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A New Model of Trend Inflation Using Disaggregates, Survey Expectations, and Uncertainty

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Online Appendix

A New Model of Trend Inflation Using Disaggregates, Survey Expectations, and Uncertainty*

Ellis Tallman† Saeed Zaman‡

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A1. Prior Elicitation

In the table below, the notation $N(a, b)$ denotes Normal distribution with mean a, and variance b; and $IG(\nu, S)$ denotes Inverse Gamma distribution with degrees of freedom parameter ν, and scale parameter S.

Table 1: Prior settings for the Baseline Model

Parameter	**Parameter Description**	**Prior**
σ^2_{hS}	Var. of the Volatility – Services π measurement eq.	$IG(10, 0.25^2 \times 9)$
σ^2_{hG}	Var. of the Volatility – Goods π measurement eq.	$IG(10, 0.25^2 \times 9)$
$\sigma^2_{h\tau S}$	Var. of the Volatility – Trend Services π eq.	$IG(10, 0.05^2 \times 9)$
$\sigma^2_{h\tau G}$	Var. of the Volatility – Trend Goods π eq.	$IG(10, 0.05^2 \times 9)$
σ^2_{λ}	Var. of the shocks to TVP λ (Phillips curve slope)	$IG(10, 0.05^2 \times 9)$
$\sigma^2_{\alpha S}$	Var. of the shocks to TVP α^S	$IG(10, 0.1^2 \times 9)$
$\sigma^2_{\alpha G}$	Var. of the shocks to TVP α^G	$IG(10, 0.1^2 \times 9)$
$\sigma^2_{c\pi}$	Var. of the shocks to TVP C^{π}	$IG(10, 0.1^2 \times 9)$
$\sigma^2_{z\pi}$	Var. of the shocks to the eq. linking survey to π^*	$IG(10, 0.447^2 \times 9)$
β^{π}	Link between π^* and survey	$N(1, 0.05^2)$
λ_0	Initial value of λ, $t = 0$	$TN(-1, 0; 0, 1)$
α^S_0	Initial value of α^S, $t = 0$	$N(0, 1)$
α^G_0	Initial value of α^G, $t = 0$	$N(0, 1)$
C^{π}_0	Initial value of C^{π}, $t = 0$	$N(0, 0.1)$
h^{Ser}_0	Initial value of h^{Ser}, $t = 0$	$N(0, 1)$
h^{Goods}_0	Initial value of h^{Goods}, $t = 0$	$N(0, 1/5)$
$h^{\tau S}_0$	Initial value of $h^{\tau S}$, $t = 0$	$N(0, 1)$
$h^{\tau G}_0$	Initial value of $h^{\tau G}$, $t = 0$	$N(0, 1)$
τ^G_0	Initial value of τ^G, $t = 0$	$N(1, 100)$
τ^S_0	Initial value of τ^S, $t = 0$	$N(3, 100)$

Table 2: Prior settings for the Single-sector Model

Parameter	**Parameter Description**	**Prior**
σ^2_{hS}	Var. of the Volatility – aggregate π measurement eq.	$IG(10, 0.25^2 \times 9)$
$\sigma^2_{h\tau S}$	Var. of the Volatility – aggregate Trend π eq.	$IG(10, 0.05^2 \times 9)$
σ^2_{λ}	Var. of the shocks to TVP λ (Phillips curve slope)	$IG(10, 0.05^2 \times 9)$
$\sigma^2_{\alpha S}$	Var. of the shocks to TVP α	$IG(10, 0.1^2 \times 9)$
$\sigma^2_{c\pi}$	Var. of the shocks to TVP C^{π}	$IG(10, 0.1^2 \times 9)$
$\sigma^2_{z\pi}$	Var. of the shocks to the eq. linking survey to π^*	$IG(10, 0.447^2 \times 9)$
β^{π}	Link between π^* and survey	$N(1, 0.05^2)$
λ_0	Initial value of λ, $t = 0$	$TN(-1, 0; 0, 1)$
α_0	Initial value of α, $t = 0$	$N(0, 1)$
C^{π}_0	Initial value of C^{π}, $t = 0$	$N(0, 0.1)$
h_0	Initial value of h, $t = 0$	$N(0, 1)$
h^{τ}_0	Initial value of h^{τ}, $t = 0$	$N(0, 1)$
τ_0	Initial value of τ, $t = 0$	$N(2, 100)$

A2. MCMC Algorithm

The Baseline model is estimated by sampling from its joint posterior distribution, which entails sequentially drawing from a set of conditional posterior densities, some standard and some non-standard.

Collect all the time-invariant model parameters into θ:
$\theta = (\sigma^2_{hS}, \sigma^2_{hG}, \sigma^2_{h\tau S}, \sigma^2_{h\tau G}, \sigma^2_{\lambda}, \sigma^2_{\alpha S}, \sigma^2_{\alpha G}, \sigma^2_{c\pi}, \sigma^2_{z\pi}, \beta^{\pi})$

Let's denote $\bullet$ as representing all other model parameters. Then, the sequential steps to draw from the conditional posterior densities are:
1. $p(\alpha^S|Y, \bullet)$
2. $p(\alpha^G|Y, \bullet)$
3. $p(\lambda|Y, \bullet)$
4. $p(\tau^S|Y, \bullet)$
5. $p(\tau^G|Y, \bullet)$
6. $p(h^{Ser}|Y, \bullet)$
7. $p(h^{Goods}|Y, \bullet)$
8. $p(h^{\tau S}|Y, \bullet)$
9. $p(h^{\tau G}|Y, \bullet)$
10. $p(C^{\pi}|Y, \bullet)$
11. $p(\theta|Y, \bullet)$

In the steps below, $Y = \pi^S$ when sampling parameters defining dynamics of services inflation, and $Y = \pi^G$ when sampling parameters defining the dynamics of goods inflation.

Step 1. Derive the conditional distribution $p(\alpha^S|Y, \bullet)$

The information about α^S comes from two sources. Below, we derive an expression for each of these two sources.

First, some notation,

$\tilde{\pi}^S_t = \pi^S_t - \tau^S_t$
$\tilde{u}_t = U_t - U^*_t$
$\tilde{v}_t = e^{h^{Ser}_t} - e^{\bar{h}^{Ser}_t}$

The first source is the services inflation measurement equation. Rewrite it in a matrix notation,

$$N^{\alpha s} = X_v \alpha^S + \varepsilon^S \qquad \varepsilon^S \sim N(0, \Omega_s), \quad where \ \Omega_s = diag(e^{h_1^{Ser}}, e^{h_2^{Ser}}, ..., e^{h_T^{Ser}}) \tag{1}$$

where,
$X_v = diag(\tilde{v}_1, ..., \tilde{v}_T)$
$N^{\alpha s} = (\tilde{\pi}_1^S - \lambda_1 \tilde{u}_1, ..., \tilde{\pi}_T^S - \lambda_T \tilde{u}_T)'$

Ignoring any terms not involving α^S, the likelihood is

$$log \ \ p(\pi^S | \alpha^S, \bullet) \propto -\tfrac{1}{2}(N^{\alpha s} - X_v \alpha^S)' \Omega_s^{-1} (N^{\alpha s} - X_v \alpha^S)$$

The second source comes from the state equation for α^S. We rewrite it in a matrix notation as follows,

$$H\alpha^S = \varepsilon^{\alpha s} \qquad \varepsilon^{\alpha s} \sim N(0, \Omega_{\alpha s}), \quad where \ \Omega_{\alpha s} = diag(\omega_{\alpha s}^2, \sigma_{\alpha s}^2, ..., \sigma_{\alpha s}^2) \tag{2}$$

Ignoring any terms not involving α^S, the prior density for α^S is given by

$$log \ \ p(\alpha^S | \sigma_{\alpha s}^2, \Omega_{\alpha s}) \propto -\tfrac{1}{2}(\alpha^S)' H' \Omega_{\alpha s}^{-1} H(\alpha^S)$$

Combining the above two conditional densities we obtain,

$$log \ \ p(\alpha^S | Y, \bullet) \propto -\tfrac{1}{2}(\alpha^S - \hat{\alpha}^S)' D_{\alpha s}^{-1} (\alpha^S - \hat{\alpha}^S)$$

where,

$$D_{\alpha s} = (H' \Omega_{\alpha s}^{-1} H + X_v' \Omega_s^{-1} X_v)^{-1}$$

$$\hat{\alpha}^S = D_{\alpha s}(X_v' \Omega_s^{-1} N^{\alpha s})$$

The candidate draws are sampled from $N(\hat{\alpha}^S, D_{\alpha s})$ using the precision-based algorithm.

Step 2. Derive the conditional distribution $p(\alpha^G | Y, \bullet)$

The information about α^G comes from two sources. Below, we derive an expression for each of these two sources.

First, some notation,

$\tilde{\pi}_t^G = \pi_t^G - \tau_t^G$
$\tilde{u}_t = U_t - U_t^*$
$\tilde{vg}_t = e^{h_t^{Goods}} - e^{\bar{h}_t^{Goods}}$

The first source is the goods inflation measurement equation. Rewrite it in a matrix notation,

$$N^{\alpha g} = X_{vg}\alpha^G + \varepsilon^G \qquad \varepsilon^G \sim N(0, \Omega_g), \quad where\ \ \Omega_g = diag(e^{h_1^{Goods}}, e^{h_2^{Goods}}, ..., e^{h_T^{Goods}}) \quad (3)$$

where,
$X_{vg} = diag(\tilde{vg}_1, ..., \tilde{vg}_T)$
$N^{\alpha g} = (\tilde{\pi}_1^G, ..., \tilde{\pi}_T^G)'$

Ignoring any terms not involving α^S, the likelihood is

$$log\ \ p(\pi^G|\alpha^G, \bullet) \propto -\tfrac{1}{2}(N^{\alpha g} - X_{vg}\alpha^G)'\Omega_g^{-1}(N^{\alpha g} - X_{vg}\alpha^G)$$

The second source comes from the state equation for α^G. We rewrite it in a matrix notation as follows,

$$H\alpha^G = \varepsilon^{\alpha g} \qquad \varepsilon^{\alpha g} \sim N(0, \Omega_{\alpha g}), \quad where\ \ \Omega_{\alpha g} = diag(\omega_{\alpha g}^2, \sigma_{\alpha g}^2, ..., \sigma_{\alpha g}^2) \qquad (4)$$

Ignoring any terms not involving α^G, the prior density for α^G is given by

$$log\ \ p(\alpha^G|\sigma_{\alpha g}^2, \Omega_{\alpha g}) \propto -\tfrac{1}{2}(\alpha^G)'H'\Omega_{\alpha g}^{-1}H(\alpha^G)$$

Combining the above two conditional densities we obtain,

$$log\ \ p(\alpha^G|Y, \bullet) \propto -\tfrac{1}{2}(\alpha^G - \hat{\alpha}^G)'D_{\alpha g}^{-1}(\alpha^G - \hat{\alpha}^G)$$

where,
$D_{\alpha g} = (H'\Omega_{\alpha g}^{-1}H + X_{vg}'\Omega_g^{-1}X_{vg})^{-1}$

$$\hat{\alpha}^G = D_{\alpha g}(X'_{vg}\Omega_g^{-1}N^{\alpha g})$$

The candidate draws are sampled from $N(\hat{\alpha}^G, D_{\alpha g})$ using the precision-based algorithm.

Step 3. Derive the conditional distribution $p(\lambda|Y, \bullet)$

The information about λ comes from two sources. Below, we derive an expression for each of these two sources.

First, some notation,

$\tilde{\pi}_t^S = \pi_t^S - \tau_t^S$
$\tilde{u}_t = U_t - U_t^*$
$\tilde{v}_t = e^{h_t^{Ser}} - e^{\overline{h}_t^{Ser}}$
$NW = (\tilde{\pi}_1^S - \alpha_1^S\tilde{v}_1, ..., \tilde{\pi}_T^S - \alpha_T^S\tilde{v}_T)'$

The first source is the services inflation measurement equation. Rewrite it in a matrix notation,

$$NW = X_u\lambda + \varepsilon^S \qquad \varepsilon^S \sim N(0, \Omega_s) \tag{5}$$

where,
$X_u = diag(\tilde{u}_1, ..., \tilde{u}_T)$

Ignoring any terms not involving λ, the likelihood is

$$log\ \ p(\pi^S|\lambda, \bullet) \propto -\tfrac{1}{2}(NW - X_u\lambda)'\Omega_s^{-1}(NW - X_u\lambda)$$

The second source comes from the state equation for λ. We rewrite it in a matrix notation as follows,

$$H\lambda = \varepsilon^\lambda \qquad \varepsilon^\lambda \sim N(0, \Omega_\lambda), \quad where\ \ \Omega_\lambda = diag(\omega_\lambda^2, \sigma_\lambda^2, ..., \sigma_\lambda^2) \tag{6}$$

$-1 < \lambda_t < 0$ for t=1,....,T

Ignoring any terms not involving λ, the prior density for λ is given by

$$log\ \ p(\lambda|\sigma_\lambda^2, \Omega_\lambda) \propto -\tfrac{1}{2}(\lambda)'H'\Omega_\lambda^{-1}H(\lambda) + g_\lambda(\lambda, \sigma_\lambda^2)$$

where,

$$g_\lambda(\lambda, \sigma_\lambda^2) = -\sum_{t=2}^{T} log\left(\Phi\left(\frac{0-\lambda_{t-1}}{\sigma_\lambda}\right) - \Phi\left(\frac{-1-\lambda_{t-1}}{\sigma_\lambda}\right)\right)$$

Combining the above two conditional densities we obtain,

$$log\ \ p(\lambda|Y, \bullet) \propto -\tfrac{1}{2}(\lambda - \hat{\lambda})' D_\lambda^{-1}(\lambda - \hat{\lambda}) + g_\lambda(\lambda, \sigma_\lambda^2)$$

where,

$D_\lambda = (H'\Omega_\lambda^{-1}H + X_u'\Omega_s^{-1}X_u)^{-1}$

$\hat{\lambda} = D_\lambda(X_u'\Omega_s^{-1}NW)$

The addition of the term $g_\lambda(\lambda, \sigma_\lambda^2)$ leads to a non-standard density. Accordingly, we sample λ using an independence-chain Metropolis-Hastings (MH) procedure. This involves first generating candidate draws from $N(\hat{\lambda}, D_\lambda)$ using the precision-based algorithm that are then accepted or rejected based on the accept-reject Metropolis-Hastings (ARMH) algorithm (discussed in Chan and Strachan, 2012).

Step 4. Derive the conditional distribution $p(\tau^S|Y, \bullet)$

The information about τ^S comes from three sources. Below, we derive an expression for each of these sources.

The first source is the services inflation measurement equation. Rewrite it in a matrix notation as,

$$\pi^S = \Phi_{\tau s} + \tau^S + \varepsilon^S \qquad \varepsilon^S \sim N(0, \Omega_S), \quad where\ \ \Omega_S = diag(e^{h_1^{Ser}}, e^{h_2^{Ser}}, ..., e^{h_T^{Ser}}) \qquad (7)$$

where, $\Phi_{\tau s} = \begin{pmatrix} \lambda_1(U_1 - U_1^*) + \alpha_1^S(e^{h_1^{Ser}} - e^{\overline{h}_1^{Ser}}) \\ \lambda_1(U_2 - U_2^*) + \alpha_2^S(e^{h_2^{Ser}} - e^{\overline{h}_2^{Ser}}) \\ \vdots \\ \lambda_1(U_T - U_T^*) + \alpha_T^S(e^{h_T^{Ser}} - e^{\overline{h}_T^{Ser}}) \end{pmatrix}$,

The likelihood is

$log\ \ p(\pi^S|U, U^*, \bullet) \propto -\frac{1}{2}\iota_T h^{Ser} - \frac{1}{2}(\pi^S - \Phi_{\tau s})'\Omega_S^{-1}(\pi^S - \Phi_{\tau s})$

where ι_T is a $T \times 1$ column vector of ones.

