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Robinson Meets Roy: Monopsony Power and Comparative Advantage*†

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Abstract

We provide a number of insights into the nature and consequences of monopsony power through the lens of comparative advantage, where employers' power in wage setting stems from match-specific rents. Chief among them is that employers will apply larger wage markdowns to workers with greater comparative advantage at their firm. This leads to stronger monopsony power over more productive workers, provided the workers' comparative advantage aligns with their absolute advantage. Using Brazilian administrative data, we confirm this prediction: monopsony disproportionately affects high-wage workers within firms and workers at high-paying firms. The model, calibrated to our estimates for Brazil, predicts that minimum wages increase both wages and formal employment for more productive workers while pushing less productive workers out of formal employment.

Keywords: Oligopsony, Comparative Advantage, Wages, Markdowns, Roy Model
JEL Classification: E2, J4

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1 Introduction

Research on labor markets is increasingly focused on the role of monopsony power, examining its impact on income inequality and labor's allocation, as well as its implications for policy interventions, specifically minimum legal wages. While traditional approaches associate monopsony power with firm characteristics—especially a firm's employment share—this paper proposes a novel perspective by attributing monopsony power to comparative advantage at the level of a worker-firm match.

Our study integrates Robinson's (1933) model of non-competitive firm wage setting with Roy's (1951) model of worker comparative advantage. The reasoning is as follows. Monopsony power emerges when a firm faces an imperfectly elastic labor supply, implying the firm has workers it could employ at lower wages. The differential between wage received and wage required represents a rent for those inframarginal workers. The Roy model associates rents with a worker's comparative advantage in their job, i.e., their productivity at their chosen job relative to alternatives. As a result, we can associate monopsony power in wage setting with workers who have a comparative advantage at their firm.

This synthesis directs attention to match characteristics in looking for monopsony power in the data. Our model predicts that employers will discriminate in wage setting, imposing larger wage markdowns on workers who exhibit greater comparative advantage at their firm. When workers' comparative advantage aligns with their absolute advantage, employers apply larger markdowns to their most productive workers. Hence, monopsony power promotes wage *equality* within firms. Intuitively, monopsony taxes worker rents; so, presuming higher-wage workers command larger employer rents, they will face steeper markdowns.

We study an oligopsony equilibrium where employers compete for workers, initially focusing on symmetric firms to analytically characterize employers' wage strategies over more and less productive workers. The allocation of workers across firms is efficient because firms' markdowns are symmetric. Firms offer wages that reflect a worker's productivity and the distribution of second-best productivities for workers of that productivity. In turn, markdowns at each productivity reflect a weighted average of workers' comparative advantage with their employer—greater comparative advantage means steeper markdowns. We also show how comparative advantage, and hence markdowns, are affected by the number of firms competing for a worker and the correlation of that worker's productivity across firms. Finally, we show that a minimum wage, through the strategic interaction of firms in oligopsony, acts to raise equilibrium wages throughout the wage distribution.

We next examine an asymmetric duopsony where one firm exhibits higher productivity. The more productive firm pays higher wages, is larger, but consequently has more inframarginal workers with rents, leading it to impose larger wage markdowns. These predictions align with those of previous models that associate monopsony power with large employers and predict that monopsony reduces aggregate productivity by distorting labor away from the largest, most productive firms (Berger et al., 2022).

We also consider a duopsony where one firm exhibits comparative advantage primarily for skilled workers (its technology is more skill-biased). This firm is high-wage because it predominantly employs high-skill workers, but especially marks down wages for those workers because comparative advantage is greatest there. Conversely, low-wage firms apply larger markdowns to less-skilled workers. These asymmetric markdown strategies impair skill sorting by diverting workers away from their comparative-advantage employer, beyond their distortion of relative firm sizes.

We test our predictions using data from Brazil's formal labor market through the Relação Anual de Informações Sociais (RAIS). The RAIS tracks establishment-worker matches, providing information on location, industry, employment, earnings, hours, and occupations. We segment workers into markets based on their metropolitan area and broad occupation. We then further stratify workers by their relative wages at their employer to see how an employer's rate of employment growth affects both wages and employment throughout the firm's wage distribution. The inverse elasticity of an employer's labor supply curve — which determines wage markdowns — is revealed through the relative responses of wages versus employment at different points in the wage distribution.

