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Optimal Short-Time Work Policy in Recessions

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Abstract

Short-time work (STW) is a subsidy program linked to a reduction in working hours that has been widely used across Europe and partly used in some US states to combat job losses in the Great Recession and the COVID-19 pandemic. Although typically used alongside an unemployment insurance (UI) system, the interaction between STW and UI remains conceptually unclear. To close this gap in the literature, I develop a search and matching model of the labor market with risk-averse workers, flexible hours choice, endogenous separations, and generalized Nash bargaining. Deriving closed-form expressions for the optimal policy mix, I demonstrate that while the UI system provides income insurance to workers, the STW system mitigates the fiscal externality of UI-induced separations. Notably, STW only exists due to the UI system. Consistent with often observed policy practice, I allow the STW system to adjust over the business cycle while keeping the UI system constant. In line with the actual policy, my findings indicate that optimal STW benefits have to increase in recessions, while in contrast to the actual policy, optimal eligibility criteria have to be tightened. Using UI with an optimal STW system is fiscally less expensive than the UI system on its own.

JEL Classification: E24, E32, H21, J63, J64, J65

Key Words: Short-time work, unemployment insurance, optimal policy, labor markets, search and matching, business cycles

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1 Introduction

The Great Recession and the COVID-19 pandemic have reignited interest in two core questions of labor market policy: How can we prevent unemployment? And how can we protect workers from income loss due to unemployment? Policymakers in most European countries,[1] and to a lesser extent in some US states,[2] relied on a combination of two policy tools: unemployment insurance (UI) and short-time work (STW). UI replaces part of workers' income when workers become unemployed, while STW offers partial income compensation to employees when employers temporarily reduce working hours. In recent crises, policymakers in many countries, including Germany and France, expanded their STW system, in terms of both generosity and accessibility,[3] while largely leaving the UI system unchanged. This raises the question of how STW and UI should be used together, and why we should adjust STW over the business cycle.

Although an extensive literature exists on optimal UI, the optimal use of STW and its interaction with UI are less well understood. In a recent literature overview, Cahuc (2024) argues that the interplay between UI and STW remains conceptually vague and that clarifying the optimal interplay is pivotal for formulating effective policy recommendations. Further, despite its extensive use in recessions, there exists no theory yet on how to use STW optimally over the business cycle.

To address this gap in the literature, I develop a real business cycle model incorporating Mortensen and Pissarides (1994) type matching frictions in the labor market. The model accounts for risk-averse workers who cannot save, flexible working hours, and endogenous separations. Workers' contracts about income, working hours, and separations are determined within a generalized Nash bargaining framework. The UI and STW systems are chosen optimally and are financed by lump-sum income taxes, and STW can be adjusted over the business cycle.

My findings indicate that, consistent with often observed policy practice, optimal STW benefits should increase in recessions. However, in contrast to common practices, eligibility conditions should be tightened rather than relaxed. Moreover, I show that optimally integrating STW with an UI system is fiscally less costly than relying on UI only.

To build intuition for these results, I derive closed-form solutions for the optimal policy mix between UI and STW. These expressions highlight a clear division of roles: the UI system provides income insurance to workers, while STW internalizes the fiscal externality that UI creates on separations. Further, STW policy must trade off employment stabilization against distortions in working hours.

[1] For an overview of job-retention systems in Europe see Mueller, Schulten, and Drahokoupil (2022)

[2] In the US, short-time work is referred to as short-time compensation (STC). For an overview of the use of STC in recent crises, see Krolikowski and Weixel (2020).

[3] See OECD (2020) and Boeri and Bruecker (2011) for adjustments of STW in the COVID-19 pandemic and Great Recession.

In this set-up, search frictions become crucial for understanding why STW policy should be adjusted over the business cycle. A decline in the job-finding rate increases the social cost of separations by prolonging expected durations on the UI system, which raises the optimal level of STW benefits. However, more generous STW benefits also strengthen firms' incentives to enter the STW scheme. To limit excessive participation and contain distortions in working hours, eligibility conditions must therefore become stricter.

In a quantitative exercise, I find that the optimal cyclical adjustment of STW is sizable. The model is calibrated to the US as frequent changes in STW policies across European countries make it difficult to credibly disentangle movements in separation rates driven by economic conditions from those induced by STW policy. Following a 1 percent negative productivity shock, optimal STW benefits increase by 12.5 percent, while the minimum hours reduction threshold to participate in the STW system rises by an additional 6 percent. Overall, the optimal STW policy closes 33 percent of the gap in inefficient aggregate consumption fluctuations and 57.5 percent of the gap in inefficient employment fluctuations at the cost of increasing the volatility of mean working hours by 36.5 percent.

The remaining parts of the introduction discuss the mechanisms behind the results in more detail and connect the paper to the related literature. Section 2 introduces the model, explores the decentralized economy, and solves for the social planner economy. Section 3 derives analytical expressions for optimal STW policy and explores its theoretical implications. Section 4 describes the calibration of the model. Section 5 briefly describes the steady state. Section 6 applies the optimal STW policy to a supply-side recession. Finally, Section 7 concludes the paper.

Mechanisms To better understand the main results, let us examine the model in more detail. The framework entails two potential reasons for government intervention. First, risk-averse workers cannot insure themselves via savings or on the financial market. Second, congestion externalities in the Diamond-Mortensen-Pissarides model generate inefficiencies in vacancy posting. Individual firms do not internalize that posting an additional vacancy reduces the probability that other firms fill their vacancies. To counter these inefficiencies, the Ramsey planner can choose the parameters of the UI and STW system.

The systems in the model work as follows. The UI system pays unemployed workers UI benefits as their only source of income while they are unemployed. STW consists of two instruments: the eligibility condition and STW benefits. Firms and workers qualify for STW when they are willing to reduce working hours below a specific participation threshold. Hijzen and Martin (2013) show that most STW systems in practice use this type of hours reduction as an eligibility criterion. Under STW, the government compensates workers for every hour they work less than usual. Note that STW thereby subsidizes reductions in working hours, leading to suboptimally low hours, as emphasized by Burdett and Wright (1989).

The UI system addresses workers' inability to insure themselves against income losses upon job loss. Optimal UI benefits must balance the classical trade-off: higher benefits provide additional income insurance for risk-averse workers, but they also distort vacancy posting and separations (cf. Pissarides 2000). In Nash bargaining, workers effectively choose between higher salaries with greater unemployment risk and lower salaries with lower unemployment risk. By increasing the worker's outside option, UI shifts this trade-off toward riskier contracts, thereby raising separation rates inefficiently. Further, higher UI benefits raise salary demands more generally, reducing firms' surplus and weakening hiring incentives. Together, these channels generate inefficiently high unemployment levels.

This raises the question of whether STW can offset the depressed job-finding and inflated separation rates. First, consider job-finding rates. Studies such as Balleer et al. (2016), Giupponi and Landais (2018), and Cahuc, Kramarz, and Nevoux (2021) suggest that STW increases vacancy posting. In contrast, once financing is taken into account, my model does not support this prediction. When posting vacancies, firms do not yet know whether they will operate at full hours and contribute to financing the system or whether they will enter STW and benefit from it. As a result, a more generous STW system does not raise vacancy posting, as it must be financed by higher taxes. Surprisingly, even when abstracting from the fiscal costs of financing STW, the Ramsey planner still chooses not to stabilize job-finding rates, as the distortionary effects of STW on working hours remain too large.

However, STW does effectively reduce inefficient separations by lowering firms' salary costs during downturns. The higher the STW benefits, the more firms are able to reduce salaries. Optimal STW benefits are designed to offset the fiscal externality created by the UI system. In essence, STW should reduce firms' salary costs by the amount that an additional unemployed worker increases the government's fiscal burden. In spirit, this follows the same logic as the optimal layoff tax derived by Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008). However, optimal STW benefits must also internalize their own distortionary effects. They must trade off the social value of retaining workers against the social costs of further distorting working hours. As a result, the Ramsey planner does not fully internalize the fiscal externality; instead, he limits the generosity of STW to keep distortions in hours low.

It is important to note that while STW secures jobs, it does not provide direct income insurance. Income insurance for unemployed workers is offered by the UI system. Firms, by contrast, provide income insurance against idiosyncratic productivity shocks as long as the worker remains employed, a finding consistent with the implicit contract literature (cf. Rosen 1985 and Braun and

Brügemann 2017).[4] In my model, this result arises because Nash bargaining occurs before idiosyncratic productivity shocks are realized, allowing firms and workers to write contracts contingent upon the realization of these shocks.

The second instrument the Ramsey planner can choose is the eligibility condition. Recall that the optimal STW benefits show that STW operates like a subsidy system. A key challenge of standard subsidy schemes, however, is to effectively target firms that genuinely require support, since productivity levels are typically not directly observable. The eligibility condition addresses this problem. It requires firms to reduce working hours below a certain threshold to qualify for the program. Since it is costly for more productive firms to cut working hours substantially, only less-productive firms are willing to reduce hours enough to enter STW. This creates a natural screening mechanism for productivity. The minimum hours reduction requirement can be adjusted so that only firms below a certain target productivity level qualify for STW. Teichgräber, Žužek, and Hensel (2022) discuss hours reduction as a screening instrument in a mechanism design model.

The optimal eligibility condition sets the threshold at the productivity level at which firms and workers would choose to separate if they did not have access to STW. A looser eligibility condition would reinforce the distortionary effects of the STW system without saving additional jobs, resulting in pure windfall effects as in Cahuc, Kramarz, and Nevoux (2021). A tighter eligibility condition would risk losing matches that could have been saved with STW.

In the model, recessions are triggered by a negative aggregate productivity shock. Salaries are assumed to be rigid in order to address the Shimer (2005) puzzle. I allow STW to vary over the business cycle while keeping UI benefits constant. Treating the UI benefits as unchanged seems to be a reasonable assumption not only for Europe, but also for the United States, where the US Department of Labor documents little variation in the UI replacement rate over time.[5]
When analyzing STW during recessions, I focus on two potential cases. First, I calibrate the model to the US economy,[6] where the current UI system replaces 45 percent of income, and look at how STW should optimally adjust over the business cycle. In a second policy experiment, I implement the optimal UI rate in the steady state and repeat the exercise. In the model, the optimal UI replacement rate is at roughly 34 percent of income.

In both cases, the reaction of the optimal STW system remains consistent. A decline in the job-

[4] Note that firms provide income insurance with or without STW. STW just reduces the costs of providing income insurance by replacing part of the income of the worker.

[5] Note that the US UI system is often described as countercyclical, as benefit durations can be extended during economic downturns. My model abstracts from such extensions and focuses solely on benefit generosity. Incorporating countercyclical UI duration would likely reinforce the business cycle results, but I omit this feature for the sake of simplicity.

[6] The US economy has the advantage that no nationwide STW scheme is in place and that only a few states exhibit considerable take-up rates (see Krolikowski and Weixel 2020). Thus, we can assume that its impact on national job-finding and separation rates is very limited, making the calibration more straightforward.

finding rate increases unemployment durations. Workers do not fully internalize the social costs of separation, as they can stay on the unemployment system longer. To internalize the additional social costs from separations, STW benefits need to be increased. However, higher STW benefits incentivize more firms and workers to enter the STW system. Thus, the eligibility criteria must be tightened to prevent windfall effects and limit distortions in working hours.

Depending on the UI set-up in the steady state, STW contributes to welfare through different channels. Broadly speaking, the Ramsey planner has three main objectives: First, he wants to reduce efficiency losses from business cycle fluctuations. These can be examined by the amount of inefficient fluctuations in aggregate consumption. Second, he wants to smooth individual consumption across households. STW can achieve this indirectly by reducing unemployment risk, and thereby stabilizing employment fluctuations. Therefore, employment stabilization serves both an efficiency and an insurance role. Finally, the planner aims to keep distortions in working hours minimal.

When UI benefits are calibrated to the current US UI system, STW stabilizes both aggregate consumption and employment at the cost of higher fluctuations in mean hours worked. Surprisingly, with optimal UI in the steady state, the optimal STW policy no longer needs to address aggregate consumption fluctuations. The reason for this is that optimal UI nearly maximizes aggregate consumption in the model. Excessively high UI benefits lead to high unemployment rates and low output, while overly low benefits discourage workers from leaving unproductive jobs, thus keeping inefficient firms afloat. Consequently, marginal job matches contribute little to output, and their loss or preservation has minimal impact on aggregate consumption stabilization. Nonetheless, employment rates still fluctuate inefficiently. STW indirectly improves workers' income insurance by shifting contracts toward lower unemployment risk but also lower wages, thereby expanding the role of firms as providers of insurance.

Literature The paper contributes to two strands of the literature. First, it adds to the extensive literature on optimal UI policy. Several studies, including Baily (1978), Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), and Chetty (2006), analyze the optimal design of UI systems with a focus on workers' job search incentives. Landais, Michaillat, and Saez (2018) extend this framework by incorporating firms' incentive to posts vacancies. Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008) examine the role of layoff taxes in mitigating UI-induced separations, while Jung and Kuester (2015) and Michau (2015) study optimal UI design using combinations of vacancy subsidies and layoff taxes. This paper contributes to this literature by examining optimal UI design in the presence of STW and showing that STW effectively internalizes the fiscal externality of the UI system. STW therefore has the same spirit as the layoff taxes of Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008).

Second, it contributes to the theoretical and quantitative literature on STW. STW in combination with UI was first studied in an implicit contract framework (see Burdett and Wright (1989), Van Audenrode (1994), and Braun and Brügemann (2017)). Within this literature, the paper is most closely related to Braun and Brügemann (2017), who also examine the optimal joint design of STW and UI. Extending the analysis to a search and matching framework proves crucial, as matching frictions, absent in the implicit contract literature, govern the logic behind business cycle adjustments of STW. When the job-finding rate varies over the cycle, the optimal STW system should adjust accordingly.

A second wave of research embeds STW into a search and matching framework to study business cycle implications, including Balleer et al. (2016), Gehrke, Lechthaler, and Merkl (2017), Dengler, Gehrke, and Zessner-Spitzenberg (2024), and Cooper, Meyer, and Schott (2017). These papers generally assume that working hours outside STW are inflexible and that STW's main role is to provide flexibility in working hour adjustment. In contrast, I follow the spirit of the implicit contract literature and Cahuc, Kramarz, and Nevoux (2021), where working hours are flexible, and STW acts as a subsidy. This approach has the advantage that STW benefits directly affect separation decisions. Assuming that changes in STW benefits have no impact on separations seems implausible. Finally, Niedermayer and Tilly (2017) and Diaz et al. (2025) look at short-time work and its interaction with human capital in a search and matching framework with heterogeneous firms.

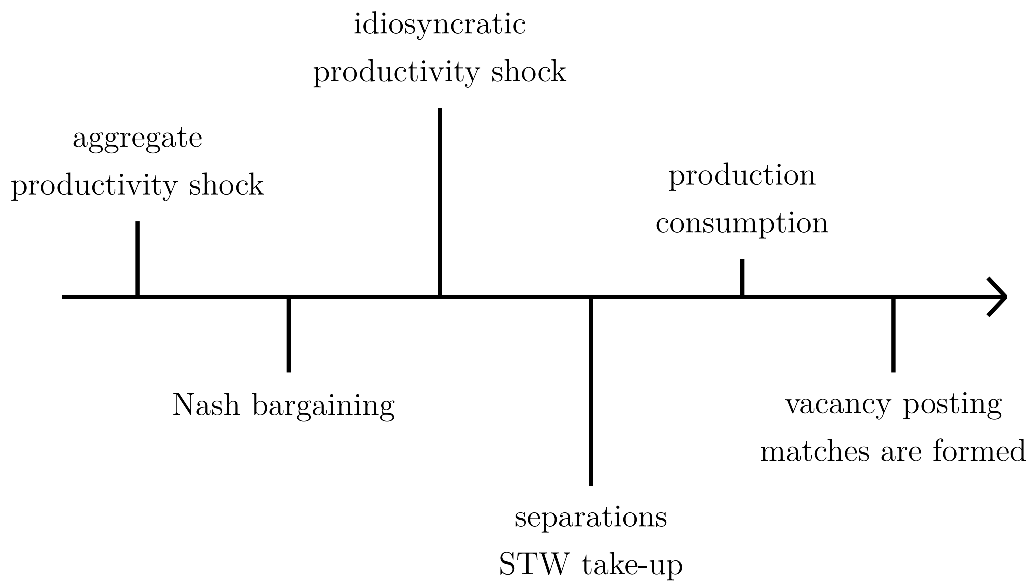
Within the literature, Balleer et al. (2016) push the idea that STW serves as an automatic stabilizer: by reducing separations, STW stabilizes employment, output, and consumption without requiring cyclical adjustments of the system. In my model, similar results arise for employment but not for consumption. Because the eligibility condition is fixed, even firms that could survive without STW enter the program, amplifying distortion in working hours and offsetting the consumption-stabilizing effects of employment stabilization. This result highlights the necessity of adjusting STW over the business cycle.

Further, Cooper, Meyer, and Schott (2017) show in a search and matching framework that STW can keep workers in unproductive matches, reducing aggregate productivity and output. In my model, the planner must weigh the costs of reallocating a worker through the labor market against the costs of retaining a worker in a less-productive firm. If STW benefits are too generous, the concern raised by Cooper, Meyer, and Schott (2017) materializes and workers stay in unproductive firms. Conversely, if benefits are too low, output is lost through excessive separations and high unemployment. Setting STW benefits optimally realigns private and social incentives, thereby avoiding misallocation.

2 Model

The economy is populated by a continuum of workers of measure one $\nu_t^w = 1$, infinitely many one-worker firms, and a continuum ν_t^f of firm owners. Each firm produces a homogeneous and non-storable good. The economy is closed. Each period, firms and workers are subject to aggregate and idiosyncratic shocks. Nonetheless, firms are ex-ante homogeneous in their match-efficiency.

Figure 1: Period Timeline



The timeline of the period is structured as follows: At the start, firms experience an aggregate productivity shock. Before the idiosyncratic productivity shocks occur, generalized Nash bargaining takes place. Firms and workers write a contract specifying income, separations, hours of work, and STW take-up for the current period, all contingent on the realization of the idiosyncratic productivity shock. Following this, the idiosyncratic productivity is drawn. Separations and STW take-up take place. Then, output is produced based on working hours, and households consume. At the end of the period, vacancies are posted and new matches are formed. New matches don't produce until next period.[7]

2.1 Decentralized Economy

In the decentralized economy, separations, vacancy postings, and working time are determined by firms and workers.

[7]For the model, symbols are summarized in Appendix A.

Firm Side Each firm that enters a match with a worker can either produce or separate from the worker. Productivity contains an aggregate component a_t that is common to all matches and an idiosyncratic component ϵ_j that is, for analytical tractability, i.i.d. across time and matches with the distribution function $G(\epsilon)$.[8]
Firm-specific output $y_t(\epsilon, h_t(\epsilon))$ depends on the productivity $a_t \cdot \epsilon$ of the firm, the number of hours worked $h_t(\epsilon)$ and the resource costs of the firm $(\mu_\epsilon - \epsilon) \cdot k_f$:[9]

$$y_t(\epsilon, h_t(\epsilon)) = a_t \cdot \epsilon \cdot h_t(\epsilon)^\alpha - (\mu_\epsilon - \epsilon) \cdot k_f \quad \text{with} \quad E[(\mu_\epsilon - \epsilon_j) \cdot k_f] = 0$$

In line with Krause and Lubik (2007), I assume that the idiosyncratic shock ϵ_j follows a log-normal distribution $\ln(\epsilon_j) \sim \mathcal{N}(\mu_{\ln(\epsilon)}, \sigma^2_{\ln(\epsilon)})$. The expected value of the idiosyncratic productivity shock thus is $\mu_\epsilon = E[\epsilon_j] = \exp(\mu_{\ln(\epsilon)} + \frac{1}{2} \cdot \sigma^2_{\ln(\epsilon)})$. Furthermore, I assume that aggregate productivity follows an AR(1) process with mean μ_a and persistence ρ_a.

$$a_t = \mu_a + \rho_a \cdot (a_{t-1} - \mu_a) + \iota_t, \quad \rho_a \in [0, 1), \quad \iota_t \sim \mathcal{N}(0, \sigma_a^2)$$

Firms are assumed to be owned by firm owners. As a result, future cash flows are discounted using a stochastic discount factor, reflecting how firm owners weigh the future marginal utility of consumption against today's:

$$Q^f_{t,t+1} = \beta \cdot \frac{u'(c^f_{t+1})}{u'(c^f_t)}$$

Here, c^f_t denotes the consumption of firm owners, $u(.)$ the utility function, and $\beta \in (0, 1)$ the time discount factor.

The value of a worker for a firm that is not on STW, and whose idiosyncratic shock has realized to ϵ, is:

$$J_t(\epsilon) = y_t(\epsilon, h_t(\epsilon)) - w_t(h_t(\epsilon)) + E_t \left[Q^f_{t,t+1} \mathcal{J}_{t+1} \right]$$

[8] If we had persistent idiosyncratic shocks, we would need a state vector to keep track of the productivity distribution of the firms. This would make computing the Ramsey policy very difficult. k_f can also be interpreted as a measure for the persistence of the idiosyncratic productivity shocks.

[9] Note that the cost shock of the firm is important, if we want to have a quantitatively realistic impact of the UI system on unemployment, endogenous separations, and time-independent idiosyncratic shocks in an otherwise analytically tractable model. It is a well-known problem that search and matching models overstate the importance of the UI system (see Costain and Reiter (2008)). To have a sensible impact of the UI system, we need a large surplus calibration. The bigger the surplus, the smaller the relative impact of a change in UI benefits. However, large surpluses lead to small separation incentives. Since the cost shock has an expectation value of zero, it allows for a large surplus calibration. At the same time, it affects the marginal firms the most, allowing for endogenous separations.

The firm gets the production value of the match $y_t(\epsilon, h_t(\epsilon))$ but pays the salary $w_t(h_t(\epsilon))$ dependent on the total working hours of the worker. $\mathcal{J}_t$ denotes the expected value of a firm at the beginning of period t, before the idiosyncratic productivity shock is realized, while $E_t\left[Q^f_{t,t+1}\mathcal{J}_{t+1}\right]$ denotes the expected discounted continuation value of the match for the firm. Note that firms are risk neutral.

The value of a worker for a firm on STW with idiosyncratic productivity ϵ can be written as:

$$J_{stw,t}(\epsilon) = y_t(\epsilon, h_{stw,t}(\epsilon)) - w_t(h_{stw,t}(\epsilon)) + E_t\left[Q^f_{t,t+1}\mathcal{J}_{t+1}\right]$$

Since working hours fall on STW, firms have to pay a smaller salary.

Access to the STW system is restricted by the government via the eligibility condition. As in Cahuc, Kramarz, and Nevoux (2021), firms and workers have the option to access STW when the number of hours worked falls below a specific threshold set by the government, denoted as D_t. This threshold serves as a criterion for determining eligibility for STW:

$$h_{stw,t}(\epsilon) \leq h_{stw,t}(\epsilon_{stw,t}) = D_t$$

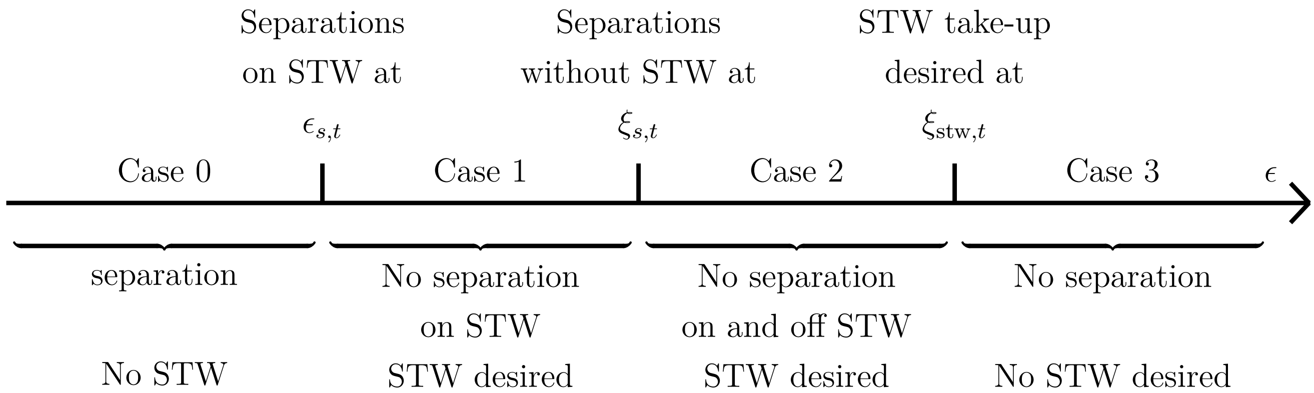
It essentially means that firms and workers are eligible to participate in the STW system if they reduce their hours worked by a certain percentage below their normal level, $\frac{D_t - \bar{h}}{\bar{h}} \cdot 100 percent$, where $\bar{h}$ represents the mean hours worked in the steady state at the firm. This eligibility condition is consistent with findings by Hijzen and Martin (2013), who identify that 15 out of 24 OECD countries with STW programs in place employ this minimum hours' reduction as an eligibility criterion. In subsequent sections, we say that the eligibility condition becomes looser when D_t increases, indicating that it becomes easier to enter into STW. The participation threshold, denoted by $\epsilon_{stw,t}$, is implicitly determined by the equation $D_t = h_{stw,t}(\epsilon_{stw,t})$ and indicates the productivity level below which matches enter the STW system. In the spirit of Teichgräber, Žužek, and Hensel (2022), the hours reduction criterion can be used as an instrument to screen for productivity and jobs at risk. Firms and workers are willing to reduce hours to qualify for STW only if their productivity has fallen sufficiently.

Conditional on the values of D_t the participation threshold $\epsilon_{stw,t}$ may or may not be binding and can have various impacts on the economy. We need to consider four distinct cases, as illustrated in Figure 2. In the figure, $\epsilon_{s,t}$ denotes the separation threshold for firms and workers with access to STW, $\xi_{s,t}$ denotes the separation threshold without access to STW, and $\xi_{stw,t}$ denotes the threshold at which firms and workers are willing to take up STW.[10] All these thresholds will be determined

[10] Note that just because firms and workers want to take up STW it does not mean they can. Firms and workers are only allowed to take up STW if $\epsilon < \epsilon_{stw.t}$.

by Nash bargaining.

Figure 2: Thresholds and employment outcomes across productivity levels



Case 0: $\epsilon_{stw,t} < \epsilon_{s,t}$. In case 0, the participation threshold is lower than the separation threshold for firms and workers with access to STW. Consequently, separations occur despite access to STW benefits, rendering the STW system irrelevant. For notational simplicity, I exclude this case from the analysis, as it does not restrict the planner's choice set. Throughout, I impose $\epsilon_{stw,t} \geq \epsilon_{s,t}$. By choosing $\epsilon_{stw,t} = \epsilon_{s,t}$, the Ramsey planner can still shut down the STW system.

Case 1: $\epsilon_{s,t} \leq \epsilon_{stw,t} < \xi_{s,t}$. Case 1 describes a situation in which matches with productivity levels $\epsilon \in [\epsilon_{s,t}, \epsilon_{stw,t}]$ have access to the STW system and are retained, whereas matches with higher productivity $\epsilon \in (\epsilon_{stw,t}, \xi_{s,t})$ do not have access and separate.

Case 2: $\xi_{s,t} \leq \epsilon_{stw,t} < \xi_{stw,t}$. In case 2, all firms and workers that would separate in the absence of STW are able to enter the STW system. At the same time, the participation threshold excludes matches with productivity $\epsilon \in (\epsilon_{stw,t}, \xi_{stw,t}]$ from receiving STW. Note that matches with $\epsilon \in (\xi_{s,t}, \epsilon_{stw,t}]$ would like to take up STW but are not at risk of separation. Following Cahuc, Kramarz, and Nevoux (2021), I refer to these matches as windfall effects.

Case 3: $\xi_{stw,t} \leq \epsilon_{stw,t}$. In case 3, the eligibility condition becomes so loose that it no longer binds, and firms and workers do not want to take up STW. Without loss of generality, I assume that the planner chooses $\epsilon_{stw,t} \leq \xi_{stw,t}$ and exclude this case from further consideration. Setting $\epsilon_{stw,t} = \xi_{stw,t}$ replicates the outcome of $\xi_{stw,t} < \epsilon_{stw,t}$.

Without loss of generality and for notational brevity, I therefore require the participation threshold to be at least as large as the separation threshold for firms and workers with access to STW, but not larger than the STW take-up threshold:

$$\epsilon_{s,t} \leq \epsilon_{stw,t} \leq \xi_{stw,t}$$

From this, we can derive the separation rate. The separation rate consists of the fraction of matches that separate despite having access to STW (case 0), plus the fraction of matches that experience a productivity shock strong enough to trigger separation but not strong enough to make them eligible for STW (case 1):

$$\rho_t = G(\epsilon_{s,t}) + \max\{G(\xi_{s,t}) - G(\epsilon_{stw,t}), 0\}$$

The expected value of a worker for a firm right before the idiosyncratic shock has realized can be denoted as:

$$\mathcal{J}_t = \int_{\max\{\epsilon_{stw,t},\xi_{s,t}\}}^{\infty} J_t(\epsilon)dG(\epsilon) + \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} J_{stw,t}(\epsilon)dG(\epsilon) - \rho_t \cdot (w_{eu,t} + F) \tag{1}$$

When the idiosyncratic productivity exceeds both the participation threshold and the separation threshold for firms without access to STW, $\epsilon \geq \max\{\epsilon_{stw,t}, \xi_{s,t}\}$, the firm continues to operate regularly. If productivity falls within the interval $\epsilon \in [\epsilon_{s,t}, \epsilon_{stw,t}]$, firms shift to production under STW. Further, when firms decide to separate from a worker, they incur two types of costs. First, they must pay severance payments, denoted as $w_{eu,t}$. Severance payments compensate the worker for the loss of employment and are part of the contract that firms and workers bargain over at the beginning of the period. Second, firms face fixed costs of job destruction, represented by F.

Firms post vacancies v_t until the expected costs of recruiting a worker equal the discounted expected value of a worker for the firm.

$$\frac{k_v}{q_t} = E_t\left[Q^f_{t+1}\mathcal{J}_{t+1}\right] \tag{2}$$

Here, q_t denotes the probability of filling a vacancy and k_v the costs of posting a vacancy.

Firm Owners There exists a continuum ν^f_t of firm owners in the economy that have positive but decreasing utility of consumption $u(.)$. Firm owners consume the profits Π_t produced in the firm sector. Profits are spread equally among all firm owners. Firm owners do not make any decisions. The value of a firm owner is denoted as:

$$V^f_t = u\left(c^f_t\right) + \beta \cdot E_t\left[V^f_{t+1}\right] \quad \text{with} \quad c^f_t = \frac{\Pi_t}{\nu^f_t}$$

Profits equal total output minus the wage bill, separation, and vacancy posting costs:

$$\begin{aligned}\Pi_t = n_t \cdot \Bigg(&\int_{\max\{\epsilon_{stw,t},\xi_{s,t}\}}^{\infty} (y_t(\epsilon, h_t(\epsilon)) - w_t(h_t(\epsilon)))\, dG(\epsilon) \\ &+ \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} (y_t(\epsilon, h_{stw,t}(\epsilon)) - w_t(h_{stw,t}(\epsilon)))\, dG(\epsilon)\Bigg) \\ &- \rho_t \cdot n_t \cdot (w_{eu,t} + F) - k_v \cdot v_t\end{aligned}$$

Here, n_t denotes employment in the economy.

Worker Side The value of an employed worker with idiosyncratic productivity ϵ can be written as:

$$V_t^w(\epsilon) = u\Big(w_t(h_t(\epsilon)) - \tau_t - v(h_t(\epsilon))\Big) + \beta \cdot E_t\left[\mathcal{V}_{t+1}^w\right]$$
$$\text{with} \quad u'(.) > 0, \quad u''(.) < 0, \quad v(0) = 0$$

Workers derive utility from consumption and disutility from working $v(h)$. Each period, workers consume their after-tax salary $w_t(h_t(\epsilon)) - \tau_t$. Further, workers are risk averse. Note that I use a quasi-linear utility function. A quasi-linear utility function excludes income effects, thereby making the theoretical results cleaner.[11] $\mathcal{V}_t^w$ denotes the value of an employed worker at the beginning of the period before the idiosyncratic shock is realized. The expected discounted value of being employed next period is denoted by $\beta \cdot E_t\left[\mathcal{V}_{t+1}^w\right]$.

The value of an employed worker on STW can be denoted as:

$$V_{stw,t}^w(\epsilon) = u\Bigg(\underbrace{w_t(h_{stw,t}(\epsilon))}_{\text{Reduced income by firm}} + \underbrace{s_t \cdot (\bar{h} - h_{stw,t}(\epsilon))}_{\text{net transfer STW}} - \tau_t - v(h_{stw,t}(\epsilon))\Bigg) + \beta \cdot E_t[\mathcal{V}_{t+1}^w]$$

During STW, firms and workers will agree on reducing working hours. Consequently, the income of workers falls. The government now steps in and compensates the worker for every working hour less than usual. $\bar{h}$ denotes the average hours worked in the steady state. Note that only firms experiencing an idiosyncratic drop in productivity will reduce hours enough to become eligible for STW. Consequently, STW effectively subsidizes matches with temporarily low productivity that are at risk of separation.

[11] Mathematically, it allows me to map a model with flexible working hours into a standard search and matching model with risk aversion.

The expected value of a worker at the beginning of the period is:

$$\mathcal{V}_t^w = \int_{\max\{\epsilon_{stw,t},\xi_{s,t}\}}^{\infty} V_t^w(\epsilon)\, dG(\epsilon) + \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} V_{stw,t}^w(\epsilon)\, dG(\epsilon)$$
$$+ \rho_t \cdot \Big(u(w_{eu,t} - \tau_t) - u(b_t) + U_t\Big)$$

As in equation 1 for the expected value of the firm, households work normally if the idiosyncratic productivity is large $\epsilon > \max\{\epsilon_{stw,t}, \epsilon_{s,t}\}$, they go on STW if $\epsilon \in [\epsilon_{s,t}, \epsilon_{stw,t}]$, and they become unemployed for $\epsilon < \epsilon_{s,t}$, and $\epsilon \in (\epsilon_{stw,t}, \xi_{s,t})$ if the participation threshold is set too strict. When workers become unemployed, they receive severance payments $w_{eu,t}$. Workers still have to pay taxes τ_t on the severance payment. As in Jung and Kuester (2015), workers get no unemployment insurance in the period when they receive the severance payment.[12]

The value of an unemployed worker at the beginning of the period can be written as:

$$U_t = u(b_t) + \beta \cdot E_t \Big[f_t \cdot \mathcal{V}_{t+1}^w + (1 - f_t) \cdot U_{t+1}\Big]$$

Being unemployed, a worker receives unemployment benefits b_t. With probability f_t, the worker finds a job and gets the value of being employed at the beginning of the next period. Otherwise, the worker stays unemployed.

Nash bargaining Bargaining takes place at the beginning of each period, before the idiosyncratic productivity ϵ is realized. Firms and workers negotiate salaries $w_t(h)$, severance payments $w_{eu,t}$, hours worked under STW $h_{stw,t}(\epsilon)$ and outside STW $h_t(\epsilon)$, the voluntary STW take-up threshold $\xi_{stw,t}$, and the separation decisions with STW $\epsilon_{s,t}$ and without STW $\xi_{s,t}$. Contracts may be conditioned on the realization of each productivity state ϵ. Let η_{t-1} denote the worker's bargaining power. Formally, the generalized Nash bargaining problem is given by:[13]

$$\max_{w_t(h), w_{eu,t}, h_t(\epsilon), h_{stw,t}(\epsilon), \xi_{s,t}, \xi_{stw,t}, \epsilon_{s,t}} \mathcal{J}_t^{1-\eta_{t-1}} \cdot (\mathcal{V}_t^w - U_t)^{\eta_{t-1}}$$

The risk-neutral firm decides to offer the risk-averse worker a contract that insures the worker against any idiosyncratic productivity shock within that period:[14]

$$u'\Bigg(\underbrace{w_t(h_t(\epsilon)) - \tau_t - v(h_t(\epsilon))}_{\tilde{c}_t^w(\epsilon)}\Bigg) = u'\Bigg(\underbrace{w_t(h_{stw,t}(\epsilon)) - \tau_t - v(h_{stw,t}(\epsilon))}_{\tilde{c}_{stw,t}(\epsilon)}\Bigg) = u'\Bigg(\underbrace{w_{eu,t} - \tau_t}_{\tilde{c}_{eu,t}}\Bigg)$$

[12] This reduces the elasticity of the separation rate on movements in the UI benefits, helping to solve the puzzle of Costain and Reiter (2008).

[13] Derivations can be found in Appendix E.

[14] Derivations of the optimality conditions implied by the Nash bargaining can be found in Appendix E.

Within the period, it guarantees the same consumption equivalent and therefore utility regardless of whether workers work regularly, are on STW, or get laid off. When workers are separated, they receive severance payments for that period:

$$\tilde{c}_t^w = \tilde{c}_t^w(\epsilon) = \tilde{c}_{stw,t}(\epsilon) = \tilde{c}_{eu,t}$$

When firms and workers separate and the worker does not find new employment within the same period, the worker transitions to the UI system. To ensure that utility is equal on and off STW, firms must cover the portion of the worker's lost salary not compensated by STW. Note that, with utility equalized, there is some room for salaries to adjust in response to changes in working hours.

The effect of risk aversion on salary can also be seen in the salary formula, that is the formula for the consumption equivalents:

$$\begin{aligned} \tilde{c}_t^w = & \underbrace{\eta_{t-1} \cdot \left(z_t - \frac{1-n_t}{n_t} \cdot b_t - \rho_t \cdot F + (1-\rho_t) \cdot \theta_t \cdot k_v \right) + (1-\eta_{t-1}) \cdot b_t}_{\text{Resource Share of Joint Surplus}} \\ & - \underbrace{(1-\eta_{t-1}) \cdot \int_{b_t}^{\tilde{c}_t} \left[\frac{u'(x)}{u'(\tilde{c}_t^w)} - 1 \right] dx}_{\text{Penalty for Risk Aversion of Workers } > \ 0} \end{aligned} \tag{3}$$

The formula for consumption equivalents consists of two parts. The first part describes how the additional resources generated by a match, beyond what the UI system provides, are shared between firms and workers. The outcome is identical to that under risk neutrality. In the formula, z_t represents the expected consumption equivalents produced by the match:

$$\begin{aligned} z_t = & \int_{\max\{\epsilon_{stw,t}, \xi_{s,t}\}}^{\infty} z_t(\epsilon, h_t(\epsilon)) dG(\epsilon) + \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} z_t(\epsilon, h_{stw,t}(\epsilon)) dG(\epsilon), \\ \text{with} \quad & z_t(\epsilon, h) = y_t(\epsilon, h) - v(h) \end{aligned}$$

The second part of the formula captures the effect of risk aversion on workers' income. Risk-averse workers prefer a smooth and predictable income stream. The risk-neutral firm internalizes this preference and provides income insurance against idiosyncratic shocks to the worker. In exchange for this insurance, workers are willing to forgo part of their salary. At the same time, incomplete unemployment insurance weakens workers' outside options, as the marginal utility of consumption is high when workers rely on UI benefits. Firms exploit this by threatening to walk away from negotiations, which allows them to secure lower salary levels. The effectiveness of this threat increases with the gap between the consumption equivalents and UI benefits.

Note that firms only offer income insurance against idiosyncratic shocks but not against aggregate shocks. Aggregate shocks have a full pass-through to the salary of the worker. This flexibility in salary causes the so-called Shimer puzzle (see Shimer 2005), which states that search and matching models struggle to generate sufficient cyclical fluctuations. The puzzle is commonly resolved by introducing wage rigidity (see Hall 2005) or, in my case, rigid salaries. In the implementation of rigid salaries, I follow Jung and Kuester (2015) and assume procyclical bargaining power of the firms.

$$(1-\eta_t) = \exp(\gamma_w \cdot a_t), \quad \gamma_w > 0$$

We can relate the expression to rigid salaries as follows: If productivity falls in recessions, but salaries are rigid, a larger share of the joint surplus is claimed by the workers. In a model with Nash bargaining, this is equivalent to reducing the firms' or increasing the workers' bargaining power. Fahr and Abbritti (2011), for instance, show that the existence of wage adjustment costs leads to the procyclical bargaining power of the firm.

As in the efficient bargaining set-upof Trigari (2006), hours are chosen to maximize the joint surplus. As a result, outside STW, the marginal product of hours worked needs to equal its marginal disutility. This is the solution the social planner would choose as well (see equation 7):

$$\underbrace{\frac{\partial y_t(\epsilon, h_t(\epsilon))}{\partial h_t(\epsilon)}}_{\text{Marginal Product of Labor}} = \underbrace{v'(h_t(\epsilon))}_{\text{Marginal Disutility of Work}} \tag{4}$$

Workers with low idiosyncratic productivity work less to save disutility of hours worked, while those with high idiosyncratic productivity will work more to make use of the extra productivity boost. This result can be interpreted as some kind of perfect working time account. A reduction in aggregate productivity will reduce the working hours of every worker in the economy.

