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Is There a Puzzle in Underwater Mortgage Default?

Lara Loewenstein, Paul Willen, Yuxi Yao, and David Zhang*

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Abstract

A recurring question in the mortgage default literature is why underwater default is rare relative to model predictions. We find that one answer is miscalibration of flow payoffs. We build a novel, detailed quantitative model of mortgage default and find that realistic rent dynamics plus mild levels of default costs are sufficient to eliminate negative-equity strategic default. We present further empirical results supporting our model's focus on flow payoffs. Our model addresses the underwater mortgage default puzzle, offers more realistic interpretations of policy consequences, and reinforces the theoretical effectiveness of cash-flow-based interventions.

Keywords: Mortgage Default, Strategic Default, Household Balance Sheets, Household Decision Making.

JEL Codes: D15, G51, R30.

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1 Introduction

In an influential paper, Foote et al. (2008) examine more than 100,000 homeowners in Massachusetts who had negative equity during the early 1990s and find that less than 10 percent of these owners eventually lost their home to foreclosure. Calculations made for the Great Recession yield a similarly low number. However, economists have struggled to explain these results.

Economists can explain why a rational borrower with negative equity might not default. Using option theory, they argue that borrowers might rationally continue making payments even when they are underwater. The logic is that the default option effectively insures borrowers against the downside of future losses if prices fall further while preserving the upside potential of regaining positive equity in the future. Therefore, defaulting immediately could cause them more financial harm in the long run.

Yet, option-theoretic models from the 1980s and 1990s based on this reasoning predicted far too many defaults. For instance, Kau et al. (1994) and Kau and Keenan (1999) suggested that *all* borrowers who are underwater by at least 25 percent would default, contradicting real-world data showing that, at the time, even borrowers with 50 percent negative equity typically continued making their mortgage payments.

More recently, economists have built dynamic models comprising many realistic features relevant to the default decision, such as stochastic labor income, borrowing and collateral constraints, and nonpayment penalties. These models can generate realistic default levels but only by assuming that borrowers face enormous psychological costs for defaulting. For example, the calibration of Campbell and Cocco (2015) used in Ganong and Noel (2023) requires a utility cost worth 25 percent of lifetime consumption. Hembre (2018) and Laufer (2018) estimate this cost to be even higher, at 50 to 70 percent. These utility penalties are staggeringly high—equivalent to about \$625,000 for a household spending \$50,000 annually over 50 years.

We argue that this “strategic default puzzle” arises not because households fail to act

optimally with respect to their financial interests, but because economists have historically miscalibrated the core incentive behind mortgage default. Conceptually, theoretical analyses typically emphasize stock variables—house prices, mortgage balances, and home equity—as the difference between home prices and mortgage balances. Yet, for households making decisions under uncertainty, the relevant tradeoff is inherently about flows, and therefore flow variables relevant to households’ default decision must be carefully calibrated. What is the utility of staying in the home versus exiting into the rental market, conditional on income and housing cost shocks? Given that the price of maintaining a call option on a home is the mortgage payment and that foreclosure necessitates paying rent, when is it rational to default? In this paper, we estimate a novel, detailed model of mortgage default to evaluate household strategic default incentives given realistic calibrations of flow costs.

Suppose that, as in the benchmark Campbell and Cocco (2015) model, there is only a single type of housing, that homeowners are exogenously assigned their homeownership and mortgage, and that defaulters are forced to become permanent renters. Under these conditions, the financial incentive to default depends entirely on the calibration of rent in the model because the “punishment” for default is having to rent permanently. However, while house prices declined significantly during the 2007–2008 housing crash, rent remained relatively flat (Loewenstein and Willen, 2023). When compared to a stable price–rent ratio, this relative rise in the cost of renting should have mitigated homeowners’ incentive to default, even if they had negative equity.

We illustrate this effect in Figure 1 by comparing output from different calibrations of the Campbell and Cocco (2015) model to empirical estimates of the change in household income for defaulters from Ganong and Noel (2023) by levels of loan-to-value ratios (LTVs). If equity is the main driver of default, then we would expect that income declines become less vital to the default decision for more underwater households. However, Ganong and Noel (2023) find that households that are substantially underwater require an equally large decline in income as households with lower LTVs, implying that household liquidity concerns

are more salient to default decisions than previously appreciated.

In Panel 1a we plot household income change conditional on default in the Campbell and Cocco (2015) model’s baseline calibration next to the estimates from Ganong and Noel (2023). In the model, high-LTV borrowers require a significantly smaller decline in income to default relative to the data because negative equity is the main driver of default in the baseline calibration, which assumes that rents move in parallel with house prices. In Panel 1b we show that adding a high nonpecuniary utility cost of default, on the order of 25 percent of lifetime consumption, enables the model to better match the data. These panels replicate the findings in Ganong and Noel (2023).

Panel 1c illustrates the novel contribution of our paper. We show that the same fit can be achieved by holding real rent fixed at 2001 levels, consistent with Loewenstein and Willen (2023), but without any nonpecuniary utility cost of default.¹ Thus, the high implied value of strategic default in previous calibrations of the Campbell and Cocco (2015) model is largely due to the high assumed financial attractiveness of renting relative to owning during housing crashes, an assumption that no longer holds under a more realistic rent-to-price process.

[Figure 1 inserted here]

Why are rents more downwardly stable than house prices, especially during times of house-price declines? There are several possible explanations. First, during financial crises, household demand for rental units may increase relative to the demand for owner-occupied units due to the execution of income-driven foreclosures, which may generate upward pressure on rents. Indeed, Foote et al. (2018) find significant household inflows into rental units as well as a rise in the number of vacant units during the 2008 housing crisis, both of which could increase rents. Second, under a standard Rosen–Roback model, rents are related to the utility of being in a location, including factors such as wages and amenities, and these factors may be relatively stable in the aggregate (Roback, 1982; Rosen, 1979). On the other

¹Appendix A discusses this model in more detail, and Appendix Figure A.1 shows that the income declines conditional on default are not different when we add back the nonpecuniary costs worth 25 percent of lifetime consumption, suggesting that such nonpecuniary costs have little additional explanatory power.

hand, house prices may be more prone to speculative dynamics (DeFusco et al., 2022) and may respond to credit supply shocks (Adelino et al., 2025), both of which would explain their greater volatility. Regardless of the reason for the lower volatility of rent, we take this empirical pattern, as reproduced in Figure 2, as a primitive in our model.

[Figure 2 inserted here]

While our calibration of the benchmark Campbell and Cocco (2015) model is revealing, it is possible that rents play an outsized role only due to the specific assumptions made in the model. Specifically, households cannot downsize their homes, yet if a household suffers a permanent income shock, one natural reason to default would be to move to a smaller, more affordable house. In fact, the model feature that households cannot downsize implies that as long as rent is higher than the household’s mortgage payment plus taxes and maintenance, households would never default regardless of the severity of income shocks or the level of negative equity, making the model-implied default rates near zero and the model-implied nonpecuniary default penalties *negative* with realistic rent processes. Second, defaulters are permanently excluded from homeownership, whereas, in reality, they can regain access to mortgage credit after several years, which further artificially reduces the appeal of negative-equity default.²

To fully understand how flow costs influence households’ default decisions, we build a novel, detailed model of mortgage default that incorporates stochastic income, savings, housing tenure choice, and mortgage choice. Specifically, our model features three improvements over those in the previous literature. First, following Kaplan et al. (2020), we incorporate heterogeneous property sizes so that households have an incentive to downsize following income

²In macroeconomic modeling, researchers may avoid these problems by assuming a lower counterfactual rent in the Campbell and Cocco (2015) model and by having the lower rent proxy for the additional incentives to default, such as being able to downsize, calibrating the lower counterfactual rent required to match actual default rates and thus making Campbell and Cocco (2015) a useful and computationally efficient input into larger models. However, the lower rent necessary to fit default rates would capture both the additional incentives to default and any household reluctance from doing so, and a more detailed calibration of the value of the additional incentives to default is necessary for our purposes of understanding whether the underwater mortgage default puzzle exists.

shocks, including a restriction that the smallest houses can only be rented (not purchased). This assumed market segmentation does not prevent strategic default in our model, as owners can still rent their preferred sized home, but is instead important for fitting the life-cycle profile of homeownership, as in Kaplan et al. (2020). While this significantly increases the computational complexity of our model by expanding the state space to include property sizes, mortgage sizes, and housing tenure choice, it is necessary for capturing the downsizing incentive of default. Second, we add realistic options that increase the attractiveness of default. We allow foreclosed households to re-buy a home after seven years in expectation after the foreclosure flag is removed from their credit profile.³ Furthermore, we allow households to live in their homes for free for one year while defaulting on their mortgage. Third, we give defaulters the realistic option to “cure” their default before foreclosure completion, thus making default in our model a two-stage problem. This adds another positive benefit to default, as it allows households to miss a year of mortgage payments to smooth consumption in response to an income shock.

We calibrate this model to match homeownership rates over the life cycle and distributions of owner and renter house values in the 2001 American Community Survey (ACS); the LTV, payment-to-income (PTI), and debt-to-income (DTI) ratios in the 2001 Survey of Consumer Finances (SCF); and delinquency rates by LTV during the 2005–2023 period in the Equifax Credit Risk Insights McDash (CRISM) data set. Importantly, we hold real rent constant, which captures the more realistic rent dynamics described in Loewenstein and Willen (2023). Regarding income, we follow Campbell et al. (2021) in allowing the income process in the model to vary by business cycle recessions and expansions and calibrate to those found in Guvenen et al. (2014). We further conduct a novel calibration of the dynamics of house prices during recessions and expansions by using zip code-level price indexes. Despite having only

³Such a transaction could be done earlier with cash for households that have the liquidity, but that is an unlikely possibility for most households. Furthermore, even households that have enough cash may have better alternative investment opportunities during the period of large house-price declines and rising rental yields, relative to defaulting and re-buying their home, which is a cash-intensive investment with zero immediate net worth gains.

seven free parameters, our model reasonably matches our targeted distributions. In addition, our model closely matches nontarget moments, PTI by LTV in the cross section of mortgage borrowers, and the house-size distribution among owners and renters.

Our calibration does require a nonpecuniary default penalty, but at 0.7 percent of lifetime consumption, it is substantially smaller than the 25 to 50 percent of lifetime income found in the previous literature. There are two ways to interpret our 0.7 percent of lifetime consumption mortgage default penalty. First, it can capture the nonmonetary costs of mortgage default, including losses of reputation in the labor market, limits from renting certain properties, and other nonpecuniary costs. These costs may be heterogeneous and larger for households that face higher reputation costs. Second, it can be viewed as an upper bound of the rational incentives to default in our life-cycle model, irrespective of the true nonpecuniary costs of default, which for many households may indeed be greater than what we calibrate.⁴ This second interpretation does not require readers to accept our model as a positive description of what households do, in the sense of Campbell (2006), but rather as a normative description of what rational optimizing households should do. The second interpretation implies that the financial incentives for rational households to strategically default, while still present, are significantly more limited compared with the estimates in the earlier literature. This in turn allows for more realistic policy analysis and adds to the theoretical generalizability for cash-flow-based interventions.

Our model also implies income changes conditional on default that are consistent with the empirical finding in Ganong and Noel (2023) described above. In fact, we note declining (as opposed to flat) income change conditional on default by LTV in our model, which is possible due to the positive correlation between income and house-price growth. That significant income declines in our model are required for negative-equity default implies that

⁴There is some evidence that this cost is larger than 0.7 percent for homeowners who choose to repay their mortgages while moving. Brueckner et al. (Forthcoming) find a median lower bound of \$28,871 to \$52,448 depending on the credit score quintile. Our model implies that these amounts are more than sufficient to largely eliminate strategic default incentives. Guiso et al. (2013) show that many survey respondents feel a moral obligation not to default.

the financial benefits to defaulting are limited and that few borrowers default purely because they have negative equity.

A key insight of our paper is that correctly calibrating flow payoffs is central to modeling household strategic behavior. Households may not be particularly incentivized to default when they are underwater if their alternative of renting has not fallen drastically in cost. On the other hand, if flow payoffs truly matter for household decisionmaking, households may be more incentivized to default if their expected rent declines permanently. We illustrate this possibility via a model counterfactual analysis in which real rents fall permanently by 10 percent. We show that this results in default rates that up to 5 percentage points higher depending on households’ LTV, with stronger effects concentrated in higher LTV households. We also show that such a permanent decline in rent results in a 2 percentage point increase in foreclosure rates conditional on defaulting. While the earlier literature focuses on “double trigger” models of mortgage default driven by negative equity and income declines, our model instead supports a “triple trigger” model of default in which income declines, negative equity, and changes to the relative costs of owning versus renting together trigger default.⁵

We also present empirical results supporting our model’s emphasis on the flow payoff differences between owning and defaulting. To identify the effect of flow payoffs, we condition on borrowers who experience 90-day default and examine differences in their foreclosure likelihoods as a function of the change in their nominal rent growth since mortgage origination. By conditioning on 90-day default, we isolate a sample of households that were likely hit with income shocks (Ganong and Noel, 2023) then examine their foreclosure behavior related to flow payoff changes. We find that, in agreement with our model, foreclosure probabilities conditional on default rise by 1.5 percentage points for every standard deviation decline in rent, where a one standard deviation decline in rent is 8.5 percent, consistent with

⁵In addition to declining rent, the flow benefit of homeownership can change due to factors such as climate catastrophes, which may increase the attractiveness of renting relative to owning, thus making the “triple trigger” benchmark model potentially relevant in the face of rising climate risk accompanied by house-price declines. For a discussion of how climate risk can affect the value of homeownership, see Benjamin Keys, “Climate Change Should Make You Rethink Homeownership,” *New York Times*, October 29, 2024, available [here](#).

our model’s predictions.