The second source of information is from the state equation of τ^S. Rewrite it in a matrix notation,

$$H\tau^S = \mu_s + \varepsilon^{\tau S} \qquad \varepsilon^{\tau S} \sim N(0, \Omega_{\tau s}), \quad where\ \ \Omega_{\tau s} = diag(\omega_{\tau s}^2, e^{h_1^{\tau S}}, ..., e^{h_{T-1}^{\tau S}}) \tag{8}$$

where, $\mu_s = \begin{pmatrix} \tau_0^S \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

That is, the prior density for τ^S is given by

$$p(\tau^S|h^{\tau S}) \propto -\tfrac{1}{2}(\tau^S - H^{-1}\mu_s)' H' \Omega_{\tau s}^{-1} H(\tau^S - H^{-1}\mu_s)$$

The third source of information comes from the measurement equation that links surveys to τ^*. Rewrite the equation in a matrix notation,

$$F^{\pi s} = \beta^\pi \tau^S + \varepsilon^{z\pi} \qquad \varepsilon^{z\pi} \sim N(0, \Omega_{z\pi}) \tag{9}$$

where,

$f_t^{\pi s} = \frac{Z_t^\pi - C^\pi - \beta^\pi(w_t^G \tau^G)}{w_t^S}$
$F^{\pi s} = (f_1^{\pi s}, f_2^{\pi s}, ..., f_T^{\pi s})'$
$\Omega_{z\pi} = diag(\sigma_{z\pi}^2, ..., \sigma_{z\pi}^2)$
Ignoring any terms not involving τ^S, we have

$$log\ \ p(Z^\pi|\tau^S, \bullet) \propto -\tfrac{1}{2}(F^{\pi s} - \beta^\pi \tau^S)' \Omega_{z\pi}^{-1}(F^{\pi s} - \beta^\pi \tau^S)$$

Combining the above three conditional densities we obtain,

$$log\ \ p(\tau^S|Y,\bullet) \propto -\tfrac{1}{2}(\tau^S-\hat{\tau}^S)'D_{\tau s}^{-1}(\tau^S-\hat{\tau}^S)$$

where,

$$D_{\tau s} = (\Omega_s^{-1} + H'\Omega_{\tau s}^{-1}H + (\beta^\pi)^2\Omega_{z\pi}^{-1})^{-1}$$

$$\hat{\tau}^S = D_{\tau s}(\Omega_s^{-1}(\pi^S - \Phi_{\tau s}) + H'\Omega_{\tau s}^{-1}\mu_s + \beta^\pi\Omega_{z\pi}^{-1}F^{\pi s})$$

The candidate draws are sampled from $N(\hat{\tau}^S, D_{\tau s})$ using the precision-based algorithm.

Step 5. Derive the conditional distribution $p(\tau^G|Y,\bullet)$

The information about τ^G comes from three sources. Below, we derive an expression for each of these sources.

The first source is the services inflation measurement equation. Rewrite it in a matrix notation as,

$$\pi^S = \Phi_{\tau g} + \tau^G + \varepsilon^G \qquad \varepsilon^G \sim N(0,\Omega_G), \quad where\ \ \Omega_G = diag(e^{h_1^{Goods}}, e^{h_2^{Goods}}, ..., e^{h_T^{Goods}}) \tag{10}$$

where, $\Phi_{\tau g} = \begin{pmatrix} \alpha_1^G(e^{h_1^{Goods}} - e^{\bar{h}_1^{Goods}}) \\ \alpha_2^G(e^{h_2^{Goods}} - e^{\bar{h}_2^{Goods}}) \\ \vdots \\ \alpha_T^G(e^{h_T^{Goods}} - e^{\bar{h}_T^{Goods}}) \end{pmatrix}$,

The likelihood is

$$log\ \ p(\pi^G|U,U^*,\bullet) \propto -\tfrac{1}{2}\iota_T h^{Goods} - \tfrac{1}{2}(\pi^G-\Phi_{\tau g})'\Omega_G^{-1}(\pi^G-\Phi_{\tau g})$$

where ι_T is a $T\times 1$ column vector of ones.

The second source of information is from the state equation of τ^G. Rewrite it in a matrix notation,

$$H\tau^G = \mu_g + \varepsilon^{\tau G} \qquad \varepsilon^{\tau G} \sim N(0,\Omega_{\tau g}), \quad where\ \ \Omega_{\tau g} = diag(\omega_{\tau g}^2, e^{h_1^{\tau G}}, ..., e^{h_{T-1}^{\tau G}}) \tag{11}$$

where, $\mu_g = \begin{pmatrix} \tau_0^G \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

That is, the prior density for τ^G is given by

$$p(\tau^G|h^{\tau G}) \propto -\tfrac{1}{2}(\tau^G - H^{-1}\mu_g)' H' \Omega_{\tau g}^{-1} H(\tau^G - H^{-1}\mu_g)$$

The third source of information comes from the measurement equation that links surveys to τ^*. Rewrite the equation in a matrix notation,

$$F^{\pi g} = \beta^\pi \tau^G + \varepsilon^{z\pi} \qquad \varepsilon^{z\pi} \sim N(0, \Omega_{z\pi}) \tag{12}$$

where,

$f_t^{\pi g} = \frac{Z_t^\pi - C^\pi - \beta^\pi (w_t^S \tau^S)}{w_t^G}$
$F^{\pi g} = (f_1^{\pi g}, f_2^{\pi g}, ..., f_T^{\pi g})'$
$\Omega_{z\pi} = diag(\sigma_{z\pi}^2, ..., \sigma_{z\pi}^2)$
Ignoring any terms not involving τ^G, we have

$$log\ \ p(Z^\pi|\tau^G, \bullet) \propto -\tfrac{1}{2}(F^{\pi g} - \beta^\pi \tau^G)' \Omega_{z\pi}^{-1} (F^{\pi g} - \beta^\pi \tau^G)$$

Combining the above three conditional densities we obtain,

$$log\ \ p(\tau^G|Y, \bullet) \propto -\tfrac{1}{2}(\tau^G - \hat{\tau}^G)' D_{\tau g}^{-1} (\tau^G - \hat{\tau}^G)$$

where,

$$D_{\tau g} = (\Omega_G^{-1} + H' \Omega_{\tau g}^{-1} H + (\beta^\pi)^2 \Omega_{z\pi}^{-1})^{-1}$$

$$\hat{\tau}^G = D_{\tau g}(\Omega_G^{-1}(\pi^G - \Phi_{\tau g}) + H' \Omega_{\tau g}^{-1} \mu_s + \beta^\pi \Omega_{z\pi}^{-1} F^{\pi g})$$

The candidate draws are sampled from $N(\hat{\tau}^G, D_{\tau g})$ using the precision-based algorithm.

Step 6. Derive the conditional distribution $p(h^{Ser}|Y, \bullet)$

This is a high-dimensional and non-standard density. To derive the conditional density $p(h^{Ser}|Y, \bullet)$, we follow the steps detailed in Chan (2017).

a. Obtain the gradient and Hessian of the log-density $log\ \ p(h^{Ser}|Y, \alpha^S, \sigma^2_{hs}, \bullet)$

According to the Bayes's theorem

$$log\ \ p(h^{Ser}|Y, \alpha^S, \sigma^2_{hs}, h^{Ser}_0) \propto p(Y|\alpha^S, h^{Ser}) \quad p(h^{Ser}|\sigma^2_{hs}, h^{Ser}_0)$$

The prior density of h^{Ser}, $p(h^{Ser}|\sigma^2_{hs}, h^{Ser}_0)$ is Gaussian with log-density

$$log\ \ p(h^{Ser}|\sigma^2_{hs}, h^{Ser}_0) = \tfrac{-1}{2\sigma^2_{hs}}\ \ (h^{Ser'}H'Hh^{Ser} - 2h^{Ser'}H'Hh^{Ser}_0\iota_T) + c_1$$

where c_1 is a constant independent of h^{Ser}.

The gradient and Hessian of the above log conditional density is

$$\tfrac{d}{dh^{Ser}}\ \ log\ \ p(h^{Ser}|\sigma^2_{hs}, h^{Ser}_0) = \tfrac{-1}{\sigma^2_{hs}}H'H(h^{Ser} - h^{Ser}_0\iota_T)$$

$$\tfrac{d^2}{dh^{Ser}dh^{Ser'}}\ \ log\ \ p(h^{Ser}|\sigma^2_{hs}, h^{Ser}_0) = \tfrac{-1}{\sigma^2_{hs}}H'H$$

Next, let f and G denote, respectively, the gradient and the negative Hessian of $log\ \ p(Y|\alpha^S, h^{Ser})$

$f = \tfrac{d}{dh^{Ser}}\ \ log\ \ p(Y|\alpha^S, h^{Ser})$
$G = \tfrac{d^2}{dh^{Ser}dh^{Ser'}}\ \ log\ \ p(Y|\alpha^S, h^{Ser})$

Note that the log conditional likelihood $log\ \ p(Y|\alpha^S, h^{Ser})$ has the form
$log\ \ p(Y|\alpha^S, h^{Ser}) = \sum_{t=1}^{T} log\ \ p(\pi^S_t|\alpha^S_t, h^{Ser}_t)$.

The t-th element of f is simply $f_t = \frac{d\ \ log\ \ p(\pi^S_t|\alpha^S_t, h^{Ser}_t)}{dh^{Ser}_t}$

And the negative Hessian G is diagonal, where the t-th diagonal element is
$G_t = \frac{-d^2\ \ log\ \ p(\pi^S_t|\alpha^S_t, h^{Ser}_t)}{(dh^{Ser}_t)^2}$

Putting all these together, the gradient and Hessian of $log\ \ p(h^{Ser}|Y, \alpha^S, \sigma^2_{hs}, h^{Ser}_0)$ are

$$S(h^{Ser}) = \frac{d}{dh^{Ser}} log\ \ p(h^{Ser}|Y, \alpha^S, \sigma^2_{hs}, h_0^{Ser}) = \frac{-1}{\sigma^2_{hs}} H'H(h^{Ser} - h_0^{Ser}\iota_T) + f$$

$$F(h^{Ser}) = \frac{d^2}{dh^{Ser} dh^{Ser\prime}} log\ \ p(h^{Ser}|Y, \alpha^S, \sigma^2_{hs}, h_0^{Ser}) = -(\frac{1}{\sigma^2_{hs}} H'H + G)$$

Solve the above using the Newton-Raphson method:

1. Initialize the algorithm with $h^{Ser} = h^{Ser(1)}$ for some constant vector $h^{Ser(1)}$

2. For t=1,2,..., use $h^{Ser} = h^{Ser(t)}$ in the evaluation of $S(h^{Ser})$ and $F(h^{Ser})$, and compute the Newton-Raphson recursion $h^{Ser(t+1)} = h^{Ser(t)} - F(h^{Ser(t)})S(h^{Ser(t)})$

3. Repeat this procedure until some convergence criterion is reached, e.g., when $||h^{Ser(t+1)} - h^{Ser(t)||} < \epsilon$ for some prefixed tolerance level ϵ.

The convergence is very fast because the Hessian is globally negative definite.

Let $\hat{h}^{Ser}$ be the mode obtained from the Newton-Raphson method and let K_{hs} denote the negative Hessian evaluated at the mode, i.e., $K_{hs} = \frac{H'H}{\sigma^2_{hs}} + G$

Then use $N(\hat{h}^{Ser}, K_{hs}^{-1})$ as the proposal density in an independence-chain MH step. Since the precision matrix is again a band matrix, we rely on the precision sampler to obtain candidate draws from $N(\hat{h}^{Ser}, K_{hs}^{-1})$ efficiently.

The candidate draw is then accepted with a probability, which is computed in the MH step.

Step 7. Derive the conditional distribution $p(h^{Goods}|Y, \bullet)$

Same procedure as discussed in the previous step, but applied to goods inflation.

Step 8. Derive the conditional distribution $p(h^{\tau S}|Y, \bullet)$

Following, Chan, Koop, and Potter (2013; 2016), we draw $h^{\tau S}$ using the accept-reject independence-chain Metropolis Hastings (ARMH) algorithm of Chan and Strachan (2012; page 32-34).

Step 9. Derive the conditional distribution $p(h^{\tau G}|Y, \bullet)$

As done in the previous step, we follow Chan, Koop, and Potter (2013; 2016) to draw $h^{\tau G}$ using the accept-reject independence-chain Metropolis Hastings (ARMH) algorithm of Chan and Strachan (2012; page 32-34).

Step 10. Derive the conditional distribution $p(C^{\pi}|Y, \bullet)$

The information about C^{π} comes from two sources. Below, we derive an expression for each of these two sources.

The first source is the measurement equation linking survey to τ^*. Rewrite it in a matrix notation,

$$N^{z\pi} = C^{\pi} + \varepsilon^{z\pi} \qquad \varepsilon^{z\pi} \sim N(0, \Omega_{z\pi}) \tag{13}$$

where,

$n_t^{z\pi} = Z_t^{\pi} - \beta^{\pi}\tau^*$
$N^{z\pi} = (n_1^{z\pi}, n_2^{z\pi}, ..., n_T^{z\pi})'$
$\Omega_{z\pi} = diag(\sigma_{z\pi}^2, ..., \sigma_{z\pi}^2)$

Ignoring any terms not involving C^{π}, the likelihood is

$$log\ \ p(Z^{\pi}|C^{\pi}, \bullet) \propto -\tfrac{1}{2}(N^{z\pi} - C^{\pi})'\Omega_{z\pi}^{-1}(N^{z\pi} - C^{\pi})$$

The second source comes from the state equation for C^{π}. We rewrite it in a matrix notation as follows,

$$HC^{\pi} = \mu_{c\pi} + \varepsilon^{c\pi} \qquad \varepsilon^{c\pi} \sim N(0, \Omega_{c\pi}), \quad where\ \ \Omega_{c\pi} = diag(\omega_{c\pi}^2, \sigma_{c\pi}^2, ..., \sigma_{c\pi}^2) \tag{14}$$

where,

$$\mu_{c\pi} = \begin{pmatrix} C_0^{\pi} \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Ignoring any terms not involving C^{π}, the prior density for C^{π} is given by

$$log\ \ p(C^{\pi}|\sigma^2_{c\pi}, \Omega_{c\pi}) \propto -\tfrac{1}{2}(C^{\pi} - H^{-1}\mu_{c\pi})' H' \Omega^{-1}_{c\pi} H (C^{\pi} - H^{-1}\mu_{c\pi})$$

Combining the above two conditional densities we obtain,

$$log\ \ p(C^{\pi}|Y, \bullet) \propto -\tfrac{1}{2}(C^{\pi} - \hat{C}^{\pi})' D^{-1}_{C^{\pi}} (C^{\pi} - \hat{C}^{\pi})$$

where,

$D_{C^{\pi}} = (H' \Omega^{-1}_{c\pi} H + \Omega^{-1}_{z\pi})^{-1}$

$\hat{C}^{\pi} = D_{C^{\pi}} (H' \Omega^{-1}_{c\pi} \mu_{c\pi} + \Omega^{-1}_{z\pi} N^{z\pi})$

The candidate draws are sampled from $N(\hat{C}^{\pi}, D_{C^{\pi}})$ using the precision-based algorithm.