Our chief finding is that employment growth generates larger wage changes for an employer's high-wage workers, while employment responds more strongly at the bottom of its wage distribution. This indicates a less elastic labor supply for employers' higher-wage workers. Our estimates imply markdowns on the order of 25 percent for those in the top wage quartile at an employer while showing minimal monopsony power for employers' low-wage workers.

In light of our asymmetric examples of duopsony, we stratify employers both by relative wages and by market shares of employment. We find that high-wage employers face a less elastic labor supply, implying larger markdowns for their workers. This holds true even for their lower-wage workers, suggesting, in terms of the model, that they have a productivity advantage throughout their wage distribution, not just with high-skill workers. However, firms' relative wages and market shares only weakly align for Brazil, unlike in models with general productivity differences across firms. In fact, we do not find that large employers face a less elastic labor supply, even though high-wage firms do. Therefore, while monopsony's effects are often emphasized with respect to lower-wage workers, particularly in minimum wage analyses,[1] our estimates suggest that the most substantial markdowns occur for high-wage workers at high-wage firms, thereby promoting wage equality.

We end by examining the impact of an informal labor market and a minimum wage—both important features of the Brazilian labor market—in a model calibrated to match our markup estimates. The literature suggests that minimum wages can reduce monopsony power by turning employers into wage-takers at the legal minimum. Our model provides two key insights. In the oligopsony equilibrium, a legal minimum drives up wages throughout the wage distribution. For this reason, it cuts markdowns and expands formal-sector employment for workers at wages above and beyond the legal minimum. At the same time, our model suggests little monopsony power vis-a-vis low-wage workers. As a result, any sizable minimum wage will drive a large number of lower-wage workers out of the formal sector into lower-paying informal jobs.

After we discuss some of the most relevant literature, Section 2 presents our model and its predictions. Sections 3 and 4 describe our data, empirical specifications, and findings. Section 5 examines minimum wage policy in our model extended to include a competitive informal sector where minimum wages go unenforced. Section 6 concludes.

Related Literature. Our paper unites two separate streams of economic theory: Roy's treatment of worker sorting and Robinson's analysis of employer market power. The Roy model is traditionally used to address comparative advantage and selection across industries or occupations under competitive wage setting. Our paper offers a novel application of the Roy model by shifting its focus from broad occupational or industry sorting to firm-level labor allocation. Framing the Roy model at the firm level naturally leads to monopsonistic wage setting where employers' market power varies across its workers.

As in much of the recent literature, our treatment of employers' market power follows in Robinson's footsteps by using heterogeneous worker rents to trace out firms' labor supply curves, e.g., Card et al. (2018), Berger et al. (2022), Azkarate-Askasua and Zerecero (2023), Lamadon et al. (2022), Chan et al. (2024), and Volpe (2024).[2] However, these rents are typically modeled via differences in workers' outside options in wages or job amenities, unknown to the firm.[3] We allow worker productivity to vary within and across firms, enabling employers

[1] References include Dube et al. (2019), Godoey and Reich (2021), and Jardim et al. (2022).

[2] Search frictions as in Burdett and Mortensen (1998) provide another way to motivate rents. A number of papers have used such rents to generate monopsony power; examples include Manning (2003), Hurst et al. (2022), Jarosch et al. (2024), and Cheremukhin and Restrepo-Echavarria (2025) These papers relate wage setting to employer characteristics, especially market share. Their implications are less clear for how monopsony power varies at the match level.

[3] Volpe (2024) allows workers to differ in their marginal rates of substitution (MRS) of wages versus employer amenities. This generates a novel implication that high-wage firms systematically attract workers who care less

to base their wage markdowns on worker productivity. Importantly, we model oligopsony in a strategic general equilibrium, with each employer recognizing the wage strategies of competing employers as in Berger et al. (2022) and Chan et al. (2024).