A potential confounder of this result is that rent growth may be correlated with economic conditions and therefore with the likelihood of income recovery by the defaulted household. We address this endogeneity in three ways. First, we note that this correlation is not necessarily positive, as worsening economic conditions drive both an increase in foreclosures and an increase in the demand for rental units, leading to higher rents. Second, we find similar coefficients when controlling for local unemployment rate and wage-growth. Third, we use the instrument from Gete and Reher (2018), which, the authors argue, predicts rent growth independent of local economic conditions, and find larger effects in the instrumental variables analysis, suggesting that the correlation between rent growth and unobserved economic conditions may indeed be negative.

2 Existing Literature on Mortgage Default

On the theoretical front, early research, including Foster and Van Order (1984) and Riddiough (1991), develops option-theoretic models of mortgage default. More recent models of mortgage default, starting with the seminal work of Campbell and Cocco (2015), use a life-cycle framework. We build extensively on the Campbell and Cocco (2015) model. Hembre (2018), Laufer (2018), and Li et al. (2022) require similar or even larger nonpecuniary costs in their life-cycle models compared with the Campbell and Cocco (2015) model. Schelkle (2018) requires more reasonable nonpecuniary costs and also requires both a low discount factor ($\beta = 0.9$) and low elasticity of intertemporal substitution ($\gamma = 5$). The absence of either makes the model similar to earlier models.⁶ Ganong and Noel (2023) provide a calibration of the Campbell and Cocco (2015) model.⁷

⁶In cutting-edge work, Kalikman and Scally (2022) introduce a model that fits the average cumulative default rates by cohort in the data by using heterogeneous default penalties, but the authors do not discuss the required magnitude of those penalties. A large macro-finance literature that includes Chatterjee and Eyigungor (2015), Corbae and Quintin (2015), Elenev et al. (2021), Diamond and Landvoigt (2022), Diamond et al. (2025), and Elenev and Liu (2025) incorporates models of mortgage default in the analyses but does not focus on default costs.

⁷Available here.

An important complementary contribution to the theory of mortgage default is Low (2023a), who introduces a model that fits the average level of above-water and underwater default by introducing empirically motivated large and time-varying psychic moving costs on the order of \$200,000 on average. However, the model still predicts unrealistically high levels of default for highly underwater households, and the paper reiterates that underwater default poses a “puzzle” for the literature. Low (2023a) emphasizes that it is important for a mortgage default model to (1) fit both above-water and underwater default rates and (2) generate reasonable moving rates, which we do in our model, as shown in Figure 6a and Appendix Figure A.2. Nevertheless, our goal is not to produce an alternative definitive model of mortgage default or to take a position about the size and prevalence of nonpecuniary default costs. We use our model primarily to show, in a rigorous quantitative framework, that underwater households lack a strong theoretical incentive to strategically default, which allows for more realistic interpretations of policy and suggests conditions under which cash-flow-based interventions are more theoretically generalizable.

On the empirical side, a large earlier literature reviewed in Foote and Willen (2018) finds low rates of underwater default even among households with negative equity. Bhutta et al. (2017) examine LTV cutoffs for defaulting during the Great Recession and find that borrowers are unlikely to default on their mortgages until their LTVs become significantly higher than earlier models’ predictions. The authors reasonably attribute the borrowers’ high LTV cutoffs to “emotional and behavioral factors.” We do not rule out the importance of emotional and behavioral factors but instead show that a modest value for these factors—0.7 percent of lifetime consumption—sufficiently addresses the “strategic default puzzle” in a rational model of mortgage default, unlike the earlier literature, which requires much larger values.⁸ Gupta and Hansman (2022) study the role of adverse selection and moral hazard in explaining the correlation between leverage and mortgage default by using interest rate

⁸Brueckner et al. (Forthcoming) find a higher lower bound for default costs based on repaying homeowners than what is required by our model, at a median lower bound of \$28,871 to \$52,448 depending on the credit score quintile, which, based on our model, is sufficient to largely eliminate strategic default.

indexes as a source of exogenous variation and find that both forces are important. An important contribution that we rely heavily on is Ganong and Noel (2023), who find that most defaults are driven by income loss or a combination of income loss and negative equity rather than purely by negative equity. A regression kink design by Indarte (2023) suggests that liquidity is the primary driver of personal bankruptcy among mortgage borrowers. Survey evidence from Low (2023b) confirms the importance of liquidity shocks in household default. Hazard instrumental variable estimates from Palmer (2024) as well as evidence from large principal reductions from randomly assigned mortgage cramdowns in Cespedes et al. (Forthcoming) suggest that negative equity is relevant for foreclosures. Generally, the literature identifies a central role for liquidity in driving mortgage default, and it finds that large negative equity is also relevant for foreclosures. Our model is consistent with these empirical findings.

3 Data

We use several data sources to calibrate our model. In particular, we obtain the PTI and LTV ratios from the 2001 SCF and the homeownership rates over the life cycle from the 2001 ACS. We also calculate default and foreclosure rates as the share of loans that go into default or foreclose, respectively, in a given year using data on owner-occupied, first-lien mortgages from 2001 through 2023 in the Equifax/McDash CRISM data set. We define default as the first month a mortgage transitions to 90-day delinquency, following the definition in Ganong and Noel (2023).

For the rent-to-price ratio, we take the average of that ratio in 2001 and hold real rent constant, as shown in Loewenstein and Willen (2023). Price-to-rent ratios used in this calibration as well as in Figure 2a come from a data set of property-level rent-to-price ratios derived from Multiple Listing Service (MLS) data, as described in Loewenstein and Willen (2023). This is a sample of renter-occupied single-family homes and condos for which we

observe contract rents and sale prices within a one-year span. Importantly, this allows us to obtain rents and prices for the same property and thereby measure rent-to-price ratios that are more realistic compared with using average rent and average sale prices in a metro area.

To calibrate zip code-level house-price dynamics, we use the CoreLogic zip code house-price index and deflate it with the CPI deflator. The CoreLogic zip code index improves on the Federal Housing Finance Agency (FHFA) zip code price index by using a larger sample of properties; the FHFA index tracks only properties financed via conforming loans. We obtain monthly zip code-level house-price index values from January 1987 through January 2008 and use them to estimate a mixture model of house-price movements.

For our empirical analysis of the relationship between nominal rent growth and default, we use the Equifax/McDash CRISM data set, which is mortgage performance data matched with anonymous credit records from Equifax, along with market-level data on changes in effective asking rents (rents net of concessions) from CoStar. We take CRISM loans originated before 2008 that had a 90-day delinquency at some point during the 2010–2019 period and track them until the end of 2024. We include the summary statistics on this data set in Table 2.

4 Model

Our life-cycle model of mortgage default includes several important features that are more realistic than those of Campbell and Cocco (2015), which allows us to more quantitatively benchmark households’ utility incentives to default. First, we endogenize housing tenure choice about owning or renting, house size, and mortgage balance, all of which allow households to downsize after being hit by a liquidity shock. While this feature significantly expands the state space of the model, it is essential for obtaining positive default costs after more realistic rent processes are calibrated. Second, we allow households to re-buy their homes after an expected seven years. Third, we incorporate a default period that precedes foreclosure and allow households to cure their default before foreclosure. By incorporating these realistic

features, we are able to more quantitatively assess the benefit of defaulting for underwater households. The options of different types of households are summarized in Figure 3. We describe them in more detail later, with time parameters and preferences in Section 4.1, income and house prices in Section 4.2, and mortgage contracts in Section 4.3. We also provide a full mathematical description of households' recursive problem in Appendix Section D.

[Figure 3 inserted here]

4.1 Time parameters and preferences

Household utility is constant relative risk aversion (CRRA) over nonhousing consumption c_{it} and housing consumption h_{it} :

$$\max E_1 \sum_{t=1}^T \beta^{t-1} \frac{((1-\eta)c_{it}^{1-\phi} + \eta h_{it}^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1-\sigma} + \beta^T b_0 \left(\frac{(b_1 + w_{i,T+1})^{1-\sigma}}{1-\sigma} \right), \quad (1)$$

where β is the discount factor, σ is the coefficient of relative risk aversion, ϕ captures the substitutability between housing h_{it} and nonhousing consumption c_{it} , and η is the relative importance of housing consumption. Housing service can be obtained by owning or renting. b_0, b_1 are parameters for the bequest motive. Specifically, b_0 measures the relative importance of utility derived from bequest, and b_1 captures the extent to which bequest is a luxury good.

Households receive income y_{it} in each period and optimize over nonhousing consumption c_{it} , housing consumption h_{it} and housing tenure choice o_{it} subject to buying and selling frictions, mortgage size m_{it} subject to cash-out refinancing frictions, and mortgage default. These choices affect liquid savings $w_{it} \geq 0$.

The homeownership and rental markets are partially segregated. The size of houses available for rent is $\{h_1^R, h_2^R, h_3^R, \dots, h_{j_R}^R\}$, and the size of houses available for purchase is $\{h_1^O, h_2^O, h_3^O, \dots, h_j^O\}$. We assume that $\exists j_R$ such that $h_j^R \leq h_1^O$ for all $j \leq j_R$. That is, some smaller units are available only for rent rather than purchase. This segregation allows us to fit the life-cycle ownership profile of households.

In each period, homeowners derive utility based on owned-housing size h . On the other hand, they pay a real maintenance cost $\delta\bar{P}h$ and property insurance and tax $\tau_h\bar{P}h$ charged as a fraction of stationary house prices. Renters do not pay these costs but instead pay real rent $\bar{R}h$ in each period. For the initial $\frac{\bar{R}}{\bar{P}}$, we apply the 2001 rent-to-price ratio estimated at a unit level from Loewenstein and Willen (2023) and keep real rents fixed at their 2001 levels, consistent with the findings in Loewenstein and Willen (2023) for the evolution of rent over the period from 2001 to the Great Recession.

Entry into homeownership involves a buyer cost τ_b , while exit from homeownership involves a seller cost τ_s . Households can also adjust their owned-housing size h while remaining owners by paying τ_b on their existing home and τ_s on their new home. Renters can freely adjust their housing size h^r .

4.2 Income and house prices

Our income process follows that of Guvenen et al. (2014), which features a mixed normal distribution whose means differ during expansions and recessions. Our process includes the addition of a life-cycle age profile χ_j , since we match on homeownership rates over the life cycle. Specifically, household i of age j receives real labor income y_{ijt} at time t given by

$$\log(y_{ijt}) = z_{it} + \chi_j + \epsilon_{it}, \quad (2)$$

$$z_{it} = \rho z_{it-1} + \eta_{it}, \quad (3)$$

where η_{it} is given by a mixed normal distribution:

$$\eta_{it} = \begin{cases} \eta_{it}^1 \sim N(\mu_{1s(t)}, \sigma_1) \text{ with prob. } p_1, \\ \eta_{it}^2 \sim N(\mu_{2s(t)}, \sigma_2) \text{ with prob. } 1 - p_1 \end{cases}, \quad (4)$$

and aggregate $s(t) \in \{E, R\}$ indicates whether the economy is in an expansion (E) or recession (R).

sion (R) year. Following Kaplan et al. (2020), our life-cycle age profile χ_j is approximated with a quadratic function of age. ϵ_{it} is a transitory shock that follows normal probability distribution with mean 0 and standard deviation σ_ϵ .

Labor income is taxed. Adopting the functional form in Heathcote et al. (2017), our model features a progressive income tax,

$$\tau(y_{ijt}) = \tau_0(y_{ijt})^{1-\tau_1}, \quad (5)$$

where τ_0 captures the average level and τ_1 captures the degree of progressivity of the income tax.

In our model, real house prices per unit of housing service h are decomposed into an average price $\bar{P}$ and a shock ζ , such that $P_t = \bar{P}\zeta_t$. Real house prices follow a random walk with drift that depends on business cycles whose shocks ϵ_s^p follow a mixed normal distribution:

$$\begin{aligned} \log(P_t) - \log(P_{t-1}) &= \underbrace{\log(\bar{P}) - \log(\bar{P})}_{=0} + \log(\zeta_t) - \log(\zeta_{t-1}) = \epsilon_s^p, \\ \epsilon_s^p &= \begin{cases} \epsilon_{s1}^p \sim N(\mu_{1s(t)s(t-1)}^P, \sigma_{1s(t)s(t-1)}^P) & \text{with prob. } \pi_{s(t)s(t-1)}, \\ \epsilon_{s2}^p \sim N(\mu_{2s(t)s(t-1)}^P, \sigma_{2s(t)s(t-1)}^P) & \text{with prob. } 1 - \pi_{s(t)s(t-1)}, \end{cases} \end{aligned} \quad (6)$$

and aggregate state $s(t) \in \{E, R\}$, which captures how average price growth and price growth volatility may differ depending on whether the economy is entering an expansion (E) or a recession (R). Note that the shared aggregate state $s(t)$ between income and house-price processes implies that they are positively correlated.

4.3 Mortgage contracts, home equity extraction, and default

Our model features the most popular mortgage contracts: 30-year fixed-rate contracts with interest rate r^m and term $N = 30$, which involves constant nominal payments over time as

given by the standard amortization formula. Note that real mortgage payments are deflated by the rate of inflation. The borrower is subject to two constraints when originating or refinancing a mortgage (that is, when mortgage age $n = 1$). First, the leverage is limited by a loan-to-value constraint:

$$D_{it} \leq (1 - d)P_t h, \quad (7)$$

where $\xi = 5\%$ implies a down-payment requirement of 5 percent. Second, there is a cap on the scheduled payment-to-income ratio at mortgage origination, which is set at 50 percent based on the analysis in Greenwald (2018) of the pre-2000 period.