Step 11. Derive the conditional distribution $p(\theta|Y, \bullet)$

There are 10 parameters in the vector θ. These parameters are drawn in 10 separate blocks using standard regression procedures.

Substep 11.1 Derive the conditional distribution $p(\beta^{\pi}|Y, \bullet)$

Candidate draws are sampled from $N(\hat{\beta}^{\pi}, D_{\beta\pi})$ using the precision-based algorithm.

where,

$D_{\beta\pi} = (V^{-1}_{\beta\pi} + X^{z\pi\prime} \Omega^{-1}_{z\pi} X^{z\pi})^{-1}$
$\hat{\beta}^{\pi} = D_{\beta\pi} (V^{-1}_{\beta\pi} \beta^{\pi}_0 + X^{z\pi\prime} \Omega^{-1}_{z\pi} J^{z\pi})$

$j_t^{z\pi} = Z_t^{\pi} - C_t^{\pi}$
$J^{z\pi} = (j_1^{z\pi}, ..., j_T^{z\pi})'$
$x_t^{z\pi} = w_t^S \tau_t^S + w_t^G \tau_t^G$
$X^{z\pi} = (x_1^{z\pi}, ..., x_T^{z\pi})'$

$V_{\beta\pi}^{-1}$ is the prior variance and β_0^{π} is the prior mean for β^{π}

Substep 11.2 Derive the conditional distribution $p(\sigma_{c\pi}^2|Y, \bullet)$

$p(\sigma_{c\pi}^2|Y, \bullet)$ is a standard inverse-Gamma density,

Candidate draws are sampled from

$$p(\sigma_{c\pi}^2|Y, \bullet) \sim IG(\nu_{c\pi 0} + \tfrac{T-1}{2}, S_{c\pi 0} + \tfrac{1}{2}\sum_{t=2}^{T}(C_t^{\pi} - C_{t-1}^{\pi})^2)$$

Substep 11.3 Derive the conditional distribution $p(\sigma_{z\pi}^2|Y, \bullet)$

$p(\sigma_{z\pi}^2|Y, \bullet)$ is a standard inverse-Gamma density,

Candidate draws are sampled from

$$p(\sigma_{z\pi}^2|Y, \bullet) \sim IG(\nu_{z\pi 0} + \tfrac{T}{2}, S_{z\pi 0} + \tfrac{1}{2}\sum_{t=1}^{T}(Z_t^{\pi} - C_t^{\pi} - \beta^{\pi}\tau^*)^2)$$

Substep 11.4 Derive the conditional distribution $p(\sigma_{\alpha s}^2|Y, \bullet)$

$p(\sigma_{\alpha s}^2|Y, \bullet)$ is a standard inverse-Gamma density,

Candidate draws are sampled from

$$p(\sigma_{\alpha s}^2|Y, \bullet) \sim IG(\nu_{\alpha s0} + \tfrac{T-1}{2}, S_{\alpha s0} + \tfrac{1}{2}\sum_{t=2}^{T}(\alpha_t^S - \alpha_{t-1}^S)^2)$$

Substep 11.5 Derive the conditional distribution $p(\sigma_{\alpha G}^2|Y, \bullet)$

$p(\sigma^2_{\alpha G}|Y, \bullet)$ is a standard inverse-Gamma density,

Candidate draws are sampled from

$$p(\sigma^2_{\alpha G}|Y, \bullet) \sim IG(\nu_{\alpha g0} + \tfrac{T-1}{2}, S_{\alpha g0} + \tfrac{1}{2}\sum_{t=2}^{T}(\alpha^G_t - \alpha^G_{t-1})^2)$$

Substep 11.6 Derive the conditional distribution $p(\sigma^2_{hs}|Y, \bullet)$

$p(\sigma^2_{hs}|Y, \bullet)$ is a standard inverse-Gamma density,

Candidate draws are sampled from

$$p(\sigma^2_{hs}|Y, \bullet) \sim IG(\nu_{hs0} + \tfrac{T-1}{2}, S_{hs0} + \tfrac{1}{2}\sum_{t=2}^{T}(h^{Ser}_t - h^{Ser}_{t-1})^2)$$

Substep 11.7 Derive the conditional distribution $p(\sigma^2_{hG}|Y, \bullet)$

$p(\sigma^2_{hG}|Y, \bullet)$ is a standard inverse-Gamma density,

Candidate draws are sampled from

$$p(\sigma^2_{hG}|Y, \bullet) \sim IG(\nu_{hg0} + \tfrac{T-1}{2}, S_{hg0} + \tfrac{1}{2}\sum_{t=2}^{T}(h^{Goods}_t - h^{Goods}_{t-1})^2)$$

Substep 11.8 Derive the conditional distribution $p(\sigma^2_{h\tau s}|Y, \bullet)$

$p(\sigma^2_{h\tau s}|Y, \bullet)$ is a standard inverse-Gamma density,

Candidate draws are sampled from

$$p(\sigma^2_{h\tau s}|Y, \bullet) \sim IG(\nu_{h\tau s0} + \tfrac{T-1}{2}, S_{h\tau s0} + \tfrac{1}{2}\sum_{t=2}^{T}(h^{\tau S}_t - h^{\tau S}_{t-1})^2)$$

Substep 11.9 Derive the conditional distribution $p(\sigma^2_{h\tau G}|Y, \bullet)$

$p(\sigma^2_{h\tau G}|Y, \bullet)$ is a standard inverse-Gamma density,

Candidate draws are sampled from

$$p(\sigma^2_{h\tau G}|Y, \bullet) \sim IG(\nu_{h\tau g0} + \tfrac{T-1}{2}, S_{h\tau g0} + \tfrac{1}{2}\sum_{t=2}^{T}(h_t^{\tau G} - h_{t-1}^{\tau G})^2)$$

Substep 11.10 Derive the conditional distribution $p(\sigma^2_\lambda|Y, \bullet)$

$p(\sigma^2_\lambda|Y, \bullet)$ is a non-standard density because of the constraints on λ,

$$log\ \ p(\sigma^2_\lambda|Y, \bullet) \propto -(\nu_{\lambda 0}+1)\ \ log\ \ \sigma^2_\lambda - \tfrac{S_{\lambda 0}}{\sigma^2_\lambda} - \tfrac{T-1}{2}\ \ log\ \ \sigma^2_\lambda - \tfrac{1}{2\sigma^2_\lambda}\textstyle\sum_{t=2}^{T}(\lambda_t - \lambda_{t-1})^2 + g_\lambda(\lambda, \sigma^2_\lambda)$$

The candidate draws from $p(\sigma^2_\lambda|Y, \bullet)$ are obtained via the MH step with the proposal density $IG(\nu_{\lambda 0} + \frac{T-1}{2}, S_{\lambda 0} + \frac{1}{2}\sum_{t=2}^{T}(\lambda_t - \lambda_{t-1})^2)$

A3. MCMC Convergence Diagnostics

This section documents the diagnostic properties of the Base model's MCMC algorithm. Following Koop, Leon-Gonzalez, and Strachan (2010), among others, we report the autocorrelation functions of the posterior draws (10th and 50th order sample autocorrelation), inefficiency factors (IFs), and convergence diagnostic (CD) of Geweke (1992) of the model parameters.[1]

One of the most common metrics examined to assess the efficiency of the MCMC sampler is to look at the autocorrelation function of the draws, which indicates how well the chain is mixing. Low autocorrelations are preferred to higher because the lower the autocorrelation, the closer the draws are to being independent and the higher the efficiency of the algorithm. The plots shown in the left panels of Figures 1,...,5 correspond to the 10th and 50th order autocorrelations in the draws, and as can be seen, they indicate very low autocorrelation. In the case of the 50th order autocorrelation, most indicate correlation less than 0.1, and in the case of 10th order except for a few, most indicate correlation below 0.3.

To assess whether the MCMC sampler has converged, a rough rule of thumb is to look at the CDs and see whether 95% of them are less than 2. If they are, then convergence is likely achieved. Based on the plots in the right panels of Figures 1,...,5, majority of CDs are within +/- 2.

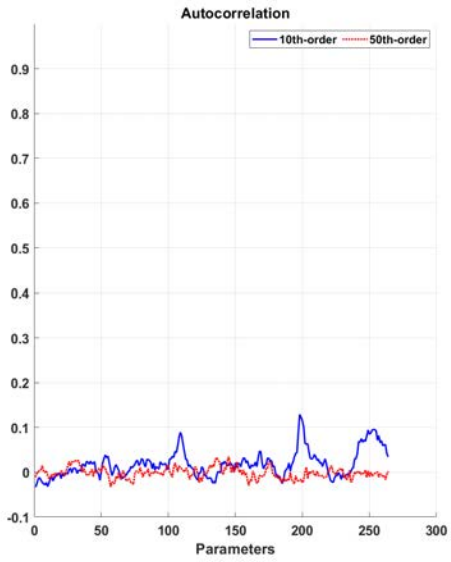
The inefficiency factor related to the autocorrelation functions is the inverse of Geweke's (1992) relative numerical efficiency measure (RNE). It is computed using the following formula, $(1 + 2\sum_{i=1}^{\infty} \rho_i)$, where ρ_i refers to the $k - th$ order autocorrelation of the chain. Figure 6 plots the IF for all the model parameters of the Baseline model. The values lower than or close to 20 are considered desirable. As shown, most of the IFs are below 20. (Note: IFs are computed using the default setting in LeSage's toolbox: estimation of spectral density at frequency zero uses a tapered window of 4%.)

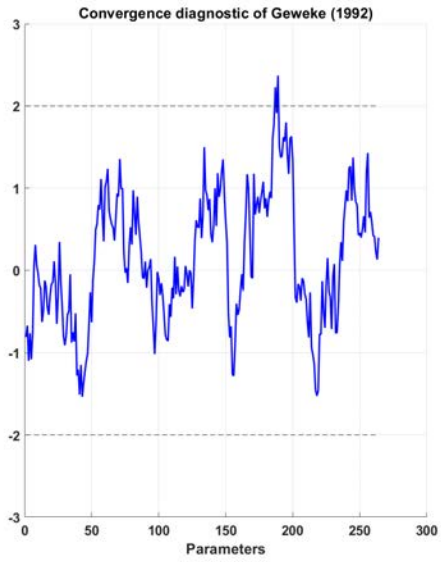
Overall, these diagnostic measures suggest good convergence properties of the MCMC algorithm developed for the Baseline model.

[1] In computing some of these metrics, we have benefitted from the Matlab toolbox developed by James P. LeSage. A detailed explanation, including intuition for these convergence diagnostics, is provided in Koop (2003; page 67-68) and Chan et al. (2019; page 209).

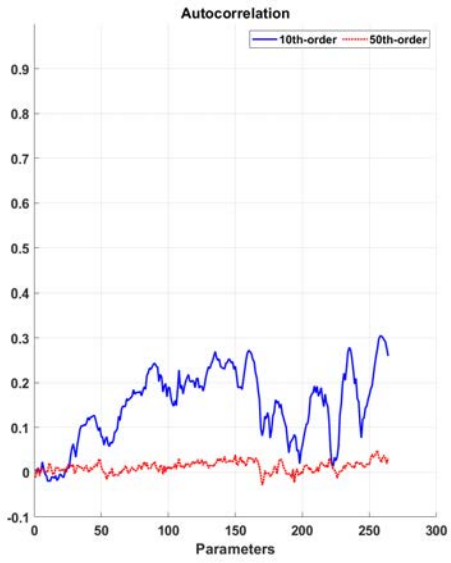
Figure 1: MCMC Diagnostics: Autocorrelation and CDs

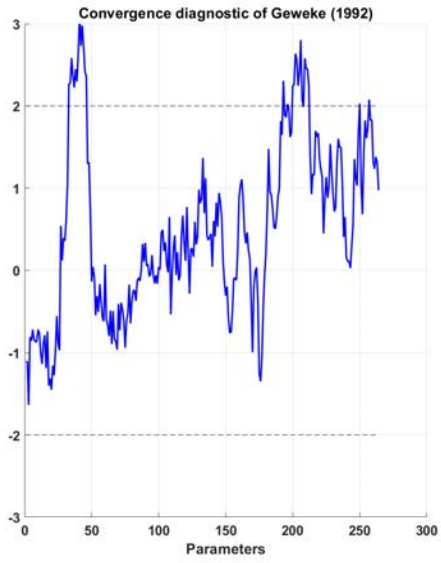
SV parameter: h^{Ser}





SV parameter: h^{Goods}

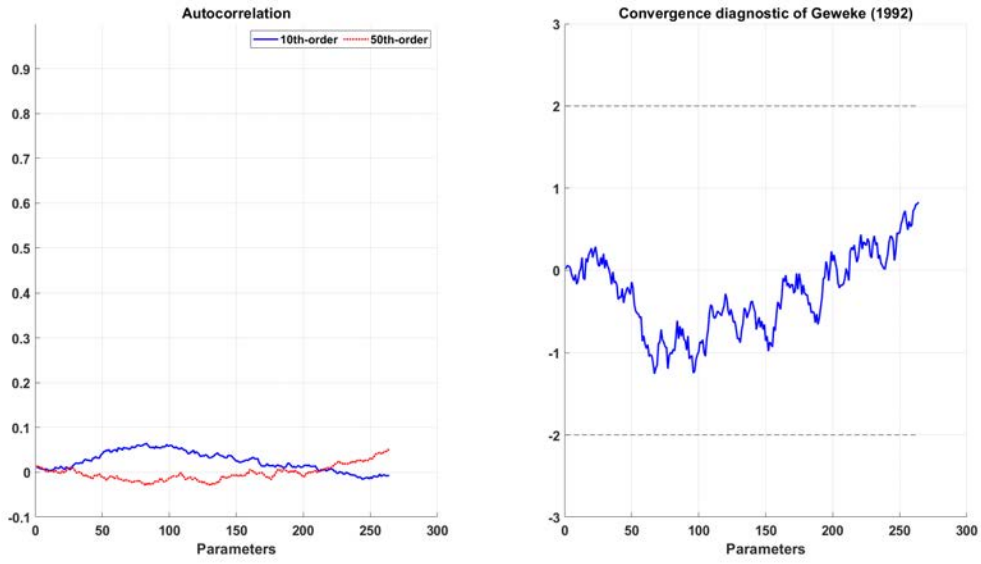




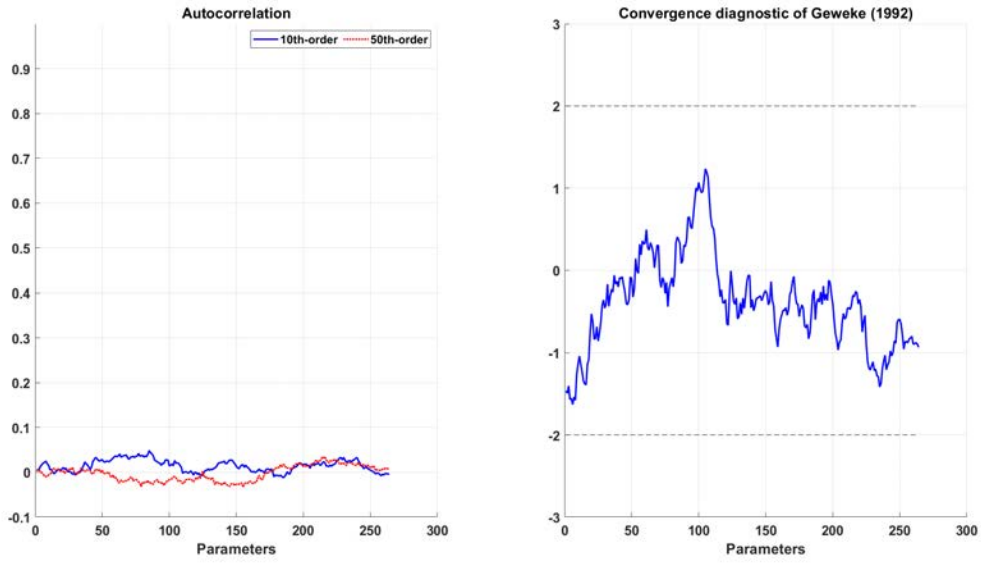
Note: Diagnostics based on estimating data from 1959Q3 through 2025Q2.