Empirically, if we do *not* stratify workers by their relative wages, we estimate a markdown for Brazil of 13 percent. That roughly aligns with a number of estimates for the US and elsewhere. Felix (2024) and Lobel (2024), like us, estimate labor supply elasticities for firms in Brazil's formal sector. Lobel, using payroll tax changes as an instrument, finds an implied markdown of 19 percent, whereas Felix, instrumenting with tariff changes, reports a much higher markdown of 50 percent. Among empirical studies of the US labor market, Staiger et al. (2010) estimate 10 percent markdowns in nursing; Dube et al. (2019) estimate 20 percent markdowns in retailing; Yeh et al. (2022) estimate 35 percent markdowns in manufacturing; and Lamadon et al. (2022), Seegmiller (2023), and Berger et al. (2022) respectively estimate 15 percent, 17 percent, and 25 percent markdowns economy-wide.

We provide evidence that markdowns vary considerably by the wages of workers at an employer, with roughly 25 percent markdowns for higher-wage workers but near zero for lower-wage workers. These results, and our model predictions, align with those of recent papers that find higher earnings responses within firms to tax changes, (Carbonnier et al., 2022; Lobel, 2024), to new patents, (Kline et al., 2019), and to firm stock price (Seegmiller, 2023).[4] They also align with Han's (2024) findings that imperfect human capital transferability strengthens monopsony power.

A number of papers have examined minimum wages in light of monopsony power. Engbom and Moser (2022), Machado Parente (2024), and Luduvice et al. (2024), like us, consider minimum wages in the presence of both market power and an informal sector. Hurst et al. (2022) and Berger et al. (2025) point out that minimum wages have limited impact on monopsony if most workers' wages are well above a plausible legal minimum. Our model reinforces this limitation because monopsony power is skewed to higher-wage workers. But, at the same time, our model provides a further rationale for minimum wages: because a minimum wage reduces markups throughout the wage distribution, it draws workers back to the formal sector even if their wages are above the minimum wage.

about amenities, implying that high-wage firms face a more elastic labor supply. Volpe also allows for the MRS of wages versus amenities to correlate with certain observable worker characteristics, e.g., schooling. As a result, a firm will vary the markdown across its workers if they differ in those observables.

[4] Seegmiller (2023) estimates a less elastic supply for workers with higher observable skills, e.g., more schooling. We find that supply is less elastic more generally for high-wage workers within a firm.

2 Model

We analyze wage setting and employment allocations for a symmetric oligopsony and for an asymmetric duopsony. We first present the general features of the economy and its equilibrium.

2.1 Economic Environment and Preliminaries

Consider a labor market consisting of K firms and a unit mass of heterogeneous workers. Each worker is endowed with a set of skills that translate to $Y_k \geq 0$ units of labor at firm $k \in \{1, \cdots, K\}$. Firm k's output aggregates its workers' productivities linearly; so its total output is $\int y n_k(y) dy$, where $n_k(y)$ denotes the measure of workers k employs of productivity y. As a result, we can express a worker's productivity as efficiency units or levels at firm k.

Workers' efficiency levels can be correlated across firms. Let $\mathscr{F}(y_1, \cdots, y_K) = P(Y_1 \leq y_1, \cdots, Y_K \leq y_K)$ denote the joint distribution of efficiency units in the market with marginal distributions denoted by $F_k(y_k) = P(Y_k \leq y_k)$. We assume that F_k are strictly increasing and continuous. Under these conditions, G admits a unique copula representation (Sklar, 1959): $\mathscr{F}(y_1, \cdots, y_K) = C(u_1, ..., u_K)$, where C is a copula and $u_k = F_k(y_k)$.

We define comparative advantage by comparing a worker's productivity across firms to the distribution of efficiency levels in the population.