Furthermore, mortgage origination and refinancing involve an origination cost of 1 percent of the loan amount plus \$2,000 in 2001 dollars, following Agarwal et al. (2013). Mortgage refinancing allows households to extract their home equity in times of house-price increases and thereby increase their LTV.

At the beginning of each period t , mortgage borrowers can choose to default by stopping their mortgage payments for a year. If they have sufficient liquidity, borrowers in default can cure their mortgage and become current again by paying the amount they owe plus a late fee of 5 percent of the payments they missed. They can also pay off the entire mortgage and the late fee, sell their home, and rent. If they choose neither of these options, they will fall into foreclosure. Defaulting incurs an immediate utility cost of ψ , which captures the impact of default on the borrower's credit score, any impact on future credit access, and/or any moral or reputational concerns.

A foreclosed borrower loses all their home equity, becomes a renter, and cannot buy a home as long as their foreclosure flag, Ω , remains on their record. In each period, there is probability q that their record of default will be removed in the following period, giving them the option to become owners again. Appendix Section D provides a mathematical description of our model's features.

5 Calibration and Implications

A key question for our model involves the implications for the utility incentives to default after the model’s parameters are calibrated to key moments of the US housing market. We describe the calibration process in Section 5.1, the model’s fit to both targeted and nontargeted moments in Section 5.2, the model’s implications for income changes given default in Section 5.3, and the model’s implications for a counterfactual drop in rent in Section 5.4.

5.1 Parameter calibration

We calibrate the model to match the age profile of homeownership rates, loan-to-value ratios, payment-to-income ratios, debt-to-income ratios, and default rates by LTV ratios. We conduct the calibration in two stages. In the first stage, we determine the parameters that are directly estimated using the data or taken from the literature. In the second stage, we calibrate key parameters by minimizing the absolute distance between the model’s moments and those in the data. Table 1 summarizes both sets of parameters.

5.1.1 Parameters determined outside the model

Each period in our model corresponds to one year in the data.

Demographics: Households are born at age 23 and live to age 85. Households retire at age 64.

Preference: Following Guren et al. (2021), we set intertemporal elasticity of substitution σ to 2. Discount rate β , housing share in the utility η , substitution between housing and nonhousing consumption ϕ , bequest motives b_0, b_1 , and the utility cost of default ψ are to be calibrated.

Asset: The risk-free interest r is set as 2 percent per year. The average annual inflation rate has been stable at about 2 percent since the late 1990s. The mortgage spread is $\zeta_m =$

1.5%, which is the average difference between contract mortgage interest rates and the market yield on 30-year Treasury bonds based on data from the Federal Housing Finance Agency and Federal Reserve Bank of St. Louis. Annual inflation π is 2 percent, which gives us an annual nominal mortgage interest of $r_m = 5.5\%$.

We construct the initial asset distribution using data from the 2001 Survey of Consumer Finances for households aged 20 to 22. We use the value of all their assets to construct the initial asset distribution.

Housing: We set the annual property tax and maintenance cost at 1.5 percent and 1 percent, respectively, which match the average property taxes and owner costs reported in the 2001 American Community Survey. Transaction costs for buyers and sellers are $k_s = 6\%$ and $k_b = 2\%$ (see, for example, Sommer et al. (2013)). The variable mortgage closing cost is set as $\omega_1 = 1\%$, which matches the average initial fees reported by the FHFA. The constant mortgage closing cost is set as $\omega_0 = 2000$ in 2001 dollars, following Agarwal et al. (2013).

For the baseline calibration in 2001, we set the down-payment requirement ξ as 5 percent, which is standard in the literature. The constraint on the payment-to-income ratio is set as 50 percent.

Aggregate State: There are two aggregate states: recessions R and expansions E . The transition probabilities between recessions and expansions are taken from Campbell et al. (2021). The annual probability of transitioning from an expansion to a recession is 0.18, and the probability of transitioning from a recession to an expansion is 0.63. Both income and house prices depend on this aggregate state, and therefore income and house-price shocks are correlated in the model.

Income: Following Guvenen et al. (2014), we set the annual autocorrelation of persistent earnings shocks ρ as 0.979 and the standard deviation of transitory shock σ_ϵ as 0.186. The mixed normal distribution has a first mixture probability of $p_1 = 0.49$. The two mixture components have innovation terms during expansions E and recessions R that are $\mu_{1E} = 0.119, \mu_{2E} = -0.026, \mu_{1R} = -0.102, \mu_{2R} = 0.094, \sigma_1 = 0.325, \sigma_2 = 0.001$ for the parameters

in Equation 4, with subscripts E and R corresponding to the aggregate state subscript $s(t)$ in Equation 4. The deterministic age-income profile $\bar{w}_j$ is calibrated to match the average household income for different age groups using data from the Panel Study of Income Dynamics (PSID). Parameters in the tax schedule $\tau_0 = 4.787$ and $\tau_1 = 0.151$ come from Kaplan et al. (2020).

Average House Prices and Rent: The average house-price level, $\bar{P} = \$162,169$, is the median house value reported by owners, and the annual rent, $\bar{R} = \bar{P} \times 0.1019$, is calibrated to the sample’s average rent-to-price ratio for a single-family home or condo that was both sold and rented on the MLS in 2001, following Loewenstein and Willen (2023). In the model, house prices are allowed to move dynamically, while real rents are assumed to stay constant, capturing the 2001–2012 period when house prices rose then crashed. As shown in Figure 2a, real rents were relatively flat from 2001 through 2012, while house prices were volatile.

Important to this calibration is that rent-to-price ratios fell about 30 percent from 2001 to the 2020s due to house-price growth (Loewenstein and Willen, 2023), which made renting more financially attractive regarding flow costs in recent years compared with our model period. Whether this implies a greater strategic incentive to default should another house-price crash occur depends on households’ reasons for choosing to own rather than rent despite a lower rent-to-price ratio. If households choose to own a home despite a lower rent-to-price ratio due to an ownership premium or concerns about rent-growth risk (Sinai and Souleles, 2005), and if those factors are relatively stable, then strategic default incentives may continue to be moderated. Our model can be adapted to include an ownership premium and to address concerns about rent-growth risk if it is used to evaluate a crash in the 2020s housing market.⁹

House-Price Dynamics: We estimate the house-price dynamics using CoreLogic zip code house-price index growth for the 1987–2008 period deflated with the CPI deflator. When the economy stays in expansion (that is, an aggregate state of E in the preceding

⁹Our model does not currently include an ownership premium, as it is not needed to fit ownership rates in our setting. Many models in the literature do include such a premium and use it to fit homeownership rates.

period and the current period), we estimate that house-price shocks have mean $\mu_{EE}^P = 0.02$ and standard deviation $\sigma_{EE}^P = 0.057$ and model these shocks as a normal distribution with $\pi_{EE} = 1$. When the economy stays in recession (that is, an aggregate state of R in the preceding period and the current period), we estimate a mean $\mu_{RR}^P = -0.017$ and standard deviation $\sigma_{RR}^P = 0.09$, which we also model as a normal distribution with $\pi_{RR} = 1$. We note that our estimated mean real house-price growth is larger during expansions than during recessions, at 2 percent versus -1.7 percent, respectively, while the standard deviation of house-price growth is smaller in expansions than in recessions, at 5.7 percent and 9 percent, respectively.

When the economy falls from expansion to recession (that is, an aggregate state that transitions from E in the preceding period to R in the current period), we use a mixed normal distribution to fit house-price shocks and capture the fact that some recessions are more likely than others to be associated with house-price declines. We use a maximum likelihood approach to estimate these mixtures. We estimate the probability of the first mixture as $\pi_{ER} = 0.2$, the mean and standard deviation of the first mixture as $\mu_{1,ER}^P = -0.0925$, $\sigma_{1,ER} = 0.0758$, and the mean and standard deviation of the second mixture as $\mu_{2,ER}^P = -0.021$, $\sigma_{2,ER} = 0.0708$. Similarly, when the economy rises from recession to expansion (that is, an aggregate state that transitions from R in the preceding period to E this period), we use a mixed normal distribution to calibrate house-price shocks and estimate the probability of the first mixture as $\pi_{RE} = 0.22$, the mean and standard deviation of the first mixture as $\mu_{1,RE}^P = 0.0013$, $\sigma_{1,RE} = 0.0293$, and the mean and standard deviation of the second mixture as $\mu_{2,RE}^P = 0.021$, $\sigma_{2,RE} = 0.0611$. We note that the means of the two mixtures are more different and the standard deviations are higher during transitions from recession to expansion than during transitions from expansion to recession. This suggests that in our model, there is more dispersion in house-price growth as the economy enters into a recession. The relatively low levels of mean house-price growth as the economy emerges from recessions to expansions $\mu_{1,RE}^P, \mu_{2,RE}^P$ also point to limited mean reversion in the house-price

dynamics.

In summary, our house-price dynamics exhibit lower expected price growth and limited expected mean reversion during recessions. This estimated dynamic results mostly from our estimation period ending in 2008, before the period of house-price increases that followed the Great Recession. We chose this estimation period to be conservative. The limited mean reversion is likely to increase the financial incentives for negative-equity default in our model, thus making our finding of limited financial incentives on negative-equity default more stark. In addition, the estimation period may also correctly capture the expectations of negative-equity households during the housing-crash period if they were making similar inferences based on historical data.

[Table 1 inserted here]

5.1.2 Parameters calibrated within the model

We calibrate the discount rate β , minimum house size h^{min} , housing share η , substitutability between housing and nonhousing consumption ϕ , the two parameters that shape the bequest motive (b_0 and b_1), and the default utility cost ψ to the 2001 ACS age profile (for 58 age groups) of the homeownership rate, 2001 SCF loan-to-value ratios, and mortgage payment-to-income ratios by age group. We also calibrate to default rates by LTV among owners from our Equifax/McDash CRISM data set. Importantly, when calibrating to Equifax/McDash CRISM default rates, we assume that their mark-to-market LTVs are measured with a 14 percent standard deviation error, which is consistent with the exploration in Bogin et al. (2019) of the accuracy of the zip code-level price indexes we use to calculate mark-to-market LTVs. We estimate these parameters using the simulated method of moments. Specifically, we choose the parameters that minimize the distance between simulated moments in the stationary equilibrium and the data.

Although we jointly calibrate seven parameters, each parameter is most closely related to one moment. The minimum house size available to buy, h^{min} , is most sensitive to the home-

ownership rates of young and old households with relatively low wealth, as these households are more likely to be constrained by the minimum house size. The housing share and the substitutability between housing and nonhousing consumption in the utility function, η and ϕ , are most closely related to the payment-to-income ratios. The discount factor, β , influences how much households care about future consumption and is related to loan-to-value ratios. The parameters of the bequest motive, b_0 and b_1 , have the largest impact on the ownership rates and mortgage debts of older households. A stronger bequest motive increases older households' likelihood of continuing to own their home and reduces their likelihood of using mortgage refinancing to extract equity. In particular, b_1 mainly captures the extent to which a bequest is a luxury good, and ψ is mainly relevant for the default rate.

The calibrated values of our parameters are also shown in Table 1. We calibrate an $h^{min} = 0.8$, which implies that houses larger than 80 percent of the median house size are available to buy. We calibrate a housing share in utility η of 0.2. The elasticity of substitution between housing and nonhousing consumption $\frac{1}{\phi}$ is calibrated to $\frac{1}{1.5} = 0.67$, close to Li et al. (2016). Our calibrated discount factor β is 0.92, within the range provided by previous studies (see, for example, Guren et al. (2021) and Athreya et al. (2018)). The two parameters for the bequest motive are $b_0 = 20$ and $b_1 = 1$. Importantly, the calibrated value for the default utility cost is $\psi = 0.15$, which is equivalent to a 0.7 percent lifetime consumption loss. This number is low compared with the 25 to 50 percent loss of lifetime consumption estimated in the earlier literature.

5.2 Fit to life-cycle moments

Figure 4 examines the fit of our calibration regarding life-cycle moments by comparing the model-generated moments to data from the 2001 Survey of Consumer Finances. Panel (a) of Figure 4 plots life-cycle homeownership rates, defined as the share of all households in the model that are homeowners. Panel (b) of Figure 4 plots life-cycle loan-to-value ratios. Panel (c) of Figure 4 plots life-cycle payment-to-income ratios. The payment-to-income

ratios are defined as the ratio of the sum of a borrower’s mortgage, property tax, and insurance payments divided by their income in the model. When comparing them to the 2021 SCF data, we define these ratios as the amount of a borrower’s mortgage payment (which, for most households, includes property taxes and insurance as escrow) divided by their reported household income. Panel (d) of Figure 4 plots life-cycle debt-to-income ratios, defined as a borrower’s mortgage balance divided by their income. Given the limited number of free parameters, our model matches these life-cycle moments well. Importantly, it captures the increase in the homeownership rate and decreasing mortgage balances by age, thereby capturing the joint household housing-tenure choice and mortgage debt choice.

[Figure 4 inserted here]

We use moments that are not targeted in the calibration to further assess the performance of our baseline. Specifically, we compare the distribution of PTI by LTV at mortgage origination, house values, and annual rents simulated in our model to the data for all owners. As emphasized in Ganong and Noel (2023) and Low (2023b), income shocks play a major role in driving mortgage default; matching the payment-to-income ratio by LTV is important in predicting mortgage default; and fitting the distribution of house values and annual rents is important for capturing the downsizing motive of default. Figure 5 plots model fit in relation to these nontargeted moments. Although not directly targeted in the calibration, our model matches the distribution of PTI by LTV, house values, and annual rents well.

