)^2}{D_{F_s}}, \qquad \bar{\tau}^k = \tau_u + \tau_s + \bar{\tau}_i^k \tag{B.21}$$

$$\alpha_0^k = \frac{\tau_u}{\bar{\tau}^k}, \quad \alpha_s^k = \frac{\tau_s}{\bar{\tau}^k}, \quad \alpha_i^k = \frac{\bar{\tau}_i^k}{\bar{\tau}^k} \tag{B.22}$$

3. Update policy coefficients (Equations (B.17) to (B.19)).

$$F_p^{k+1} = \frac{\sigma\kappa\Gamma\alpha_0^k}{\kappa^2+\lambda} \tag{B.23}$$

$$F_s^{k+1} = \frac{\sigma\kappa\Gamma\alpha_s^k}{\kappa^2+\lambda} \tag{B.24}$$

$$F_u^{k+1} = \frac{\sigma\kappa(1+\Gamma\alpha_i^k)}{\kappa^2+\lambda} \tag{B.25}$$

4. Check convergence.

   If $|F_u^{k+1} - F_u^k| < \varepsilon$ (e.g. $\varepsilon = 10^{-10}$), stop. Otherwise return to Step 2.

# C Solution Method for the Dynamic Model

This section provides details of the solution to the equilibrium inflation and output in the dynamic model.

First, we need to re-write the Phillips curve in terms of expected current shocks and expected future inflation and output. To do so, take $\mathbb{E}_t^p$ on both sides of (2):

$$\mathbb{E}_t^p \pi_t = \beta\theta\, \mathbb{E}_t^p \pi_{t+1} + (1-\theta)\, \mathbb{E}_t^p \pi_t + \kappa\theta\, y_t + \mathbb{E}_t^p u_t \tag{C.1}$$

This equation implies that firms are able to observe current output, as the actual $y_t$ enters $\mathbb{E}_t^p \pi_t$. This is because the IS curve is a function of expected future equilibrium and the interest rate. Expectations are the same between households and firms, and both households and firms are able to observe the interest rate.

Collecting $E_t^p \pi_t$ and substituting back into the Phillips curve yields

$$\pi_t = \beta\, \mathbb{E}_t^p \pi_{t+1} + \kappa y_t + \Gamma\, \mathbb{E}_t^p u_t + u_t \tag{C.2}$$

where $\Gamma \equiv (1-\theta)/\theta$.

There are 10 coefficients to determine. We separate them into two groups. First, $(\phi_u \phi_o, \psi_u, \psi_p)$ are involved in the expected future equilibrium, and will be solved as a fixed-point solution to a $4 \times 4$ system by matching coefficients on $u_t$ and $\mathbb{E}_{t-1}^p u_{t-1}$ between the structural equations and the conjectured equations of inflation and output. The remaining 6 parameters are solved in closed-form by matching coefficients on $\varepsilon_t^s$, $\xi_t$, and $e_t$.

### C.1 The 4 × 4 system

First, in the IS equation, substitute $i_t$ by $i_t = F_u u_t + F_l \mathbb{E}^p_{t-1} u_{t-1} + F_s s_t + e_t$, substitute expected future output and inflation by $\mathbb{E}_t u_t$, and substitute $\mathbb{E}_t u_t$ by the belief-updating process. Matching the coefficients on $u_t$ yields

$$\phi_u = (\phi_u \rho + \phi_o) \cdot \alpha^p \alpha - \frac{1}{\sigma}\Big[(F_u + F_s) - (\psi_u \rho + \psi_o) \cdot \alpha^p \alpha\Big] \tag{C.3}$$

Rearrange and get:

$$\phi_u(1 - \rho\alpha^p\alpha) - \phi_o \alpha^p \alpha - \frac{\rho\alpha^p\alpha}{\sigma}\psi_u - \frac{\alpha^p\alpha}{\sigma}\psi_o = -\frac{F_u + F_s}{\sigma} \tag{IS-$u$}$$

Then, match the coefficients on $\mathbb{E}_{t-1} u_{t-1}$ and get

$$\phi_o = (\phi_u \rho + \phi_o) \cdot \rho(1 - \alpha^p) - \frac{1}{\sigma}\Big[F_l - (\psi_u \rho + \psi_o) \cdot \rho(1 - \alpha^p)\Big] \tag{C.4}$$