Firms and workers want to access the STW system when the surplus gain from the STW subsidy exceeds the loss from working hours reduction:

$$\underbrace{s_t \cdot \left(\bar{h} - h_{stw,t}(\xi_{stw,t})\right)}_{\text{Surplus Gain from STW Subsidy}} = \underbrace{z_t(\xi_{stw,t}, h_{stw,t}(\xi_{stw,t})) - z_t(\xi_{stw,t}, h_t(\xi_{stw,t}))}_{\text{Surplus Loss from suboptimal low Working Hours}}$$

If firms and workers are highly productive, they will decide not to enter the STW system, as they can only attract benefits if they reduce the working hours below the usual level $\bar{h}$. Instead, they want to work more than usual to exploit the benefits of the extra productivity. All matches with an hours choice of $h_t(\epsilon) < \bar{h}$ will want to enter the STW system to exploit benefits. Naturally, the government would want to set a stricter eligibility condition, as otherwise more than 50 percent of the workforce would want to enter the STW system.

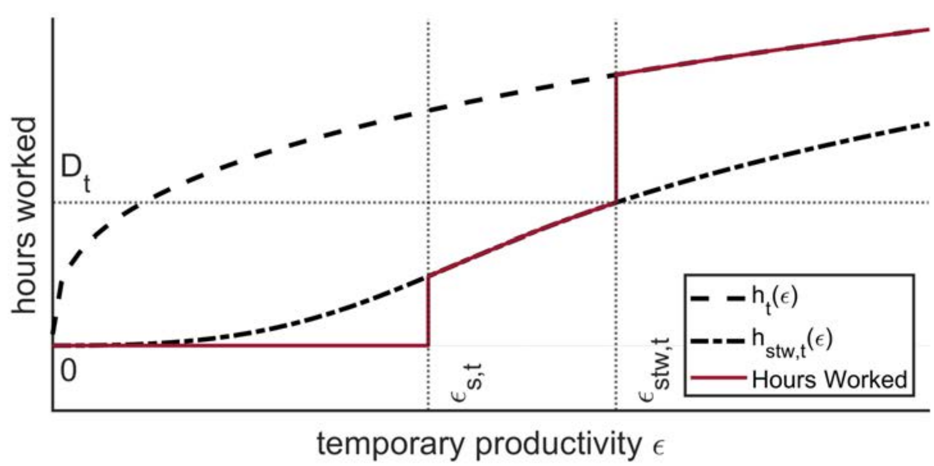
Working hours on STW are chosen sub optimally low. By reducing the number of hours worked, firms and workers can not only reduce disutility from work but they can also attract more STW benefits (see Cahuc, Kramarz, and Nevoux 2021):

$$\underbrace{\frac{\partial y_t(\epsilon, h_{stw,t}(\epsilon))}{\partial h_{stw,t}(\epsilon)}}_{\text{Marginal Product of Labor}} = \underbrace{v'(h_{stw,t}(\epsilon))}_{\text{Marginal Disutility of Work}} + \underbrace{s_t}_{\text{STW Benefits}} \tag{5}$$

To provide a visual representation of the hours distortion effect of STW for the case $\xi_{s,t} \leq \epsilon_{stw,t}$, Figure 3 illustrates the relationship between hours worked and idiosyncratic productivity. The dashed line represents the hypothetical working hours without the STW, while the dot-dashed line shows working hours under the STW system. The red line indicates the actual hours worked.

When productivity is high, workers tend to work their normal hours. As productivity and thus working hours decline, both firms and workers have the option to access the STW program, which reduces working hours below the optimal level. We refer to this reduction as the hours distortion effect of STW. The loss in working hours is represented by the area between the hypothetical hours worked without STW and the actual hours worked under STW. Its implications for the optimal provision of STW are discussed extensively in subsequent sections. If productivity declines further, separations occur, and working hours drop to zero.

Figure 3: Hours Distortion Effect of STW



Notes: The figure illustrates how STW influences the hours choice decision. $h_t(\epsilon)$ denotes the hours that firms and workers would choose if they were not on STW. $h_{stw,t}(\epsilon)$ denotes the hours choice if they were on STW. The red line shows the actual hours choice dependent on being on regular production $\epsilon > \epsilon_{stw,t}$, on STW $\epsilon \in [\epsilon_{s,t}, \epsilon_{stw,t}]$ or separated $\epsilon < \epsilon_{s,t}$. The differences between the red line and the dashed line on STW show the hours distortion effect of STW.

Separations occur if the joint surplus, after the idiosyncratic shock has been realized, becomes negative. The separation threshold without access to STW can thus be determined as:

$$z_t(\xi_{s,t}, h_t(\xi_{s,t})) + F + \frac{1 - \eta_t \cdot f_t}{1 - \eta_t} \cdot \frac{k_v}{q_t} = 0$$

The separation threshold with STW can be determined as:

$$z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + s_t \cdot (\bar{h} - h_{stw,t}(\epsilon_{s,t})) + F + \frac{1 - \eta_t \cdot f_t}{1 - \eta_t} \cdot \frac{k_v}{q_t} = 0 \tag{6}$$

Firms and workers choose to separate when period output minus the disutility of work $z_t(\epsilon, h)$ becomes negative. Separation costs F act as a deterrent to separations. Further, firms have an incentive to hoard workers to save on search costs for replacements, while workers face a loss in the expected value of employment upon separation. This expected value is partially offset by the worker's chance of finding a new job, captured by $1 - \eta_t \cdot f_t < 1$.

Notably, STW increases the joint surplus and thus discourages separations. Firms remain committed to insuring workers against income fluctuations, even when they are temporarily unproductive. Higher STW benefits reduce the salary commitment required from temporarily low-productive firms, thereby lowering separation incentives.

Government's Budget Constraint I assume that the government must balance its budget every period. Income taxes finance the UI system and the STW system.

$$n_t \cdot \tau_t = (1 - n_t) \cdot b_t + n_t \cdot \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon)\right) \cdot dG(\epsilon)$$

The government will determine the UI and the STW system endogenously. The tax is adjusted accordingly.

Labor Market Flows Based on the timing of the economy, we can formulate the law of motion of employment n_t:

$$n_t = (1 - \rho_t) \cdot n_{t-1} + m_{t-1}$$

Here, n_t denotes the number of employed workers at the beginning of the period. m_t denotes the number of newly formed matches. $1 - n_t + \rho_t \cdot n_t$ denotes the number of unemployed workers after separations took place. Unemployed workers are matched with vacancies v_t according to a Cobb-Douglas matching function:

$$m_t = \bar{m} \cdot v_t^{1-\gamma} \cdot (1 - n_t + \rho_t \cdot n_t)^{\gamma}$$

The parameter $\bar{m}$ determines the matching efficiency, and $\gamma \in (0,1)$ denotes the elasticity of the matching function for unemployment. Labor market tightness is defined as the ratio of vacancies to unemployed $\theta_t = \frac{v_t}{1-n_t+\rho_t \cdot n_t}$. Based on the matching function and the labor market tightness, we can derive the probability of finding a job f_t and the probability of filling a vacancy q_t:

$$f_t = \bar{m} \cdot \theta_t^{1-\gamma}, \qquad q_t = \bar{m} \cdot \theta_t^{-\gamma}$$

Market Clearing Market clearing is defined via consumption equivalents. These can be used to pay the aggregate costs of vacancy posting $v_t \cdot k_v$, separation costs $\rho_t \cdot n_t \cdot F$, and consumption equivalents of employed $\tilde{c}_t^w$ and unemployed c_t^u workers as well as firm owners c_t^f:

$$n_t \cdot z_t = v_t \cdot k_v + \rho_t \cdot n_t \cdot F + n_t \cdot \tilde{c}_t^w + (1-n_t) \cdot c_t^u + \nu_t^f \cdot c_t^f$$

2.2 Social Planner

The social planner serves as a reference framework for determining the efficient allocation in the economy, given its technological constraints. It will be used to compare the decentralized outcome with the efficient one. The planner is assumed to weight the utility of all households equally. Given the production technology of the economy (I) and the matching technology incorporated in the law of motion for employment (II), the planner can freely choose consumption, working hours, separations, and the job-finding rate.

$$\begin{aligned} W_t^P = \max_{\theta_t, \epsilon_{s,t}, h_t(\epsilon)} \; & n_t \cdot \int_0^\infty u(\tilde{c}_t^w(\epsilon)) \, dG(\epsilon) + (1-n_t) \cdot u(c_t^u) + \nu_t^f \cdot u(c_t^f) \\ & + \beta \cdot E_t \left[W_{t+1}^P \right] \end{aligned}$$

subject to

$$\begin{aligned} \text{(I)} \quad n_t \cdot \int_0^\infty \tilde{c}_t^w(\epsilon) \, dG(\epsilon) + (1-n_t) \cdot c_t^u + \nu_t^f \cdot c_t^f & \\ = n_t \cdot \int_{\epsilon_{s,t}}^\infty \left[y(\epsilon, h_t(\epsilon)) - v_t(h_t(\epsilon)) \right] \, dG(\epsilon) & \\ - \theta_t \cdot (1 - n_t + G(\epsilon_{s,t}) \cdot n_t) \cdot k_v - n_t \cdot G(\epsilon_{s,t}) \cdot F & \\ \text{(II)} \quad n_{t+1} = (1 - G(\epsilon_{s,t})) \cdot n_t + f(\theta_t) \cdot (1 - n_t + G(\epsilon_{s,t}) \cdot n_t) & \end{aligned}$$

Since workers and firm owners are risk averse, the social planner wants to offer the same consumption equivalents and thus utility, regardless of whether a worker is employed or unemployed.

$$\tilde{c}_t = \tilde{c}_t^w(\epsilon) = c_t^u = c_t^f$$

Note that in the decentralized economy, firms can insure employed workers against idiosyncratic productivity shocks but cannot insure unemployed workers. Unemployed workers need to resort to the UI system. As in the decentralized economy, the social planner cannot insure households against aggregate shocks.

Since the planner can allocate resources freely, he maximizes output minus disutility from work and reallocation costs of a worker via the labor market. Just like firms and workers outside STW (see equation 4), the planner selects working hours such that the marginal productivity of hours worked is equal to the marginal disutility derived from work:[15]

$$\underbrace{\frac{\partial y_t(\epsilon, h_t(\epsilon))}{\partial h_t(\epsilon)}}_{\text{Marginal Product of Labor}} = \underbrace{v'(h_t(\epsilon))}_{\text{Marginal Disutility of Work}} \tag{7}$$

As a result, working hours are optimally determined in the absence of STW intervention. The social planner discounts future welfare using a stochastic discount factor reflecting how households weigh the future marginal utility of consumption against today's:

$$Q_{t,t+1} = \beta \cdot \frac{u'(\tilde{c}_{t+1})}{u'(\tilde{c}_t)}$$

The optimal hiring condition can be written as:

$$\underbrace{\frac{1}{1-\gamma}}_{\text{Congestion Externality}} \cdot \underbrace{\frac{k_v}{q_t}}_{\text{Recruitment Costs}} = \underbrace{E_t\left[Q_{t,t+1}\left(\int_{\epsilon_{s,t+1}}^{\infty} z_{t+1}(\epsilon, h_{t+1}(\epsilon))dG(\epsilon) - G(\epsilon_{s,t+1}) \cdot F\right)\right]}_{\text{Expected Increase in Welfare}}$$
$$+ E_t\left[Q_{t,t+1} \underbrace{(1-\gamma \cdot f_{t+1})}_{\substack{\text{Dynamic} \\ \text{Congestion Externality}}} \cdot \underbrace{\frac{(1-G(\epsilon_{s,t+1}))}{1-\gamma} \cdot \frac{k_v}{q_{t+1}}}_{\text{Saved Social Recruitment Costs}}\right]$$

In a given period, the social planner hires workers up to the point where the social cost of hiring an additional worker equals its social gain. When the social planner fills a vacancy, it increases the current output and reduces recruitment costs in the subsequent period. The social costs of filling a vacancy are twofold:
First, the planner incurs the resource cost of posting a vacancy, k_v, which takes $1/q_t$ periods on average to be filled. Second, posting and filling a vacancy generates congestion externalities in two ways. On the one hand, posting an additional vacancy reduces the probability that other firms fill their vacancies, thereby increasing overall recruitment costs. This static congestion externality is captured by the term $\frac{1}{1-\gamma} > 1$. On the other hand, retaining a worker reduces the pool of

[15] The derivations of the optimality conditions of the planner can be found in Appendix F.

unemployed workers, which raises firms' hiring costs in the next period. This dynamic congestion externality discounts the recruitment cost savings in the subsequent period by $(1 - \gamma f_{t+1}) < 1$.

From the perspective of a social planner, a separation should occur whenever the cost of keeping an unproductive match alive exceeds the social cost of employee turnover:

$$\underbrace{a_t \cdot \epsilon_{s,t} \cdot h_t(\epsilon_{s,t})^\alpha - (\mu_\epsilon - \epsilon_{s,t}) \cdot c_f - v\left(h_t(\epsilon_{s,t})\right)}_{\text{Social costs from keeping unproductive matches alive}} = \underbrace{-F - \frac{1 - \gamma \cdot f_t}{1 - \gamma} \cdot \frac{k_v}{q_t}}_{\text{Social costs from reallocating a worker via the labor market}}$$

The social costs of employee turnover consist of the resource cost of separating from a worker, F, and the social cost of replacing that worker. The social replacement cost again reflects three components. First, there is the resource cost of posting a vacancy k_v. Second, the creation of a vacancy generates a static congestion externality, captured by the factor $\frac{1}{1-\gamma} > 1$, which reflects other firms' lower job-filling probabilities. Third, separations give rise to a dynamic congestion externality: by increasing the pool of unemployed workers, a separation makes future recruitment easier for firms, reflected in the discount factor $(1 - \gamma \cdot f_t) < 1$.

3 Optimal STW Policy

Next, we turn to the second-best economy to analyze how closely STW can realign private incentives with the social planner's allocation. The following sections derive closed-form solutions for the optimal STW policy combined with a UI in the steady state via a Ramsey planner. This analysis highlights which inefficiencies STW can address in the economy, how the eligibility condition, STW benefits, and UI benefits optimally interact, and how the optimal policy relates to aggregate variables and to the business cycle.

3.1 Ramsey Planner

The Ramsey planner chooses the available policy instruments in the economy to bring the decentralized allocation as close as possible to the social planner's allocation. As the social planner, the Ramsey planner weights the utility of all workers equally. The available instruments are unemployment benefits b_t and the parameters of the STW system, namely, the eligibility condition D_t and the STW benefit rate s_t. All policy choices are constrained by the decentralized labor market equilibrium.

By adjusting UI benefits, the planner can influence the degree of income insurance provided to unemployed workers. Through STW, the planner can affect the separation rate. Note that the use of STW also introduces a distortion in the choice of working hours, which is captured by the welfare cost term $n_t \cdot \Omega_t$ defined in definition 1. The Ramsey problem can therefore be written as:

$$\begin{aligned} W_t^G = \max_{D_t, s_t, b_t} \ & (1-n_t) \cdot u(b_t) + n_t \cdot u(\tilde{c}_t^w) \\ & + \nu_t^f \cdot u\Bigg(\Big[n_t \cdot \int_{\mathcal{B}_t} \Big(y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon))\Big) \, dG(\epsilon) \\ & \qquad - n_t \cdot \Omega_t - n_t \cdot \tilde{c}_t^w - (1-n_t) \cdot b_t - n_t \cdot \rho_t \cdot F - v_t \cdot k_v\Big] / \nu_t^f\Bigg) \\ & + \beta \cdot E_t \left[W_{t+1}^G\right] \end{aligned}$$

$$\text{s.t. decentralized equilibrium}$$

The full problem is displayed in Appendix H. Integration takes place over all productivity states with employment $\mathcal{B}_t = [\epsilon_{s,t}, \epsilon_{stw,t}] \cup [\xi_{s,t}, \infty)$. Since the primary focus of the paper is not on distributional conflicts between firm owners and employed workers, I set the number of firm owners ν_t^f such that firms and workers have the same amount of consumption units $c_t^f = \tilde{c}_t^w$. Inclusion of distributional conflicts between firm owners and workers would not fundamentally change the subsequent analysis, but would increase the complexity of the expressions.

Definition 1, Welfare Costs of Hours Distortions

The aggregate difference between output minus disutility of work with and without hours distortions of STW is defined as the welfare costs of hours distortions $n_t \cdot \Omega_t$. Here, Ω_t can be denoted as:

$$\Omega_t = \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \Big[\underbrace{z_t(\epsilon, h_t(\epsilon))}_{\text{No Hours Distortion}} - \underbrace{z_t(\epsilon, h_{stw,t}(\epsilon))}_{\text{With Hours Distortion}} \Big] dG(\epsilon)$$

Definition 1 defines the welfare costs of hours distortions and, therefore, the welfare costs of using STW as the difference between output minus the disutility of work with and without the hours distortion effect of the STW system. The optimal implementation of STW is significantly influenced by how the participation threshold and benefits impact the hours distortion problem associated with STW. The results are summarized in Lemma 1.

Lemma 1, Welfare Costs of Hours Distortions
The total welfare costs of STW $n_t \cdot \Omega_t$ are positive, and increase, if the participation threshold becomes looser or the STW benefits become more generous or both as:

$$\Omega_t \geq 0, \quad \frac{\partial \Omega_t}{\partial s_t} > 0, \quad \frac{\partial \Omega_t}{\partial \epsilon_{stw,t}} > 0, \quad \frac{\partial^2 \Omega_t}{\partial s_t \partial \epsilon_{stw,t}} > 0$$

Note, when the eligibility condition loosens, the participation threshold loosens: $\frac{\partial \epsilon_{stw,t}}{\partial D_t} > 0$

PROOF: *Appendix I*

First, the welfare costs must be positive. In the absence of STW, firms and workers would naturally choose the optimal number of hours worked. However, under STW, working hours are distorted downward, resulting in inefficiently low production levels.
Second, more generous STW benefits create stronger incentives for workers on STW to reduce their hours, thus strengthening the hours distortion effect.
Third, the welfare costs increase as the participation threshold becomes looser. A looser participation threshold allows more firms and workers to access STW, growing the size of the system. Consequently, a greater number of them will choose sub optimally low working hours, leading to a larger loss in output.
Finally, if the eligibility condition becomes looser and the STW benefits become more generous, the hours distortion effects are further exacerbated.
Note that when the eligibility condition D_t becomes looser, the participation threshold must also be relaxed, making their effects on hours distortions interchangeable.

3.2 Inefficiencies in the Economy

To identify the main inefficiencies in the economy that STW is intended to address, it is useful to first understand the rationale and trade-offs of the UI system and how STW can mitigate these inefficiencies. This analysis also sheds light on how the STW system interacts with inefficient job postings arising from deviations from the Hosios condition (see Hosios (1990)). Accordingly, I derive the optimality condition for UI benefits given an STW system and illustrate how STW affects it.

The core inefficiency in the economy is that financial markets are incomplete: workers cannot fully insure themselves against income losses during unemployment. This motivates the existence of a UI system. Through UI, the Ramsey planner seeks to insure workers against income losses due to job loss. Ideally, as shown by the social planner, the Ramsey planner would like to fully insure workers against income shocks, such that $\tilde{c}^w = b$. However, it is well known that UI systems generate fiscal externalities, creating a wedge between $\tilde{c}^w$ and b, so that $\tilde{c}^w > b$.

Proposition 1, Optimal UI Given STW in Steady State

Suppose the economy is in its non-stochastic steady state and a non-optimized STW system exists where its participation threshold is set so that $\epsilon_{stw} > \xi_s$. Then, the optimal UI benefits can be determined by

$$\underbrace{(1-n)\cdot\frac{u'(b)-u'(\tilde{c}^w)}{u'(\tilde{c}^w)}}_{\text{Provide Income Insurance}} = L_V \cdot \underbrace{\left(-\frac{\partial f^{ge}}{\partial b}\cdot u\right)}_{\text{Reduction Hiring by UI}} + L_S \cdot \underbrace{\left(n\cdot g(\epsilon_s)\cdot\frac{\partial \epsilon_s^{ge}}{\partial b}\right)}_{\text{Increased Separations by UI}}$$

where L_V determines the social value from hiring one additional worker, while L_S denotes the social value of the marginal match.

$$L_V = \underbrace{\frac{\eta-\gamma}{(1-\gamma)\cdot(1-\eta)}\cdot\beta\cdot\mathcal{J}}_{\text{Deviation from Hosios Condition}} + \underbrace{FE}_{\text{Fiscal Externality UI on Hiring}}$$

$$L_S = \underbrace{(1-f)\cdot FE}_{\text{Fiscal Externality UI on Separations}} - \underbrace{s\cdot(\bar{h}-h_{stw}(\epsilon_s))}_{\text{net transfer STW}}$$

The fiscal externality (FE) of the UI system is defined as

$$FE = \frac{\beta}{1-\beta\cdot(1-G(\epsilon_s))\cdot(1-f)}\cdot\left[\underbrace{b}_{\text{UI benefits}} + \underbrace{\frac{1-n}{n}\cdot b}_{\text{Avoid Income Tax}}\right]$$

PROOF: *Appendix H*

Proposition 1 shows that the wedge in this model arises because UI enhances workers' outside options. As a result, workers and firms negotiate contracts with higher wages and higher unemployment risk, leading to inefficiently low vacancy posting and inefficiently high separation rates. Notably, Proposition 1 also shows that STW cannot correct the inefficiency associated with insufficient hiring efforts. However, STW can, in theory, eliminate all inefficient separations.

To understand this outcome, we examine the efficiency costs of UI in detail. The term $\left(-\frac{\partial f^{ge}}{\partial b}\cdot u\right)$ captures the reduction in hiring efforts and thus job-finding rates in response to higher UI benefits, with the superscript *ge* indicating that all general equilibrium effects are considered. The term L_V

represents the social value of hiring an additional worker. Together, these terms reflect the welfare loss from reduced hiring due to increased UI benefits.

The social value of hiring an additional worker depends on the deviation from the Hosios (1990) condition and the fiscal externality (FE) of UI on hiring. When workers' bargaining power is too high ($\eta > \gamma$), firms face elevated wages, reducing the private value of hiring below the social value. The UI system reinforces this effect, as unemployment benefits, b, and the avoidance of income taxes in unemployment, $\frac{1-n}{n} \cdot b$, increase the workers' outside option. Both effects together indicate inefficiently low hiring rates, making reductions in hiring socially costly $L_V > 0$. Notably, the absence of STW in this expression shows that STW cannot realign private and social incentives for hiring. Any increase in STW's generosity is offset by a tax increase required to fund it, leaving the firm's value and hiring incentives unchanged.

The term $\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial b}\right)$ represents the number of additional separations resulting from larger UI benefits. L_S denotes the social value of the marginal match to the economy. Together, the terms represent the welfare loss from inflated separations due to UI benefits.

The social value of the marginal match depends on the fiscal externality of the UI system and the STW benefits. The option to rely on the UI system enables workers to negotiate higher salaries in exchange for accepting greater unemployment risk. Consequently, matches may break up despite having a positive social value, making it costly to increase separations. STW can reduce the number of inefficient separations by reducing salary commitments for firms in bad times. The expression indicates that sufficiently high STW benefits can eliminate all inefficient separations, setting the social value of the marginal match equal to zero: $L_S = 0$.

This result offers a new perspective on the concern of Cooper, Meyer, and Schott (2017) that STW reduces allocative efficiency by impeding the reallocation of workers between unproductive and productive firms. If STW benefits are set too generously, the argument of Cooper, Meyer, and Schott (2017) is valid, suggesting that the social value of maintaining a marginal match is negative $L_S < 0$; in this scenario, it would be socially efficient for the worker to leave an unproductive firm and seek new employment. However, if STW benefits are sufficiently low, this concern becomes less relevant, as the social value of the match remains positive, $L_S > 0$.

3.3 Optimal STW Benefits, Eligibility Condition and UI Benefits

We are now well-positioned to analyze the joint determination of the STW and UI systems. We begin by examining how STW benefits can be optimally set to mitigate inefficient separations in the economy.

Proposition 2, Optimal STW Benefits in Steady State
Consider the economy as previously described. Assume that it has converged to its non-stochastic steady state. Then, the optimal STW benefits, s, are determined by:

$$\underbrace{s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right)}_{\text{net transfer STW}} = \underbrace{(1-f) \cdot FE}_{\substack{\text{A: Fiscal Externality UI on Separations} \\ >\ 0}} - \underbrace{\frac{MWL_s}{MMS_s}}_{\substack{\text{B: Hours Distortion Cost of Worker Retention} \\ >\ 0}} - \underbrace{\tilde{BE}}_{C\text{: Bargaining Effect}}$$

Here, $MWL_s = n \cdot \frac{\partial \Omega}{\partial s}$ *denotes the total welfare loss from hours distortions induced by a marginal increase in STW benefits, while* $MMS_s = n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s}{\partial s}$ *denotes the total number of workers retained by marginally increasing STW benefits.*

PROOF: *Appendix H*

Proposition 2 shows that the determination of optimal STW benefits, denoted s, follows a two-step procedure.
First, the Ramsey planner calculates the optimal net transfer to the least productive matches (right-hand side of the equation). This represents the amount of resources the planner intends to allocate to the marginal match.
Second, the planner accounts for how the STW system affects the reduction in working hours by firms and workers (left-hand side of the equation), represented by $\bar{h} - h_{stw}(\epsilon_s)$. This reduction determines the effective amount of resources transferred to the match for given STW benefits s. Higher STW benefits lead to a larger reduction in working hours, which in turn increases the transfer of resources to the match. Using this information, the Ramsey planner adjusts the STW benefits to achieve the optimal net transfer of resources.

The optimal net transfer of resources to the least productive matches comprises three components. Part A explains the rationale for STW's existence: the fiscal externality of the UI system. Parts B and C discuss the trade-offs of using STW, namely its potential to distort working hours and its influence on the bargaining process. First, let us look at the reason to use STW:

Part A states that the optimal transfer under STW must equal the fiscal externality of UI benefits on separations, $(1-f) \cdot FE$. The term $(1-f)$ arises because a separated worker immediately finds a new job, and thus does not enter the UI system, with probability f. From Proposition 1,

we know that applying this rule eliminates all inefficient separations.
But what exactly is the fiscal externality? The expression FE can be interpreted as the net present value of the transfers a worker receives from the government upon becoming unemployed (UI benefits plus income tax savings). Equivalently, it represents the expected fiscal cost imposed on the government when a worker enters unemployment. In spirit, this corresponds to the rule derived by Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008) in their analysis of optimal layoff taxes. To make this interpretation more transparent, Corollary 1 presents a simplified expression for the fiscal externality under the assumption of no discounting:

Corollary 1, Fiscal Externality UI on Separations Under No Discounting
Assume no discounting $\beta = 1$. Then, the fiscal externality of UI on separations is

$$(1-f) \cdot FE = T \cdot b \quad \text{with} \quad T = \frac{1-f}{f}$$

where T denotes the expected duration a worker spends on the UI system after separation.

PROOF: *Appendix J*

Under this assumption, the fiscal externality of the UI system equals the expected duration a worker spends on UI when laid off T, multiplied by the UI benefits b. In other words, it measures the total cost an unemployed worker imposes on the UI system.
This has important policy implications: larger UI benefits or longer unemployment spells increase the social and fiscal costs of separations, and consequently raise the optimal STW benefits. This idea for prolonged unemployment spells is formalized in Corollary 2.

Corollary 2, Higher Job-Finding Rates Decrease Optimal STW Benefits
Assume risk neutrality and that the STW benefits are set optimally according to Proposition 2. Then, ceteris paribus, the welfare costs of the UI system and, thus, STW benefits increase if the job-finding rate decreases:

$$\frac{\partial s}{\partial f} < 0$$

PROOF: *Appendix J*

The link to the duration of an unemployment spell is particularly relevant for understanding how STW should optimally respond to the business cycle. Section 4 below shows that recessions are characterized by significant declines in the job-finding rate.

Part B of the formula addresses the objection of Burdett and Wright (1989) that STW distorts working hours. The planner recognizes that the use of STW is costly as it downward distorts the hours choice in the economy. Consequently, the planner faces a trade-off between preventing socially undesirable separations and minimizing the hours distortions introduced by the STW system. To minimize distortions, optimal STW benefits do not internalize the full fiscal externality of the STW system. Instead, optimal benefits are reduced by the social cost of retaining an additional worker with higher STW benefits.

The welfare costs of retaining an additional worker with STW depend on two factors: the total marginal welfare loss from increasing STW benefits, MWL_s, and the number of additional workers that can be retained by a marginal increase in STW benefits, MMS_s.
MWL_s primarily depends on the size of the STW system, that is, the number of workers it supports. Raising STW benefits distorts not only the working hours of newly retained workers but also those already in the system, which makes worker retention in larger systems more costly.
MMS_s can be seen as a measure of how effective STW is in retaining workers. If an increase in STW benefits retains many workers, the distortion per rescued match is low. However, if only a few workers can be retained, the Ramsey planner is discouraged from using STW to avoid excessive distortions. In the calibrated model, the costs of retaining an additional worker with STW are substantial, reducing the optimal net transfer by approximately 30 percent.[16]

Corollary 3, Looser Eligibility Decreases Optimal STW Benefits
Assume risk neutrality and that the eligibility condition D is exogenous such that $\epsilon_{stw} \geq \xi_s$. Otherwise, STW benefits are chosen according to Proposition 2. Then, a looser eligibility condition increases the social costs of hiring an additional worker with STW and reduces the optimal net transfer and, ceteris paribus, the optimal STW benefits.

$$\frac{\partial s}{\partial D} < 0$$

PROOF: *Appendix J*

[16] An interesting effect related to the welfare costs of rescuing an additional worker with STW arises in the case of zero STW (i.e., no work on STW). In this scenario, the additional distortions from STW benefits are minimal ($\frac{\partial \Omega}{\partial s} \to 0$). Thus, the net transfer equals the benefits provided without any hours distortion effects $L_S^{\star} = 0$.

Corollary 3 underscores a key implication of the welfare costs of retaining an additional worker through STW: a looser eligibility condition reduces the optimal STW benefits. Expanding eligibility increases the number of workers on STW, thereby raising the marginal cost of worker retention, $\frac{\partial^2 \Omega}{\partial s, \partial D} > 0$. This yields two implications. First, eligibility should remain strict to limit the size of the STW system and its associated distortions. Second, because the share of workers on STW typically rises in recessions, the marginal cost of retention increases in downturns, potentially weakening STW's stabilizing role over the business cycle.

Part C of the formula highlights the fact that Nash bargaining under risk aversion is not fully efficient in the sense that it cannot perfectly distribute utility between firm owners and workers with salaries. In fact, due to decreasing marginal utility, workers lose more from a salary cut than firms gain. Therefore, salary cuts lead to a fall in the joint surplus and thus less vacancy posting and more separations. Appendix B offers a detailed discussion of the formula.

In the calibrated model, the effect reduces optimal net transfers by roughly 8 percent to get a sense of the size of the effect. The size itself depends on the degree of risk aversion and the difference between the consumption equivalent and UI benefits.

Putting Parts A, B, and C together, a surprising implication of the formula for optimal STW benefits is that STW provides no direct income insurance. Its sole purpose is to offset the fiscal externality of the UI system. Thereby, STW indeed takes the role of an optimal layoff tax in the sense of Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008). We can see this more clearly by setting unemployment benefits to zero and imposing the Hosios condition. In this case, optimal STW benefits are equal to zero as formally stated in Corollary 4. STW itself provides no direct income insurance as firms write a contract that insures workers against any idiosyncratic productivity shocks to the firm. Appendix D derives the optimal layoff tax within the model and shows its similarity to the STW system.

Corollary 4, STW Has No Direct Insurance Role
Suppose that $b = 0$ and the Hosios condition is met $\eta = \gamma$. Then, STW benefits are zero. This implies that STW itself is not used to provide direct income insurance.

PROOF: *Appendix J*

Optimal Eligibility Condition Part B of the formula for STW benefits highlights a key drawback of STW: it distorts working hours. The eligibility condition serves as the main instrument for mitigating these distortionary effects.

The primary goal of STW is to provide support to firm-worker matches that would otherwise break up. This could be done with a simple subsidy scheme. However, a key challenge for governments with regular subsidy programs is the difficulty in identifying matches at the risk of breaking up. The government would need to know their productivity, which is hard to observe in practice. STW addresses this issue by using reductions in working hours as a screening tool to reveal a firm's productivity level. The more unproductive a firm is at its current level, the more inclined it will be to reduce working hours. This allows the government to elicit productivity with the minimum hours reduction requirement as the eligibility condition.[17]

To determine the optimal eligibility condition, we must, therefore, consider which productivity level the Ramsey planner wants to set as the participation threshold ϵ_{stw}, and adjust the minimum hours reduction threshold D accordingly. The logic behind the eligibility condition is to minimize the number of firms and workers accessing STW to keep distortions in working hours low. Proposition 3 states that this is achieved when the participation threshold equals the separation threshold of firms and workers without access to STW ($\epsilon_{stw} = \xi_s$, equation 8).

Proposition 3, Optimal Eligibility Condition in Steady State

Consider the economy as previously described and assume that it has converged to its non-stochastic steady state. Then, the optimal eligibility condition $D = h_{stw}(\epsilon_{stw})$ is implicitly defined by the separation threshold of a firm without STW ($\epsilon_{stw} = \xi_s$):

$$z(\epsilon_{stw}, h(\epsilon_{stw})) + F + \frac{1-\eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0 \tag{8}$$

as long as the welfare costs of a looser participation threshold are positive:

$$\underbrace{n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}}_{\text{More hours distortion}} + L_V \cdot \underbrace{\left(-\frac{\partial f^{ge}}{\partial \epsilon_{stw}} \cdot u\right)}_{\text{Less hiring}} + L_S^{\star} \cdot \underbrace{\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial \epsilon_{stw}}\right)}_{\text{More Separations on STW}} + L_{\xi} \cdot \underbrace{\left(n \cdot g(\xi_s) \cdot \frac{\partial \xi_s^{ge}}{\partial \epsilon_{stw}}\right)}_{\text{More Workers need STW}} > 0 \tag{9}$$

Here, $L_s = L_s^{\star} = \frac{MWL_s}{MMS_s}$ denotes the social value of the marginal match, which equals the social cost of retaining an additional worker through STW. The term L_{ξ} denotes the social cost of increasing separation incentives among workers without access to STW, discussed in the Appendix H.

PROOF: *Appendix H*

[17]For a detailed discussion of this screening mechanism from a mechanism design perspective and its trade-off with working hours distortions, see Teichgräber, Žužek, and Hensel (2022).

To understand why the participation threshold should be set equal to the separation threshold of firms and workers without access to STW ($\epsilon_{stw} = \xi_s$), it is useful to consider the consequences of choosing an participation threshold that is either too loose ($\epsilon_{stw} > \xi_s$) or too strict ($\epsilon_{stw} < \xi_s$).

First, let us consider a too strict participation threshold ($\epsilon_{stw} < \xi_s$). In this case, some less productive matches qualify for STW, while some more productive matches are excluded and separate. Retaining less-productive matches while allowing more productive matches to break up is clearly inefficient. Thus, the participation threshold must be at least as loose as the separation threshold without STW ($\epsilon_{stw} \geq \xi_s$).

Next, let us consider a too loose participation threshold ($\epsilon_{stw} > \xi_s$). The costs of choosing a participation threshold above the separation cutoff wihtout STW are summarized in Equation 9. A looser participation threshold increases welfare losses from STW ($\frac{\partial \Omega}{\partial \epsilon_{stw}} > 0$), because the hours distortion effect spreads to more firms without preventing additional separations. Moreover, it reduces job-finding rates $\left(-\frac{\partial f^{ge}}{\partial \epsilon_{stw}} \cdot u\right)$ and raises separation incentives both for firms with access to STW $\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial \epsilon_{stw}}\right)$ and for those without access $\left(n \cdot g(\xi_s) \cdot \frac{\partial \xi_s^{ge}}{\partial \epsilon_{stw}}\right)$ through hours distortions. The terms $L_S^\star$ and L_ξ capture the respective welfare costs of the additional separation incentives.

At first glance, easier access to STW appears to increase the surplus of firms and workers. However, this effect is offset by the corresponding rise in income taxes required to finance the system. Worse, the hours distortion effect lowers expected output and thus reduces joint surplus. It must therefore be optimal to choose an eligibility condition no looser than the separation threshold without STW ($\epsilon_{stw} \leq \xi_s$), eliminating all windfall effects.

To summarize, if the eligibility condition is too strict, the planner fails to save matches worth saving. If it is too loose, the distortionary effects of STW become excessive. Consequently, the optimal eligibility condition D equates the participation threshold with the separation threshold of firms and workers without access to STW: $\epsilon_{stw} = \xi_s$

Equation 9 also provides a sufficient condition for this result. For $\eta \geq \gamma$, the condition is clearly satisfied. Even for $\eta < \gamma$, it is difficult to construct scenarios in which it would not hold, as this would require firms to post so many vacancies that reducing output to lower firm profits would become desirable. Using STW in such a case would be implausible.

Corollary 5, Eligiblity and STW Benefits

Further, assume that the government wants to implement the participation threshold ϵ_{stw}. Then the government has to react with a stricter eligibility to implement larger STW benefits.

$$\frac{\partial D}{\partial s} < 0$$

PROOF: *Appendix J*

There is one additional mechanism that is important for understanding how to set the optimal eligibility condition. Recall that the government first determines the optimal participation threshold ϵ_{stw} and then chooses the required hours-reduction threshold $D = h_{stw}(\epsilon_{stw})$. Note that this threshold depends not only on the productivity cutoff, but also on the generosity of STW benefits.

Corollary 5 shows that an increase in STW benefits tightens the optimal eligibility condition for any given productivity threshold. Higher STW benefits encourage matches to reduce working hours in order to qualify for the program. To prevent workers who would have been retained anyway from entering the system, the eligibility condition must become stricter. This mechanism is particularly relevant when considering how the eligibility condition should adjust as optimal STW benefits vary over the business cycle.

Corollary 6, STW Saves Fiscal Costs

Suppose $\beta = 1$ and $\eta \geq \gamma$. Then, ceteris paribus, combining the UI system with an optimal STW system is less fiscally expensive than having a UI system only.

PROOF: *Appendix J*

Combining the optimal STW benefits and the optimal eligibility condition yields an interesting implication for the fiscal cost of the system. Corollary 6 states that, ceteris paribus, combining UI with an optimal STW system is fiscally less costly than operating a UI only system. First, the optimal participation threshold $\epsilon_{stw} = \xi_s$ ensures that only those workers who would otherwise separate gain access to STW; thus, the government supports the same fraction of workers under both systems. Second, retaining workers through STW is fiscally cheaper than allowing them to enter UI: the hours distortion effect and the bargaining effect both reduce optimal STW benefits below the fiscal cost of separations. Consequently, an optimal STW system lowers overall fiscal costs relative to a UI-only system.

Optimal UI Benefits For completeness, let us now derive the optimal UI benefits, given that the STW system is set optimally. Proposition 4 offers an expression. Unaltered, the Ramsey planner faces a trade-off between providing additional income insurance for workers and the economic distortions introduced by higher UI benefits. The negative impact of UI benefits on vacancy posting has not changed.

Compared to the case with a given STW system, the interpretation shifts slightly. An increase in UI benefits still inefficiently raises separations and is captured by $\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial b}\right)$. However, under an optimal STW policy, the government can offset this by retaining workers through raising STW benefits. This approach is costly, as increasing STW benefits also amplifies the system's distortionary effects. These costs are captured by L_S^*, which represents both the expense of rescuing an additional worker with STW and the social value of the marginal match.[18] Since L_S^* is positive, the social planner chooses not to prevent all inefficient separations, to avoid additional hours distortions with STW.

Proposition 4, Optimal UI with Optimal STW in Steady State
Suppose the economy is in its non-stochastic steady state and the STW system is set optimally. Then, the optimal UI benefits can be determined by

$$\underbrace{(1-n) \cdot \frac{u'(b) - u'(\tilde{c}^w)}{u'(\tilde{c}^w)}}_{\text{Provide Income Insurance}}$$
$$= L_V \cdot \underbrace{\left(-\frac{\partial f^{ge}}{\partial b} \cdot u\right)}_{\text{Less Hiring}} + L_S^\star \cdot \underbrace{\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial b}\right)}_{\text{More Separations}} + L_{STW}^\star \cdot \underbrace{\left(n \cdot g(\epsilon_{stw}) \cdot \frac{\partial \epsilon_{stw}^{ge}}{\partial b}\right)}_{\text{More Workers on STW}}$$

Here, $L_{STW}^\star = \frac{MWL_{\epsilon_{stw}}}{MMS_{\epsilon_{stw}}}$ *denotes the social cost of adding a worker to STW by loosening the participation threshold. Here,* $MWL_{\epsilon_{stw}} = n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}$ *is the associated total welfare loss and* $MMS_{\epsilon_{stw}} = n \cdot g(\epsilon_{stw})$ *the number of additional matches entering STW.*

PROOF: *Appendix J*

Further, the government must account for its impact on the optimal eligibility condition. When UI benefits rise, firms without access to STW will separate at higher productivity levels. In response,

[18]The costs of rescuing a worker with STW must be equivalent to the social value of the marginal match as the Ramsey planner will want to rescue workers on STW until the additional social benefit of rescuing them is equivalent to the additional social costs of the STW system.

the government loosens the eligibility condition, increasing the share of workers in the STW system. The resulting welfare costs per additional worker are captured by $L^{\star}_{STW}$. They depend on two factors. MWL_s represents the total welfare loss from marginal loosening of the participation threshold. This loss is divided by $MMS_{\epsilon_{stw}}$, the number of additional workers entering the STW system when the participation threshold is loosened.