Figure 2: MCMC Diagnostics: Autocorrelation and CDs

SV parameter: $h^{\tau S}$



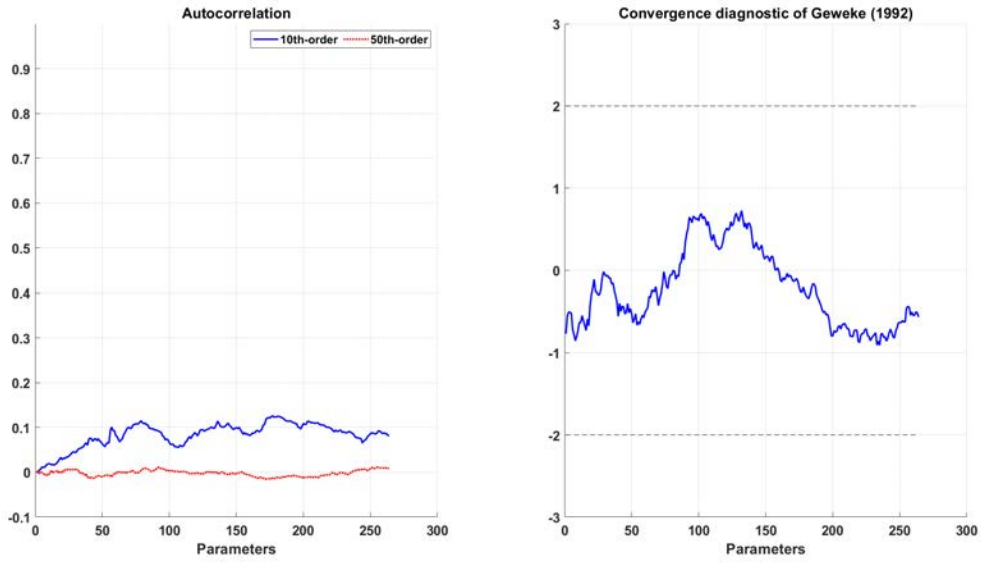
SV parameter: $h^{\tau G}$



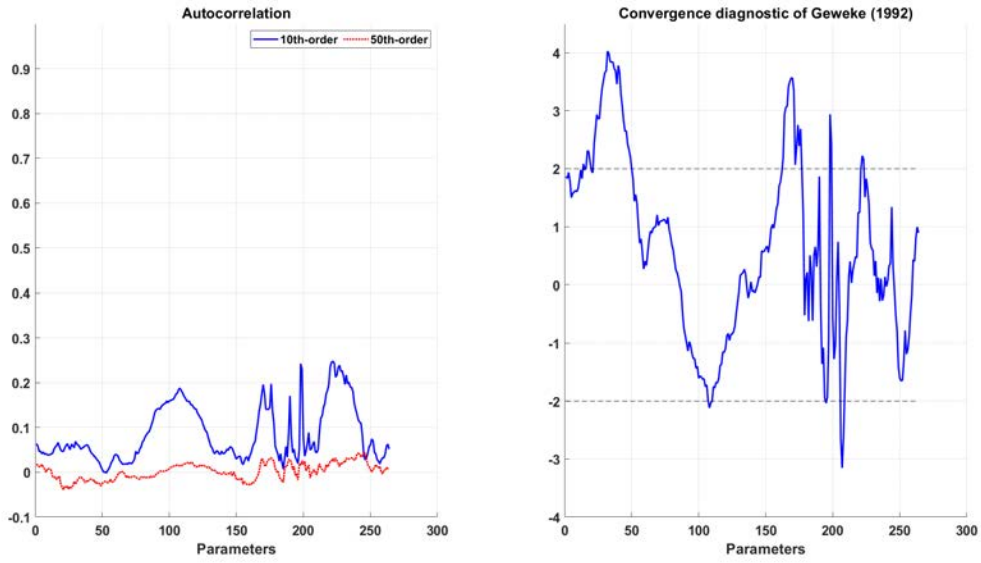
Note: Diagnostics based on estimating data from 1959Q3 through 2025Q2.

Figure 3: MCMC Diagnostics: Autocorrelation and CDs

TVP parameter: α^S



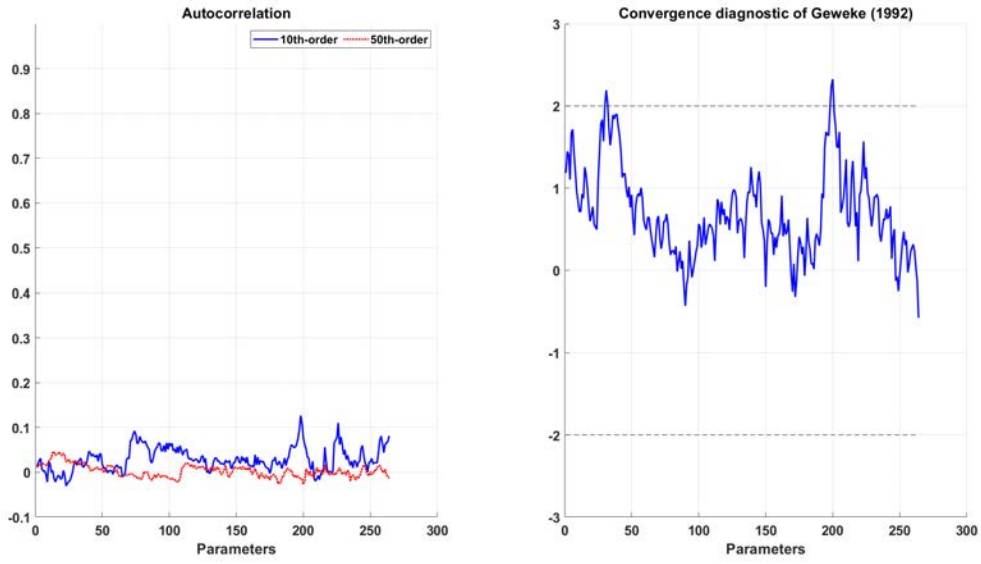
TVP parameter: α^G



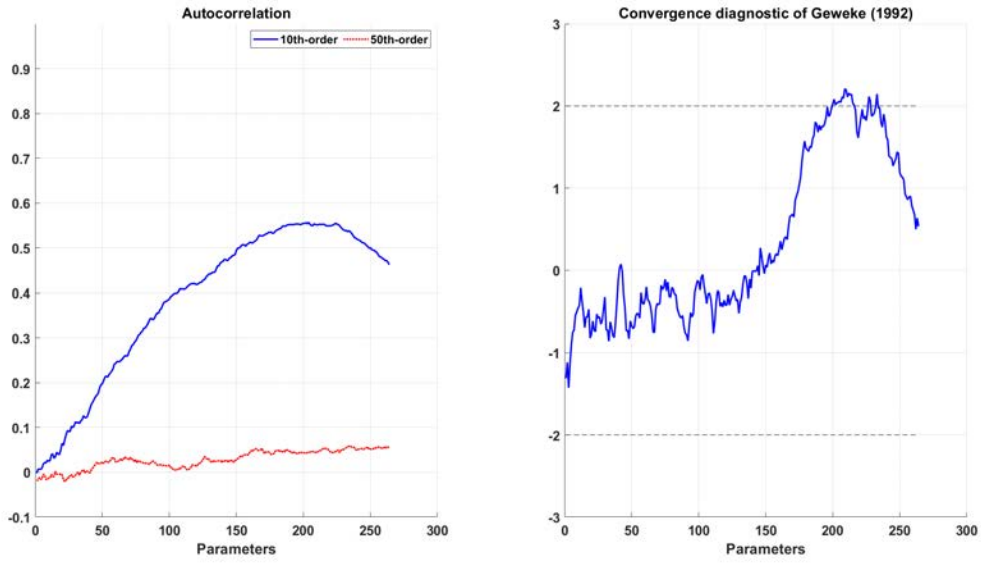
Note: Diagnostics based on estimating data from 1959Q3 through 2025Q2.

Figure 4: MCMC Diagnostics: Autocorrelation and CDs

TVP parameter: λ



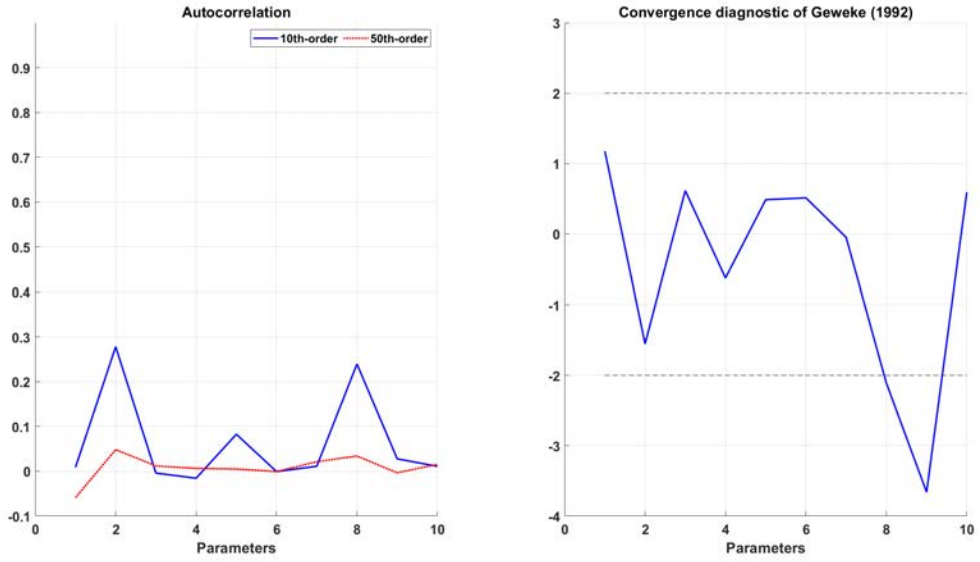
TVP parameter: C^{π}



Note: Diagnostics based on estimating data from 1959Q3 through 2025Q2.

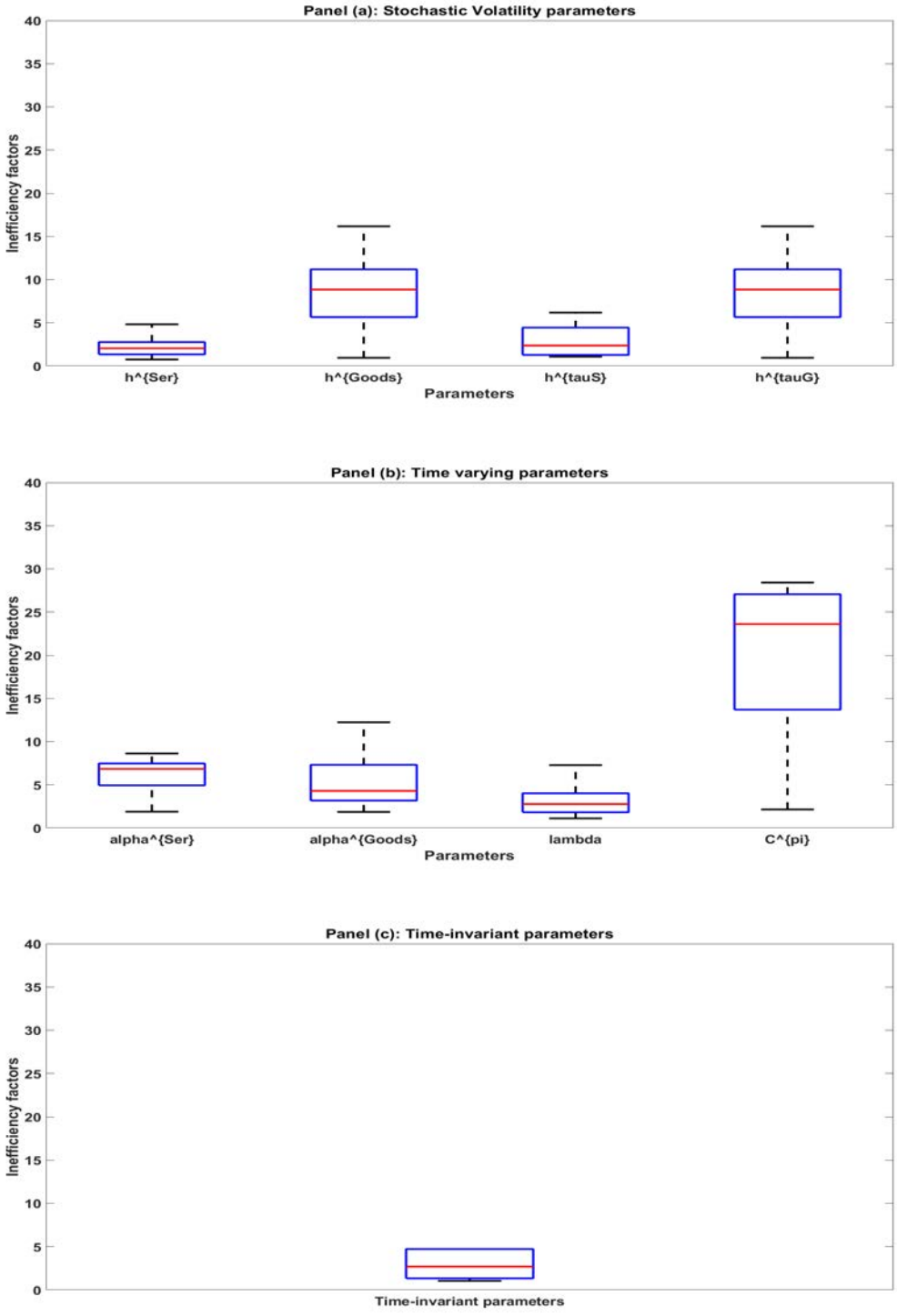
Figure 5: MCMC Diagnostics: Autocorrelation and CDs

Time-invariant parameters



Note: Diagnostics based on estimating data from 1959Q3 through 2025Q2.