Rearrange and get

$$-\rho^2(1-\alpha^p)\phi_u + (1 - \rho(1-\alpha^p))\phi_o - \frac{\rho^2(1-\alpha^p)}{\sigma}\psi_u - \frac{\rho(1-\alpha^p)}{\sigma}\psi_o = -\frac{F_l}{\sigma} \tag{IS-$E$}$$

Next, in the Phillips curve equation, substitute the expected future output and inflation with $\mathbb{E}_t u_t$, substitute $y_t$ with its conjectured form, and substitute $\mathbb{E}_t u_t$ with the belief-updating process. Matching the coefficients on $u_t$:

$$\psi_u = \beta(\psi_u \rho + \psi_o)\alpha^p \alpha + \kappa\phi_u + \Gamma\alpha^p\alpha + 1 \tag{C.5}$$

Rearrange and get

$$-\kappa\theta\phi_u + \psi_u\Big[1 - (1-\theta)\alpha^p\alpha - \beta\theta\rho\alpha^p\alpha\Big] - \beta\theta\alpha^p\alpha\,\psi_o = 1 \tag{PC-$u$}$$

Match the coefficient on $\mathbb{E}_{t-1} u_{t-1}$ and get

$$\psi_o = \beta(\psi_u \rho + \psi_o)\rho(1-\alpha^p) + \kappa\phi_o + \Gamma\rho(1-\alpha^p) \tag{C.6}$$

Rearrange and get

$$-\kappa\phi_o - \frac{\rho(1-\alpha^p)(\beta\theta\rho + (1-\theta))}{\theta}\psi_u + \left(1 - \frac{\beta\theta\rho(1-\alpha^p)}{\theta}\right)\psi_o = 0 \tag{PC-$E$}$$

Finally, $(\phi_u\phi_o, \psi_u, \psi_p)$ is the solution to the system of equations of (IS-$u$)–(PC-$E$) The matrix form is given by

$$\underbrace{\begin{bmatrix} 1-\rho\alpha^p\alpha & -\alpha^p\alpha & -\frac{\rho\alpha^p\alpha}{\sigma} & -\frac{\alpha^p\alpha}{\sigma} \\ -\rho^2(1-\alpha^p) & 1-\rho(1-\alpha^p) & -\frac{\rho^2(1-\alpha^p)}{\sigma} & -\frac{\rho(1-\alpha^p)}{\sigma} \\ -\kappa\theta & 0 & 1-(1-\theta)\alpha^p\alpha-\beta\theta\rho\alpha^p\alpha & -\beta\theta\alpha^p\alpha \\ 0 & -\kappa & -\frac{\rho(1-\alpha^p)(\beta\theta\rho+(1-\theta))}{\theta} & 1-\frac{\beta\theta\rho(1-\alpha^p)}{\theta} \end{bmatrix}}_{A} \begin{bmatrix} \phi_u \\ \phi_o \\ \psi_u \\ \psi_o \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{F_u+F_s}{\sigma} \\ -\frac{F_l}{\sigma} \\ 1 \\ 0 \end{bmatrix}}_{\mathbf{b}} \tag{C.7}$$

The solution is $[\phi_u, \phi_o, \psi_u, \psi_o]' = A^{-1}\mathbf{b}$.

### C.2 Auxiliary coefficients

Once $(\phi_u, \phi_o, \psi_u, \psi_o)$ are known, all remaining coefficients are computed in closed form, one at a time. To simplify notations, first define the forward term:

$$\mathcal{F} \equiv \phi_u\rho + \phi_o + \frac{\psi_u\rho + \psi_o}{\sigma} \tag{C.8}$$

This collects all of the forward-looking IS terms that recur across the $\varepsilon_t^s$, $\xi_t$, and $e_t$ channels. Also define the Phillips curve forward multiplier:

$$\Gamma_{pc} \equiv \beta\theta(\psi_u\rho + \psi_o) + (1-\theta)\psi_u \tag{C.9}$$

which collects the inflation expectation and current-period PC terms.