4 Calibration and Solution Procedure

This section calibrates the model to the US economy, using a period length of one month. As a baseline for the US economy, I assume rigid salaries, UI benefits that remain unchanged over the business cycle, and the absence of an STW system. Treating UI benefits as constant seems reasonable, as the US Department of Labor documents little variation in the UI replacement rate over time. Excluding STW reflects the fact that there is no nationwide STW program in the US. Only a few states have exhibited considerable take-up rates in the past (see Krolikowski and Weixel 2020), implying a limited impact on aggregate separation rates.[19] To analyze STW, I introduce an STW system into the baseline economy and allow its parameters to vary over the business cycle.

Data Used for Calibration I calibrate the model to data from 1952:I to 2020:I. The unemployment rate is taken from the US Bureau of Labor Statistics. Following Shimer (2005), the job-finding rate and separation rate are calculated using data on the absolute number of unemployed u^a, newly unemployed[20] u^s and employed e^a workers from the US Bureau of Labor Statistics: $f_t = 1 - \frac{u^a_{t+1} - u^s_{t+1}}{u^a_t}$, $s_t = \frac{u^s_{t+1}}{e_t \cdot (1 - \frac{1}{2} \cdot f_t)}$. For vacancies, I use the composite help-wanted index from Barnichon (2010). Average weekly hours $\bar{h}_t/4 = E_t[h_t(\epsilon)|\epsilon \geq \epsilon_{s,t}]/4$ and average labor productivity per worker $p_t = E[a_t \cdot \epsilon_j \cdot h_t(\epsilon_j)^\alpha | \epsilon_j \geq \epsilon_{s,t}]$ are retrieved for the non-farm business sector from the US Bureau of Labor Statistics.

The business cycle properties are reported in Table 1. Following Shimer (2005), the table reports log-deviations from an HP trend with smoothing parameter 10^5. The properties of the business cycle data are well known. Vacancies, unemployment, and labor market tightness are very volatile. The job-finding rate and the average hours worked are pro-cyclical, while separations are counter-cyclical. Separations are less volatile than the job-finding rate.

[19] The limited use of STW in the US may be an advantage for calibrating the model to the US rather than to the European economy, as the existence, introduction, and frequent modification of STW schemes in many European countries make it difficult to disentangle movements in separation rates driven by economic conditions from those induced by STW policy.

[20] Unemployed for less than 5 weeks

Table 1: Business Cycle Properties US Data

		v_t	f_t	ρ_t	u_t	θ_t	$\bar{h}_t$	p_t
Standard Deviation		20.13	14.31	8.2	20.49	39.67	0.81	1.91
Autocorrelation		0.95	0.95	0.77	0.95	0.96	0.92	0.9
	v	1	0.85	-0.55	-0.92	0.98	0.55	0.19
	f	-	1	-0.29	-0.93	0.91	0.38	0.09
	ρ	-	-	1	0.6	-0.59	-0.63	-0.4
Correlation	u	-	-	-	1	-0.98	-0.55	-0.23
	θ	-	-	-	-	1	0.57	0.22
	$\bar{h}$	-	-	-	-	-	1	0.46
	p	-	-	-	-	-	-	1

Notes: The table lists the second moments of the data. u_t, v_t, f_t, $\bar{h}_t$ and ρ_t are expressed as quarterly averages of monthly series. p is the seasonally adjusted average labor productivity in the non-farm business sector. All variables are reported as log-deviations from an HP trend with smoothing parameter 10^5.

Calibrated Parameters Table 6 summarizes the chosen parameter values, and Table 3 the respective business cycle properties of the model. Following Jung and Kuester (2015), I set the discount factor to $\beta = 0.996$. As target steady states, I choose the monthly steady state job-finding rate of $f = 0.41$ and separation rate $\rho = 0.03$ from the data. To implement the job-finding rate, I set vacancy posting costs to $k_v = 0.209$. To implement the separation rate, I set the strength of the resource cost shock to $c_f = 14.214$. The matching efficiency parameter $\bar{m} = 0.383$ is determined by targeting a monthly vacancy filling rate of $q = 0.338$. This is the monthly equivalent of the quarterly job-filling rate of 0.71 reported in Haan, Ramey, and Watson (2000). I set the bargaining power of the worker to $\eta = 0.65$, which is, according to Petrongolo and Pissarides (2001), within the reasonable set of parameter estimates. In order to ensure that inefficiencies in the steady state are only driven by the UI system, the Hosios condition (see Hosios 1990) is implemented by setting the elasticity of the matching function with respect to unemployment equal to the bargaining power of the firm: $\gamma = \eta$. The unemployment benefits are set to $b = 0.45$, which ensures a replacement rate of 45*percent* of the wage, which is the empirical value reported by Engen and Gruber (2001).[21] The parameter $\bar{h}$ represents the mean monthly hours worked in a firm and is set to its steady state value in the baseline economy: $\bar{h} = 0.839$. Similar to Christoffel and Linzert (2010), I set the labor elasticity of the production function to $\alpha = 0.65$. The disutility of work has the common functional form of v(h) $= \frac{h^{1+\psi}}{1+\psi}$, $\psi > 0$. The utility function is assumed to be CRRA $u(\tilde{c}_t) = \frac{\tilde{c}_t^{1-\phi}-1}{1-\phi}$ with constant relative risk aversion parameter $\phi = 2$. Following Domeij and Floden (2006), I set the

[21] The US Department of Labor reports that replacement rates of UI benefits in the United States have remained virtually unchanged over time. However, the US UI system extends the maximum duration of benefits during periods of high unemployment. Since my framework focuses on benefit levels rather than benefit duration, I abstract from these extensions.

Table 2: Parameters

Parameter	Description	Value
Preferences		
β	Discount rate	0.996
ψ	Inverse Frisch elasticity	1.5
ϕ	Coefficient relative risk aversion	2
Vacancies, Matching, Bargaining		
k_v	Vacancy posting costs	0.209
$\bar{m}$	Matching parameter	0.383
γ	Elasticity matching function w.r.t. unemployment	0.65
η	Bargaining power worker in steady state	0.65
γ_w	Degree of cyclicality of bargaining power of worker	15.779
Production and Separations		
α	Labor elasticity production function	0.65
μ_a	Mean aggregate productivity	1.0
$\sigma_a \cdot 100$	s.d. aggregate productivity	0.25
ρ_a	Autocorr. productivity shock	0.985
$\mu_{\ln(\epsilon)}$	Steers mean of log-normal distribution	0.094
$\sigma_{\ln(\epsilon)}$	Steers variance of log-normal distribution	0.12
k_f	Strength resource cost shock	14.214
F	Separation costs	1.441
Labor Market Policy		
b	UI benefits	0.45
$\bar{h}$	"Normal" hours worked	0.839

Frish-elasticity to 0.66, which implies $\psi = 1.5$. As in Krause and Lubik (2007), I set the parameter for the variance of the log-normal distribution of the idiosyncratic shock to $\sigma = 0.12$. In order to normalize the wage to 1, the parameter that steers the mean of the log-normal distribution is set to $\mu = 0.094$.

In order to reach a standard deviation (s.d.) of 0.02 of labor productivity over the business cycle, I set the standard deviation of the aggregate productivity shock to $\sigma_a = 0.0025$ and follow Jung and Kuester (2015) in setting the autocorrelation to $\rho_a = 0.985$. To match the standard deviation of the job-finding rate of 0.0143, I set the degree of cyclicality of bargaining power of the firm to $\gamma_w = 15.5$. Further, I target a standard deviation of the separation rate of 0.0082, by setting the separation costs to F = 1.441.[22]

[22]This is consistent with the value used in Silva and Toledo (2009) for the US economy as severance payments plus the wasteful separation costs account for roughly 8 weeks of the annual salary of a worker. Silva and Toledo (2009) and Ahr and Ahr (2000) report that turnover costs vary between 25 percent and 200 percent of the annual salary. In this model, turnover costs would be at the lower end, with roughly 25 percent accounting for recruitment and wasteful separation costs as well as severance payments.

Compare the business cycle volatilities from the baseline economy from Table 3 to the US business cycle facts in table 1. With the calibration chosen above, we can closely replicate the business cycle properties from the data. Note that a large chunk of the fluctuations is driven by our assumption of the procyclical bargaining power of the firms. Therefore, a lot of these fluctuations must be inefficient, which gives room for the policymaker to intervene.

To solve the model, I rely on first-order perturbation using the code of Schmitt-Grohe and Uribe (2004) based on the symbolic toolbox of Matlab.

Table 3: Business Cycle Properties: Baseline Model

		v_t	f_t	ρ_t	u_t	θ_t	$\bar{h}_t$	p_t
Standard Deviation		19.94	14.31	8.2	21.37	40.88	0.76	1.91
Autocorrelation		0.95	0.97	0.97	0.98	0.97	0.97	0.97
	v	1	1	-0.99	-0.98	1	1	1
	f	-	1	-1	-1	1	1	1
	ρ	-	-	1	1	-1	-1	-1
Correlation	u	-	-	-	1	-1	-1	-1
	θ	-	-	-	-	1	1	1
	$\bar{h}$	-	-	-	-	-	1	1
	p	-	-	-	-	-	-	1

Notes: The table reports the second moments of the model. As in the data of Shimer (2005), all variables are quarterly averages of monthly series and reported as log-deviations. p_t denotes the average output per worker, that is $p_t = E[a_t \cdot \epsilon_j \cdot h_t(\epsilon_j)^\alpha | \epsilon_j \geq \epsilon_{s,t}]$.

5 Steady State

Before turning to the business cycle results, let us first highlight a few steady state findings to provide a sense of the magnitude of the effects derived in the previous sections. Table 4 presents the steady-state results of the model. Columns 1 and 2 compare the baseline economy to an economy that includes an optimal STW system. As predicted by the theory, STW has virtually no effect on the job-finding rate and operates almost entirely by reducing separation rates. The separation rate falls from 3 percent to 2.1 percent, which leads to a substantial decline in the unemployment rate. Consistent with the theoretical predictions, the introduction of STW also reduces government fiscal expenditures, which decline by approximately 10 percent.

Column 3 shows the steady state results with the optimal UI system but no STW system. Quanti-

Table 4: Steady State Results

	baseline	optimal STW	optimal UI	optimal UI, STW
Job-finding rate	0.41	0.41	0.507	0.479
Separation rate	0.03	0.021	0.015	0.014
Fraction of workers on STW	-	0.009	-	0.004
Unemployment	0.07	0.049	0.027	0.03
Average hours worked	0.839	0.834	0.837	0.836
Consumption	0.979	0.984	0.992	0.991
UI benefits	0.45	0.45	0.34	0.37
STW benefits	-	0.735	-	0.543
Minimum hours reduction	-	-66%	-	-55%
Lump-Sum income tax	0.019	0.017	0.005	0.007

Notes: The table compares the steady state results between the baseline model, the model of optimal STW benefits with given UI benefits, the model with optimal UI benefits, and the model where both STW benefits and UI benefits are chosen optimally. Further, note that minimum hours' reduction denotes how far working hours have to fall below their normal level to enter the STW system: $\frac{D-\bar{h}}{\bar{h}}$.

tatively, the model suggests that the optimal UI benefits are lower than the current US UI benefits. Reducing UI benefits would significantly raise employment and expand consumption possibilities. Column 4 shows the optimal policy mix between UI and STW. When STW is introduced as part of the optimal policy mix, it directly addresses the fiscal externality that UI creates through its effect on separations. Proposition 4 suggests that this enables the government to offer more generous UI benefits. Quantitatively, optimal UI benefits rise by approximately 9 percent. The reduced fiscal externality of the UI system is used entirely to increase the generosity of the UI system, rather than to further reduce unemployment.

Finally, note the difference in the eligibility condition between the economy with only optimized STW and the economy with both optimal UI and STW benefits. In the latter, a roughly 16 percent smaller reduction in working hours is required for a match to become eligible for STW. The reason is straightforward: lower UI benefits reduce the optimal STW benefits significantly. This weakens the incentive for matches to reduce hours, making fewer matches eligible for STW.[23] Further, the fraction of workers on the STW system is slashed in half. Lower STW benefits and higher job-finding rates lead to a smaller number of inefficient separations and thus a smaller fraction of workers who need to enter the STW system.

[23]Note that two effects are at play. First, lower UI benefits make workers without access to STW more willing to cling to their jobs, allowing matches to survive at lower productivity levels. From this perspective, the optimal eligibility condition should be set more strictly. However, lower STW benefits simultaneously reduce firms' incentives to enter the STW system, dominating the first effect.

6 Optimal STW Policy in Recessions

The steady-state results show that optimal UI benefits are lower than in the calibrated economy. I begin by analyzing optimal STW policy in a recession under the calibrated economy. I then examine the optimal STW response when UI benefits are set at their steady-state optimal level.

6.1 Inefficiencies in the Business Cycle

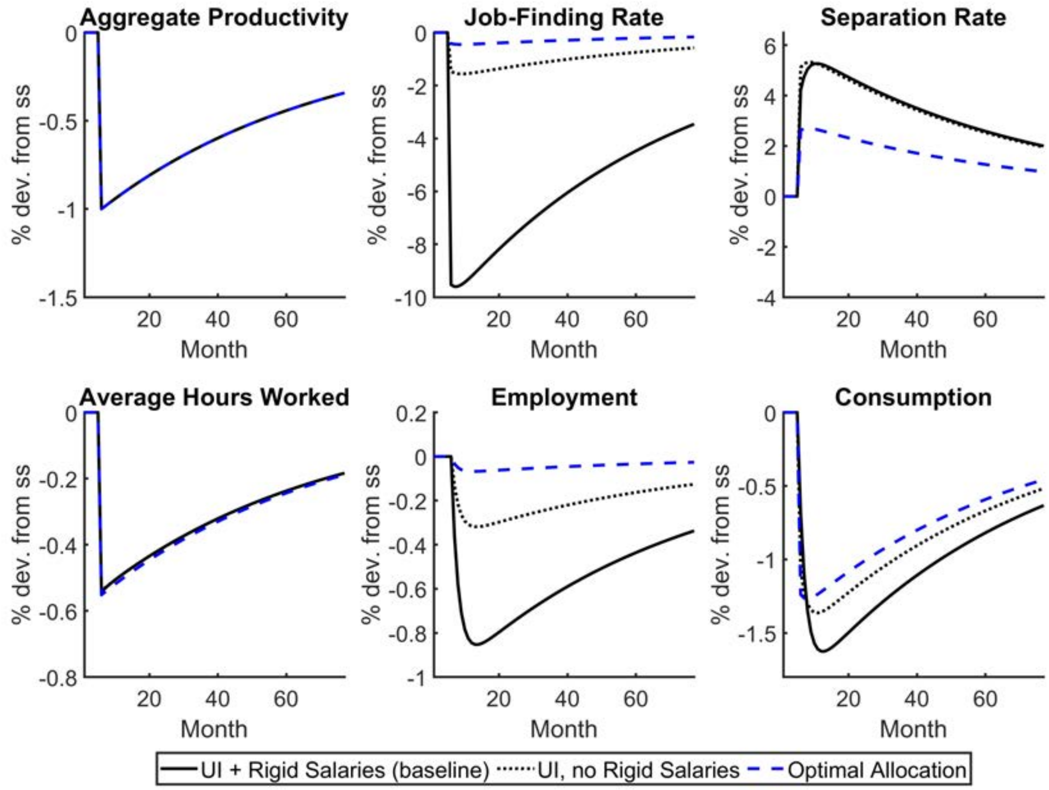
The business cycle is driven by aggregate productivity shocks. Figure 4 shows the response of the economy to a 1 percent negative productivity shock and illustrates the resulting inefficiencies over the business cycle. The solid black line corresponds to the calibrated decentralized economy with rigid salaries and UI benefits but no STW system. UI benefits in this economy are not chosen optimally but are calibrated at the current US rate of 45 percent. We refer to this set-up as the baseline economy. The dashed blue line represents the response of the social planner's economy, that is, the socially optimal response to the aggregate productivity shock. Finally, the dashed black line depicts an economy with UI benefits but flexible salaries. Comparing the baseline economy to the planner's economy allows us to assess the magnitude of business cycle inefficiencies, while comparing the economy with UI but flexible salaries to both the baseline and the social planner's economy helps disentangle the respective roles of salary rigidity and UI benefits.

A negative aggregate productivity shock lowers the joint surplus of matches. Firms thus reduce vacancy posting, causing the job-finding rate to fall. At the same time, a larger share of firm-worker matches generates a negative surplus, raising separation rates. Lower job-finding rates combined with higher separations reduce employment. Output and consumption decline due to a fall in productvity, employment and average working hours.

These fluctuations are partly efficient (see the social planner allocation in dashed blue). Similar to the baseline economy, the social planner increases separations to eliminate low-productivity matches (a cleansing effect) and reduces vacancy posting when additional workers contribute less to output. However, the presence of UI and rigid salaries inefficiently amplifies these fluctuations, as seen by the gap between the planner allocation and the baseline economy.

To assess the impact of UI-induced inefficiencies, compare the dotted black line with the dashed blue line. Job-finding rates are depressed and separation rates inflated. This occurs because a decline in the job-finding rate raises the fiscal externality of UI (see Corollary 1). Although longer unemployment spells worsen workers' outside options, the UI system partly cushions this effect by providing benefits over a longer horizon. This keeps salaries elevated, causing the optimal and decentralized job-finding and separation rates to diverge. Moreover, higher unemployment raises fiscal costs, forcing income taxes upward. This reduces the joint surplus of firms and workers and

Figure 4: Inefficiencies in the Business Cycle



Notes: The figure shows impulse response functions for a 1 percent negative productivity shock. The black line shows the response of the baseline economy, that is, the economy with the distortionary effects of the UI system and rigid salaries as inefficiencies, but without STW system. The black dashed line shows the reaction of an economy without rigid salaries but with the distortionary effects of the UI system. The blue dashed line shows the response of the social planner's economy.

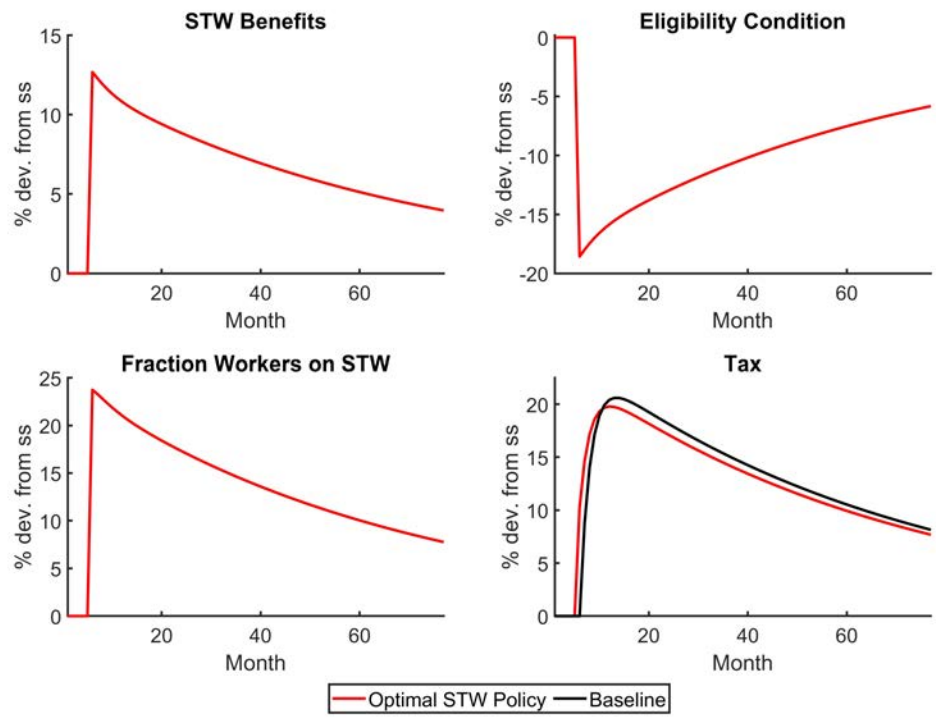
further amplifies the effect.

Introducing rigid salaries further exacerbates these distortions (solid vs. dotted black). In recessions, salary rigidity drives the economy away from the Hosios-condition: the firm's share of the joint surplus falls. Firms respond by cutting vacancies sharply, causing the job-finding rate to fall. This leads to a large increase in inefficient unemployment and intensifies the distortionary effects of the UI system.

6.2 Optimal STW Policy

Figure 5 shows the response of the optimal STW policy to a 1 percent negative productivity shock (red line) and compares the associated tax adjustment with those in the baseline economy (solid black line). Figure 6 displays the response of the economy to the same shock, comparing the reaction under optimal STW benefits (solid red) with the social planner's allocation (dashed blue) and the baseline economy (solid black).

Figure 5: Optimal STW Policy - Instruments



Notes: The figure shows the response of the economy with an optimal STW system (red line) to a 1 percent negative productivity shock, and compares it to the baseline economy (black line).

A striking result from Figure 6 is that STW cannot stabilize the job-finding rate, even though the job-finding rate deviates substantially from its efficient level. This outcome is driven by the balanced-budget assumption and is consistent with Proposition 1. Firms do not know at the beginning of the period whether they will receive STW support or contribute to its financing, eliminating any positive effect that higher STW benefits might otherwise have on vacancy posting.

The inability of STW to stabilize the job-finding rate in recessions may seem surprising in light of studies such as Balleer et al. (2016), Giupponi and Landais (2018), and Cahuc, Kramarz, and Nevoux (2021), which argue that STW could potentially stabilize hiring. However, the result is robust even beyond the balanced-budget assumption. Even if the Ramsey planner could use STW to influence vacancy posting, he would choose not to, as the resulting distortion in working hours would outweigh any gains. A detailed discussion is provided in Appendix C.

Figure 6: Optimal STW Policy - Allocation

Notes: The figure shows the optimal response of the STW system (red line) to a 1 percent negative productivity shock and compares it to the baseline economy (black line) and the social planner (blue dashed line).

Since STW cannot prevent the decline in the job-finding rate, it becomes increasingly difficult for workers to find new employment during recessions. As unemployment spells lengthen, the social costs of separations rise. In response, Figure 5 shows that the Ramsey planner increases STW benefits, as implied by Corollary 2, mirroring actual STW policy. The rise in STW benefits reduces separations significantly. Indeed, the Ramsey planner even allows separations to fall during recessions to offset the sharp drop in job-finding rates, thereby preventing a severe decline in employment.

In contrast to many real-world STW implementations, I find that the eligibility condition does not need to be loosened during a recession. In fact, it must be tightened (Figure 5). Higher STW benefits incentivize matches to reduce working hours, making more matches eligible. To prevent windfall gains among firms that could retain workers without STW, the eligibility condition must be tightened, as described in Corollary 5. This ensures that only matches in genuine need of support access the program.

Stabilizing the economy by raising STW benefits comes at two costs. First, by preventing separations of unproductive matches, STW weakens the cleansing effect of recessions and reduces average firm productivity. Second, larger STW benefits and thus a larger share of workers on STW amplify the distortionary effects of the system, lowering average hours worked. These mechanisms explain why the optimal STW policy does not stabilize output (Figure 6).

The impact on consumption, however, comes closer to the planner allocation. By hoarding labor, STW reduces costly worker reallocation through the labor market, lowering firing and recruiting costs. As a result, 33 percent of inefficient consumption fluctuations can be eliminated despite distortions in working hours.

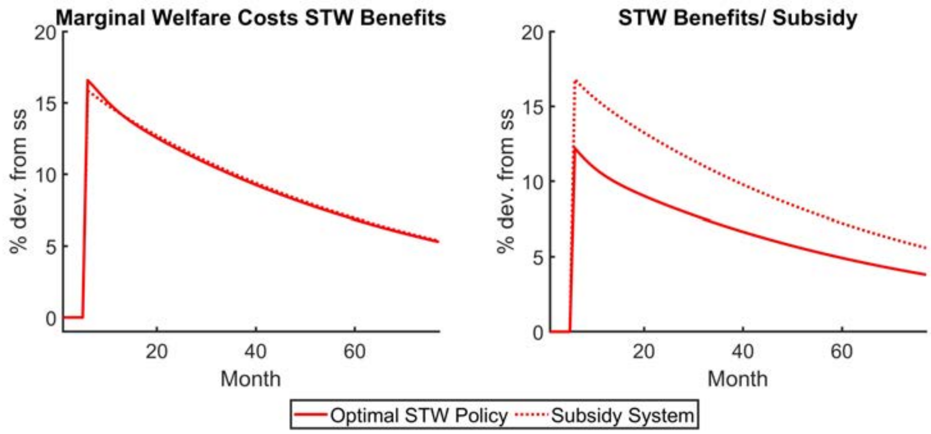
Importantly, the lower right panel of Figure 5 shows that the tax rate differs little between an optimal STW system and a system without STW. By stabilizing employment, STW prevents workers from entering the UI system, containing fiscal costs during recessions. After the downturn, STW should revert to its steady-state level.

In conclusion, STW can stabilize employment and consumption but not output during recessions. Its inability to influence the job-finding rate and its destabilizing effect on average hours worked prevent it from replicating the planner allocation.

6.3 Welfare Costs of STW and Optimal STW Policy Adjustment

Proposition 2 states that the optimal STW benefits depend on two main effects. Part A shows that STW exists to offset the UI system's distortionary effects on separations. Section 6.2 discusses the impact of the fiscal externality on optimal STW policy over the business cycle in detail. Part B of the formula looks at how the distortionary effects of the STW system influence the optimal provision of benefits. This section investigates Part B's impact on the business cycle. Figures 7 and 8 compare the response of the optimal STW system to a hypothetical subsidy system that pays benefits exactly when STW does, but without distorting working hours. This will give us a measure of how important working hours distortions are for the stabilization of the business cycle. Note that such a subsidy system is not implementable in practice. An optimal STW system would only subsidize matches in which the worker could not be retained otherwise, which requires knowledge of the productivity states in the economy. STW circumvents this problem as it can elicit productivity by linking the subsidy to reduced working hours, as exploited in Proposition 3.

Figure 7: Optimal STW Policy, Influence of Hours Distortion - Instruments

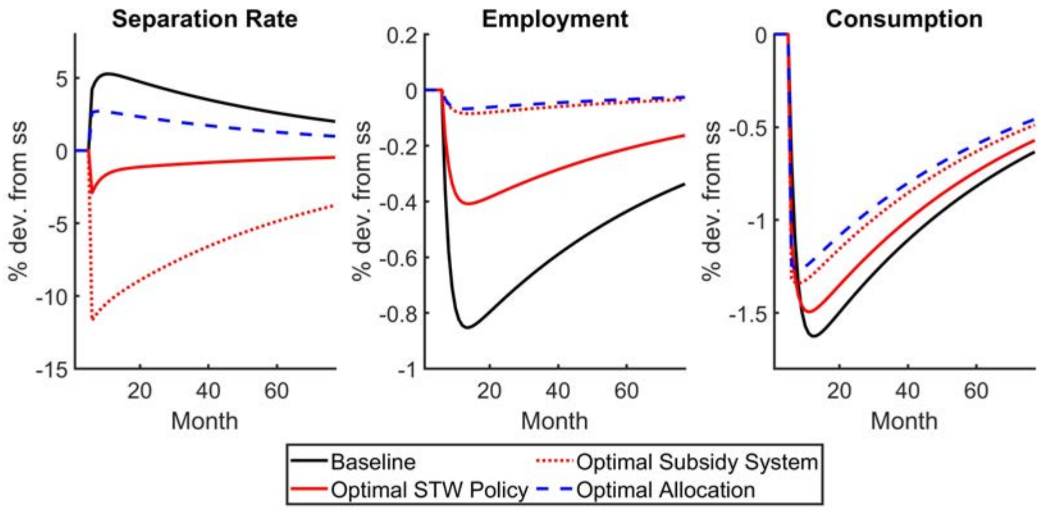


Notes: The figure compares the optimal adjustment of the STW system (solid line) to a hypothetical subsidy system without hours distortion (dashed line) to a 1 percent negative productivity shock.

Corollary 3 suggests that the distortion of working hours, and thus the costs of retaining a worker with STW, must increase during a recession as the fraction of workers on STW expands (see Figure 5). Figure 7 shows that the marginal welfare costs of STW benefits $\frac{\partial \Omega_t}{\partial s_t}$ rise indeed significantly in a recession. As more workers take advantage of the STW system during economic downturns, any increase in the STW benefits affects a larger number of firms, which significantly raises the overall distortionary effects and costs of STW benefits. Faced with higher marginal welfare costs of STW benefits, the planner responds by limiting the increase in STW benefits during recessions. The red dashed line shows how much the optimal transfer would increase if STW benefits did not have distortionary effects.

Consequently, the planner is unable to implement the optimal separation rate. Figure 8 shows that in the absence of distortions, the planner could reduce separations by more than 10 percent, bringing the economy close to the optimal employment level. This, in turn, brings the consumption response close to the optimal level. However, even then full stabilization of consumption remains unattainable, as stabilizing employment solely by reducing separations leads to a decrease in the mean productivity of the economy. In contrast, when distortions are present, the planner allows separations to drop only by 3 percent, resulting in more pronounced fluctuations in employment. This, coupled with greater fluctuations in working hours, limits the planner's ability to implement the optimal consumption response.

Figure 8: Optimal STW Policy, Influence of Hours Distortion - Allocation



Notes: The figure shows the impulse response functions of an economy with optimal STW system (solid red line) to a hypothetical subsidy system without hours distortion (dashed red line) for a 1 percent negative productivity shock. Further, it shows the response of the social planner economy (blue dashed line) and the baseline economy (black line).

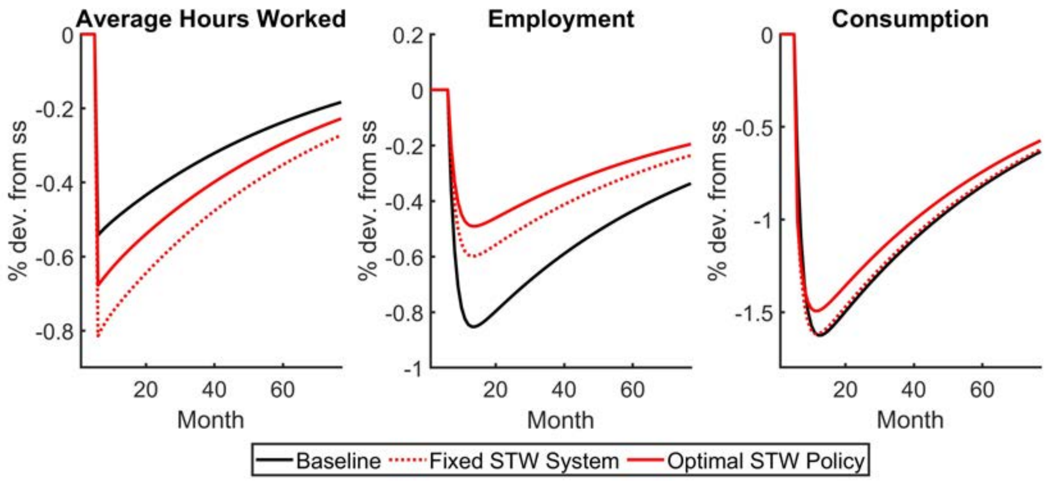
From this section, we can conclude that the distortion of working hours significantly reduces STW's capacity to stabilize both employment and consumption during recessions. In the case of employment, it is responsible for almost all inefficient fluctuations, while in the case of consumption, it is responsible for almost 73 percent, while roughly 27 percent stems from a reduction in average productivity.

6.4 Importance of a Dynamic STW System

The last sections have shown that optimal STW policy requires the eligibility condition and STW benefits to be adjusted in the business cycle. Balleer et al. (2016) argue that STW acts as an automatic stabilizer. The system can stabilize employment and consumption without the need to be adjusted. Therefore, the question can be raised: how important is a dynamic STW system for business cycle stabilization? To answer the question, I implement the optimal STW system in the steady state, but keep the eligibility condition and STW benefits constant in a recession. Figure 9 shows the results. The dotted red line displays the fixed STW system, while the red line shows the response of the optimal STW system and the black line the baseline economy.

Employment is still stabilized compared to the baseline economy. During recessions, firms and workers opt to reduce working hours, allowing them to qualify for additional STW benefits, which increases the net transfer to the least productive matches. This mechanism mitigates the decline

Figure 9: Fixed STW System



Notes: The figure shows the impulse response function of an economy with fixed STW system (dashed line, red) to the optimal STW system (red line) and the baseline economy (black line) for a 1 percent negative productivity shock. The fixed STW system sets STW optimally in the steady state but does not let the system adjust over the business cycle.

in the joint surplus of matches, leading to fewer separations.[24] While employment is stabilized, it is less effectively so than under optimal STW policy, as benefits cannot be increased to reduce separations during downturns.

However, despite not adjusting STW benefits, average hours worked decline more in the fixed STW system. This is due to the fact that the government does not tighten the eligibility conditions. Firms and workers that could continue working without STW opt to enter the system.[25] These windfall effects intensify the distortionary impact of STW and show how important the optimal implementation of the eligibility condition is. The combination of less stabilized employment and greater fluctuation in average working hours undermines the STW program's ability to stabilize consumption during recessions.

In conclusion, the analysis supports the findings of Balleer et al. (2016) that STW automatically stabilizes employment. It does not find stabilization of consumption. The main reason is that the eligibility condition of Balleer et al. (2016) does not allow for windfall effects.

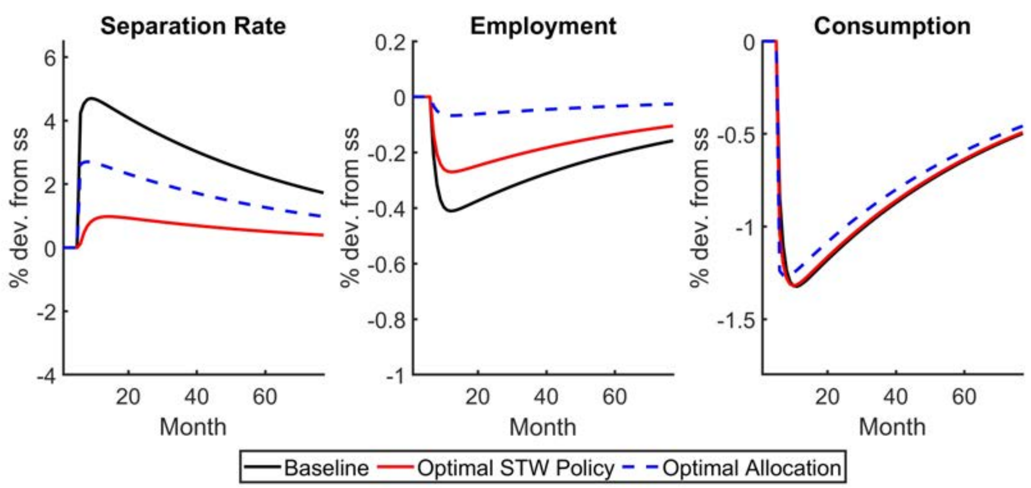
[24]Note that this mechanism differs from that in Balleer et al. (2016). Outside STW, their model does not allow working hours to adjust. Only STW allows for a reduction in hours and, thus, the wage bill. In recessions, firms can lower hours worked in response to a negative productivity shock. Thanks to the STW system, they consolidate wage expenditures and stabilize separations.

[25]After a negative productivity shock, the job-finding rate decreases, and workers' unemployment prospects worsen, making it more challenging to find new employment. Consequently, workers are more inclined to stay with their current employer, working fewer hours for a reduced salary. However, due to the reduced working hours, they become eligible for the STW system despite not being in need of support, making their entrance into STW inefficient (see also Corollary 5).

6.5 Optimal STW Policy in Recessions with Optimal UI in SS

Up to this point, we have analyzed optimal STW policy within an economy calibrated to the current US UI system. We now turn to an environment in which both UI and STW are set optimally in the steady state. Figure 10 reports the aggregate responses in this setting. As before, the dashed blue line shows the Ramsey allocation, the solid black line depicts the response in the baseline economy, and the red line illustrates the dynamics under the optimal STW system. Figure 11 presents the corresponding optimal adjustment of STW policy over the cycle.

Figure 10: Optimal STW Policy with Optimal UI in Steady State - Allocation

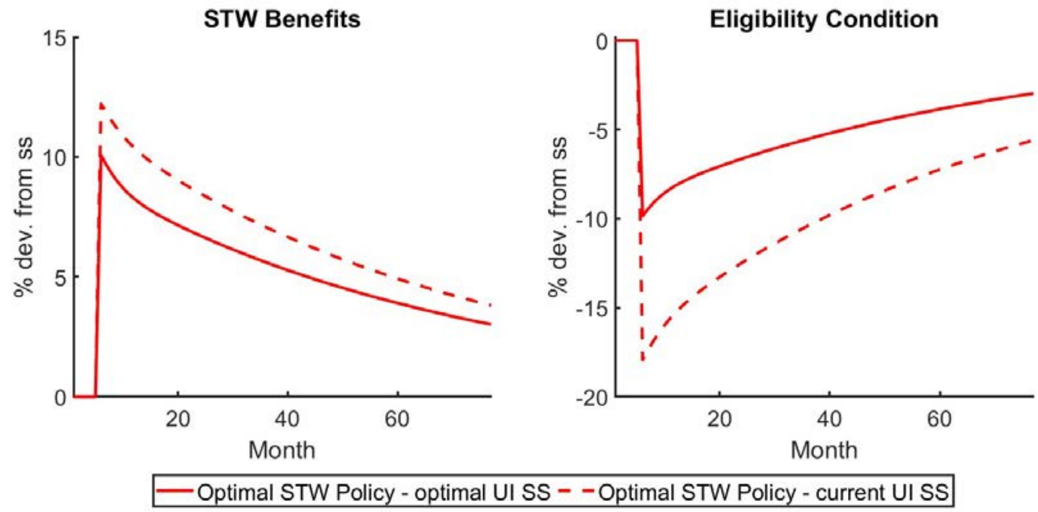


Notes: The figure shows the optimal response of the STW system with optimal UI in the steady state (red line) to a 1 percent negative productivity shock and compares it to the baseline economy with corresponding UI benefits in the steady state (black line) and the social planner economy (blue dashed line).

Figure 11, shows that the optimal STW system, when paired with optimal steady state UI benefits, responds qualitatively very similarly to the case without optimal UI. However, this similarity does not extend to the allocation responses. While the separation rate and employment remain stabilized at levels comparable to those under inefficient UI, the consumption response of the decentralized economy (black line) now closely tracks the social planner's allocation. As a result, there is relatively little scope for further improving consumption stabilization through cyclical adjustments in STW.

This raises the question of why STW is used at all in this environment. Besides ensuring that working hours remain efficient, the planner has two primary objectives: (i) allocating resources efficiently (maximizing output net of the disutility of work), and (ii) providing income insurance to workers. While the scope for improving efficiency is limited when UI benefits are already chosen optimally in the steady state, the planner can still use STW to enhance workers' income insurance.

Figure 11: Optimal STW Policy with Optimal UI in Steady State - Instruments



Notes: The figure shows the optimal response of the STW system with optimal UI in the steady state (red line) to a 1 percent negative productivity shock and compares it to the response of the STW system with the current UI system (dashed red line).

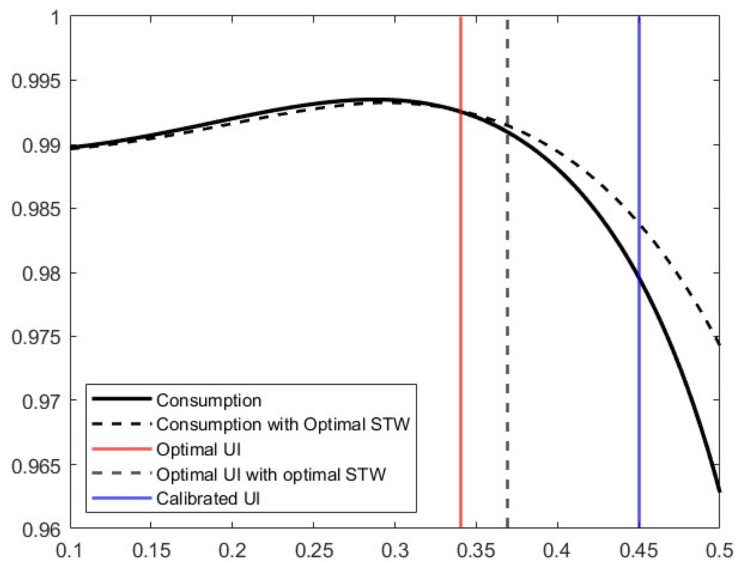
The Ramsey planner provides insurance along two margins: an intensive margin, through the level of UI benefits, and an extensive margin, by influencing the unemployment rate directly. STW operates through the extensive margin by lowering unemployment risk. By reducing inefficient fluctuations in separations and employment, STW helps smooth workers' income and thereby improves consumption insurance, even when the intensive margin (UI) is already set optimally in the steady state.

A remaining question is why consumption responds so closely to its optimal level when UI benefits are set optimally. Figure 12 shows that the optimal replacement rate lies very close to the level that maximizes consumption in the decentralized economy. The value of a marginal match must therefore be close to zero. At first glance, this might seem surprising, as higher UI benefits are traditionally associated with excessive unemployment and therefore lower output. However, the figure shows that very low UI benefits can depress output and thus consumption as well.

The mechanism is as follows. When UI benefits are low, unemployment becomes extremely costly for risk-averse workers. As a consequence, the value of maintaining employment rises, and in Nash bargaining workers prefer contracts that minimize unemployment risk, even if this implies lower salaries. As shown in equation 3, firms obtain a substantial salary discount when workers are poorly insured against unemployment.

