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**Online Appendix to:**

**Improving the Median CPI: Maximal Disaggregation Isn't  
Necessarily Optimal**

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# Online Appendix

## Improving the Median CPI: Maximal Disaggregation Isn't Necessarily Optimal\*

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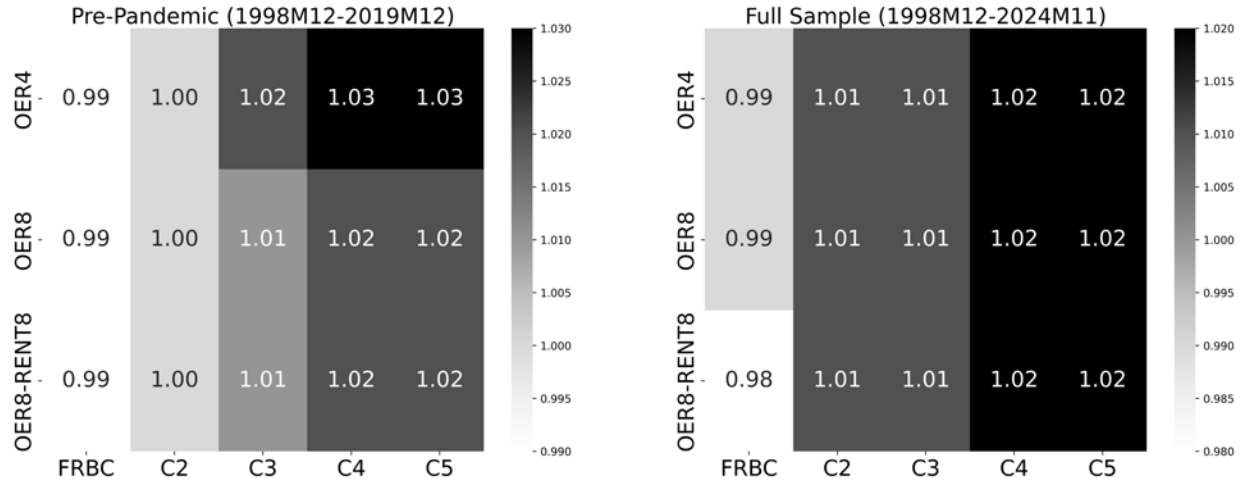
## A Appendix: Trimmed-Mean CPI Results

Performance of the trimmed-mean measures are generally less sensitive to the level of disaggregation (along either dimension) than are median measures. Differences across alternative series are often quite small.

### A.0.1 Accuracy in Mean

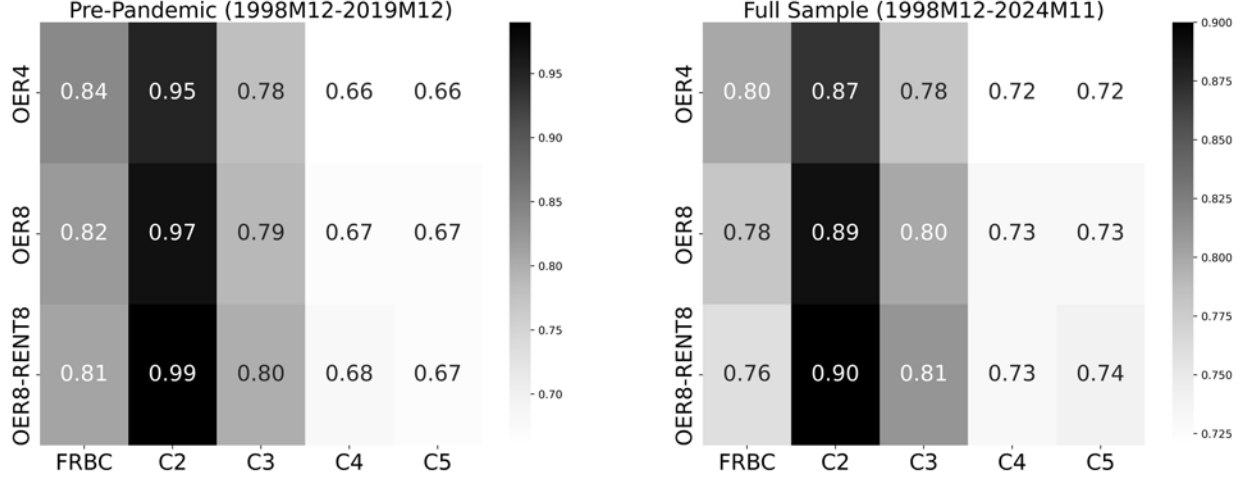
In Figure A.1, for each trimmed-mean inflation candidate, we report the ratio of the average trimmed-mean inflation rate and the average CPI inflation rate. All ratios are approximately equal to 1. In Figure A.2 we report  $p$ -values for  $t$ -tests of equality of means. Results show that the observed differences between average inflation in each candidate trimmed-mean measure and headline CPI inflation are not statistically significant

Figure A.1: Mean of Trimmed-Mean Inflation Measures Relative to Mean of CPI Inflation



Notes: Reported figures are the ratio of the average of the trimmed-mean inflation rate and the average of CPI inflation. Both averages are computed as the mean of 12-month inflation rates, measured by percent changes, over the indicated period. Darker shading indicates higher values, while lighter shading indicates lower values.

Figure A.2:  $p$ -Values of a Statistical Test of Equal Mean Relative to CPI Inflation

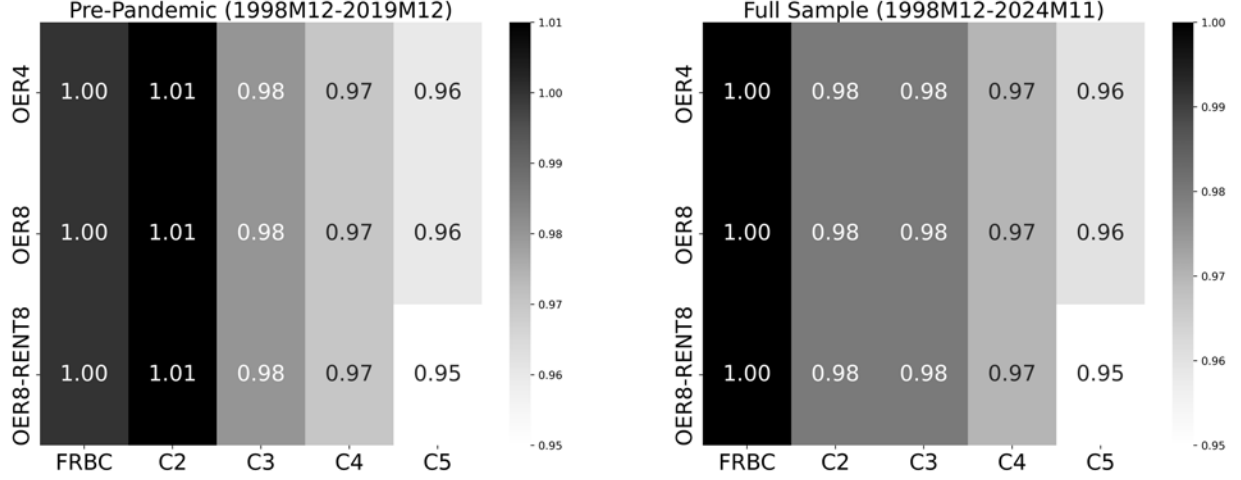


Notes: Reported figures are the  $p$ -values of a  $t$ -test of  $H_0 : \mathbb{E}[\pi_j^c] = \mathbb{E}[CPI]$ , where  $\pi_j^c$  denotes the  $j$ th candidate trimmed-mean inflation measure. The  $p$ -value is obtained by taking the difference of each trimmed-mean inflation measure from CPI inflation, and regressing this against a constant. The test statistic of the constant term is calculated using heteroskedasticity-and-autocorrelation-consistent (HAC) standard errors with a small sample correction and  $\lfloor 4[T/100]^{2/9} \rfloor$  lags, where  $T$  refers to the size of the estimation sample. Darker shading indicates higher values, while lighter shading indicates lower values.

### A.0.2 Accuracy versus a Standard Ex-post MTT Estimate

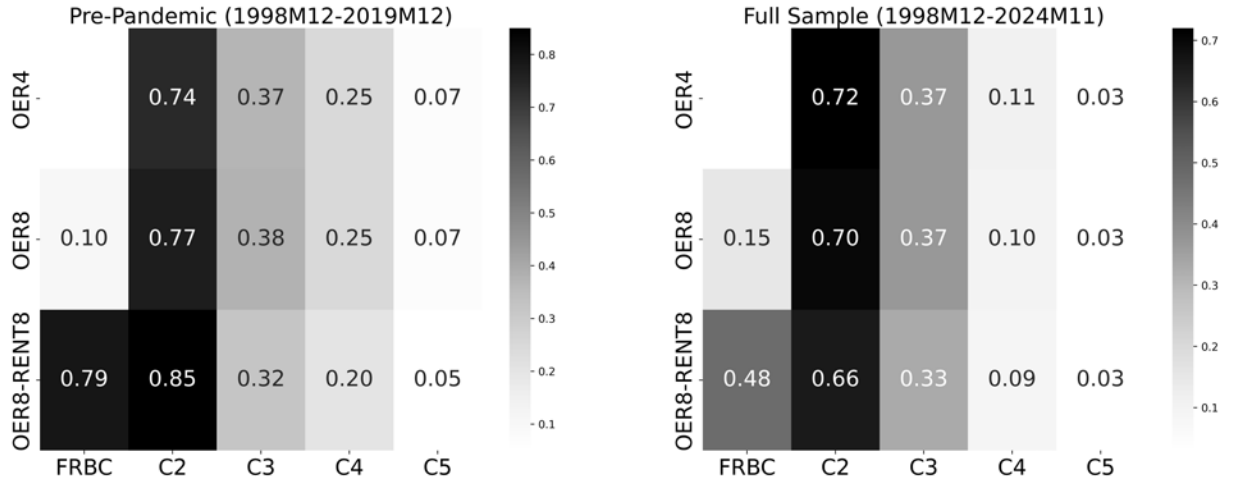
In Figure A.3, we report the RMSE of each measure of trimmed-mean inflation against a 37-month centered moving average (37MMA) of 12-month CPI inflation relative to the RMSE of the baseline trimmed-mean FRBC-OER4 inflation measure. C5-OER8-RENT8 outperforms, with an RMSE that is 5% lower than that of trimmed-mean FRBC-OER4 inflation pre-pandemic and in the full sample. DM  $p$ -values reported in Figure A.4 show that the observed reduction in the RMSE of C5-OER8-RENT8 relative to FRBC-OER4 is statistically significant at the 10% level in the pre-pandemic sample and at the 5% level in the full sample.