[Figure 5 inserted here]

5.3 Implications for mortgage default

We use our model to examine its implications for mortgage default. Figure 6 presents the results. Panel (a) of Figure 6 plots the fit of the model regarding default rates. We find that the model fits well in terms of default rates by LTV bins for both above-water and underwater households.

[Figure 6 inserted here]

Panel (b) of Figure 6 plots the average income change among defaulters as a fraction of their mortgage payments. We find significant average income loss among the defaulters, comparable to the size of mortgage payments for all underwater borrowers, consistent with Ganong and Noel (2023) and with limited strategic default behavior. Notably, Panel (b) of Figure 6 suggests that the income change before default decreases with the LTV, which is in sharp contrast to pure negative-equity-driven strategic default. This implies that defaulter income changes increase with the LTV. The reason for this is the positive correlation between income and house prices induced by our aggregate state s : periods with house-price declines coincide with periods of income declines, leading to greater observed income declines for defaulting underwater households.

Panel (c) of Figure 6 plots the foreclosure/sale/cure rates by LTV. Positive-equity defaulters are more likely to sell or cure their mortgages, whereas negative-equity defaulters are more likely to fall into foreclosure. Therefore, while there is limited strategic default in the sense of pure negative-equity-driven default in our model, the model still implies a relationship between negative equity and eventual foreclosure completion conditional on default, which is consistent with Palmer (2024) and Cespedes et al. (Forthcoming).

We also examine the model-implied average income change conditional on the type of default resolution (that is, foreclosure, cure, or sale) in the period following default in Appendix Figure A.3. Curing typically requires a positive income shock after defaulting, consistent with a borrower being hit with a negative income shock before default and being unable to afford their mortgage payment without an income recovery. Lower LTV households experience a larger variation in income change relative to mortgage payments due to their lower mortgage payments. When borrowers whose LTV is lower than 60 percent (positive equity) are hit with a further negative income shock after defaulting, they sell their homes, which is consistent with their potential demand to downsize. For borrowers whose LTV is higher than 60 percent, the decision to sell or foreclose after defaulting is related less to income

changes following default and more to feasibility, whereas the decision to cure continues to rely on a large positive income shock, consistent with the central role of liquidity in driving the initial default decision.

5.4 Analysis of flow payoffs

5.4.1 Lower cost of renting

Our model suggests that negative equity by itself does not necessarily create a large financial incentive to default if the alternative to renting remains unattractive and that flow utility is central to a household’s incentive to default. To examine this effect more closely in a counterfactual, we simulate a one-time 10 percent permanent decrease in real rent and examine the resulting increases in default rates across the LTV distribution. As shown in Figure 2 and Appendix Figure A.4, a 10 percent decline in real rent is roughly the maximum that borrowers experienced when house prices crashed during the 2007–2012 period, though as we know *ex post*, those declines were not permanent.

Panel (a) of Figure 7 shows that in this counterfactual, default rates rise by about 5 percentage points for households with an LTV of 130 to 140 percent and for households with an LTV that is higher than 140 percent. The effect declines with the LTV, and the default rates of households with an LTV under 80 percent are unaffected by rent declines. This suggests that flow utility considerations may be more important for default behavior when households are already underwater and potentially hit by an income shock. It also suggests a “triple trigger” model of default consisting of negative equity, income shocks, and flow utility shocks. As flow utility shocks become more common—due, for example, to climate catastrophes making homes less attractive places to live—this model of default may become more relevant.

[Figure 7 inserted here]

Panel (b) of Figure 7 shows that foreclosure rates conditional on default rise about 2

percentage points when households expect real rents to have fallen 10 percent permanently. This rise in foreclosure rates conditional on default is relatively uniform for households with a 70 percent or higher LTV. As defaulting households in our time period typically face severe liquidity shocks (Ganong and Noel, 2023), the effort they subsequently exert to prevent eventual foreclosure as it relates to rent declines is indicative of flow utility considerations being relevant to household decisionmaking. We test this model prediction in Section 6.

5.4.2 Adjustment of house size

An important component of the flow payoff comparison between continuing to own and defaulting is the ability to downsize after defaulting, thereby saving on rent. That is, the relevant counterfactual may not be renting the same size house, but rather renting a smaller unit. This downsizing possibility may be optimal for households that experience an income shock and therefore have a lower demand for space. We examine this possibility in Figure 8, which plots the change in house size among households that underwent foreclosure.

[Figure 8 inserted here]

As Figure 8 shows, most households that undergo foreclosure in our model do downsize, which is consistent with the negative income shock they experienced. Approximately 74 percent of foreclosed households downsize to 66 to 33 percent of their original house size, whereas 25 percent of foreclosed households downsize to less than or equal to 66 percent of their original house size. Only 1 percent has a house size that is more than 33 percent of their original house size. The significant downsizing following foreclosure is consistent with the negative income shock that was experienced by the defaulters in our model.

6 Further External Validity

Our model implies that household foreclosure decisions should be correlated with the flow utility value of renting versus owning and that there should be fewer foreclosures conditional

on default in areas with greater rent growth. We study this prediction empirically using a measure, at the core-based statistical area (CBSA) level from CoStar, of effective rents per square foot, which are asking rents for new tenants net of concessions such as a free month of rent. We also use mortgage servicer data from CRISM, which include information on mortgage performance matched with credit bureau data that contain information on any outstanding second liens. We limit our sample to first-lien, owner-occupied mortgages. Summary statistics for our main sample are in Table 2.

Since many loans with moderate and even severe delinquency eventually cure, we limit our sample to loans that experience 90-day default and test whether the local rent growth experienced by those borrowers has any statistical effect on whether they eventually fall into foreclosure. The sample therefore contains one observation per loan, at the time of the 90-day default, and the dependent variable is an indicator of foreclosure completion. This exercise is similar to the counterfactual exercise discussed earlier (and shown in Figure 7b) in which rents fall 10 percent. To facilitate the use of an instrument for rent growth, which we discuss later, we also limit the sample to loans originated before 2008.

For our baseline analyses, we run the following linear probability model:

$$D_i = \beta_1 \Delta \ln(\text{rent}_{ot(i)}) + \beta_2 LTV_i + \zeta \mathbf{X}_i + \gamma_{t(i)} + \psi_{o(i)} + \delta_i + \epsilon_i,$$

where D_i is an indicator of whether loan i is eventually foreclosed upon multiplied by 100.¹⁰ The main right-hand-side variable ($\Delta \ln(\text{Rent}_{ot(i)})$) is the effective rent growth per square foot associated with loan i from that loan's origination date (o) to the date of 90-day default (t(i)). Our measure of the current LTV_i is created using the sum of the outstanding first-lien mortgage balance and any outstanding balances on second liens in the numerator and the purchase price updated using a county-level house-price index. The $\mathbf{X}_i$ vector of additional characteristics includes the monthly mortgage payment at the time of default (including

¹⁰We consider a foreclosure completed if the loan status subsequently becomes "R" or "L," which stand for real estate owned or liquidated, respectively.

escrow payments for property taxes and insurance), an indicator for whether the loan is fixed rate, and two measures of local economic conditions: the average wages in that county and year and the county-level annual unemployment rate. Lastly, we include fixed effects for the year of default ($\gamma_{t(i)}$), the year the loan was originated ($\psi_{o(i)}$), and state where the property is located (δ_i). The year fixed effects capture any countrywide macroeconomic events; the origination year fixed effects control for time since origination; and state fixed effects capture any fixed factors related to the state, such as state-level recourse laws. Standard errors are clustered by the year of 90-day default. Our main coefficient of interest is β_1 , which captures the degree to which the probability of eventual foreclosure is affected by changes in the cost of renting following loan origination. We expect β_1 to be negative, indicating that the more rental costs increase, the lower the probability of eventual foreclosure.

The results are presented in Table 3. All the regressors are normalized to be mean zero, standard deviation of one. In column (1), we include only loan-level covariates. In columns (2) and (3), we add measures of local economic conditions. The addition of these variables has little impact on the coefficient on rent growth, which is statistically significant and negative. The coefficient on rent growth implies that a one standard deviation increase in rent growth (which, according to Table 2, is an increase of about 8.5 percentage points) results in a 1.5 to 1.7 percentage point decline in the probability of eventual foreclosure. In other words, a 10 percentage point decline in rent growth would increase foreclosure rates by 1.7 to 2 percentage points. This is relative to an average foreclosure rate of 33.6 percent in this sample of defaulted loans. This is on par with the results from the model displayed in Figure 7b. The other significant coefficient is on LTV, whereby a higher LTV is positively correlated with the probability of eventual foreclosure conditional on default, also consistent with our model prediction in Figure 6c.

An important potential confounder to our results is that rent growth may be correlated with local economic conditions beyond those captured by our controls. To address this issue, we instrument for rent growth using a measure, created by Gete and Reher (2018), of the

share of banks by CBSA that became subject to stress testing after the Great Recession. Gete and Reher (2018) show that as mortgage supply contracted in these markets, rent growth increased while demand for rental properties grew. The paper also shows that the change in rents due to this channel are unrelated to local economic conditions and unrelated to origination conditions for loans originated before 2008.

The result in column (4) indicates that once we instrument for rent growth using the Gete and Reher (2018) instrument, our estimated effect of rent growth on foreclosure completion is larger. Specifically, a one standard deviation increase in rent growth reduces foreclosure completions by more than 6 percentage points. That this effect is larger than our OLS estimates in columns (1) and (3) suggests that rent growth may actually be negatively correlated with unobserved positive economic conditions, perhaps due to unobserved negative economic conditions increasing rental demand relative to owning over the time period (Foote et al., 2018), making our OLS estimates downwardly biased in terms of magnitude.

7 Discussion

Our paper emphasizes the central role of flow payoffs in evaluating models of household behavior. While the earlier literature tends to focus the discussion of strategic behavior on the role of negative equity and income, the combination of which gives rise to “double trigger” default caused by a combination of negative equity and income shocks, our analysis highlights that flow payoffs are also an important component of strategic behavior. This suggests a “triple trigger” framework in which home equity, income shocks, and changes to the flow costs of owning versus renting all need to be considered when evaluating empirical evidence on mortgage default.

Our results also have implications for understanding the 2020s housing market. Rent-to-price ratios for 2001 are comparable to those for 2009 (Loewenstein and Willen, 2023), so our calibration accurately captures the housing-crash period, during which underwater

default was most salient. In more recent years, rent-to-price ratios have fallen about 30 percent relative to the 2001 level (Loewenstein and Willen, 2023). Whether another housing crash will result in greater strategic incentives to default will depend on why households continue to choose to own despite a lower rent-to-price ratio. If households choose to own due to an ownership premium or to avoid the risk of rent growth (Sinai and Souleles, 2005), then strategic default incentives may continue to be moderated even with a low initial rent-to-price ratio as long as the reasons households choose to own (for example, ownership premium and rent growth risk) are unchanged.

8 Conclusion

The lack of ubiquitous strategic mortgage default during the mid-2000s house-price declines has posed a challenge to those studying mortgage default, with many commentators suggesting nonpecuniary costs, such as shame and social management, as reasons (White, 2010b). Using a detailed quantitative model of mortgage default, we show that factoring in a more realistic process for rent largely eliminates the household strategic default incentives. Therefore, in sharp contrast to the earlier literature, we estimate the strategic benefits of defaulting on a mortgage to be limited for underwater households when rents are downwardly stable in times of steep house-price declines.

Our paper has two important policy implications. Our finding that the financial benefits of defaulting to underwater households are dramatically lower than previously estimated has implications for policy design. If the financial cost of defaulting was as high as previously thought, then we would have expected a foreclosure to improve a household’s financial position. In fact, such high costs would imply that a policy preventing foreclosure (such as forbearance or principal reduction) would be welfare reducing. But, in fact, higher default rates during the 2008 housing crash are considered to be the primary driver of lower housing returns among specific demographic groups (Kermani and Wong, 2021). Our model instead

suggests that defaulting over the Great Recession period was ex ante costly and that liquidity policies meant to prevent foreclosure were ex ante financially beneficial, thus offering a more realistic interpretation of the effects of policy such as the Home Affordable Modification Program (HAMP).

Second, our research contributes to the theoretical foundation supporting the efficacy of cash-flow-based policies, such as mortgage forbearance, in reducing household defaults. Empirical evidence suggests that these policies are more cost-effective than alternatives focused on home equity, like loan modifications, in preventing defaults (Ganong and Noel, 2020). Our model enhances the generalizability of these findings by indicating that their effectiveness does not depend on significant costs driven by moral, emotional, or other non-financial factors. In fact, concerns that moral considerations might lead households to act against their financial interests have prompted discussions on the acceptability of strategic default.¹¹ Our study suggests that by accurately assessing the ongoing costs associated with renting versus homeownership, the debate over morality becomes unnecessary.

¹¹White (2010a) argued prominently that strategic default is morally acceptable. See a description by Roger Lowenstein, “Walk Away from Your Mortgage!” *New York Times*, January 10, 2020, available [here](#).

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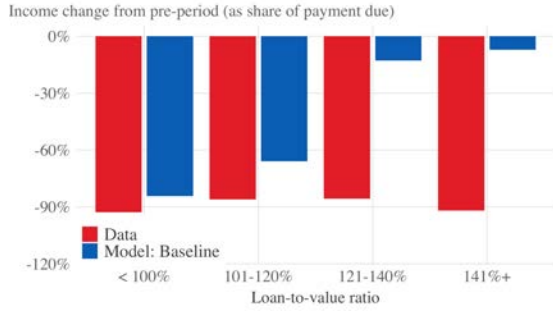
Sommer, K., Sullivan, P., Verbrugge, R., 2013. The equilibrium effect of fundamentals on house prices and rents. *Journal of Monetary Economics* 60, 854–870. doi:10.1016/j.jmoneco.2013.04.017.

