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A Behavioral Foundation for the Investment Wedge

Jean-Paul L’Huillier Pierlauro Lopez Sanjay R. Singh*

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Abstract

Motivated by behavioral evidence, we develop a tractable method for incorporating competition neglect into a general equilibrium firm investment problem. Competition neglect causes firms to systematically underestimate the investment of their competitors. When we introduce competition neglect into a canonical RBC model, this friction acts like an investment wedge that causes over-investment at first, and under-investment later on. In contrast to a model with exogenous investment shocks, these dynamics are accompanied by a realistic variation in equity premia, even in the absence of financial frictions. Investment booms raise stock prices in general equilibrium, predicting periods of low excess returns going forward. The model can generate realistic comovement of real and financial variables.

Keywords: Animal spirits, boom-bust cycles, behavioral macroeconomics.

JEL codes: E22, E32, D21, D91.

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1 Introduction

Despite Keynes' influential idea that animal spirits and investment-driven business cycles are crucial to understanding major macroeconomic events such as the Great Depression and the Great Recession, modeling these dynamics in standard DSGE frameworks has remained a challenging exercise. To this day, the concept of investment-driven fluctuations remains difficult to operationalize. Efforts to study the effects of exogenous investment shocks, such as those by Justiniano, Primiceri, and Tambalotti (2010), have been influential and have successfully replicated business cycles in quantitative models, but with a significant shortcoming: These models fail to generate realistic comovement across macroeconomic and financial aggregates.

This paper develops a behavioral framework to microfound the sources of investment fluctuations. Expressed in terms of the business cycle accounting methodology introduced by Chari, Kehoe, and McGrattan (2007), our framework gives rise to an *endogenous* investment wedge (without the addition of exogenous shocks to investment dynamics). The cornerstone of our approach is a behavioral friction known as *competition neglect* (CN), introduced by Greenwood and Hanson (2015), whereby firms systematically neglect their competitors' investment decisions. This causes distortions in firms' expectations about their own returns to investment.

We explore the implications of these distortions in otherwise efficient general equilibrium economies. In response to a productivity shock, our endogenous investment wedge displays boom-bust dynamics. The investment wedge is initially negative, acting like an investment subsidy, and then turns positive, acting like an investment tax. Relative to the frictionless benchmark, the model therefore produces over-investment initially followed by under-investment. Remarkably, when we turn to financial variables, competition neglect is able to produce realistic stock price dynamics, even in the absence of other ingredients. Specifically, the investment boom generates a stock market boom that is predictably followed by low excess returns. Moreover, a standard Campbell-Shiller decomposition reveals that the amount of return predictability produced by the model matches empirical moments.

Under CN, firms underestimate the investment responses of other firms. We build on Greenwood and Hanson (2015), who study a partial equilibrium Q theory investment model. Neglect causes firms to overinvest following a positive shock, because they overestimate the returns to investment. These returns are

a negative function of aggregate investment, driven by other firms’ investment, which is *underestimated* by firms. In this paper, we introduce this insight into a fully fledged DSGE framework. We show how to operationalize the concept of CN in nonlinear general equilibrium settings. By doing so, we derive an explicit form for the investment wedge and demonstrate its quantitative implications for macroeconomic fluctuations.

The psychological foundation of CN is rooted in well-documented cognitive biases. Camerer and Lovallo (1999) demonstrate that overconfidence leads individuals to overestimate their relative abilities, resulting in excessive market entry, a phenomenon closely related to firms’ overinvestment. These biases align with broader evidence from social psychology, where individuals consistently exhibit overconfidence in various domains, such as the well-documented “better-than-average” effect (Svenson 1981).¹ (Kahneman, Egan, et al. 2011) further emphasizes that this kind of behavioral error is prevalent in contexts that feature delayed feedback, such as real-world investment problems. By this theoretical mechanism, delayed feedback reinforces firms’ systematic errors in forecasting investment returns. Moreover, the findings of Greenwood and Hanson (2015) emphasize that CN can interact with extrapolation (Bordalo, Gennaioli, and Shleifer 2022) to generate powerful amplification. By incorporating these psychological insights into macroeconomic models, we can better understand their impact on investment distortions and cyclical fluctuations.

As a first step, we develop a tractable solution method that allows for the incorporation of CN into general equilibrium models. We write down the distorted problem of the behavioral firm, making explicit the bounds on rationality that the firm faces. When doing this, we suitably split the firm, thereby carefully isolating the friction to the branch of the firm that chooses capital investment, to align with the evidence provided by Greenwood and Hanson (2015). As a result, our approach provides a bounded rationality equivalence to the behavioral friction, making it both analytically convenient and broadly applicable to a range of macroeconomic settings.

Conventional real business cycle (RBC) and dynamic stochastic general equilibrium (DSGE) models typically assume that firms make investment decisions under rational expectations (RE). This means, in particular, that they understand the competitive responses of other firms in the market when evaluating the returns to investment. By changing this assumption to the more realistic

¹This classic study in social psychology shows that a majority of drivers regard themselves as more skillful and less risky than the average driver in each respective group, which is statistically impossible.

CN assumption, we obtain the following qualitative and quantitative results.

Qualitatively, we obtain a closed-form solution for the wedges implied by the behavioral friction in a benchmark RBC setting, connecting CN to the broader literature on business cycle accounting. Interestingly, CN drives only an investment wedge, with the efficiency, labor, and government spending wedge at zero. We show that the investment wedge introduced by CN does not lead to simple mean-reverting dynamics. Instead, it generates endogenous investment booms and busts, even in response to standard AR(1) productivity shocks. In particular, firms' misperceptions cause them to overinvest following positive shocks, which leads to an overaccumulation of capital. As these misperceptions are later corrected, investment falls below its efficient level, generating a bust phase. This mechanism provides a novel explanation for observed patterns of investment economic shocks, and adds significant structure to Keynes' idea of animal spirits.

Quantitatively, the financial implications of CN are examined through its impact on stock prices, earnings, and asset returns. Overall, the model produces time variation in equity premia. Specifically, the model predicts that stock prices initially surge due to excessive investment, but later decline as firms adjust their expectations. These dynamics lead to predictable excess returns, where high valuations are followed by lower-than-expected stock returns, a pattern consistent with empirical findings on return predictability. We contrast these results with an alternative model featuring exogenous investment shocks, demonstrating that such a specification fails both to produce suitable financial comovement and to generate the sign-switching behavior of the investment wedge observed under CN. By endogenizing the investment wedge through competition neglect, the paper provides a novel explanation for cyclical investment patterns and excess return predictability.

1.1 Related Literature

This paper belongs to a literature in behavioral macroeconomics and finance that uses behavioral distortions to explain the comovement between macroeconomic and financial variables (see Barberis, Greenwood, Jin, and Shleifer 2015, Bordalo, Gennaioli, La Porta, O'Brien, and Shleifer 2024, and Bastianello and Fontanier 2024, among others). The investment problem we analyze is connected to the analysis in Angeletos, Huo, and Sastry (2021), who organize systematic biases in forecasts around alternative notions of imperfect information

and extrapolation. In closely related work, Angeletos and Lian (2023) provide a comprehensive analysis of “dampened GE,” a close cousin of the workings of CN in GE models. We study the application of this dampening of GE on a fully specified environment that leads to strategic substitutes. Our framework is an RBC model, featuring capital (an endogenous state variable). Because of the interaction of these behavioral frictions with the endogenous state, our framework gives rise to novel boom-bust dynamics in the investment wedge.

Closely related to the workings of CN, Bastianello and Fontanier (2024,2025) explore the notion of partial equilibrium thinking in financial markets, to show that it generates bubbles and amplification. Bordalo, Gennaioli, Shleifer, and Terry (2021) apply diagnostic expectation to a framework with heterogeneous firms and credit frictions. They obtain countercyclical credit spreads and aggregate fragility due to extrapolative behavior in good times. Ilut and Valchev (2022) study the effects of bounded rationality at the level of problem solving, a notion that is related to the bounded rationality on the perception of the firms’ competitors’ actions. Flynn and Sastry (2024a) explore how limited firm attention generates cycles. Flynn and Sastry (2024b) explore contagion dynamics due to behavioral belief formation. By embedding CN into a GE framework, our paper contributes to this growing literature on how behavioral frictions shape macroeconomic fluctuations.²

A classic problem in business cycle research concerns the ability of dynamic GE models to generate realistic business cycle fluctuations. A common challenge is the one of achieving the right kind of comovement among real variables (Barro and King 1984). This issue has been discussed in multiple setups. Close connections to our work are found in the context of news shocks (Beaudry and Portier 2007; Jaimovich and Rebelo 2009), and in the context of two-sector models (Basu, Fernald, and Liu 2012; Basu, Fernald, Fisher, and Kimball 2013). Remarkably, it is straightforward to generate the right comovement with the CN model, even in the absence of other frictions, given that the underlying be-

²Farhi and Werning (2019) and Gabaix (2020) introduce behavioral frictions into New Keynesian models, demonstrating how cognitive limitations and myopia alter macroeconomic dynamics. In related work, Bianchi-Vimercati, Eichenbaum, and Guerreiro (2024) explore level- k agents and the effects of fiscal spending. Garcia-Schmidt and Woodford (2019) explore the related notion of reflective equilibrium. Adam, Marcet, and Beutel (2017) consider stock price extrapolation in the context of learning among internally consistent agents. Bordalo, Gennaioli, and Shleifer (2018) and Bordalo, Gennaioli, Ma, and Shleifer (2020) pioneer the modeling of extrapolation based on the diagnostic expectations distortion (see Gabaix 2019 for a re-interpretation in terms of attention, and for further analysis). See also Bianchi, Ilut, and Saijo (2024), L’Huillier, Singh, and Yoo (2023), and Maxted (2023), who study diagnostic expectation in general equilibrium.

havioral dynamics inherit the properties of TFP shocks, which are the drivers of fluctuations.

Our paper is also related to the literature studying the importance of the investment wedge in business cycle fluctuations. See, for example, Greenwood, Hercowitz, and Huffman (1988), Greenwood, Hercowitz, and Krusell (1997), Fisher (2006), Justiniano, Primiceri, and Tambalotti (2010), Schmitt-Grohé and Uribe (2011). Basu, Candian, Chahrour, and Valchev (2025) connect risky investment to financial aggregates. We contribute to this literature by micro-founding an endogenous source of an investment wedge that is grounded in a behavioral limitation of investment firms. The dynamics implied by our endogenous wedge are less prone to the co-movement problem generated by exogenous investment wedges. Also, the endogenous wedge is able to generate comovement of financial variables.

1.2 Organization

The paper is organized as follows. Section 2 presents our solution method for embedding CN into DSGE models. Section 3 develops a real business cycle model with CN and characterizes the investment wedge analytically. Section 4 provides a quantitative analysis, exploring the empirical relevance of the investment wedge and its implications for financial moments. Section 5 concludes.