Figure A.3:  $RMSE(\hat{\pi}^{37MMA} - \hat{\pi}_j)$  of Trimmed-Mean Inflation Measures Relative to FRBC-OER4



Notes: Reported figures are the RMSE of deviations of the trimmed-mean inflation measure from a 37-month centered moving average of CPI Inflation, divided by the same for trimmed-mean FRBC-OER4 inflation. In the pre-pandemic sample, the moving average is computed using CPI inflation through December 2019 only. Darker shading indicates higher values, while lighter shading indicates lower values.

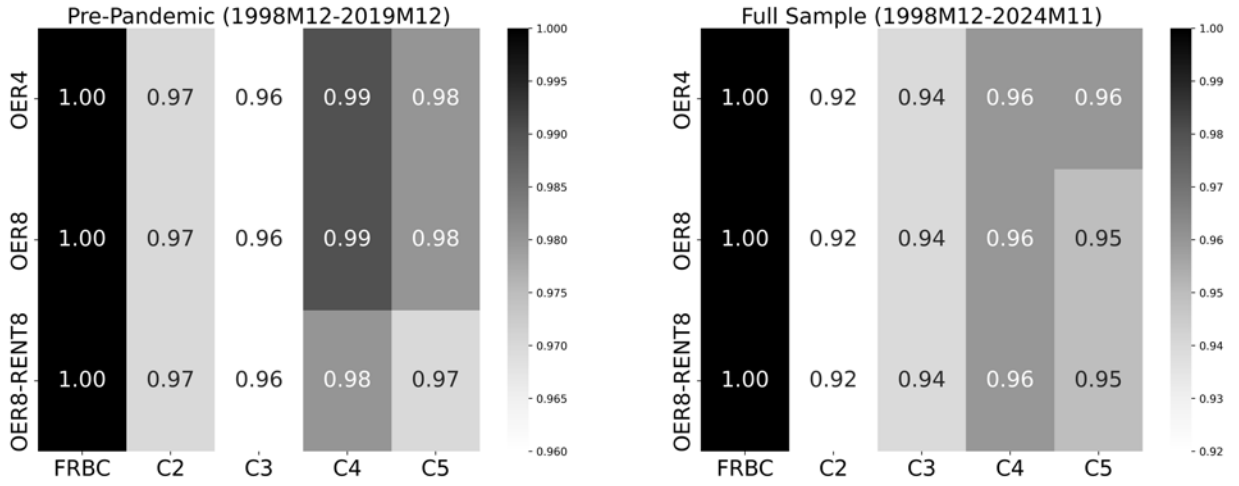
Figure A.4:  $p$ -Values of a Statistical Test of Equal Ability in Tracking  $\hat{\pi}^{37MMA}$  for Trimmed-Mean Inflation Measures, Relative to FRBC-OER4



Notes: Reported figures are the  $p$ -values of a Diebold-Mariano (1995) test that  $RMSE(\hat{\pi}^{37MMA} - \pi_{FRBC-OER4}^c)$  and  $RMSE(\hat{\pi}^{37MMA} - \pi_j^c)$  are equal, where  $j$  denotes the  $j$ th candidate trimmed-mean inflation measure. The  $p$ -value is obtained by taking the difference of the two squared errors series  $e_{j,t}^{37MMA}$  and  $e_{FRBC-OER4,t}^{37MMA}$ , and regressing the resulting series against a constant. The test statistic of the constant term is then calculated using HAC standard errors with a small sample correction and  $\lfloor 4[T/100]^{2/9} \rfloor$  lags, where  $T$  refers to the size of the estimation sample. Darker shading indicates higher values, while lighter shading indicates lower values.

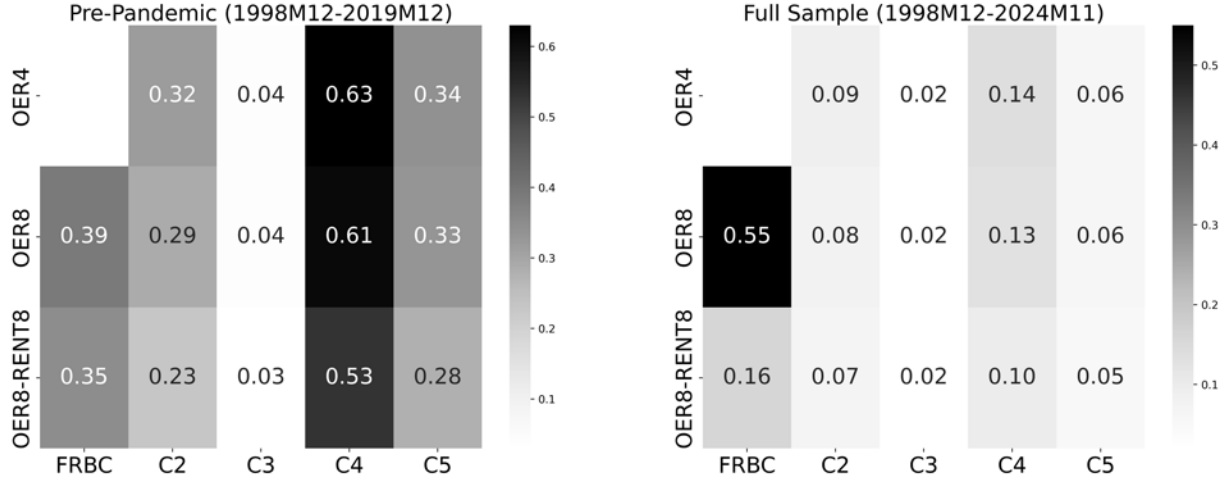
In Figure A.5, we report the RMSE of each trimmed-mean inflation measure against the 2SMA trend relative to the same for trimmed-mean FRBC-OER4 inflation, and in Figure A.6 we report for each trimmed-mean candidate the  $p$ -values of the DM test of equal ability in tracking the 2SMA trend estimate against the FRBC-OER4 baseline. In contrast to results using the 37MMA trend, C3 splits perform best in the pre-pandemic sample, achieving a 4% reduction in RMSE relative to the FRBC-OER4 baseline, while C2 splits perform best in the full sample, achieving an 8% reduction in RMSE, both of which are statistically significant at the 5% level.

Figure A.5:  $RMSE(\hat{\pi}^{2SMA} - \hat{\pi}_j)$  of Trimmed-Mean Inflation Measures Relative to FRBC-OER4



Notes: Reported figures are the RMSE of deviations of the trimmed-mean inflation measure from a two-stage centered moving average (2SMA) of CPI inflation, divided by the same for trimmed-mean FRBC-OER4 inflation. In the pre-pandemic sample, the moving average is computed using CPI inflation through December 2019 only. Darker shading indicates higher values, while lighter shading indicates lower values.

Figure A.6:  $p$ -Values of a Statistical Test of Equal Ability in Tracking  $\hat{\pi}^{2SMA}$  for Trimmed-Mean Inflation Measures, Relative to FRBC-OER4



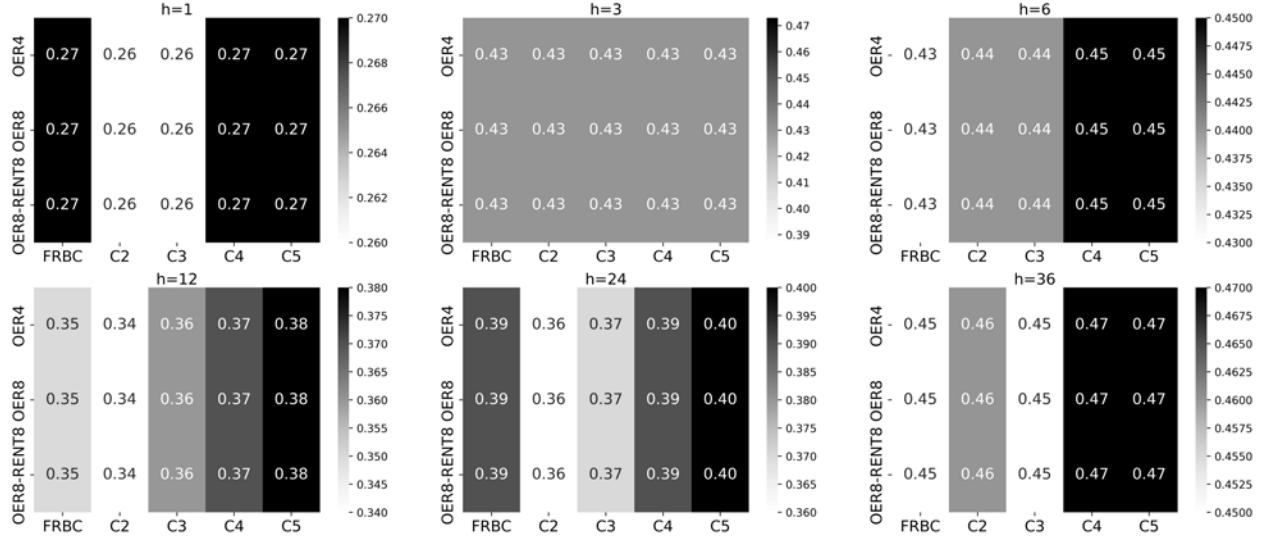
Notes: Reported figures are the  $p$ -values of a Diebold-Mariano (1995) test that  $RMSE(\hat{\pi}^{2SMA} - \hat{\pi}_{FRBC-OER4})$  and  $RMSE(\hat{\pi}^{2SMA} - \hat{\pi}_j)$  are equal, where  $j$  denotes the  $j$ th candidate trimmed-mean inflation measure. The  $p$ -value is obtained by taking the difference of the two squared errors series  $\hat{e}_{j,t}^{2SMA}$  and  $\hat{e}_{FRBC-OER4,t}^{2SMA}$ , and regressing the resulting series against a constant. The test statistic of the constant term is then calculated using HAC standard errors with a small sample correction and  $\lfloor 4[T/100]^{2/9} \rfloor$  lags, where  $T$  refers to the size of the estimation sample. Darker shading indicates higher values, while lighter shading indicates lower values.