Definition 1 *A worker with efficiency levels y_k and y_j at firms k and j has a comparative advantage at firm k over firm j if $\frac{y_j}{y_k} \leq \frac{\mathbb{E}[Y_j]}{\mathbb{E}[Y_k]}$.*

The definition above is ordinal. The (cardinal) strength of comparative advantage is determined by how much better a worker is at firm k versus j relative to an average worker. Accordingly, we quantify that strength by the distance between the ratio of a worker's efficiency levels at the two firms and the corresponding ratio of population means:

$$R(y_j/y_k) = \frac{\mathbb{E}[Y_j]}{\mathbb{E}[Y_k]} - \frac{y_j}{y_k},$$

for $y_j \leq y_k$. Note that when $\mathbb{E}[Y_j] = \mathbb{E}[Y_k]$, the strength of the comparative advantage varies between 0 and 1.

Each worker can be hired by a single employer. An allocation (of workers to firms) is therefore a K-dimensional partition $\Pi = \{\pi_1, \cdots, \pi_K\}$ of $\mathbb{R}_+^K$. Employment at firm k is given by the measure of workers allocated to that firm: $n_k(\pi_k) = P(\{Y_1, \cdots, Y_K\} \in \pi_k)$. Naturally, $\sum_k n_k(\pi_k) = 1$. We consider an allocation to be efficient if workers are allocated where they are most productive. Formally,

Definition 2 *An allocation* Π *is efficient if* $\mathbf{y} = \{y_1, \cdots, y_K\} \in \pi_k$ *if and only if* $y_k \geq y_j$ *for all* $j, k \in \{1, \cdots, K\}$ *and for all* $\mathbf{y} \in \mathbb{R}_+^K$.

2.2 Oligopsonistic Competition

Firms compete over workers by simultaneously posting wage offers that depend on a worker's productivity at their firm: $w_k(y) : \mathbb{R}_+ \to \mathbb{R}_+$. We assume that firm k observes worker's productivity y_k, but not their productivity at other firms $y_{j \neq k}$. Workers see their wage offers across firms. Based on those offers, a worker chooses the employer that pays them the highest wage:

$$\max_{k \in \{1, \cdots, K\}} w_k(y_k). \tag{1}$$

Thus a firm attracts a worker only if its wage offer tops all other firms. In turn, the labor supplied to firm k at wage w by workers with efficiency $Y_k = y$ comes from those whose efficiency levels elsewhere do not garner a better wage:

$$n_k(w; y) = P(w_j(y_j) \leq w, \forall j \neq k | Y_k = y) f_k(y), \tag{2}$$

where $f_k(y) = F_k'(y)$ is the density of workers with $Y_k = y$. The probability that w is the worker's highest wage offer is conditional on $Y_k = y$, since workers' productivities are generally correlated across firms.

The labor supply function faced by firm k is non-decreasing in w because the probability of w being the highest offer is increasing in w by definition. Recognizing this, firms set their wage schedules to maximize profits:

$$\max_{0 \leq w_k(y) \leq y} \int (y - w_k(y)) n_k(w_k(y); y) dy\,. \tag{3}$$

The following defines the equilibrium in this labor market:

Definition 3 *An oligopsonistic equilibrium is a set of wage schedules* $w_k(y)$ *and an employment allocation* Π *such that:*

1. *For each* $k = \{1, \cdots, K\}$, $w_k(y)$ *maximizes firm k's profits given the labor supply function* $n_k(w; y)$ *and wage schedules of other firms* $\{w_j(y)\}_{j \neq k}$.

2. *Workers choose the highest wage offer given the set of wage schedules,* $\{w_k(y)\}_{k=1}^K$.

3. *The labor market clears* $n_k(\pi_k) = n_k(w_k(y); y)$ *for all* $k = \{1, \cdots, K\}$.

The equilibrium in our oligopsony setting is similar to that in first-price, sealed-bid auctions. The existence of an equilibrium in that context with monotone bidding strategies is addressed in Athey (2001) and Reny and Zamir (2004). To characterize the equilibrium, note that firm k's profit-maximizing wage offer for workers with efficiency level y satisfies:

$$(y-w)\frac{\partial n_k(w;y)}{\partial w} = n_k(w;y). \tag{4}$$

By marginally raising the wage, firm k raises employment by $\partial n_k(w;y)/\partial w$. Each added worker contributes $y-w$ to profits. The optimal wage offer balances this gain against the transfers it entails to inframarginal workers, those for whom firm k's wage offer is already their best. That cost, per dollar higher wage, is $n_k(w;y)$. Rearranging the optimality condition gives the well-known representation in monopsonistic labor markets:

$$\frac{w}{y} = \frac{\eta_k(w;y)}{1+\eta_k(w;y)},$$

where $\eta_k(w;y) = \partial n_k(w;y)/\partial w \times w/n_k(w;y)$ is the elasticity of labor supplied by workers of efficiency level y to firm k at wage level w. A more elastic labor supply schedule, higher $\eta_k(w;y)$, calls for a higher w/y and, therefore, a lower wage markdown $1-w/y$.

2.3 Symmetric Equilibria

We begin our analysis with a symmetric market where firm productivities are identically drawn. This allows us to analytically characterize the equilibrium with symmetric wage strategies. Formally,

Definition 4 *A market is symmetric if C is exchangeable and has common marginals $F_k(y) = F(y)$ for all $k = \{1,\cdots,K\}$.*

For any arbitrary distribution function $\tilde{F}(x|y)$ over the interval $x \in [0,y]$, define $\mathbb{E}_{\tilde{F}}[x|x \leq y] = \int_0^y x d\tilde{F}(x|y)$ as the weighted upper-truncated mean of x with weights given by $\tilde{F}'(x|y)$.

Theorem 1 (Symmetric Equilibrium) *Consider a symmetric market with K firms. The symmetric oligopsony equilibrium is characterized by the following wage offer policy and employment allocation.*

1. *Wage offer policy*

$$w(y) = \int_0^y x d\tilde{F}(x|y) = \mathbb{E}_{\tilde{F}}(x|x \leq y),$$

where $\tilde{F}(x|y) = \exp[-\int_x^y \sum_{j\neq k} \frac{C_{kj}(F(z),\ldots,F(z)}{C_k(F(z),\ldots,F(z))} F'(z)dz]$ and C_{kj} is the cross-partial derivative of the C w.r.t. $u_k = F_k(y)$ and $u_j = F_j(y)$ for all $j \neq k \in \{1,\ldots,K\}$.

2. *For any* $\mathbf{y} = \{y_1, ..., y_K\} \in \mathbb{R}^K_+$, $\mathbf{y} \in \pi_k$ *if and only if* $k = argmax_k\{y_1, ..., y_K\}$*, and employment at firm* k *is given by:*

$$n_k(\pi_k) = C_k\left[F(y), \cdots, F(y)\right] F'(y) \quad \forall k = \{1, ..., K\}\,.$$

Theorem 1 characterizes firm k's optimal wage offer as a weighted average of efficiency levels that are inferior to the worker's productivity at k. The weight assigned to each efficiency level, $x < y$, reflects the probability that x represents the worker's second-best productivity.[5] To see this, note the following (heuristic) identity.

$$\frac{\sum_{j\neq k} C_{jk}(F(x), ..., F(x))}{C_k(F(x), ..., F(x))} \approx \frac{P[\max_{j\neq k} Y_j \simeq x \mid Y_k = x]}{P[\max_{j\neq k} Y_j \leq x \mid Y_k = x]},$$

where $Y_j \simeq x$ means $Y_j \in [x - \epsilon, x + \epsilon]$ for some small $\epsilon > 0$. The denominator represents the fraction of workers, with efficiency $Y_k = x$ at firm k, who are less productive elsewhere. All such workers are employed at firm k by Theorem 1. The numerator is the relative density of workers with $Y_k = x$ who are (almost) equally productive elsewhere. So the above ratio can be viewed as the probability that a worker at k is equally productive elsewhere.[6] If that probability is higher, then $\tilde{F}(x|y)$ is lower at any $x < y$, effectively raising the wage offer. Because $C_k(F(x), ..., F(x))F'(x) = n_k(x)$, the ratio can also be interpreted as the density of marginal employees to total employment at firm k of workers with efficiency level $Y_k = x$. Two factors that could naturally raise that probability are a larger number of competitors, K, or a higher correlation of worker productivity across firms. We discuss these further below.