The problem is that unemployment can play a productive role in the economy by allowing workers to leave unproductive firms and reallocate to more productive ones. When UI benefits are too low, workers tend to remain in unproductive firms, pulling down average productivity. If separation rates fall sufficiently, the resulting drop in aggregate productivity can outweigh the gains from higher employment, ultimately reducing output and, in turn, consumption.

Figure 12: Consumption and UI



Notes: The figure shows the steady state consumption levels in response to different UI replacement rates when there is only a UI system in place and when STW optimally reacts to the UI system.

In conclusion, when UI benefits are set optimally in the steady state, there remains little room for STW to improve on efficiency considerations. However, it still improves welfare as it helps to smooth consumption among households by stabilizing unemployment risk.

7 Discussion and Conclusion

In conclusion, the model presented in this paper demonstrates that STW can be a valuable complement to the UI system. While the UI system offers income insurance to workers, the STW system helps mitigate its distortionary effects. In the model, STW itself does not provide direct income insurance, as firms insure workers against idiosyncratic productivity shocks while employed. To reduce the distortionary impacts of the UI system during recessions, STW is adjusted by offering more generous benefits and tightening the eligibility condition. The model finds that the current

US UI benefits are too generous. As a result, STW plays an important role in stabilizing inefficient fluctuations in consumption and employment. When UI benefits are set optimally in the steady state, business cycles do not result in substantial efficiency losses in production. However, they still face difficulties in insuring workers against income loss in unemployment. STW smooths workers' income indirectly by reducing unemployment risk and, thereby, employment fluctuations.

Despite its benefits, STW has two main shortcomings that prevent it from fully implementing the planner's solution. First, STW distorts working hours. When setting optimal STW benefits, the planner faces a trade-off between implementing the optimal separation rate and minimizing the distortion of working hours. This makes it crucial to adjust STW over the business cycle. If STW is not adjusted, the distortion of working hours can exacerbate fluctuations, undermining STW's ability to stabilize total working hours.

Second, unlike as alluded to in papers such as Balleer et al. (2016), Giupponi and Landais (2018), or Cahuc, Kramarz, and Nevoux (2021), STW cannot stabilize the job-finding rate. If STW benefits are financed through a tax on salaries, any increase in STW benefits will be offset by a corresponding increase in the tax rate, nullifying the impact on the joint surplus of firms and workers. Even when considering lump-sum taxes, allowing STW to influence vacancy creation directly, the planner would refrain from stabilizing the job-finding rate, as the additional distortions to working hours would be too costly.

The paper suggests four avenues for further research. First, an intriguing result from the theoretical section is that STW functions similarly to a layoff tax, in the sense of Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008). The key difference, however, is that STW distorts working hours, making it less effective than layoff taxes. Despite this, layoff taxes and STW are fundamentally different instruments: STW is a subsidy, while layoff taxes are a penalty. In the model, their similarity arises because firms are assumed to never be financially constrained, enabling them to offer insurance to workers and pay layoff costs regardless of circumstances. But what if firms do face financial constraints? In such a scenario, STW might provide an insurance component for workers that layoff taxes cannot. Additionally, paying penalties could become infeasible for financially constrained firms. A comparison of STW and layoff taxes under financial constraints would be a valuable area of exploration.

Second, Cooper, Meyer, and Schott (2017) argue that a major drawback of STW is its potential to reduce allocative efficiency by incentivizing workers to remain in less productive firms, thereby hindering their reallocation to more productive firms. However, I find that STW can strike a balance between reducing allocative inefficiency and minimizing the costs of reallocating workers through the labor market, effectively leaving no room for allocative inefficiencies. This outcome is based on the assumption that the shock duration is uniform for all workers. But what happens if firms and workers experience shocks of varying durations? Investigating how optimal STW policy would respond to such differences in shock duration could provide new insights into how STW should deal with allocative inefficiencies.

Third, in the model, STW is an instrument to reduce separations and should not be used to influence job-finding rates because of its distortionary effects. However, job-finding rates can be inefficiently low due to the fiscal externality of the UI system and due to wage rigidities during downturns. It would therefore be interesting to analyze the joint use of STW with an instrument aimed at stimulating job creation, such as a hiring subsidy.

Finally, the US UI system is often considered to be countercyclical. While at the national level, the UI replacement rate does not vary much according to the US Department of Labor, the system provides extended benefit durations during downturns. For simplicity, my model abstracts from UI benefit duration. It would be interesting to incorporate this feature into the model. My expectation is that this countercyclical extension of UI benefits would further strengthen the case for adjusting STW over the business cycle

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A List of Important Variables and Functions

Table 5: Important Variables and Functions

Parameter	Description
Policy	
D_t	minimum hours reduction threshold (Eligibility condition)
s_t	STW benefits
b_t	UI benefits
τ_t	Income tax worker
Productivity Thresholds	
$\epsilon_{s,t}$	Separation threshold with access to STW
$\epsilon_{stw,t}$	Participation threshold
$\xi_{s,t}$	Separation threshold without access to STW
$\xi_{stw,t}$	Threshold at which firms and workers are willing to take up STW
Production	
a_t, ϵ	Aggregate and idiosyncratic productivity
μ_a	Mean aggregate productivity
$g(\epsilon), G(\epsilon)$	Pfd and cdf of idiosyncratic productivity shock
$h_t(\epsilon), h_{stw,t}(\epsilon)$	Working hours on and off STW
$y_t(\epsilon, h)$	Production function
$v(h)$	Disutility of work
$z_t(\epsilon, h), z_t$	Realized and expected output net of disutility (consumption equivalent)
$S_t(\epsilon), S_{stw,t}(\epsilon)$	Realized joint surplus on and outside STW
$\mathcal{S}_t$	Expected jont surplus
Firms and Firm Owner	
$J_t(\epsilon), J_{stw,t}(\epsilon)$	Realized value of firm on and off STW
$\mathcal{J}_t$	Expected value of the firm
Π_t	Dividends
$w_t(h), w_{eu,t}$	Salaray and severance payment
ν_t^f, ν_t^w	Number of firm owners, labor force
c_t^f	Consumption firm owners
$Q_{t,t+1}^f$	Stochastic discount factor firm owners

Table 6: Important Variables and Functions

Parameter	Description
Worker	
$V_t^w(\epsilon), V_{stw,t}^w(\epsilon)$	Realized Value of a Worker on and off STW
$\mathcal{V}_t$	Expected Value of a Worker
U_t	Value of an unemployed worker
$\tilde{c}_t^w$	Consumption equivalent employed worker
c_t^u	Consumption unemployed worker
$\tilde{c}_t(\epsilon)$	Consumption equivalent function
Labor Market	
n_t, u_t	Employment, unemployment
m_t	Matches
v_t	Vacancies
θ_t	Labor market tightness
f_t, q_t	Job finding rate and vacancy filling rate
ρ_t	Separation rate
From Propositions	
Ω_t	Welfare Costs of Hours Distortions
T	Expected time a workers spends on the UI system when separated
FE	Fiscal Externality
BE	Bargaining Effect
L_v	Social value of a new hire
L_s	Social value of the marginal worker
$L_s^\star$	Social value of marginal worker with optimal STW
$L_{stw}^\star$	Social cost of adding a worker to STW
L_ξ	Social cost of increasing separation incentives among matches without access to STW
MMS_s	Marginal number of matches preserved with STW benefits
MWL_s	Marginal welfare loss caused by STW benefits
$MMS_{\epsilon_{stw}}$	Marginal number of matches added to STW with looser eligibility
$MWL_{\epsilon_{stw}}$	Marginal welfare loss caused by looser eligibility

B The Bargaining Effect

As described in the main text, the bargaining effect illustrates how STW influences vacancy posting and separation behavior of firms via the salary channel. Diminishing returns to consumption make it harder for firms to reduce salaries. However, the introduction of STW necessitates salary reductions to offset firms' losses during bad idiosyncratic productivity draws. These adjustments are imperfect, resulting in relatively high wages, which, in turn, leads to fewer vacancies being posted and an increase in separations. Mathematically, this effect can be expressed as:

$$\tilde{BE} = \frac{\frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}}{1 + \frac{\beta \cdot (1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}} \cdot \frac{1 - f(\theta)}{1 - \eta} \cdot \frac{k_v}{q(\theta)}$$

with

$$BE = \frac{\left(-\frac{u''(\tilde{c}^w)}{u'(\tilde{c}^w)}\right) \cdot \frac{u(\tilde{c}^w) - u(b)}{u'(\tilde{c}^w)}}{1 + (1-\eta) \cdot \left(-\frac{u''(\tilde{c}^w)}{u'(\tilde{c}^w)}\right) \cdot \frac{u(\tilde{c}^w) - u(b)}{u'(\tilde{c}^w)}} < \frac{1}{1 - \eta}$$

Note that λ_θ is the Lagrange multiplier of the job-creation condition which can also be interpreted as the Lagrange multiplier of the value of the firm. It captures welfare losses by posting insufficient vacancies and experiencing excessive separations. The imperfect reduction in wages leads to more separations, not only in firms utilizing STW but also in those without access to STW. To mitigate the additional loss of these matches, a looser eligibility condition must be implemented, increasing the hours distortion effects of STW.

BE captures the effect of risk aversion. Under risk aversion, BE must be positive as:

$$\text{risk-aversion} \quad \Rightarrow \quad -u''(\tilde{c}^w_t) > 0$$

Note that the bargaining effect is zero under risk-neutrality:

$$\text{risk-neutrality} \quad \Rightarrow \quad u''(\tilde{c}^w_t) = 0 \quad \Rightarrow \quad BE = 0 \quad \Rightarrow \quad \tilde{BE} = 0$$

C What If STW Can Directly Influence Vacancy Posting?

The paper identifies the inability to stabilize the job-finding rate as one of the core problems of STW. In contrast, other authors, such as Balleer et al. (2016), Giupponi and Landais (2018), and Cahuc, Kramarz, and Nevoux (2021), highlight the ability of STW to increase vacancy postings as a strength of STW. The difference lies in the financing of the STW system. While my model finances the STW system by income taxes, their models rely on lump-sum taxes or do not consider the financing of STW at all. This makes a big difference: any increase in the joint surplus of firms and workers from higher expected STW benefits is in my model offset by a corresponding rise in the production tax, making it impossible for the government to steer job-finding rates with STW.

Nonetheless, while the balanced-budget assumption certainly is a good assumption for the long run, it might not need to hold in the short run. In fact, governments may choose to borrow in the short run to avoid raising taxes during recessions. Therefore, an expansion of the STW system financed by a deficit might still help stabilize the job-finding rates in recessions.

To explore this conjecture, I replace the production tax with a lump-sum tax on all households. For simplicity, I assume risk neutrality. Lump-sum taxes in a model with risk aversion suffer from distributional consequences that I do not want to discuss here. Under risk neutrality, lump-sum taxes do not influence vacancy posting or separation decisions, allowing STW to directly influence vacancy posting. By increasing the generosity of the STW system, the government can now stimulate the expected joint surplus of firms and workers, encouraging vacancy creation.

Lemma 2
Under lump-sum taxation, the social value of hiring an additional worker changes to L'_V

$$L'_V = \underbrace{\frac{\eta-\gamma}{(1-\eta)\cdot(1-\gamma)}}_{\text{Congestion Externality}} + \underbrace{\frac{\beta}{1-\beta\cdot(1-G(\epsilon_s))\cdot(1-f)}\cdot b}_{\text{Fiscal Externality UI on Hiring (FE')}}$$
$$+ \underbrace{\frac{\beta}{1-\beta\cdot(1-G(\epsilon_s))\cdot(1-f)}\cdot\left(\int_{\epsilon_s}^{\epsilon_{stw}}(\bar{h}-h_{stw}(\epsilon))\cdot s\cdot dG(\epsilon)\right)}_{\text{STW increases vacancy posting}}$$

while the social value of the marginal match $L^\star_S$ *and the social cost of adding a worker to STW* $L^\star_{STW}$ *stay the same.*

PROOF: Appendix K

This becomes clear analytically in Lemma 2, which derives the social value of hiring an additional worker. A more generous STW system, whether through a looser participation threshold or higher benefits, encourages vacancy posting, thereby lowering the social value of hiring an additional worker. Under an optimal STW policy, the social value of the marginal match, $L^\star_S$, remains unchanged because the social planner hires until the cost of an additional worker with STW equals its welfare gain. The same applies to the welfare cost of adding a worker to STW $L^\star_{STW}$.

Proposition 5 outlines the optimal eligibility condition in an economy with a lump-sum tax. In determining this condition, the government faces a new trade-off, as described in equation C.2. On one side, a looser eligibility condition raises the likelihood that a firm can use the STW program. This increases the expected benefits from the system, enhancing the joint surplus of firms and workers, which in turn raises job-finding rates and lowers separation rates and thus also the need for STW. On the other side, relaxing the eligibility condition also spreads the distortionary effects on working hours across more firms, heightening the welfare costs associated with the STW system.

Proposition 5, Optimal Eligibility Condition in Steady-State - Lump-Sum Tax
Consider the economy described in Section 2.1 and replace the income tax with a lump-sum tax. Further, assume that the economy has converged to its non-stochastic steady state and $\epsilon_{stw,t} \geq \xi_{s.t}$. *Then, the optimal eligibility condition* $D = h_{stw}(\epsilon_{stw})$ *is implicitly defined by the separation threshold of a firm without STW*

$$z(\epsilon_{stw}, h(\epsilon_{stw})) + F + \frac{1 - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} = 0 \tag{C.1}$$

as long as the welfare costs of a looser eligibility outweigh its welfare gains:

$$\underbrace{n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}}_{\text{Welfare Costs}} > L'_V \cdot \underbrace{\frac{\partial f^{ge}}{\partial \epsilon_{stw}}}_{\text{More jobs}} \cdot u + L^\star_S \cdot \underbrace{\left(-n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon^{ge}_s}{\partial \epsilon_{stw}}\right)}_{\text{Less separations}} + L^\star_\xi \cdot \underbrace{\left(-n \cdot g(\xi_s) \cdot \frac{\partial \xi^{ge}_s}{\partial \epsilon_{stw}}\right)}_{\text{Less workers need STW}} \tag{C.2}$$

Otherwise it is determined by setting the welfare gains of a looser eligibility condition equal to its welfare costs. $L^\star_\xi = \mathbb{1}\{\epsilon_{stw} = \xi_s\} \cdot L^\star_{STW}$ *denotes the social costs of additional separation incentives in matches without access to STW.*

PROOF: Appendix K

There are two possible outcomes. In the first, the welfare gains from posting more vacancies by loosening the eligibility criteria outweigh the distortionary effects of the STW system. Here, the optimal eligibility condition is achieved by balancing the welfare gains of a looser eligibility condition against its welfare losses. In the second outcome, the welfare costs from increased working-hour distortions outweigh the benefits of additional vacancy postings. In this case, the eligibility condition should be set as strictly as possible. The optimal participation threshold in this case would match the separation threshold of a firm without access to STW in this period

($\epsilon_{stw,t} = \xi_{s,t}$). Determining which effect dominates is a quantitative question.[26]

Proposition 6 establishes the optimal STW benefits under lump-sum taxes. In contrast to the previous section, the Ramsey planner's objective is to implement the optimal expected net transfer of the STW system, as indicated by equation C.3. Potential future STW benefits are priced in the expected benefits. In the absence of income taxes, the STW system operates not only by increasing the period surplus, but also by raising the expected surplus of firms and workers.

Proposition 6, Optimal STW Subsidy in Steady State - Lump-Sum Tax

Consider the economy described in section 2.1 and replace the income tax with a lump-sum tax. Further, assume that the economy has converged to its non-stochastic steady state and the eligibility condition is set according to Proposition 5. Then, the optimal STW subsidy s is implicitly determined by the optimal expected net transfer s^{net}*:*

$$\begin{aligned} s^{net} &= s \cdot (\bar{h} - h_{stw}(\epsilon_s)) \\ &\quad + \frac{\beta \cdot (1-f)}{1 - \beta \cdot (1 - G(\epsilon_s)) \cdot (1-f)} \cdot \int_{\epsilon_s}^{\epsilon_{stw}} s \cdot (\bar{h} - h_{stw}(\epsilon)) \cdot dG(\epsilon) \end{aligned} \tag{C.3}$$

The optimal expected net transfer s^{net} *is pinned down by:*

$$s^{net} = \underbrace{(1-f) \cdot FE'}_{\substack{\text{A: Influence Distortionary Effect UI on Separations} \\ > 0}} - \underbrace{\frac{MWL_s}{MMS_s}}_{\substack{\text{B: Hours Distortion Cost of Worker Retention} \\ > 0}} + \underbrace{\frac{MWG_s}{MMS_s}}_{\text{C: Welfare Gain increasing joint Surplus}}$$

The welfare gain from increasing the joint surplus is defined as:

$$MWG_s = n \cdot \left[\underbrace{L'_V \cdot \frac{\partial f^{ge}}{\partial s} \cdot u}_{\text{More jobs}} + \underbrace{L^\star_S \cdot \left(-n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon^{ge}_s}{\partial s} \right)}_{\text{Fewer separations}} + \underbrace{L^\star_{STW} \cdot \left(-n \cdot g(\epsilon_{stw}) \cdot \frac{\partial \epsilon^{ge}_{stw}}{\partial s} \right)}_{\text{Fewer workers on STW}} \right]$$

PROOF: Appendix K

[26] Note that I excluded $\epsilon_{stw} \geq \xi_s$ from consideration. As established in the main text, it cannot be optimal to preserve matches in lower-productive firms while denying support to more productive matches that would break up without support.

The optimal expected net transfer depends on three factors (see equation H.1). Similar to Proposition 2, the expected net-tansfer should reduce the salary paid borne by the firm by the financial burden the worker would impose on the UI system (A) minus a penalty for the welfare costs of using the STW system (B). Different from Proposition 2, larger STW benefits can now increase the expected value of firms, stimulating vacancy posting and lessening separations even more. The incentive of smaller separations also reduces the number of workers who need to be rescued with STW. Therefore, the planner adjusts the STW benefits upward (C). The Ramsey planner weighs the benefits of reducing inefficient separations and increasing suboptimally low vacancy posting efforts against distorting working hours in the economy. Whether a larger transfer of STW benefits indeed helps to stabilize the job-finding rate remains a quantitative question.

Propositions 5 and 6 highlight that STW has the potential to stabilize job-finding rates as suggested by Balleer et al. (2016), Giupponi and Landais (2018), and Cahuc, Kramarz, and Nevoux (2021). Loosening the eligibility condition and increasing STW benefits both increase the joint surplus of firms and workers and thus incentivize vacancy posting. The prerequisite is that we do not have to worry about financing the system, that is, the budget balance assumption. However, both propositions also stress that increasing vacancy posting incentives always has to be traded off against the additional distortions that STW introduces into the system.

To evaluate which effect dominates, I use the same calibration strategy as described in Section 4. However, due to the risk-neutrality assumption, I need to re-calibrate the model. For brevity, Table 7 lists all calibrated parameters with calibration strategy. Table 8 shows that the model closely replicates US business cycle facts.

The calibration reveals that the distortionary effects of the STW system clearly outweigh the benefits of additional vacancy postings. First, to evaluate how to set the eligibility condition, we can look at its Lagrange multiplier. If the Lagrange multiplier is negative, then the distortionary effects of STW dominate, and we want to set the eligibility condition to be as strict as possible:

$$\lambda_{stw} = \left[L_V' \cdot \underbrace{\frac{\partial f^{ge}}{\partial \epsilon_{stw}} \cdot u}_{\text{More jobs}} + L_S^\star \cdot \underbrace{\left(-n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial \epsilon_{stw}} \right)}_{\text{Less separations}} + L_{STW}^\star \cdot \underbrace{\left(-n \cdot g(\epsilon_{stw}) \cdot \frac{\partial \epsilon_{stw}^{ge}}{\partial \epsilon_{stw}} \right)}_{\text{Less Workers Need STW}} \right] - n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}$$

Figure 13 shows that the Lagrange multiplier is indeed negative. In fact, it becomes even more negative in recessions. As benefits increase in recessions, it makes adding workers to the system increasingly costly.

Table 7: Parameters for Model with Lump-Sum Tax

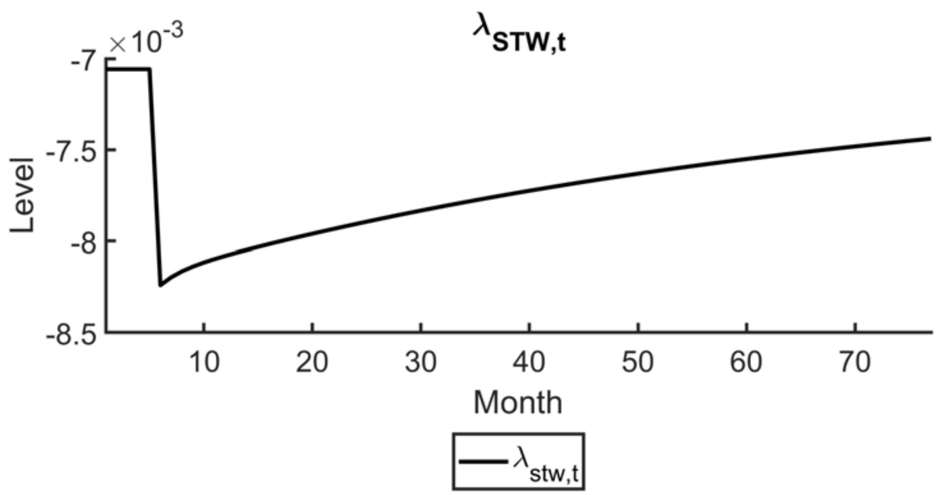
Parameter	Description	Value	Reason
ρ	Target ss separation rate	0.03	Data
f	Target ss job-finding rate	0.41	Data
q	Target ss vacancy filling rate	0.338	Haan, Ramey, and Watson (2000)
β	Discount rate	0.996	Jung and Kuester (2015)
ψ	Inverse Frisch elasticity	1.5	Domeij and Floden (2006)
γ	Elasticity matching function w.r.t. unemployment	0.65	Shimer (2005)
η	Bargaining power worker	0.65	Implements Hosios condition
γ_w	Coeff. reaction bargaining power to productivity shock	15.5	s.d. job-finding rate 14.31 in data
F	Separation costs	1.01	s.d. separation rate of 8.2 in data
b	UI benefits	0.4	40 percent replacement rate of wage
α	Labor elasticity in production	0.65	Christoffel and Linzert (2010)
$\bar{h}$	"Normal" hours worked	0.834	Mean hours in baseline
ρ_a	Autocorr. productivity shock	0.985	Jung and Kuester (2015)
μ_a	Mean aggregate productivity	1.0	Normalization
$\sigma_a \cdot 100$	s.d. aggregate productivity	0.259	s.d. labor prod. of 1.91 in data
$\mu_{\ln(\epsilon)}$	Steers mean of log-normal distribution	0.082	Normalizes wage to 1
$\sigma_{\ln(\epsilon)}$	Steers variance of log-normal distribution	0.12	Krause and Lubik (2007)
$\bar{m}$	Matching parameter	0.383	Calculated by target ss
k_v	Vacancy posting costs	0.139	Calculated by target ss
k_f	Strength of resource cost shock	10.441	Calculated by target ss

Table 8: Business Cycle Properties: Baseline Model with Lump-Sum Tax

		v_t	f_t	ρ_t	u_t	θ_t	$\bar{h}_t$	p_t
Standard Deviation		19.8	14.31	8.2	21.26	40.88	0.76	1.91
Autocorrelation		0.95	0.97	0.97	0.98	0.97	0.97	0.97
	v	1	1	-0.99	-0.98	1	1	1
	f	-	1	-1	-1	1	1	1
	ρ	-	-	1	1	-1	-1	-1
Correlation	u	-	-	-	1	-1	-1	-1
	θ	-	-	-	-	1	1	1
	$\bar{h}$	-	-	-	-	-	1	1
	p	-	-	-	-	-	-	1

Notes: The table reports the second moments of the model. As in the data of Shimer (2005), all variables are quarterly averages of monthly series and reported as log-deviations. p_t denotes the average output per worker, that is $E[a_t \cdot \epsilon_j \cdot h_t(\epsilon_j)^\alpha | \epsilon_j \geq \epsilon_{s,t}]$.

Figure 13: STW Must Be Set As Strict As Possible

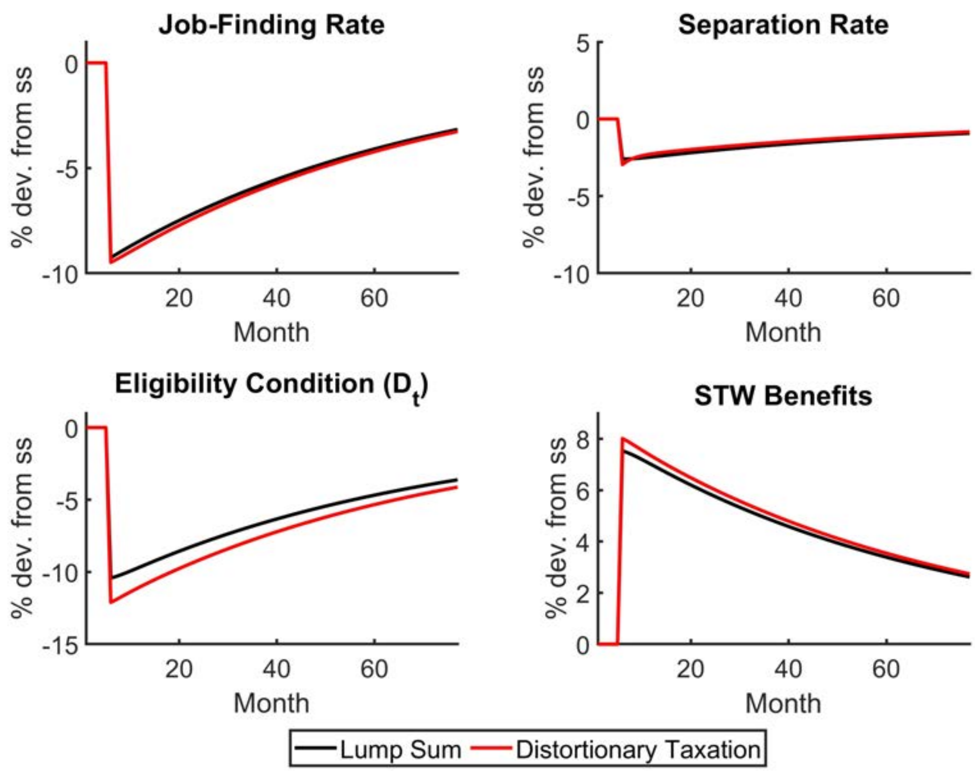


Notes: The figure shows the impulse response functions of a 1 percent negative productivity shock. It shows the reaction of the Lagrange multiplier for the eligibility condition.

Further, Figure 14 shows that it is too costly to increase STW benefits sufficiently to stabilize the job-finding rate. In fact, the impulse response functions of the job-finding rate under a STW system with lump-sum tax or distortionary taxation look almost the same, despite the ability of the Ramsey planner to stimulate vacancy postings with STW.

We can conclude that even when the Ramsey planner has the ability to directly influence vacancy postings with STW, he refrains from stabilizing the job-finding rate as the distortionary effects of the STW system are too costly.

Figure 14: Optimal STW Policy, lump-sum vs Distortionary Taxation



Notes: The figure shows the impulse response functions of a 1 percent negative productivity shock. It compares the response of the economy with optimal STW policy, financed by a lump-sum income tax (red line) against a lump-sum tax on households (black line).

D Optimal Layoff Tax vs Optimal STW

The paper explores the optimal design of short-time work (STW) policy and concludes that STW addresses the fiscal externalities of the UI system. In the US, a system with similar purpose already exists. The UI system is experience-rated. This means that firms' contributions to the UI system increase when workers are laid off, effectively functioning as a layoff tax that firms must pay when they separate from employees.

In this policy experiment, I exchange the STW system for a layoff tax. Firms are required to pay a tax to the government when they separate a worker. The revenue from the tax can be used to finance the UI system. Proposition 7 derives the optimal layoff tax, showing that it serves the same purpose as the STW system within the model.

Proposition 7, Optimal STW Benefits in Steady State

Consider the economy as previously described. Assume that it has converged to its non-stochastic steady state. Then, the optimal layoff tax τ_S is determined by:

$$\tau_S = \underbrace{(1-f)\cdot FE}_{\text{A: Fiscal Externality UI } > \ 0} - \underbrace{\tilde{BE}}_{C\text{: Bargaining Effect}}$$

PROOF: Appendix L

The key distinction between STW and layoff taxes in the model is that layoff taxes do not distort working hours. This difference is reflected in the comparison between the optimal layoff tax and the optimal net transfer of STW benefits (as discussed in Proposition 2 versus Proposition 7). The planner reduces the optimal STW benefits to minimize the distortionary effects associated with the STW system. As a result, we can conclude that, within the model, layoff taxes are superior to STW benefits due to their ability to avoid these distortions.

This result might be surprising, as STW and layoff taxes should be two fundamentally different instruments. STW functions as a subsidy, while layoff taxes operate as a penalty. Given this, one might wonder how they can lead to similar outcomes within the model. There are two main reasons for this.

First, neither STW nor layoff taxes directly influence the job-finding rate. The budget constraint for the economy with the STW system is:

$$n_t \cdot \tau_t = (1-n_t)\cdot b_t + n_t \cdot \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon)\right)\cdot dG(\epsilon)$$

The budget constraint for the economy with layoff taxes is:

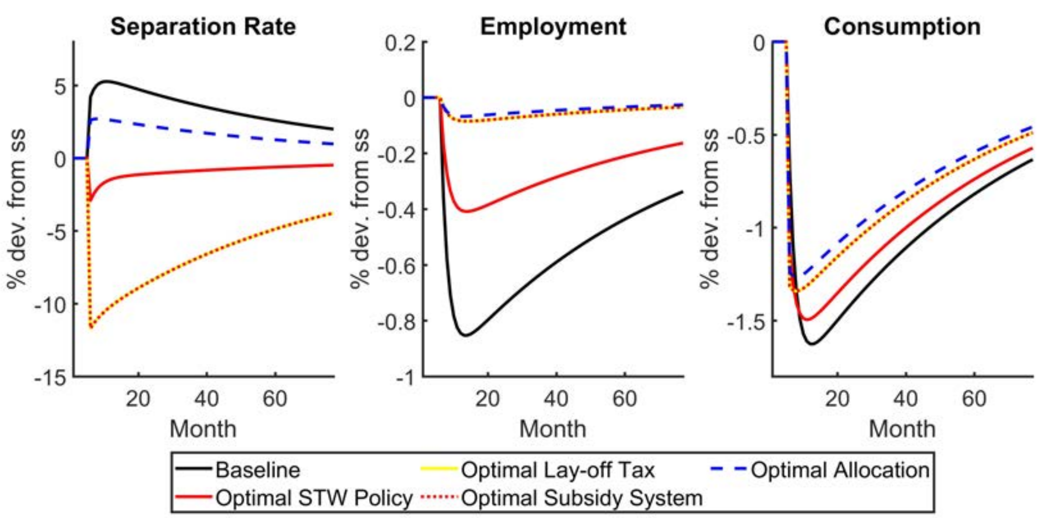
$$n_t \cdot \tau_t = (1-n_t)\cdot b_t - n_t \cdot \rho_t \cdot \tau_{S,t}$$

Increasing the generosity of STW can enhance the joint surplus of firms and workers; however, the subsequent rise in income taxes required to finance the STW system counteracts this benefit. Conversely, raising the layoff tax reduces the joint surplus of firms and workers. Nevertheless, an increase in the layoff tax also lowers the amount of income taxes needed to fund the UI system, which offsets the negative impact of the layoff tax on the joint surplus. If the surplus is not altered, job-finding rates do not alter.

Second, firms in the model are never financially constrained, which allows them to offer insurance to workers and pay layoff costs regardless of their financial situation. Under these circumstances, layoff taxes are clearly superior to STW. With financial constraints, things might be different. STW could be a tool to provide imperfect income insurance for the firm. Further, layoff taxes might become ineffective under financial constraints. A comparison of STW and layoff taxes in such circumstances would be interesting.

To see the quantitative difference between the optimal STW policy and optimal layoff taxes in the model, I calculated the impulse response functions to a 1 percent negative productivity shock. Figure 15 shows that, due to the absence of working hours distortions, layoff taxes are much better suited to stabilizing the business cycle. In fact, it replicates the results of the hypothetical subsidy system from Section 5.4. Almost all inefficient fluctuations in employment and consumption can be solved.

Figure 15: Optimal Lay-off Taxes - Allocation



Notes: The figure shows the impulse response function of an economy with an optimal STW system (red solid line) to a hypothetical subsidy system without hours distortion (red dashed line) and optimal layoff taxes (yellow line) for a 1 percent negative productivity shock and compares it to the response of the social planner economy (blue dashed line) and the baseline economy (black line).

E Nash bargaining

Wage and Severance Payment

The FOC for salary outside STW:

$$\eta_{t-1} \cdot u'(\tilde{c}_t(\epsilon)) \cdot g(\epsilon) \cdot \mathcal{J}_t = (1 - \eta_{t-1}) \cdot g(\epsilon) \cdot (\mathcal{V}_t^w - U_t)$$

The FOC for salary on STW:

$$\eta_{t-1} \cdot u'(\tilde{c}_{stw,t}(\epsilon)) \cdot g(\epsilon) \cdot \mathcal{J}_t = (1 - \eta_{t-1}) \cdot g(\epsilon) \cdot (\mathcal{V}_t^w - U_t)$$

The FOC for severance pay:

$$\eta_{t-1} \cdot u'(\tilde{c}_{eu,t}) \cdot g(\epsilon) \cdot \mathcal{J}_t = (1 - \eta_{t-1}) \cdot g(\epsilon) \cdot (\mathcal{V}_t^w - U_t)$$

Rearranging gives:

$$u'(\tilde{c}_t(\epsilon)) = u'(\tilde{c}_{stw,t}(\epsilon)) = u'(\tilde{c}_{eu,t})$$

Workers are perfectly insured against idiosyncratic shocks in the period:

$$\tilde{c}_t^w = \tilde{c}_t(\epsilon) = \tilde{c}_{stw,t}(\epsilon) = \tilde{c}_{eu,t}$$

Using the full insurance result, we get the optimal wage equation:

$$\eta_{t-1} \cdot \mathcal{J}_t = (1 - \eta_{t-1}) \cdot \frac{\mathcal{V}_t^w - U_t}{u'(\tilde{c}_t^w)}$$

Using the optimal wage equation, we gain expressions for the value of the firm and the surplus of employed workers, dependent on the joint surplus weighted by the marginal utility for firms and workers:

$$\mathcal{J}_t = (1 - \eta_{t-1}) \cdot \left(\mathcal{J}_t + \frac{\mathcal{V}_t^w - U_t}{u'(\tilde{c}_t^w)} \right)$$
$$\mathcal{V}_t^w - U_t = \eta_{t-1} \cdot \left(\mathcal{J}_t + \frac{\mathcal{V}_t^w - U_t}{u'(\tilde{c}_t^w)} \right)$$

The surplus of workers is denoted as:

$$\mathcal{V}_t^w = \int_{\max\{\epsilon_{stw,t}, \xi_{s,t}\}}^{\infty} V_t^w(\epsilon) dG(\epsilon) + \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} V_{stw,t}^w(\epsilon) dG(\epsilon)$$
$$+ \rho_t \cdot (u(c_{eu,t}) - u(b_t) + U_t)$$

$$
\begin{aligned}
&= \int_{\max\{\epsilon_{stw,t},\xi_{s,t}\}}^{\infty} u(\tilde{c}_t(\epsilon))dG(\epsilon) + \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} u(\tilde{c}_{stw,t}(\epsilon))dG(\epsilon) + \rho_t \cdot u(c_{eu,t}) \\
&\quad + \rho_t \cdot (U_t - u(b_t)) + (1-\rho_t) \cdot E_t \left[\beta \cdot \frac{\mathcal{V}^w_{t+1} - U_{t+1}}{u'(\tilde{c}^w_t)}\right]
\end{aligned}
$$

Insert value of unemployed worker:

$$
\begin{aligned}
\mathcal{V}^w_t &= \int_{\max\{\epsilon_{stw,t},\xi_{s,t}\}}^{\infty} \left[u(\tilde{c}_t(\epsilon)) - u(b_t)\right] dG(\epsilon) + \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \left[u(\tilde{c}_t(\epsilon)) - u(b_t)\right] dG(\epsilon) \\
&\quad + \rho_t \cdot \left[u(c_{eu,t}) - u(b_t)\right] \\
&\quad + (1-\rho_t) \cdot (1-f_t) \cdot \beta \cdot E_t \left[\mathcal{V}^w_{t+1} - U_{t+1}\right]
\end{aligned}
$$

Using the insurance result, we get:

$$
\mathcal{V}^w_t = u(\tilde{c}^w_t) - u(b_t) + (1-\rho_t) \cdot (1-f_t) \cdot \beta \cdot E_t \left[\mathcal{V}^w_{t+1} - U_{t+1}\right]
$$

Using the insurance result and dividing the expression by the marginal utility of consumption gives:

$$
\begin{aligned}
\frac{\mathcal{V}^w_t - U_t}{u'(\tilde{c}^w_t)} &= \frac{u(\tilde{c}^w_t) - u(b_t)}{u'(\tilde{c}^w_t)} + (1-\rho_t) \cdot (1-f_t) \cdot E_t \left[\beta \cdot \frac{\mathcal{V}^w_{t+1} - U_{t+1}}{u'(\tilde{c}^w_t)}\right] \\
&= \frac{u(\tilde{c}^w_t) - u(b_t)}{u'(\tilde{c}^w_t)} + (1-\rho_t) \cdot (1-f_t) \cdot E_t \left[\beta \cdot \frac{u'(\tilde{c}^w_{t+1})}{u'(\tilde{c}^w_t)} \cdot \frac{\mathcal{V}^w_{t+1} - U_{t+1}}{u'(\tilde{c}^w_{t+1})}\right] \\
&= \frac{u(\tilde{c}^w_t) - u(b_t)}{u'(\tilde{c}^w_t)} + (1-\rho_t) \cdot (1-f_t) \cdot E_t \left[Q^w_{t,t+1} \cdot \frac{\mathcal{V}^w_{t+1} - U_{t+1}}{u'(\tilde{c}^w_{t+1})}\right]
\end{aligned}
$$

The value of a firm can be denoted as:

$$
\begin{aligned}
\mathcal{J}_t &= \int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))dG(\epsilon) + \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon)\right) dG(\epsilon) - \Omega_t - \tilde{c}^w_t \\
&\quad - \tau_t - \rho_t \cdot F + (1-\rho_t) \cdot E_t \left[Q^f_{t,t+1} \cdot \mathcal{J}_{t+1}\right]
\end{aligned}
$$

Insert government budget constraint:

$$
\begin{aligned}
\mathcal{J}_t &= \int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))dG(\epsilon) + \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon)\right) dG(\epsilon) - \Omega_t - \tilde{c}^w_t \\
&\quad - \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon)\right) dG(\epsilon) - \frac{1-n_t}{n_t} \cdot b_t \\
&\quad - \rho_t \cdot F + (1-\rho_t) \cdot E_t \left[Q^f_{t,t+1} \cdot \mathcal{J}_{t+1}\right]
\end{aligned}
$$

STW cancels out:

$$\mathcal{J}_t = \int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))dG(\epsilon) - \Omega_t - \tilde{c}_t^w \\ - \frac{1-n_t}{n_t} \cdot b_t - \rho_t \cdot F + (1-\rho_t) \cdot E_t \left[Q_{t,t+1}^f \cdot \mathcal{J}_{t+1} \right]$$

Insert value of the firm and surplus of workers into wage equation:

$$\eta_{t-1} \cdot \Bigg(\int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))dG(\epsilon) - \Omega_t - \tilde{c}_t^w \\ - \frac{1-n_t}{n_t} \cdot b_t - \rho_t \cdot F + (1-\rho_t) \cdot E_t \left[Q_{t,t+1}^f \cdot \mathcal{J}_{t+1} \right] \Bigg) \\ = (1-\eta_{t-1}) \cdot \left(\frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} + (1-\rho_t) \cdot E_t \left[Q_{t,t+1}^w \cdot \frac{\mathcal{V}_{t+1} - U_{t+1}}{u'(\tilde{c}_{t+1}^w)} \right] \right)$$

Rearrange for income:

$$\eta_{t-1} \cdot \tilde{c}_t^w + (1-\eta_{t-1}) \cdot \frac{u(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)} \\ = \eta_{t-1} \cdot \Bigg(\int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))\, dG(\epsilon) - \Omega_t - \tilde{c}_t^w - \frac{1-n_t}{n_t} \cdot b_t \\ - \rho_t \cdot F + (1-\rho_t) \cdot E_t \left[Q_{t,t+1}^f \cdot \mathcal{J}_{t+1} \right] \Bigg) \\ + (1-\eta_{t-1}) \cdot \left(\frac{u(b_t)}{u'(\tilde{c}_t^w)} - (1-\rho_t)(1-f_t) \cdot E_t \left[Q_{t,t+1}^w \cdot \frac{\mathcal{V}_{t+1} - U_{t+1}}{u'(\tilde{c}_{t+1}^w)} \right] \right)$$

Insert optimal wage equation again:

$$\eta_{t-1} \cdot \tilde{c}_t^w + (1-\eta_{t-1}) \cdot \frac{u(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)} \\ = \eta_{t-1} \cdot \Bigg(\int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))\, dG(\epsilon) - \Omega_t - \tilde{c}_t^w \\ - \frac{1-n_t}{n_t} \cdot b_t - \rho_t \cdot F + (1-\rho_t) \cdot E_t \left[Q_{t,t+1}^f \cdot \mathcal{J}_{t+1} \right] \Bigg) \\ + (1-\eta_{t-1}) \cdot \left(\frac{u(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)} - (1-\rho_t)(1-f_t) \cdot E_t \left[Q_{t,t+1}^w \cdot \frac{\eta_t}{1-\eta_t} \cdot \mathcal{J}_{t+1} \right] \right)$$

Rearrange:

$$
\begin{aligned}
&\eta_{t-1} \cdot \tilde{c}_t^w + (1-\eta_{t-1}) \cdot \frac{u(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)} \\
&\quad = \eta_{t-1} \cdot \Bigg(\int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))\, dG(\epsilon) - \Omega_t - \tilde{c}_t^w \\
&\qquad\quad - \frac{1-n_t}{n_t} \cdot b_t - \rho_t \cdot F + (1-\rho_t) f_t \cdot E_t \left[Q_{t,t+1}^f \cdot \mathcal{J}_{t+1} \right] \Bigg) \\
&\qquad + (1-\eta_{t-1}) \cdot \frac{u(b_t)}{u'(\tilde{c}_t^w)} \\
&\qquad - \frac{\eta_{t-1} - \eta_t}{1-\eta_t} \cdot (1-\rho_t)(1-f_t) \cdot E_t \left[Q_{t,t+1}^w \cdot \mathcal{J}_{t+1} \right]
\end{aligned}
$$

Insert free-entry condition and use $\tilde{c}_t^w = c_t^f$:

$$
\begin{aligned}
&\eta_{t-1} \cdot \tilde{c}_t^w + (1-\eta_{t-1}) \cdot \frac{u(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)} \\
&\quad = \eta_{t-1} \cdot \Bigg(\int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))\, dG(\epsilon) - \Omega_t - \tilde{c}_t^w \\
&\qquad\quad - \frac{1-n_t}{n_t} \cdot b_t - \rho_t \cdot F + (1-\rho_t) \cdot \theta_t \cdot k_v \Bigg) \\
&\qquad + (1-\eta_{t-1}) \cdot \frac{u(b_t)}{u'(\tilde{c}_t^w)} \\
&\qquad - \frac{\eta_{t-1} - \eta_t}{1-\eta_t} \cdot (1-\rho_t)(1-f_t) \cdot \frac{k_v}{q_t}
\end{aligned}
$$

Hours Worked

FOC of hours worked outside STW:

$$
\begin{aligned}
&\alpha \cdot a_t \cdot \epsilon \cdot (h_t(\epsilon))^{\alpha-1} \cdot g(\epsilon) \cdot (1-\eta_{t-1}) \cdot (\mathcal{V}_t^w - U_t) \\
&\quad = v'(h_t(\epsilon)) \cdot u'(\tilde{c}_t^w) \cdot g(\epsilon) \cdot \eta_{t-1} \cdot \mathcal{J}_t
\end{aligned}
$$

Inserting optimality condition of the wage gives:

$$
\alpha \cdot a_t \cdot \epsilon \cdot (h_t(\epsilon))^{\alpha-1} = v'\left(h_t(\epsilon)\right)
$$

FOC of hours worked on STW:

$$
\alpha \cdot a_t \cdot \epsilon \cdot (h_{stw,t}(\epsilon))^{\alpha-1} \cdot g(\epsilon) \cdot (1-\eta_{t-1}) \cdot (\mathcal{V}_t^w - U_t)
$$

$$= (v'(h_{stw,t}(\epsilon)) + s_t) \cdot u'(\tilde{c}_t^w) \cdot g(\epsilon) \cdot \eta_{t-1} \cdot \mathcal{J}_t$$

Inserting the optimality condition for the wage gives:

$$\alpha \cdot a_t \cdot \epsilon \cdot h_{stw,t}(\epsilon)^{\alpha-1} = v'(h_{stw,t}(\epsilon)) + s_t$$

This is the condition for the optimal hours' choice.