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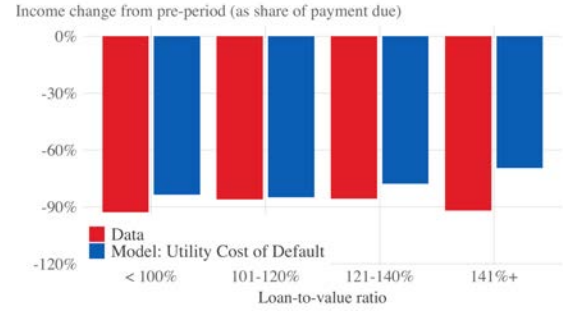
White, B.T., 2010b. Underwater and not walking away: Shame, fear, and the social management of the housing crisis. *Wake Forest L. Rev.* 45, 971.

Figure 1: Income Changes Conditional on Default Implied by Campbell and Cocco (2015)'s Model, Compared with Ganong and Noel (2023)'s Data

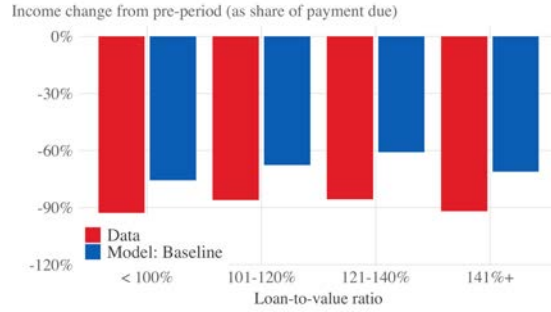
Note: In this figure, we present the results of the income change conditional on default implied by Campbell and Cocco (2015)'s model, as compared with the income declines in Ganong and Noel (2023)'s bank account data. In Panel 1a, we plot the income changes conditional on default from the model without any nonpecuniary default stigmas in blue, as compared with the data in red. In Panel 1b, we add a high default stigma to the model worth 25 percent of lifetime consumption, with results plotted in blue, compared with the same data in red. In Panel 1c we plot the results of the Campbell and Cocco (2015) model without any stigma but with real rent fixed at constant 2001 levels.



(a) Campbell and Cocco (2015)'s Model



(b) Campbell and Cocco (2015)'s Model with High Default Stigma



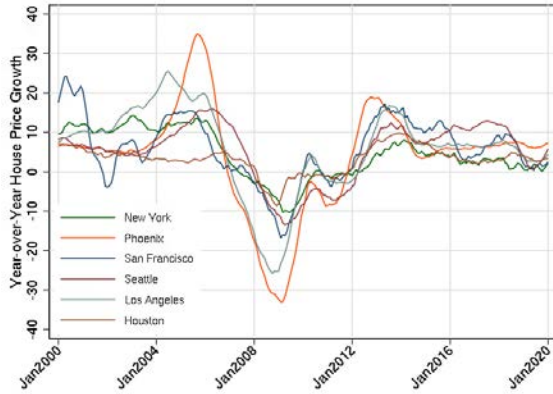
(c) Campbell and Cocco (2015)'s Model with Constant Real Rent

Figure 2: House Prices, Rents, and New-tenant Rents in a Cross Section of Cities

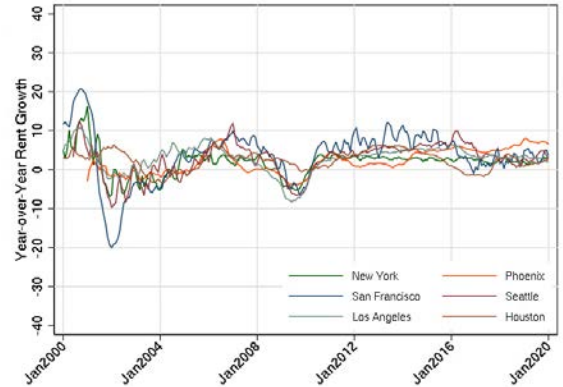
Note: Panel 2a of this figure is reproduced from Loewenstein and Willen (2023). It is a depiction of real house-price indexes (Case-Shiller) and rent (BLS) over time. Panels 2b and 2c are plots of nominal year-over-year house price and rent growth, respectively, for a selection of cities. House-price growth is measured using the CoreLogic HPIs and rent growth is measured using the CoreLogic Single-Family Rent Index. The cities included were chosen because they have a single-family rent index from 2024 going back to 2000. Adams et al. (2024) show that CoreLogic SFRI inflation rates are a good approximation of new-tenant rent inflation from the BLS Housing Survey, which is a representative sample of renter-occupied housing units. The percentage of households with negative equity and more than 10 percent decline in rents is plotted in Appendix Figure A.4.



(a) Real House Prices and Rents (Loewenstein and Willen, 2023)



(b) Nominal House-price Growth in a Cross Section of Cities (CoreLogic HPI)



(c) Nominal New-tenant Rent Growth in a Cross Section of Cities (CoreLogic SFRI)

Figure 3: Dynamic Options of Households

Note: This is a depiction of the dynamic options of households as described in Section 4. Flagged Renters are renters with a foreclosure flag.

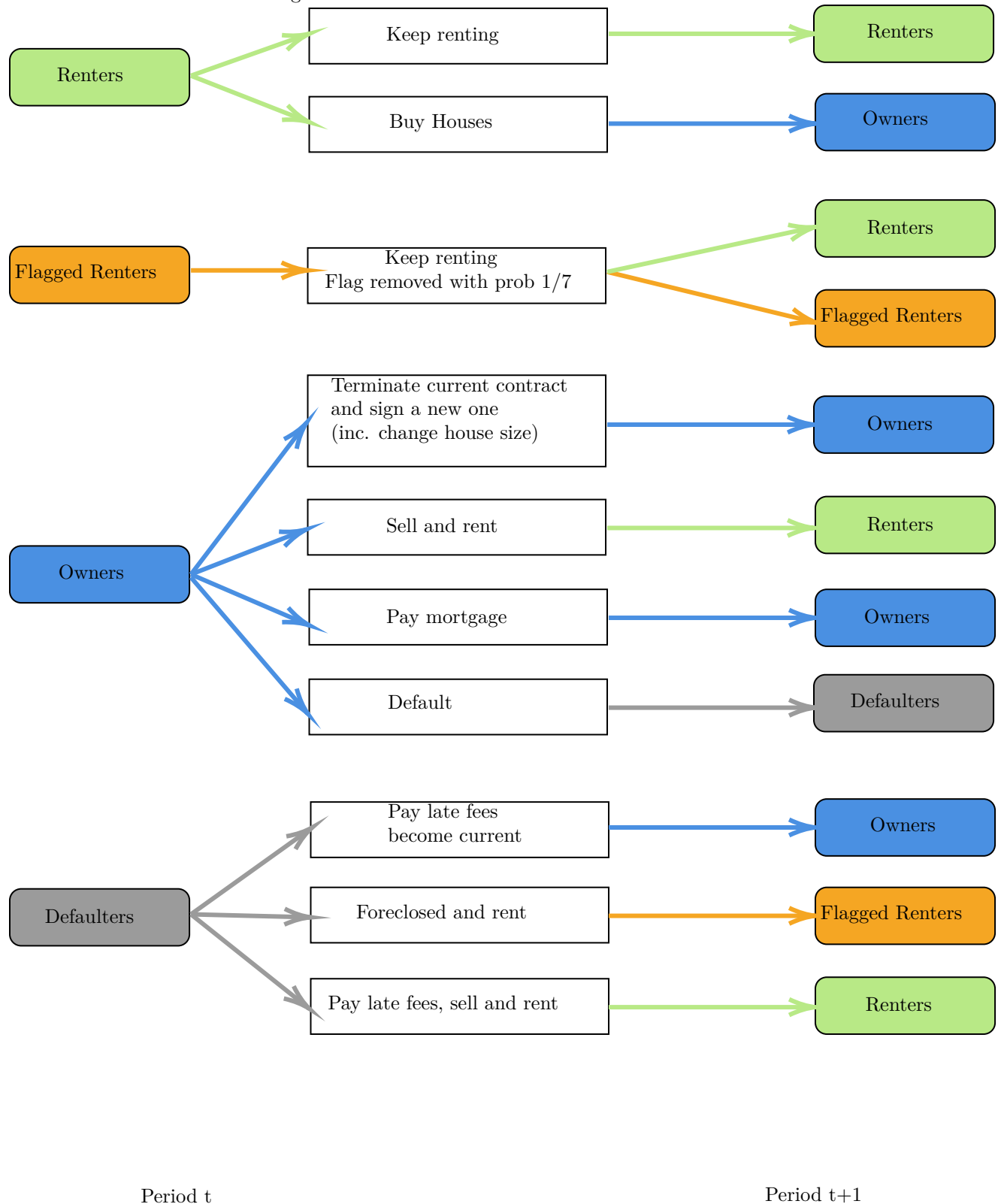
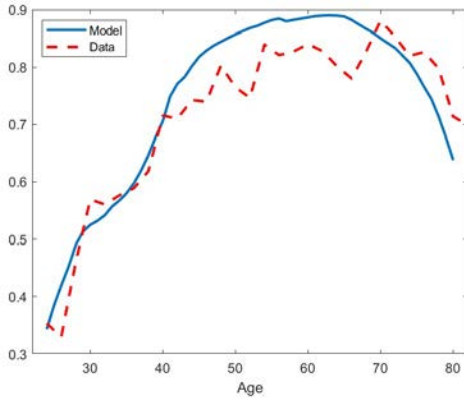
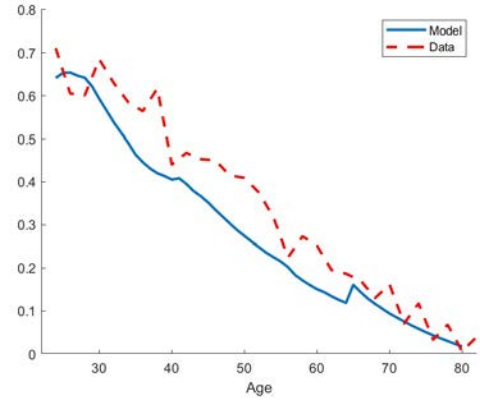


Figure 4: Calibration Results: Life-cycle Moments

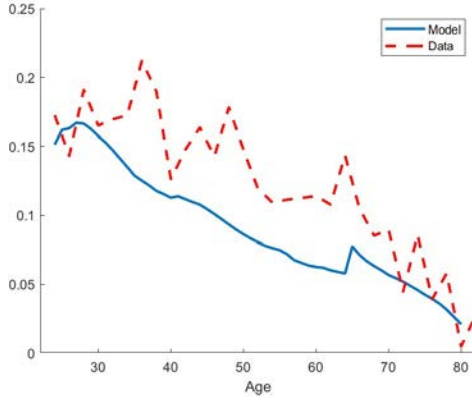
Note: In this figure, we present the model fit of our targeted moments to the data given the parameters in Table 1. Homeownership rates are in Panel 4a; LTVs by borrower age are in Panel 4b; average PTIs by borrower age are in Panel 4c; and average DTIs by borrower age are in Panel 4d. Sources: PTIs and LTVs are from the 2001 SCF; homeownership rates are from the 2001 ACS; default rates are from CRISM.



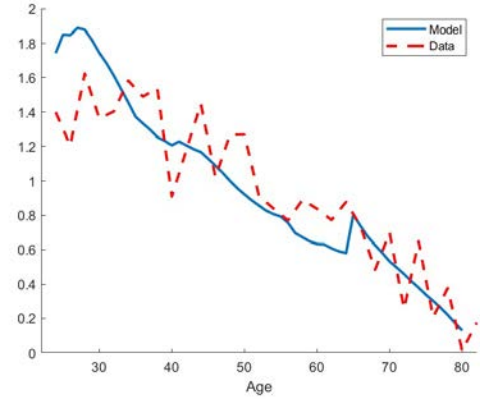
(a) Homeownership Rates



(b) Loan-to-Value Ratios



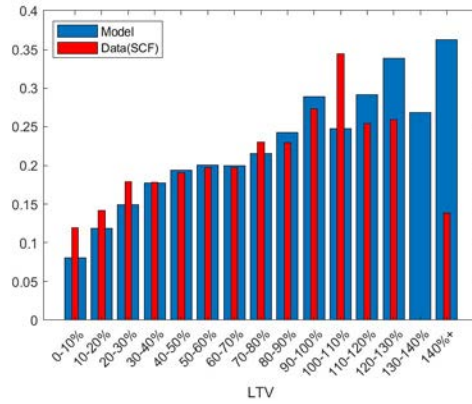
(c) Payment-to-Income Ratios



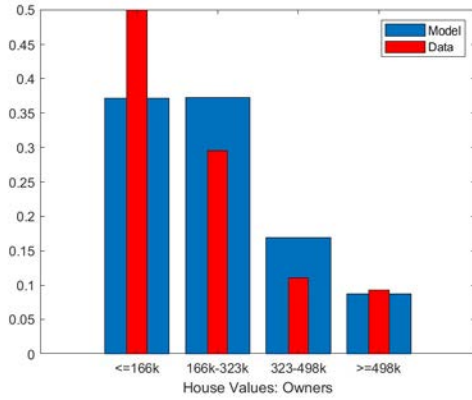
(d) Debt-to-Income Ratios

Figure 5: Model Fit in Nontargeted Moments

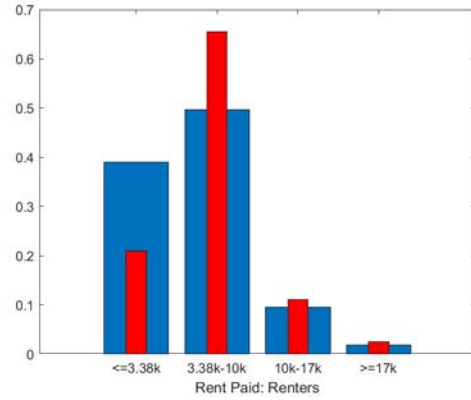
Note: We compare nontargeted model moments to empirical estimates from the 2001 SCF. Panel 5a is a comparison of payment-to-income ratios; Panel 5b is a comparison of the distribution of home values among homeowners; and Panel 5c is a comparison of annual rents paid by renters.