## A.1 Predictive Power over Future Inflation

### A.1.1 In-sample Explanatory Power

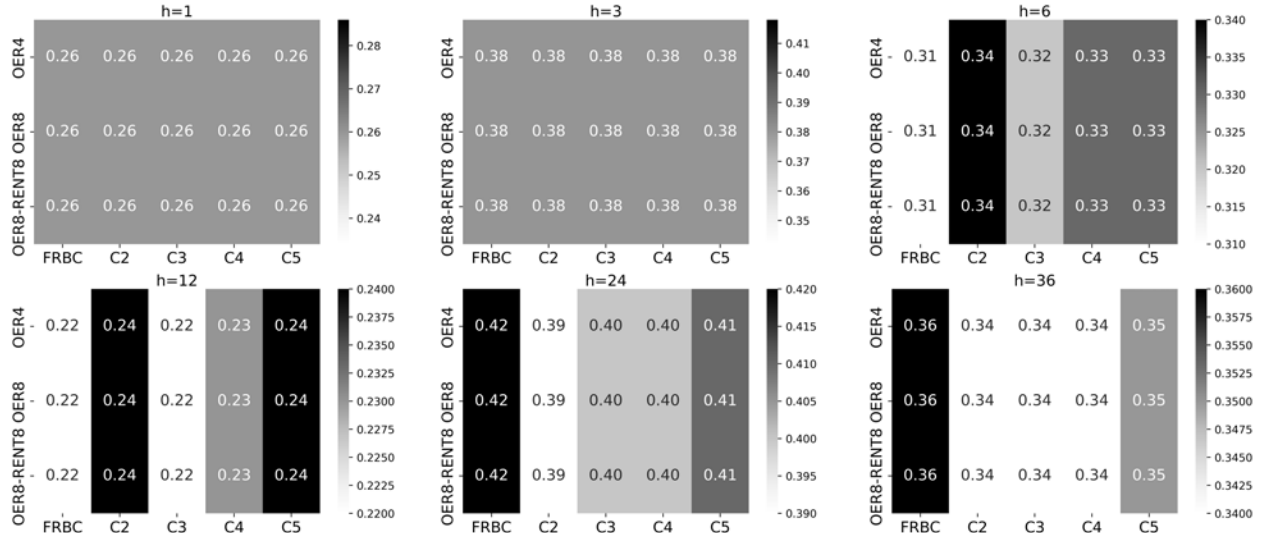
In Figure A.7 and Figure A.8 we report the adjusted  $R^2$  from fitting Equation 2 for each measure of trimmed-mean inflation and for horizons  $h \in \{1, 3, 6, 12, 24, 36\}$  in the pre-pandemic and full samples, respectively. Pre-pandemic, C5 measures match or exceed the rest, although differences between measures are small. Results are more mixed in the full sample, with small differences between measures once again.

Figure A.7: In-Sample Adjusted  $R^2$  of Equation 2, Pre-Pandemic Sample (1998M12-2019M12)



Notes: Reported figures are the adjusted  $R^2$  from fitting Equation 2 for each  $j$ th candidate trimmed-mean inflation measure.  $h$  denotes the horizon in months. Darker shading indicates higher values, while lighter shading indicates lower values.

Figure A.8: In-Sample Adjusted  $R^2$  of Equation 2, Full Sample (1998M12-2024M11)



Notes: Reported figures are the adjusted  $R^2$  from fitting Equation 2 for each  $j$ th candidate trimmed-mean inflation measure.  $h$  denotes the horizon in months. Darker shading indicates higher values, while lighter shading indicates lower values.



### A.1.2 Out-of-Sample Forecasting Ability

To measure trimmed-mean CPI's predictive accuracy over headline CPI, we report in Table A.1 and Table A.2 out-of-sample-forecast RMSFEs relative to the RMSFE of forecasts made using the FRBC-OER4 benchmark for the pre-pandemic and full samples, respectively.

In the pre-pandemic sample, the largest statistically significant gains in forecast accuracy occur at the 6-month (C4-OER8-RENT8 at the 5% level and C5-OER8-RENT8 at the 10% level) and 12-month horizons (C5 measures, at the 10% level). In the full sample, the largest statistically significant gains in forecast accuracy occur at the 6-month horizon (C2 measures, at the 10% level).

Table A.1: Relative RMSFEs of Out-of-Sample Forecasts Using Equation 2, Pre-Pandemic Sample (1998M12-2019M12)

Forecast Horizon (Months)	1	3	6	12	24	36
FRBC-OER4 RMSFE	2.25	1.8	1.53	1.24	1.21	0.98
FRBC-OER8	1.0	1.0	1.0	1.0	1.0	1.01
FRBC-OER8-RENT8	1.0	1.0	1.0*	1.0*	1.0	1.0
C2-OER4	1.0	0.99	0.99	0.97	1.04	0.98
C2-OER8	1.0	0.99	0.99	0.97	1.04	0.99
C2-OER8-RENT8	1.0	0.99	0.99	0.97	1.03	0.98
C3-OER4	1.0	0.99	0.99	0.99	1.02	0.99
C3-OER8	1.0	0.99	0.99	0.99	1.02	0.99
C3-OER8-RENT8	1.0	0.99	0.99*	0.99	1.01	0.99
C4-OER4	1.0	1.0	0.98**	0.98	1.01	1.0
C4-OER8	1.0	1.0	0.98**	0.98	1.01	1.0
C4-OER8-RENT8	1.0	1.0	0.97**	0.97	1.0	1.0
C5-OER4	1.0	0.99	0.97	0.96*	0.99	1.02
C5-OER8	1.0	0.99	0.97	0.96*	0.99	1.02
C5-OER8-RENT8	1.0	0.99	0.97*	0.96*	0.99	1.02

Notes: RMSFE is the root mean squared forecast error. The row "FRBC-OER4 RMSFE" reports the raw RMSFE of the FRBC-OER4 benchmark for each forecast horizon. All other rows report relative RMSFEs, with the RMSFE of FRBC-OER4 for forecast horizon  $h$  taken as the denominator for relative RMSFEs in column forecast horizon  $h$ . Relative RMSFEs less than 1 are highlighted in green. For each relative RMSFE, we calculate the Diebold and Mariano (1995) (DM) test with the small-sample correction of Harvey, Leybourne, and Newbold (1997) for equal predictive accuracy between a given forecast and the forecast from the FRBC-OER4 benchmark. Relative RMSFEs in bold and with \*, \*\*, or \*\*\* denote rejections of the null hypothesis of equal predictive accuracy of the alternative median CPI measure and the FRBC-OER4 benchmark at the 10%, 5%, or 1% level, respectively, based on the DM test.

Table A.2: Relative RMSFEs of Out-of-Sample Forecasts Using Equation 2, Full Sample (1998M12-2024M11)

Forecast Horizon (Months)	1	3	6	12	24	36
FRBC-OER4 RMSFE	2.71	2.35	2.17	2.18	2.26	2.03
FRBC-OER8	1.0*	1.0	1.0	1.0	1.0	1.0*
FRBC-OER8-RENT8	1.0	1.0	1.0**	1.0**	1.0	1.0
C2-OER4	1.0	0.99	<b>0.96*</b>	0.97	1.03*	1.02
C2-OER8	1.0	0.99	<b>0.96*</b>	0.97	1.03*	1.02
C2-OER8-RENT8	1.0	0.99	<b>0.96*</b>	0.97	1.03	1.02
C3-OER4	1.0	1.0	0.99	1.0	1.02	1.02
C3-OER8	1.0	1.0	0.99	1.0	1.02	1.02
C3-OER8-RENT8	1.0	0.99	0.99	1.0	1.02	1.02
C4-OER4	1.01	0.99	<b>0.98*</b>	1.0	1.02	1.02
C4-OER8	1.01	0.99	<b>0.98*</b>	1.0	1.02	1.03
C4-OER8-RENT8	1.01	0.99	<b>0.98*</b>	0.99	1.02	1.02
C5-OER4	1.01	1.0	0.98	0.99	1.02	1.02
C5-OER8	1.01	1.0	0.98	0.99	1.02	1.02
C5-OER8-RENT8	1.01	1.0	0.98	0.99	1.02	1.02

Notes: RMSFE is the root mean squared forecast error. The row "FRBC-OER4 RMSFE" reports the raw RMSFE of the FRBC-OER4 benchmark for each forecast horizon. All other rows report relative RMSFEs, with the RMSFE of FRBC-OER4 for forecast horizon  $h$  taken as the denominator for relative RMSFEs in column forecast horizon  $h$ . Relative RMSFEs less than 1 are highlighted in green. For each relative RMSFE, we calculate the Diebold and Mariano (1995) (DM) test with the small-sample correction of Harvey, Leybourne, and Newbold (1997) for equal predictive accuracy between a given forecast and the forecast from the FRBC-OER4 benchmark. Relative RMSFEs in bold and with \*, \*\*, or \*\*\* denote rejections of the null hypothesis of equal predictive accuracy of the alternative median CPI measure and the FRBC-OER4 benchmark at the 10%, 5%, or 1% level, respectively, based on the DM test.

## A.2 Summary of Results

We find that trimmed-mean CPI is insensitive to increasing shelter disaggregation, but can benefit from increasing the level of non-shelter disaggregation beyond FRBC. However, the optimal level of non-shelter disaggregation is unclear, as it appears to shift depending on the criteria and the sample period over which performance on the criteria is evaluated.

### A.3 Empirical Application: Phillips Curve Relationship for Trimmed-Mean CPI

We estimate Equation (3) over two different sample periods for Trimmed-Mean CPI series. There is very little variation in coefficient estimates across different levels of disaggregation in either dimension.