2 Solution Method: Competition Neglect in General Equilibrium

We outline our solution method for competition neglect in general equilibrium (GE). As in Greenwood and Hanson (2015) (henceforth, GH), we focus on firms, which make investment decisions in anticipation of future returns. When forming their expectations, firms are subject to the competition neglect bias. This bias leads them to underestimate the investment responses of their competitors, thereby overestimating the returns that good opportunities bring.

It is important to recall that the original GH model is specified in partial equilibrium. Their setup exploits tractability under linear decision rules, where CN can be introduced by the direct application of simple rules on the expectation operator.³ In contrast, a crucial implication of GE is that, in a canonical

³See GH, p. 87.

production structure, the investment firm problem is no longer linear in aggregate capital. Hence, the same direct approach to modifying the expectation operator is no longer available. In nonlinear settings, one needs to write the general form of the distorted firm objective, an undertaking that is the main goal of this section. The distorted objective of the firm leads to an “as if” problem, which can then be optimized to find the CN solution. Our equilibrium definition shows how this solution can be made consistent with GE. We fully characterize this approach, and also establish an equivalence result by applying it to the original GH problem.

2.1 Model Setup: Physical Environment

In this setup, we focus on the problem of the investment firm. This firm invests with the goal of accumulating capital, which is then rented out. For this reason we also refer to this firm, as is common in the DSGE literature, as a “capital-goods producer.” We write this framework with the goal of embedding it in a standard RBC model, which is fully set up in Section 3.

A continuum of competitive investment firms, indexed by $f \in [0, 1]$, accumulate capital by investing units of an undifferentiated final good. The firm’s law of motion of capital is

$$K_{f,t+1} = K_{ft} + I_{ft} \quad (1)$$

where the investment choice I_{ft} is expressed net of depreciation (i.e., if gross investment is I_{ft}^G , $I_{ft} = I_{ft}^G - \delta K_{ft}$, where δ is the capital depreciation rate). Each firm starts from a common initial condition of its own capital, $K_{f0} = K_0$. All firms are identical, and using symmetry, we can obtain the aggregate law of motion of capital

$$K_{t+1} = K_t + I_t \quad (2)$$

Firms choose a sequence of investment decisions to maximize profits. Profits are a function of the market rental rate for capital services. Taking standard steps in the profit maximization program of competitive final-good producers with a Cobb-Douglas production function, we obtain the following expression for this market rental rate:

$$H_t = \alpha \left(\frac{A_t N_t}{K_t} \right)^{1-\alpha} \quad (3)$$

where α is the capital share and N_t is the labor input. TFP follows the exogenous law of motion

$$\log A_t = (1 - \rho_a) \log \bar{A} + \rho_a \log A_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim N(0, \sigma^2)$.

Profits are discounted using the consumers' stochastic discount factor

$$M_{0,t} \equiv \prod_{s=1}^t M_s \quad (4)$$

which discounts date- t payoffs to date 0, where

$$M_t = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\gamma}, \quad (5)$$

C_t is consumption at time t , γ is the inverse of the intertemporal elasticity of substitution, and β is the consumer time preference factor.

Firms are subject to capital adjustment costs, which we denote by $\Phi(K_{ft}, I_{ft})$. We assume that in the steady state $\Phi(\bar{K}, \bar{I}) = \Phi_1(\bar{K}, \bar{I}) = \Phi_2(\bar{K}, \bar{I}) = 0$, and $\Phi_{22}(\bar{K}, \bar{I}) < 0$.

2.2 Model Setup: Distorted Firm Problem

Consider the problem of firm f at the initial period $t = 0$. The firm is subject to bounded rationality, operating with a degree of CN denoted by ω , $0 \leq \omega < 1$. Due to CN, firm f distorts its perception of aggregate investment by biasing it toward a *default value* of investment, which the firm believes to be the aggregate response when it fully neglects the reaction of its competitors. Here, we assume such a default value is the steady-state value of aggregate investment.

Accordingly, denote by I_t^ω the perceived date t , $t \geq 0$, of aggregate investment. It is given by

$$I_t^\omega \equiv \omega I_{ft} + (1 - \omega)I \quad (6)$$

where I_{ft} is the investment policy of firm f at date t , and I is steady-state investment. All firms are identical (we will formally impose symmetry in the solution below), and therefore, according to this equation, the behavioral firm underestimates how much its competitors' choices react to shocks. The bias reflects the firm's failure to understand the symmetric behavior of all com-

petitors, and hence its failure to incorporate it into its beliefs about aggregate investment.⁴ $\omega = 0$ means that CN is full: The firm believes that aggregate investment is equal to its steady-state value I . $\omega = 1$ is the RE benchmark of no CN, where the firm fully realizes that its competitors will react symmetrically.

Using equation (6), the perceived period t aggregate capital is given by

$$K_{t|0}^\omega \equiv K_0 + \sum_{l=0}^t I_l^\omega \quad (7)$$

where the subindex $t|0$ signifies that this is perceived period t capital, with perception formed at $t = 0$. (The perception of aggregate investment I_t^ω , instead, is formed state-by-state, as in equation (6).)

The decision is then repeated at date 1 and at every subsequent date. Following GH, we assume that, at the beginning of every t , firms (accurately) observe the current aggregate capital stock K_t (but have distorted beliefs about future capital), so

$$K_{t|t}^\omega = K_t \quad (8)$$

Naturally, firms do not suffer direct behavioral distortions when evaluating their *own* future capital K_{ft} , $t > 0$, in that they understand their individual investment choices. Therefore, they use the correct law of motion (1). Only their assessment of the evolution of aggregate capital is distorted.

We assume that this boundedly rational firm receives two informational inputs required for profit maximization. These inputs are given in terms of mappings from the perceived endogenous state (aggregate capital) and exogenous state (TFP) into rental rate and discount factor values. We write these mappings as $\mathcal{H}(K_{t|0}^\omega, A_t)$ and $\mathcal{M}_0(K_{t|0}^\omega, A_t)$, where the subscript “0” in the second mapping denotes dependence on date-0 aggregate states. The idea is that capital-producing firms economize on thinking costs by receiving these mappings as ready-to-use functions of their perceptions. Investment firms might lack the expertise or knowledge to compute these general equilibrium mappings, but have close contacts that are able to supply them. These contacts could emerge from consultation with final-goods producers in the case of the

⁴A quote from Joe Roth, former chairman of Walt Disney Studios, cited in Camerer and Lovo (1999), reflects this well (the question is about why so many big-budget movies are released on the same weekends): “Hubris. Hubris. If you only think about your own business, you think: ‘I’ve got a good story department, I’ve got a good marketing department, we’re going to go out and do this.’ And you don’t think that everybody else is thinking the same way. In a given weekend in a year you’ll have five movies open, and there’s certainly not enough people to go around.”

rental rate, or shareholders (households) in the case of discount factors.⁵ To maximize their profits, firms evaluate future scenarios by plugging in hypothetical values of future TFP A_t and perceived aggregate capital $K_{t|0}^\omega$, and obtain values for the rental rate H_t and discount factor $M_{0,t}$. This approach preserves bounded rationality and allows for a well-defined notion of general equilibrium. Namely, our definition of general equilibrium that we provide in Section 3 requires that these informational inputs are correct, in the sense that they are consistent with the equilibrium values of rental rates and discount factor values that actually occur in equilibrium.

Given the above definitions, we are now in a position where we can write the distorted problem that the investment firm solves. Our setup casts the problem of the firm that is subject to CN into the “as if” problem of a firm subject to bounded rationality, rationally choosing the degree of neglect. Specifically, firm f chooses a path of investment I_{ft} to solve:

$$\max_{\{I_{ft}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \mathcal{M}_0(K_{t|0}^\omega, A_t) \left\{ \mathcal{H}(K_{t|0}^\omega, A_t) K_{ft} - I_{ft} - \delta K_{ft} - \Phi(K_{ft}, I_{ft}) \right\} \quad (9)$$

subject to the individual capital accumulation equation (1). In this problem, perceived aggregate capital is given by equations (6) and (7). The expectation is taken over the objective distribution of shocks ε_t , which is known (undistorted) by the firm. The firm solves this problem separately for each period, i.e., at $t = 1$ the firm solves the problem anew.

The following technical remark is in order. In general problems, such as the nonlinear problem presented here, CN enters primitively as a distortion to the state-contingent aggregate policies, as in (6). Neglect ω makes such a distortion state specific, an important difference compared to the linear setting of GH.

2.2.1 Example: Greenwood and Hanson (2015)

We take a slight detour and first prove equivalence to the original GH problem of the above distorted problem. GH postulate the following linear capital rental

⁵Alternatively, these mappings could emerge from a within-firm segmentation, where one floor is charged with forecasting GE variables as functions of perceived capital, and another is charged with choosing investment given these forecasts. These mappings encompass what Kahneman and Lovallo (1993) call “inside view” of forecasts, according to which decision makers focus on local information (in this case, the firm’s own capital stock and internal cost determinants), but have scant knowledge of outside developments (such as the determination of GE variables).

rate:

$$H_t = A_t - BK_t \quad (10)$$

where B is a constant slope parameter. This function is perfectly known by the firm, and hence $\mathcal{H}(A_t, K_t) = H_t$. The discount factor is a constant and equal to β . The adjustment costs function is quadratic in investment, $\Phi(I_{ft}) = \phi I_{ft}^2/2$. As shown by GH substituting these variables into problem (9) and taking first-order conditions leads to the GH solution

$$I_{ft} = xK_t + yA_t$$

with $x = \frac{B\omega + r\phi}{2\phi\omega} - \sqrt{\left(\frac{B\omega + r\phi}{2\phi\omega}\right)^2 + \frac{B}{\phi\omega}}$ and $y = \frac{\rho_a}{\phi(1-\rho_a) - \frac{B}{x}}$ if $\omega > 0$, or $x = -\frac{B}{r\phi}$ and $y = \frac{\rho_a}{\phi(1-\rho_a) + r\phi}$ if $\omega = 0$. By symmetry, all firms choose the same level of investment and $I_t = I_{ft}$.

It can be verified that, under CN ($\omega < 1$), this solution leads to overinvestment when a positive shock $\varepsilon_0 > 0$ hits. That is, the behavioral firm overreacts compared to the rational firm ($\omega = 1$). This is because the behavioral firm fails to anticipate that an increase in investment by other firms increases aggregate investment I_0 and future capital K_t , $t > 0$, lowering returns H_t going forward. Notice that, in the steady state, investment is zero, $I = 0$, a property that extends to the general problem above. We discuss this feature below.