(a) Average Payment-to-Income Ratio by LTV



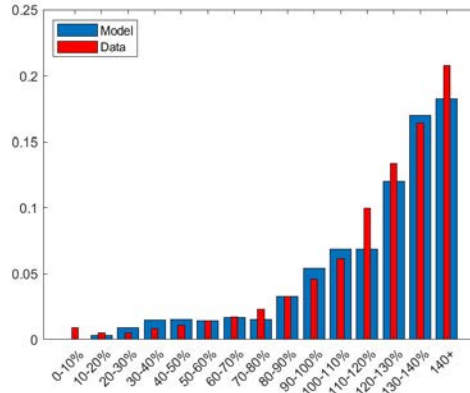
(b) Distribution of House Values among Owners



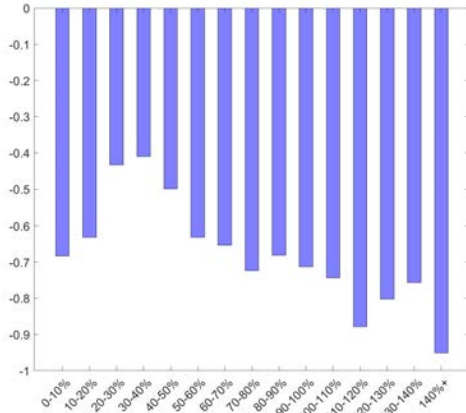
(c) Distribution of Annual Rents among Renters

Figure 6: Model Implications for Mortgage Default

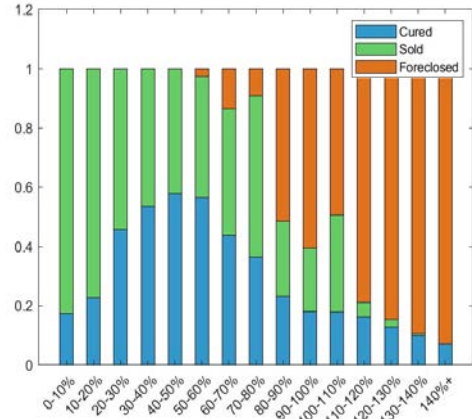
Note: Panel 6a is a depiction of the model-implied default rate by LTV and compared with our CRISM data. In Panel 6b, we plot the average one-year income change among defaulters relative to their mortgage payment. In Panel 6c, we present the foreclosure/sale/cure rates among defaulters by LTV. In all panels, the model-implied LTVs are assumed to be measured with error with a standard deviation of 14 percent.



(a) Default Rate



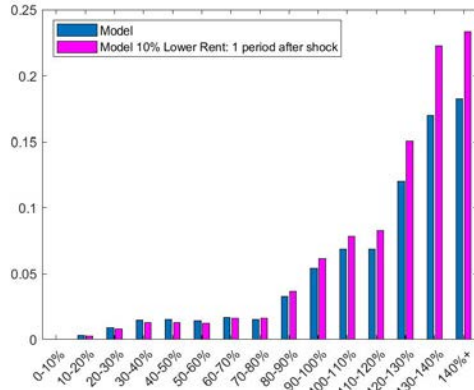
(b) Income Change before Default as a Fraction of Mortgage Payment



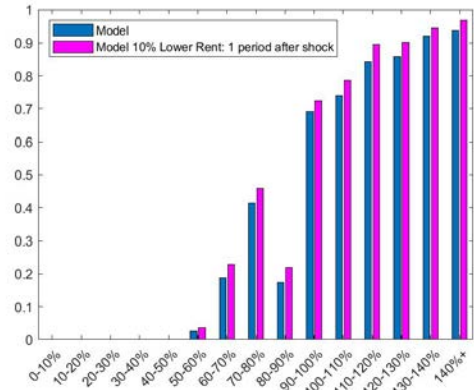
(c) Foreclosure/Sale/Cure Rates by LTV among Defaulted Loans

Figure 7: Implication of a Permanent 10 Percent Decline in Real Rent

Note: All panels are a depiction of the short-run impact of a 10 percent decline in real rent on default and foreclosure one year after the rent shock. In Panel 7a, the default rate is defined as the number of households that become delinquent within the LTV bin by the total number of households within the LTV bin. In Panel 7b, the foreclosure rate among defaulters is defined as the fraction of delinquent households that undergo foreclosure by their mark-to-market LTV. For Panels 7a and 7b, model-implied LTVs are assumed to be measured with error with a standard deviation of 14 percent.



(a) Default Rate by LTV



(b) Foreclosure Rate Conditional on Default

Figure 8: Housing Consumption Adjustment among Foreclosed Households

Note: This is a plot of the distribution of housing consumption adjustment among foreclosed households. Housing consumption adjustment is measured by the ratio of the difference of the size of the house that a household rents after foreclosure to that of the home it previously owned.

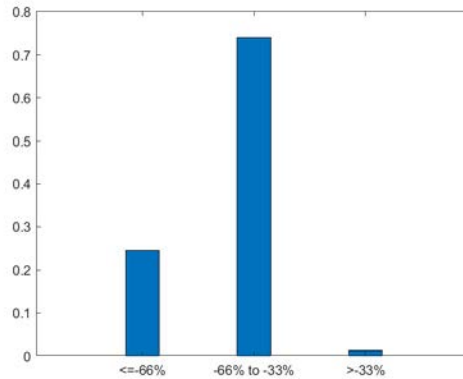


Table 1: List of Parameters

Note: These are the model parameters that are either directly calibrated by grid search via moment matching or taken from the literature. Calibrated parameters are described as “Calibrated,” and a source is provided otherwise. Borrower birth, retirement, and exit ages are assumed and labeled as “Assumed.”

Parameter	Value	Source
Demographics		
Household born age	23	Assumed
Household retirement	64	Assumed
Household exit age	85	Assumed
Preferences		
Intertemporal elasticity of substitution σ	2	Guren et al. (2021)
Discount rate β	0.92	Calibrated
Housing share in utility η	0.2	Calibrated
Substitution between housing and nonhousing consumption ϕ	1.5	Calibrated
Bequest motive b_0	20	Calibrated
Bequest motive constant term b_1	1	Calibrated
Utility cost of default ψ	0.15 (CEV 0.7%)	Calibrated
Asset		
Annual risk-free interest rate	2%	Assumed
Annual inflation rate π	2%	Inflation target
Mortgage spread ζ_m	1.5%	FHFA & Federal Reserve Bank
Housing		
Annual property tax τ_h	1.5%	American Community Survey
Annual maintenance cost δ	1%	American Community Survey
Seller transaction cost k_s	6%	Sommer et al. (2013)
Buyer transaction cost k_b	2%	Sommer et al. (2013)
Mortgage origination cost (fixed) ω_0	\$2000 2001 dollars	Agarwal et al. (2013)
Mortgage origination cost (variable) ω_1	1%	Agarwal et al. (2013)
Downpayment requirement χ	5%	LTV distribution
Cap on PTI PTI^{limit}	0.50	Pre-2000 standard
Term N	30	Assumed
Minimum purchase size $h^{\min}$	0.8	Calibrated
Foreclosure flag removal probability	1/7	Flag stays for 7 years in expectation
Average price $\bar{P}$ per unit of h	\$162,169	Estimated
Annual rent $\bar{R}$ per unit of h^r	\$16,525	Estimated
Aggregate State		
$P(recession recession)$	0.37	Campbell et al. (2021)
$P(recession expansion)$	0.18	Campbell et al. (2021)
Income		
Income profile $\bar{w}_j$		PSID
Tax schedule τ_0	4.787	Kaplan et al. (2020)
Tax schedule τ_1	0.151	Kaplan et al. (2020)
Income process ρ	0.979	Güvenen et al. (2014)
Income process p_1	0.49	Güvenen et al. (2014)
Income process μ_{1E}, μ_{2E}	0.119, -0.026	Güvenen et al. (2014)
Income process μ_{1R}, μ_{2R}	-0.102, 0.094	Güvenen et al. (2014)
Income process $\sigma_1, \sigma_2, \sigma_\epsilon$	0.325, 0.001, 0.186	Güvenen et al. (2014)
House-price shocks		
Expansion to expansion $\mu_{EE}^P, \sigma_{EE}^P$	0.02, 0.057	Estimated
Recession to recession $\mu_{RR}^P, \sigma_{RR}^P$	-0.017, 0.09	Estimated
Expansion to recession $\pi_{ER}, \mu_{1,ER}^P, \sigma_{1,ER}^P, \mu_{2,ER}^P, \sigma_{2,ER}^P$	0.2, -0.0925, 0.0758, -0.021, 0.0708	Estimated
Recession to expansion $\pi_{RE}, \mu_{1,RE}^P, \sigma_{1,RE}^P, \mu_{2,RE}^P, \sigma_{2,RE}^P$	0.22, 0.0013, 0.0293, 0.021, 0.0611	Estimated

Table 2: Summary Statistics

Note: These are summary statistics for our empirical analysis. The data are from the sample used for the regressions in Table 3, so they are limited to loans originated before 2008 in CBSAs for which we have the independent variable from Gete and Reher (2018). Rent growth is from the year of mortgage origination to the year of the first 90-day default. There is one observation per loan. *Sources:* Authors' calculations using Equifax Credit Risks Insight Servicing" and ICE, McDash; CoStar; BLS Local Area Unemployment Statistics; Census Annual County Population Estimates; and the Quarterly Census of Employment and Wages.

	Mean	Std
Foreclosure 90-Day Default	33.6	47.2
Current LTV (%)	91.4	34.2
Monthly Mortgage Payment (\$)	1560.2	1146.5
Fixed Rate (%)	73.4	44.2
$\Delta \ln(\text{Rent/Sq. Ft.})_{t,o}$	6.3	8.5
Unemployment Rate (%)	8.2	3.0
Average Wage (\$,000)	46.9	10.2
N	250,435	

Table 3: Probability of Foreclosure Conditional on 90-day Default

Note: The dependent variable is an indicator of 90-day default conditional on future foreclosure completion multiplied by 100. The sample is limited to owner-occupied first-lien mortgages for which we have non-missing information for effective rents, wages, unemployment, and local population and that experienced a 90-day default. Each loan has one observation. Rent growth is the log change in the CBSA-level effective rent per square foot from CoStar at the time of loan origination relative to the time of the 90-day default. The monthly mortgage payment includes escrow payments for that loan. The average wage is for all covered employees in a given county and year, and county employment is an annual count of total employees. Standard errors are clustered by year. All regressors are normalized to have mean zero and a standard deviation of one. Current LTV is a combined LTV measure calculated using the sum of the primary principal balance outstanding and any outstanding debt on home equity lines of credit and closed-end seconds as the numerator, and a house value updated using county-level house-price indexes in the denominator. The independent variable is from Gete and Reher (2018). *Sources:* Authors' calculations using Equifax Credit Risks Insight Servicing" and ICE, McDash; CoStar; BLS Local Area Unemployment Statistics; Census Annual County Population Estimates; and the Quarterly Census of Employment and Wages.

	Foreclosure (in percentage points)			
	(1)	(2)	(3)	(4)
$\Delta(\text{Rent})$	-1.72*** (0.41)	-1.60*** (0.41)	-1.52*** (0.41)	-6.13** (2.27)
Current LTV	5.25*** (0.29)	5.21*** (0.29)	5.17*** (0.28)	4.65*** (0.43)
$\ln(\text{Monthly Payment})$	-0.44 (0.68)	-0.35 (0.69)	-0.32 (0.71)	-0.37 (0.69)
$\ln(\text{Average Wages})$		-0.51*** (0.15)	-0.44*** (0.12)	-0.19 (0.19)
Unemployment Rate			0.37 (0.42)	-0.82 (0.74)
R_a^2	0.054	0.054	0.054	0.007
Observations	250,435	250,435	250,435	250,435
Year FEs	Y	Y	Y	Y
Close Year FEs	Y	Y	Y	Y
State FEs	Y	Y	Y	Y
FRM Dummy	Y	Y	Y	Y
Mean(Y)	33.6	33.6	33.6	33.6
Std Dev(Y)	47.2	47.2	47.2	47.2
Model	OLS	OLS	OLS	IV
F-Stat				1868

Internet Appendix

This appendix supplements the analysis in this paper. The following is a list of the appendix sections.