Table A.3: Estimated Phillips Curve Slope

Series	Trimmed-Mean CPI			
	Sample: 2000-2019		Sample: 2000-2024	
	$\hat{\beta}$	$p$ -value	$\hat{\beta}$	$p$ -value
FRBC-OER4	-0.195	0.00	-0.310	0.00
FRBC-OER8	-0.193	0.00	-0.308	0.00
FRBC-OER8-RENT8	-0.193	0.00	-0.308	0.00
C2-OER4	-0.182	0.00	-0.292	0.00
C2-OER8	-0.181	0.00	-0.290	0.00
C2-OER8-RENT8	-0.181	0.00	-0.290	0.00
C3-OER4	-0.190	0.00	-0.308	0.00
C3-OER8	-0.190	0.00	-0.307	0.00
C3-OER8-RENT8	-0.190	0.00	-0.307	0.00
C4-OER4	-0.178	0.00	-0.299	0.00
C4-OER8	-0.178	0.00	-0.298	0.00
C4-OER8-RENT8	-0.178	0.00	-0.298	0.00
C5-OER4	-0.180	0.00	-0.300	0.00
C5-OER8	-0.179	0.00	-0.299	0.00
C5-OER8-RENT8	-0.179	0.00	-0.299	0.00

Note: The estimates shown are for two different estimation samples: 2000M1 through 2019M12 (denoted Sample: 2000-2019) and 2000M1 through 2024M11 (denoted Sample: 2000-2024). The data from 1999M1 through 1999M12 are used to compute the lagged value of the unemployment rate gap. Standard errors are computed using heteroskedasticity-and-autocorrelation-consistent (HAC) standard errors with a small sample correction and  $\lfloor 4[T/100]^{2/9} \rfloor$  lags, where  $T$  refers to the size of the estimation sample.

## B Appendix: Nonlinear Phillips Curve, Median CPI

### B.1 Nonlinearities in Phillips Curve: Threshold Regression

Recent work has highlighted nonlinearities in the Phillips curve. We extend Equation (3) to allow for nonlinearities in the Phillips curve by using threshold regression, i.e., approximating the curvature of a nonlinear function by a piecewise-linear function. Following Doser et al. (2023), we estimate the model with a continuity constraint. In particular, we construct the variable  $z_t = \max(x_t - \bar{x}_t, 0)$ , where  $x_t$  is defined in equation (4), the average of the unemployment gap over the preceding 12 months, and selection of  $\bar{x}_t$  is described momentarily. The term  $z_t$  is included as a regressor in Equation (3), to yield

$$\pi_{j,t} = \alpha_j + \beta_{1,j}x_t + \beta_{2,j}z_t + e_{j,t} \quad (1)$$

If the Phillips curve is linear, then  $\hat{\beta}_2$  will be statistically indistinguishable from 0. If not, then the slope of the Phillips curve is different when the unemployment gap exceeds  $\bar{x}_t$ . We select  $\bar{x}_t$  as a round number that minimizes the residual sum of squares (see Hansen, 1996) for the FRBC-OER4 case, with a constraint that each regime must contain at least 15% of the sample. The estimation procedure does not restrict the overall shape of the Phillips curve: it could be convex – a steeper Phillips curve when the labor market is tight, pushing up inflation – or concave – a steeper Phillips curve when the labor market is slack, pushing down inflation.

Our search procedure yielded  $\bar{x}_t = 3\frac{1}{4}$ . Linearity is fairly convincingly rejected: in these data and over this time period, the main departure from linearity relates to high unemployment gap periods. In particular, the Phillips curve is concave, i.e., steeper when the unemployment gap is above  $3\frac{1}{4}$ . Slope estimates are modestly sensitive to the degree of disaggregation: as in the simpler linear specification in the main text, for these Median CPI series, Phillips curve coefficients generally become smaller – and for  $\hat{\beta}_2$ , sometimes statistically insignificant (at the 5% level) – with increased disaggregation of the shelter indexes. ( $\hat{\beta}_2$  is smallest at the C2 level of disaggregation for non-shelter indexes.)

Table B.1: Estimated Phillips Curve Slope (Kinked)

Series	Median CPI			
	$x_t$		$z_t$	
	$\hat{\beta}_1$	$p$ -value	$\hat{\beta}_2$	$p$ -value
FRBC-OER4	-0.189	0.00	-0.540	0.01
FRBC-OER8	-0.187	0.00	-0.510	0.02
FRBC-OER8-RENT8	-0.170	0.00	-0.474	0.03
C2-OER4	-0.206	0.00	-0.473	0.02
C2-OER8	-0.185	0.00	-0.454	0.02
C2-OER8-RENT8	-0.185	0.00	-0.311	0.13
C3-OER4	-0.213	0.00	-0.525	0.01
C3-OER8	-0.207	0.00	-0.511	0.01
C3-OER8-RENT8	-0.193	0.00	-0.400	0.06
C4-OER4	-0.217	0.00	-0.581	0.01
C4-OER8	-0.192	0.00	-0.567	0.00
C4-OER8-RENT8	-0.180	0.00	-0.465	0.02
C5-OER4	-0.223	0.00	-0.569	0.01
C5-OER8	-0.200	0.00	-0.555	0.01
C5-OER8-RENT8	-0.185	0.00	-0.438	0.02

Note: The estimates shown are for 2000M1 through 2019M12.  $\beta_1$  and  $\beta_1$  are from Equation (5). As  $\beta_2$  is statistically significant and negative, this means that the Phillips curve is concave: the slope is steeper when the unemployment gap exceeds  $3\frac{1}{4}$ . Standard errors are computed using heteroskedasticity-and-autocorrelation-consistent (HAC) standard errors with a small sample correction and  $\lfloor 4[T/100]^{2/9} \rfloor$  lags, where  $T$  refers to the size of the estimation sample.

### B.1.1 Nonlinearities in Phillips Curve: Frequency-Dependent Regression

Ashley and Verbrugge (2025) found that inflation responds differently to persistent (low-frequency) versus moderately persistent (or transient) fluctuations in the unemployment gap. The nature of the frequency-dependence uncovered aligns with business-cycle stages. Inflation responds strongly to “overheating,” i.e., to the unemployment gap when that gap is persistently negative, and to “recessions,” i.e., to the positive part of the moderate-frequency gap – which becomes positive when the unemployment gap *is rising sharply*, and falls back to zero shortly after the unemployment rate peaks.<sup>1</sup> This frequency-dependent specification solves numerous “inflation puzzles” — e.g.,

<sup>1</sup>Appendix H in Ashley and Verbrugge (2025) relates these frequency-dependent findings to existing economic theory.

missing inflation/disinflation — that have been noted in the literature. Importantly, its forecasting performance is on par with conventional benchmarks in the forecasting literature; see the original paper, Verbrugge and Zaman (2023), and Verbrugge and Zaman (2024). Inspired by these papers, we here estimate

$$\pi_{j,t} = \alpha_j + \gamma_{1,j} \left( gap_{low-freq.,t-12}^- \right) + \gamma_{2,j} \left( gap_{med-freq.,t-12}^+ \right) + \lambda \pi_{j,t-12} + e_{j,t}$$

where  $\left( gap_{low-freq.,t-12}^- \right)$  is the negative part of the low-frequency component of the unemployment gap, and  $\left( gap_{med-freq.,t-12}^+ \right)$  is the positive part of the medium-frequency component of the unemployment gap. Note that, as explained in Ashley and Verbrugge (2025), one must use a one-sided frequency decomposition of the unemployment gap.

Coefficient estimates are quantitatively large, and linearity is rejected in favor of frequency-dependence.<sup>2</sup> There is some sensitivity to the degree of disaggregation. The medium-frequency (“recession”) Phillips curve coefficients,  $\hat{\gamma}_2$ , are smallest at the C2 level of disaggregation for non-shelter indexes, and – as in the results in the main text, — are generally smaller with more disaggregation of shelter indexes.

Conversely, the low-frequency Phillips curve coefficients,  $\hat{\gamma}_1$ , generally become *larger* with more disaggregation of shelter indexes. Further, after *peaking* at the C2 level, these coefficients become *smaller* – and have declining *p*-values – with more disaggregation of other indexes.

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<sup>2</sup>A linear data generating process yields the same coefficient estimate across all frequencies.

Table B.2: Estimated Phillips Curve Slope (Frequency-Dependent)

Series	Median CPI			
	$\left(gap_{low-freq.,t-12}^{-}\right)$		$\left(gap_{med-freq.,t-12}^{+}\right)$	
	$\hat{\gamma}_1$	$p$ -value	$\hat{\gamma}_2$	$p$ -value
FRBC-OER4	-0.872	0.01	-2.705	0.00
FRBC-OER8	-0.926	0.00	-2.636	0.00
FRBC-OER8-RENT8	-0.947	0.00	-2.493	0.00
C2-OER4	-0.939	0.04	-2.576	0.00
C2-OER8	-0.950	0.04	-2.427	0.00
C2-OER8-RENT8	-1.080	0.02	-2.190	0.00
C3-OER4	-0.870	0.04	-2.772	0.00
C3-OER8	-0.904	0.02	-2.682	0.00
C3-OER8-RENT8	-0.985	0.02	-2.439	0.00
C4-OER4	-0.774	0.06	-2.912	0.00
C4-OER8	-0.779	0.05	-2.677	0.00
C4-OER8-RENT8	-0.747	0.04	-2.481	0.00
C5-OER4	-0.606	0.09	-2.870	0.00
C5-OER8	-0.584	0.08	-2.727	0.00
C5-OER8-RENT8	-0.682	0.04	-2.404	0.00

Note: The estimates shown are for 2000M1 through 2019M12.  $\hat{\gamma}_1$  and  $\hat{\gamma}_2$  are from regression specification immediately above. This is a frequency-dependent specification.  $\hat{\gamma}_1$  is the “overheating” coefficient, i.e., the coefficient on the *negative* low-frequency gap; this term is nonzero when  $u_t < u_t^*$  persistently.  $\hat{\gamma}_2$  is the “recession” coefficient, i.e., the *positive* part of the medium-frequency component; this term is nonzero when the unemployment rate is *climbing rapidly* and becomes zero shortly after the unemployment rate peaks. Standard errors are computed using heteroskedasticity-and-autocorrelation-consistent (HAC) standard errors with a small sample correction and  $\lfloor 4[T/100]^{2/9} \rfloor$  lags, where  $T$  refers to the size of the estimation sample.

## C Appendix: Median Versus Mean, for Finding the Center of the Cross-sectional Distribution of Monthly Inflation Rates

The median of a sample is its middle value, i.e., half of the points in the sample are higher than the median and half are lower.

But some samples have weights, i.e., each point has an associated weight. This is the case in the CPI, where each point represents the percentage change in the index of a given category of goods or services, and the weights correspond to the aggregation weights in the CPI. The CPI in a given

month is the weighted mean of this sample.