First work on the coefficient on $\varepsilon_t^s$ on the IS curve. $\varepsilon_t^s$ enters the IS curve first through $\mathbb{E}_t^p u_t$, which propagates through $E_t^p y_{t+1}$ and $E_t^p \pi_{t+1}$ with coefficients $(\phi_u\rho + \phi_o)\alpha^p\alpha_s$ and $(\psi_u\rho + \psi_o)\alpha^p\alpha_s$ respectively. Net IS contribution: $\alpha^p\alpha_s\mathcal{F}$. In addition, since $i_t$ reacts to $\varepsilon_t^s$ and $i_t$ directly affects $y_t$, $\varepsilon_t^s$ also enters IS with coefficient $-F_s/\sigma$.

Therefore:

$$\phi_s = \alpha^p\alpha_s\mathcal{F} - \frac{F_s}{\sigma} \tag{C.10}$$

Next work on the coefficient on $\varepsilon_t^s$ on the Phillips curve. Comparing the structural Phillips curve and the conjectured inflation equilibrium yields

$$\psi_s = \Gamma_{pc} \cdot \alpha^p\alpha_s + \kappa\theta\phi_s \tag{C.11}$$

$\xi_t = \zeta_s s_t$ is the private sector's misperception, not the actual rule, so there is no direct contribution from the interest rate, only through beliefs. Equating coefficients on $\xi_t$ in the IS curve and the Phillips curve yields:

$$\phi_\xi = c\mathcal{F} \tag{C.12}$$

$$\psi_\xi = \Gamma_{pc} \cdot c + \kappa\theta\phi_\xi \tag{C.13}$$

$e_t$ enters beliefs with the same coefficient $c$ as $\xi_t$ and it also enters the IS curve through $i_t$, which gives an additional $-1/\sigma$ term in the IS curve. Comparing coefficients on $e_t$ in the IS curve yields:

$$\phi_e = \phi_\xi - \frac{1}{\sigma} \tag{C.14}$$

$e_t$ affects $\pi_t$ both through $\kappa y_t$ and through expectations. Comparing the coefficient on $e_t$ in the Phillips curve yields:

$$\psi_e = \psi_\xi - \frac{\kappa\theta}{\sigma} \tag{C.15}$$

# D Proofs

## D.1 Proof of Lemma 3

I re-write the interest rate in terms of the total response to $u_t$: The interest rate is:

$$i_t = F_u u_t + F_s s_t = \underbrace{(F_u + F_s)}_{\equiv A} u_t + F_s \varepsilon_t^s \tag{D.1}$$

This re-parameterization is one-to-one. $A$ is the total response to $u_t$: it governs how aggressively the central bank stabilizes cost-push shocks. $F_s$ is the response to signal noise $\varepsilon_t^s$: it governs the central bank's use of the noisy signal.

The Kalman precision of the interest rate signal depends on $F_u = A - F_s$ through:

$$\bar{\tau}_i(A, F_s) = \frac{(A - F_s)^2}{\sigma_{F_s}^2(\sigma_u^2 + \sigma_s^2)} \tag{D.2}$$

Raising $A$ holding $F_s$ fixed is equivalent to raising $F_u$, which raises $\bar{\tau}_i$ and hence $\alpha_i$. This is the key monotonicity used in both proofs below.

Substituting the equilibrium inflation rate and output level into the loss function:

$$\mathcal{L}(A, F_s; \sigma_{F_s}^2) = \underbrace{(\psi_u^2 + \lambda\phi_u^2)\sigma_u^2 + (\psi_s^2 + \lambda\phi_s^2)\sigma_s^2}_{\mathcal{L}_0(A, F_s)} + \underbrace{(\psi_\xi^2 + \lambda\phi_\xi^2)\sigma_{F_s}^2(\sigma_u^2 + \sigma_s^2)}_{\mathcal{C}(A, F_s; \sigma_{F_s}^2)} \tag{D.3}$$

The term $\mathcal{C}$ is the *information cost*. It is strictly positive with the information effect and is zero in the no-information-effect benchmark. Note that $\mathcal{L}_0 \neq \mathcal{L}^{NI}$: both have the same functional form but with the information effect, the Kalman weights $(\alpha_s, \alpha_i, \alpha^p)$ differ from their *NI* benchmark values, so does the equilibrium coefficients $(\psi_u, \psi_s, \phi_u, \phi_s)$ as well.