Separations on STW

The FOC of the separation threshold is:

$$\begin{aligned}
&\eta_{t-1} \cdot g(\epsilon_{s,t}) \cdot (1 - f_t) \cdot \beta \cdot E_t \left[\mathcal{V}_{t+1}^w - U_{t+1}\right] \cdot \mathcal{J}_t \\
&+ (1 - \eta_{t-1}) \cdot g(\epsilon_{s,t}) \cdot \Big(z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right) \\
&+ F + E_t \left[Q_{t,t+1}^w \cdot \mathcal{J}_{t+1}\right] \Big) \cdot (\mathcal{V}_t - U_t) = 0
\end{aligned}$$

Insert optimal wage equation:

$$\begin{aligned}
&(1 - f_t) \cdot \beta \cdot E_t \left[\frac{\mathcal{V}_{t+1}^w - U_{t+1}}{u'(\tilde{c}_t^w)}\right] + F \\
&+ z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right) + E_t \left[Q_{t,t+1}^w \cdot \mathcal{J}_{t+1}\right] = 0
\end{aligned}$$

Account for stochastic multiplier of the workers:

$$\begin{aligned}
&(1 - f_t) \cdot E_t \left[Q_{t,t+1}^w \cdot \frac{\mathcal{V}_{t+1}^w - U_{t+1}}{u'(\tilde{c}_t^w)}\right] + F \\
&+ z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right) + E_t \left[Q_{t,t+1}^w \cdot \mathcal{J}_{t+1}\right] = 0
\end{aligned}$$

Use optimal wage equation again:

$$\begin{aligned}
&(1 - f_t) \cdot \frac{\eta_t}{1 - \eta_t} \cdot E_t \left[Q_{t,t+1}^w \cdot \mathcal{J}_{t+1}\right] + F \\
&+ z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right) + E_t \left[Q_{t,t+1}^w \cdot \mathcal{J}_{t+1}\right] = 0
\end{aligned}$$

Rearrange:

$$\begin{aligned}
&z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right) + F \\
&+ (1 - f_t) \cdot \frac{1 - \eta_t \cdot f_t}{1 - \eta_t} \cdot E_t \left[Q_{t,t+1}^w \cdot \mathcal{J}_{t+1}\right] = 0
\end{aligned}$$

Insert free-entry condition:

$$\begin{aligned}
&z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right) + F \\
&+ (1 - f_t) \cdot \frac{1 - \eta_t \cdot f_t}{1 - \eta_t} \cdot \frac{k_v}{q_t} = 0
\end{aligned}$$

Separations outside STW

The FOC of the separation threshold is:

$$\begin{aligned}
&\eta_{t-1} \cdot g(\xi_{s,t}) \cdot (1 - f_t) \cdot \beta \cdot E_t\left[\mathcal{V}^w_{t+1} - U_{t+1}\right] \cdot \mathcal{J}_t \\
&+ (1 - \eta_{t-1}) \cdot g(\xi_{s,t}) \cdot \left(z_t(\xi_{s,t}, h_t(\xi_{s,t})) + F + E_t\left[Q^w_{t,t+1} \cdot \mathcal{J}_{t+1}\right]\right) \cdot (\mathcal{V}_t - U_t) \\
&= 0
\end{aligned}$$

Insert optimal wage equation:

$$\begin{aligned}
&z_t(\xi_{s,t}, h_t(\xi_{s,t})) + F + E_t\left[Q^w_{t,t+1} \cdot \mathcal{J}_{t+1}\right] \\
&+ (1 - f_t) \cdot \beta \cdot E_t\left[\frac{\mathcal{V}^w_{t+1} - U_{t+1}}{u'(\tilde{c}^w_t)}\right] = 0
\end{aligned}$$

Account for stochastic multiplier of the workers:

$$\begin{aligned}
&z_t(\xi_{s,t}, h_t(\xi_{s,t})) + F + E_t\left[Q^w_{t,t+1} \cdot \mathcal{J}_{t+1}\right] \\
&+ (1 - f_t) \cdot E_t\left[Q^w_{t,t+1} \cdot \frac{\mathcal{V}^w_{t+1} - U_{t+1}}{u'(\tilde{c}^w_{t+1})}\right] = 0
\end{aligned}$$

Use optimal wage equation again:

$$\begin{aligned}
&z_t(\xi_{s,t}, h_t(\xi_{s,t})) + F + E_t\left[Q^w_{t,t+1} \cdot \mathcal{J}_{t+1}\right] \\
&+ (1 - f_t) \cdot \frac{\eta_t}{1 - \eta_t} \cdot E_t\left[Q^w_{t,t+1} \cdot \mathcal{J}_{t+1}\right] = 0
\end{aligned}$$

Rearrange:

$$\begin{aligned}
&z_t(\xi_{s,t}, h_t(\xi_{s,t})) + F \\
&+ (1 - f_t) \cdot \frac{1 - \eta_t \cdot f_t}{1 - \eta_t} \cdot E_t\left[Q^w_{t,t+1} \cdot \mathcal{J}_{t+1}\right] = 0
\end{aligned}$$

Insert free-entry condition:

$$z_t(\xi_{s,t}, h_t(\xi_{s,t})) + F + (1 - f_t) \cdot \frac{1 - \eta_t \cdot f_t}{1 - \eta_t} \cdot \frac{k_v}{q_t} = 0$$

F Derivation Social Planner

$$W_t^P = \max_{\theta_t, \epsilon_{s,t}, h_t(\epsilon)} n_t \cdot \int_0^\infty u(\tilde{c}_t^w(\epsilon)) dG(\epsilon) + (1-n_t) \cdot u(c_t^u) + \nu_t^f \cdot u(c_t^f) + \beta \cdot E_t \left[W_{t+1}^P \right]$$

subject to

$$\begin{aligned}
(I) \quad & n_t \cdot \int_0^\infty \tilde{c}_t^w(\epsilon) dG(\epsilon) + (1-n_t) \cdot c_t^u + \nu_t^f \cdot c_t^f = \\
& \qquad n_t \cdot \int_{\epsilon_{s,t}}^\infty y(\epsilon, h_t(\epsilon)) - v_t(h_t(\epsilon)) dG(\epsilon) \\
& \qquad - \theta \cdot (1 - n_t + G(\epsilon_{s,t}) \cdot n_t) \cdot k_v - n_t \cdot G(\epsilon_{s,t}) \cdot F \\
(II) \quad & n_{t+1} = (1 - G(\epsilon_{s,t})) \cdot n_t + f(\theta_t) \cdot (1 - n_t + G(\epsilon_{s,t}) \cdot n_t)
\end{aligned}$$

I solve the optimization problem using the Lagrangian method. For brevity, I do not spell out the full Lagrangian but introduce the relevant notation. The Lagrangian is denoted by $\mathcal{L}$. The Lagrange multiplier associated with the resource constraint is denoted by $\lambda_{m,t}$, and the multiplier associated with the law of motion of employment by $\lambda_{n,t}$.

First-Order Conditions (FOC) of the Planner

FOC for consumption of employed workers:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tilde{c}_t^w(\epsilon)} &= \beta^t \cdot n_t \cdot g(\epsilon) \cdot u'(c_t(\epsilon)) - \beta^t \cdot n_t \cdot g(\epsilon) \cdot \lambda_{m,t} = 0 \\
&\Leftrightarrow \lambda_{m,t} = u'(\tilde{c}_t(\epsilon))
\end{aligned}$$

FOC for consumption of unemployed workers:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tilde{c}_t^w(\epsilon)} &= \beta^t \cdot (1-n_t) \cdot u'(c_t^u) - \beta^t \cdot (1-n_t) \cdot \lambda_{m,t} = 0 \\
&\Leftrightarrow \lambda_{m,t} = u'(c_t^u)
\end{aligned}$$

FOC for consumption of firm owners:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tilde{c}_t^w(\epsilon)} &= \beta^t \cdot \nu_t^f \cdot u'(c_t^f) - \beta^t \cdot \nu_t^f \cdot \lambda_{m,t} = 0 \\
&\Leftrightarrow \lambda_{m,t} = u'(c_t^f)
\end{aligned}$$

FOC for hours worked:

$$\frac{\partial \mathcal{L}}{\partial h_t(\epsilon)} = \beta^t \cdot \lambda_{m,t} \cdot n_t \cdot \left(\frac{\partial y_t(\epsilon, h_t(\epsilon))}{\partial h_t(\epsilon)} - v'(h_t(\epsilon)) \right) \cdot g(\epsilon) = 0$$

$$\Leftrightarrow \frac{\partial y_t(\epsilon, h_t(\epsilon))}{\partial h_t(\epsilon)} = v'(h_t(\epsilon))$$

This is equation 7.

FOC for employment:

$$\frac{\partial \mathcal{L}}{\partial n_t} = \beta^t \cdot \lambda_{m,t} \cdot \left(\int_{\epsilon_{s,t}}^{\infty} [y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon))] \, dG(\epsilon) - G(\epsilon_{s,t}) \cdot F + \theta_t \cdot k_v \right) - \beta^{t-1} \cdot \lambda_{n,t-1} + \beta^t \cdot (1 - \theta_t \cdot q(\theta_t)) \cdot (1 - G(\epsilon_{s,t})) \cdot \lambda_{n,t} = 0$$

$$\Leftrightarrow \quad \lambda_{n,t-1} = \beta \cdot \lambda_{m,t} \cdot \left(\int_{\epsilon_{s,t}}^{\infty} [y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon))] \, dG(\epsilon) - G(\epsilon_{s,t}) \cdot F + \theta_t \cdot k_v \right) + \beta \cdot (1 - \theta_t \cdot q(\theta_t)) \cdot (1 - G(\epsilon_{s,t})) \cdot \lambda_{n,t}$$

$$\Leftrightarrow \quad \frac{\lambda_{n,t-1}}{\lambda_{m,t-1}} = \beta \cdot \frac{\lambda_{m,t}}{\lambda_{m,t-1}} \cdot \left(\int_{\epsilon_{s,t}}^{\infty} [y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon))] \, dG(\epsilon) - G(\epsilon_{s,t}) \cdot F + \theta_t \cdot k_v \right) + \beta \cdot \frac{\lambda_{m,t}}{\lambda_{m,t-1}} \cdot (1 - \theta_t \cdot q(\theta_t)) \cdot (1 - G(\epsilon_{s,t})) \cdot \frac{\lambda_{n,t}}{\lambda_{m,t}}$$

FOC for the labor market tightness:

$$\frac{\partial \mathcal{L}}{\partial \theta_t} = -\beta^t \cdot \lambda_{m,t} \cdot k_v \cdot (1 - n_t + G(\epsilon_{s,t}) \cdot n_t) + \beta^t \cdot \lambda_{n,t} \cdot (q(\theta_t) + \theta_t \cdot q'(\theta_t)) \cdot (1 - n_t + G(\epsilon_{s,t}) \cdot n_t) = 0$$

$$\Leftrightarrow \quad \lambda_{n,t} = \frac{\lambda_{m,t}}{1 + \theta_t \cdot \frac{q'(\theta_t)}{q(\theta_t)}} \cdot \frac{k_v}{q(\theta_t)}$$

Note that we can express the elasticity of the matching function with respect to unemployment as:

$$\theta_t \cdot \frac{q'(\theta_t)}{q(\theta_t)} = -\gamma \cdot \theta_t \cdot \frac{\chi \cdot \theta_t^{-\gamma-1}}{\chi \cdot \theta_t^{-\gamma}} = -\gamma$$

Using this expression gives:

$$\frac{\lambda_{n,t}}{\lambda_{m,t}} = \frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)}$$

FOC separation threshold:

$$-\frac{\partial \mathcal{L}}{\partial \epsilon_{s,t}} = \beta^t \cdot \lambda_{m,t} \cdot (y_t(\epsilon_{s,t}, h_t(\epsilon_{s,t})) - v(h_t(\epsilon_{s,t})) + F + \theta \cdot k_v) \cdot g(\epsilon_{s,t})$$
$$+ \beta^t \cdot \lambda_{n,t} \cdot (1 - f(\theta_t)) \cdot g(\epsilon_{s,t}) = 0$$

This is equivalent to:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{s,t}} = y_t(\epsilon_{s,t}, h_t(\epsilon_{s,t})) - v(h_t(\epsilon_{s,t})) + F + \theta_t \cdot k_v + (1 - f(\theta_t)) \cdot \frac{\lambda_{n,t}}{\lambda_{m,t}} \cdot = 0$$

Distribution of Income It must hold:

$$\lambda_{m,t} = \lambda_{m,t} = \lambda_{m,t} \quad \Leftrightarrow \quad u'(\tilde{c}_t(\epsilon)) = u'(\tilde{c}_t^u) = u'(c_t^f)$$
$$\Leftrightarrow \quad \tilde{c}_t := \tilde{c}_t(\epsilon) = \tilde{c}_t^u = c_t^f$$

Planner's Job-Creation Condition
Insert Lagrange multiplier:

$$\frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta_{t-1})} = \beta \cdot \frac{u'(\tilde{c}_t)}{u'(\tilde{c}_{t-1})} \cdot \Bigg(\int_{\epsilon_{s,t}}^{\infty} [y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon))] \, dG(\epsilon) - G(\epsilon_{s,t}) \cdot F$$
$$+ (1 - G(\epsilon_{s,t})) \cdot \theta_t \cdot k_v \Bigg)$$
$$+ \beta \cdot \frac{u'(\tilde{c}_t)}{u'(\tilde{c}_{t-1})} \cdot (1 - \theta_t \cdot q(\theta_t)) \cdot (1 - G(\epsilon_{s,t})) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)}$$

Using that $\theta_t = \frac{\theta_t \cdot q(\theta_t)}{q(\theta_t)}$ gives:

$$\frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta_{t-1})} = \beta \cdot \frac{u'(\tilde{c}_t)}{u'(\tilde{c}_{t-1})} \cdot \Bigg(\int_{\epsilon_{s,t}}^{\infty} [y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon))] \, dG(\epsilon) - G(\epsilon_{s,t}) \cdot F$$
$$+ (1 - G(\epsilon_{s,t})) \cdot \theta_t \cdot k_v \Bigg)$$
$$+ \beta \cdot \frac{u'(\tilde{c}_t)}{u'(\tilde{c}_{t-1})} \cdot (1 - G(\epsilon_{s,t})) \cdot \left[\frac{1 - \theta_t \cdot q(\theta_t)}{1-\gamma} + \theta_t \cdot q(\theta_t) \right] \cdot \frac{k_v}{q(\theta_t)}$$

Rearranging:

$$\frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta_{t-1})} = \beta \cdot \frac{u'(\tilde{c}_t)}{u'(\tilde{c}_{t-1})} \cdot \left(\int_{\epsilon_{s,t}}^{\infty} \left[y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon)) \right] dG(\epsilon) - G(\epsilon_{s,t}) \cdot F \right) + \beta \cdot \frac{u'(\tilde{c}_t)}{u'(\tilde{c}_{t-1})} \cdot (1 - G(\epsilon_{s,t})) \cdot \frac{1 - \gamma \cdot \theta_t \cdot q(\theta_t)}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)}$$

Planner's Separation Decision
Insert FOC for employment into FOC for separation threshold:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{s,t}} = y_t(\epsilon_{s,t}, h_t(\epsilon_{s,t})) - v(h_t(\epsilon_{s,t})) + F + \theta_t \cdot k_v + (1 - f(\theta_t)) \cdot \frac{k_v}{q(\theta_t)} = 0$$

Using that $\theta_t = \frac{\theta_t \cdot q(\theta_t)}{q(\theta_t)}$ again gives:

$$[y_t(\epsilon_{s,t}, h_t(\epsilon_{s,t})) - v(h_t(\epsilon_{s,t}))] + F + \frac{1 - \gamma \cdot \theta_t \cdot q(\theta_t)}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)} = 0$$

With $f_t = \theta_t \cdot q(\theta_t)$ and $q_t = q(\theta_t)$, this is equivalent to the separation decision of the planner expressed in the paper.

G Derivation Decentralized Economy

Derivation Job-Creation Condition

Joint surplus:

$$\mathcal{J}_t + \frac{\mathcal{V}_t - U_t}{u'(\tilde{c}_t^w)} = \int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon)) dG(\epsilon) - \Omega_t - \frac{1-n_t}{n_t} \cdot b_t - \tilde{c}_t^w + \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} - \rho_t \cdot F + (1-\rho_t) \cdot E_t \left[Q_{t+1}^f \cdot \mathcal{J}_{t+1} + \beta \cdot \frac{\mathcal{V}_{t+1} - U_{t+1}}{u'(\tilde{c}_t^w)} \right]$$

Insert optimal wage equation:

$$\frac{\mathcal{J}_t}{1-\eta_{t-1}} = \int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon)) dG(\epsilon) - \Omega_t - \frac{1-n_t}{n_t} \cdot b_t - \tilde{c}_t^w + \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} - \rho_t \cdot F + (1-\rho_t) \cdot E_t \left[Q_{t+1}^f \cdot \mathcal{J}_{t+1} + (1-f_t) \cdot Q_{t+1}^w \cdot \frac{\eta_t}{1-\eta_t} \cdot \mathcal{J}_{t+1} \right]$$

Since we assumed that ν_t^f endogenously adjusts so that consumption equivalents for firms and workers are equal, $\tilde{c}_t^f = \tilde{c}_t^w$ it must apply: $Q_{t+1}^f = Q_{t+1}^w$.

$$\frac{\mathcal{J}_t}{1-\eta_{t-1}} = \int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))dG(\epsilon) - \Omega_t - \frac{1-n_t}{n_t} \cdot b_t - \tilde{c}_t^w + \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)}$$
$$- \rho_t \cdot F + (1-\rho_t) \cdot \frac{(1-\eta_t \cdot f_t)}{1-\eta_t} \cdot E_t\left[Q_{t+1}^w \cdot \mathcal{J}_{t+1}\right]$$

Insert free-entry condition:

$$\frac{1}{1-\eta_{t-1}} \cdot \frac{1}{Q_t^w} \cdot \frac{k_v}{q_{t-1}} = \int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))dG(\epsilon) - \Omega_t - \frac{1-n_t}{n_t} \cdot b_t - \tilde{c}_t^w + \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)}$$
$$- \rho_t \cdot F + (1-\rho_t) \cdot \frac{(1-\eta_t \cdot f_t)}{1-\eta_t} \cdot \frac{k_v}{q_t}$$

Rearrange:

$$\frac{1}{1-\eta_{t-1}} \cdot \frac{k_v}{q_{t-1}} = Q_t^w \cdot \left(\int_{\mathcal{B}_t} z_t(\epsilon, h_t(\epsilon))dG(\epsilon) - \Omega_t - \frac{1-n_t}{n_t} \cdot b_t - \tilde{c}_t^w + \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} \right)$$
$$- \rho_t \cdot F + Q_t^w \cdot \left((1-\rho_t) \cdot \frac{1-\eta_t \cdot f_t}{1-\eta_t} \cdot \frac{k_v}{q_t} \right)$$

Thus:

$$\frac{1}{1-\eta_t} \cdot \frac{k_v}{q_t} = E_t\Bigg[Q_{t+1}^w \cdot \Bigg(\int_{\mathcal{B}_{t+1}} z_{t+1}(\epsilon, h_t(\epsilon))dG(\epsilon) - \Omega_{t+1} - \frac{1-n_{t+1}}{n_{t+1}} \cdot b_t - \tilde{c}_{t+1}^w$$
$$+ \frac{u(\tilde{c}_{t+1}^w) - u(b_{t+1})}{u'(\tilde{c}_{t+1}^w)} \Bigg) \Bigg]$$
$$+ E_t\Bigg[- \rho_{t+1} \cdot F + Q_{t+1}^w \cdot \left((1-\rho_{t+1}) \cdot \frac{1-\eta_{t+1} \cdot f_{t+1}}{1-\eta_{t+1}} \cdot \frac{k_v}{q_{t+1}} \right) \Bigg]$$

H Derivation Optimal Policy

The Ramsey planner chooses the eligibility condition D_t, the STW benefits s_t, and UI benefits b_t subject to the labor market equilibrium. Check Appendices E and G for a derivation of the core labor market equilibrium conditions.

$$W_t^G = \max_{D_t, s_t, b_t} (1-n_t) \cdot u(b_t) + n_t \cdot u(\tilde{c}_t^w)$$
$$+ \nu_t^f \cdot u\Bigg(\Bigg[n_t \cdot \int_{\mathcal{B}_t} \left(y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon))\right) dG(\epsilon) - n_t \cdot \Omega_t - n_t \cdot \tilde{c}_t^w$$

$$
- (1-n_t)\cdot b_t - n_t\cdot\rho_t\cdot F - \theta_t\cdot(1-n_t+\rho_t\cdot n_t)\cdot k_v\Big]/\nu_t^f\Big)
$$

$$
+\beta\cdot E_t W_{t+1}^G
$$

Set:

$$
\mathcal{B}_t = [\epsilon_{s,t}, \epsilon_{stw,t}] \cup [\xi_{s,t}, \infty)
$$

Note that we have assumed that ν_t^f endogenously adjusts so that consumption equivalents for firms and workers are equal, $\tilde{c}_t^f = \tilde{c}_t^w$. This rules out any distributional conflict between firm owners and workers. To ensure that the Ramsey planner cannot exploit this relationship, we must treat it as exogenous from the planner's perspective. This implies that the planner must view the stochastic discount factor as exogenous, since the equality of consumption equivalents was used to derive the stochastic discount factor appearing in the job-creation condition.

Job-creation condition:

$$
\begin{aligned}
\frac{1}{1-\eta_t}\cdot\frac{k_v}{q(\theta_t)} = E_t\Bigg[Q_{t,t+1}^w\Bigg(\int_{\mathcal{B}_t} y_{t+1}(\epsilon, h_{t+1}(\epsilon)) - v(h_{t+1}(\epsilon))dG(\epsilon) - \Omega_{t+1} - \rho_t\cdot F \\
- \frac{1-n_{t+1}}{n_{t+1}}\cdot b_{t+1} - \tilde{c}_{t+1}^w + \frac{u(\tilde{c}_{t+1}^w) - u(b_{t+1})}{u'(\tilde{c}_{t+1}^w)}\Bigg)\Bigg] \\
+ E_t\left[Q_{t,t+1}^w\left((1-\rho_{t+1})\cdot\frac{1-\eta_{t+1}\cdot f(\theta_{t+1})}{1-\eta_{t+1}}\cdot\frac{k_v}{q(\theta_{t+1})}\right)\right]
\end{aligned}
$$

Income insurance contract:

$$
\begin{aligned}
&\eta_{t-1}\cdot\tilde{c}_t^w + (1-\eta_{t-1})\cdot\frac{u(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)} \\
&\quad = \eta_{t-1}\cdot\left(\int_{\mathcal{B}_t}\left[y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon))\right]dG(\epsilon) - \rho_t\cdot F - \Omega_t - \frac{1-n_t}{n_t}\cdot b_t + (1-\rho_t)\cdot\theta_t\cdot k_v\right) \\
&\qquad + (1-\eta_{t-1})\cdot\frac{u(b_t)}{u'(\tilde{c}_t^w)} \\
&\qquad - \frac{\eta_{t-1}-\eta_t}{1-\eta_t}\cdot(1-\rho_t)\cdot(1-f_t)\cdot\frac{k_v}{q_t}
\end{aligned}
$$

Separation conditions without access to STW:

$$
y_t(\xi_{s,t}, h_t(\xi_{s,t})) - v(h_t(\xi_{s,t})) + F + \frac{1-\eta_t\cdot f_t}{1-\eta_t}\cdot\frac{k_v}{q_t} = 0
$$

Separation conditions on STW:

$$y_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - v(h_{stw,t}(\epsilon_{s,t})) + (\bar{h} - h_{stw,t}(\epsilon_{s,t})) \cdot s_t \\ + F + \frac{1-\eta_t \cdot f_t}{1-\eta_t} \cdot \frac{k_v}{q_t} = 0$$

Working hours outside STW:

$$\alpha \cdot a_t \cdot \epsilon \cdot h_t(\epsilon)^{\alpha-1} = h_t(\epsilon)^{\psi}$$

Working hours on STW:

$$\alpha \cdot a_t \cdot \epsilon \cdot h_{stw,t}(\epsilon)^{\alpha-1} = h_{stw,t}(\epsilon)^{\psi} + s_t$$

Welfare costs of STW:

$$\Omega_t = \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \Big[y_t(\epsilon, h_t(\epsilon)) - v\left(h_t(\epsilon)\right) - y_t(\epsilon, h_{stw,t}(\epsilon)) + v\left(h_{stw,t}(\epsilon)\right) \Big] dG(\epsilon)$$

Stochastic discount factor:

$$Q^w_{t,t+1} = \beta \cdot \frac{u'(\tilde{c}^w_{t+1})}{u'(\tilde{c}^w_t)}$$

Law of motion of employment:

$$n_{t+1} = (1-\rho_t) \cdot n_t + f(\theta_t) \cdot (1 - n_t + \rho_t \cdot n_t)$$

Separation rate:

$$\rho_t = \Big[\max\{G(\xi_{s,t}) - G(\epsilon_{stw,t}), 0\} + G(\epsilon_{s,t}) \Big]$$

I solve the optimization problem using the Lagrangian method. For brevity, I do not spell out the full Lagrangian but introduce the relevant notation. The Lagrangian is denoted by $\mathcal{L}$. The Lagrange multiplier associated with the job-creation condition is denoted by $\lambda_{\theta,t}$; the multiplier associated with the income insurance contract by $\lambda_{c,t}$; the multiplier associated with the separation condition without access to STW by $\lambda_{\xi,t}$; the multiplier associated with the separation condition under STW by $\lambda_{stw,t}$; the multiplier associated with working hours outside STW by $\lambda_{h,t}(\epsilon)$; the multiplier associated with working hours under STW by $\lambda_{h_{stw},t}(\epsilon)$; the multiplier associated with the welfare costs of STW by $\lambda_{\Omega,t}$; the multiplier associated with the stochastic discount factor by $\lambda_{Q,t}$; the multiplier associated with the law of motion of employment by $\lambda_{n,t}$; and the multiplier associated with the separation rate by $\lambda_{\rho,t}$.

H.1 Constrained Efficient Separation and Job-Creation Conditions

First, let us derive the constrained efficient separation and job-creation conditions. To do this, we need to form the FOCs for consumption, labor market tightness, employment and the separation threshold on STW.

FOC for the consumption equivalent:

$$\frac{\partial \mathcal{L}}{\partial \tilde{c}_t^w} = \underbrace{n_t \cdot \left[u'(\tilde{c}_t^w) - \frac{\nu_t^f}{\nu_t^f} \cdot u'(\tilde{c}_t^w) \right]}_{=\,0} - \lambda_{\theta,t-1} \cdot Q_t^w \cdot \frac{u''(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)} \cdot \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)}$$
$$- \lambda_{c,t} \cdot \left[1 - (1-\eta_{t-1}) \cdot \frac{u''(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)} \cdot \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} \right] = 0$$

Changes in consumption influence the value of the joint surplus and thus hiring incentives:

$$\Leftrightarrow \lambda_{c,t} = \lambda_{\theta,t-1} \cdot Q_t \cdot BE_t$$
$$\text{with} \quad BE_t = \frac{\left(-\frac{u''(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)}\right) \cdot \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)}}{1 + (1-\eta_{t-1}) \cdot \left(-\frac{u''(\tilde{c}_t^w)}{u'(\tilde{c}_t^w)}\right) \cdot \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)}} < \frac{1}{1-\eta_{t-1}}$$

FOC for labor market tightness:

$$\frac{\partial \mathcal{L}}{\partial \theta_t} = -\underbrace{(1 - n_t + G(\epsilon_{s,t}) \cdot n_t)}_{u_t} \cdot k_v \cdot \underbrace{\frac{\nu_t^f}{\nu_t^f} \cdot u'(c_t^f)}_{u'(\tilde{c}_t^w)}$$
$$+ \underbrace{(1 - n_t + G(\epsilon_{s,t}) \cdot n_t)}_{u_t} \cdot (1-\gamma) \cdot q(\theta_t) \cdot E_t\left[\lambda_{n,t+1}\right]$$
$$- \frac{1}{1-\eta} \cdot k_v \cdot \gamma \cdot \frac{\lambda_{\theta,t}}{f(\theta_t)}$$
$$+ \frac{1}{1-\eta} \cdot \left(\frac{\gamma}{f(\theta_t)} - \eta \right) \cdot (\beta \cdot (1 - G(\epsilon_{s,t})) \cdot \lambda_{\theta,t-1} - \lambda_{\epsilon,t} - \lambda_{stw,t}) \cdot k_v$$
$$+ \lambda_{c,t} \cdot \left(\eta \cdot (1 - G(\epsilon_{s,t})) \cdot k_v + \frac{\eta_t - \eta_{t-1}}{1-\eta_t} \cdot (1-\rho_t) \cdot \left(\frac{\gamma}{f(\theta_t)} - 1 \right) \cdot k_v \right) = 0$$

Rearranging gives:

$$\Leftrightarrow \frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)} + \frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)}$$

$$
\begin{aligned}
&\cdot \frac{1}{1-\eta} \cdot \frac{1}{m(\theta_t)} \cdot \frac{\gamma \cdot \lambda_{\theta,t} - (\gamma - f(\theta_t) \cdot \eta) \cdot (\beta \cdot (1 - G(\epsilon_{s,t})) \cdot \lambda_{\theta,t-1} - \lambda_{\epsilon,t} - \lambda_{stw,t})}{u'(\tilde{c}_t^w)} \\
&- \frac{\lambda_{c,t}}{u'(\tilde{c}_t^w)} \cdot \left(\eta \cdot (1 - G(\epsilon_{s,t})) \cdot k_v + \frac{\eta_t - \eta_{t-1}}{1 - \eta_t} \cdot (1 - \rho_t) \cdot \left(\frac{\gamma}{f(\theta_t)} - 1 \right) \cdot k_v \right) \\
&\cdot \frac{\eta \cdot (1 - G(\epsilon_{s,t}))}{u_t} \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)} \\
= \beta \cdot \frac{E_t\left[\lambda_{n,t+1}\right]}{u'(\tilde{c}_t^w)}
\end{aligned}
$$

Define χ_t as:

$$
\begin{aligned}
&\chi_t = \\
&\frac{1}{1-\eta} \cdot \frac{1}{m(\theta_t)} \cdot \frac{\gamma \cdot \lambda_{\theta,t} - (\gamma - f(\theta_t) \cdot \eta) \cdot (\beta \cdot (1 - G(\epsilon_{s,t})) \cdot \lambda_{\theta,t-1} - \lambda_{\epsilon,t} - \lambda_{stw,t})}{u'(\tilde{c}_t^w)} \\
&- \frac{\lambda_{c,t}}{u'(\tilde{c}_t^w)} \cdot \left(\eta \cdot (1 - G(\epsilon_{s,t})) \cdot k_v + \frac{\eta_t - \eta_{t-1}}{1 - \eta_t} \cdot (1 - \rho_t) \cdot \left(\frac{\gamma}{f(\theta_t)} - 1 \right) \cdot k_v \right) \cdot \frac{\eta \cdot (1 - G(\epsilon_{s,t}))}{u_t}
\end{aligned}
$$

This simplifies the expression and gives the first building block for the constrained optimal job-creation and separation condition:

$$
\beta \cdot \frac{E_t\left[\lambda_{n,t+1}\right]}{u'(\tilde{c}_t^w)} = \frac{1 + \chi_t}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)}
$$

FOC for employment:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial n_t} = u(\tilde{c}_t^w) - u(b_t) \\
&+ \underbrace{\frac{\nu_t^f}{\nu_t^f} \cdot u'(c_t^f)}_{u'(\tilde{c}_t^w)} \cdot \left(\int_{\epsilon_{s,t}}^{\infty} -y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon)) dG(\epsilon) - \Omega_t - \rho_t \cdot F + b_t + (1 - G(\epsilon_{s,t})) \cdot \theta_t \cdot k_v \right) \\
&+ (\beta \cdot \lambda_{\theta,t-1} + \eta \cdot \lambda_{c,t}) \cdot \frac{b_t}{n_t^2} \\
&- \lambda_{n,t} + \beta \cdot (1 - f(\theta_t)) \cdot (1 - G(\epsilon_{s,t})) \cdot E_t\left[\lambda_{n,t+1}\right]
\end{aligned}
$$

Rearranging gives the second building block of the constrained efficient job-creation condition:

$$
\begin{aligned}
\Leftrightarrow &\frac{u'(\tilde{c}_{t-1}^w)}{u'(\tilde{c}_t^w)} \cdot \frac{\lambda_{n,t}}{u'(\tilde{c}_{t-1}^w)} \\
&= \int_{\epsilon_{s,t}}^{\infty} y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon)) dG(\epsilon) - \Omega_t - \rho_t \cdot F + b_t + (1 - G(\epsilon_{s,t})) \cdot \theta_t \cdot k_v \\
&+ \frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} + \frac{(\beta \cdot \lambda_{\theta,t-1} + \lambda_{c,t})}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{b_t}{n_t}
\end{aligned}
$$

$$+\beta \cdot (1-f(\theta_t)) \cdot (1-G(\epsilon_{s,t})) \cdot \frac{E_t[\lambda_{n,t+1}]}{u'(\tilde{c}_t^w)}$$

Inserting the first building block of the constrained efficient job-creation condition gives:

$$\begin{aligned}
&\frac{1+\chi_{t-1}}{1-\gamma} \cdot \frac{k_v}{q(\theta_{t-1})} = \\
&\beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot \left(\int_{\epsilon_{s,t}}^{\infty} y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon)) dG(\epsilon) - \Omega_t - \rho_t \cdot F + b_t + (1-G(\epsilon_{s,t})) \cdot \theta_t \cdot k_v \right) \\
&+ \beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot \left(\frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} + \frac{(\beta \cdot \lambda_{\theta,t-1} + \lambda_{c,t})}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{b_t}{n_t} \right) \\
&+ \beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot (1-G(\epsilon_{s,t})) \cdot (1-f(\theta_t)) \cdot \frac{1+\chi_t}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)}
\end{aligned}$$

Here, $\frac{1+\chi_{t-1}}{1-\gamma} \cdot \frac{k_v}{q(\theta_{t-1})}$ can be interpreted as the social value of creating a new match.