Corresponding to a weighted mean, one can also construct a weighted median. A weighted median of a sample is constructed using the same logic as an unweighted median: the weighted median is that value such that half of the weight of the data lies below it. In that sense, like the median, it identifies the middle value (i.e., the center of the distribution).

A weighted median is very insensitive to outliers, as the following thought experiment demonstrates. Consider taking just a single point in the sample that lies above the median, and replacing it with a point that is arbitrarily large. The weighted median will not change its value (at all): this outlier has not contaminated the information in the weighted median.

Meanwhile, as noted above, the CPI is a weighted *average*. If a distribution is Normally distributed, a weighted average provides a very good signal of the center of the sample. But a weighted mean has a serious drawback. Unlike the weighted median, a weighted mean is extremely sensitive to outliers. For instance, one may increase the weighted mean by an arbitrary amount by taking just one single point that lies above the weighted mean—no matter how small its weight—and increasing it by a sufficiently large amount.

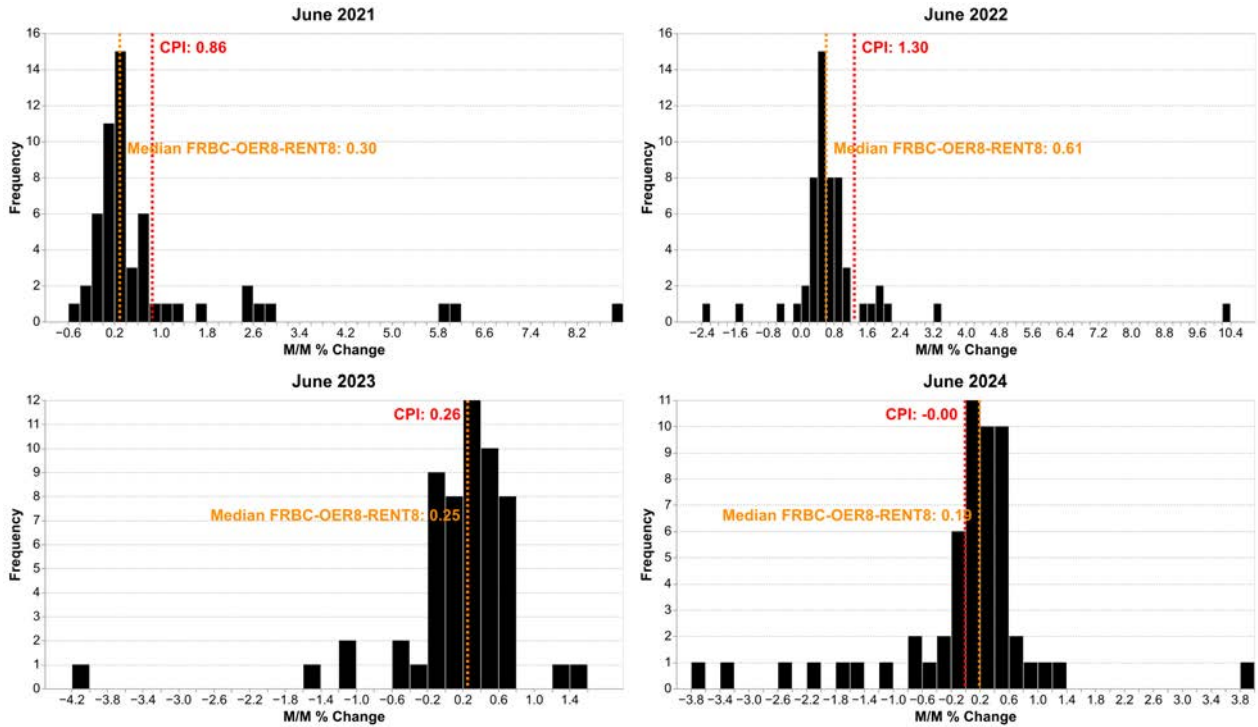
The cross-sectional distribution of CPI components is quite leptokurtic, i.e., has fat tails. This means that frequently, a given sample will have unusually big, or unusually small, points. Particularly if those points are associated with non-negligible weights, the sample mean will be pulled very strongly in the direction of the unusual points, and fail to provide an efficient signal of the center of the distribution. Indeed, this provides the rationale for limited-influence estimators, such as the median or trimmed-mean, which perform much better when the distribution is quite leptokurtic.

This behavior is well-illustrated for the four months of the CPI’s underlying component growth rates depicted in Figure B.1: June 2021, June 2022, June 2023 and June 2024. Depicted in the four panels of the figure are month-over-month percentage changes in the CPI components, along with the official CPI reading of that month, and the corresponding Median CPI reading (corresponding to our FRBC-OER8-Rent8 variant). In June 2021, for instance, there are three extreme positive outliers (as discussed in the main text). They are massively larger than the bulk of the inflation observations that month. The median CPI (in M/M terms), depicted with an orange dotted line, reads 0.3 for that month. It is easy to see that it is providing an accurate read of the center of



the bulk of the inflation component readings. Conversely, the CPI (in red) – pulled as it was by those outliers – came in at more than double that amount, namely, at 0.86. In this month, the CPI was clearly not a reasonable signal of the center of the distribution. In June 2022, we see something qualitatively similar: there is a mass of observations lying between -0.4 and +1.3, and it is unsurprising to see that the median is 0.61. The CPI, conversely, read +1.30 that month, more than double the value of the median, and well above the bulk of the observations, pulled as it was by a few outliers (including one extreme outlier). Conversely, we see that in June 2023, the sample distribution of CPI component growth rates was only modestly negatively skewed with only two outliers that were not nearly as extreme. The CPI ended up close to the Median value (0.26 vs. 0.25); but in this month, unlike the previous two cases we just looked at, the CPI seems a reasonable reading of the center of the distribution. Finally, in June 2024, the sample distribution is clearly displaying a more pronounced negative skew, with a fairly large number of observations in the deeply negative range. These negative outliers pulled the CPI down to a reading of 0.0, while the Median appears to be providing a more reasonable signal of the center of the distribution, at +0.19.

Figure C.1: Cross-sectional Distribution of Inflation in CPI Components



## D Appendix: A Brief History of Improvements to the FRBC Median and Trimmed-Mean CPIs

As noted in the main text, the FRBC Median and 16% Trimmed-Mean CPI inflation originated from the seminal work of Bryan and Pike (1991) and Bryan and Cecchetti (1994), who were the first to propose a theoretical and statistical justification for the use of the median or trimmed mean as measures of “core” (or trend) inflation (Dolmas and Wynne 2008).<sup>3</sup> While the FRBC has published these limited-influence estimators of the MTT in CPI inflation for decades, the components of CPI inflation from which these measures are calculated have evolved over time.<sup>4</sup>

Prior to 1998, the FRBC calculated the Median and Trimmed-Mean CPI using 36 CPI components. In 1998, the Bureau of Labor Statistics (BLS) carried out its sixth comprehensive revision of the CPI, leading the FRBC to modify its component basket, for a revised total of 41 components.<sup>5</sup> Importantly, prior to 1998, Median and Trimmed-Mean CPI used the Shelter component, whereas after 1998, Shelter was split into: Rent of primary residence (Rent); Lodging away from home; Owners’ Equivalent Rent of primary residence (OER); and Tenants’ and household insurance.<sup>6</sup>

In July 2007, the FRBC again revised the Median and Trimmed-Mean CPI. Under this new “Revised Methodology,” OER was split into four regional OER subindexes, one each for the Northeast, Midwest, South, and West. This change was prompted by research by Brischetto and Richards (2007)—later confirmed by the FRBC (2007)—that found that breaking up OER improved the ability of trimmed-mean measures to track the trend in CPI inflation. Concurrently, the FRBC added the component Leased Cars and Trucks, bringing the total to 45 CPI components.<sup>7</sup>

Since its introduction in 2007, small methodological adjustments have since been made to the “Revised Methodology” Median and Trimmed-Mean CPI to ensure that it reflects the most recent

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<sup>3</sup>The earliest precursor to today’s Median CPI in Bryan and Pike (1991) was derived from just seven CPI components.

<sup>4</sup>The FRBC updates the Median and Trimmed-Mean CPI each month immediately following a new CPI data release by the Bureau of Labor Statistics (BLS) and makes these data available at <https://www.clevelandfed.org/en/indicators-and-data/median-cpi>

<sup>5</sup>For more on this and other revisions, see <https://www.bls.gov/cpi/additional-resources/historical-changes.htm>

<sup>6</sup>We refer to measures calculated from either set of components as the “Old Methodology” Median and Trimmed-Mean CPI. The “Old Methodology” data begin in 1967 through July 2007.

<sup>7</sup>Data for the “Revised Methodology” measures begin in 1983, as this is when the BLS introduced the rental equivalence method of measuring the cost of owner-occupied shelter. The list of the components used under the “Revised Methodology” is available on request from the authors.

statistical techniques and data availability. For example, the BLS does not seasonally adjust the four regional OER subindices despite the presence of seasonality in each (FRBC 2007). Since the FRBC Median and Trimmed-Mean CPI indices use seasonally adjusted (SA) data (see Higgins and Verbrugge 2015 for a discussion), the FRBC seasonally adjusts the regional OER series. Whereas the FRBC originally used the Census Bureau's X-12-ARIMA procedure to do this, it has since switched to the newer X-13-ARIMA-SEATS procedure.