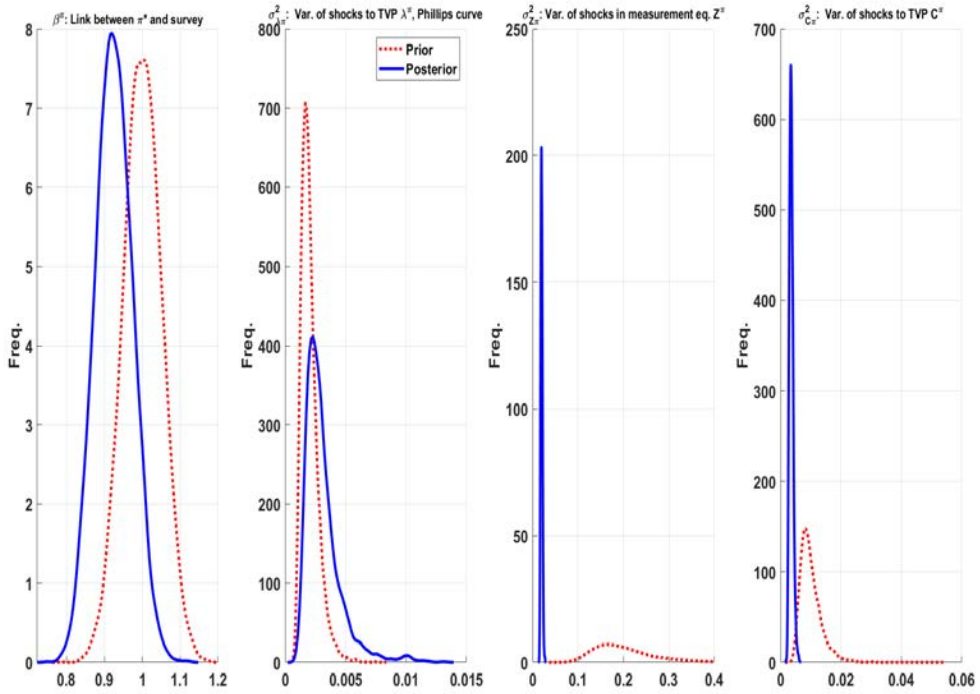
Figure 6: MCMC Diagnostics: Inefficiency Factors



Note: Diagnostics based on estimating data from 1959Q3 through 2025Q2. The red line represents the median, the lower and upper lines represent the 25th and 75th percentiles, respectively. The whiskers extend to the maximum and minimum.

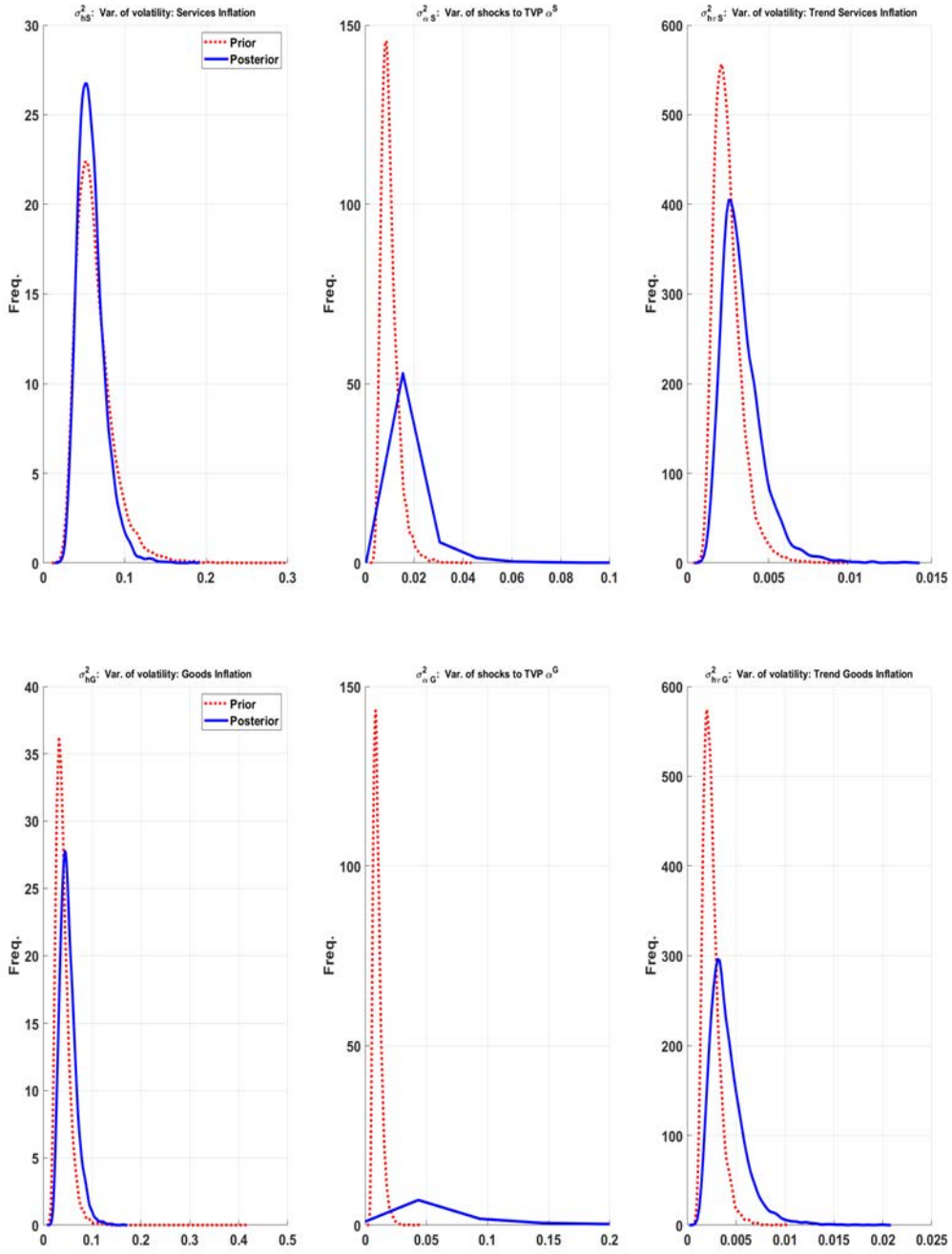
A4. Prior and Posterior Distributions of the Parameters: Baseline Model

Figure 7: Prior and Posterior Distributions of the Parameters: Baseline Model



Note: Plotted in dotted red lines are the prior distributions, in blue are the posterior distributions based on estimating data from 1959Q3 through 2025Q2.

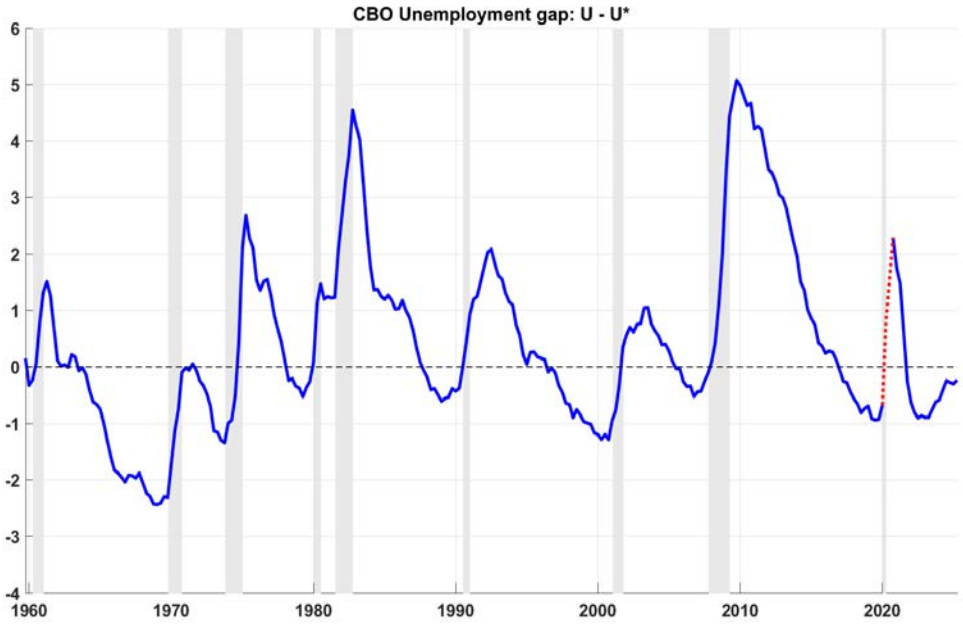
Figure 8: Prior and Posterior Distributions of the Parameters: Baseline Model



Note: Plotted in dotted red lines are the prior distributions, in blue are the posterior distributions based on estimating data from 1959Q3 through 2025Q2.

A5. CBO Unemployment Rate Gap

Figure 9: Congressional Budget Office estimate of the Unemployment Gap



Note: The plot in blue represents the Congressional Budget Office (CBO) estimate of the Unemployment gap, defined as the difference between the unemployment rate and the CBO estimate of the natural rate of unemployment. The plot in red represents the inferred value of the unemployment gap for 2020Q2 and 2020Q3, the COVID-pandemic quarters; the inferred value for 2020Q2 is based on averaging the actual unemployment gap values for 2020Q1 and 2020Q4, and for 2020Q3 is based on averaging the inferred 2020Q2 value and the actual 2020Q4 value. The shaded bars represent the NBER-defined recession bars.

A6. Additional Baseline Model Specifications

We estimate many additional variants of the Baseline model to assess the empirical importance of time-variation in the parameters, capturing the effects of uncertainty on inflation, and the strength of the Phillips curve relationship. We then compare the model fit of these model variants to the Baseline model that allows for the time-variation in all the parameters. The comparison provides strong empirical support for the inclusion of time-variation in the parameters, confirming findings of the earlier papers.

M8: Baseline but no TVP in α^S. To assess the empirical support of time variation in the parameter α^S, which captures the effect of uncertainty on the level of services inflation, we estimate a variant of the baseline model with a time-invariant parameter α^S.

M9: Baseline but no TVP in α^S and α^G . To assess the empirical support for time variation in the parameter α^G, which captures the effect of uncertainty on the level of goods inflation, we take the model M2 and shut down time variation in parameter α^G. Comparing the model fit of M2 with M3 provides an assessment of the usefulness of time variation in α^G, and comparing the model fit of M3 with the baseline M1 provides a joint assessment of the usefulness of allowing for time variation in both parameters α^S and α^G.

M10: Baseline but no TVP in λ. To assess the empirical support for time variation in the parameter λ, which captures the business cycle effects on the level of services inflation (Phillips curve slope), we estimate a variant of the baseline model with a time-invariant parameter λ.

M11: Baseline but no TVP in α^S, α^G, and λ . To jointly assess the empirical support for time variation in the parameters α^S, α^G, and λ, we estimate a variant of the baseline model that shuts off time variation for all three parameters.

M12: Two-Sector + Uncertainty + PC (Baseline but No Survey data). To assess the usefulness of survey expectations, we estimate a variant of the baseline model that excludes the equation linking survey data to trend inflation.

M13: Single-Sector + Survey + Uncertainty + PC. This variant models inflation dynamics using aggregate PCE inflation rather than the two-sector goods-and-services inflation breakdown used in the baseline model. It keeps all other features of the

baseline model: uses survey data, allows time-varying uncertainty effects on the level of aggregate inflation, allows a time-varying Phillips curve relationship, and includes SV in the shocks to trend inflation and in the measurement equation defining inflation. (See section below for model details.) Comparing this model's fit to aggregate inflation data with that of the baseline model (M1) assesses the usefulness of modeling aggregate PCE inflation using sectoral inflation data (goods and services). It is worth pointing out that, even though this model does not use sector-level data, the fact that it incorporates three other features makes it a novel empirical model.

M14: Single-Sector + Survey. To assess the usefulness of allowing for uncertainty and business cycle effects on the level of aggregate PCE inflation, we estimate a variant of the single-sector model that zeroes out α^S, α^G, and λ for all time periods.

M15: Single-Sector + Uncertainty + PC (Single-Sector but No Survey data). To assess the usefulness of survey expectations, we estimate a variant of the single-sector model that excludes the equation linking survey data to trend inflation.

A6.1 Single-Sector Empirical Model

The specification of our single-sector model closely follows that of the two-sector model, except that, instead of using sector-level data, it directly models the dynamics of aggregate inflation.

Aggregate PCE inflation is modeled as

$$\pi_t = \underbrace{\tau_t}_{\text{Trend}} + \underbrace{\lambda_t \ (U_t - U_t^*)}_{\text{Business cycle effect}} + \underbrace{\alpha_t \ (e^{h_t} - e^{\overline{h}_t})}_{\text{Uncertainty effect}} + \varepsilon_t^{\pi}, \ \ \varepsilon_t^{\pi} \sim N(0, e^{h_t}) \qquad (15)$$

where $\overline{h}_t = \frac{\sum_{i=1}^{t-1} h_i}{t-1}$; U_t^* is informed from outside the model (CBO estimate)

Like before, the time-varying parameter, λ, the slope of the price Phillips curve is constrained between (-1,0), and the parameter α, which captures the volatility feedback, evolves according to an RW process.

Trend in aggregate PCE inflation, τ_t is modeled as a driftless random walk, with a shock term, whose variance is allowed to vary over time,

$$\tau_t = \tau_{t-1} \;\; + \;\; \varepsilon_t^{\tau}, \;\; \varepsilon_t^{\tau} \sim N(0, e^{h_t^{\tau}}) \tag{16}$$

The aggregate trend, τ, is linked to long-run survey expectations Z^{π},

$$Z_t^{\pi} = C_t^{\pi} \;\; + \;\; \beta^{\pi}\tau_t \;\; + \;\; \varepsilon_t^{z\pi}, \;\; \varepsilon_t^{z\pi} \sim N(0, \sigma_{z\pi}^2) \tag{17}$$

Like before, C_t^{π} is a time-varying intercept assumed to evolve as an RW process. Lastly, the SV processes are modeled as a driftless RW in the log-variance,

$$\begin{aligned} h_t &= h_{t-1} \;\; + \;\; \varepsilon_t^{h}, \;\; \varepsilon_t^{h} \sim N(0, \sigma_h^2) \\ h_t^{\tau} &= h_{t-1}^{\tau} \;\; + \;\; \varepsilon_t^{h\tau}, \;\; \varepsilon_t^{h\tau} \sim N(0, \sigma_{h\tau}^2) \end{aligned} \tag{18}$$

A6.2 Results: Comparison Model Fit

Table 3: Bayesian Model Comparison: Baseline Model vs. Variants
Panel A: Baseline (Two-sector model) vs. Variants

	1960-2025Q2, MDD		1960-2019, MDD	
Model	Services π	Goods π	Services π	Goods π
M1: Baseline	-310.4	-592.6	-276.1	-536.1
M8: Baseline but no TVP in α^S	-322.1	-600.1	-286.1	-545.6
M9: Baseline but no TVP in α^S and α^G	-319.7	-626.3	-283.5	-567.6
M10: Baseline but no TVP in λ	-312.4	-596.5	-282.5	-551.6
M11: Baseline but no TVP in α^S, α^G, and λ	-324.5	-628.4	-288.1	-569.5
M12: Two-Sector + Uncertainty + PC	-282.8	-583.0	-254.7	-527.3