Under the information effect, $\alpha_i > 0$, so $\mathcal{C} > 0$ for all $\sigma^2_{F_s} > 0$. The concentrated loss is:

$$\tilde{\mathcal{L}}(A; \sigma^2_{F_s}) = \tilde{\mathcal{L}}_0(A; \sigma^2_{F_s}) + \tilde{\mathcal{C}}(A; \sigma^2_{F_s}) \tag{D.4}$$

where $\tilde{\mathcal{C}}(A; \sigma^2_{F_s}) = \mathcal{C}(A, F^*_s(A); \sigma^2_{F_s}) > 0$. It is easy to see that $\partial \tilde{\mathcal{L}} / \partial \sigma^2_{F_s} > 0$, and therefore the loss — and hence its minimizer $A^{*,H1}$ — depends on $\sigma^2_{F_s}$.

In addition, at fixed $(A, F_s)$:

$$\frac{\partial \mathcal{L}}{\partial \sigma^2_{F_s}} = \frac{\partial \mathcal{L}_0}{\partial \sigma^2_{F_s}} + (\psi^2_\xi + \lambda \phi^2_\xi)(\sigma^2_u + \sigma^2_s) \tag{D.5}$$

The second term is strictly positive since $\psi^2_\xi + \lambda \phi^2_\xi > 0$ under the information effect. The first term $\partial \mathcal{L}_0 / \partial \sigma^2_{F_s}$ captures how the Kalman weights $(\alpha_s, \alpha_i, \alpha^p)$ change with $\sigma^2_{F_s}$: as $\sigma^2_{F_s}$ rises, $\bar{\tau}_i$ falls, $\alpha_i$ falls, and $\alpha_s$ rises back toward 1. By the envelope theorem applied to the Kalman filter — the private sector optimally aggregates signals — this reweighting has only a second-order welfare effect on $\mathcal{L}_0$ at the margin (the private sector was already optimally weighting the signals). Hence the first-order effect of $\sigma^2_{F_s}$ on the loss is dominated by the direct cost in the second term. Therefore:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \sigma^2_{F_s}} > 0 \tag{D.6}$$

The concentrated loss is strictly increasing in $\sigma^2_{F_s}$, so its minimizer $A^*$ depends on $\sigma^2_{F_s}$. Certainty equivalence fails.

### D.2 Proof of Proposition 1

The *NI* benchmark concentrated first-order condition gives:

$$\left. \frac{d \tilde{\mathcal{L}}^{NI}}{dA} \right|_{A^{*,NI}} = 0 \tag{D.7}$$

By the envelope theorem, under the information effect, decompose the concentrated derivative at $A^{*,NI}$ into the *NI* component and the additional information distortion under the

information effect:

$$\frac{\partial \mathcal{L}}{\partial A} = \frac{\partial \mathcal{L}_0}{\partial A} + \frac{\partial \mathcal{C}}{\partial A} \tag{D.8}$$

Since the Kalman weight on $s_t$ is smaller under the information effect,

$$\left.\frac{\partial \mathcal{L}_0}{\partial A}\right|_{A^{*,NI}} \geq \left.\frac{d\tilde{\mathcal{L}}^{NI}}{dA}\right|_{A^{*,NI}} = 0 \tag{D.9}$$

with the inequality strict whenever $\alpha_i > 0$.

Conditional on the optimal allocation between $F_u$ and $F_s$, Lemma 2 implies that

$$\left.\frac{\partial \mathcal{C}}{\partial A}\right|_{A^{*,NI}, F_s^*(A^{*,NI})} > 0 \tag{D.10}$$

Combine all terms ((D.9) and (D.10)):

$$\left.\frac{d\tilde{\mathcal{L}}}{dA}\right|_{A^{*,NI}} = \underbrace{\left.\frac{\partial \mathcal{L}_0}{\partial A}\right|_{A^{*,NI}}}_{\geq 0} + \underbrace{\left.\frac{\partial \mathcal{C}}{\partial A}\right|_{A^{*,NI}}}_{> 0} > 0 \tag{D.11}$$

In the baseline case, the concentrated loss is strictly increasing in $A$ at $A^{*,NI}$. The CB reduces its loss by setting $A < A^{*,NI}$. The first-order condition:

$$\left.\frac{d\tilde{\mathcal{L}}}{dA}\right|_{A^*} = 0 \tag{D.12}$$

is satisfied at some $A^* < A^{*,NI}$, completing the proof of Proposition 1.