## E Appendix: Procedures for Weights, Seasonal Adjustment, and Computing the Median and Trimmed-Mean CPI

### E.1 Computing Expenditure Weights and Seasonal Adjustment

As noted above, expenditure weights are set in December, then updated every month based upon price movements. Let us consider a concrete example. Suppose we are in March and are given: (1) the values of the non-seasonally-adjusted (NSA) price index of component  $x$  for December ( $I_{Dec}^x$ ) through March ( $I_{Mar}^x$ ); (2) the values of the same for the headline CPI-U ( $I_{Dec}^{CPI}$  through  $I_{Mar}^{CPI}$ ); and (3) the annual (December) relative importance of  $x$ ,  $R_{Dec}^x$ . We wish to compute  $R_{Mar}^x$ . The current BLS method to construct the non-normalized weight  $R_{Mar}^x$  is given by:

$$R_{Mar}^x = R_{Dec}^x * \left( \frac{I_{Mar}^x}{I_{Dec}^x} \right)$$

To construct the normalized weight, one adjusts all the relative weights so as to ensure that they all add up to 100 by simply dividing the non-normalized weight by the analogous “updated relative importance” for the entire CPI – which has an initial “relative importance” of 100 in December – which is given by:

$$R_{Mar}^{CPI} = 100 * \left( \frac{I_{Mar}^{CPI}}{I_{Dec}^{CPI}} \right)$$

Hence the normalized weight for  $x$ ,  $\Phi_{Mar}^x$ , equals:

$$\Phi_{Mar}^x = \frac{R_{Mar}^x}{R_{Mar}^{CPI}}$$

One can also rewrite this as a recursive formula. Clearly:

$$R_{Mar}^x = R_{Dec}^x * \left( \frac{I_{Mar}^x}{I_{Dec}^x} \right) = R_{Dec}^x * \left( \frac{I_{Feb}^x}{I_{Dec}^x} \right) \left( \frac{I_{Mar}^x}{I_{Feb}^x} \right) = R_{Feb}^x \left( \frac{I_{Mar}^x}{I_{Feb}^x} \right)$$

Similarly:

$$R_{Mar}^{CPI} = R_{Feb}^{CPI} \left( \frac{I_{Mar}^{CPI}}{I_{Feb}^{CPI}} \right)$$

This implies:

$$\Phi_{Mar}^x = \frac{R_{Mar}^x}{R_{Mar}^{CPI}} = \frac{R_{Feb}^x}{R_{Feb}^{CPI}} * \frac{(I_{Mar}^x/I_{Feb}^x)}{(I_{Mar}^{CPI}/I_{Feb}^{CPI})} = \Phi_{Feb}^x \frac{(I_{Mar}^x/I_{Feb}^x)}{(I_{Mar}^{CPI}/I_{Feb}^{CPI})}$$

With this methodology, we compute for each month  $t$  the weights  $\Phi_t^c$  for each component  $c$  in Northeast OER, Midwest OER, South OER, and West OER.

This leaves the monthly CPI indices for each component. The BLS produces both SA and NSA versions of the headline CPI-U index and most components.<sup>8</sup> However, the BLS does not publish SA price indices for the four regional OER indices despite the presence of seasonality in each series. As a result, FRBC seasonally adjusts these series using the BLS methodology described above.

## E.2 Calculating the Median and Trimmed-Mean CPI Inflation

For a given collection of CPI components  $C$ , denote as

$$\pi_t^c = 100\left(\frac{I_t^c}{I_{t-1}^c} - 1\right)$$

the monthly inflation rate of component  $c$  in month  $t$ . For a regional OER component,  $I_t^c$  is the corresponding NSA CPI index that has been seasonally adjusted, as explained in the previous section. For other components,  $I_t^c$  is the SA index published by the BLS, if available. If a given index  $I_t^c$  is only available from the BLS in NSA form, then since that component does not display significant seasonality, we use the NSA index for that component. For each component  $c$ , in addition to  $\pi_t^c$ , we have available  $\Phi_t^c$ , calculated as explained in the previous section for each regional OER component and taken from the BLS otherwise. To calculate the median and trimmed-mean in month  $t$ :

1. For each  $c \in C$ , if either  $\pi_t^c$  or  $\Phi_t^c$  is missing, component  $c$  is dropped from any further calculations. Denote as  $\tilde{C}$  the set of components  $C$  excluding components with missing data.<sup>9</sup>
2. Renormalize the weights such that for each  $c \in \tilde{C}$ :  $\tilde{\Phi}_t^c = 100(\Phi_t^c / \sum_{c \in \tilde{C}} \Phi_t^c)$ .

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<sup>8</sup>As the BLS explains: “Seasonally adjusted data are computed using seasonal factors derived by the X-13 ARIMA-SEATS Seasonal Adjustment Method. These factors are updated with the release of January data in February and reflect price movements from the previous calendar year. The new factors are used to revise the previous 5 years of seasonally adjusted data; older seasonally adjusted indexes are considered to be final.” For more information on seasonal adjustment in the CPI, see <https://www.bls.gov/cpi/seasonal-adjustment/>.

<sup>9</sup>Missing data are rare, but can happen if the BLS has insufficient source data to publish the component.

3. For all  $c \in \tilde{C}$ , sort  $\pi_t^c$  from smallest to largest. More formally, define a one-to-one mapping  $c \leftrightarrow j$ ,  $j = 1, \dots, J$ , where  $j$  denotes the relative position of  $\pi_t^c$ . For example,  $c \leftrightarrow j = 1$  if  $\pi_t^c$  is the smallest monthly inflation rate, and  $c \leftrightarrow j = J$  if  $\pi_t^c$  is the largest.
4. For each  $j$ , compute cumulative weight  $w(j) = \sum_{k=1}^j \tilde{\Phi}_t^k$  where  $\tilde{\Phi}_t^j \equiv \tilde{\Phi}_t^c$  if and only if  $c \leftrightarrow j$ .
5. Find the first  $j$  for which  $50 \leq w(j)$ . Denoting this index as  $j^{MED}$ , the median component is the component  $c^{MED}$  satisfying  $c^{MED} \leftrightarrow j^{MED}$ , and the median inflation rate is  $\pi_t^{c^{MED}}$ .
6. To calculate the 16% trimmed-mean:
  - (a) Find the first  $j$  for which  $8 < w(j)$ . Denote this index as  $j_S$  and set its normalized relative importance to  $\tilde{\Phi}_t^{j_S} \equiv \tilde{\Phi}_t^j - 8$ .
  - (b) Find the first  $j$  for which  $92 \leq w(j)$ . Denote this index as  $j_E$  and set its normalized relative importance to  $\tilde{\Phi}_t^{j_E} \equiv \tilde{\Phi}_t^j - \tilde{\Phi}_t^{j-1}$ .
  - (c) Calculate the trimmed-mean:

$$\pi_t^{TM} = \frac{\sum_{j \in [j_S, j_E]} \pi_t^j \tilde{\Phi}_t^j}{\sum_{j \in [j_S, j_E]} \tilde{\Phi}_t^j} = \frac{\sum_{j \in [j_S, j_E]} \pi_t^j \tilde{\Phi}_t^j}{84}$$

## F Appendix: Full Statement and Proof of Proposition 1

We begin with some definitions. Consider a discrete collection of  $N$  random variables:  $A = \{X_j : j = 1, \dots, N\}$ , each with an associated non-negative weight  $w_j : j \in A$  with  $\sum_{j=1}^N w_j = 1$ . Denote a member of the set  $A$  by  $V$ . We define the weighted sample median as follows. After the random variables are realized, sort the random variables from smallest to largest, indexed by  $k$ , so that  $v_k$  is the  $k^{\text{th}}$  largest realization. The cumulative sum weight through index  $l$  is defined by  $\sum_{k=1}^l w_k$ . Then the weighted median of the sample of random variables  $A$  is defined as follows:  $wmed(A) = v_l : \sum_{k=1}^l w_k \leq 0.5$  and  $\sum_{k=1}^{l+1} w_k > 0.5$ .

**Proposition 1.** *Suppose that there is a collection of  $N$  random variables:  $B = \{X_j : j = 1, \dots, N\}$ , each with an associated non-negative weight  $w_j : j \in B$  with  $\sum_{j=1}^N w_j = 1$ . Suppose that there exists a set  $C \subset B$ , of cardinality  $r$ , whose elements are unobserved; instead, what is observed is their weighted mean  $Y = \sum_{j \in C} w_j X_j$ . Without loss of generality, assume that the indexes of the random variables in  $C$  are  $\{M - r, M - r + 1, \dots, M\}$ . Moreover, there exists a second set  $D \subset B$ , of cardinality  $s$ , with  $C \cap D = \emptyset$ , with a weighted mean  $Z = \sum_{j \in D} w_j X_j$ . Without loss of generality, assume that the indexes of the random variables in  $D$  are  $\{M - r - s, M - r - s + 1, \dots, M - s - 1\}$ .*

Let

$$G = \{Y, X_j : j = 1, \dots, M - r - 1\}$$

and let

$$H = \{Y, Z, X_j : j = 1, \dots, M - r - s\}$$

Then the following inequality need not hold:

$$E[wmed(G) - wmed(B)]^2 \leq E[wmed(H) - wmed(B)]^2$$

*Proof.* We prove this via a counterexample. We consider a collection of 7 random variables  $B = \{X_i, i = 1, \dots, 7\}$  with the associated collection of aggregation weights  $W = \{w_1, \dots, w_7\}$  given by  $W = \{0.025, 0.025, 0.19, 0.19, 0.19, 0.19, 0.19\}$ ; thus  $X_1$  and  $X_2$  each have a weight of 0.02, and the

other variables each have a weight of 0.19.

The range of  $X_i$  is denoted  $R_i = R_{i,0} \cup R_{i,1}$ , where realizations of  $X_i$  occur in  $R_{i,0}$  if the realization of a binary variable  $Y = 0$ , and occur in  $R_{i,1}$  if  $Y = 1$ . The ranges of the variables satisfy the following:

$$R_{1,0} = [267, 306]; R_{1,1} = [-297, -257]$$

$$R_{2,0} = R_{1,1}; R_{2,1} = R_{1,0}$$

$$R_{3,0} = R_{3,1} = [1.99, 2.01]$$

$$R_{4,0} = [1.4, 1.6]; R_{4,1} = [2.4, 2.6]$$

$$R_{5,0} = [2.2, 2.3]; R_{5,1} = [1.7, 1.8]$$

$$R_{6,0} = R_{5,1}; R_{6,1} = R_{5,0}$$

$$R_{7,0} = R_{4,1}, R_{7,1} = R_{4,0}$$

Given these distributions, the weighted sample median  $wmed(B)$  is always  $x_3$ , near 2.0, and  $X_3$  is the population weighted median.

But suppose that  $X_1$  and  $X_5$  are unobserved; instead, only their weighted average  $Z_1$  is observed. This variable has an aggregation weight of 0.215. Realizations of  $Z_1$  satisfy  $R_{Z_1,0} = [7, 8]$  and  $R_{Z_1,1} = [-6, -7]$  – thus in any sample, if  $Y = 0$ ,  $Z_1$  is ordered last, and when  $Y = 1$ , in any sample,  $Z_1$  is ordered first. Thus when  $Y = 0$ , the weighted sample median  $wmed(B)$  is always  $x_6$ , with a realization between 2.2 and 2.4; and when  $Y = 1$ , the weighted sample median is also always  $x_6$ , with a realization between 1.7 and 1.8. In this example, the median of the most disaggregated *observed* collection of random variables will always deviate by at least 0.2 from the median of the underlying distribution. Depending upon the underlying distributions, this median might be unbiased on average, but in any given month it is never closer than 0.2 to its estimation goal.