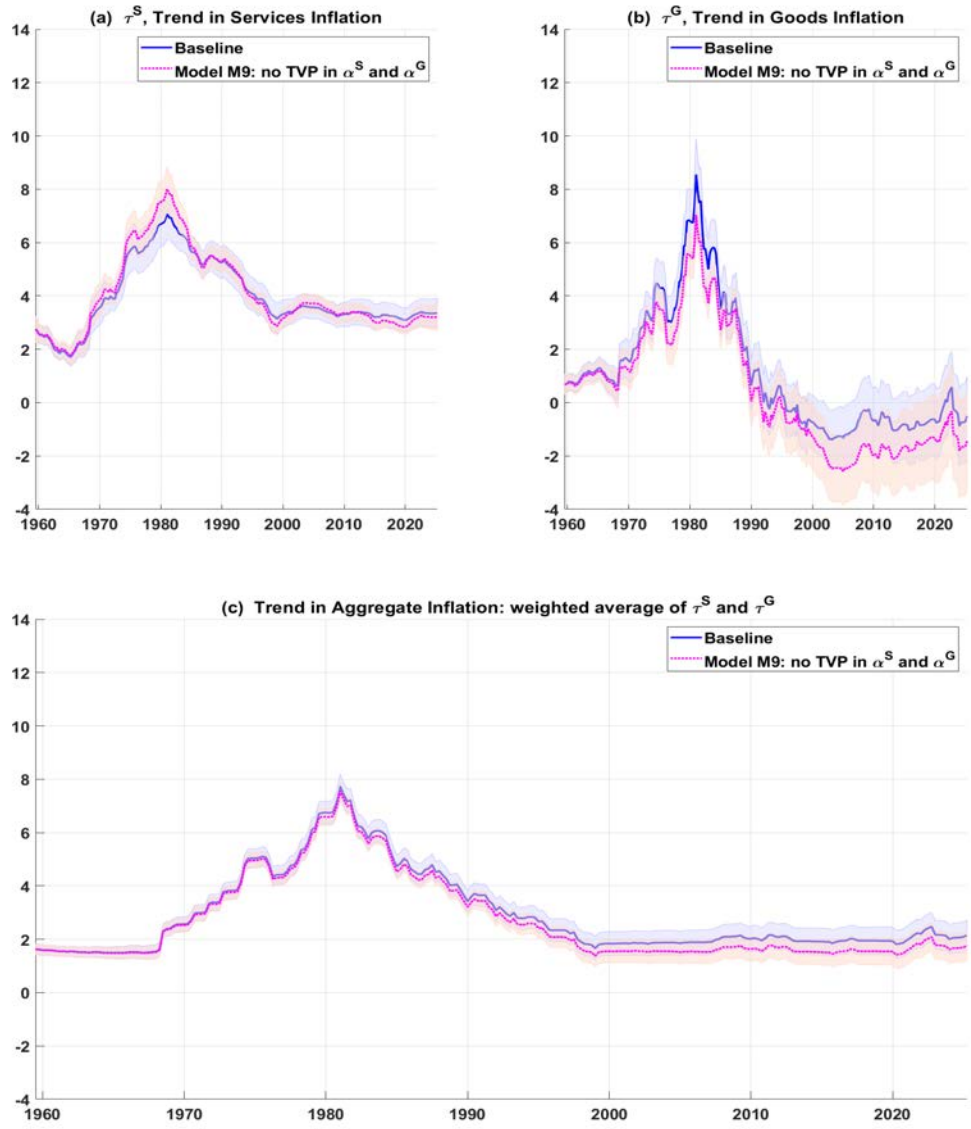
Panel B: Baseline (Two-sector model) vs. Single-sector model and its Variants

	MDD Aggregate PCE π	
Model	1960-2025Q2 sample	1960-2019 sample
M1: Baseline	-398.0	-358.4
M12: Two-Sector + Uncertainty + PC	-386.9	-349.2
M13: Single-Sector + Survey + Uncertainty + PC	-415.8	-372.4
M14: Single-Sector + Survey	-455.4	-407.1
M15: Single-Sector + Uncertainty + PC	-393.7	-355.3

Note: MDD refers to the log of the marginal data density, which measures the model's fit to the services inflation data and to the goods inflation data. The top panel reports the model's fit to the data over the entire sample (1960-2025Q2) and the pre-COVID sample (1960-2019Q4); to reduce sensitivity to the prior, in computing the MDD, we discard the initial two predictive likelihoods corresponding to 1959Q3 and 1959Q4. The bottom panel reports the model fit to the aggregate PCE inflation for the single-sector model (which models directly the aggregate PCE inflation) and its variants, to facilitate comparison, the implied model fit for the Baseline model and Baseline model without survey data (Two-Sector + Uncertainty + PC) are also reported. The term "no TVP" means no time variation is permitted in the parameter of interest.

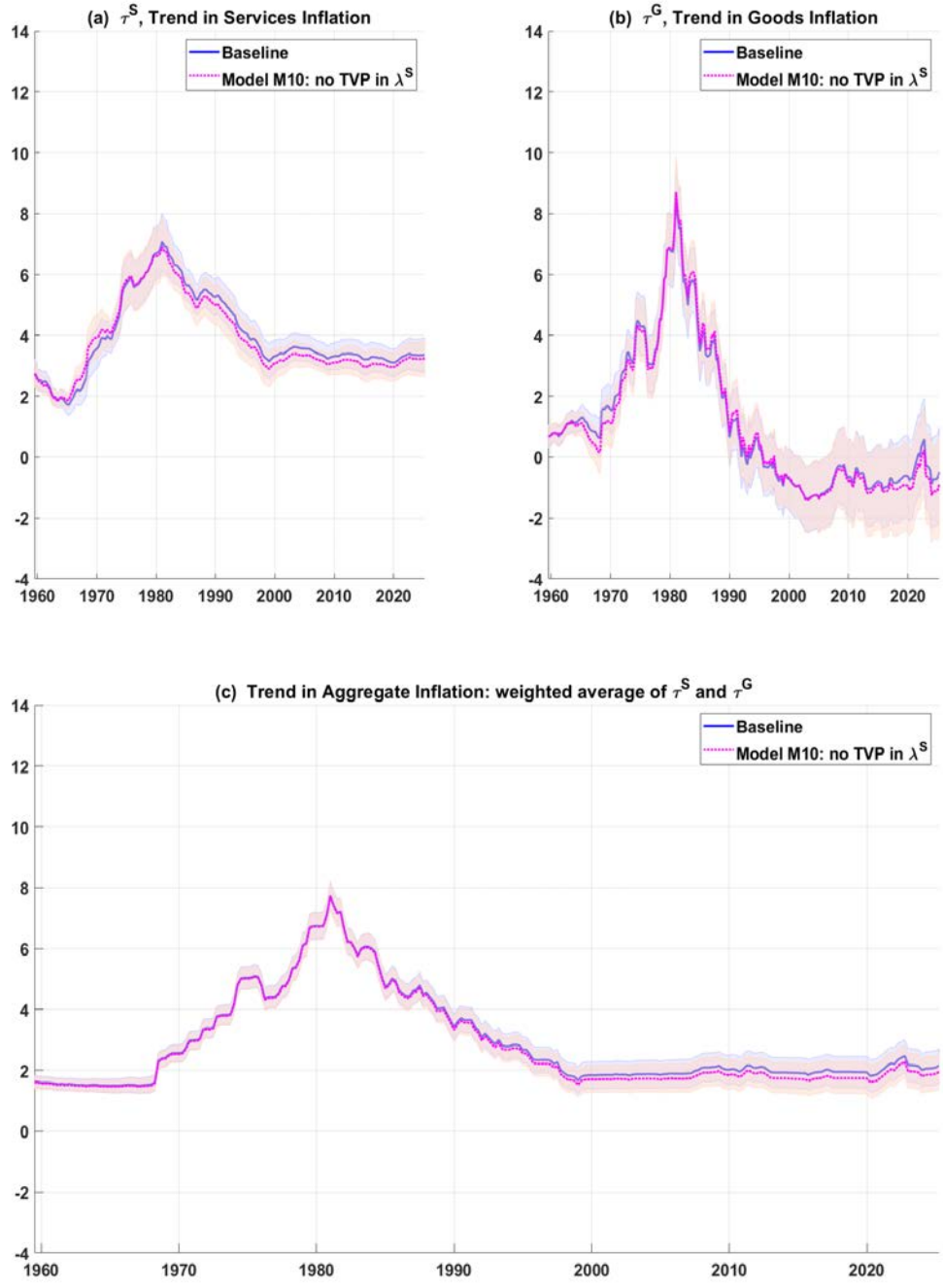
A6.2 Results: Trend Comparison between Baseline and Variants

Figure 10: Trend Inflation: Baseline vs. Variant with time-invariant Uncertainty effects



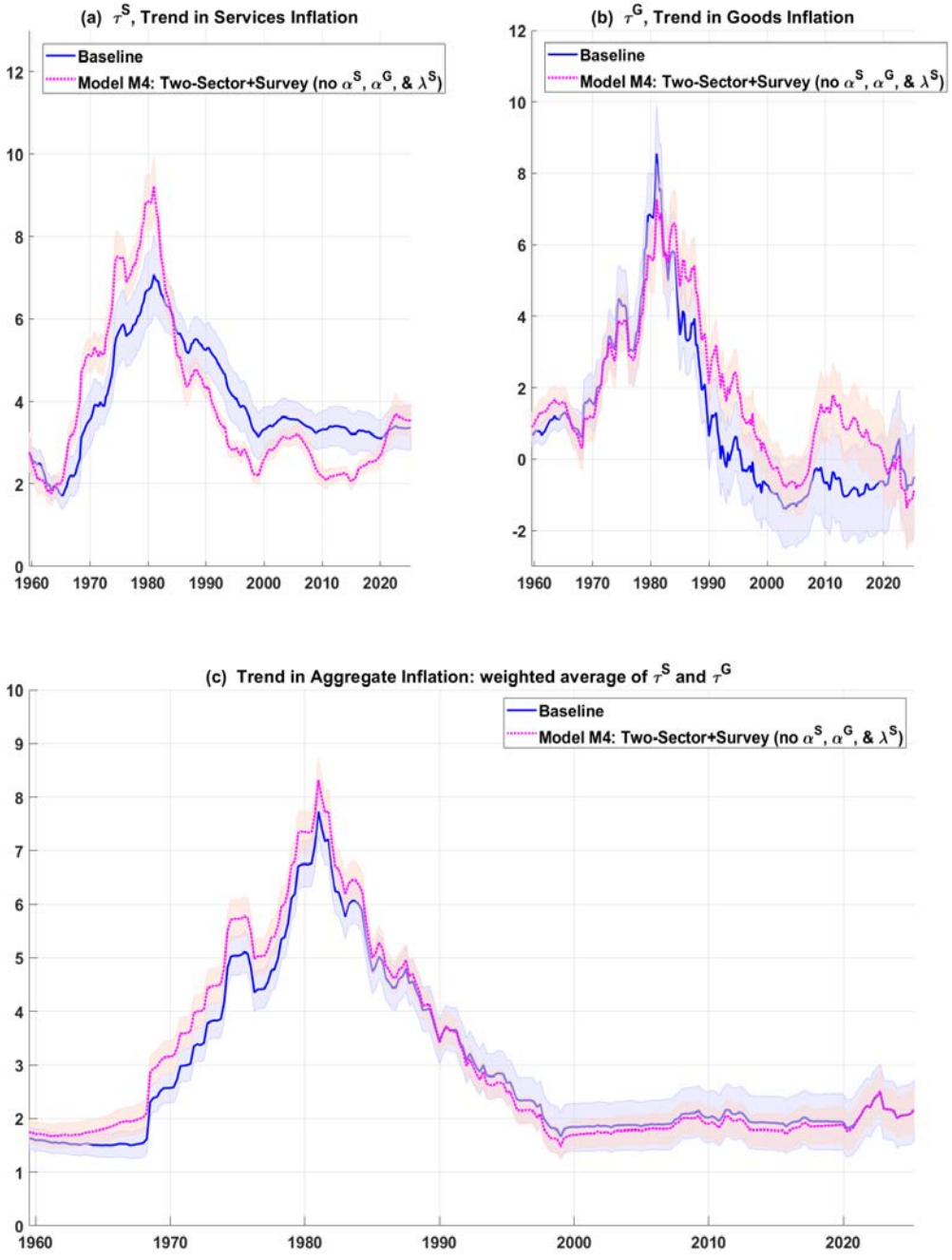
Note: The posterior estimates are based on the full sample (from 1959Q3 through 2025Q2). The shaded area represents the 68% posterior coverage interval. Panel (a), posterior estimates of the trend in services PCE inflation. Panel (b), posterior estimates of the trend in goods PCE inflation. Panel (c), posterior estimates of the trend in aggregate PCE inflation, constructed as a weighted average of the trends in services and goods PCE inflation.

Figure 11: Trend Inflation: Baseline vs. Variant with time-invariant Phillips curve



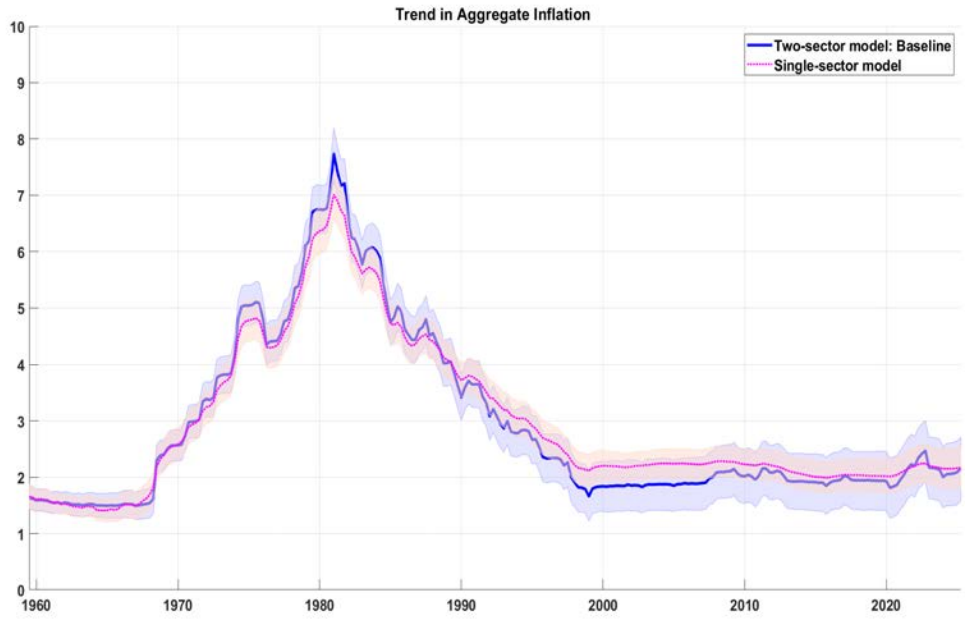
Note: The posterior estimates are based on the full sample (from 1959Q3 through 2025Q2). The shaded area represents the 68% posterior coverage interval. Panel (a), posterior estimates of the trend in services PCE inflation. Panel (b), posterior estimates of the trend in goods PCE inflation. Panel (c), posterior estimates of the trend in aggregate PCE inflation, constructed as a weighted average of the trends in services and goods PCE inflation.