Rearranging gives the **constrained efficient job-creation condition:**

$$\begin{aligned}
\Leftrightarrow &\frac{1+\chi_{t-1}}{1-\gamma} \cdot \frac{k_v}{q(\theta_{t-1})} = \\
&\beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot \left(\int_{\epsilon_{s,t}}^{\infty} y_t(\epsilon, h_t(\epsilon)) - v(h_t(\epsilon)) dG(\epsilon) - \Omega_t - \rho_t \cdot F + b_t \right) \\
&+ \beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot \left(\frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} + \frac{(\beta \cdot \lambda_{\theta,t-1} + \lambda_{c,t})}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{b_t}{n_t} \right) \\
&+ \beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot (1-G(\epsilon_{s,t})) \cdot \frac{(1-\gamma \cdot f(\theta_t)) + (1-f(\theta_t)) \cdot \chi_t}{1-\gamma} \cdot \frac{k_v}{q(\theta_t)}
\end{aligned}$$

The FOC for the optimal separation threshold is:

$$\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_{s,t}} = &\, n \cdot g(\epsilon_{s,t}) \cdot [y_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - v(h_{stw,t}(\epsilon_{s,t})) + F + k_v \cdot \theta_t] \cdot \underbrace{\frac{\nu_t^f}{\nu_t^f} \cdot \tilde{u}'(c_t^F)}_{u'(\tilde{c}_t^w)} \\
&+ n_t \cdot g(\epsilon_{s,t}) \cdot (1-f(\theta_t)) \cdot E_t\left[\lambda_{n,t+1}\right] \\
&+ \lambda_{\theta,t-1} \cdot g(\epsilon_{s,t}) \\
&\quad \cdot \beta \cdot \left[y_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - v(h_{stw,t}(\epsilon_{s,t})) + F + \frac{1-f(\theta_t) \cdot \eta}{1-\eta} \cdot \frac{k_v}{q(\theta_t)} \right] \\
&+ \lambda_{c,t} \cdot g(\epsilon_{s,t}) \cdot [y_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - v(h_{stw,t}(\epsilon_{s,t})) + F + \theta_t \cdot k_v] \\
&- \lambda_{c,t} \cdot \frac{\eta_{t-1} - \eta_t}{1-\eta_t} \cdot g(\epsilon_{s,t}) \cdot (1-f_t) \cdot \frac{k_v}{q_t} \\
&+ \lambda_{\epsilon,t} \cdot \Big[a_t \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha} + c_f
\end{aligned}$$

$$+\frac{\partial h_{stw,t}(\epsilon_{s,t})}{\partial \epsilon_{s,t}} \cdot \underbrace{\left(\alpha \cdot a_t \cdot \epsilon_{s,t} \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha-1} - v'(h_{stw,t}(\epsilon_{s,t})) - s_t\right)}_{=0} \Bigg]$$

$$= 0$$

Rearranging gives the second building block for the constrained efficient separation condition:

$$\Leftrightarrow y_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - v(h_{stw,t}(\epsilon_{s,t})) + F + k_v \cdot \theta_t + (1 - f(\theta_t)) \cdot \frac{E_t\left[\lambda_{n,t+1}\right]}{u'(\tilde{c}_t^w)}$$
$$- \frac{\eta_{t-1} - \eta_t}{1 - \eta_t} \cdot (1 - \rho_t) \cdot (1 - f_t) \cdot \frac{k_v}{q_t}$$
$$- \frac{\beta \cdot \lambda_{\theta,t-1} + \lambda_{c,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot s_t - \frac{\eta \cdot \lambda_{c,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{1 - f(\theta_t)}{1 - \eta} \cdot \frac{k_v}{q(\theta_t)}$$
$$+ \frac{1}{g(\epsilon_{s,t})} \cdot \frac{\lambda_{\epsilon,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot (a_t \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha} + c_f) = 0$$

Inserting the first building block of the constrained efficient separation condition gives:

$$\Leftrightarrow y_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - v(h_{stw,t}(\epsilon_{s,t})) + F + \theta_t \cdot k_v + (1 - f(\theta_t)) \cdot \frac{1 + \chi_t}{1 - \gamma} \cdot \frac{k_v}{q(\theta_t)}$$
$$- \frac{\beta \cdot \lambda_{\theta,t-1} + \eta \cdot \lambda_{c,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot s_t - \frac{\eta \cdot \lambda_{c,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{1 - f(\theta_t)}{1 - \eta} \cdot \frac{k_v}{q(\theta_t)}$$
$$+ \frac{1}{g(\epsilon_{s,t})} \cdot \frac{\lambda_{\epsilon,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot (a_t \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha} + c_f) = 0$$

Rearranging gives the **constrained efficient optimal separation condition:**

$$\Leftrightarrow y_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - v(h_{stw,t}(\epsilon_{s,t})) + F$$
$$+ \frac{1 - \gamma \cdot f(\theta_t) + (1 - f(\theta_t)) \cdot \chi_t}{1 - \gamma} \cdot \frac{k_v}{q(\theta_t)}$$
$$- \frac{(1 + \eta \cdot BE_t) \cdot \lambda_{\theta,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot s_t - \frac{BE_t \cdot \lambda_{\theta,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{1 - f(\theta_t)}{1 - \eta} \cdot \frac{k_v}{q(\theta_t)}$$
$$= -\frac{1}{g(\epsilon_{s,t})} \cdot \frac{\lambda_{\epsilon,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot (a_t \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha} + c_f)$$

H.2 Constrained Efficient vs. Decentralized Conditions Steady State

This section subtracts the decentralized job-creation and separation conditions from the constrained efficient conditions, giving us a sense of wether job-creation and destruction is inefficiently high or low. Inserting the Lagrange multiplier for the consumption equivalent lets the constrained efficient job-creation condition in the steady state simplify to:

$$\begin{aligned}\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q(\theta)} &= \beta\cdot\left(\int_{\epsilon_s}^{\infty} y(\epsilon,h(\epsilon)) - v(h(\epsilon))dG(\epsilon) - \Omega - \rho\cdot F + b\right)\\ &+ \beta\cdot\left(\frac{u(\tilde{c}^w)-u(b)}{u'(\tilde{c}^w)} + \frac{\beta\cdot(1+\eta\cdot BE)\cdot\lambda_\theta}{n\cdot u'(\tilde{c}^w)}\cdot\frac{b}{n}\right)\\ &+ \beta\cdot(1-G(\epsilon_s))\cdot\frac{(1-\gamma\cdot f(\theta))+(1-f(\theta))\cdot\chi}{1-\gamma}\cdot\frac{k_v}{q(\theta)}\end{aligned}$$

Next, we subtract the decentralized separation condition from the constrained efficient separation condition. To do this, recognize two auxiliary calculations. First, subtract the joint surplus of firms and workers from the social value of creating a new match:

$$\begin{aligned}\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q(\theta)} - \frac{1}{1-\eta}\cdot\frac{k_v}{q(\theta)} &= \frac{(1-\eta)\cdot(1+\chi)-(1-\gamma)}{(1-\gamma)\cdot(1-\eta)}\cdot\frac{k_v}{q(\theta)}\\ &= \left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)}\end{aligned}$$

Second, do this for the continuation value:

$$\begin{aligned}&\frac{(1-\gamma\cdot f(\theta))+(1-f(\theta))\cdot\chi}{1-\gamma}\cdot\frac{k_v}{q(\theta)} - \frac{1-\eta\cdot f(\theta)}{1-\eta}\cdot\frac{k_v}{q(\theta)}\\ &= \frac{(1-\eta)\cdot((1-\gamma\cdot f(\theta))+(1-f(\theta))\cdot\chi)-(1-\gamma)\cdot(1-\eta\cdot f(\theta))}{(1-\eta)\cdot(1-\gamma)}\cdot\frac{k_v}{q(\theta)}\\ &= \frac{(1-\eta)\cdot((1-\gamma\cdot f(\theta))+(1-f(\theta))\cdot\chi)-(1-\gamma)\cdot(1-\eta\cdot f(\theta))}{(1-\eta)\cdot(1-\gamma)}\cdot\frac{k_v}{q(\theta)}\\ &= (1-f(\theta))\cdot\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)}\end{aligned}$$

Putting both together gives the deviation of the constrained efficient separation condition from the decentralized:

$$\begin{aligned}\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)} &= \beta\cdot\left(1+\frac{\beta\cdot(1+\eta\cdot BE)\cdot\lambda_\theta}{n\cdot u'(\tilde{c}^w)}\right)\cdot\frac{b}{n}\\ &+ \beta\cdot(1-G(\epsilon_s))\cdot(1-f(\theta))\cdot\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)}\end{aligned}$$

Rearranging, the **deviation from the optimal job-creation condition** can be expressed as:

$$\begin{aligned}&\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)}\\ &= \frac{\beta}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))}\cdot\left(1+\frac{\beta\cdot(1+\eta\cdot BE)\cdot\lambda_\theta}{n\cdot u'(\tilde{c}^w)}\right)\cdot\frac{b}{n}\end{aligned}$$

The formula describes wether the social value of creating a new job exceeds the social cost of creating it. If $\left(\chi - \frac{\eta-\gamma}{1-\eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta)} > 0$ then firms and workers undervalue the social value of a new match, leading to too low job-creation.

The constrained optimal separation condition in the steady state can be expressed as:

$$\begin{aligned} \Leftrightarrow y(\epsilon_s, h_{stw}(\epsilon_s)) - v(h_{stw}(\epsilon_s)) + F + (1-\gamma \cdot f(\theta)) \cdot \frac{1+\chi}{1-\gamma} \cdot \frac{k_v}{q(\theta)} \\ - \frac{(1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot s - \frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot \frac{1-f(\theta)}{1-\eta} \cdot \frac{k_v}{q(\theta)} \\ + \frac{1}{g(\epsilon_s)} \cdot \frac{\lambda_\epsilon}{n \cdot u'(\tilde{c}^w)} \cdot (a \cdot h(\epsilon_s)^\alpha + c_f) = 0 \end{aligned}$$

Next, subtract the decentralized separation condition:

$$\begin{aligned} & - s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right) + (1-f(\theta)) \cdot \left(\chi - \frac{\eta-\gamma}{1-\eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta)} \\ & - \frac{(1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right) - \frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot \frac{1-f(\theta)}{1-\eta} \cdot \frac{k_v}{q(\theta)} \\ & + \frac{1}{g(\epsilon_s)} \cdot \frac{\lambda_\epsilon}{n \cdot u'(\tilde{c}^w)} \cdot (a \cdot h(\epsilon_s)^\alpha + c_f) = 0 \end{aligned}$$

Rearrange for the **optimal STW subsidy.**

$$\begin{aligned} \Leftrightarrow \ s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right) = & (1-f(\theta)) \cdot \left(\chi - \frac{\eta-\gamma}{1-\eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta)} \\ & - \frac{(1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right) \\ & + \frac{1}{g(\epsilon_s)} \cdot \frac{\lambda_\epsilon}{n \cdot u'(\tilde{c}^w)} \cdot (a \cdot h(\epsilon_s)^\alpha + c_f) \\ & - \frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot \frac{1-f(\theta)}{1-\eta} \cdot \frac{k_v}{q(\theta)} \end{aligned}$$

The subsidy is there to realign the optimal and decentralized separation condition. We can see this when we look at the **deviation from the optimal separation condition**:

$$\begin{aligned} & \left(1 + \frac{\beta \cdot (1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right) \\ & \cdot \left(\frac{\beta \cdot (1-f(\theta))}{1-\beta \cdot (1-f(\theta)) \cdot (1-G(\epsilon_s))} \cdot \frac{b}{n} - \left(\bar{h} - h_{stw}(\epsilon_s)\right) \cdot s\right) = \\ & - \frac{1}{g(\epsilon_s)} \cdot \frac{\lambda_\epsilon}{n \cdot u'(\tilde{c}^w)} \cdot (a \cdot h(\epsilon_s)^\alpha + c_f) + \frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot \frac{1-f(\theta)}{1-\eta} \cdot \frac{k_v}{q(\theta)} \end{aligned}$$

When the right hand side of the equation is positive, then the decentralized economy sheds too many workers.

H.3 Optimal UI Given STW - Steady State

To calculate the optimal UI system given a non-optimized STW system in the steady state, assume that the participation threshold is looser than the separation threshold without access to STW: $\epsilon_{stw,t} > \xi_{s,t}$. First, we have to form the FOC for the optimal UI benefits:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial b_t} &= (1-n_t)\cdot(u'(b_t)-u'(\tilde{c}_t^w)) \\ &\quad - \lambda_{\theta,t}\cdot\beta\cdot\left[\left(\frac{u'(b_t)}{u'(\tilde{c}_t^w)}+\frac{1-n_t}{n_t}\right)-BE_t\cdot\left((1-\eta)\cdot\frac{u'(b_t)}{u'(\tilde{c}_t^w)}-\eta\cdot\frac{1-n_t}{n_t}\right)\right]=0\end{aligned}$$

Note that the expression depends on the Lagrange multiplier of the job-creation condition λ_θ. λ_θ is pinned down by in χ. We can find two different expressions for χ.

First, note that the deviation of the private value of the match from the social value of the match contains χ:

$$\begin{aligned}&\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)}= \\ &\qquad\frac{\beta}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))}\cdot\left(1+\frac{\beta\cdot(1+\eta\cdot BE)\cdot\lambda_\theta}{n\cdot u'(\tilde{c}^w)}\right)\cdot\frac{b}{n}\end{aligned}$$

Rearranging for χ gives the right-hand side of the formula for calculating the Lagrange multiplier for the job-creation condition λ_θ:

$$\begin{aligned}\chi = &\frac{\eta-\gamma}{1-\eta} \\ &+\frac{\beta}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))} \\ &\quad\cdot\left(1+\frac{\beta\cdot(1+\eta\cdot BE)\cdot\lambda_\theta}{n\cdot u'(\tilde{c}^w)}\right)\cdot\frac{b}{n}/\left(\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)}\right)\end{aligned}$$

The left-hand side of the equation can be expressed by the steady state value of χ.

$$\begin{aligned}\chi = &\frac{1}{1-\eta}\cdot\frac{\gamma-\beta\cdot(\gamma-(1-(1-\eta)\cdot BE)\cdot\eta\cdot f(\theta))\cdot(1-G(\epsilon_s))}{m(\theta)\cdot u'(\tilde{c}^w)}\cdot\lambda_\theta \\ &-\frac{1}{1-\eta}\cdot\frac{(\gamma-\eta\cdot f(\theta))\cdot(-\lambda_\epsilon-\lambda_\xi)}{m(\theta)\cdot u'(\tilde{c}^w)}\end{aligned}$$

Note that the Lagrange multiplier for the separation condition without access to STW benefits is zero $\lambda_\xi = 0$. To see this, we first have to derive the FOC of the separation threshold of firms without access to STW:

$$\frac{\partial \mathcal{L}}{\partial \xi_{s,t}} =$$

$$-\lambda_{\xi,t} \cdot \left(a_t \cdot h_t(\xi_{s,t})^\alpha + c_f + \frac{\partial h_t(\xi_{s,t})}{\partial \xi_{s,t}} \cdot \underbrace{\left(\alpha \cdot a_t \cdot h_t(\xi_{s,t})^{\alpha-1} - v'(h_t(\xi_{s,t}))\right)}_{=0} \right) \cdot u'(\tilde{c}^w) = 0$$

$$\Leftrightarrow \lambda_{\xi,t} = 0$$

The result can be explained intuitively by the fact that changing the separation threshold of firms without access to STW will not influence the separation rate as long as we assume that $\epsilon_{stw} > \xi_s$. $\lambda_\xi = 0$ simplifies our expression for χ to:

$$\chi = \frac{1}{1-\eta} \cdot \frac{\gamma - \beta \cdot (\gamma - (1-(1-\eta) \cdot BE) \cdot \eta \cdot f(\theta)) \cdot (1 - G(\epsilon_s))}{m(\theta) \cdot u'(\tilde{c}^w)} \cdot \lambda_\theta$$
$$- \frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot (-\lambda_\epsilon)}{m(\theta) \cdot u'(\tilde{c}^w)}$$

Now let us set both expressions for χ equal so that we can pin down λ_θ.

$$\frac{1}{1-\eta} \cdot \frac{\gamma - \beta \cdot (\gamma - (1-(1-\eta) \cdot BE) \cdot \eta \cdot f(\theta)) \cdot (1 - G(\epsilon_s))}{m(\theta) \cdot u'(\tilde{c}^w)} \cdot \lambda_\theta =$$
$$+ \frac{\eta - \gamma}{1-\eta}$$
$$+ \frac{\beta}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))}$$
$$\cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right) \cdot \frac{b}{n} / \left(\frac{1}{1-\gamma} \cdot \frac{k_v}{q(\theta)}\right)$$
$$+ \frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot (-\lambda_\epsilon)}{m(\theta) \cdot u'(\tilde{c}^w)}$$

Rearranging the equation gives:

$$\frac{1}{1-\eta} \cdot \frac{\gamma - \beta \cdot (\gamma - (1-(1-\eta) \cdot BE) \cdot \eta \cdot f(\theta)) \cdot (1 - G(\epsilon_s))}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot k_v \cdot \lambda_\theta =$$
$$+ (1-\gamma) \cdot u \cdot q(\theta) \cdot \frac{\eta - \gamma}{(1-\gamma) \cdot (1-\eta)} \cdot \frac{k_v}{q(\theta)}$$
$$+ (1-\gamma) \cdot u \cdot q(\theta) \cdot \frac{\beta}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))}$$

$$
\begin{aligned}
&\cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right) \cdot \frac{b}{n} \\
&+ \frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot k_v}{f(\theta)} \cdot \frac{-\lambda_\epsilon}{u'(\tilde{c}^w)}
\end{aligned}
$$

We still have to calculate the Lagrange multiplier for the separation condition λ_ϵ from the formula for the deviation from the optimal separation condition:

$$
\begin{aligned}
-\lambda_\epsilon = &\left(1 + \frac{\beta \cdot (1 + \eta \cdot BE)}{n \cdot u'(\tilde{c}^w)} \cdot \lambda_\theta\right) \cdot \left(\frac{\beta \cdot (1 - f(\theta))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n}\right. \\
&\left. - s \cdot (\bar{h} - h_{stw}(\epsilon_s))\right) \cdot \frac{g(\epsilon_s) \cdot n \cdot u'(\tilde{c}^w)}{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f} \\
&- BE \cdot \lambda_\theta \cdot \frac{1 - f(\theta)}{1 - \eta} \cdot \frac{k_v}{q(\theta)} \cdot \frac{g(\epsilon_s)}{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}
\end{aligned}
$$

Inserting the Lagrange multiplier for separations into the formula above gives:

$$
\begin{aligned}
&\frac{1}{1-\eta} \cdot \frac{\gamma - \beta \cdot \left(\gamma - \left(1 - (1-\eta) \cdot BE\right) \cdot \eta \cdot f(\theta)\right) \cdot (1 - G(\epsilon_s))}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot k_v \cdot \lambda_\theta \\
&= f'(\theta) \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma) \cdot (1-\eta)} \cdot \frac{k_v}{q(\theta)} + \frac{\beta}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))}\right. \\
&\quad \left. \cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right) \cdot \frac{b}{n}\right) \\
&\quad + \left(-\frac{\partial \epsilon_s}{\partial \theta}\right) \cdot g(\epsilon_s) \cdot n \cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE)}{n \cdot u'(\tilde{c}^w)} \cdot \lambda_\theta\right) \\
&\quad \cdot \left(\frac{\beta \cdot (1 - f(\theta))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n} - s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right)\right) \\
&\quad - BE \cdot \lambda_\theta \cdot \frac{1 - f(\theta)}{1-\eta} \cdot \frac{k_v}{q(\theta)} \cdot \frac{g(\epsilon_s)}{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f} \cdot \frac{1}{1-\eta} \cdot \frac{\gamma - \eta \cdot f(\theta)}{f(\theta)} \cdot k_v
\end{aligned}
$$

with

$$
\begin{aligned}
f'(\theta) &= (1-\gamma) \cdot q(\theta) \\
\left(-\frac{\partial \epsilon_s}{\partial \theta}\right) &= \frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot k_v}{f(\theta)} \cdot \frac{1}{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}
\end{aligned}
$$

Note that according to the implicit function theorem it must hold:

$$
\left(-\frac{\partial \epsilon_s}{\partial \theta}\right) = \frac{\frac{\partial S_{stw}(\epsilon_s)}{\partial \theta}}{\frac{\partial S_{stw}(\epsilon_s)}{\partial \epsilon_s}} = \frac{\frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot k_v}{f(\theta)}}{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f} \tag{H.1}
$$

Next, solve for λ_θ:

$$\begin{aligned}\lambda_\theta = & \frac{u'(\tilde{c}^w)}{M} \cdot f'(\theta) \cdot u \\ & \cdot \left[\frac{\eta - \gamma}{(1-\eta)(1-\gamma)} \cdot \frac{k_v}{q(\theta)} + \frac{\beta}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n} \right] \\ & + \frac{u'(\tilde{c}^w)}{M} \cdot \left(- \frac{\partial \epsilon_s}{\partial \theta} \right) \cdot g(\epsilon_s) \cdot n \\ & \cdot \left[\frac{\beta \cdot (1 - f(\theta))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n} - s \cdot (\bar{h} - h_{stw}(\epsilon_s)) \right]\end{aligned}$$

I call M the inverse multiplier of the match:

$$\begin{aligned}M = & \underbrace{\frac{\gamma - \beta \cdot (\gamma - (1 - (1-\eta) \cdot BE) \cdot \eta \cdot f(\theta)) \cdot (1 - G(\epsilon_s)) + (\gamma - \eta \cdot f(\theta)) \cdot BE \cdot \frac{1 - f(\theta)}{1-\eta} \cdot \frac{k_v}{q(\theta)} \cdot \frac{g(\epsilon_s)}{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}}{(1-\eta) \cdot f(\theta)}}_{\text{regular multiplier effect}} \\ & - \underbrace{\frac{\beta \cdot (1 - G(\epsilon_s))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{\beta \cdot (1 + \eta \cdot BE)}{n} \cdot \frac{b}{n} \cdot f'(\theta) \cdot u}_{\text{tax effect via vacancy posting}} \\ & - \underbrace{\frac{\beta \cdot (1 - G(\epsilon_s))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{\beta \cdot (1 + \eta \cdot BE)}{n} \cdot \left(\frac{b}{n} - s \cdot (\bar{h} - h_{stw}(\epsilon_s)) \right) \cdot \left(- \frac{\partial \epsilon_s}{\partial \theta} \right) \cdot g(\epsilon_s) \cdot n}_{\text{tax effect via separations}}\end{aligned}$$

M captures the feedback effects that altering the value of the firm in one period has on the labor market tightness. The feedback effects can be divided into three channels, which can be illustrated by considering an increase in UI benefits. The first channel is the regular channel, where an increase in UI benefits depresses the joint surplus of firms and workers, not only in the present but also in the future.

The second channel emerges through the free-entry condition. As the joint surplus declines, firms reduce vacancy postings. This reduction leads to higher unemployment, which, in turn, increases the fiscal burden of the UI system. To cover these additional costs, higher taxes are required, which further depresses the joint surplus, thereby amplifying the initial impact.

Finally, the third channel arises through the separation condition. A decrease in the joint surplus increases the separation rate, as the continuation value of the match between firms and workers diminishes. More separations result in higher unemployment, which once again raises the costs of the UI system, leading to the need for higher taxes. This creates a reinforcing loop that further reduces the joint surplus of firms and workers.

It's important to note that the last effect is mitigated by the STW system. Larger STW benefits reduce separation incentives, thereby lowering the number of workers entering the UI system. As a result, the increase in unemployment is smaller, which in turn lessens the distortionary tax effect caused by the UI system. STW itself has a neutral impact on the joint surplus of firms and workers. On the one hand, an increase in STW benefits raises the joint surplus through the subsidy effect. On the other hand, this increase is offset by the need to raise taxes to finance the system, which decreases the joint surplus. Ultimately, these two effects cancel each other out.

In the following, I require the inverse multiplier to be positive, $M > 0$. Otherwise, the model would not converge to its steady state. We can abbreviate the inverse multiplier by recognizing that it captures the general equilibrium effect that a change in the surplus or value of the firm has on the labor market tightness:

$$\frac{\partial \theta^{ge}}{\partial S} = \frac{1}{M} \quad \text{or} \quad \frac{\partial \theta^{ge}}{\partial \mathcal{J}} = \frac{1}{1-\eta} \cdot \frac{1}{M}$$

We can rewrite the formula for the Lagrange multiplier as:

$$\begin{aligned}\lambda_\theta = & \frac{u'(\tilde{c}^w)}{1-\eta} \cdot \frac{\partial \theta^{ge}}{\partial \mathcal{J}} \cdot f'(\theta) \cdot u \\ & \cdot \left[\frac{\eta - \gamma}{(1-\eta)\cdot(1-\gamma)} \cdot \frac{k_v}{q(\theta)} + \frac{\beta}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))} \cdot \frac{b}{n} \right] \\ & + \frac{u'(\tilde{c}^w)}{1-\eta} \cdot \frac{\partial \theta^{ge}}{\partial \mathcal{J}} \cdot \left(-\frac{\partial \epsilon_s}{\partial \theta} \right) \cdot g(\epsilon_s) \cdot n \\ & \cdot \left[\frac{\beta\cdot(1-f(\theta))}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))} \cdot \frac{b}{n} - s\cdot(\bar{h} - h_{stw}(\epsilon_s)) \right]\end{aligned}$$

Remember that the optimal UI benefits can be characterized by:

$$\begin{aligned}(1-n)\cdot & (u'(b) - u'(\tilde{c}^w)) = \\ & \lambda_\theta \cdot \beta \cdot \left[\left(\frac{u'(b)}{u'(\tilde{c}^w)} + \frac{1-n}{n} \right) - BE \cdot \left((1-\eta)\cdot \frac{u'(b)}{u'(\tilde{c}^w)} - \eta \cdot \frac{1-n}{n} \right) \right]\end{aligned}$$

Let L_V and L_S denote the social value of an additional hire and from retaining the marginal match:

$$\begin{aligned}L_V &= \frac{\eta - \gamma}{(1-\eta)\cdot(1-\gamma)} \cdot \frac{k_v}{q(\theta)} + \frac{\beta}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))} \cdot \frac{b}{n} \\ L_S &= \frac{\beta\cdot(1-f(\theta))}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))} \cdot \frac{b}{n} - s\cdot(\bar{h} - h_{stw}(\epsilon_s))\end{aligned}$$

Further, note that most of the right-hand side of the equation for optimal UI benefits denote the first derivative of the value of the firm to an increase in UI benefits:

$$-\frac{\partial \mathcal{J}}{\partial b} = \beta \cdot (1-\eta) \cdot \left[\left(\frac{u'(b)}{u'(\tilde{c}^w)} + \frac{1-n}{n} \right) - BE \cdot \left((1-\eta)\cdot \frac{u'(b)}{u'(\tilde{c}^w)} - \eta \cdot \frac{1-n}{n} \right) \right]$$

Inserting λ_θ into the formula for optimal UI benefits, we can express the equation for the optimal UI benefits as:

$$
\begin{aligned}
(1-n)\cdot\frac{u'(b)-u'(\tilde{c}^w)}{u'(\tilde{c}^w)} &= \\
& L_V\cdot\left(-\frac{\partial\mathcal{J}}{\partial b}\right)\cdot\frac{\partial\theta^{ge}}{\partial\mathcal{J}}\cdot f'(\theta)\cdot u + L_S\cdot\left(-\frac{\partial\mathcal{J}}{\partial b}\right)\cdot\frac{\partial\theta^{ge}}{\partial\mathcal{J}}\cdot\left(-\frac{\partial\epsilon_s}{\partial\theta}\right)\cdot g(\epsilon_s)\cdot n
\end{aligned}
$$

Or simplified as:

$$
(1-n)\cdot\frac{u'(b)-u'(\tilde{c}^w)}{u'(\tilde{c}^w)} = L_V\cdot\left(-\frac{\partial f^{ge}}{\partial b}\right)\cdot u + L_S\cdot\frac{\partial\epsilon_s^{ge}}{\partial b}\cdot g(\epsilon_s)\cdot n
$$

H.4 Optimal STW Benefits - Steady State

To calculate the optimal STW benefits, we first derive the the FOC for STW benefits:

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial s_t} &= -\int_{\epsilon_{s,t}}^{\epsilon_{stw,t}}\lambda_{h_{stw},t}(\epsilon)dG(\epsilon) \\
&\quad -\lambda_{\epsilon,t}\cdot\Bigg[\left(\bar{h}-h_{stw,t}(\epsilon_{s,t})\right) \\
&\qquad +\frac{\partial h_{stw,t}(\epsilon_{s,t})}{\partial s_t}\cdot\underbrace{\left(\alpha\cdot a_t\cdot\epsilon_{s,t}\cdot h_{stw,t}(\epsilon_{s,t})^{\alpha-1}-v'(h_{stw,t}(\epsilon_{s,t}))-s_t\right)}_{=0}\Bigg] \\
&= 0
\end{aligned}
$$

Note that the FOC depends on the Lagrange multiplier for working hours $\lambda_{h_{stw},t}(\epsilon)$. To calculate that, we derive the FOC of working time on STW:

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial h_{stw,t}(\epsilon)} &= -\left(n_t\cdot\underbrace{\frac{\nu_t^f}{\nu_t^f}\cdot\tilde{u}(c_t^f)}_{\tilde{u}'(\tilde{c}_t^w)}+\lambda_{\theta,t-1}+\eta\cdot\lambda_{c,t}\right)\cdot\frac{\partial\Omega_t}{\partial h_{stw,t}(\epsilon)} \\
&\quad -\lambda_{h_{stw},t}(\epsilon)\cdot\left(\alpha\cdot(\alpha-1)\cdot a_t\cdot\epsilon\cdot h_{stw,t}(\epsilon)^{\alpha-2}-v''(h_{stw,t}(\epsilon))\right)=0
\end{aligned}
$$

Rearranging gives:

$$
\lambda_{h_{stw},t}(\epsilon) = -n_t\cdot u'(\tilde{c}_t^w)\cdot\frac{\left(1+\frac{(1+\eta\cdot BE)\cdot\lambda_{\theta,t-1}}{n_t\cdot u'(\tilde{c}_t^w)}\right)}{(\alpha-1)\cdot\alpha\cdot a_t\cdot\epsilon\cdot h_{stw.t}(\epsilon)^{\alpha-2}-v''(h_{stw,t}(\epsilon))}\cdot\frac{\partial\Omega_t}{\partial h_{stw,t}(\epsilon)}
$$

Next, we can insert the Lagrange multiplier from the optimal hours choice on STW into the FOC for the optimal STW benefits:

$$\begin{aligned}\Leftrightarrow -n_t \cdot u'(\tilde{c}_t^w) \cdot \left(1 + \frac{(1+\eta \cdot BE) \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)}\right) \cdot \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \frac{\partial \Omega_t}{\partial h_{stw,t}(\epsilon)} \cdot \frac{h_{stw,t}(\epsilon)}{s_t} dG(\epsilon) \\ - \lambda_{\epsilon,t} \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right) = 0\end{aligned}$$

Here, the first derivative of working hours for STW benefits can be denoted as:

$$\frac{h_{stw,t}(\epsilon)}{s_t} = \frac{1}{(\alpha-1) \cdot \alpha \cdot a_t \cdot \epsilon \cdot h_{stw,t}(\epsilon)^{\alpha-2} - v''(h_{stw,t}(\epsilon))} < 0$$

Rearranging gives us the Lagrange multiplier for the separation condition.

$$\Leftrightarrow \frac{\lambda_{\epsilon,t}}{n_t \cdot u'(\tilde{c}_t^w)} = -\frac{1 + \frac{\beta \cdot (1+\eta \cdot BE_t) \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)}}{\bar{h} - h_{stw,t}(\epsilon_{s,t})} \cdot \frac{\partial \Omega_t}{\partial s_t}$$

The planner decides to set the STW benefits so that they do not prevent all inefficient separations to save on the costs of hours distortion. A negative λ_ϵ implies that not all matches worth saving were retained.

Next, insert the Lagrange multiplier into the deviation from the constrained efficient separation condition and rearrange for the optimal STW subsidy:

$$\begin{aligned}\Leftrightarrow \quad & s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right) \\ & = \frac{\beta \cdot (1-f(\theta))}{1 - \beta \cdot (1-f(\theta)) \cdot (1-G(\epsilon_s))} \cdot \left(1 + \frac{\beta \cdot (1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right) \cdot \frac{b}{n} \\ & - \frac{(1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right) \\ & - \frac{1}{g(\epsilon_s)} \cdot \left(1 + \frac{\beta \cdot (1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right) \cdot \frac{\partial \Omega}{\partial s} \cdot \frac{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}{\bar{h} - h_{stw}(\epsilon_s)} \\ & - \frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot \frac{1-f(\theta)}{1-\eta} \cdot \frac{k_v}{q(\theta)}\end{aligned}$$

Note that STW benefits reduce separation incentives by keeping unproductive workers employed, reducing the surplus of a match. At the same time, keeping these workers employed reduces the fiscal costs of the UI system, reduces taxes and thus increases the joint surplus. We can see this in the following equation by the term $\left(1 + \frac{\beta \cdot (1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right)$.

$$\Leftrightarrow \quad \left(1 + \frac{\beta \cdot (1+\eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right) \cdot s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right)$$

$$= \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right) \cdot \left(\frac{\beta \cdot (1 - f(\theta))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n} - \frac{1}{g(\epsilon_s)} \cdot \frac{\partial \Omega}{\partial s} \cdot \frac{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}{\bar{h} - h_{stw}(\epsilon_s)}\right) - \frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot \frac{1 - f(\theta)}{1 - \eta} \cdot \frac{k_v}{q(\theta)}$$

Both effects are equally strong, and the term crosses out, affecting only the bargaining effect:

$$\Leftrightarrow \; s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right) = \frac{\beta \cdot (1 - f(\theta))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n} - \frac{1}{g(\epsilon_s)} \cdot \frac{\partial \Omega}{\partial s} \cdot \frac{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}{\bar{h} - h_{stw}(\epsilon_s)} - \frac{\frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}}{1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}} \cdot \frac{1 - f(\theta)}{1 - \eta} \cdot \frac{k_v}{q(\theta)}$$

Again, note that according to the implicit function theorem, it must hold:

$$\frac{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}{\bar{h} - h_{stw}(\epsilon)} = \frac{\frac{\partial S_{stw}(\epsilon_s)}{\partial \epsilon_s}}{\frac{\partial S_{stw}(\epsilon_s)}{\partial s}} = -\frac{1}{-\frac{\frac{\partial S_{stw}(\epsilon_s)}{\partial s}}{\frac{\partial S_{stw}(\epsilon_s)}{\epsilon_s}}} = -\frac{1}{\frac{\partial \epsilon_s}{\partial s}} > 0$$

and define the bargaining effect as:

$$\tilde{BE} = \frac{\frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}}{1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}} \cdot \frac{1 - f(\theta)}{1 - \eta} \cdot \frac{k_v}{q(\theta)}$$

Then we get the formula for the optimal STW benefits as in Proposition 2:

$$s \cdot \left(\bar{h} - h_{stw}(\epsilon_s)\right) = \frac{\beta \cdot (1 - f(\theta))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n} - \frac{1}{g(\epsilon_s)} \cdot \frac{\partial \Omega}{\partial s} \cdot \left[-\frac{1}{\frac{\partial \epsilon_s}{\partial s}}\right] - \tilde{BE}$$

H.5 Optimal Eligibility Condition Steady State

This section derives the optimal eligibility condition. It aims to show, using proof by contradiction, that $\epsilon_{stw,t} = \xi_{s,t}$ as long as the sufficient condition of Proposition 3 holds:

$$\underbrace{n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}}_{\text{More hours distortion}} + L_V \cdot \underbrace{\left(-\frac{\partial f^{ge}}{\partial \epsilon_{stw}} \cdot u\right)}_{\text{Less hiring}} + L_S^\star \cdot \underbrace{\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial \epsilon_{stw}}\right)}_{\text{More Separations}} + L_\xi^\star \cdot \underbrace{\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \xi_s^{ge}}{\partial \epsilon_{stw}}\right)}_{\text{More Workers need STW}} > 0$$

The section will show that each possible value of $\epsilon_{stw,t}$, except for $\epsilon_{stw,t} = \xi_{s,t}$ itself, cannot constitute an optimum.

case 0: $\epsilon_{stw,t} < \epsilon_{s,t}$

Case 0 examines the case where the participation threshold is stricter than the separation threshold of a firm with access to STW. As a result, STW does not exist in this economy. By excluding any $\epsilon_{stw,t} < \epsilon_{s,t}$ from the optimization problem of the Ramsey planner, we do not restrict the choice set of the Ramsey planner. The Ramsey planner can always choose $\epsilon_{stw,t} = \epsilon_{s,t}$ or $s_t = 0$ to eliminate the STW system from the economy.

case 1: $\epsilon_{s,t} \leq \epsilon_{stw,t} < \xi_{s,t}$;

Case 1 describes the situation where the eligibility condition is so strict that matches with $\epsilon_p \in (\epsilon_{stw,t}, \xi_{s,t})$ are not allowed to go onto the STW system and subsequently dissolve. At the same time, matches with $\epsilon_u \in [\epsilon_{s,t}, \epsilon_{stw,t}]$ are allowed to enter the STW system and are rescued. The paragraph wants to show that it cannot be optimal to rescue less productive matches while letting more productive matches dissolve $\epsilon_u < \epsilon_p$. To verify the claim, it has to be shown that the Ramsey planner would always want to loosen the participation threshold as long as we are in case 1. The Ramsey planner always wants to loosen the eligibility condition if the following condition is always met:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{stw,t}} \overset{!}{>} 0$$

Showing this would mean that no $\epsilon_{stw,t} \in [\epsilon_{s,t}, \xi_{s,t})$ is optimal. So we start by calculating the first-order derivative of the Lagrange problem:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{stw,t}} = n_t \cdot \underbrace{\frac{\nu_t^f}{\nu_t^f} \cdot \tilde{u}'(c_t^F)}_{u'(\tilde{c}_t^w)} \cdot g(\epsilon_{stw,t}) \cdot \Big(y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F + \theta_t \cdot k_v\Big)$$

$$
\begin{aligned}
&+ n_t \cdot g(\epsilon_{stw,t}) \cdot (1 - f(\theta_t)) \cdot E_t\left[\lambda_{n,t+1}\right] \\
&+ \lambda_{\theta,t-1} \cdot g(\epsilon_{stw,t}) \cdot \beta \cdot \Big(y_t(\epsilon, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F \\
&\quad + \frac{1 - f(\theta_t) \cdot \eta_t}{1 - \eta_t} \cdot \frac{k_v}{q(\theta_t)} \Big) \\
&+ \lambda_{c,t} \cdot g(\epsilon_{stw,t}) \cdot \Big(y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F + \theta_t \cdot k_v \Big) \overset{!}{>} 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F + \theta_t \cdot k_v + (1 - f(\theta_t)) \cdot \frac{E_t\left[\lambda_{n,t+1}\right]}{u'(\tilde{c}_t)} \\
&+ \frac{\lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \beta \cdot \Big(y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F \\
&\quad + \frac{1 - f(\theta_t) \cdot \eta_t}{1 - \eta_t} \cdot \frac{k_v}{q(\theta_t)} \Big) \\
&+ \frac{\lambda_{c,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \Big(y_t(\epsilon, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F + \theta_t \cdot k_v \Big) \overset{!}{>} 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F + \theta_t \cdot k_v + (1 - f(\theta_t)) \cdot \frac{E_t\left[\lambda_{n,t+1}\right]}{u'(\tilde{c}_t^w)} \\
&+ \left(\frac{\beta \cdot (1 + \eta_{t-1} \cdot BE_t) \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)} \right) \cdot \Big(y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F \\
&\quad + \frac{1 - f(\theta_t) \cdot \eta_t}{1 - \eta_t} \cdot \frac{k_v}{q(\theta_t)} \Big) \\
&- \frac{\beta \cdot BE_t \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{1 - f(\theta_t)}{1 - \eta_t} \cdot \frac{k_v}{q(\theta_t)} \overset{!}{>} 0
\end{aligned}
$$

Next, we insert the Lagrange multiplier for the law of motion of employment $E_t[\lambda_{n,t+1}]$:

$$
\begin{aligned}
& y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F + \theta_t \cdot k_v \\
&+ (1 - f(\theta_t)) \cdot \frac{1 + \chi_t}{1 - \gamma} \cdot \frac{k_v}{q(\theta_t)} \\
&+ \left(\frac{\beta \cdot (1 + \eta_{t-1} \cdot BE_t) \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t)} \right) \cdot \Big(y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F \\
&\quad + \frac{1 - f(\theta_t) \cdot \eta_t}{1 - \eta_t} \cdot \frac{k_v}{q(\theta_t)} \Big) \\
&- \frac{\beta \cdot BE_t \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t)} \cdot \frac{1 - f(\theta_t)}{1 - \eta_t} \cdot \frac{k_v}{q(\theta_t)} = 0
\end{aligned}
$$

$$
\Leftrightarrow \quad y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F
$$

$$
\begin{aligned}
&+ \frac{(1-\gamma\cdot f(\theta_t)) + (1-f(\theta_t))\cdot \chi_t}{1-\gamma}\cdot\frac{k_v}{q(\theta_t)}\\
&+ \left(\frac{\beta\cdot(1+\eta_{t-1}\cdot BE_t)\cdot\lambda_{\theta,t-1}}{n_t\cdot u'(\tilde{c}_t)}\right)\cdot\Big(y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) + F\\
&\quad + \frac{1-f(\theta_t)\cdot\eta_t}{1-\eta_t}\cdot\frac{k_v}{q(\theta_t)}\Big)\\
&- \frac{\beta\cdot BE_t\cdot\lambda_{\theta,t-1}}{n_t\cdot u'(\tilde{c}_t)}\cdot\frac{1-f(\theta_t)}{1-\eta_t}\cdot\frac{k_v}{q(\theta_t)} = 0
\end{aligned}
$$

To reduce the notational burden, let us denote the difference in the surplus between a match at productivity level $\epsilon_{stw,t}$ and a match at the separation threshold without STW $\epsilon_{s,t}$ as in the main text:

$$
\begin{aligned}
y_t(\epsilon_{stw,t}, h_{stw,t}(\epsilon_{stw,t})) - v(h_{stw,t}(\epsilon_{stw,t})) - y_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + v(h_{stw,t}(\epsilon_{s,t}))&\\
= z_{stw,t}(\epsilon_{stw,t}) - z_{stw,t}(\epsilon_{s,t}) > 0&
\end{aligned}
$$

The surplus of a match with higher productivity $\epsilon_{stw,t} > \epsilon_{s,t}$ is, of course, larger than a match with smaller productivity. Next, we subtract the socially optimal separation condition on STW from the expression above. We can interpret the operation as deducting the social value of rescuing a match with productivity level $\epsilon_{s,t}$ from the social value of rescuing a match with productivity level $\epsilon_{s,t}$:

$$
\begin{aligned}
&\left(1+\frac{\beta\cdot(1+\eta_{t-1}\cdot BE_t)\cdot\lambda_{\theta,t-1}}{n_t\cdot u'(\tilde{c}^w_t)}\right)\cdot\left(z_{stw,t}(\epsilon_{stw,t}) - z_{stw,t}(\epsilon_{s,t})\right)\\
&- \frac{1}{g(\epsilon_{s,t})}\cdot\frac{\lambda_\epsilon}{n\cdot u'(\tilde{c}^w_t)}\cdot\left(a_t\cdot h_{stw,t}(\epsilon_{s,t})^\alpha + c_f\right) \overset{!}{>} 0
\end{aligned}
$$

From the FOC of the eligibility condition, we get the Lagrange multiplier for the decentralized separation condition. Inserting into the equation above gives:

$$
\begin{aligned}
&\left(1+\frac{\beta\cdot(1+\eta_{t-1}\cdot BE_t)\cdot\lambda_{\theta,t-1}}{n_t\cdot u'(\tilde{c}^w_t)}\right)\cdot\left(z_{stw,t}(\epsilon_{stw,t}) - z_{stw,t}(\epsilon_{s,t})\right)\\
&+ \frac{1}{g(\epsilon_{s,t})}\cdot\left(1+\frac{\beta\cdot(1+\eta\cdot BE_t)\cdot\lambda_{\theta,t-1}}{n_t\cdot u'(\tilde{c}^w_t)}\right)\cdot\frac{\partial\Omega_t}{\partial s_t}\cdot\frac{a_t\cdot h_{stw,t}(\epsilon_{s,t})^\alpha + c_f}{\bar{h} - h_{stw,t}(\epsilon_{s,t})} \overset{!}{>} 0
\end{aligned}
$$