However, consider aggregating  $X_2$  and  $X_6$  into a variable  $Z_2$ . Realizations of  $Z_2$  satisfy  $R_{Z_2,0} = [-7, -8]$  and  $R_{Z_2,1} = [6, 7]$ . This variable has an aggregation weight of 0.215. The weighted median of the set  $H = \{X_3, X_4, X_7, Z_1, Z_2\}$  is always  $x_3$ , exactly equal to the underlying sample weighted median.



In this example,

$$E[wmed(G) - wmed(B)]^2 > E[wmed(H) - wmed(B)]^2 = 0$$

Hence, the most disaggregated set available,  $G$ , need not yield the most accurate weighted median estimate.  $\square$

One might object that a fixed bias is easy to adjust for. But the simpler example in the main paper demonstrates that even if a median estimator is unbiased (on average), using more aggregated indexes may enhance accuracy.

## G Appendix: Construction of Density Forecasts

### G.1 Computation of Density Forecasts

To construct density forecasts using equation 2, we use a parametric block wild bootstrap algorithm identical to that used by Knotek and Zaman (2023) to construct density forecasts for their single-equation model. The approach accounts for both the parameter and the shock uncertainty.

We illustrate the approach using a general representation for a multivariate regression model (e.g., equation 2 in this paper), which can be written as follows,

$$y_t = \beta_0 + \alpha X_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2) \quad (2)$$

Assume that  $\hat{\beta}_0$ ,  $\hat{\alpha}$ , and  $\hat{\sigma}^2$  are the OLS estimates obtained through the estimation of equation (2) over the sample  $1, \dots, T$ . The least squares residuals  $\hat{\varepsilon}_t$  have mean 0 and variance  $\hat{\sigma}^2$ .

#### Algorithm: Wild Block Bootstrap

For  $d = 1, \dots, D$  do the following:

1. Construct a transformed series of residuals  $\{\ddot{\varepsilon}_t\}_{t=1}^T$  from the OLS residuals  $\{\hat{\varepsilon}_t\}_{t=1}^T$ , where

$$\ddot{\varepsilon}_t = h(\hat{\varepsilon}_t)u_t, \quad u_t \sim N(0, 1),$$

and  $h(\cdot)$  is a transformation function that modifies the original least squares residuals to correct for possible heteroscedasticity. Following Chernick and LaBudde (2011, Ch. 6, Sec. 6.6), we set

$$h(\hat{\varepsilon}_t) = \frac{\hat{\varepsilon}_t}{1 - H}, \quad \text{where } H = X(X'X)^{-1}X'.$$

We also tried  $h(\hat{\varepsilon}_t) = \frac{\hat{\varepsilon}_t}{(1-H)^{1/2}}$ , another widely used transformation.

2. Sampling from  $\tilde{\varepsilon}$ :

(a) To correct for possible serial correlation (following Aastveit et al., 2014), draw blocks of consecutive errors from  $\tilde{\varepsilon}$ . Define the block size as  $b_{\text{size}} = 4$  (commonly set greater than or equal to the forecast horizon). Let  $T$  be the number of observations, and define the number of non-overlapping blocks as

$$b_{\text{number}} = \text{ceil}\left(\frac{T}{b_{\text{size}}}\right).$$

(b) For  $l = 1, \dots, b_{\text{size}}$  and  $j = 1, \dots, b_{\text{number}}$ , construct the bootstrap sample for  $y^*$ :

$$y_{(j-1)b_{\text{size}}+l}^* = \hat{\beta}_0 + \hat{\alpha}X_{(j-1)b_{\text{size}}+l} + \varepsilon_{(j-1)b_{\text{size}}+l}^*,$$

where  $\varepsilon_{(j-1)b_{\text{size}}+l}^* = \tilde{\varepsilon}_{(j-1)b_{\text{size}}+l} \cdot \delta_j$ , and  $\delta_j$  is a Rademacher variable, following Davidson and Flachaire (2008) and Aastveit et al. (2014):

$$\delta_j = \begin{cases} +1, & \text{with probability 0.5,} \\ -1, & \text{with probability 0.5.} \end{cases}$$

3. Based on the bootstrap sample  $y^*$  (constructed in the previous step), re-estimate the model in equation (2) to obtain updated estimates  $\hat{\beta}_0^{(d)}$ ,  $\hat{\alpha}^{(d)}$ , and  $\hat{\sigma}^{2(d)}$ .

4. Use  $\hat{\beta}_0^{(d)}$  and  $\hat{\alpha}^{(d)}$  in equation (2) to generate  $h$  step-ahead forecast  $\hat{y}_{t+h}^{(d)}$ .

5. Repeat for all  $d = 1, \dots, D$ .

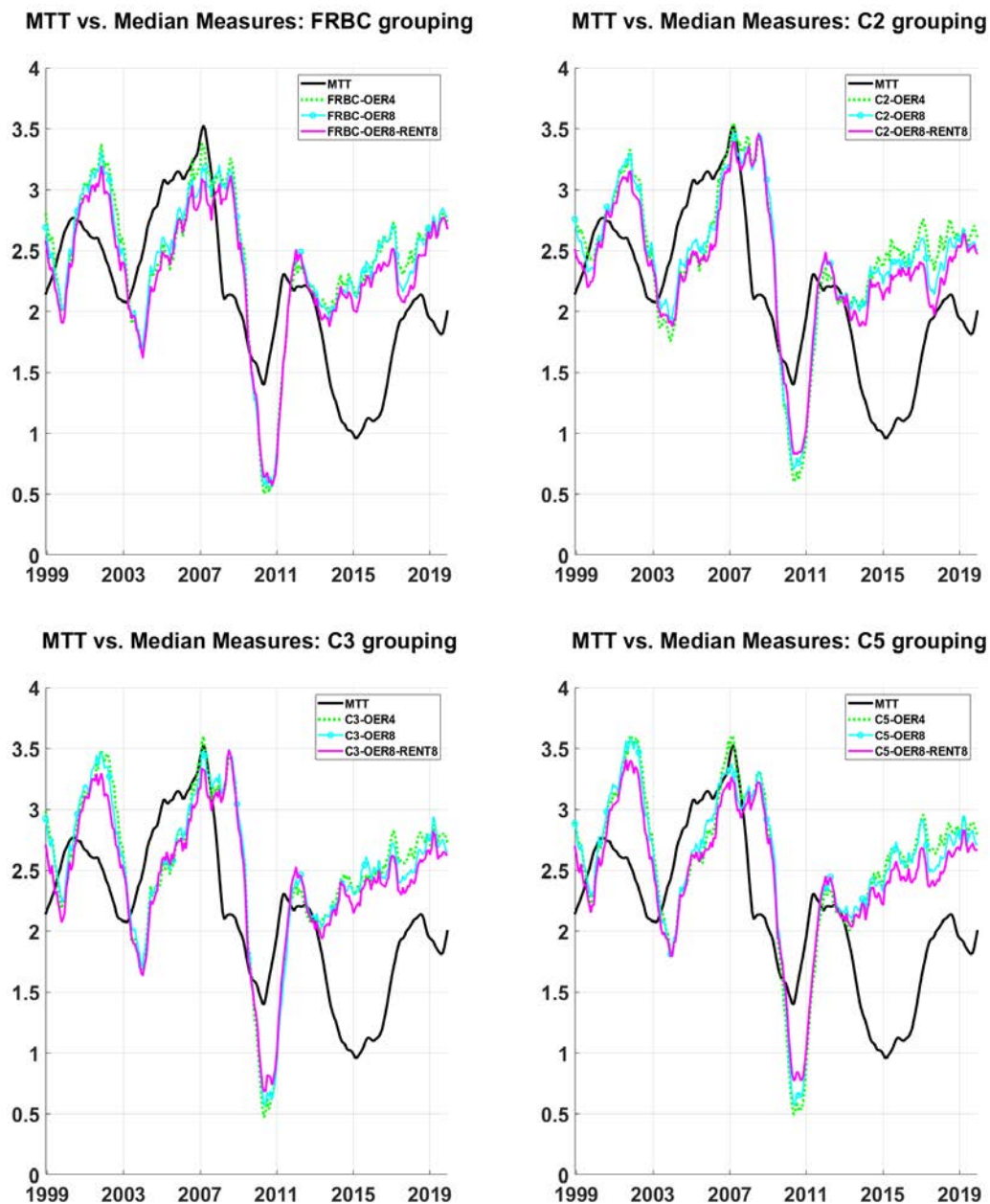
The empirical distribution of  $\{\hat{y}_{t+h}^{(d)}\}_{d=1}^D$  constitutes our estimate of the  $h$ -step-ahead density.

## H Appendix: Various Uses of Median and Trimmed Mean CPIs

- Forecasting: Smith, 2004; Meyer, Venkatu and Zaman, 2013; Liu and Smith, 2014; Meyer and Zaman 2019; Verbrugge and Zaman, 2024a; Ocampo, Schoenle and Smith 2023.
- Inputs into more sophisticated estimates of medium-term trend inflation: Mertens, 2016.
- Scrutinizing stylized inflation facts: Bryan and Cecchetti, 1999; Verbrugge, 1999; Fang, Miller and Yeh, 2010.
- Understanding inflation uncertainty: Metiu and Prieto, 2023.
- Discriminating between models of price adjustment: Ashley and Ye, 2012.
- Locating a stable Phillips curve: Ball and Mazumder, 2011; Ball and Mazumder, 2019a,b; Stock and Watson, 2020; Ashley and Verbrugge, 2025.
- Studying the effects of oil supply shocks: Kilian, 2008.
- Understanding inflation expectations and their relationship to inflation: Verbrugge and Zaman, 2021.
- Understanding post-Great Recession and post-COVID inflation dynamics: Ball and Mazumder, 2011; Mazumder, 2018; Ball et al., 2021; Ball, Leigh, and Mishra 2022; Verbrugge and Zaman 2023, 2024; Cotton et al., 2023; Verbrugge, 2024.