Figure 12: Trend Inflation: Baseline vs. Variant with No Uncertainty and Phillips Curve



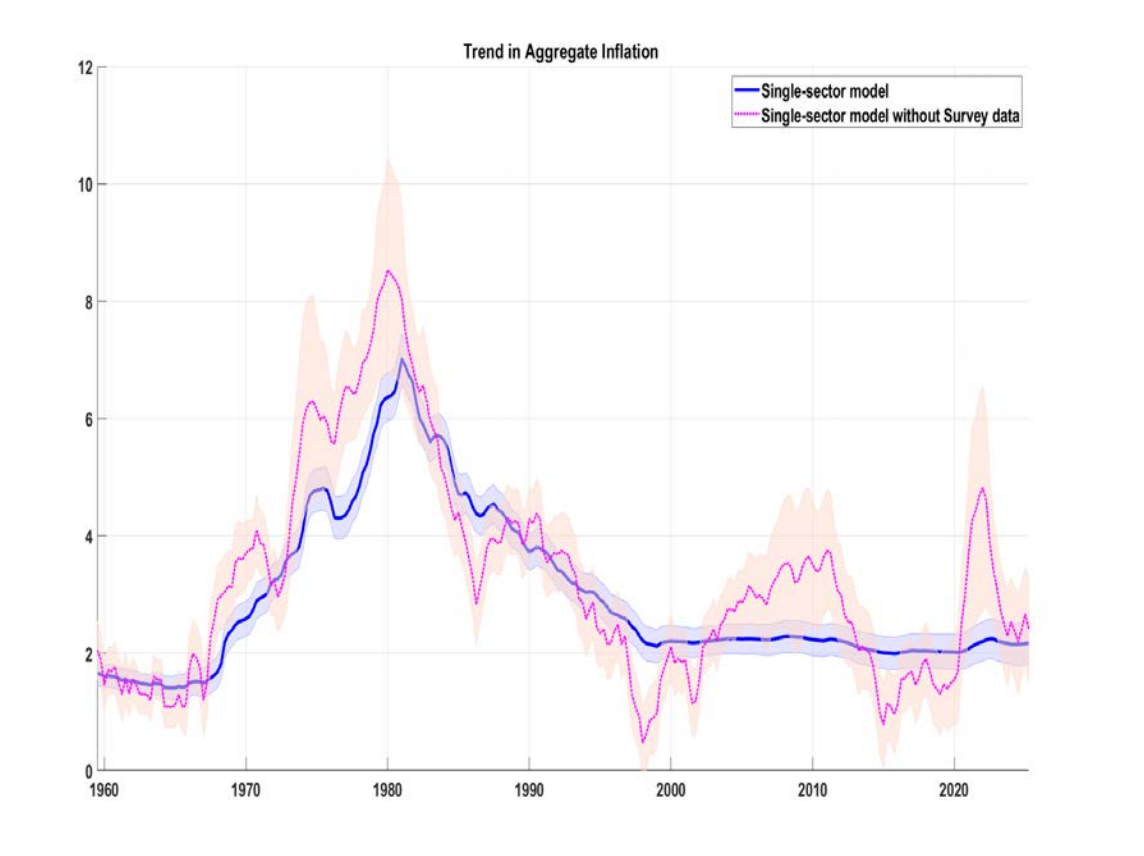
Note: The posterior estimates are based on the full sample (from 1959Q3 through 2025Q2). The shaded area represents the 68% posterior coverage interval. Panel (a), posterior estimates of the trend in services PCE inflation. Panel (b), posterior estimates of the trend in goods PCE inflation. Panel (c), posterior estimates of the trend in aggregate PCE inflation, constructed as a weighted average of the trends in services and goods PCE inflation.

Figure 13: Trend Inflation: Baseline vs. Single-Sector Model



Note: The posterior estimates are based on the full sample (from 1959Q3 through 2025Q2). The shaded area represents the 68% posterior coverage interval. The solid and dashed lines represent the posterior mean estimates from the (two-sector) baseline model and single-sector models, respectively (posterior estimates of the trend in services PCE inflation.

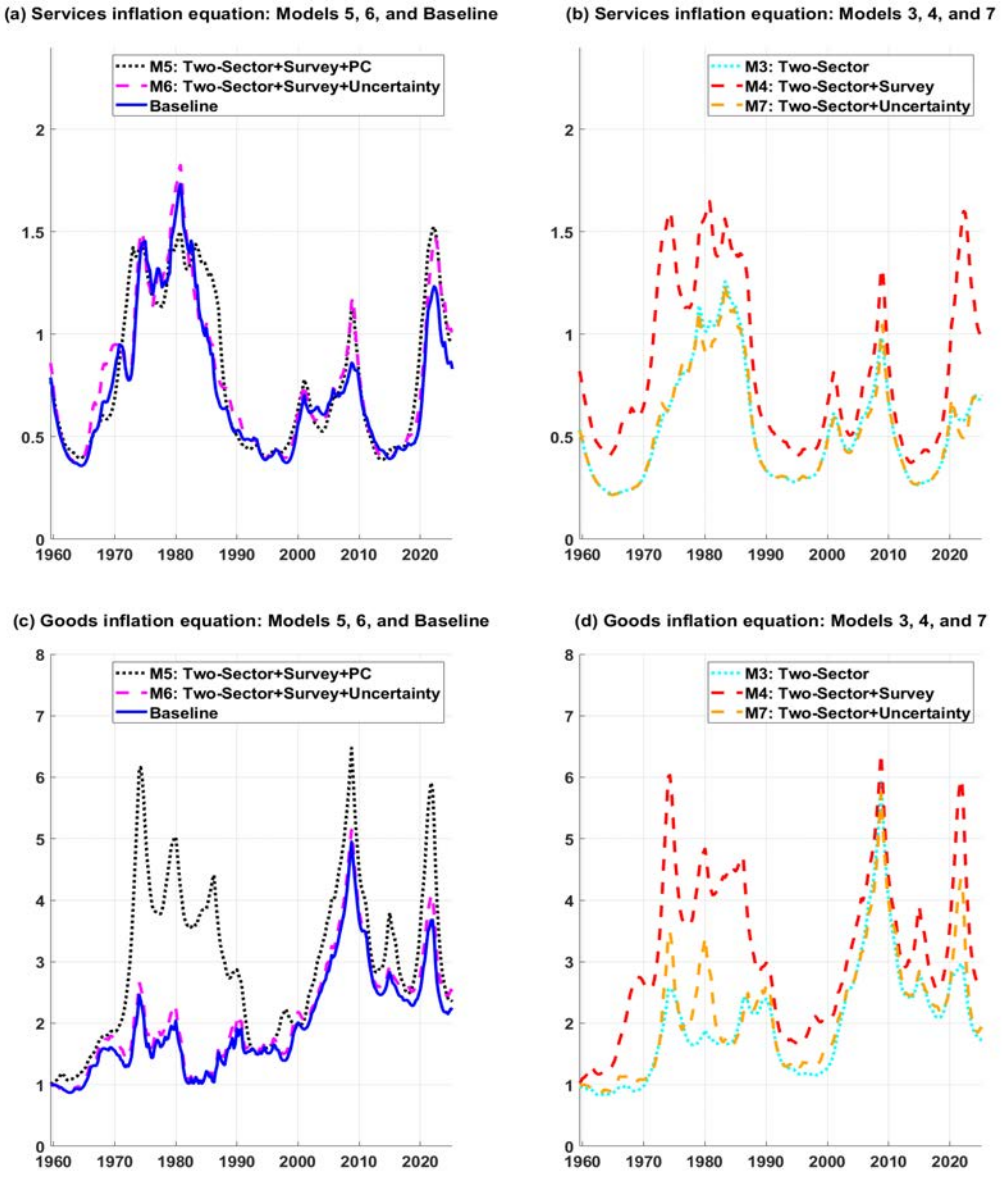
Figure 14: Trend Inflation: Single-Sector Model



Note: The posterior estimates are based on the full sample (from 1959Q3 through 2025Q2). The shaded area represents the 68% posterior coverage interval.

A7. Uncertainty Estimates: Sensitivity to Modeling Assumptions

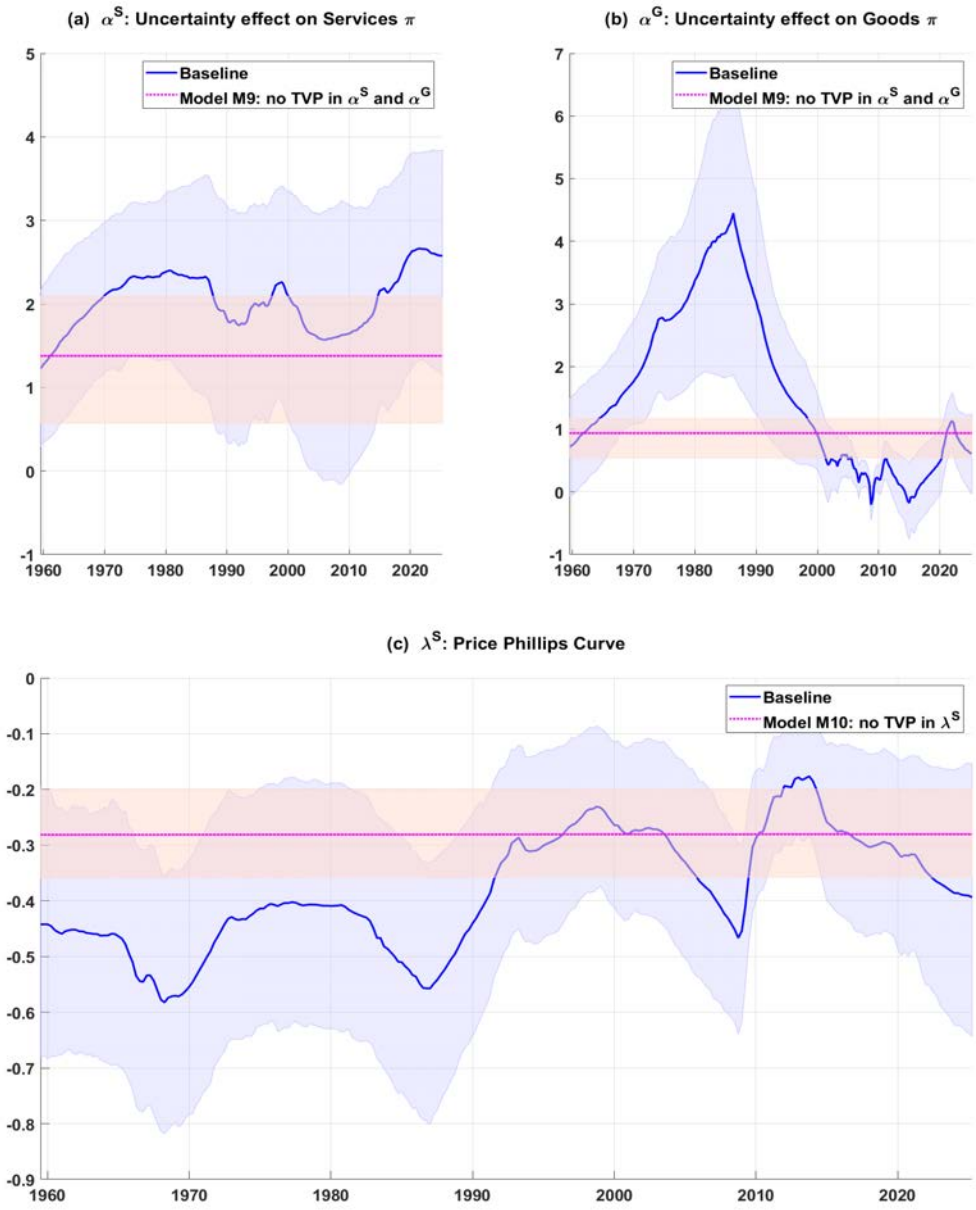
Figure 15: Estimates of Stochastic Volatility



Note: Panel (a) and Panel (b) present the uncertainty estimates associated with services inflation, i.e., the posterior median estimates of the standard deviation (sd) of the shocks defining the services inflation measurement equation. Panel (c) and Panel (d) present the uncertainty estimates associated with goods inflation. The estimates are computed using the full sample (from 1959Q3 through 2025Q2).

A8. Comparing Parameter Estimates: Baseline vs. Model with time-invariant parameters

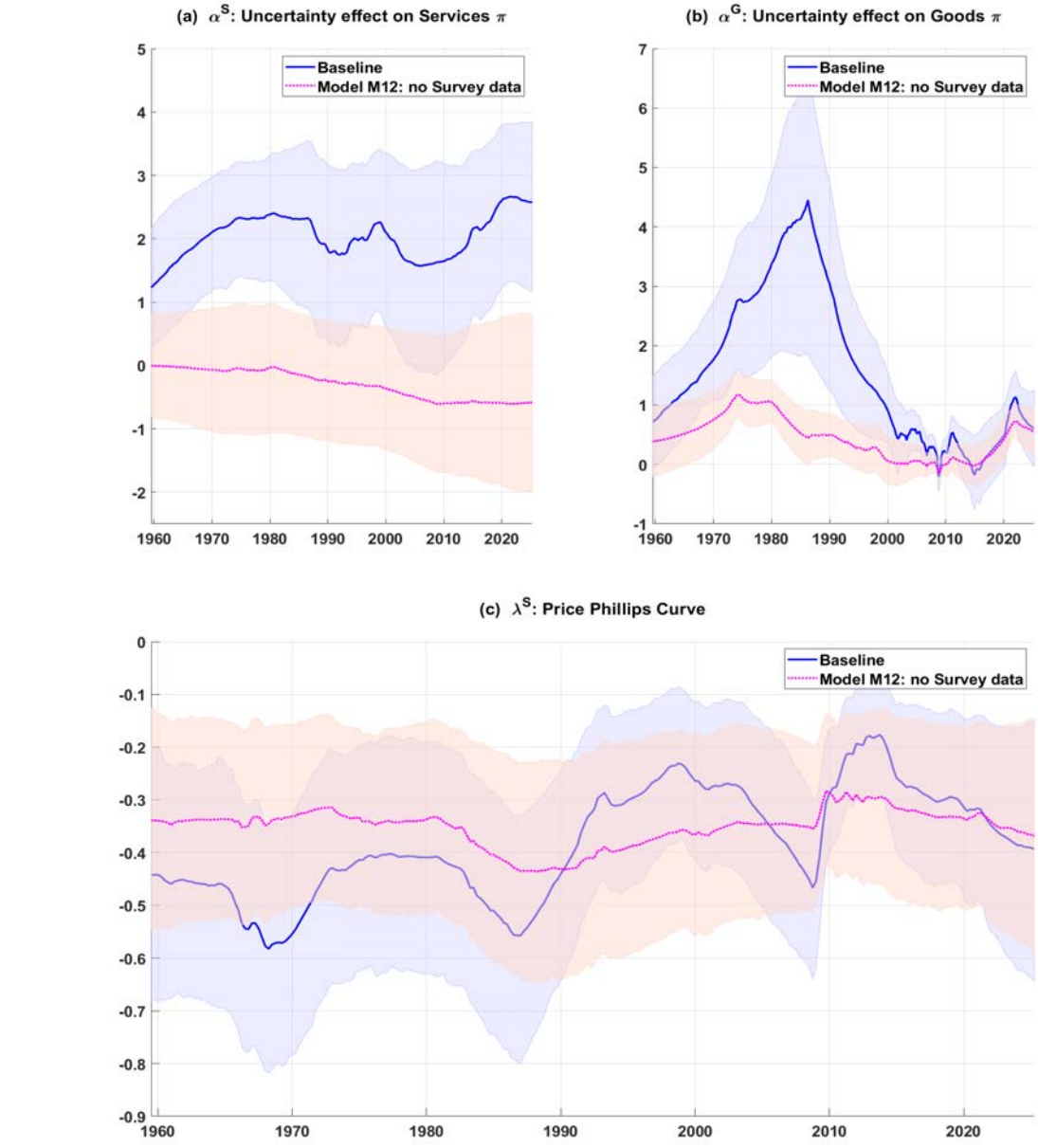
Figure 16: Estimates of TVPs: Base vs. Variant with time-invariant parameters



Note: Plotted are the posterior estimates of the parameters computed using the full sample (from 1959Q3 through 2025Q2). The solid lines represent the posterior mean, and the shaded area represents the 68% credible intervals.