Rearranging, we see that the Ramsey planner always wants to choose a looser eligibility condition

$$
\left(1+\frac{\beta\cdot(1+\eta_{t-1}\cdot BE_t)\cdot\lambda_{\theta,t-1}}{n_t\cdot u'(\tilde{c}^w_t)}\right)\cdot\Bigg(z_{stw,t}(\epsilon_{stw,t}) - z_{stw,t}(\epsilon_{s,t})
$$

$$+\frac{1}{g(\epsilon_{s,t})}\cdot\frac{\partial\Omega_t}{\partial s_t}\cdot\frac{a_t\cdot h_{stw,t(\epsilon_{s,t})^\alpha+c_f}}{\bar{h}-h_{stw,t}(\epsilon_{s,t})}\Bigg)>0$$

as long as:

$$\left(1+\frac{\beta\cdot(1+\eta_{t-1}\cdot BE_t)\cdot\lambda_{\theta,t-1}}{n_t\cdot u'(\tilde{c}_t^w)}\right)\overset{!}{>}0$$

Multiplying both sides of the equation with $\frac{\partial\Omega}{\partial\epsilon_{stw}}$ gives us an expression that resembles the sufficient condition:

$$n\cdot u'(\tilde{c}^w)\cdot\frac{\partial\Omega}{\partial\epsilon_{stw}}+(1+\eta\cdot BE)\cdot\lambda_\theta\cdot\frac{\partial\Omega}{\partial\epsilon_{stw}}\overset{!}{>}0$$

However, we need to determine λ_θ. Remember the expression for χ :

$$\begin{aligned}\chi=&\frac{1}{1-\eta}\cdot\frac{\gamma-\beta\cdot(\gamma-(1-(1-\eta)\cdot BE)\cdot\eta\cdot f(\theta))\cdot(1-G(\epsilon_s))}{m(\theta)\cdot u'(\tilde{c}^w)}\cdot\lambda_\theta\\&-\frac{1}{1-\eta}\cdot\frac{(\gamma-\eta\cdot f(\theta))\cdot(-\lambda_\epsilon)}{m(\theta)\cdot u'(\tilde{c}^w)}\\&-\frac{1}{1-\eta}\cdot\frac{(\gamma-\eta\cdot f(\theta))\cdot(-\lambda_\xi)}{m(\theta)\cdot u'(\tilde{c}^w)}\end{aligned}$$

We know λ_ϵ but we still need to determine λ_ξ to determine λ_θ. To do so, we have to derive the FOC of λ_ξ:

$$\begin{aligned}-\frac{\partial\mathcal{L}}{\partial\xi_{s,t}}=\;&n_t\cdot g(\xi_{s,t})\cdot[y_t(\xi_{s,t},h_t(\xi_{s,t}))-v(h_t(\xi_{s,t}))+F+k_v\cdot\theta_t]\cdot\underbrace{\Lambda_t\cdot\tilde{u}'(c_t^F)}_{u'(\tilde{c}_t^w)}\\&+n_t\cdot g(\xi_{s,t})\cdot(1-f(\theta_t))\cdot E_t\left[\lambda_{n,t+1}\right]\\&+\lambda_{\theta,t-1}\cdot g(\xi_{s,t})\cdot\beta\cdot\left[y_t(\xi_{s,t},h_t(\xi_{s,t}))-v(h_t(\xi_{s,t}))+F+\frac{1-f(\theta_t)\cdot\eta}{1-\eta}\cdot\frac{k_v}{q(\theta_t)}\right]\\&+\lambda_{c,t}\cdot g(\xi_{s,t})\cdot[y_t(\xi_{s,t},h_t(\xi_{s,t}))-v(h_t(\xi_{s,t}))+F+\theta_t\cdot k_v]\\&+\lambda_{\xi,t}\cdot\Bigg[a_t\cdot h_t(\xi_{s,t})^\alpha+c_f\\&\quad+\frac{\partial h_t(\xi_{s,t})}{\partial\xi_{s,t}}\cdot\underbrace{\left(\alpha\cdot a_t\cdot\xi_{s,t}\cdot h_{stw,t}(\xi_{s,t})^{\alpha-1}-v'(h_{stw,t}(\xi_{s,t}))-s_t\right)}_{=0}\Bigg]=0\end{aligned}$$

Rearranging gives the social value for rescuing matches with productivity $\xi_{s,t}$.

$$y_t(\xi_{s,t},h_{stw,t}(\xi_{s,t}))-v(h_{stw,t}(\xi_{s,t}))+F+k_v\cdot\theta_t+(1-f(\theta_t))\cdot\frac{E_t\left[\lambda_{n,t+1}\right]}{u'(\tilde{c}_t^w)}$$

$$
\begin{aligned}
&- \frac{\beta \cdot (1 + \eta \cdot BE_t) \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \Bigg[y_t(\xi_{s,t}, h_t(\xi_{s,t})) - v(h_t(\xi_{s,t})) + F \\
&\qquad + \frac{1 - f(\theta_t) \cdot \eta}{1 - \eta} \cdot \frac{k_v}{q(\theta_t)} \Bigg] \\
&- \frac{\beta \cdot \eta \cdot BE_t \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{1 - f(\theta_t)}{1 - \eta} \cdot \frac{k_v}{q(\theta_t)} \\
= &- \frac{1}{g(\xi_{s,t})} \cdot \frac{\lambda_{\xi,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot (a_t \cdot h_t(\xi_{s,t})^\alpha + c_f)
\end{aligned}
$$

We can express the social value of rescuing a worker with productivity $\xi_{s,t}$ conditional on the social value of rescuing the marginal matches $\xi_{s,t}$:

$$
\begin{aligned}
&- \frac{1}{g(\xi_{s,t})} \cdot \frac{\lambda_{\xi,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot (a_t \cdot h_t(\xi_{s,t})^\alpha + c_f) = \\
&\quad - \frac{1}{g(\epsilon_{s,t})} \cdot \frac{\lambda_{\epsilon,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot (a_t \cdot h_{stw,t}(\epsilon_{s,t})^\alpha + c_f) \\
&\quad + \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE_t) \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)}\right) \cdot (z_t(\xi_{s,t}) - z_{stw,t}(\epsilon_{s,t}))
\end{aligned}
$$

More productive matches must be socially more valuable than less productive matches. We can see this by the fact: $(z_t(\xi_{s,t}) - z_{stw,t}(\epsilon_{s,t})) > 0$. Inserting the Lagrange multiplier for the separation threshold with STW gives:

$$
\begin{aligned}
&- \frac{1}{g(\xi_{s,t})} \cdot \frac{\lambda_{\xi,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot (a_t \cdot h_t(\xi_{s,t})^\alpha + c_f) = \\
&+ \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE_t) \cdot \lambda_{\theta,t-1}}{n_t \cdot u'(\tilde{c}_t^w)}\right) \cdot \Bigg(\frac{1}{g(\epsilon_s)} \cdot \frac{\partial \Omega}{\partial s} \cdot \frac{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}{\bar{h} - h_{stw}(\epsilon_s)} \\
&\quad + (z_t(\xi_{s,t}) - z_{stw,t}(\epsilon_{s,t})) \Bigg)
\end{aligned}
$$

Using the expression for $\lambda_{\xi,t}$ we can use the same procedure as in Section H.3 to derive the Lagrange multiplier λ_θ and get the following statement:

$$
\begin{aligned}
&\underbrace{n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}}_{\text{More hours distortion}} + L_V \cdot \underbrace{\left(-\frac{\partial f^{ge}}{\partial \epsilon_{stw}} \cdot u\right)}_{\text{Less hiring}} + L_S^\star \cdot \underbrace{\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial \epsilon_{stw}}\right)}_{\text{More Separations}} \\
&+ \underbrace{(L_S^\star + z_t(\xi_{s,t}) - z_{stw,t}(\epsilon_{s,t})) \cdot \left(n \cdot g(\xi_s) \cdot \frac{\partial \xi_s^{ge}}{\partial \epsilon_{stw}}\right)}_{\text{Loose additional workers that could have been employed}} > 0
\end{aligned}
$$

With $L_\xi = L_S^\star + z_t(\xi_{s,t}) - z_{stw,t}(\epsilon_{s,t})$ we recover the assumption made in the proposition, proving the argument.

case 2: $\xi_{s,t} < \epsilon_{stw,t} < \xi_{stw,t}$ Assume now that we choose a looser eligibility condition. The first derivative for the eligibility condition is:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{stw,t}} = -\underbrace{\frac{\nu_t^f}{\nu_t^f} \cdot \tilde{u}'(c_t^f)}_{u'(\tilde{c}_t^w)} \cdot n_t \cdot \frac{\partial \Omega_t}{\partial \epsilon_{stw,t}} - \beta \cdot (1 + \eta \cdot BE_t) \cdot \lambda_{\theta,t-1} \cdot \frac{\partial \Omega_t}{\partial \epsilon_{stw,t}}$$

If we can show that

$$\tilde{u}'(\tilde{c}^w) \cdot n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} + \beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} > 0$$

then it must always be better to set a stricter eligibility condition. This is practically the sufficient condition of Proposition 3. To show this, we have to calculate the Lagrange multiplier for the job-creation condition again. Note that we can show that the Lagrange multiplier for the separation threshold for STW is zero:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \xi_{s,t}} &= -\lambda_{\xi,t} \cdot \left(a_t \cdot h_t^\alpha(\xi_{s,t}) + c_f + \frac{\partial h_t(\xi_{s,t})}{\partial \xi_{s,t}} \cdot \underbrace{\left(\alpha \cdot a_t \cdot \xi_{s,t} \cdot h_t(\xi_{s,t})^{\alpha-1} - v'(h_t(\xi_{s,t}))\right)}_{=0} \right) \\
&= 0 \\
&\Leftrightarrow \lambda_{\xi,t} = 0
\end{aligned}$$

Any increase in the separation condition without STW does not affect separations. $\lambda_\xi = 0$ simplifies the expression for χ to:

$$\begin{aligned}
\chi = &\frac{1}{1-\eta} \cdot \frac{\gamma - \beta \cdot (\gamma - (1 - (1-\eta) \cdot BE) \cdot \eta \cdot f(\theta)) \cdot (1 - G(\epsilon_s))}{m(\theta) \cdot u'(\tilde{c}^w)} \cdot \lambda_\theta \\
&- \frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot (-\lambda_\epsilon)}{m(\theta) \cdot u'(\tilde{c}^w)}
\end{aligned}$$

Using the same steps as in Section H.3, we can derive the Lagrange multiplier λ_θ and show that if

$$\underbrace{n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}}_{\text{More hours distortion}} + L_V \cdot \underbrace{\left(-\frac{\partial f^{ge}}{\partial \epsilon_{stw}} \cdot u \right)}_{\text{Less hiring}} + L_S^\star \cdot \underbrace{\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial \epsilon_{stw}} \right)}_{\text{More Separations}} > 0$$

then the Ramsey planner always wants to set a stricter eligibility condition, which is the sufficient condition from Proposition 3. Note that from $\lambda_\xi = 0$ it follows that $L_\xi = 0$.

case 3: $\xi_{stw,t} \leq \epsilon_{stw,t}$

Case 3 describes the situation in which the participation threshold is so loose that it is no longer binding. Firms and workers decide when to enter the STW system. A looser eligibility condition does not have an impact on the economy anymore. However, in case 2, we have seen that a stricter eligibility condition will reduce welfare losses from the use of STW. As a result, case 3 cannot be optimal.

H.6 Optimal UI Considering Optimal STW in Steady State

The FOC for the UI benefits for the UI system with optimal STW is the same as for the UI system given UI:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial b_t} &= (1-n_t)\cdot(u'(b_t)-u'(\tilde{c}_t^w))\\ &\quad -\lambda_{\theta,t}\cdot\beta\cdot\left[\left(\frac{u'(b_t)}{u'(\tilde{c}_t^w)}+\frac{1-n_t}{n_t}\right)-BE_t\cdot\left((1-\eta)\cdot\frac{u'(b_t)}{u'(\tilde{c}_t^w)}-\eta\cdot\frac{1-n_t}{n_t}\right)\right]\end{aligned}$$

What changes again is the definition for λ_θ. To calculate λ_θ consider first how we calculate χ:

$$\begin{aligned}\chi\cdot\left(\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)}\right) &= \frac{\eta-\gamma}{1-\eta}\cdot\left(\frac{1}{1-\gamma}\cdot\frac{k_v}{q(\theta)}\right)\\ &\quad+\frac{\beta}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))}\cdot\left(1+\frac{\beta\cdot(1+\eta\cdot BE)\cdot\lambda_\theta}{n\cdot u'(\tilde{c}^w)}\right)\cdot\frac{b}{n}\end{aligned}$$

With an optimal set STW system, χ can be denoted as:

$$\begin{aligned}\chi &= \frac{1}{1-\eta}\cdot\frac{\gamma-\beta\cdot(\gamma-(1-(1-\eta)\cdot BE)\cdot\eta\cdot f(\theta))\cdot(1-G(\epsilon_s))}{m(\theta)\cdot u'(\tilde{c}^w)}\cdot\lambda_\theta\\ &\quad-\frac{1}{1-\eta}\cdot\frac{(\gamma-\eta\cdot f(\theta))\cdot(-\lambda_\epsilon)}{m(\theta)\cdot u'(\tilde{c}^w)}\\ &\quad-\frac{1}{1-\eta}\cdot\frac{(\gamma-\eta\cdot f(\theta))\cdot(-\lambda_{stw})}{m(\theta)\cdot u'(\tilde{c}^w)}\end{aligned}$$

The Lagrange multiplier for the decentralized separation condition in the steady state is:

$$\frac{\lambda_\epsilon}{n\cdot u'(\tilde{c}^w)} = -\frac{1+\frac{\beta\cdot(1+\eta\cdot BE)\cdot\lambda_\theta}{n\cdot u'(\tilde{c}^w)}}{\bar{h}-h_{stw}(\epsilon_s)}\cdot\frac{\partial\Omega}{\partial s}$$

Further, we need to calculate the Lagrange multiplier for the optimal separation condition $\xi_{s,t} = \epsilon_{stw,t}$. To do that, consider the FOC for the participation threshold:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{stw,t}} &= -\underbrace{\Lambda_t \cdot \tilde{u}(c_t^F)}_{u'(\tilde{c}_t^w)} \cdot n_t \cdot \frac{\partial \Omega_t}{\partial \epsilon_{stw,t}} - \beta \cdot (1 + \eta \cdot BE_t) \cdot \lambda_{\theta,t-1} \cdot \frac{\partial \Omega_t}{\partial \epsilon_{stw,t}} \\
&- \lambda_{stw,t} \cdot \left[a_t \cdot h_t(\epsilon_{s,t})^\alpha + c_f + \frac{\partial h_t(\epsilon_{s,t})}{\partial \epsilon_{s,t}} \cdot \underbrace{\left(\alpha \cdot a_t \cdot \epsilon_{s,t} \cdot h_t(\epsilon_{s,t})^{\alpha-1} - v'(h_t(\epsilon_{s,t})) \right)}_{=0} \right] \\
&= 0
\end{aligned}$$

Rearranging gives an expression for the FOC of the Lagrange multiplier:

$$\begin{aligned}
\lambda_{stw} &= -\frac{n \cdot u'(\tilde{c}^w) + \beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{a \cdot h(\epsilon_{stw})^\alpha + c_f} \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} \\
&= -n \cdot u'(\tilde{c}^w) \cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE)}{n \cdot u'(\tilde{c}^w)} \cdot \lambda_\theta \right) \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} / (a \cdot h(\epsilon_{stw})^\alpha + c_f)
\end{aligned}$$

Using our expression for ξ_s and inserting it into the expression for the deviation of the private and social value of the match gives:

$$\begin{aligned}
&\frac{1}{(1-\eta)} \cdot \frac{\gamma - \beta \cdot (\gamma - (1 - (1-\eta) \cdot BE) \cdot \eta \cdot f(\theta)) \cdot (1 - G(\epsilon_s))}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot \lambda_\theta = \\
&+ f'(\theta) \cdot u \cdot \Bigg(\frac{\eta - \gamma}{(1-\eta) \cdot (1-\gamma)} \cdot \mathcal{J} \\
&\qquad + \frac{\beta}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \right) \cdot \frac{b}{n} \Bigg) \\
&+ \frac{1}{(1-\eta)} \cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \right) \cdot \frac{\partial \Omega}{\partial s} \cdot \frac{\partial s}{\partial \epsilon_s} \cdot \frac{\partial \epsilon_s}{\partial \theta} \cdot n \\
&+ \frac{1}{(1-\eta)} \cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \right) \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} \cdot \frac{\partial \epsilon_{stw}}{\partial \theta} \cdot n
\end{aligned}$$

with

$$\begin{aligned}
f'(\theta) &= (1-\gamma) \cdot q(\theta) \\
\left(-\frac{\partial \epsilon_{stw}}{\partial \theta} \right) &= \frac{n \cdot (\gamma - \eta \cdot f(\theta))}{f(\theta)} / (a \cdot h_{stw}(\epsilon_{stw})^\alpha + c_f) \\
\left(-\frac{\partial s}{\partial \epsilon_s} \right) &= \frac{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f}{\bar{h} - h_{stw}(\epsilon_s)} \\
\left(-\frac{\partial \epsilon_s}{\partial \theta} \right) &= \frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot k_v}{f(\theta)} \cdot \frac{1}{a \cdot h_{stw}(\epsilon_s)^\alpha + c_f} \cdot k_v
\end{aligned}$$

Next, solve for the Lagrange multiplier of the social value of the match:

$$\begin{aligned}\lambda_\theta = \frac{u'(\tilde{c}^w)}{M'} \cdot \frac{\partial n}{\partial \theta} \cdot \Bigg[& \frac{\eta - \gamma}{(1-\eta)\cdot(1-\gamma)} \cdot \frac{k_v}{q(\theta)} \\ & + \frac{\beta}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))} \cdot \frac{b}{n} \Bigg] \\ + \frac{u'(\tilde{c}^w)}{M'} \cdot \frac{\partial \Omega}{\partial s} & \cdot \left(-\frac{\partial s}{\partial \epsilon_s}\right) \cdot \left(-\frac{\partial \epsilon_s}{\partial \theta}\right) \cdot g(\epsilon_s) \cdot n \\ + \frac{u'(\tilde{c}^w)}{M'} \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} & \cdot \left(-\frac{\partial \epsilon_{stw}}{\partial \theta}\right) \cdot g(\epsilon_{stw}) \cdot n\end{aligned}$$

Define M' as the inverse multiplier in the case of an optimal STW system.

$$\begin{aligned}M' = & \underbrace{\frac{\gamma - \beta\cdot(\gamma - (1-(1-\eta)\cdot BE)\cdot\eta\cdot f(\theta))\cdot(1-G(\epsilon_s))}{(1-\eta)\cdot f(\theta)}}_{\text{regular multiplier}} \\ & - \underbrace{\frac{\beta}{1-\beta\cdot(1-f(\theta))\cdot(1-G(\epsilon_s))} \cdot \frac{(1+\eta\cdot BE)}{n} \cdot \frac{b}{n} \cdot f'(\theta) \cdot u}_{\text{Amplification over tax}} \\ & - \underbrace{\frac{\beta\cdot(1+\eta\cdot BE)}{n} \cdot \frac{\partial \Omega}{\partial s} \cdot \left(-\frac{\partial s}{\partial \epsilon_s}\right) \cdot \left(-\frac{\partial \epsilon_s}{\partial \theta}\right) \cdot n}_{\text{Amplification over reaction of STW benefits}} \\ & - \underbrace{\frac{\beta\cdot(1+\eta\cdot BE)}{n} \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} \cdot \left(-\frac{\partial \epsilon_{stw}}{\partial \theta}\right) \cdot n}_{\text{Amplification over reaction of eligibility condition}}\end{aligned}$$

Compared to the inverse multiplier of the exogenous STW system, the regular channel and the influence of vacancy posting on the costs of the UI system remain largely unchanged. However, the separation channel now accounts for the reaction of the STW system. When UI benefits increase, the joint surplus of firms and workers decreases, leading to a higher separation rate. In response, the STW system offers more generous benefits, but these benefits distort working hours, thereby reducing production. The subsidy effect of STW is effectively nullified by the corresponding increase in taxes, further decreasing the joint surplus of firms and workers.

Not only does the separation rate rise for firms with access to STW, but it also increases for firms without access to STW. As a result, the eligibility condition must be loosened, spreading the distortionary effects of reduced working hours across more firms. This, similar to an increase in STW benefits, leads to a decline in production, which in turn reduces the joint surplus of firms and workers.

Again, the paper assumes the inverse multiplier to be positive to guarantee convergence to the steady state. We can abbreviate the inverse multiplier by recognizing the fact that it captures the general equilibrium effect a change in the surplus or value of the firm has on the labor market tightness:

$$\frac{\partial \theta^{ge}}{\partial \mathcal{S}} = \frac{1}{M'} \quad \text{or} \quad \frac{\partial \theta^{ge}}{\partial \mathcal{J}} = \frac{1}{1-\eta} \cdot \frac{1}{M'}$$

Plugging this and the expression for λ_θ into the FOC for the optimal UI gives:

$$\underbrace{(1-n) \cdot \frac{u'(b) - u'(\tilde{c}^w)}{u'(\tilde{c}^w)}}_{\text{Provide additional Income Insurance}} =$$

$$+ L_V \cdot \underbrace{\left(-\frac{\partial f^{ge}}{\partial b} \cdot u\right)}_{\text{Less Hiring}} + \underbrace{n \cdot \frac{\partial \Omega}{\partial s} \cdot \frac{\partial s}{\partial \epsilon_s} \cdot \frac{\partial \epsilon_s^{ge}}{\partial b}}_{\text{STW Benefits increase to counter Separations}} + \underbrace{n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} \cdot \frac{\partial \epsilon_{stw}^{ge}}{\partial b}}_{\text{Looser eligibility condition}}$$

Here, L_V denotes the social value of an additional hire:

$$L_V = \left[\frac{\eta - \gamma}{(1-\eta)\cdot(1-\gamma)} \cdot \frac{k_v}{q(\theta)} + \frac{\beta}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n}\right]$$

Further, note that most of the right-hand side of the optimal UI benefits denotes the first derivative of the value of the firm to an increase in UI benefits:

$$-\frac{\partial \mathcal{J}}{\partial b} = \beta \cdot (1-\eta) \cdot \left[\left(\frac{u'(b)}{u'(\tilde{c}^w)} + \frac{1-n}{n}\right) - BE \cdot \left((1-\eta) \cdot \frac{u'(b)}{u'(\tilde{c}^w)} - \eta \cdot \frac{1-n}{n}\right)\right]$$

Finally, denote:

$$(-\frac{\partial f^{ge}}{\partial b}) \cdot u = f'(\theta) \cdot u \cdot \frac{\partial \theta^{ge}}{\partial \mathcal{J}} \cdot \frac{\partial \mathcal{J}}{\partial b}$$
$$\frac{\partial \epsilon_s^{ge}}{\partial b} = \frac{\partial \mathcal{J}}{\partial b} \cdot \frac{\partial \theta^{ge}}{\partial \mathcal{J}} \cdot \left(-\frac{\partial \epsilon_s}{\partial \theta}\right)$$
$$\frac{\partial \epsilon_{stw}^{ge}}{\partial b} = \frac{\partial \mathcal{J}}{\partial b} \cdot \frac{\partial \theta^{ge}}{\partial \mathcal{J}} \cdot \left(-\frac{\partial \epsilon_{stw}}{\partial \theta}\right)$$

We can also reformulate the optimality condition for UI benefits under an optimal STW policy to align it with the expression for the optimality condition of UI with a non-optimized STW system. To do this, note that

$$n \cdot \frac{\partial \Omega}{\partial s} \cdot \frac{\partial s}{\partial \epsilon_s} \cdot \frac{\partial \epsilon_s^{ge}}{\partial b} = \frac{n \cdot \frac{\partial \Omega}{\partial s}}{n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s}{\partial s}} \cdot n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial b}$$
$$= L_S^\star \cdot n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial b}$$

and

$$n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} \cdot \frac{\partial \epsilon_{stw}^{ge}}{\partial b} = \frac{n \cdot \frac{\partial \Omega}{\partial s}}{n \cdot g(\epsilon_s)} \cdot n \cdot g(\epsilon_{stw}) \cdot \frac{\partial \epsilon_s^{ge}}{\partial b}$$
$$= L_{STW}^{\star} \cdot n \cdot g(\epsilon_{stw}) \cdot \frac{\partial \epsilon_{stw}^{ge}}{\partial b}$$

Therefore, we can rewrite the condition as:

$$\underbrace{(1-n) \cdot \frac{u'(b) - u'(\tilde{c}^w)}{u'(\tilde{c}^w)}}_{\text{Provide Income Insurance}}$$
$$= L_V \cdot \underbrace{\left(-\frac{\partial f^{ge}}{\partial b} \cdot u\right)}_{\text{Less Hiring}} + L_S^{\star} \cdot \underbrace{\left(n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial b}\right)}_{\text{More Separations}} + L_{STW}^{\star} \cdot \underbrace{\left(n \cdot g(\epsilon_{stw}) \cdot \frac{\partial \epsilon_{stw}^{ge}}{\partial b}\right)}_{\text{More Workers on STW}}$$

I Lemma 1

(i) The welfare costs of STW can be denoted as:

$$\Omega_t = \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \Big[y_t(\epsilon) - v\left(h_t(\epsilon)\right) - y_{stw,t}(\epsilon) + v\left(h_{stw,t}(\epsilon)\right) \Big] dG(\epsilon)$$

From Nash bargaining, we can infer:

$$y_t(\epsilon_{stw,t}) - v(h_t(\epsilon_{stw,t})) - [y_{stw,t}(\epsilon_{stw,t}) - v(h_{stw,t}(\epsilon_{stw,t}))] > 0$$

Note that in Nash bargaining $h_t(\epsilon_{stw,t})$ is chosen to maximize $y_t(\epsilon_{stw,t}) - v(h_t(\epsilon_{stw,t}))$. And $h_{stw,t}(\epsilon_{stw,t})$ is chosen to maximize $y_t(\epsilon_{stw,t}) - v(h_t(\epsilon_{stw,t})) + (\bar{h} - h_{stw,t}(\epsilon_{stw,t})) \cdot s_t$. As long as $s_t \neq 0$ we get $h_t(\epsilon_{stw,t}) \neq h_{stw,t}(\epsilon_{stw,t})$. Thus, the first part of the equation must be larger than the second, which fulfills the condition. This implies that $\Omega_t > 0$

(ii) From the derivative of the welfare costs for the STW benefits:

$$\frac{\partial \Omega_t}{\partial s_t} = -\int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \frac{\partial h_{stw,t}(\epsilon)}{\partial s_t} \cdot \Big(\alpha \cdot a_t \cdot \epsilon \cdot h_{stw,t}(\epsilon)^{\alpha-1} - v'(h_{stw,t}(\epsilon))\Big)\, dG(\epsilon) \overset{!}{>} 0$$

Using the optimality condition for the hours choice on STW, we can rewrite the equation to:

$$\frac{\partial \Omega_t}{\partial s_t} = -\int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \frac{\partial h_{stw,t}(\epsilon)}{\partial s_t} \cdot s_t \cdot dG(\epsilon) \overset{!}{>} 0$$

Since the derivation of the number of hours worked for the STW benefits is negative for $\alpha < 1$

$$\frac{\partial h_{stw,t}(\epsilon)}{\partial s_t} = \frac{1}{\alpha \cdot (\alpha - 1) \cdot a_t \cdot \epsilon \cdot h_{stw,t}(\epsilon)^{\alpha-2} - \psi \cdot h_{stw,t}^{\psi-1}} < 0$$
$$\text{for} \quad \alpha \in (0,1) \text{ and } \psi, \epsilon, h_{stw,t}(\epsilon) > 0$$

and the STW benefits s_t need to be positive, we can conclude that the derivation for the welfare costs of STW must be positive.

(iii) From the derivative of the welfare costs for the eligibility condition:

$$\frac{\partial \Omega_t}{\partial \epsilon_{stw,t}} = \Big(y_t(\epsilon_{stw,t}) - v(h_t(\epsilon_{stw,t})) - [y_{stw,t}(\epsilon_{stw,t}) - v(h_{stw,t}(\epsilon_{stw,t}))] \Big) \cdot g(\epsilon_{stw,t})$$
$$\overset{!}{>} 0$$
$$\Leftrightarrow y_t(\epsilon_{stw,t}) - v(h_t(\epsilon_{stw,t})) - [y_{stw,t}(\epsilon_{stw,t}) - v(h_{stw,t}(\epsilon_{stw,t}))] \overset{!}{>} 0$$

As argued in (i), this equation must hold due to the Nash bargaining set-up.

(iv) Using our result from (iii), it follows:

$$\frac{\partial^2 \Omega_t}{\partial s_t \partial \epsilon_{stw,t}} = - \underbrace{\frac{\partial h_{stw,t}(\epsilon_{\epsilon_{stw,t}})}{\partial s_t}}_{>\,0} \cdot s_t \cdot g(\epsilon_{stw,t}) > 0$$

Note: If we loosen the eligibility condition, then we loosen the participation threshold. The relationship is implicitly defined as:

$$D_t = h_{stw,t}(\epsilon_{stw,t})$$

Using the implicit function theorem gives:

$$\frac{\partial \epsilon_{stw,t}}{\partial D_t} = \frac{1}{h'_{stw,t}(\epsilon_{stw,t})} > 0$$

The first derivative for working hours must be positive: $h'_{stw,t}(\epsilon_{stw,t}) = a_t \cdot h_{stw,t}(\epsilon_{stw,t})^{\alpha} + c_f > 0$.

J Corollaries

All corollaries are under the ceteris paribus assumption. For the corollaries, ceteris paribus implies that separation rates and job-finding rates are seen as exogenous. Therefore, all results have to be interpreted as partial equilibrium results.

Corollary 1 The fiscal externality on hiring is defined as:

$$FE = \frac{1}{1-\beta\cdot(1-G(\epsilon_s))\cdot(1-f)}\cdot\left[b+\frac{1-n}{n}\cdot b\right]$$

Using the law of motion of employment, we can derive the steady state employment rate:

$$n = \frac{f}{f+G(\epsilon_s)\cdot(1-f)}$$

Rearranging the fiscal externality and setting the discount factor to 1 gives:

$$FE = \frac{1}{f+G(\epsilon_s)\cdot(1-f)}\cdot\frac{b}{n}$$

Multiplying and dividing by f recovers employment in the employment rate in the equation:

$$FE = \frac{1}{f}\cdot\frac{f}{f+G(\epsilon_s)\cdot(1-f)}\cdot\frac{b}{n}$$

Inserting the employment rate gives:

$$FE = \frac{b}{f}$$

This proves Corollary 1.

Corollary 2 The optimal STW benefits can be denoted as:

$$s = \frac{1}{\bar{h}-h_{stw}(\epsilon_s)}\cdot\Bigg(\frac{1-f}{1-\beta\cdot(1-G(\epsilon_s))\cdot(1-f)}\cdot\frac{b}{n} - \frac{n\cdot\frac{\partial\Omega}{\partial s}}{n\cdot g(\epsilon_s)\cdot\left(-\frac{\partial\epsilon_s}{\partial s}\right)} - \tilde{BE}\Bigg)$$

Without risk aversion, the bargaining effect is zero: $\tilde{BE} = 0$.

$$s = \frac{1}{\bar{h}-h_{stw}(\epsilon_s)}\cdot\Bigg(\frac{1-f}{1-\beta\cdot(1-G(\epsilon_s))\cdot(1-f)}\cdot\frac{b}{n} - \frac{n\cdot\frac{\partial\Omega}{\partial s}}{n\cdot g(\epsilon_s)\cdot\left(-\frac{\partial\epsilon_s}{\partial s}\right)}\Bigg)$$

Note that the working hours distortions do not depend on the job-finding rate. Thus, we can simplify notation to:

$$s = \frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \cdot \frac{\beta \cdot (1-f)}{1 - \beta \cdot (1 - G(\epsilon_s)) \cdot (1-f)} \cdot \frac{b}{n} - C$$

C denotes a constant that does not depend on f. Note that the steady state employment rate gives:

$$s = \frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \cdot \frac{1-f}{1 - \beta \cdot (1 - G(\epsilon_s)) \cdot (1-f)} \cdot \frac{b}{n(f)} - C$$

Taking the first derivatives gives:

$$\begin{aligned} -\frac{\partial s}{\partial f} = & \frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \cdot \frac{1 + \beta \cdot (1 - G(\epsilon_s)) \cdot f}{\left[1 - \beta \cdot (1 - G(\epsilon_s)) \cdot (1-f)\right]^2} \cdot \frac{b}{n(f)} \\ & \frac{\beta \cdot (1-f)}{1 - \beta \cdot (1 - G(\epsilon_s)) \cdot (1-f)} \cdot \frac{b}{n(f)^2} \cdot n'(f) > 0 \end{aligned}$$

The first term reflects how much longer a worker separated in this period will remain unemployed due to the decline in the job-finding rate. The second term highlights that lower job-finding rates not only decrease employment but also increase the fiscal burden on the UI system, as more workers remain dependent on UI benefits for extended periods:

$$n'(f) = \frac{G(\epsilon_s)}{(f + G(\epsilon_s) \cdot (1-f))^2}$$

We can conclude that UI benefits must rise after a fall in job-finding rates.

Corollary 3 Without risk aversion, the bargaining effect is zero: $\tilde{BE} = 0$. The optimal STW benefits can thus be expressed as:

$$\begin{aligned} s = & \frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \cdot \left(\frac{1-f}{1 - \beta \cdot (1 - G(\epsilon_s)) \cdot (1-f)} \cdot \frac{b}{n} \right. \\ & \left. - \frac{n \cdot \frac{\partial \Omega}{\partial s}}{n \cdot g(\epsilon_s) \cdot \left(-\frac{\partial \epsilon_s}{\partial s}\right)} \right) \end{aligned}$$

The first derivative is:

$$\frac{\partial s}{\partial D} = -\frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \underbrace{\frac{\partial^2 \Omega}{\partial s \partial D}}_{>0} / \underbrace{\left[-g(\epsilon_s) \cdot \frac{\partial \epsilon_s}{\partial s}\right]}_{>0} < 0$$

Increasing both STW benefits and the eligibility condition increases the distortionary effects of the STW system. STW benefits have to be stricter.

Corollary 4 T.b.s.: $b = 0$ and $\eta = \gamma \Rightarrow s = 0$

We proceed by guess and verify. Guess that $s = 0$. Then we know that the welfare costs of STW are zero, and so are its derivatives for the STW benefits

$$\frac{\partial \Omega}{\partial s} = -\int_{\epsilon_s}^{\epsilon_{stw}} \frac{\partial h_{stw}(\epsilon)}{\partial s} \cdot s \cdot dG(\epsilon) = 0$$

and the eligibility condition:

$$\frac{\partial \Omega}{\partial D} = \frac{\partial \epsilon_{stw}}{\partial D} \cdot \Big(y(\epsilon_{stw}) - v(h(\epsilon_{stw})) - [y_{stw}(\epsilon_{stw}) - v(h_{stw}(\epsilon_{stw}))] \Big) \cdot g(\epsilon_{stw}) = 0$$

as $h_{stw}(\epsilon_s) = h(\epsilon_s)$. Therefore, we can imply that the welfare cost penalty of STW must be equal to zero:

$$\left[n \cdot \frac{\partial \Omega}{\partial s} \right] / \left[n \cdot g(\epsilon_s) \cdot \left(-\frac{\partial \epsilon_s}{\partial s} \right) \right] = 0$$

Further, it is trivial that the fiscal externality of the UI system on separations with $b = 0$ is equal to zero:

$$(1 - f) \cdot FE = 0$$

Finally, we have to look at the bargaining effect. Appendix B shows that the bargaining effect depends on the Lagrange multiplier of the value of the firm. We can write the Lagrange multiplier as

$$\lambda_\theta = L_V + L_S^\star + L_{STW}^\star$$

with

$$\begin{aligned} L_V &= \underbrace{\frac{\eta - \gamma}{(1-\gamma) \cdot (1-\eta)} \cdot \beta \cdot \mathcal{J}}_{\text{Deviation from Hosios Condition}} + \underbrace{FE}_{\text{Fiscal Externality UI on Hiring}} \\ L_S^\star &= \frac{MWL_s}{MMS_s} \\ L_{STW}^\star &= \frac{MWL_{\epsilon_{stw}}}{MMS_{\epsilon_{stw}}} \end{aligned}$$

When $\eta = \gamma$, then the Hosios condition is fulfilled. Further, $b = 0$ implies that $FE = 0$; thus, we can conclude that vacancy posting must be efficient, $L_V = 0$. Further, we already established that for $s = 0$ STW does not distort the economy: $MWL_s = n \cdot \frac{\partial \Omega}{\partial s} = 0$ and $MWL_s = n \cdot \frac{\partial \Omega}{\partial s} = 0$ implying that there are no inefficient separations $LS = 0$ and the eligibility condition does not cause any inefficiencies $L_{STW} = 0$. From this, we can infer that

$$\tilde{BE} = 0$$

Looking at the formula for the optimal STW benefits

$$\begin{aligned} s &= \frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \cdot \left((1-f) \cdot FE - \left[n \cdot \frac{\partial \Omega}{\partial s} \right] / \left[n \cdot g(\epsilon_s) \cdot \left(-\frac{\partial \epsilon_s}{\partial s} \right) \right] - \tilde{BE} \right) \\ &= 0 \end{aligned}$$

and given the fact that the fiscal externality on separations, the welfare cost penalty of STW and $\tilde{BE}$ are zero, we can infer that s is zero, verifying the initial guess.

Corollary 5 Let us look at what happens when we offer more generous STW benefits. Suppose the government wants to impose the participation threshold ϵ_{stw}. Then we know that we need to set the following minimum hours reduction threshold:

$$D = h_{stw}(\epsilon_{stw})$$

When the government decides to reduce working, we need to keep the eligibility condition stricter:

$$\frac{\partial D}{\partial s} = \frac{\partial h_{stw}(\epsilon_{stw})}{\partial s} < 0$$

We have already shown that in Lemma 1.

Corollary 6 The steady state budget constraint is:

$$n \cdot \tau = (1-n) \cdot b + n \cdot \int_{\epsilon_s}^{\epsilon_{stw}} s \cdot \left(\bar{h} - h_{stw}(\epsilon) \right) \cdot dG(\epsilon)$$

Rearrange for the lump-sum income tax:

$$\tau = \frac{(1-n)}{n} \cdot b + \int_{\epsilon_s}^{\epsilon_{stw}} s \cdot \left(\bar{h} - h_{stw}(\epsilon) \right) \cdot dG(\epsilon)$$

Insert optimal eligibility condition and STW benefits:

$$\tau = \frac{(1-n)}{n} \cdot b + \int_{\epsilon_s}^{\xi_s} \frac{\bar{h} - h_{stw}(\epsilon)}{\bar{h} - h_{stw}(\epsilon_s)} \cdot \left((1-f) \cdot FE - \tilde{\Omega} - \tilde{BE}\right) dG(\epsilon)$$

with

$$\tilde{\Omega} = \left[n \cdot \frac{\partial \Omega}{\partial s}\right] / \left[n \cdot g(\epsilon_s) \cdot \left(-\frac{\partial \epsilon_s}{\partial s}\right)\right]$$

Set $\beta = 1$:

$$\tau = \frac{(1-f)}{f} \cdot G(\epsilon_s) \cdot b + \int_{\epsilon_s}^{\xi_s} \frac{\bar{h} - h_{stw}(\epsilon)}{\bar{h} - h_{stw}(\epsilon_s)} \cdot \left(\frac{1-f}{f} \cdot b - \tilde{\Omega} - \tilde{BE}\right) dG(\epsilon)$$

Note that

$$\frac{\bar{h} - h_{stw}(\epsilon)}{\bar{h} - h_{stw}(\epsilon_s)} < 1$$

Less hours reduction makes rescuing a job at risk less expensive than rescuing the marginal match. It must therefore apply:

$$\begin{aligned}
\tau &< \frac{1-f}{f} \cdot G(\epsilon_s) \cdot b + (G(\xi_s) - G(\epsilon_s)) \cdot \left(\tilde{\Omega} + \tilde{BE}\right) \\
&= \underbrace{G(\xi_s) \cdot \frac{1-f}{f} \cdot b}_{\text{Costs of UI system without STW}} - \underbrace{(G(\xi_s) - G(\epsilon_s)) \cdot \left(\tilde{\Omega} + \tilde{BE}\right)}_{\text{Costs saved with STW}}
\end{aligned}$$

Ceteris paribus UI combined with optimal STW must be less expensive then a UI system only.