2.3 Recursive Formulation, Equilibrium, and Solution

Problem (9) can be written recursively. In fact, at date t , the firm knows the entire history of aggregate shocks up to time t , and will make current and future decisions based on a distorted aggregate investment policy function. At date $t+1$, the only thing that changes in the firm's problem is that it will have acquired information about the realized aggregate state (K_{t+1}, A_{t+1}) . But, other than the firm evaluating its optimal individual policy for date $t+1$ with the correct information (K_{t+1}, A_{t+1}) rather than with $(K_{t+1|t}^\omega, A_{t+1})$, the mapping of aggregate states into the optimal decision remains the same. The recursive problem is

$$\begin{aligned} V(K_{ft}, K_{t|t}^\omega, A_t) = \max_{I_{ft}} & \mathcal{H}(K_{t|t}^\omega, A_t)K_{ft} - I_{ft} - \delta K_{ft} - \frac{\phi}{2}I_{ft}^2 \\ & + \mathbb{E}_t \mathcal{M}_t(K_{t+1|t}^\omega, A_{t+1})V(K_{ft} + I_{ft}, K_{t|t}^\omega + I_t^\omega, A_{t+1}) \end{aligned}$$

or, by equation (8),

$$V(K_{ft}, K_t, A_t) = \max_{I_{ft}} \mathcal{H}(K_t, A_t) K_{ft} - I_{ft} - \delta K_{ft} - \frac{\phi}{2} I_{ft}^2 \\ + \mathbb{E}_t \mathcal{M}_t(K_{t+1|t}^\omega, A_{t+1}) V(K_{ft} + I_{ft}, K_t + I_t^\omega, A_{t+1})$$

subject to the individual capital accumulation equation (1).

The optimal choice is characterized by an investment Euler equation:

$$1 + \phi I_{ft} = \mathbb{E}_t \mathcal{M}_t(K_{t+1|t}^\omega, A_{t+1}) [\mathcal{H}(K_{t+1|t}^\omega, A_{t+1}) + 1 - \delta + \phi I_{f,t+1}] \quad (11)$$

In order to solve for the equilibrium strategy in the aggregate investment sector, we need to first impose symmetry. Care needs to be exercised at this step due to the belief distortion and its impact on aggregation. We first write equilibrium strategies using explicit policy function notation, to make explicit how the distortion of beliefs about future aggregate capital enters (11). For the investment choice I_{ft} that solves equation (11), we define the function $G_I(\cdot)$, which maps the individual state (capital), perceived aggregate capital, and TFP, into individual optimal choices

$$I_{ft} = G_I(K_{ft}, K_{t|t}^\omega, A_t)$$

which, given (8), boils down to $I_{ft} = G_I(K_{ft}, K_t, A_t)$. Notice that, one period ahead (that is, for the optimal individual investment on the right-hand side of (11)), we have

$$I_{f,t+1} = G_I(K_{f,t+1}, K_{t+1|t}^\omega, A_{t+1})$$

This equation highlights an additional effect of the belief distortion, which is that the firm misperceives aggregate investment going forward. This distortion comes on top of the main distortion on the perceived returns on investment through $\mathcal{H}(K_{t+1|t}^\omega, A_{t+1})$.

Symmetry is imposed by the requirement that aggregate and individual investment choices coincide:

$$I_t = G_I(K_t, K_t, A_t)$$

By the law of motion of capital and the common initial condition K_0 , this implies $K_{ft} = K_t$ for all t .

We define the equilibrium as follows.

Definition 1 (Capital-Good Sector Equilibrium) *For a given sequence of realizations of the exogenous process $\{A_t\}$ and given mappings of the rental rate $\mathcal{H}(\cdot, \cdot)$ and the stochastic discount factor $\mathcal{M}_t(\cdot, \cdot)$, a symmetric competitive equilibrium in the capital-good sector is given by a sequence of state-contingent individual choices $\{I_{ft}, K_{ft}\}$, perceived aggregate quantities $\{I_t^\omega, K_{t+\tau|t}^\omega\}$, for each firm f and each future date $t + \tau$, $\tau \geq 0$; and by a sequence of aggregate allocations $\{I_t, K_t\}$ such that, at each date $t \in \{0, \dots, \infty\}$, (a) the individual choice investment solves the individual firm's problem, (b) the aggregate capital-good market clears, and, (c) capital-goods producers' choices are symmetric.*

This definition pins down the equilibrium among behavioral firms. In order to construct a *general* equilibrium (GE), we will require the mappings used by firms to be *correct* in the sense that if, at period t , the firm plugs in the true value of the capital stock K_t and TFP A_t into, say, $\mathcal{H}(\cdot, \cdot)$, it obtains the true value of the rental rate, $\mathcal{H}(K_t, A_t) = H_t$. By this requirement, therefore, the firm is behavioral *only* to the extent that it misperceives K_t . In fact, the mappings $\mathcal{H}(\cdot, \cdot)$ and $\mathcal{M}_t(\cdot, \cdot)$ are simply a technical device to be able to “plug in” our behavioral firm model into any given DSGE model. Take any model that produces equilibrium values of H_t and M_t ; we can use the associated mappings $\mathcal{H}(\cdot, \cdot)$ and $\mathcal{M}_t(\cdot, \cdot)$ to embed our firm environment into that model.

Thus, our GE construction boils down to a fixed point between the behavioral firm equilibrium and the general equilibrium. At this fixed point, for the aggregate mappings that each individual firm takes as given, the investment sector produces a sequence of state-contingent values of aggregate investment and capital; for a given sequence of investment and capital, the general equilibrium structure produces values for the rental rate and the discount factor; the definition of equilibrium for the aggregate economy in the next section imposes consistency between the two. While we will apply this approach to a canonical RBC model, it should be clear that the approach is general.

We approximate the solution of the model to first order. This approach offers simplicity, remaining nevertheless sufficient to capture a realistic comovement between real and financial variables, as we will establish in Section 4.

3 A Real Business Cycle Model with Competition Neglect

In this section, we derive a real business cycle model augmented with competition neglect in the investment sector. In the spirit of business cycle accounting, we use a frictionless benchmark to isolate the endogenous investment wedge generated by this novel behavioral friction. We find that even productivity shocks, which are efficient in the frictionless benchmark, lead to an investment wedge, as well as create boom-bust dynamics in investment.

3.1 Model

Time is discrete and runs forever. There are four sets of agents in the economy: households, final-good-producing firms, capital-good-producing firms, and the government. Except for the behavioral friction in the capital-goods sector, all agents are fully rational, and other model ingredients are standard. We begin by describing each block, and then proceed to the equilibrium characterization.

3.1.1 Household

The household is modeled as a large family composed of a continuum of members, indexed by $f \in [0, 1]$, which provides perfect consumption insurance across its members. Each member owns an individual capital-good-producing firm. Individual investors manage their respective firms and are subject to CN when making investment decisions. Individual profits of such firms are paid to the respective investors, who then remit them to the household, which redistributes them equally among all members. The household makes aggregate consumption and labor supply decisions on behalf of its members and saves in a risk-free bond in zero net supply, which is priced by the household without behavioral distortions.⁶

Accordingly, at each time t , a representative household chooses consumption C_t , hours N_t , and savings in the form of real government bonds B_{t+1} , to

⁶This centralized decision-making under full insurance is a convenient device to allow for aggregation without introducing intra-household heterogeneity. See, for example, Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017).

maximize the following lifetime utility:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right] \quad (12)$$

subject to a per-period budget constraint

$$C_t + \frac{1}{1+r_t} B_{t+1} = B_t + W_t N_t + D_t + T_t$$

where χ parameterizes the disutility of labor, φ is the inverse of the Frisch elasticity of labor supply, W_t is the real wage, D_t are dividends from the ownership of investment and goods-producing firms, and T_t are lump-sum government transfers. Government bonds pay off at a $1 + r_t$ interest rate in the following period. (The remaining notation is conventional and is already defined in Section 2.)

3.1.2 Final-Good-Producing Sector

The final good is produced by a continuum of perfectly competitive producers. At each time t , identical firms produce the final good Y_t combining labor N_t and physical capital K_t , according to the technology: $Y_t = (A_t N_t)^{1-\alpha} K_t^\alpha$, where $0 < \alpha < 1$ is a Cobb-Douglas parameter. Firms hire labor and capital at wage rate W_t and rental rate H_t , respectively.

3.1.3 Capital-Good-Producing Sector

A continuum of competitive firms, indexed by f , accumulate capital goods by investing units of the undifferentiated final good. They are subject to bounded rationality in that they understand the dynamics of their own capital stock but have limited awareness of their competitors' actions, and therefore their perceived aggregate net investment is distorted. Their problem is similar to what was described in Section 2. To avoid repetition, we refer the reader to Section 2 for a detailed discussion, and only provide an outline here.

At each period t , these firms rent out their capital stock K_{ft} at perceived market rental rate $\mathcal{H}(K_{t+j|t}^\omega, A_{t+j})$, which they take as given. They spend δK_{ft} of the final good on repairing depreciated capital. Changes in net investment I_{ft} are subject to investment adjustment costs $\Phi(K_{ft}, I_{ft})$ as defined in Section 2. In this section, for analytical tractability, we assume quadratic adjustment

costs $\Phi(\cdot, \cdot) = \frac{\phi}{2} I_{it}^2$. In Section 4, we present quantitative results with a more general adjustment cost function.

Each firm chooses a path of individual investment to maximize the expected present discounted value of profits. The problem is described in Section 2. The stock of capital for each firm evolves according to equation (1).

3.1.4 Government

The government runs a balanced budget every period, spending G_t of the final good, and raising net taxes $-T_t$. We assume government bonds are in zero net supply.

3.1.5 Aggregate Resource Constraints

The final good in the economy is used for consumption, net investment, depreciation repairs, investment adjustment costs, and government spending. The final-goods market clearing is given by:

$$Y_t = C_t + I_t - \delta K_t - \frac{\phi}{2} I_t^2 + G_t \quad (13)$$

We will assume government spending is zero: $G_t = 0$. The aggregate capital stock, K_t , follows the conventional law of motion (2).

3.2 Equilibrium Characterization

Definition 2 (General Equilibrium) *A symmetric competitive general equilibrium with CN is a sequence of state-contingent allocations $\{C_t, N_t, Y_t, I_{ft}, K_{ft+1}, I_t, K_{t+1}\}$, prices $\{W_t, H_t, r_t\}$, and perceived aggregate quantities $\{I_{t+\tau}^\omega, K_{t+\tau|t}^\omega\}$, for each firm f and each future date $t + \tau$, $\tau \geq 0$ such that, at each date t , (a) the choices of consumption and hours solve the household's problem, (b) the choices of labor demand and production solve the problem of final-good producers, (c) the choices of neglect and investment solve the individual capital-goods producers' problem, (d) firms' choices are symmetric, (e) the mappings $\mathcal{H}(\cdot, \cdot)$, and $\mathcal{M}_t(\cdot, \cdot)$ satisfy consistency:*

$$\mathcal{H}(K_t, A_t) = H_t \quad (14)$$

$$\mathcal{M}_t(K_{t+1}, A_{t+1}) = M_{t,t+1} \quad (15)$$

and (f) prices clear goods, labor, and bond markets.