A9. Comparing Parameter Estimates: Baseline vs. Model without Survey Expectations Data

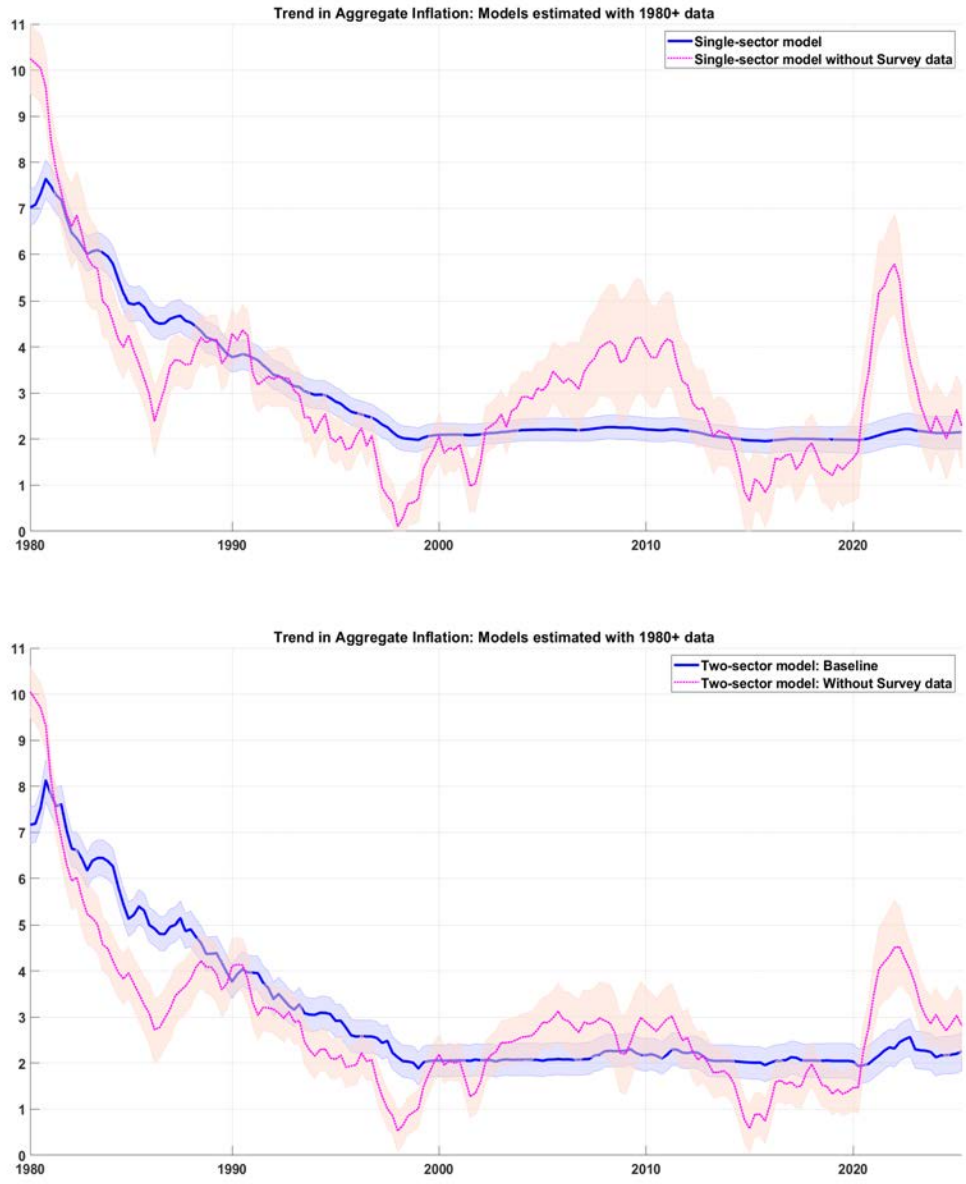
Figure 17: Estimates of TVPs: Base vs. Variant with no Survey data



Note: Plotted are the posterior estimates of the parameters computed using the full sample (from 1959Q3 through 2025Q2). The solid lines represent the posterior mean, and the shaded area represents the 68% credible intervals.

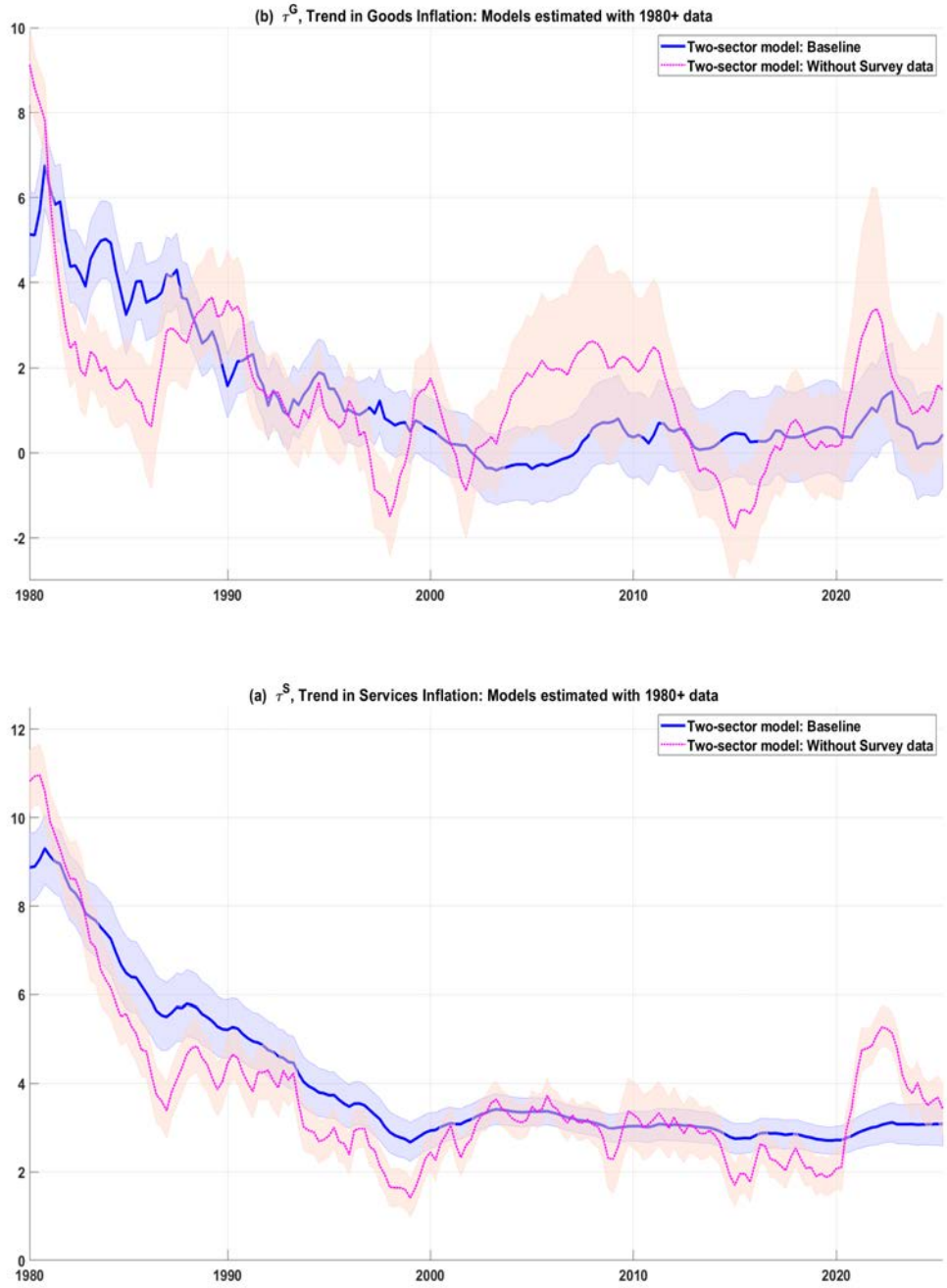
A10. Trend estimates based on estimating models with data starting from 1980 onward

Figure 18: Trend Estimates: Baseline and Single-Sector Models



Note: Plotted are the posterior estimates of the parameters computed using the full sample (from 1980Q2 through 2025Q2). The solid lines represent the posterior mean, and the shaded area represents the 68% credible intervals.

Figure 19: Goods and Services Trend Estimates: Base vs. Variant with no Survey data



Note: Plotted are the posterior estimates of the parameters computed using the full sample (from 1980Q2 through 2025Q2). The solid lines represent the posterior mean, and the shaded area represents the 68% credible intervals.

Table 4: Bayesian Model Comparison: Models estimated with data 1980+

*Panel A: Baseline (Two-sector model) vs. Variant without Survey data

	1980-2025Q2, MDD		1980-2019, MDD	
Model	Services π	Goods π	Services π	Goods π
M1: Baseline	-204.3	-437.8	-168.5	-379.8
M7: Baseline but No Survey data	-192.0	-418.0	-163.7	-362.6

*Panel B: Baseline, Single-sector, and Variants without Survey data

	MDD Aggregate PCE π	
Model	1980-2025Q2 sample	1980-2019 sample
M1: Baseline	-287.1	-246.1
M7: Baseline but No Survey data	-267.9	-230.3
M8: Single-sector	-290.2	-246.1
M10: Single-sector but No Survey data	-273.2	-236.0

Note: MDD refers to the log of the marginal data density, which measures the model's fit to the services inflation data and to goods inflation data. The top panel reports the model's fit to the data over the entire sample (1980Q4-2025Q2) and the pre-COVID sample (1980Q4-2019Q4). The bottom panel reports the model fit to the aggregate PCE inflation for the single-sector model (which models directly the aggregate PCE inflation) and its variants, to facilitate comparison, the implied model fit for the Baseline model and Baseline model without survey data are also reported.

A11. Univariate Buildout: Add AR term and Survey data

In the paper, the Univariate model is the same as the univariate UCSV model of Stock and Watson (2007). A strand of literature, including Chan et al. (2018), extends the UCSV of SW to include an autoregressive (AR1) gap term, where the gap is defined as the deviation of inflation from its trend, and the AR1 parameter varies over time to capture time-varying persistence. As discussed in Chan et al. (2013) and Chan et al. (2018), the rationale for including a time-varying AR1 gap is to allow for (possibly time-varying) persistence in the transitory component of inflation. Under this formulation, high inflation could be explained by persistent deviations of inflation from a possibly low trend inflation, reflecting monetary policymakers' greater tolerance for higher inflation than their desired low level of trend inflation for a temporary period. In our setup, the time-varying persistence in the inflation gap (i.e., the transitory component) is a function of two time-varying components: the Phillips curve and the uncertainty feedback.

An implication of allowing for a time-varying AR1 persistence parameter is that persistent movements in inflation are not only captured by the trend component but also by the AR1 persistence parameter; hence, trend estimates from such a univariate model are relatively less noisy than those from univariate UCSV formulations similar to Stock and Watson. Because of less noisy trend estimates from a univariate AR1 model compared to univariate without AR1, adding survey data influences the trend significantly less than in a model where there is no AR1; hence, the differences in the model fit across models with and without survey data are less stark than is the case for univariate models similar to SW, which omits the AR1 component. This is evident in the MDD shown in the table below. In Table 5 shown below, focusing on the columns reporting MDD for the sample ending 2019, the MDD for the model labeled "Univariate + AR [tighter priors similar to CCK]" is -389.3 compared to -396.5 for the model "Univariate + AR + Survey [tighter priors similar to CCK]", a difference that is significantly smaller than comparing models "Univariate [UCSV-SW]" and "Univariate [UCSV-SW] + Survey". Most importantly, if the AR1 univariate model specifications are estimated with data starting in 1980+ (as was the case for the featured results in Chan et al. (2018)), the AR1 model with survey data has a better fit to the model compared to the AR1 model without the survey data (see Table 6). This finding is consistent with Chan et al. (2018), who also found that for models (with AR1 component) estimated with post-1980 data, the inclusion of survey data improved model fit: In Table 6, the MDD for the model denoted "Univariate + AR [tighter priors similar to CCK]" is -306.1 (full-sample) and for model with survey data

the MDD is -305.2, so there is a slight improvement in the fit. However, a comparison across two similar model formulations with less tight priors does not yield the same conclusion: comparing "Univariate + AR" and "Univariate + AR + Survey". Note that the data prefer specifications with less tight priors. Overall, this highlights the role of the estimation sample and prior setting as additional factors influencing the empirical usefulness of survey data for model fit.

It is noteworthy that the univariate formulation similar to Stock and Watson (2007) fits the aggregate PCE inflation data better than the univariate AR1 model, which is consistent with the supplementary analysis reported in Eo et al. (2023), who found that extending the two-sector UCSV model by adding AR1 substantially worsened the fit of the model to the inflation data. (CCK refers to Chan et al. (2018)).

Lastly, we reinforce the usefulness of uncertainty and Phillips curve components by reporting the MDD of a model that extends the univariate model by adding uncertainty and PC. As can be seen, adding these two components to the univariate model of SW improves the model fit: Univariate [UCSV-SW] MDD of -398.8 vs. "Univariate [UCSV SW] + Uncertainty + PC" MDD of -393.7. This result reinforces the results of Chan (2017) and corroborates our findings for the Two-Sector model discussed in the paper.

Table 5: Bayesian Model Comparison: Univariate Models with and without Survey data estimated with data 1960+

	MDD Aggregate PCE π	
Model	1960-2025Q2 sample	1960-2019 sample
Univariate [UCSV-SW]	-398.8	-361.2
Univariate [UCSV-SW] + Survey	-455.4	-407.1
Univariate + AR	-421.7	-380.2
Univariate + AR + Survey	-432.2	-390.2
Univariate + AR [tighter priors similar to CCK]	-431.3	-389.3
Univariate + AR + Survey [tighter priors, CCK]	-439.0	-396.5

Model	1960-2025Q2 sample	1960-2019 sample
Univariate [UCSV SW] + Uncertainty + PC	-393.7	-355.3

Note: MDD refers to the log of the marginal data density, which measures the model's fit to the services inflation data and to goods inflation data. The table reports the model's fit to the data over the entire sample (1960Q1-2025Q2) and the pre-COVID sample (1960Q1-2019Q4).

Table 6: Bayesian Model Comparison: Univariate Models with and without Survey data estimated with data **1980+**

	MDD Aggregate PCE π	
Model	1980-2025Q2 sample	1980-2019 sample
Univariate + AR	-289.2	-247.7
Univariate + AR + Survey	-296.2	-254.3
Univariate + AR [tighter priors similar to CCK]	-306.1	-263.9
Univariate + AR + Survey [tighter priors, CCK]	-305.2	-262.8

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