Table of Contents

A	Details about the Campbell and Cocco (2015) Model	2
B	Additional Model Results	4
C	Additional Exhibits	5
D	Households' Recursive Problem in Our More Detailed Model of Mortgage Default	6

A Details about the Campbell and Cocco (2015) Model

The Campbell and Cocco (2015) model assumes that rent-to-price ratios $\frac{R_{it}}{P_{it}}$ evolve as:

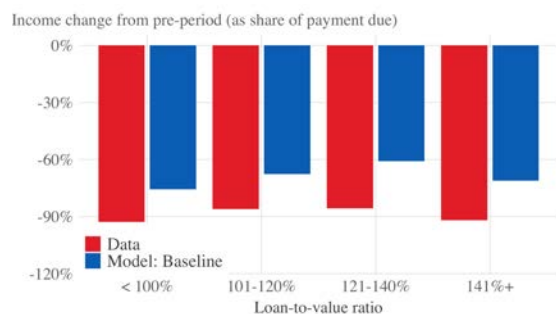
$$\frac{R_{it}}{P_{it}} = [\underbrace{i_{i,t}}_{\text{nominal rate}} - \underbrace{E_t(\exp(\Delta p_{t+1}^H + \pi_t) - 1)}_{\text{expected nominal house price growth}} + \underbrace{\tau_p}_{\text{property tax}} + \underbrace{m_p}_{\text{maintenance}}], \quad (8)$$

which implies that, to the extent the nominal rate $i_{i,t}$ is pro-cyclical, $\frac{R_{it}}{P_{it}}$ is also pro-cyclical and falls during recessions. For example, a 4 percentage point decrease in $i_{i,t}$ from 2007 to 2010 implies that the $\frac{R_{it}}{P_{it}}$ fell from approximately 7 percent in 2007 to approximately 3 percent in 2010. In reality, the rent-to-price ratio rose from approximately 7 percent to approximately 10 percent from 2007 to 2010 (Loewenstein and Willen, 2023), a three-fold difference relative to the model's assumption.

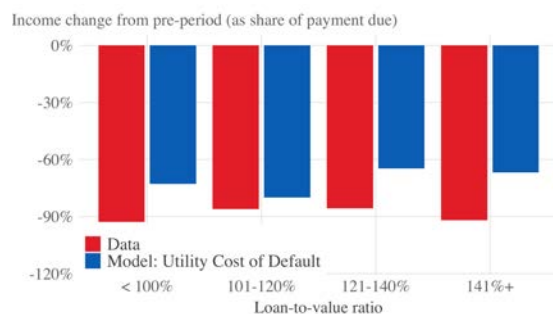
Panel (a) of Figure A.1 examines the Campbell and Cocco (2015) model with constant real rent and finds that it largely fits the data regarding income change conditional on default by borrower LTV. Panel (b) of Figure A.1 examines the Campbell and Cocco (2015) model with constant real rent as well as a high default stigma worth 25 percent of lifetime consumption, which shows that, relative to Panel (a) of Figure A.1, a high default stigma has little additional explanatory power.

Figure A.1: Income Changes Conditional on Default Implied by Campbell and Cocco (2015)'s Model, Compared with Ganong and Noel (2023)'s Data, Additional Scenarios

Note: This figure presents the results of the income change conditional on default implied by Campbell and Cocco (2015)'s model, as compared with the income declines in Ganong and Noel (2023)'s bank account data. Panel (a) presents the results of the Campbell and Cocco (2015) model without any stigma but fixes real rent at constant 2001 levels. Panel (b) adds a high default stigma to the model worth 25 percent of lifetime consumption in addition to fixing real rent at 2001 levels.



(a) Campbell and Cocco (2015)'s Model with Constant Real Rent



(b) Campbell and Cocco (2015)'s Model with Constant Real Rent and High Default Stigma

B Additional Model Results

Figure A.2: Model-implied Moving Rates

Note: This figure presents our model's implied annual share of homeowners who move by their age. The model is as described in Section 4.

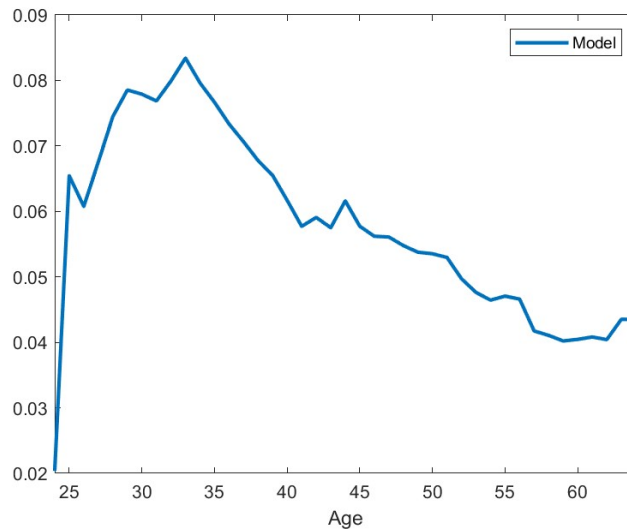
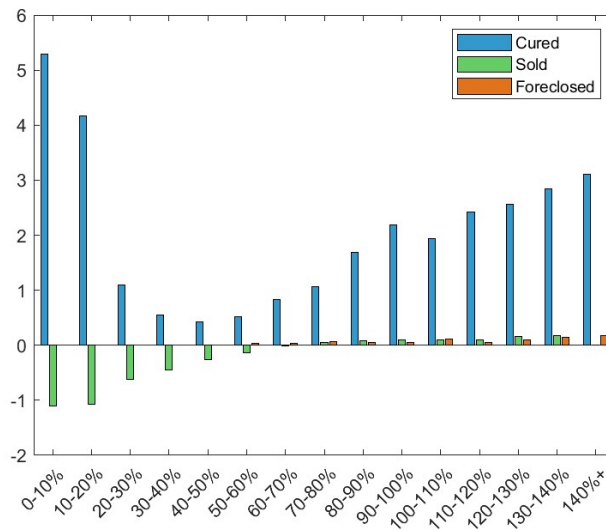


Figure A.3: Income Change Conditional on Default Resolution as a Fraction of Mortgage Payment

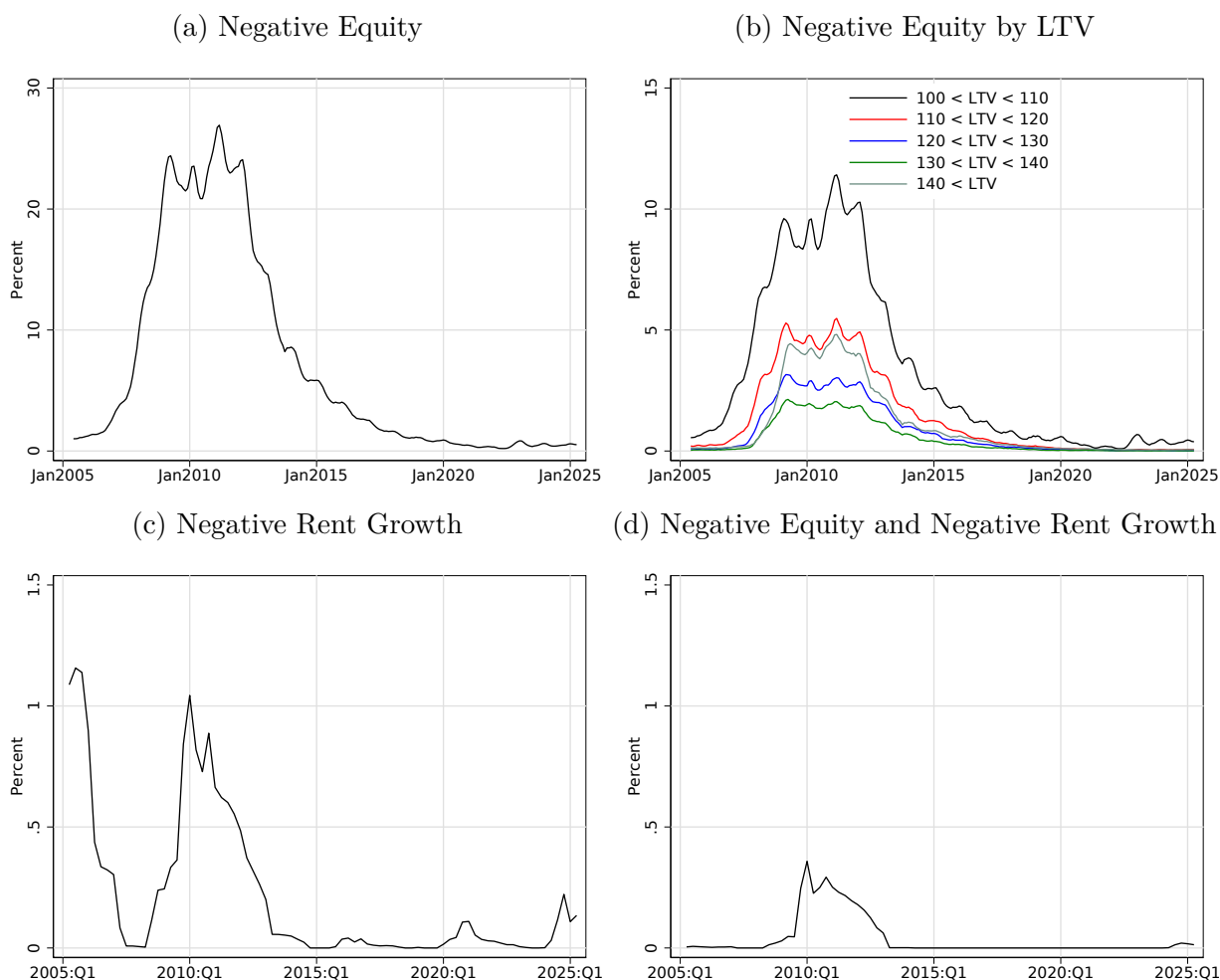
Note: In this figure, we plot the average one-year income change among defaulters relative to their mortgage payment by type of default resolution (foreclosed, cured, or sold). The model is described in Section 4. These borrowers had already experienced an average income drop when deciding to default (depicted in Figure 6b). In this figure, we plot their subsequent income change and shows that borrowers are more likely to cure when their income recovers relative to a year prior. Model-implied LTVs are assumed to be measured with error with a standard deviation of 14 percent.



C Additional Exhibits

Figure A.4: Percentage of Loans with Negative Equity and/or Rent Growth

Note: Figure A.4a shows the percentage of active loans with negative equity. Figure A.4b is the percentage of loans with negative equity in the given LTV buckets. Figure A.4c is the percentage of loans that have experienced at least a 10 percent decline in rents since origination. Figure A.4d shows the percentage of loans that have negative equity and have experienced at least a 10 percent decline in rents since origination. Rents are measured using effective rents per square foot by CBSA from CoStar. Loan data are from CRISM, and LTVs are adjusted to account for second liens. Property values are updated using zip code house-price indexes from CoreLogic.



D Households' Recursive Problem in Our More Detailed Model of Mortgage Default

The state variables (Λ) of a household are its mortgage age n , owned house size h , mortgage payment m , saving in the risk-free asset k , current income shocks ϵ , current house-price shocks ζ , current aggregate state $s \in \{E, R\}$, age j , and whether it has a default flag on file $\Omega \in \{0, 1\}$. The household value function has nine states summarized by Λ :

$$\Lambda = (n, h, m, k, \epsilon, \zeta, s, j, \Omega). \quad (9)$$

At the beginning of each period, there are four types of households: (1) a renter with a clean foreclosure record $\Omega = 0$ and no owned house $h = 0$ makes a decision on the size of house to rent and whether to become an owner starting from the current period; (2) a homeowner with positive owned housing size $h > 0$ who makes the decision to change house size, sell and rent, pay their mortgage, or default the next period; (3) a defaulted owner who is behind on their mortgage payment and who makes a decision to pay late fees and become current, get foreclosed on and rent, or pay their mortgage, sell, and rent; and (4) a flagged renter with a foreclosure record on file $\Omega = 1$ makes a decision on the size of the house to rent. Figure 3 details the options of different types of households.

D.0.1 Renters' maximization problem

Households that enter the period with no owned housing ($h = 0$) choose between getting the service through the rental market by making a decision on the size of the house to rent, $h^r \in \{h_1^R, h_2^R, h_3^R, \dots, h_J^R\}$ (option labeled as RR) and becoming owners by choosing the size of house to buy h' and mortgage contract m' (option labeled as RO). The value of renters' maximization problem is:

$$V(n = 0, h = 0, m = 0, k, \epsilon, z, \zeta, s, j, \Omega = 0) = \max_{R, O} \{V^{RR}(.), V^{RO}(.)\}, \quad (10)$$

where $V^{RR}(.)$ is the value of continuing to be a renter, and $V^{RO}(.)$ is the value of becoming an owner, with the option labels RR and RO in the superscripts. Because renters do not have a mortgage, their mortgage age state n is in the missing state represented by zero. Their house size h , mortgage payment m , and default flag Ω are also zero. We mathematically define $V^{RR}(.), V^{RO}(.)$ next.

Continue to be renters

The value of continuing to be a renter is:

$$\begin{aligned} V^{RR}(0, 0, 0, k, \epsilon, \zeta, s, j, 0) = \max_{h^r, k'} \{ & \frac{((1 - \eta)c^{1-\phi} + \eta(h^r)^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1 - \sigma} \\ & + \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} V(0, 0, 0, k', \epsilon', \zeta', s', j + 1, 0) \}, \quad (11) \\ \text{s.t.} \quad & c + k' + \bar{R}h^r = (1 - \tau(y(\epsilon, j)))y(\epsilon, j) + (1 + r)k \end{aligned}$$

where the states in $V^{RR}(.)$ follow the same order as in Equations (9) and (10), $\bar{R}h^r$ is total rental payment, $(1 - \tau(y(\epsilon, j)))y(\epsilon, j)$ is the after-tax income, and rk is the return on the risk-free asset.