There is no CN in the steady state. We write the loglinearized system of equations around the steady state using a minimum state variable representation, in which all variables are written as a function of the aggregate capital stock and the level of TFP.

For some variable $X_t = \mathcal{X}(K_t, A_t)$, we denote $\psi_{Xk} \equiv \partial \mathcal{X}(K, 1) / \partial \log(K_t)$ and $\psi_{Xa} \equiv \partial \mathcal{X}(K, 1) / \partial \log(A_t)$, evaluated at the steady state. So, for example, $\psi_{ck} = \partial \log(C_t) / \partial \log(K_t)$, $\psi_{ca} = \partial \log(C_t) / \partial \log(A_t)$, $\psi_{Ik} = \partial I_t / \partial \log(K_t)$, $\psi_{Ia} = \partial I_t / \partial \log(A_t)$, and so on. For variables inside the capital-goods producers' problem, the relevant state variable at time $t + \tau$ is the perceived capital stock $\hat{k}_{t+\tau|t}^\omega$, $\tau > 0$. In equilibrium, the solution to firm-level investment is equal to aggregate investment, since all firms are identical.

The loglinearized equations, with hats for deviations from the steady state, are

$$\psi_{rk}\hat{k}_t + \psi_{ra}\hat{a}_t = -\gamma \mathbb{E}_t \left[\psi_{ck} \left(\frac{\psi_{Ik}}{K} \hat{k}_t + \frac{\psi_{Ia}}{K} \hat{a}_t \right) + \psi_{ca}\hat{a}_{t+1} - \psi_{ca}\hat{a}_t \right] \quad (16)$$

$$(1 - \alpha)\hat{a}_t + \alpha\hat{k}_t = \gamma(\psi_{ck}\hat{k}_t + \psi_{ca}\hat{a}_t) + (\alpha + \varphi)(\psi_{nk}\hat{k}_t + \psi_{na}\hat{a}_t) \quad (17)$$

$$(1 - \alpha)(\hat{a}_t + \psi_{nk}\hat{k}_t + \psi_{na}\hat{a}_t) + \alpha\hat{k}_t = \frac{C}{Y}(\psi_{ck}\hat{k}_t + \psi_{ca}\hat{a}_t) + \frac{K}{Y} \left(\frac{\psi_{Ik}}{K} \hat{k}_t + \frac{\psi_{Ia}}{K} \hat{a}_t \right) + \frac{\delta K}{Y} \hat{k}_t \quad (18)$$

$$\begin{aligned} \phi K \left(\frac{\psi_{Ik}}{K} \hat{k}_t + \frac{\psi_{Ia}}{K} \hat{a}_t \right) &= -\gamma \mathbb{E}_t \left[\psi_{ck}\hat{k}_{t+1|t}^\omega + \psi_{ca}\hat{a}_{t+1} - \psi_{ck}\hat{k}_t - \psi_{ca}\hat{a}_t \right] \\ &+ (1 - \alpha)[1 - \beta(1 - \delta)] \mathbb{E}_t \left[\hat{a}_{t+1} + \psi_{nk}\hat{k}_{t+1|t}^\omega + \psi_{na}\hat{a}_{t+1} - \hat{k}_{t+1|t}^\omega \right] \\ &+ \beta \phi K \mathbb{E}_t \left[\frac{\psi_{Ik}}{K} \hat{k}_{t+1|t}^\omega + \frac{\psi_{Ia}}{K} \hat{a}_{t+1} \right] \end{aligned} \quad (19)$$

$$\hat{k}_{t+1|t}^\omega = \hat{k}_t + \omega \left(\frac{\psi_{Ik}}{K} \hat{k}_t + \frac{\psi_{Ia}}{K} \hat{a}_t \right) \quad (20)$$

The exogenous AR(1) process for $\hat{a}_t$ is given by :

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \epsilon_t \quad (21)$$

where ϵ_t are iid shocks $\sim N(0, \sigma_a^2)$. Equations (16) - (18) are aggregate intratemporal labor supply, intertemporal consumption, and economy resource constraint equations, respectively. Equation (19) is the investment Euler equation for the capital-goods producer, hence the dependence on the perceived capital stock $\hat{k}_{t+1|t}^\omega$. Equation (20) is the law of motion for the perceived capital stock.

The solution can be obtained by undetermined coefficients. We present the solution in Appendix B.1 and summarize it in Proposition 1.

Proposition 1 *The solution to the real business cycle CN model takes the form of policy functions for aggregate endogenous variables $\{\hat{n}_t, \hat{k}_{t+1}, \hat{I}_t, \hat{c}_t, \hat{r}_t\}$:*

$$\hat{x}_t = \psi_{xk}\hat{k}_t + \psi_{xa}\hat{a}_t \quad (22)$$

The perceived aggregate capital stock is given by the following law of motion:

$$\hat{k}_{t+\tau+1|t}^\omega = (1 + \omega \frac{\psi_{Ik}}{K})\hat{k}_{t+\tau|t}^\omega + \omega \frac{\psi_{Ia}}{K}\mathbb{E}_t\hat{a}_{t+\tau} \quad (23)$$

with $\hat{k}_{t|t}^\omega = \hat{k}_t$ at each time t .

3.3 Endogenous Investment Wedge

In order to construct the investment wedge produced by CN, we will consider a frictionless benchmark model. This model is the same as the RBC model, except that the investment-goods sector has full information rational expectations. We dub this model the “prototype model.” This prototype model will be written with four wedges: a labor wedge τ_{nt} (tax on household’s labor income), an efficiency wedge τ_{at} , an investment wedge τ_{It} (tax on net investment), and a government spending wedge τ_{gt} . These wedges will allow us to trace out the implications of CN.

We first list the loglinearized equations of the prototype model, denoting its endogenous variables with an asterisk. The loglinearized equilibrium in the prototype model is given by the sequence of $\{\hat{c}_t^*, \hat{r}_t^*, \hat{I}_t^*, \hat{n}_t^*, \hat{k}_{t+1}^*\}$ for a given exogenous shock process $\{\hat{a}_t\}$, For given processes for wedges $\{\tau_{at}, \tau_{nt}, \tau_{gt}, \tau_{It}\}$

and the initial capital stock $\hat{k}_{-1}$, these equations are:

$$\hat{c}_t^* = \mathbb{E}_t \hat{c}_{t+1}^* - \frac{1}{\gamma} \hat{r}_t^* \quad (24)$$

$$(1 - \alpha)(\hat{a}_t + \tau_{at}) + \alpha \hat{k}_t^* = \gamma \hat{c}_t^* + (\alpha + \varphi) \hat{n}_t^* + \tau_{nt} \quad (25)$$

$$(1 - \alpha)(\hat{a}_t + \hat{n}_t^*) + \alpha \hat{k}_t^* = \frac{C}{Y} \hat{c}_t^* + \frac{1}{Y} \hat{I}_t^* + \frac{\delta K}{Y} \hat{k}_t^* + \tau_{gt} \quad (26)$$

$$\begin{aligned} \tau_{It} + \phi \hat{I}_t^* = & -\gamma(\mathbb{E}_t \hat{c}_{t+1}^* - \hat{c}_t^*) + (1 - \alpha)[1 - \beta(1 - \delta)]\mathbb{E}_t(\hat{a}_{t+1} + \hat{n}_{t+1}^* - \hat{k}_{t+1}^*) \\ & + \beta \phi \mathbb{E}_t \hat{I}_{t+1}^* + \beta \mathbb{E}_t \tau_{It+1} \end{aligned} \quad (27)$$

$$\hat{k}_{t+1}^* = \hat{k}_t^* + \frac{1}{K} \hat{I}_t^* \quad (28)$$

Equation (27) recursively defines the investment wedge τ_{It} as a function of the observed variables in the prototype model. Following the standard business cycle accounting procedure (Chari, Kehoe, and McGrattan 2007), we proceed to recover the wedges such that the prototype model with wedges is observationally equivalent to the CN model. We obtain the following result.

Proposition 2 (Business Cycle Accounting) *The dynamics of output, consumption, investment, capital stock, and the real interest rate in the prototype model with wedges are observationally equivalent to that of CN when*

1. *the investment wedge is given by:*

$$\tau_{It} = \nu_k K \hat{k}_t + \nu_a K \hat{a}_t \quad (29)$$

where the coefficients are given by

$$\begin{aligned} \nu_k &= (1 - \omega) \left(-\gamma \frac{\psi_{ck}}{K} - \beta B(1 - \psi_{nk}) + \beta \phi \psi_{Ik} \right) \frac{\psi_{Ik}}{1 - \beta(1 + \psi_{Ik})} \\ \nu_a &= (1 - \omega) \left(-\gamma \frac{\psi_{ck}}{K} - \beta B(1 - \psi_{nk}) + \beta \phi \psi_{Ik} \right) \frac{1 - \beta}{1 - \beta(1 + \psi_{Ik})} \frac{\psi_{Ia}}{1 - \beta \rho_a} \end{aligned}$$

with $B \equiv (1 - \alpha)[1 - \beta(1 - \delta)]/\beta K$, and K is the steady-state level of the capital stock, and the coefficients ψ_{xy} for $x = \{\hat{c}, \hat{n}, \hat{I}\}$ and $y = \{\hat{a}, \hat{k}\}$ are defined in Proposition 1, and

2. *the labor wedge, efficiency wedge, and government wedge are set to zero in all periods.*

The investment wedge given in Equation (29) is zero when $\omega = 1$, i.e., when there is no CN. Due to CN in the investment sector, $0 \leq \omega < 1$, a shock to

productivity gives rise to an investment wedge, which is effectively a distortion in the investment Euler equation. After a productivity shock, the capital-goods-producing firms neglect their competitors' symmetrical behavior and, as a result, make an incorrect estimate of the total investment in the economy, leading to distortions in the perceived returns to their own investment. Hence, an investment wedge emerges endogenously in the model.

Notice that the behavioral friction only drives a wedge on investment, and this wedge does not lead to inefficient movements in the rest of the economy. That is, all other wedges $\{\tau_{at}, \tau_{nt}, \tau_{gt}\}$ are equal to zero. The reason is simple: Given that the behavioral distortion only distorts investment firms, and that those firms do not hire labor, the static relationship between consumption, labor and output is undistorted. This holds even if the distortion propagates in GE and changes the equilibrium path of capital, which in turn changes the wage and the marginal rate of substitution between capital and labor in the first-order condition of final-good producers. This first-order condition still holds exactly, leading to no labor wedge. A similar reasoning shows why the other wedges are zero as well.

Because the distortions affect an endogenous state variable, i.e., the stock of capital, the wedge can be alternatively represented as the following ARMA (2,1) process, as stated in the following corollary.