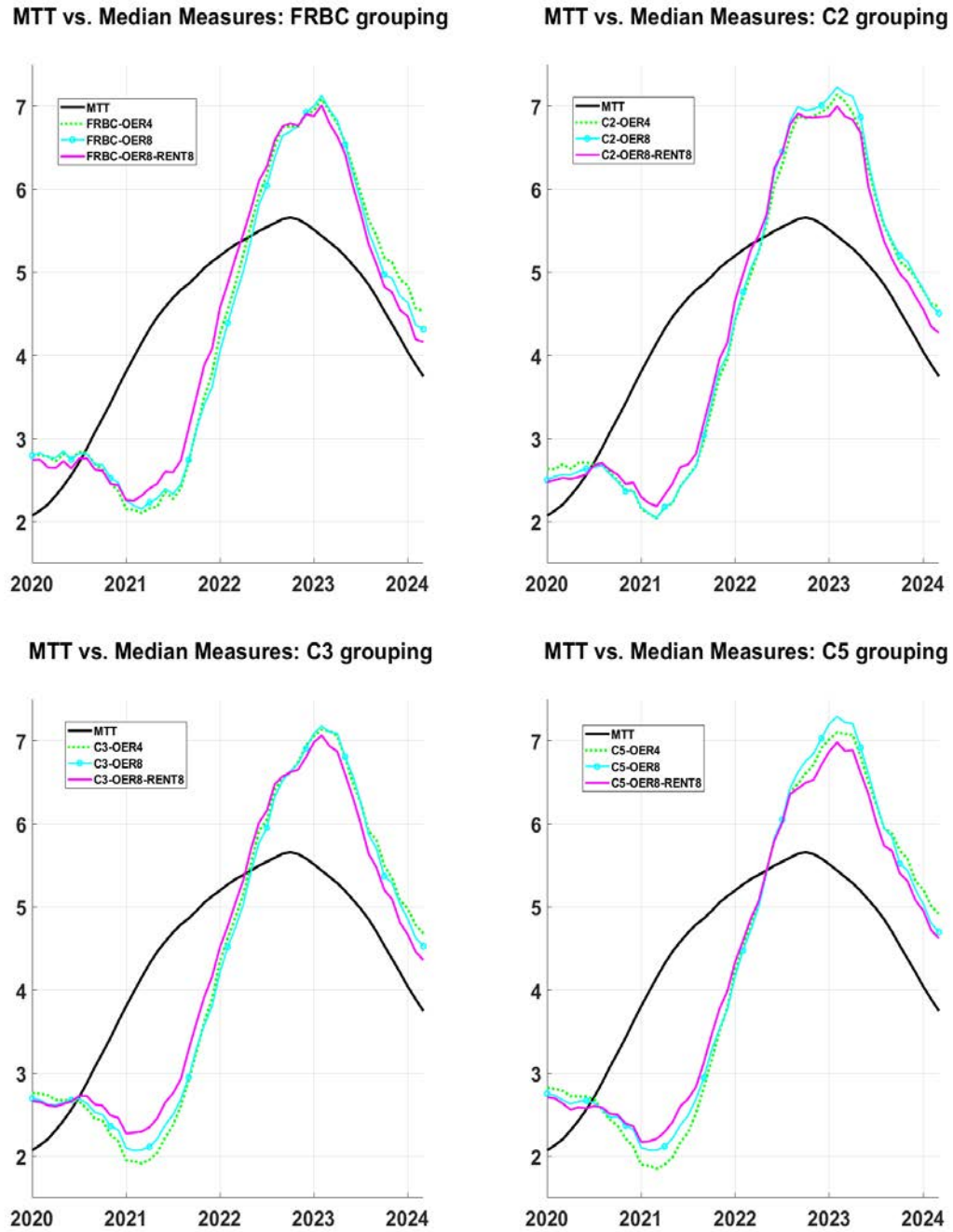
# I Appendix: Plots Comparing MTT (based on 37MMA) with Median Measures

Figure I.1: Pre-Pandemic Period



Notes: MTT plot represents the MTT proxy constructed as a 37-month centered moving average of 12-month CPI inflation, i.e., average of inflation in the current month, the preceding 18 months, and the subsequent 18 months. The plots for all the median measures are 12-month inflation rates.

Figure I.2: Post-Pandemic Period



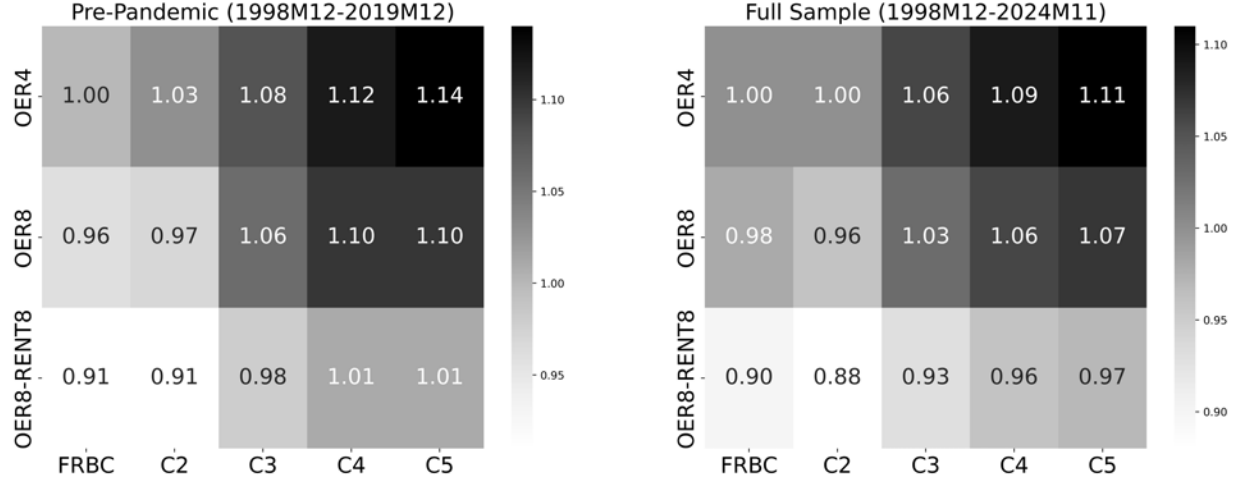
Notes: MTT plot represents the MTT proxy constructed as a 37-month centered moving average of 12-month CPI inflation, i.e., average of inflation in the current month, the preceding 18 months, and the subsequent 18 months. The plots for all the median measures are 12-month inflation rates. To compute the MTT, we use the CPI data through the month of September 2025, which means the latest value of MTT is for March 2024.

Gaps may appear very large during the 2010s, reflecting an extended period of extreme skewness

in the cross-sectional distribution of CPI component growth rates.

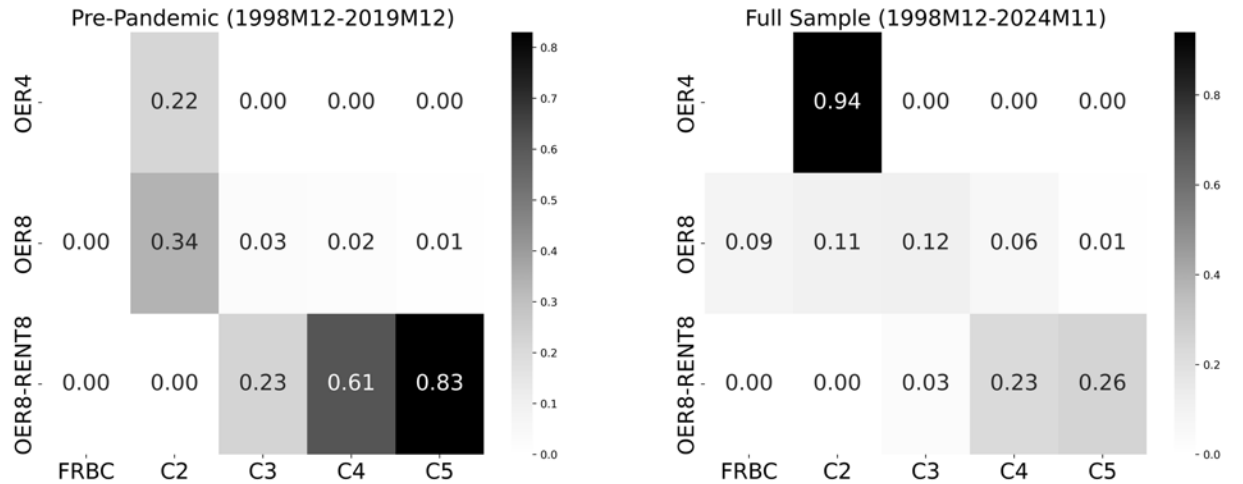
Figure I.3: Accuracy of Median Inflation Measures Relative to FRBC-OER4: 2SMA

Panel A:  $RMSE(\hat{\pi}^{2SMA} - \hat{\pi}_j)$  of Median Inflation Measures Relative to FRBC-OER4



Notes: Reported figures are the RMSE of deviations of the median inflation measure from a two-stage centered moving average (2SMA) of CPI inflation, divided by the same for median FRBC-OER4 inflation. In the pre-pandemic sample, the moving average is computed using CPI inflation through December 2019 only. Darker shading indicates higher values, while lighter shading indicates lower values.

Panel B:  $p$ -Values of a Statistical Test of Equal Ability in Tracking  $\hat{\pi}^{2SMA}$  for Median Inflation Measures, Relative to FRBC-OER4



Notes: Reported figures are the  $p$ -values of a Diebold-Mariano (1995) test that  $RMSE(\hat{\pi}^{2SMA} - \hat{\pi}_{FRBC-OER4})$  and  $RMSE(\hat{\pi}^{2SMA} - \hat{\pi}_j)$  are equal, where  $j$  denotes the  $j$ th candidate median inflation measure. The  $p$ -value is obtained by taking the difference of the two squared errors series  $\hat{e}_{j,t}^{2SMA}$  and  $\hat{e}_{FRBC-OER4,t}^{2SMA}$ , and regressing the resulting series against a constant. The test statistic of the constant term is then calculated using HAC standard errors with a small sample correction and  $[4[T/100]^{2/9}]$  lags, where  $T$  refers to the size of the estimation sample. Darker shading indicates higher values, while lighter shading indicates lower values<sup>30</sup>

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