K Derivation Optimal STW Policy under lump-sum Taxation

Under lump-sum taxation and without risk aversion, the problem of the planner changes to:

$$\begin{aligned}
W_t^P = \max_{\theta_t, \epsilon_{s,t}, \epsilon_{stw,t}, h_t(\epsilon), s_t} n_t \cdot \int_{\epsilon_{s,t}}^{\infty} z_t(\epsilon, h_t(\epsilon)) dG(\epsilon) - n_t \cdot \Omega_t \\
- n_t \cdot \rho_t \cdot F - \theta_t \cdot (1 - n_t + \rho_t \cdot n_t) \cdot k_v + \beta \cdot E_t \left[W_{t+1}^P\right]
\end{aligned}$$

Law of motion of employment:

$$n_{t+1} = (1 - \rho_t) \cdot n_t + f(\theta_t) \cdot (1 - n_t + \rho_t \cdot n_t)$$

Job-Creation Condition:

$$\frac{1}{1-\eta_t}\frac{k_v}{q(\theta_t)} = \beta \cdot E_t\Bigg(\int_{\epsilon_{stw,t+1}}^{\infty} z_{t+1}\left(\epsilon, h_{t+1}(\epsilon)\right) dG(\epsilon)$$
$$+ \int_{\epsilon_{s,t+1}}^{\epsilon_{stw,t+1}} \Big[z_{t+1}\left(\epsilon, h_{stw,t+1}(\epsilon)\right) + \tau_{stw,t+1} \cdot \left(\bar{h} - h_{stw,t+1}(\epsilon)\right) \Big] dG(\epsilon)$$
$$- b_{t+1} + \rho_{t+1} \cdot (-F) + (1-\rho_{t+1}) \cdot \frac{1-\eta_{t+1}\cdot\theta_{t+1}\cdot q(\theta_{t+1})}{1-\eta_{t+1}} \cdot \frac{k_v}{q(\theta_{t+1})} \Bigg)$$

Separation Condition with STW:

$$z_{stw,t}\left(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})\right) + s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right)$$
$$+ F + \frac{1-\eta_t\cdot\theta_t\cdot q(\theta_t)}{1-\eta_t} \cdot \frac{k_v}{q(\theta_t)} = 0$$

Separation Condition without access to STW:

$$z_t\left(\xi_{s,t}, h_t(\xi_{s,t})\right) + F + \frac{1-\eta_t\cdot\theta_t\cdot q(\theta_t)}{1-\eta_t} \cdot \frac{k_v}{q(\theta_t)} = 0$$

Welfare costs of STW:

$$\Omega_t = \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \Big[z_t\left(\epsilon, h_t(\epsilon)\right) - z_t\left(\epsilon, h_{stw,t}(\epsilon)\right) \Big] dG(\epsilon)$$

Separation rate:

$$\rho_t = G(\epsilon_{s,t})$$

The eligibility condition is set so that the participation threshold is at least as loose as the productivity threshold at which firms and workers separate without access to STW:

$$\epsilon_{stw,t} \geq \xi_{s,t}$$

I solve the optimization problem with Kuhn–Tucker. For brevity, I do not spell out the full Lagrangian but introduce the relevant notation. The Lagrangian is denoted by $\mathcal{L}$. The Lagrange multiplier associated with the law of motion of employment is denoted by $\lambda_{\theta,t}$; the multiplier associated with the job-creation condition by $\lambda_{\theta,t}$; the multiplier associated with the separation condition without access to STW by $\lambda_{\xi,t}$; the multiplier associated with the separation condition under STW by $\lambda_{\epsilon,t}$; the multiplier associated with the welfare costs of STW by $\lambda_{\Omega,t}$; and the multiplier associated with the separation rate by $\lambda_{\rho,t}$. The Lagrange multiplier associated with the inequality constraint $\epsilon_{stw} \geq \xi_{s,t}$ is denoted by $\lambda_{stw,t}$.

K.1 Derivation Optimal Separation and Job-Creation Condition

The optimality condition for employment can be written as:

$$\frac{\partial \mathcal{L}}{\partial n_t} = \int_{\epsilon_{s,t}}^{\infty} z_t(\epsilon, h_t(\epsilon)) dG(\epsilon) - \Omega_t - \rho_t \cdot F + (1-\rho_t) \cdot \theta_t \cdot k_v$$
$$- \lambda_{n,t} + \beta \cdot (1 - \theta_t \cdot q(\theta_t)) \cdot (1-\rho_t) \cdot E_t \left[\lambda_{n,t+1}\right] = 0$$

Correspondingly, the FOC for the labor market tightness is:

$$\frac{\partial \mathcal{L}}{\partial \theta_t} = - (1-n_t) \cdot k_v + \beta \cdot (1-n_t) \cdot (1-\gamma) \cdot q(\theta_t) \cdot E_t \left[\lambda_{n,t+1}\right]$$
$$- \frac{1}{1-\eta_t} \cdot \frac{k_v}{q(\theta_t)} \cdot \gamma \cdot \frac{\lambda_{\theta,t}}{\theta_t}$$
$$+ \frac{1}{1-\eta_t} \cdot \left(\frac{\gamma}{\theta_t \cdot q(\theta_t)} - \eta_t \right) \cdot k_v \cdot ((1-\rho_t) \cdot \lambda_{\theta,t-1} - \lambda_{\epsilon,t} - \lambda_{\xi,t}) = 0$$

Rewriting the optimality condition for the labor market tightness gives:

$$\beta \cdot E_t \left[\lambda_{n,t+1}\right] = \left(\frac{1+\chi_t}{1-\gamma} \right) \cdot \frac{k_v}{q(\theta_t)}$$

With

$$\chi_t = \frac{1}{1-n_t} \cdot \frac{1}{1-\eta_t} \cdot \frac{1}{\theta_t \cdot q(\theta_t)} \cdot \Bigg(\gamma \cdot \lambda_{\theta,t} - (\gamma - \theta_t \cdot q(\theta_t) \cdot \eta_t)$$
$$\cdot ((1-G(\epsilon_{s,t})) \cdot \lambda_{\theta,t-1} - \lambda_{\epsilon,t} - \lambda_{\xi,t}) \Bigg)$$

Finally, we can derive the FOC for the separation threshold:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{s,t}} = - (z_{stw,t}(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - F) \cdot g(\epsilon_{s,t}) \cdot n_t$$
$$- \lambda_{n,t} \cdot (1 - f(\theta_t)) \cdot g(\epsilon_{s,t}) \cdot n_t$$
$$- \lambda_{\theta,t-1} \cdot \Bigg(z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + (\bar{h} - h_{stw,t}(\epsilon_{s,t})) \cdot s_t$$
$$+ F + \frac{1 - \theta_t \cdot q(\theta_t) \cdot \eta_t}{1-\eta_t} \cdot \frac{k_v}{q(\theta_t)} \Bigg) \cdot g(\epsilon_{s,t})$$
$$- \lambda_{\epsilon,t} \cdot \left[a_t \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha} + c_f\right] = 0$$

$$
\begin{aligned}
\Leftrightarrow &- (z_{stw,t}(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) - F) - \lambda_{n,t} \cdot (1 - f(\theta_t)) \\
&- \frac{\lambda_{\theta,t-1}}{n_t} \cdot \Bigg(z_{stw,t}(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + (\bar{h} - h_{stw,t}(\epsilon_{s,t})) \cdot s_t \\
&\quad + F + \frac{1 - \theta_t \cdot q(\theta_t) \cdot \eta_t}{1 - \eta_t} \cdot \frac{k_v}{q(\theta_t)} \Bigg) \\
&- \frac{\lambda_{\epsilon,t}}{g(\epsilon_{s,t}) \cdot n_t} \cdot [a_t \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha} + c_f] = 0
\end{aligned}
$$

Using the optimality condition for employment and the labor market density, we can derive the optimal vacancy posting condition:

$$
\begin{aligned}
\frac{1}{1-\gamma} \cdot \frac{(1+\chi_t) \cdot k_v}{q_t} = \beta \cdot E_t &\Bigg[\int_{\epsilon_{s,t+1}}^{\infty} z_{t+1}(\epsilon, h_{t+1}(\epsilon)) dG(\epsilon) - \Omega_{t+1} + \rho_{t+1} \cdot (-F) \Bigg] \\
&+ \beta \cdot E_t \Bigg[(1 - \rho_{t+1}) \cdot \frac{1 - \gamma \cdot f_{t+1} + (1 - f_{t+1}) \cdot \chi_{t+1}}{1 - \gamma} \cdot \frac{k_v}{q_{t+1}} \Bigg]
\end{aligned}
$$

Likewise, we can derive the optimal separation decision by plugging the optimality condition for employment and the labor market density into the optimality condition for the separation threshold.

$$
\begin{aligned}
z_{t+1}(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + \lambda_{\epsilon_s,t} \cdot (a_t \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha} + c_f) + F& \\
+ \frac{1 - \gamma \cdot f_t + (1 - f_t) \cdot \chi_t}{1 - \gamma} \cdot \frac{k_v}{q_t} = 0&
\end{aligned}
\tag{K.1}
$$

Next, we deduct the decentralized job-creation condition from the optimal vacancy posting condition, giving an expression for the deviation of the decentralized job creation condition from the optimal:

$$
\begin{aligned}
\left(\chi_t - \frac{\eta_t - \gamma}{1 - \eta_t} \right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q_t} = \beta \cdot &\left[b_{t+1} - \int_{\epsilon_{s,t+1}}^{\epsilon_{stw,t+1}} (\bar{h} - h_{stw,t+1}(\epsilon)) dG(\epsilon) \cdot \tau_{stw,t+1} \right] \\
+ \beta \cdot E_t \Bigg[(1 - G(\epsilon_{s,t+1})) \cdot (1 - f_{t+1}) \cdot &\left(\chi_{t+1} - \frac{\eta_{t+1} - \gamma}{1 - \eta_{t+1}} \right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q_{t+1}} \Bigg]
\end{aligned}
$$

Likewise, we deduct the decentralized separation condition from the optimal separation condition and rearrange it for the STW subsidy:

$$
\lambda_{\epsilon,t} \cdot (a_t \cdot h_{stw,t}(\epsilon_{s,t})^{\alpha} + c_f) \cdot \frac{1 - G(\epsilon_{s,t})}{n_t \cdot g(\epsilon_{s,t})} - s_t \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t}) \right)
$$

$$+ (1 - f_t) \cdot \left(\chi_t - \frac{\eta_t - \gamma}{1 - \eta_t}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q_t} = 0 \tag{K.2}$$

K.2 Optimal Eligibility Condition

First, let us form the FOC of the participation threshold:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{stw,t}} = -\, n_t \cdot \frac{\partial \Omega_t}{\partial \epsilon_{stw,t}} + \lambda_{\theta,t-1} \cdot \Big[S_{stw}(\epsilon_{stw}) - S(\epsilon_{stw}) \Big] \cdot g(\epsilon_{stw,t}) - \lambda_{stw,t} = 0$$

where $S_{stw}(\epsilon)$ denotes the joint surplus of firms and workers on STW with idiosyncratic productivity ϵ and $S(\epsilon)$ the corresponding joint surplus outside STW. According to the Kuhn-Tucker conditions the Lagrange multiplier must be zero $\lambda_{stw,t} = 0$ if the condition is not binding $\epsilon_{stw,t} > \xi_{s,t}$ or negative $\lambda_{stw,t} < 0$ if the condition is binding $\epsilon_{stw,t} = \xi_{s,t}$:

$$\lambda_{stw,t} = \begin{cases} \lambda_{\theta,t-1} \cdot [S_{stw,t}(\epsilon_{stw,t}) - S_t(\epsilon_{stw,t})] - \frac{\partial \Omega_t}{\partial \epsilon_{stw,t}} < 0, & \text{for} \quad \epsilon_{stw,t} = \xi_{s,t} \\ 0, & \text{for} \quad \epsilon_{stw,t} > \xi_{s,t} \end{cases}$$

If the condition is binding, we get the same eligibility condition as in the model with lump-sum income taxes. If it is not, then we get a trade-off between hours distortions and increasing joint surplus of firms and workers to stimulate vacancy posting and reduce separations. To see this in the formula, we need to derive the Lagrange multiplier for the job-creation condition λ_θ. We follow the procedure from Appendix H.

The **deviation from the optimal job-creation condition** in the steady state can be denoted as:

$$\begin{aligned} \left(\chi - \frac{\eta - \gamma}{1 - \eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} = b - \int_{\epsilon_s}^{\epsilon_{stw}} (\bar{h} - h_{stw}(\epsilon)) dG(\epsilon) \cdot s \\ + \beta \cdot (1 - G(\epsilon_s)) \cdot (1 - f) \cdot \left(\chi - \frac{\eta - \gamma}{1 - \eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} \end{aligned} \tag{K.3}$$

Rearranging for χ gives:

$$\begin{aligned} \chi = & \frac{\eta - \gamma}{1 - \eta} \\ & + \frac{1}{1 - \beta \cdot (1 - \rho) \cdot (1 - f)} \cdot \left(b - \int_{\epsilon_s}^{\epsilon_{stw}} (\bar{h} - h_{stw}(\epsilon)) dG(\epsilon) \cdot s\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} \end{aligned}$$

From the optimality condition of the labor market tightness, we can derive a second expression for χ :

$$\chi = \frac{1}{1-n} \cdot \frac{1}{1-\eta} \cdot \frac{1}{\theta \cdot q(\theta)} \cdot (\gamma \cdot \lambda_{\theta,t} - (\gamma - \theta_t \cdot q(\theta) \cdot \eta) \cdot ((1 - G(\epsilon_s)) \cdot \lambda_\theta - \lambda_{\epsilon_s} - \lambda_{\xi_s}))$$

Since we want to isolate λ_θ we need to find expressions for the Lagrange multipliers of λ_{ϵ_s} and λ_{ξ_s}. First, we derive the optimality condition for the STW benefits to isolate λ_{ϵ_s}:

$$\frac{\partial}{\partial s_t} = -n_t \cdot \frac{\partial \Omega_t}{\partial s_t} + \lambda_{\theta,t-1} \cdot \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \left(\bar{h} - h_{stw,t}(\epsilon)\right) dG(\epsilon) - \lambda_{\epsilon_s,t} \cdot \left(\bar{h} - h_{stw,t}(\epsilon_{s,t})\right) = 0$$

Rearranging and imposing the steady state, the Lagrange multiplier for the separation condition of firms with access to STW can be denoted by:

$$-\lambda_\epsilon = \frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \cdot \left(n_t \cdot \frac{\partial \Omega}{\partial s} - \lambda_\theta \cdot \int_{\epsilon_s}^{\epsilon_{stw}} \left(\bar{h} - h_{stw}(\epsilon)\right) dG(\epsilon) \right)$$

Next, we can form the FOC for the separation threshold without access to STW benefits and convert it to the steady state:

$$\frac{\partial}{\partial \xi_{s,t}} = \lambda_{stw,t} - \lambda_{\xi,t} \cdot (a_t \cdot h_{stw,t}(\xi_{s,t})^\alpha + c_f) = 0$$

$$\Leftrightarrow \qquad \lambda_{stw} = \lambda_\xi \cdot (a \cdot h_{stw}(\xi_s)^\alpha + c_f)$$

This allows us to calculate the Lagrange multiplier for the separation threshold of firms without access to STW. Note that we have a Kuhn-Tucker problem, so that λ_{stw} and λ_{ξ_s} require a case distinction:

$$\lambda_{\xi_s} = \begin{cases} \frac{\lambda_\theta \cdot [S_{stw}(\epsilon_{stw}) - S(\epsilon_{stw})] - \frac{\partial \Omega}{\partial \epsilon_{stw}}}{\left(a \cdot h_{stw}(\xi_s)^\alpha + c_f\right)}, & \text{for} \quad \epsilon_{stw} = \xi_s \\ 0, & \text{for} \quad \epsilon_{stw} > \xi_s \end{cases}$$

This can be rewritten using the indicator function:

$$\lambda_{\xi_s} = \frac{\mathbb{1}\{\epsilon_{stw} = \xi_s\}}{(a \cdot h_{stw}(\xi_s)^\alpha + c_f)} \cdot \left(\lambda_\theta \cdot [S_{stw}(\epsilon_{stw}) - S(\epsilon_{stw})] - n_t \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} \right)$$

In a next step, we can set both expressions for χ equal and insert our new expressions for the Lagrange multipliers:

$$
\begin{aligned}
&\frac{1}{1-\eta} \cdot \frac{\gamma - \beta \cdot (\gamma - \eta \cdot f(\theta)) \cdot (1 - G(\epsilon_s))}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot k_v \cdot \lambda_\theta \\
&= (1-\gamma) \cdot u \cdot q(\theta) \cdot \Bigg(\frac{\eta - \gamma}{(1-\gamma) \cdot (1-\eta)} \cdot \frac{k_v}{q(\theta)} \\
&\quad + \frac{\beta}{1 - \beta \cdot (1-\rho) \cdot (1-f)} \cdot \left(b - \int_{\epsilon_s}^{\epsilon_{stw}} (\bar{h} - h_{stw}(\epsilon)) dG(\epsilon) \cdot s \right) \Bigg) \\
&\quad + \frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot k_v}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot \frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \\
&\quad \cdot \left(\lambda_\theta \cdot \int_{\epsilon_s}^{\epsilon_{stw}} \left(\bar{h} - h_{stw}(\epsilon) \right) dG(\epsilon) - n \cdot \frac{\partial \Omega}{\partial s} \right) \\
&\quad + \frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta)) \cdot k_v}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot \frac{\mathbb{1}\{\epsilon_{stw} = \xi_s\}}{a \cdot h_{stw}(\epsilon_{stw})^\alpha + c_f} \\
&\quad \cdot \left(\lambda_\theta \cdot [S_{stw}(\epsilon_{stw}) - S(\epsilon_{stw})] - n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} \right).
\end{aligned}
$$

Using the implicit function theorem, we can denote:

$$
\begin{aligned}
\frac{\partial s}{\partial \theta} &= -\frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta))}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot k_v \cdot \frac{1}{\bar{h} - h_{stw}(\epsilon_s)} \\
\frac{\partial \xi_s}{\partial \theta} &= -\frac{1}{1-\eta} \cdot \frac{(\gamma - \eta \cdot f(\theta))}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot k_v \cdot \frac{1}{(a \cdot h_{stw}(\xi_s)^\alpha + c_f)}
\end{aligned}
$$

Further, note that

$$
(1-\gamma) \cdot q(\theta) = f'(\theta)
$$

This allows us to solve for λ_θ:

$$
\begin{aligned}
\lambda_\theta = \frac{f'(\theta) \cdot u}{M} \cdot &\Bigg(\frac{\eta - \gamma}{(1-\gamma) \cdot (1-\eta)} \cdot \frac{k_v}{q(\theta)} \\
&+ \frac{\beta}{1 - \beta \cdot (1-\rho) \cdot (1-f)} \cdot \left(b - \int_{\epsilon_s}^{\epsilon_{stw}} (\bar{h} - h_{stw}(\epsilon)) \, dG(\epsilon) \cdot s \right) \Bigg) \\
&- \frac{n}{M} \cdot \frac{\partial s}{\partial \theta} \cdot \frac{\partial \Omega}{\partial s} \\
&- \mathbb{1}\{\epsilon_{stw} = \xi_s\} \cdot \frac{n}{M} \cdot \frac{\partial \epsilon_{stw}}{\partial \theta} \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}.
\end{aligned}
$$

Here, M denotes the inverse multiplier:

$$\begin{aligned}M = \frac{1}{1-\eta} \cdot \frac{\gamma - \beta \cdot (\gamma - \eta \cdot f(\theta)) \cdot (1 - G(\epsilon_s))}{f(\theta) \cdot u'(\tilde{c}^w)} \cdot k_v \\ + \frac{\partial s}{\partial \theta} \cdot \int_{\epsilon_s}^{\epsilon_{stw}} \left(\bar{h} - h_{stw}(\epsilon)\right) dG(\epsilon) \\ + \frac{\partial \xi_s}{\partial \theta} \cdot \mathbb{1}\{\epsilon_{stw} = \xi_s\} \cdot [S_{stw}(\epsilon_{stw}) - S(\epsilon_{stw})]\end{aligned}$$

This is nothing else but the general equilibrium effect that an increase in the joint surplus has on the labor market tightness:

$$\frac{\partial \theta^{ge}}{\partial \mathcal{S}} = \frac{1}{M}$$

Inserting this simplifies the expression to:

$$\begin{aligned}\lambda_\theta = \frac{\partial \theta^{ge}}{\partial \mathcal{S}} \cdot f'(\theta) \cdot u \cdot \Bigg(\frac{\eta - \gamma}{(1-\gamma) \cdot (1-\eta)} \cdot \frac{k_v}{q(\theta)} \\ + \frac{\beta}{1 - \beta \cdot (1-\rho) \cdot (1-f)} \cdot \left(b - \int_{\epsilon_s}^{\epsilon_{stw}} (\bar{h} - h_{stw}(\epsilon))\, dG(\epsilon) \cdot s \right) \Bigg) \\ - \frac{\partial \theta^{ge}}{\partial \mathcal{S}} \cdot \left[\frac{\partial s}{\partial \theta} \cdot n \cdot \frac{\partial \Omega}{\partial s} + \frac{\partial \xi_s}{\partial \theta} \cdot \mathbb{1}\{\epsilon_{stw} = \xi_s\} \cdot n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}} \right]\end{aligned}$$

Note that

$$\begin{aligned}\frac{\partial \theta^{ge}}{\partial \mathcal{S}} \cdot \frac{\partial s}{\partial \theta} &= \frac{\partial \theta^{ge}}{\partial \mathcal{S}} \cdot \frac{\partial \epsilon_s}{\partial \theta} \cdot \frac{n \cdot g(\epsilon_s)}{n \cdot g(\epsilon_s) \cdot \left(-\frac{\partial \epsilon_s}{\partial s}\right)} \\ &= \frac{n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial \mathcal{S}}}{n \cdot g(\epsilon_s) \cdot \left(-\frac{\partial \epsilon_s}{\partial s}\right))}.\end{aligned}$$

and that

$$\begin{aligned}\frac{\partial \theta^{ge}}{\partial \mathcal{S}} \cdot \frac{\partial \epsilon_{stw}}{\partial \theta} &= \frac{\partial \theta^{ge}}{\partial \mathcal{S}} \cdot \frac{\partial \epsilon_{stw}}{\partial \theta} \cdot \frac{n \cdot g(\epsilon_{stw})}{n \cdot g(\epsilon_{stw})} \\ &= \frac{n \cdot g(\epsilon_{stw}) \cdot \frac{\partial \epsilon_s^{ge}}{\partial \mathcal{S}}}{n \cdot g(\epsilon_{stw})}.\end{aligned}$$

This changes the expression to:

$$\lambda_\theta = \frac{\partial f^{ge}}{\partial \mathcal{S}} \cdot u \cdot \Bigg(\frac{\eta - \gamma}{(1-\gamma) \cdot (1-\eta)} \cdot \frac{k_v}{q(\theta)}$$

$$+\frac{\beta}{1-\beta\cdot(1-\rho)\cdot(1-f)}\cdot\left(b-\int_{\epsilon_s}^{\epsilon_{stw}}(\bar{h}-h_{stw}(\epsilon))dG(\epsilon)\cdot s\right)\Bigg)$$
$$-\left[n\cdot g(\epsilon_s)\cdot\frac{\partial\epsilon_s^{ge}}{\partial\mathcal{S}}\right]\cdot\frac{n\cdot\frac{\partial\Omega}{\partial s}}{n\cdot g(\epsilon_s)\cdot\left(-\frac{\partial\epsilon_s}{\partial s}\right))}$$
$$-\mathbb{1}\{\epsilon_{stw}=\xi_s\}\cdot\left[n\cdot g(\epsilon_{stw})\cdot\frac{\partial\epsilon_{stw}^{ge}}{\partial\mathcal{S}}\right]\cdot\frac{n\cdot\frac{\partial\Omega}{\partial\epsilon_{stw}}}{n\cdot g(\epsilon_{stw})}.$$

Define:

$$L_V'=\frac{\eta-\gamma}{(1-\gamma)\cdot(1-\eta)}\cdot\frac{k_v}{q(\theta)}$$
$$+\frac{\beta}{1-\beta\cdot(1-\rho)\cdot(1-f)}\cdot\left(b-\int_{\epsilon_s}^{\epsilon_{stw}}(\bar{h}-h_{stw}(\epsilon))dG(\epsilon)\cdot s\right),$$
$$L_S^\star=\frac{n\cdot\frac{\partial\Omega}{\partial s}}{n\cdot g(\epsilon_s)\cdot\left(-\frac{\partial\epsilon_s}{\partial s}\right)},$$
$$L_{STW}^\star=\frac{n\cdot\frac{\partial\Omega}{\partial\epsilon_{stw}}}{n\cdot g(\epsilon_{stw})}$$
$$L_\xi^\star=\mathbb{1}\{\epsilon_{stw}=\xi_s\}\cdot L_{STW}^\star$$

This simplifies the expression for the Lagrange multiplier of the job-creation condition to:

$$\lambda_\theta=\frac{\partial f^{ge}}{\partial\mathcal{S}}\cdot u\cdot L_V'-\left[n\cdot g(\epsilon_s)\cdot\frac{\partial\epsilon_s^{ge}}{\partial\mathcal{S}}\right]\cdot L_S^\star-\left[n\cdot g(\xi_s)\cdot\frac{\partial\xi_s^{ge}}{\partial\mathcal{S}}\right]\cdot L_\xi^\star$$

The Lagrange multiplier for the Kuhn-Tucker Condition can thus be expressed as:

$$\left(\frac{\partial f^{ge}}{\partial\mathcal{S}}\cdot u\cdot L_V'-\left[n\cdot g(\epsilon_s)\cdot\frac{\partial\epsilon_s^{ge}}{\partial\mathcal{S}}\right]\cdot L_S^\star-\left[n\cdot g(\xi_s)\cdot\frac{\partial\xi_s^{ge}}{\partial\mathcal{S}}\right]\cdot L_\xi^\star\right)\cdot[S_{stw}(\epsilon_{stw})-S(\epsilon_{stw})]$$
$$-n\cdot\frac{\partial\Omega}{\partial\epsilon_{stw}}<0$$

Rearranging and simplifying gives the condition of Proposition 5.

$$\frac{\partial\Omega}{\partial\epsilon_{stw}}>\frac{\partial f^{ge}}{\partial\epsilon_{stw}}\cdot u\cdot L_V'+\left[n\cdot g(\epsilon_s)\cdot\left(-\frac{\partial\epsilon_s^{ge}}{\partial\epsilon_{stw}}\right)\right]\cdot L_S^\star+\left[n\cdot g(\xi_s)\cdot\left(-\frac{\partial\xi_s^{ge}}{\partial\epsilon_{stw}}\right)\right]\cdot L_\xi^\star$$

K.3 Derivation Optimal STW Benefits

Plugging equation K.1 into equation K.2 and iterating forward gives:

$$s_t\cdot\left(\bar{h}-h_{stw,t}(\epsilon_{s,t})\right)=\lambda_{\epsilon,t}\cdot(a_t\cdot h_{stw,t}(\epsilon_{s,t})^\alpha+c_f)\cdot\frac{1-G(\epsilon_{s,t})}{n_t\cdot g(\epsilon_{s,t})}$$

$$+\sum_{j=1}^{\infty}\left[\prod_{i=1}^{j}\beta\cdot(1-f_{t+i-1})\cdot(1-G(\epsilon_{s,t+i}))\right]\cdot\left[b_{t+j}-\int_{\epsilon_{s,t}}^{\epsilon_{stw,t}}\left(\bar{h}-h_{stw,t}(\epsilon)\right)\cdot s_t\right]$$

Inserting the Lagrange multiplier $\lambda_{\epsilon,t}$ and using the implicit function theorem so that

$$\frac{a_t\cdot h_{stw,t}(\epsilon_{s,t})^{\alpha}+c_f}{\bar{h}-h_{stw,t}(\epsilon)}=\frac{\frac{\partial S_t(\epsilon_{stw,t})}{\partial\epsilon_{s,t}}}{\frac{\partial S_{stw,t}(\epsilon_{s,t})}{s_t}}=-\frac{1}{\frac{\partial\epsilon_{s,t}}{\partial s_t}}>0$$

gives:

$$\begin{aligned}
&s_t\cdot\left(\bar{h}-h_{stw,t}(\epsilon_{s,t})\right)=\\
&-\left[\frac{n_t}{1-G(\epsilon_{s,t})}\cdot\frac{\partial\Omega_t}{\partial s_t}+\lambda_{\theta,t-1}\cdot\int_{\epsilon_{s,t}}^{\epsilon_{stw,t}}\left(\bar{h}-h_{stw,t}(\epsilon)\right)dG(\epsilon)\right]\\
&\quad\cdot\left(-\frac{1}{\frac{\partial\epsilon_{s,t}}{\partial s_t}}\right)\cdot\frac{1-G(\epsilon_{s,t})}{g(\epsilon_{s,t})\cdot n_t}\\
&+\sum_{j=1}^{\infty}\left[\prod_{i=1}^{j}\beta\cdot(1-f_{t+i-1})\cdot(1-G(\epsilon_{s,t+i}))\right]\\
&\quad\cdot\left[b_{t+j}-\int_{\epsilon_{s,t+j}}^{\epsilon_{stw,t+j}}\left(\bar{h}-h_{stw,t+j}(\epsilon)\right)\cdot s_{t+j}\,d\epsilon\right]
\end{aligned}$$

Assume that the system has converged to its non-stochastic steady state, then the STW subsidy can be expressed as:

$$\begin{aligned}
&s\cdot(\bar{h}-h_{stw}(\epsilon_s))=\\
&\quad\frac{\beta\cdot(1-f)}{1-\beta\cdot(1-G(\epsilon_s))\cdot(1-f)}\cdot\left(b-\int_{\epsilon_s}^{\epsilon_{stw}}(\bar{h}-h_{stw}(\epsilon))dG(\epsilon)\cdot s\right)\\
&\quad-\frac{1}{g(\epsilon_s)}\cdot\frac{\partial\Omega}{\partial s}/\frac{\partial\epsilon_s}{\partial s}\\
&\quad+\frac{\lambda_\theta}{g(\epsilon_s)}\cdot\int_{\epsilon_s}^{\epsilon_{stw}}\left(\bar{h}-h_{stw}(\epsilon)\right)dG(\epsilon)/\frac{\partial\epsilon_s}{\partial s}
\end{aligned}$$

Inserting expression for λ_θ gives

$$\begin{aligned}
&\frac{\beta\cdot(1-f)}{1-\beta\cdot(1-G(\epsilon_s))\cdot(1-f)}\cdot\left(b-\int_{\epsilon_s}^{\epsilon_{stw}}(\bar{h}-h_{stw}(\epsilon))dG(\epsilon)\cdot s\right)\\
&\quad-\frac{1}{g(\epsilon_s)}\cdot\frac{\partial\Omega}{\partial s}/\frac{\partial\epsilon_s}{\partial s}\\
&\quad-\left[\frac{\partial f^{ge}}{\partial\mathcal{S}}\cdot u\cdot L_V-\left[n\cdot g(\epsilon_s)\cdot\frac{\partial\epsilon_s^{ge}}{\partial\mathcal{S}}\right]\cdot L_S^{\star}-\left[n\cdot g(\xi_s)\cdot\frac{\partial\xi_s^{ge}}{\partial\mathcal{S}}\right]\cdot L_\xi^{\star}\right]\cdot\frac{\partial\mathcal{S}}{\partial s}/\left[g(\epsilon_s)\cdot\frac{\partial\epsilon_s}{\partial s}\right]
\end{aligned}$$

with

$$\frac{\partial \mathcal{S}}{\partial s} = \int_{\epsilon_s}^{\epsilon_{stw}} \Big(\bar{h} - h_{stw}(\epsilon)\Big)\, dG(\epsilon)$$

Simplifying gives:

$$\begin{aligned}
&\frac{\beta \cdot (1-f)}{1-\beta \cdot (1-G(\epsilon_s)) \cdot (1-f)} \cdot \left(b - \int_{\epsilon_s}^{\epsilon_{stw}} (\bar{h} - h_{stw}(\epsilon)) dG(\epsilon) \cdot s\right) \\
&\quad - \frac{1}{g(\epsilon_s)} \cdot \frac{\partial \Omega}{\partial s} \Big/ \frac{\partial \epsilon_s}{\partial s} \\
&\quad - \left[\frac{\partial f^{ge}}{\partial s} \cdot u \cdot L_V - \left[n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial s}\right] \cdot L_S^\star - \left[n \cdot g(\xi_s) \cdot \frac{\partial \xi_s^{ge}}{\partial s}\right] \cdot L_\xi^\star\right] \Big/ \left[g(\epsilon_s) \cdot \frac{\partial \epsilon_s}{\partial s}\right]
\end{aligned}$$

With $\epsilon_{stw} = \xi_s$ we get:

$$\begin{aligned}
&\frac{\beta \cdot (1-f)}{1-\beta \cdot (1-G(\epsilon_s)) \cdot (1-f)} \cdot \left(b - \int_{\epsilon_s}^{\epsilon_{stw}} (\bar{h} - h_{stw}(\epsilon)) dG(\epsilon) \cdot s\right) \\
&\quad - \frac{1}{g(\epsilon_s)} \cdot \frac{\partial \Omega}{\partial s} \Big/ \frac{\partial \epsilon_s}{\partial s} \\
&\quad - \left[\frac{\partial f^{ge}}{\partial s} \cdot u \cdot L_V - \left[n \cdot g(\epsilon_s) \cdot \frac{\partial \epsilon_s^{ge}}{\partial s}\right] \cdot L_S^\star - \left[n \cdot g(\epsilon_{stw}) \cdot \frac{\partial \epsilon_{stw}^{ge}}{\partial s}\right] \cdot L_\xi^\star\right] \Big/ \left[g(\epsilon_s) \cdot \frac{\partial \epsilon_s}{\partial s}\right]
\end{aligned}$$

This completes the proof of Proposition 6.

L Derivation Optimal Layoff Taxes

In this exercise, we swap the STW system with a layoff tax. The Ramsey planner chooses the layoff tax τ_S subject to the labor market equilibrium.

$$\begin{aligned}
W_t^G = \max_{\tau_{S,t}} \Bigg\{ & (1-n_t) \cdot u(b_t) + n_t \cdot u(\tilde{c}_t^w) \\
& + \nu_t^f \cdot u\Bigg(\Bigg[n_t \cdot \int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} \Big(z_t(\epsilon, h_t(\epsilon dG(\epsilon) \\
& \quad - n_t \cdot \tilde{c}_t^w - (1-n_t) \cdot b_t - \theta_t \cdot (1 - n_t + \rho_t \cdot n_t) \cdot k_v\Bigg] \Big/ \nu_t^f \Bigg) \\
& + \beta \cdot E_t W_{t+1}^G \Bigg\}
\end{aligned}$$

Job-Creation Condition:

$$\frac{1}{1-\eta_t} \cdot \frac{k_v}{q(\theta_t)} = E_t\Bigg[Q^w_{t,t+1}\Bigg(\int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} z_{t+1}(\epsilon, h_{t+1}(\epsilon))dG(\epsilon) + G(\epsilon_{s,t}) \cdot F - \frac{1-n_{t+1}}{n_{t+1}} \cdot b_{t+1} - \tilde{c}^w_{t+1} + \frac{u(\tilde{c}^w_{t+1}) - u(b_{t+1})}{u'(\tilde{c}^w_{t+1})}\Bigg)\Bigg] + E_t\left[Q^w_{t,t+1}\left((1-\rho_{t+1}) \cdot \frac{1-\eta_{t+1} \cdot f(\theta_{t+1})}{1-\eta_{t+1}} \cdot \frac{k_v}{q(\theta_{t+1})}\right)\right]$$

Income Insurance contract:

$$\eta_{t-1} \cdot \tilde{c}^w_t + (1-\eta_{t-1}) \cdot \frac{u(\tilde{c}^w_t)}{u'(\tilde{c}^w_t)} = \eta_{t-1} \cdot \left(\int_{\epsilon_{s,t}}^{\epsilon_{stw,t}} z_t(\epsilon, h_t(\epsilon))dG(\epsilon) + G(\epsilon_{s,t}) \cdot F - \frac{1-n_t}{n_t} \cdot b_t + (1-\rho_t) \cdot \theta_t \cdot k_v\right) + (1-\eta_{t-1}) \cdot \frac{u(b_t)}{u'(\tilde{c}^w_t)} - \frac{\eta_{t-1}-\eta_t}{1-\eta_t} \cdot (1-\rho_t) \cdot (1-f_t) \cdot \frac{k_v}{q_t}$$

Separation conditions on STW:

$$z_t(\epsilon_{s,t}, h_{stw,t}(\epsilon_{s,t})) + \tau_{S,t} + F + \frac{1-\eta_t \cdot f_t}{1-\eta_t} \cdot \frac{k_v}{q_t} = 0$$

Working hours:

$$\alpha \cdot a_t \cdot \epsilon \cdot h_t(\epsilon)^{\alpha-1} = h_t(\epsilon)^{\psi}$$

Stochastic discount factor:

$$Q^w_{t,t+1} = \beta \cdot \frac{u'(\tilde{c}^w_{t+1})}{u'(\tilde{c}^w_t)}$$

Law of motion of employment:

$$n_{t+1} = (1-\rho_t) \cdot n_t + f(\theta_t) \cdot (1-n_t+\rho_t \cdot n_t)$$

I keep the notation of Appendix H. Similar to Appendix H, we can derive the **constrained efficient job-creation condition:**

$$\Leftrightarrow \frac{1+\chi_{t-1}}{1-\gamma} \cdot \frac{k_v}{q(\theta_{t-1})}$$

$$
\begin{aligned}
&= \beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot \left(\int_{\epsilon_{s,t}}^{\infty} z_t(\epsilon, h_t(\epsilon)) dG(\epsilon) + G(\epsilon_{s,t}) \cdot F + b_t \right) \\
&\quad + \beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot \left(\frac{u(\tilde{c}_t^w) - u(b_t)}{u'(\tilde{c}_t^w)} + \frac{(\beta \cdot \lambda_{\theta,t-1} + \lambda_{c,t})}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{b_t}{n_t} \right) \\
&\quad + \beta \cdot \frac{u'(\tilde{c}_t^w)}{u'(\tilde{c}_{t-1}^w)} \cdot (1 - G(\epsilon_{s,t})) \cdot \frac{(1 - \gamma \cdot f(\theta_t)) + (1 - f(\theta_t)) \cdot \chi_t}{1 - \gamma} \cdot \frac{k_v}{q(\theta_t)}
\end{aligned}
$$

Following Appendix H we can derive a condition that expresses the deviation of the decentralized job-creation condition from the constrained efficient job-creation condition in the steady state and call it the **deviation from the optimal job-creation condition**.

$$
\begin{aligned}
&\left(\chi - \frac{\eta - \gamma}{1 - \eta} \right) \cdot \frac{1}{1 - \gamma} \cdot \frac{k_v}{q(\theta)} \\
&= \frac{\beta}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \right) \cdot \frac{b}{n}
\end{aligned}
$$

Following Appendix H, we can also derive the **constrained efficient optimal separation condition:**

$$
\begin{aligned}
\Leftrightarrow\ & z_t(\epsilon_{s,t}, h_t(\epsilon_{s,t})) + F \\
&+ \frac{1 - \gamma \cdot f(\theta_t) + (1 - f(\theta_t)) \cdot \chi_t}{1 - \gamma} \cdot \frac{k_v}{q(\theta_t)} \\
&- \frac{(1 + \eta \cdot BE_t) \cdot \lambda_{\theta,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \tau_{S,t} \\
&- \frac{BE_t \cdot \lambda_{\theta,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot \frac{1 - f(\theta_t)}{1 - \eta} \cdot \frac{k_v}{q(\theta_t)} \\
= & -\frac{1}{g(\epsilon_{s,t})} \cdot \frac{\lambda_{\epsilon,t}}{n_t \cdot u'(\tilde{c}_t^w)} \cdot (a_t \cdot h_t(\epsilon_{s,t})^\alpha + c_f)
\end{aligned}
$$

Imposing the steady state and subtracting the decentralized job-creation condition lets us rearrange for the **optimal STW subsidy.**

$$
\begin{aligned}
\Leftrightarrow\ \tau_S = & (1 - f(\theta)) \cdot \left(\chi - \frac{\eta - \gamma}{1 - \eta} \right) \cdot \frac{1}{1 - \gamma} \cdot \frac{k_v}{q(\theta)} \\
&- \frac{(1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot \tau_S \\
&+ \frac{1}{g(\epsilon_s)} \cdot \frac{\lambda_\epsilon}{n \cdot u'(\tilde{c}^w)} \cdot (a \cdot h(\epsilon_s)^\alpha + c_f) \\
&- \frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)} \cdot \frac{1 - f(\theta)}{1 - \eta} \cdot \frac{k_v}{q(\theta)}
\end{aligned}
$$

Inserting the deviation from the optimal job-creation condition gives:

$$\tau_S = \frac{\beta \cdot (1 - f(\theta))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \frac{b}{n} + \frac{\frac{1}{g(\epsilon_s)} \cdot \frac{\lambda_\epsilon}{n \cdot u'(\tilde{c}^w)}}{\left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right)} \cdot (a \cdot h(\epsilon_s)^\alpha + c_f)$$
$$- \frac{\frac{BE \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}}{\left(1 + \frac{\beta \cdot (1 + \eta \cdot BE) \cdot \lambda_\theta}{n \cdot u'(\tilde{c}^w)}\right)} \cdot \frac{1 - f(\theta)}{1 - \eta} \cdot \frac{k_v}{q(\theta)}$$

Forming the first derivative for the optimal layoff tax gives:

$$\frac{\partial}{\partial \tau_S} = -\lambda_\epsilon = 0$$

Note that it sets the Lagrange multiplier for the decentralized separation condition to zero. This means that the layoff tax can impose the optimal number of separations. Applying this gives the result from Proposition 7.

$$\tau_S = \frac{\beta \cdot (1 - f(\theta))}{1 - \beta \cdot (1 - f(\theta)) \cdot (1 - G(\epsilon_s))} \cdot \left[1 + \frac{1 - n}{n}\right] \cdot b - \tilde{BE}$$