Corollary 1 (ARMA (2,1) Representation of the Investment Wedge)

The investment wedge admits the following ARMA(2,1) representation:

$$\tau_{It} = (1 + \psi_{Ik} + \rho_a)\tau_{It-1} - (1 + \psi_{Ik})\rho_a\tau_{It-2} + \nu_a K\epsilon_t + [\nu_k\psi_{Ia} - \nu_a(1 + \psi_{Ik})]K\epsilon_{t-1}$$

The second-order dynamic representation reveals an important implication of the model. When the coefficient on τ_{It-2} has the opposite sign from τ_{It-1} , the model can produce a boom-bust dynamic in the investment wedge. In a special case, studied in Proposition 3, we analytically prove that the model gives rise to such interesting feedback effects.

Proposition 3 (Special Case: Boom-Bust Dynamics in Investment Wedge)

Assuming full competition neglect $\omega = 0$, linear utility in consumption ($\gamma = 0$), and inelastic labor supply $\chi = 0$, the coefficients on the investment wedge de-

rived in equation 29 are now given by

$$\begin{aligned}\nu_k &= (-\beta B + \beta\phi\psi_{Ik}) \frac{\psi_{Ik}}{1 - \beta(1 + \psi_{Ik})} > 0 \\ \nu_a &= (-\beta B + \beta\phi\psi_{Ik}) \frac{1 - \beta}{1 - \beta(1 + \psi_{Ik})} \frac{\psi_{Ia}}{1 - \beta\rho_a} < 0\end{aligned}$$

where $\psi_{Ik} = -\frac{B}{r\phi} < 0$ and $\psi_{Ia} = \frac{B\rho_a}{\phi(1-\rho_a)+r\phi} \geq 0$.

After an unanticipated positive TFP shock at date t , the investment firms misperceive the expected return on capital in the following period. This is because they underestimate the amount of total investment at date t , and hence underestimate the total stock of capital available for production in the subsequent period. Under a business cycle accounting, this misperception shows up as a negative investment wedge, implying an as-if subsidy to capital investment. The coefficient on capital stock in the investment wedge (at $\omega = 0$) is unambiguously positive. As the shock abates, the dynamics are driven by the first component in equation (29), which implies that the wedge eventually becomes positive and the behavioral friction gives rise to an as if tax on capital investment relative to the frictionless prototype model.

4 Quantitative Analysis

This section investigates the quantitative implications of the model. We explore the economic effects of a positive productivity shock, focusing on both real and financial variables, and highlighting the distinct dynamics introduced by CN and the investment wedge it endogenously generates. We then assess the model's implications for stock return predictability, which, as we show, is tightly related to this investment wedge. Finally, we contrast the model's behavior with that of the one generated by exogenous shocks to the investment wedge, such as the investment shocks emphasized by Justiniano, Primiceri, and Tambalotti (2010). In contrast to a model driven by investment shocks, our model successfully produces comovement among macroeconomic and financial variables.

4.1 Calibration

Table 1 reports the parameter values of the model. We choose a standard rate of time preference $\beta = 0.99$ that implies a risk-free rate of around 1 percent

Parameter	Value
β Time preference	0.99
γ Inverse EIS = $1/\gamma$	1
χ Labor disutility parameter	1.0324
φ Inverse Frisch elasticity = $(1 - N)/N/\varphi$	1
μ Mean growth rate	0.02/4
δ Capital depreciation rate	.025
α Capital share in value added	1/3
ϕ Capital adjustment costs	20
ρ_a Serial correlation productivity shock	$0.95^{1/4}$
σ_a Standard deviation productivity shock	0.005
G Steady-state government spending	0
ω Steady-state awareness	0.1

Table 1: Parameter Values (Quarterly Frequency)

p.a. The inverse elasticity of intertemporal substitution is set to 1. We choose a labor disutility parameter $\chi = 1.03$ to imply steady-state hours of 0.5, so half of the available time endowment is devoted to production, and the curvature of leisure in the utility function $\varphi = 1$ to imply a Frisch elasticity of 1.

The economy grows deterministically along a balanced-growth path with average growth μ of 2 percent p.a. Capital depreciates at a value of 10 percent p.a., and the capital share in value added has a standard value of $\alpha = 1/3$. We choose a capital adjustment cost parameter of 20, a value roughly in the middle of the range found in the literature – and close to the value used by Brinca, Chari, Kehoe, and McGrattan (2016). The exogenous process for productivity is a persistent $AR(1)$ process with an annualized autoregressive coefficient of 0.95 and an annualized standard deviation for its shocks of 0.01.

Finally, we set the competition awareness parameter to $\omega = 0.1$, which implies that, when considering the aggregate investment policy, individual firms place a weight of 10 percent on the true response of aggregate investment and of 90 percent on the default response, the steady state of aggregate investment along the balanced-growth path. This value for the behavioral friction produces a realistic amount of excess return predictability, as discussed below.

4.2 Real Effects of a Productivity Shock

We begin by examining how the economy responds to a positive productivity shock in our model. Figure 1 shows the dynamic responses of the main real variables to a one-standard-deviation productivity shock under two versions of

the model: the frictionless benchmark (dashed blue lines) and the version with competition neglect (solid red lines).

Several notable patterns emerge. Investment rises strongly on impact in the frictional model, overshooting the frictionless benchmark. Over time, investment converges back to the steady state and even falls below the frictionless path for a period—what we term underinvestment.

Consumption exhibits a modest hump-shaped pattern, with a higher peak under frictions, while output exhibits a stronger near-term expansion than in the absence of frictions. Eventually both begin reverting toward their balanced-growth path, with a slightly faster reversion in the case with behavioral frictions. Hours worked increase at the outset in both models, and more under frictions, then undershoot the frictionless path before returning to the steady state. The path of hours mimics the path of investment.

To understand the mechanism, it is useful to interpret the results through the lens of the observationally equivalent prototype model with wedges described above. Here, as show in the figure, the endogenous investment wedge produced by competition neglect responds non-trivially and changes sign after the productivity shock, acting like an investment subsidy at first and like an investment tax later. Indeed, the response of investment reflects the subsidy-like effect of the endogenous investment wedge when productivity first improves, with firms over-investing relative to the frictionless case, and the tax-like effect later.

Overall, these results illustrate how a positive productivity shock, mediated by behavioral frictions, can generate a short-lived *boom-bust response* relative to the frictionless model.

4.3 Financial Market Implications of a Productivity Shock

We now introduce stock prices, dividends, and returns into the picture. Because final-good producers realize no profits, the aggregate stock market value and dividend can be defined by aggregating the prices and dividends of capital-good producers. Assuming a dividend flow equal to the free cash flows generated each period by the firm,

$$D_t = H_t K_t - I_t - \delta K_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \frac{I}{K} \right)^2 K_t$$

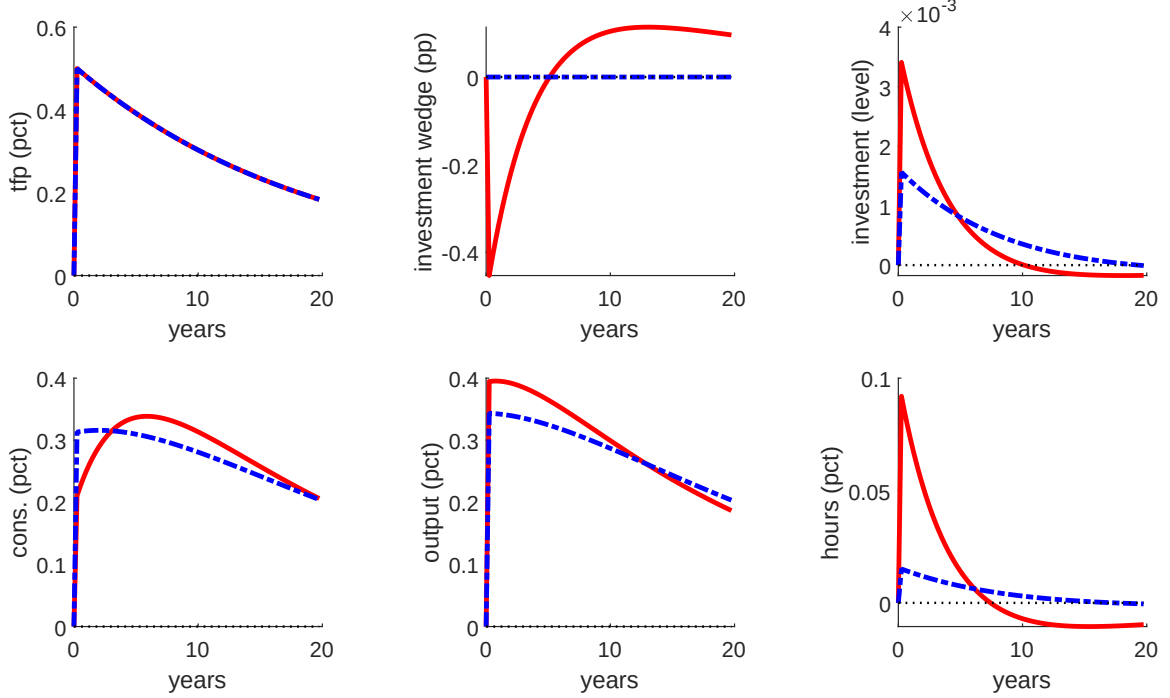


Figure 1: Impulse responses of real variables to a +1 s.d. productivity shock. Red (solid): with competition neglect. Blue (dashed): frictionless model. Deviations from balanced-growth path.

we can write the cum-dividend value of an individual firm (where we omit the i notation because of symmetry) as

$$V_t = D_t + \mathbb{E}_t M_{t+1|t} V_{t+1} \quad (30)$$

where $M_{t+1|t}$ is the one-period stochastic discount factor used by capital-good producers. Here, we compute the value of the firm from the point of view of entrepreneurs, who own the firm and therefore are also the marginal investors in the stock. That is, and consistent with our earlier notation, the expressions in equation (30) follow the distorted problem setup in Section 2.