Because the wage offer is effectively a truncated mean, it follows that $0 < w(y) < y$ whenever $y > 0$.[7] This implies that markdowns (or profits) are always positive. Furthermore, $w(y)$ is (strictly) increasing in y. The strict monotonicity of the wage function delivers the following two results:

Theorem 2 *The symmetric oligopsony equilibrium is efficient.*

A worker with efficiency levels y_k and y_j at firms k and j works for k if and only if $w(y_k) > w(y_j)$, which only happens if $y_k > y_j$ since $w(y)$ is strictly increasing. Because firms are identical, their offers exhibit the same wage markdown at a given y. Therefore, a worker can receive a better offer elsewhere only if they are more efficient at that firm. This result does not extend to equilibria with asymmetric firms as we illustrate in the next subsection.

[5] See the proof for Theorem 1 and other results in Appendix A.

[6] We mean this heuristically. Since the distribution of $\max_{j\neq k} Y_j$ is continuous, the density to distribution ratio may generally exceed one.

[7] Because $\tilde{F}'(x|y) > 0$ for all $x \leq y$, non-zero weights are given to efficiency levels that are strictly below y.

Lemma 1 *The joint distribution of wage offers $w(y_k)$ is described by the copula $C^w = C$, and the common marginal distribution $F^w(x) = F(w^{-1}(x))$.*

Lemma 1 states that the joint distribution of wage offers inherits the dependency structure of the productivity distribution, but with different marginal distributions. The result follows from the stability of copulas under (strict) monotonic transformations (Nelsen, 2006).[8] Because workers take their highest wage offer, the equilibrium distribution of wages is defined by the distribution of $\max\{w(y_1), \cdots, w(y_k)\}$, which is given by the diagonal section of the C copula that describes the wage offer distribution:

Theorem 3 *The equilibrium distribution of wages in a symmetric oligopsony equilibrium is:*

$$P(w^* \leq \omega) = C(F^w(\omega), \cdots, F^w(\omega)). \tag{5}$$

2.3.1 Markdowns in an Oligopsony Equilibrium

Oligopsony wage policies tie markdowns closely to comparative advantage. For any worker with productivity y, the markdown is:

$$1 - \frac{w(y)}{y} = \mathbb{E}_{\tilde{F}}\left(1 - \frac{x}{y}\middle| x < y\right) = \mathbb{E}_{\tilde{F}}[R(x/y)], \tag{6}$$

where the last equality follows from $\mathbb{E}[Y_j] = \mathbb{E}[Y_j]$ in a symmetric market. Equation (6) makes the role of comparative advantage in wage setting explicit. For workers of efficiency y, the markdown is a weighted average of their comparative advantages at their firm, with larger markdowns if comparative advantage is stronger. Recall that $\tilde{F}$ weights each $x < y$ according to the density of marginally employed workers conditional on employment: $\delta'(x|Y_k = x)/\delta(x|Y_k = x)$. So, equivalently, the markdown is determined by weighting comparative advantage at firm k versus all competitors. If the joint distribution of productivity at k and best productivity elsewhere is dense along its diagonal (where equal), then markdowns are small. This could reflect either productivities that are highly correlated across firms, with little room for comparative advantage, or many competitors, raising a worker's outside option.

How markdowns vary by worker productivity will generally reflect the marginal distribution of efficiencies, as well as the dependence structure of workers' efficiencies across employers. The following proposition gives the necessary and sufficient condition for markdowns to increase with productivity.

[8] Because the stability of copulas to monotonic transformations does not require the transformations to be identical for all y_k, this result extends to asymmetric equilibria with strictly increasing wage-offer strategies.

Theorem 4 *Let $G(y) = \int_0^y \tilde{F}(x|y)dx$. Then for any $y > 0$, $1 - w(y)/y$ is increasing in y if and only if the elasticity of $G(y)$ with respect to y exceeds 1.*

At this level of generality, markdowns can be increasing, decreasing, or non-monotonic in productivity, depending on the joint distribution of efficiencies across employers. We illustrate this next via specific examples chosen to highlight the roles of competition and correlation in wage setting.