Become owners The value of becoming an owner is:

$$\begin{aligned}
V^{RO}(0, 0, 0, k, \epsilon, z, \zeta, s, j, 0) &= \max_{h', m', k'} \left\{ \frac{((1 - \eta)c^{1-\phi} + \eta(h')^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1 - \sigma} \right. \\
&\quad \left. + \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} V(1, h', m', k', \epsilon', z', \zeta, s, j, 0) \right\} \\
&\quad s.t. \\
&\quad \frac{m'(1 - (1 + r_m)^{-N})}{r_m P h'} < 1 - \xi, \quad (12) \\
c + k' + (1 + \tau_b) \bar{P} \zeta h' + (\delta + \tau_h) \bar{P} h' &= (1 - \tau(y(\epsilon, j))) y(\epsilon, j) + (1 + r) k \\
&\quad + (1 - \omega_1) \frac{m'(1 - (1 + r_m)^{-N})}{r_m} - \omega_0 \mathbb{1}_{m > 0} \\
\frac{m'}{y(\epsilon, j)} &< PTI^{limit}
\end{aligned}$$

where the states in $V^{RO}(\cdot)$ follow the same order as in Equations (9) and (10), $\frac{m'(1 - (1 + r_m)^{-N})}{r_m}$ is the amount of a new mortgage loan, $(1 + \tau_b) \bar{P} \zeta h'$ is the total cost of purchasing a new house, and $(\delta + \tau_h) \bar{P} h'$ is the cost of maintenance and property tax. Down-payment requirement ξ and payment-to-income ratio cap PTI^{limit} both apply as the household gets a new mortgage loan. Following Boar et al. (2022), we assume that there is a constant mortgage closing cost ω_0 and a variable cost that is proportional to mortgage debt ω_1 .

D.0.2 Homeowners' maximization problem

A homeowner, $h > 0$, with a mortgage contract m that was signed n years ago has four options: (1) continue with the current mortgage contract (option labeled as C); (2) get a new mortgage (refinance) without adjusting current house size or terminate the current mortgage by selling the house (and buying another house or not) (option labeled as N); (3) sell the house and rent (option labeled as OR); or (4) default on the mortgage (option labeled as D).

Thus, the value function V is given by the maximum value of these four options, with

the option labels in superscripts:

$$V(n, h, m, k, \epsilon, \zeta, s, j, 0) = \max_{C, N, OR, D} \{V^C(\cdot), V^N(\cdot), V^{OR}(\cdot), V^D(\cdot)\}. \quad (13)$$

For notation simplicity, in Equation (13) and in all subsequent value functions, we write the states in the same order as in Equation (9). In Equation (13), the last state of $V(\cdot)$, the foreclosure flag Ω , is set to zero because only defaulters can get foreclosed on and gain the foreclosure flag as flagged renters.

Continue with current mortgage contract

Owners who decide to continue with their current mortgage contract choose current consumption c and saving k' . The value of staying with the current mortgage contract is:

$$\begin{aligned} V^C(n, h, m, k, \epsilon, \zeta, s, j, 0) = & \max_{k'} \frac{((1 - \eta)c^{1-\phi} + \eta(h)^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1 - \sigma} + \\ & \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} [\mathbb{1}_{n < N} V(n + 1, h, m, k', \epsilon', \zeta', s', j + 1, 0) + \mathbb{1}_{n = N} V(0, h, 0, k', \epsilon', \zeta', s', j + 1, 0)], \\ \text{s.t. } & c + k' + (\delta + \tau_h)\bar{P}h = (1 - \tau(y(\epsilon, j)))y(\epsilon, j) + (1 + r)k - \frac{m}{(1 + \pi)^n} \end{aligned} \quad (14)$$

where $\mathbb{1}_{n=N}$ is an indicator function that takes a value of 1 if this is the last period of the mortgage contract. In other words, starting from the next period, the household has no mortgage debt. $\bar{P}$ is the stationary house price, which does not vary with business cycles, and $\delta\bar{P}h$ is the maintenance cost and $\tau_h\bar{P}h$ is the property tax, which are assumed to be proportional to the average house price $\bar{P}$. In other words, maintenance costs and property tax do not vary with business cycles. $(1 - \tau(y(\epsilon, j)))y(\epsilon, j)$ is the after-tax income, rk is the return on savings, and $\frac{m}{(1 + \pi)^n}$ is the real mortgage payment for a mortgage contract that was signed n periods ago with an inflation rate of π . Note that our model features declining real mortgage payments over time due to inflation.

Refinance or change house size

Owners who decide to refinance or adjust house size will first terminate the current mortgage contract then get a new mortgage contract (m') and a new house (h'). The value of getting a new loan is:

$$\begin{aligned}
V^N(n, h, m, k, z, \zeta, s, j, 0) &= \max_{k', h', m'} \frac{((1 - \eta)c^{1-\phi} + \eta(h')^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1 - \sigma} \\
&\quad + \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} V(1, h', m', k', z', \zeta', s', j + 1, 0) \\
&\quad s.t. \\
&\quad \frac{m'(1 - (1 + r_m)^{-N})}{r_m \bar{P}\zeta h'} < 1 - \xi \\
&\quad \frac{m'}{y(z, \epsilon, j)} < PTI^{limit} \\
c + k' + (\delta + \tau_h)Ph + (\tau_s \bar{P}\zeta h + \tau_b \bar{P}\zeta h') \mathbb{1}_{h \neq h'} &= (1 - \tau(y(\epsilon, j)))y(\epsilon, j) \\
&\quad + (1 + r)k + \bar{P}\zeta h - D(m, n) - \bar{P}\zeta h' \\
&\quad + (1 - \omega_1) \frac{m'(1 - (1 + r_m)^{-N})}{r_m} - \omega_0 \mathbb{1}_{m' > 0}
\end{aligned} \tag{15}$$

where $\mathbb{1}_{h \neq h'}$ is an indicator function that takes a value of 1 if the household adjusts its house size (that is, $h \neq h'$). Specifically, $\tau_s \bar{P}\zeta h$ is the cost of selling the current house, and $\tau_b \bar{P}\zeta h'$ is the cost of purchasing a new one. $D(m, n)$ is the loan balance on the current mortgage contract, which depends on the scheduled mortgage payment m and the number of payments households have made n , and $\bar{P}\zeta h - D(m, n)$ is home equity. $\bar{P}\zeta h'$ is the value of the new house, and $\frac{m'(1 - (1 + r_m)^{-N})}{r_m}$ is the amount of the new mortgage loan. We assume that there is a mortgage origination cost, which has a constant part ω_0 and a variable part proportional to the total loan amount, ω_1 . There are two additional constraints at mortgage origination. First, the loan-to-value ratio has to be lower than $1 - \xi$, which is equivalent to a minimum down-payment of ξ . Second, the scheduled mortgage-to-income ratio cannot exceed PTI^{limit} .

Sell and rent

Owners who decide to sell and become renters will first terminate the current mortgage contract and receive their housing service from the rental market. The value of selling and renting is:

$$\begin{aligned}
V^{OR}(n, h, m, k, z, \zeta, s, j, 0) &= \max_{k', h^r} \frac{((1 - \eta)c^{1-\phi} + \eta(h^r)^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1 - \sigma} \\
&\quad + \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} V(0, 0, 0, k', z', \zeta', s', j + 1, 0), \\
s.t. \quad c + k' + \bar{R}h^r &= (1 - \tau(y(\epsilon, j))y(\epsilon, j) + (1 + r)k \\
&\quad + (1 - \tau_s)\bar{P}\zeta h - D(m, n)
\end{aligned} \tag{16}$$

where Rh^r is rental cost. Specifically, $(1 - \tau_s)\bar{P}\zeta h - D(m, n)$ is the profit from selling the house net of transaction costs τ_s and τ_b and outstanding debt $D(m, n)$.

Default

Owners who choose to default (become delinquent) do not pay the mortgage and incur a direct utility cost ψ , which captures the potential consequence of late payments such as a decline in credit score or a potential reputation concern.

The value of default on a current mortgage contract is:

$$\begin{aligned}
V^D(n, h, m, k, \epsilon, \zeta, s, j, 0) &= \max_{k'} \frac{((1 - \eta)c^{1-\phi} + \eta(h)^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1 - \sigma} - \psi \\
&\quad + \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} VQ(n, h, m, k', \epsilon', \zeta', s', j + 1, 0) \\
s.t. \quad c + k' + (\delta + \tau_h)\bar{P}h &= (1 - \tau(y(\epsilon, j))y(\epsilon, j) + (1 + r)k
\end{aligned} \tag{17}$$

where $VQ(\cdot)$ is the value of starting the next period as delinquent owners.

Note that while we assume that defaulting homeowners do not pay their mortgage for a year, we still assume they pay the property tax and maintenance $(\delta + \tau_h)\bar{P}h$. This makes the possibility of curing their mortgage more natural in the subsequent period. To the extent

that households anticipating foreclosure also do not pay property taxes and maintenance, our default penalty may be understated by the amount $(\delta + \tau_h)\bar{P}h$, which with our calibrated $\delta = 1.5\%$ and $\tau_h = 1\%$ is a financial cost of \$4,054 for our median house size of \$162,169.¹

D.0.3 Defaulted owners' maximization problem

Defaulted owners have three options: (1) become current on their debt by paying all outstanding dues (missed payments and current payments), late fees, and applicable interest on late payments (option labeled as U); (2) pay the outstanding dues, sell the house, and rent (option labeled as S); (3) get foreclosed on and start the next period as renters with a foreclosure flag (option labeled as F).

The value of default owners Vq is given by the maximum value of these three options, with the option labels in superscripts:

$$VQ(n, h, m, k, \epsilon, \zeta, s, j, 0) = \max_{U, S, F} \{VQ^U(\cdot), VQ^S(\cdot), VQ^F(\cdot)\}. \quad (18)$$

Become current

The value of becoming current on mortgage debt is:

$$\begin{aligned} VQ^U(n, h, m, k, \epsilon, \zeta, s, j, 0) = & \max_{k'} \frac{((1 - \eta)c^{1-\phi} + \eta(h)^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1 - \sigma} + \\ & \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} [\mathbb{1}_{n < N-2} V(n+2, h, m, k', \epsilon', \zeta', s', j+1, 0) \\ & + \mathbb{1}_{n \geq N-2} V(0, h, 0, k', \epsilon', \zeta', s', j+1, 0)] \\ & s.t. \\ & c + k' + (\delta + \tau_h)\bar{P}h + \frac{m}{(1 + \pi)^{n+1}} + (1 + r_m + \kappa) \frac{m}{(1 + \pi)^{n+1}} = (1 - \tau(y(\epsilon, j)))y(\epsilon, j) \\ & + (1 + r)k \end{aligned} \quad (19)$$

¹This can be calculated as $\$162,169 * (\delta + \tau_h) = \$162,169 * 0.025 = \$4,054$.

where $(1+r_m+\kappa)\frac{m}{(1+\pi)^{n+1}}$ is the interest (r_m) and penalty (κ) for the late mortgage payment, and $\frac{m}{(1+\pi)^{n+1}}$ is the amount due in the current period.

Pay, sell, and rent

It is possible for delinquent households to sell the house, pay the outstanding amounts, and rent. Note that underwater households may prefer this option to being foreclosed on, as it allows them to get another house in the near future. The value of this option is:

$$VQ^S(n, h, m, k, \epsilon, \zeta, s, j, 0) = \max_{k', h^r} \frac{((1-\eta)c^{1-\phi} + \eta(h^r)^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1-\sigma} + \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} V(0, 0, 0, k', \epsilon', \zeta', s', j+1, 0) \\ s.t. \quad , \quad (20)$$

$$c + k' + (1+r_m+\kappa)\frac{m}{(1+\pi)^{n+1}} + \bar{R}h^r = (1-\tau(y(\epsilon, j)))y(\epsilon, j) + (1+r)k + (1-\tau_s)\bar{P}\zeta h - D(m, n+1)$$

where $(1+r_m+\kappa)\frac{m}{(1+\pi)^{n+1}}$ is the interest (r_m) and penalty (κ) for the outstanding mortgage payment carried from the preceding period, and $(1-\tau_s)\bar{P}\zeta h - D(m, n+1)$ is the profit from selling the house net of the transaction costs and mortgage debt. Note that after households pay the outstanding amount due, the remaining debt becomes $D(m, n+1)$.

Foreclosure If delinquent households decide to walk away from their debt, their household is foreclosed on, and they will start the next period with a foreclosure flag, which prevents them from becoming owners. The value of this option is:

$$VQ^F(n, h, m, k, \epsilon, \zeta, s, j, 0) = \max_{k', h^r} \frac{((1-\eta)c^{1-\phi} + \eta(h^r)^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1-\sigma} + \beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} VF(0, 0, 0, k', \epsilon', \zeta', s', j+1, 1) \\ s.t. \quad , \quad (21)$$

$$c + k' + \bar{R}h^r = (1-\tau(y(\epsilon, j)))y(\epsilon, j) + (1+r)k$$

where $VF(.)$ is the value of being renters with a foreclosure flag. Foreclosed households become renters, get housing service from the rental market, and pay rent $\bar{R}h^r$.

D.0.4 Flagged renters

Renters with a foreclosure flag cannot become owners in the current period. However, at the end of the period, with probability q , their flag will be removed, and they can start the next period as regular renters. The value of being a renter with a foreclosure flag is:

$$VF(0, 0, 0, k, \epsilon, \zeta, s, j, 1) = \max_{k', h^r} \frac{((1 - \eta)c^{1-\phi} + \eta(h^r)^{1-\phi})^{\frac{1-\sigma}{1-\phi}}}{1 - \sigma} +$$

$$\beta E_{(\epsilon', \zeta', s') | (\epsilon, \zeta, s)} [(1 - q)VF(0, 0, 0, k', \epsilon', \zeta', s', j + 1, 1) + qV(0, 0, 0, k', \epsilon', \zeta', s', j + 1, 0)] \quad (22)$$

$$s.t. \quad c + k' + \bar{R}h^r = (1 - \tau(y(\epsilon, j)))y(\epsilon, j) + (1 + r)k$$