Accordingly, we define the stock return as

$$R_{st+1} = \frac{V_{t+1}}{V_t - D_t} \quad (31)$$

As we show in the appendix, in this setup, average Tobin's Q equals marginal Tobin's Q . As a result, the equity return, R_{st+1} , is approximately equal to the firm's investment return, R_{kt+1} , reflected in the optimal investment condition,

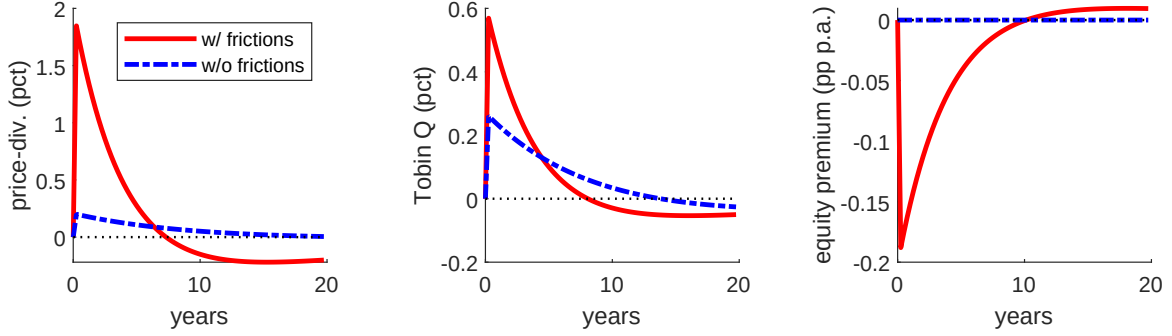


Figure 2: Impulse responses of financial variables to a +1 s.d. productivity shock. Red (solid): with competition neglect. Blue (dashed): frictionless model. Variables are in deviations from the balanced-growth path.

or

$$R_{st+1} \approx R_{kt+1} = \frac{H_{t+1} - \delta + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \frac{I}{K} \right) \frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \frac{I}{K} \right)^2 + Q_{t+1}}{Q_t}$$

In this context, we can construct the risk-free rate $r_t = -\log \mathbb{E}_t M_{t+1}$ and the equity premium $\mathbb{E}_t(r_{st+1} - r_t)$. Here, expectations are computed not from the perspective of the capital-good producers, but rather from the perspective of the household, which is not affected by behavioral distortions (and understands the correct law of motion of capital)—or, equivalently, from the perspective of an econometrician who regresses realized returns and discount rates on the current observed states.

Figure 2 displays the impulse responses for several financial variables to the same productivity shock displayed in Figure 1; namely, for aggregate stock price-dividend ratios $(V_t - D_t)/D_t$, aggregate Tobin's Q Q_t , and the equity premium $\mathbb{E}_t(r_{st+1} - r_t)$. Aggregate stock price valuations, both price-dividend ratios and Tobin's Q , exhibit an initial jump that overshoots at first, and then undershoots, the frictionless valuation ratios. Associated with these movements in stock prices is an initial investment surge and, importantly, a negative equity premium.

Indeed, the model's core mechanism, overinvestment followed by underinvestment, translates naturally into predictable movements in stock returns in excess over the risk-free rate. In the frictionless model, the increases in investment are such that the expected return on investment equals the risk-free rate, which increases as incentives to save decrease as a consequence of the transiently higher productivity. In contrast, under CN, firms fail to take into account

the response of other firms. As a consequence, they invest until their individual perceived expected return on investment equals the risk-free rate, but their evaluation of such future returns is distorted. Indeed, the resulting over-investment will push the actual future returns below the risk-free rate. Therefore, high valuations predict negative future excess returns on investment.

To understand the mechanism, we can once again turn to the interpretation of our mechanism as an endogenous investment wedge within a prototype RBC model. In the appendix we show that linearizing the firm's optimal-investment and valuation conditions around the steady state implies a tight relationship between excess stock returns and the endogenous investment wedge generated by our mechanism; namely, we can write

$$\mathbb{E}_t[r_{st+1} - r_t] = \tau_{xt} - \beta \mathbb{E}_t[\tau_{xt+1}] \quad (32)$$

Thus, the endogenous movements of the investment wedge are tightly linked to excess return predictability. This property results in time-varying measured risk premia, even though individual firms set risk premia to zero ex ante, but they do so under their distorted expectations. Thus, the model is able to generate periods with returns above and below the risk-free rate. Specifically, an investment boom is followed by negative excess returns. Intuitively, when the investment wedge subsidizes investment, the resulting overinvestment phase leads to negative future excess returns, and vice versa.

Overall, the model highlights how our behavioral distortion can give rise to cyclical investment behavior. There is a boom then a bust in investment relative to the frictionless economy, and time-varying equity premia, as high valuations are followed by disappointing returns rather than high earnings.

4.4 Predictability of Stock Returns

An implication of our model is the predictability of excess stock returns after a productivity shock. Investment booms are followed by predictably negative stock returns.

To quantitatively gauge whether our setup can produce a realistic amount of return predictability, we turn to a standard Campbell-Shiller decomposition of the variance of stock valuations. Namely, using the approximate definition

of stock returns,

$$r_{st+1} = -\log(\beta e^{(1-\gamma)\mu}) + \Delta d_{t+1} + \beta e^{(1-\gamma)\mu}(pd_{t+1} - pd) - (pd_t - pd)$$

it follows that variation in log price-dividend ratios, pd , must correspond to either variation in future dividend growth or variation in future returns, or

$$1 = \frac{\text{cov}(\sum_{j=1}^{\infty} (\beta e^{(1-\gamma)\mu})^j \Delta d_{t+j}, pd_t)}{\text{var}(pd_t)} - \frac{\text{cov}(\sum_{j=1}^{\infty} (\beta e^{(1-\gamma)\mu})^j r_{st+j}, pd_t)}{\text{var}(pd_t)}$$

Empirical estimates of such decompositions differ, depending on the construction of the equity payouts—see Larrain and Yogo (2008) and Cochrane (2011). Overall, to emphasize the ability of our mechanism to generate return predictability, we target a decomposition that attributes around 20 percent of the variation in price-dividend ratios to covariance with future dividend growth and around 80 percent to covariance with future returns. Indeed, we choose the parameter that controls the strength of the behavioral friction, ω , so our model matches this fact about return predictability.

Note that our model is able to generate movements in the equity premium, or, equivalently, predictable excess equity returns, even to a first-order approximation around the deterministic steady state, without relying on financial frictions, such as the ones used in Christiano, Motto, and Rostagno (2014), among many others. This feature follows from our behavioral distortion, which affects the expected return on investment perceived by capital-good producers, and which consequently does not coincide with the outcome of a predictive regression of realized returns on the current state. Since optimality implies that individual firms equate their distorted ex-ante investment return with the risk-free rate, it follows that a predictive regression of ex-post returns on the current state can differ from the risk-free rate. Therefore, the equity premium moves with the state of the economy.

4.5 Endogenous versus Exogenous Investment Wedges

It can be useful to compare the dynamics of our endogenous investment wedge with the response of the economy to an exogenous autoregressive shock to the investment wedge, as in the investment shocks emphasized in Justiniano, Primiceri, and Tambalotti (2010). A model with an exogenous investment wedge shock that follows typical autoregressive dynamics would not be able to pro-

duce an investment wedge that switches sign.

Figure 3 plots the response of the main macroeconomic and financial variables in our model described above, including the endogenous response of the investment wedge, as well as the response in the frictionless prototype model to a negative exogenous investment shock, $s_{xt} \sim AR(1)$, that scales the investment expenditures of capital-good producers, which becomes $(1 + s_{xt})I_t$. Such exogenous autoregressive movements in the investment wedge are associated with persistently positive investment dynamics, as the shock acts throughout the horizon exclusively like an investment subsidy, thereby missing the underinvestment phase generated by the movements in the endogenous wedge of Figure 1.

Furthermore, this shock has a tendency to generate negative comovement between investment as consumption, even though there are strategies to mitigate, and even reverse, such a tendency, as detailed in Justiniano, Primiceri, and Tambalotti (2010). However, such exogenous investment shocks can never be observationally equivalent to our model in its implications for stock prices. First, such shocks will never be able to generate excess return predictability to the extent that they move stock returns and the risk-free rate identically. Second, the investment shocks emphasized by Justiniano, Primiceri, and Tambalotti (2010) tend to produce a negative comovement between investment and certain stock price valuation measures, chiefly among them Tobin’s Q , as evident from Figure 3. See also Christiano, Ilut, Motto, and Rostagno (2008) and Christiano, Motto, and Rostagno (2014) for a discussion of such difficulties.

4.6 Discussion

The analysis in this section highlights several insights. First, behavioral frictions such as competition neglect can generate boom-bust investment cycles in response to productivity shocks, characterized by an initial phase of overinvestment followed by underinvestment relative to a frictionless economy. This behavior contrasts sharply with standard models, where investment adjusts more smoothly to fundamentals.

Second, the model produces time-varying equity premia, linking high valuations with predictably negative excess returns. This endogenous predictability arises from the strategic distortions in firms’ investment decisions, providing a novel mechanism distinct from traditional risk-based explanations.

Finally, we show that these dynamics cannot be easily mimicked by standard

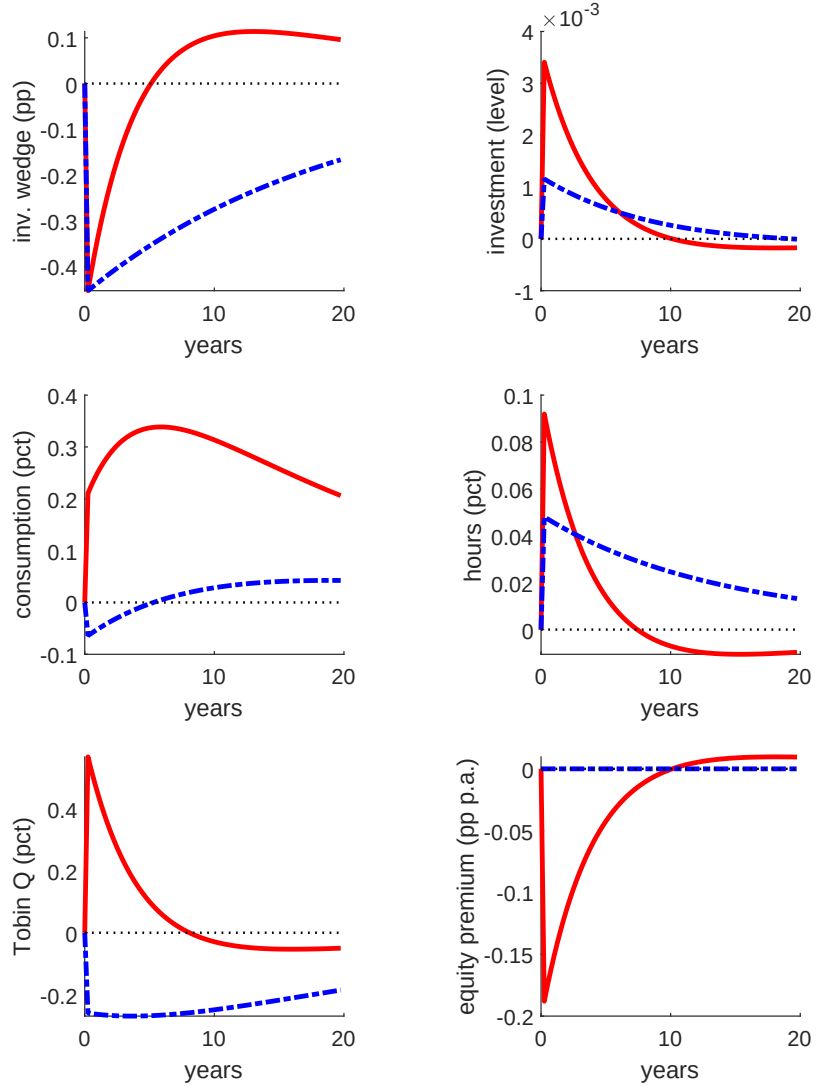


Figure 3: Impulse responses of the main variables to a shock that generates a movement in the investment wedge. Red line: response in the endogenous-investment wedge model to a +1 s.d. productivity shock. Blue line: response to an exogenous investment-wedge shock in the frictionless model. The size of the exogenous investment shock is calibrated to generate the same response on impact of the investment wedge. Variables are in deviations from the balanced-growth path.

exogenous investment shocks, underscoring the importance of modeling the endogenous origins of investment distortions for understanding macroeconomic and asset pricing fluctuations.