2.3.2 Examples

We begin with a market where a worker's efficiency levels are independent across employers and continue with three cases of co-dependence. Worker efficiencies correlate differently between employers in these cases, leading to diverse implications for markdowns.

Example 1 (Independent Copula) *If Y_k is independent across employers, then the optimal wage offer policy is $w(y) = \int_0^y x d\tilde{F}^I(x|y)$ with*

$$\tilde{F}^I(x|y) = \left[\frac{F(x)}{F(y)}\right]^{K-1}.$$

Consider the simplest case first: a duopsony where Y_1 and Y_2 are independent. Then, $\tilde{F}(x|y) = F(x)/F(y)$, which implies $w(y) = \mathbb{E}[x|x < y]$. The equilibrium wage offer for employees with productivity y reflects their average efficiency at the competing firm conditional on that efficiency being inferior to y. Equivalently, the markdown reflects the average comparative advantage of workers with efficiency level y: $1 - w(y)/y = \mathbb{E}[1 - x/y|x < y]$.

More generally, when $K > 2$, $\tilde{F}^I(x|y)$ is the conditional probability that the highest productivity across $K-1$ competitors is inferior to y. Because the equilibrium wage is the conditional expectation of the second-best productivity level, it monotonically rises with K, matching productivity in the limit, reflecting that second-best productivity stochastically increases with the number of competitors.[9]

Next, consider a duopsony where efficiencies at the two firms are co-dependent. The literature typically treats dependence via the correlation coefficient while assuming joint log-normality of efficiencies.

Example 2 (Gaussian Copula) *Let $K = 2$ and $C = C^G(\rho)$ be the Gaussian copula with correlation $\rho \in [-1, 1]$ and normal marginals $F_k(\ln x) = \Phi(\frac{\ln x - \mu}{\sigma})$ for $k \in \{1, 2\}$. The wage offer policy in a symmetric duopsony equilibrium is $w^G(y, \rho) = \int_0^y x d\tilde{F}^G(x|y;\rho)$ with*

[9] Formally, $\tilde{F}^I(x|y)$ declines with K for all $x < y$ with $\tilde{F}^I(y|y) = 1$, creating a first-order stochastic ranking with respect to K. In the extreme, $\lim_{K\to\infty} \tilde{F}^I(x|y) = 0$ for all $x < y$, implying $w(y) = y$.

$$\tilde{F}(x|y;\rho) = \left[\frac{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\ln x-\mu)}{\sigma}\right)}{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\ln y-\mu)}{\sigma}\right)}\right]^{\frac{1}{1-\rho}}.$$

Example 2 captures the distinct roles of co-dependence in wage setting. In the definition here for $\tilde{F}(x|y;\rho)$, ρ appears twice inside the bracketed term and once in the exponent. By raising the exponent, a higher ρ lowers $\tilde{F}(x|y;\rho)$ for all $x < y$, raising wage offers at all productivity levels. This mechanism is similar to the role of K above. Intuitively, competition gets stiffer either when the number of competitors increase or when workers are similarly productive at all firms. Wages equal productivity in either limit, as $K \to \infty$ or as $\rho \to 1$. But this need not happen for all workers because the bracketed term, which captures the dependence between productivity and the outside option, is not monotonic in ρ. To see this, note that if $\rho = 0$, then all workers have the same expected productivity at the competing firm. A positive ρ better aligns the competing offer with productivity, which is good news for high-productivity workers, but bad news for low-productivity workers. As a result, wage markdowns may increase or decrease with the correlation depending on the worker's productivity, y, even though the wage does converge to productivity as $\rho \to 1$.

The Gaussian copula imposes a specific structure on the distribution of comparative advantage across efficiency levels, in particular, that the degree of correlation is common at all levels. We next consider Archimedean copulas, which allow for richer co-dependence:

Definition 5 *Let $\varphi : [0,1] \to [0,\infty]$ be a continuous, strictly decreasing function with $\psi(1) = 0$ and suppose that the (pseudo) inverse of ψ is a convex function. Then copula $C(u,v)$ is Archimedean if $\psi(C(u,v)) = \psi(u) + \psi(v)$.*[10]

Archimedean copulas are symmetric and associative. The function ψ is called the generator function. Before we show some examples, let us state the equilibrium wage offer policy.