5 Conclusion

This paper contributes to the understanding of how firm expectations interact with aggregate investment outcomes. While much of the literature on bounded rationality focuses on household decision-making, our model highlights the broader implications of firm-level biases for macroeconomic variables. This approach suggests that firms' decision-making processes, particularly their forecasting of market conditions, may be just as important as traditional drivers of aggregate demand, such as preference shocks or news shocks. By explicitly modeling this behavioral friction, our framework offers a tight connection between the macroeconomic literature on firm investment and the behavioral economics literature on overconfidence.

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Appendix

A Dynamic Consistency and Recursivity

This appendix establishes a dynamic consistency result showing that, despite the distorted use of equilibrium mappings, firms' problems admit a recursive representation and equilibrium can be characterized as a stationary fixed point. This ensures that the equilibrium is well posed and can be analyzed using standard recursive methods. The result, stated in words, is that the problem is recursive (and time consistent) in the vector composed by a firm's own capital, aggregate capital, and TFP.

Consider a stationary state-contingent plan for aggregate investment, $I_t = \bar{h}(K_t, A_t)$, where K_t is aggregate capital and A_t an exogenous shock. At date t , individual firms know K_t and A_t , along with individual capital K_{ft} , and correctly understand the law of motion and distribution of exogenous shocks; however, they believe that the aggregate policy function for date- $t+j$ investment is $\omega \bar{h}(K_{t+j}, A_{t+j}) + (1 - \omega)I$ for all $j \geq 0$, which describes the firm's internal forecasting rule for future aggregate outcomes, where $I = 0$ is the steady-state value of aggregate net investment. It follows that firms think that the aggregate capital stock is $K_{t+j+1} = K_{t+j} + \omega \bar{h}(K_{t+j}, A_{t+j})$ for all $j \geq 0$, which is the firm's subjective forecasting rule for this state of the economy.

In equilibrium, the true realized law of motion for aggregate capital is $K_{t+1} = K_t + \bar{h}(K_t, A_t)$, so, holding $\bar{h}$ fixed, ω affects only the firm's internal forecast, not the contemporaneous aggregation of actions. Still, through general-equilibrium forces, ω will affect the map $\bar{h}$. To see this, note that, in equilibrium, symmetry implies that all firms choose the same action $I_{ft} = I_t$, so that the perceived law of motion can be written as in (6) as $K_{t+j+1} = K_{t+j} + \omega I_{ft+j}$. This expression illustrates how, while firms treat the mapping as given when optimizing, in equilibrium the aggregate mapping is determined by a fixed point that depends on the firms' bounded rationality. This way, behavioral distortions in individual investment affect the equilibrium aggregate mapping.

Furthermore, the firm requires additional aggregate mappings (the mappings from aggregate states to rental rates and to discount factors) to solve its individual problem. We assume that firms receive these mappings and use them without cost, thereby assessing the choices of shareholders and clients with the correct mappings. Note that, because firms do not internally solve for these aggregate mappings but instead treat them as externally given objects, they

cannot reverse-engineer the aggregate investment policy from them. This assumption also implies that firms do not update their forecasting rule based on realized forecast errors. In this precise sense, the bounded rationality assumption can be interpreted as a computational constraint faced by the firm, not a statistical misspecification of transition probabilities.

The resulting structure implies that the solution to the problem is a Markov solution, namely, a stationary policy h such that date- t optimal individual investment is $I_{ft} = h(K_{ft}, K_t, A_t)$. Because preferences are time-consistent and the firm's perceived transition law is time-invariant in the observed state and does not depend on past forecast errors or past plans, the continuation problem faced by future selves is identical to the one anticipated by the current self. In particular, if the ex-ante optimal plan prescribes actions according to a stationary policy function h , then at any future date t , conditional on the realized state, it is optimal for the firm to follow the same policy function h . This dynamic consistency result implies that the firm's problem can be written recursively.

Goal We characterize equilibrium as a fixed point in the space of stationary aggregate policy mappings, which the firm receives from its contacts. These contacts can be thought of as experts or specialized agents (see the discussion in Section 2). These contacts are able to solve for the economy's GE, whereas individual firms are boundedly rational and do not internally solve the GE, but receive the GE mapping from their contacts.

We first solve the firm's problem taking an arbitrary stationary mapping as given, establish dynamic consistency and recursive representation, and then impose equilibrium consistency to verify that the resulting aggregate mapping lies in the same stationary class and is consistent with equilibrium aggregation. This follows standard equilibrium verification arguments used in recursive general equilibrium models.

Timing Individual firms start period t by observing the realization of the current exogenous shock A_t , the current aggregate state K_t , and carry over from the previous period knowledge of their individual endogenous state K_{ft} . The contacts start the period observing the realization of the current exogenous shock A_t and carry over from the previous period knowledge of all individual endogenous states K_{ft} and of the aggregate endogenous state K_t . Individual firms then make individual decisions. Simultaneously, the contacts understand

the problem faced by individual firms as well as the definition of equilibrium, and costlessly compute the decisions of individual firms and aggregate decisions, and communicate to the individual firms the aggregate decision functions $\bar{h}$ (aggregate investment map), Λ (aggregate marginal utility), and $\mathcal{H}$ (aggregate rental rate). Aggregate capital evolves within period t , but firms observe the realized K_{t+1} only at the beginning of period $t + 1$.

Individual firm sequence problem at date τ Starting at period τ at a common state $s_\tau = (K_{f\tau}, K_\tau, A_\tau)$, the individual firm chooses state-contingent plans for net investment $\{I_{ft|\tau}\}_{t=\tau}^\infty$ to maximize

$$E_\tau \sum_{t=\tau}^\infty \beta^{t-\tau} \frac{\Lambda(K_{t|\tau}^\omega, A_t)}{\Lambda(K_{\tau|\tau}^\omega, A_\tau)} \left(\mathcal{H}(K_{t|\tau}^\omega, A_t) K_{ft|\tau} - I_{ft|\tau} - \delta K_{ft|\tau} - \frac{\phi}{2} I_{ft|\tau}^2 \right)$$

subject to

$$K_{ft+1|\tau} = K_{ft|\tau} + I_{ft|\tau} = K_{f\tau|\tau} + \sum_{s=\tau}^t I_{fs|\tau} \quad \text{with} \quad K_{f\tau|\tau} = K_{f\tau}$$

and where

$$K_{t+1|\tau}^\omega = K_{t|\tau}^\omega + \omega \bar{h}(K_{t|\tau}^\omega, A_t) = K_{\tau|\tau}^\omega + \sum_{s=\tau}^t \omega \bar{h}(K_{s|\tau}^\omega, A_s) \quad \text{with} \quad K_{\tau|\tau}^\omega = K_\tau$$

The same problem solved at a later date $t > \tau$ would yield the plans for net investment $\{I_{fs|t}\}_{s=t}^\infty$. The only difference relative to the date- τ problem is the realization of states up to date t . In particular, upon observing the realized aggregate state, the firm reinitializes its subjective forecast at $K_{t|t}^\omega = K_t$ while continuing to use the same time-invariant continuation belief rule thereafter. Importantly, at node s_t , the time- τ planner evaluates continuation values using exactly the same continuation belief rule and pricing functionals as the time- t self. Thus, we can now prove that $I_{it|\tau}(s_t) = I_{it|t}(s_t)$ for all states s_t .

Proposition 4 (Dynamic consistency) *Fix $\omega \in (0, 1)$, the mapping $\bar{h}$, and the law of motion for the exogenous process. Consider the firm's perceived aggregate capital process initialized at date τ , $\{K_{t|\tau}^\omega\}_{t \geq \tau}$, defined by $K_{t+1|\tau}^\omega = K_{t|\tau}^\omega + \omega \bar{h}(K_{t|\tau}^\omega, A_t)$ with $K_{\tau|\tau}^\omega = K_\tau$. Let $\{I_{ft|\tau}^*\}_{t \geq \tau}$ be an optimal state-contingent plan for the firm's planning problem at date τ . Then for any $t > \tau$ and any realized state $s_t = (K_{ft}, K_t, A_t)$, the time- τ optimal plan is sequentially optimal*

at t : $I_{ft|\tau}^*(s_t) = I_{ft|t}^*(s_t)$, where $\{I_{fs|t}^*\}_{s \geq t}$ solves the same planning problem starting at t with $K_{t|t}^\omega = K_t$ and $K_{ft|t} = K_{ft}$.

Proof. Fix $t > \tau$ and a realized state $s_t = (K_{ft}, K_t, A_t)$. Consider the time- τ optimal plan $\{I_{fs|\tau}^*\}_{s \geq \tau}$ and let $\{I_{fs|t}^*\}_{s \geq t}$ denote its restriction to periods $s \geq t$. This restricted plan is state-contingent and feasible for the planning problem starting at date t . In particular, this plan is feasible when the continuation problem at date t is initialized at the realized state s_t , so that the subjective aggregate state is reset to $K_{t|t}^\omega = K_t$, and the privately controlled state is $K_{ft|t} = K_{ft}$.

Next, suppose, by contradiction, that the restriction $\{I_{fs|\tau}^*\}_{s \geq t}$ is not optimal for the time- t planning problem at state s_t . Then the time- t optimal plan $\{I_{fs|t}^*\}_{s \geq t}$ evaluated from state s_t yields a strictly higher value than $\{I_{fs|\tau}^*\}_{s \geq t}$. Thus, construct a new plan $\{I_{fs|\tau}^{**}\}_{s \geq \tau}$ at date τ such that $I_{fs|\tau}^{**} = I_{fs|\tau}^*$ for $s = \tau, \dots, t-1$ and $I_{fs|\tau}^{**} = I_{fs|t}^*$ for $s \geq t$. Off the contingency s_t at date t , define $I_{fs|\tau}^{**}$ to coincide with $I_{fs|\tau}^*$, so that $\{I_{fs|\tau}^{**}\}_{s \geq \tau}$ is a well-defined state-contingent plan.

Now, because the perceived aggregate transition is recursive and time-invariant, a continuation plan that is feasible when starting fresh at time $t > \tau$ from state s_t can be embedded inside a state-contingent plan chosen at time τ . Therefore, the patched plan $\{I_{fs|\tau}^{**}\}_{s \geq \tau}$ is feasible for the original time- τ planning problem. Moreover, conditional on reaching state s_t , the continuation problem from date t onward is identical to the planning problem starting at date t because both use the same continuation belief rule and pricing functionals conditional on the realized state. Therefore, conditional on reaching s_t , the patched plan yields a strictly higher continuation value than the original plan while coinciding with it up to date $t-1$. Hence, the patched plan yields a strictly higher time- τ objective, contradicting optimality of $\{I_{fs|\tau}^*\}_{s \geq \tau}$. Therefore, the restriction $\{I_{fs|\tau}^*\}_{s \geq t}$ must be optimal for the time- t planning problem at state s_t , which implies $I_{ft|\tau}^*(s_t) = I_{ft|t}^*(s_t)$. ■

This result relies on the fact that the firm's perceived transition law and stochastic discounting are time-invariant and depend only on the current state, and not on past forecast errors or past plans. The firm may be systematically surprised by realized K_{t+1} relative to its prior subjective forecast $K_{t+1|\tau}^\omega$, but this affects only the realized argument s_t at which the continuation problem is evaluated; it does not alter the continuation problem itself, which is time-invariant and depends only on the current state.

Therefore, because the firm's problem is dynamically consistent and the perceived transition law and payoff functions are time-invariant and depend only on the current state, under standard regularity conditions ensuring the existence of the value function, the principle of optimality holds, and the firm's problem admits a recursive Bellman representation, to which we turn next.

Individual firm's dynamic program At each date t , the individual firm will choose actions contingent on the state at time t , $s_t = (K_{ft}, K_t, A_t)$.⁷ Consider the dynamic programming problem

$$V(K_{ft}, K_t, A_t) = \max_{I_{ft}} \mathcal{H}(K_t, A_t) K_{ft} - I_{ft} - \delta K_{ft} - \frac{\phi}{2} I_{ft}^2 \\ + \beta E_t \frac{\Lambda(K_t + \omega \bar{h}(K_t, A_t), A_{t+1})}{\Lambda(K_t, A_t)} V(K_{ft} + I_{ft}, K_t + \omega \bar{h}(K_t, A_t), A_{t+1})$$

This problem admits a stationary solution $I_{ft} = h(K_{ft}, K_t, A_t)$. Namely, optimality implies

$$1 + \phi h(K_{ft}, K_t, A_t) = \beta E_t \frac{\Lambda(K_t + \omega \bar{h}(K_t, A_t), A_{t+1})}{\Lambda(K_t, A_t)} \left(\begin{array}{l} \mathcal{H}(K_t + \omega \bar{h}(K_t, A_t), A_{t+1}) + 1 - \delta \\ + \phi h(K_{ft} + h(K_{ft}, K_t, A_t), K_t + \omega \bar{h}(K_t, A_t), A_{t+1}) \end{array} \right) \quad (33)$$

To solve for the optimal choice h of the individual firm we first need to determine which function $\bar{h}$ was communicated to it by its contacts, which solves for the equilibrium, to which we turn next.

Contacts problem The contacts solve for the symmetric capital-good sector equilibrium: a sequence of states $\{K_{ft}, K_t, A_t\}$ and a sequence of state-contingent individual net investment $\{I_{ft}\}$ such that, at each date t , (a) individual firms solve their problem, (b) aggregate variables are the integral of individual variables, and (c) each individual firm behaves symmetrically.

The set of equations considered by the contacts includes, at each date t , the optimal choice by firms (33) and the symmetry and aggregation conditions

$$\bar{h}(K_t, A_t) = h(K_t, K_t, A_t)$$

Thus, the equilibrium is characterized by a fixed point in the policy-function space: given firms' distorted use of the aggregate mapping, the contacts compute the mapping consistent with firms' optimal behavior and aggregation. The

⁷By the above notation, $K_{ft|t} = K_{ft}$ and $K_{t|t} = K_t$, so we omit the " $\cdot|t$ " notation in what follows.

contacts look therefore for the mapping h that satisfies

$$1 + \phi h(K_{ft}, K_t, A_t) = \beta E_t \frac{\Lambda(K_t + \omega h(K_t, K_t, A_t), A_{t+1})}{\Lambda(K_t, A_t)} \left(\begin{array}{l} \mathcal{H}(K_t + \omega h(K_t, K_t, A_t), A_{t+1}) + 1 - \delta \\ + \phi h(K_{ft} + h(K_{ft}, K_t, A_t), K_t + \omega h(K_t, K_t, A_t), A_{t+1}) \end{array} \right) \quad (34)$$

Trivially, after receiving from the contacts the function $\bar{h}$ such that $\bar{h}(K_t, A_t) = h(K_t, K_t, A_t)$ and that solves (34), the individual firm solving equation (33) will find the same optimal function h .

This verifies that the aggregate mapping is time-invariant and is consistent with equilibrium aggregation.

B Complements to Section 3

B.1 Proof for Proposition 1

Let $B \equiv (1 - \alpha)[1 - \beta(1 - \delta)]/\beta K$ and using the steady-state relations, the system of eight equations

$$\alpha = \gamma \psi_{ck} + (\alpha + \varphi) \psi_{nk} \quad (35)$$

$$1 - \alpha = \gamma \psi_{ca} + (\alpha + \varphi) \psi_{na} \quad (36)$$

$$(1 - \alpha) \psi_{nk} + \alpha = \frac{1 - \beta[1 - \delta(1 - \alpha)]}{1 - \beta(1 - \delta)} \psi_{ck} + \frac{\alpha \beta}{1 - \beta(1 - \delta)} \frac{\psi_{Ik}}{K} + \frac{\alpha \beta \delta}{1 - \beta(1 - \delta)} \quad (37)$$

$$(1 - \alpha)(1 + \psi_{na}) = \frac{1 - \beta[1 - \delta(1 - \alpha)]}{1 - \beta(1 - \delta)} \psi_{ca} + \frac{\alpha \beta}{1 - \beta(1 - \delta)} \frac{\psi_{Ia}}{K} \quad (38)$$

$$\psi_{rk} = -\gamma \psi_{ck} \frac{\psi_{Ik}}{K} \quad (39)$$

$$\psi_{ra} = -\gamma \left(\psi_{ck} \frac{\psi_{Ia}}{K} - \psi_{ca}(1 - \rho_a) \right) \quad (40)$$

$$\phi K \frac{\psi_{Ik}}{K} = -\gamma \psi_{ck} \theta_d \frac{\psi_{Ik}}{K} + \beta K \left(\phi \frac{\psi_{Ik}}{K} - B(1 - \psi_{nk}) \right) \left(1 + \theta_d \frac{\psi_{Ik}}{K} \right) \quad (41)$$

$$\begin{aligned} \phi K \frac{\psi_{Ia}}{K} &= -\gamma \left(\psi_{ck} \theta_d \frac{\psi_{Ia}}{K} - \psi_{ca}(1 - \rho_a) \right) - \beta B K \left((1 - \psi_{nk}) \theta_d \frac{\psi_{Ia}}{K} - (1 + \psi_{na}) \rho_a \right) \\ &\quad + \beta \phi K \left(\rho_a + \theta_d \frac{\psi_{Ik}}{K} \right) \frac{\psi_{Ia}}{K} \end{aligned} \quad (42)$$

identifies the eight coefficients $\{\psi_{ca}, \psi_{ck}, \psi_{na}, \psi_{nk}, \psi_{Ia}, \psi_{Ik}, \psi_{ra}, \psi_{rk}\}$ of the solution.

C Complements to Section 4

This appendix derives the theoretical results referred to in the main text.

C.1 Marginal Q $\approx$ Average Q

The value of an individual firm (where we omit the i notation because of symmetry) is the present discounted value of distributed profits

$$V_t = D_t + \mathbb{E}_t M_{t+1|t} V_{t+1}$$

where distributed profits are current output less costs, depreciation, and net investment,

$$D_t = H_t K_t - I_t - \delta K_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \frac{I}{K} \right)^2 K_t$$

Writing this expression as an infinite sum, we can rewrite it as follows:

$$\begin{aligned} V_0 &= \mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t|0} \left[H_t K_t - I_t - \delta K_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \frac{I}{K} \right)^2 K_t \right] \\ &= D_0 + \mathbb{E}_0 \sum_{t=1}^{\infty} M_{0,t|0} \left[H_t + 1 - \delta + \phi \left(\frac{I_t}{K_t} - \frac{I}{K} \right) \left(1 + \frac{I_t}{K_t} \right) - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \frac{I}{K} \right)^2 \right] K_t \\ &\quad - \mathbb{E}_0 \sum_{t=1}^{\infty} M_{0,t|0} \left\{ \left[1 - \phi \left(\frac{I_t}{K_t} - \frac{I}{K} \right) \right] K_{t+1} \right\} \\ &= D_0 + \left[1 + \phi \left(\frac{I_0}{K_0} - \frac{I}{K} \right) \right] K_1 \end{aligned}$$

where the third equality uses the optimal investment condition. Rearranging, and letting marginal Tobin's Q, $Q_t \equiv \mathbb{E}_t M_{t+1|t} V_{t+1}$, we can rewrite average Tobin's Q—the ratio of the ex-dividend market value and the replacement value of the firm's capital—as

$$\frac{V_t - D_t}{K_{t+1}} = 1 + \phi \left(\frac{I_t}{K_t} - \frac{I}{K} \right) = Q_t$$

Therefore, we have that average and marginal Tobin's Qs are approximately equal.

C.2 Excess Return Predictability and Investment Wedge

Armed with the result in the previous subsection, note that, in the alternative version of the model that included and derived the endogenous investment wedge, the optimal investment conditions can be stated as

$$\begin{aligned}
1 &= \mathbb{E}_t M_{t+1} \frac{H_{t+1} + 1 - \delta + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \frac{I}{K} \right) \left(1 + \frac{I_{t+1}}{K_{t+1}} \right) - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \frac{I}{K} \right)^2 + \tau_{xt+1}}{1 + \phi \left(\frac{I_t}{K_t} - \frac{I}{K} \right) + \tau_{xt}} \\
&= \mathbb{E}_t M_{t+1} \left(R_{kt+1} \frac{1 + \phi \left(\frac{I_t}{K_t} - \frac{I}{K} \right)}{1 + \phi \left(\frac{I_t}{K_t} - \frac{I}{K} \right) + \tau_{xt}} + \frac{\tau_{xt+1}}{1 + \phi \left(\frac{I_t}{K_t} - \frac{I}{K} \right) + \tau_{xt}} \right)
\end{aligned}$$

A linearization of this expression implies that equity premia are related to the endogenous investment wedge as

$$\mathbb{E}_t r_{kt+1} - r_{ft} = \tau_{xt} - \beta \mathbb{E}_t \tau_{xt+1} \quad (43)$$