Proposition 1 (Archimedean Copulas) *Let $C^A = \psi^{-1}(\sum_j \psi(u_j))$ be an Archimedean copula. The optimal wage offer policy is $w^A(y) = \int_0^y x d\tilde{F}(x|y)$ with*

$$\tilde{F}(x|y) = \left[\frac{\psi'(C^A(F(x),...,F(x))}{\psi'(C^A(F(y),...,F(y))}\right]^{-\frac{K-1}{K}}.$$

Two Archimedean copulas are of particular interest: the Clayton copula, which features weaker dependency at higher productivity levels, and the Gumbel copula, which features the

[10] The pseudo-inverse of ψ is a function $\psi^{[-1]}(x) : [0,\infty) \to [0,1]$ that equals $\psi^{-1}(x)$ if $0 \le t \le \psi(0)$ and 0 if $t \ge \psi(0)$. Note that because $\psi' < 0$ with a convex pseudo-inverse, ψ is also convex.

reverse pattern. The Clayton copula sets $\psi(u) = (u^{-\theta} - 1)/\theta$, where $\theta \geq -1$ measures the strength of dependence. Higher θ values imply more positive dependence, with $\theta < 0$ implying negative dependence, $\theta > 0$ positive dependence, and $\theta = 0$ independence.[11] The Gumbel copula uses the generator function $\psi(u) = -(\ln(u))^{\alpha}$ for $\alpha \geq 1$. Unlike Gaussian or Clayton copulas, the Gumbel copula is limited to positive dependencies with $\alpha = 1$ representing the independent copula.

The corresponding wage policies in an oligopsony equilibrium are as follows:

Example 3 (Clayton Copula) *Let* $C^{AC}(\theta) = \left(\sum_k (u_k^{-\theta} - 1)\right)^{-1/\theta}$ *where* $\theta \in (-1, \infty)$ *and* $u_k = F(y)$ *for all* $k \in \{1, ..., K\}$. *Then the optimal wage offer policy is* $w^{AC}(y, \theta) = \int_0^y x d\tilde{F}(x|y;\theta)$ *with*

$$\tilde{F}(x|y;\theta) = \left[\frac{F^{-\theta}(x) - 1 + 1/K}{F^{-\theta}(y) - 1 + 1/K}\right]^{-\frac{K-1}{K}\frac{(1+\theta)}{\theta}} = \left[\frac{C^{AC}(F(x), ..., F(x))}{C^{AC}(F(y), ..., F(y))}\right]^{\frac{K-1}{K}(1+\theta)}.$$

Example 4 (Gumbel Copula) *Let* $C^{AG}(\alpha) = \exp\left(-\left(\sum_k (-(\ln(u_k))^{\alpha})\right)^{1/\alpha}\right)$ *where* $\alpha \in [1, \infty)$ *and* $u_k = F(y)$ *for all* $k \in \{1, ..., K\}$. *Then the optimal wage offer policy is* $w^{AG}(y, \alpha) = \int_0^y x d\tilde{F}(x|y;\alpha)$ *with*

$$\tilde{F}(x|y;\alpha) = \left[\left(\frac{F(x)}{F(y)}\right)^{K^{1/\alpha}} \left(\frac{\log F(y)}{\log F(x)}\right)^{\alpha-1}\right]^{\frac{K-1}{K}}.$$

Figure 1 compares the dependence structure in the Gumbel and Clayton copulas with the standard Gaussian copula in a bivariate setting. Each panel shows the joint distribution of $F(y_1)$ and $F(y_2)$. The dependency parameter of each copula is calibrated to the same degree of dependency as measured by Kendall's $\tau = 0.75$.[12] Recall that the Gaussian copula (panel b) features a constant correlation across efficiency levels. Under the Clayton copula (panel a) the correlation between efficiencies is strongest at low levels and weakens with productivity. The reverse is true in panel (c) for the Gumbel copula.

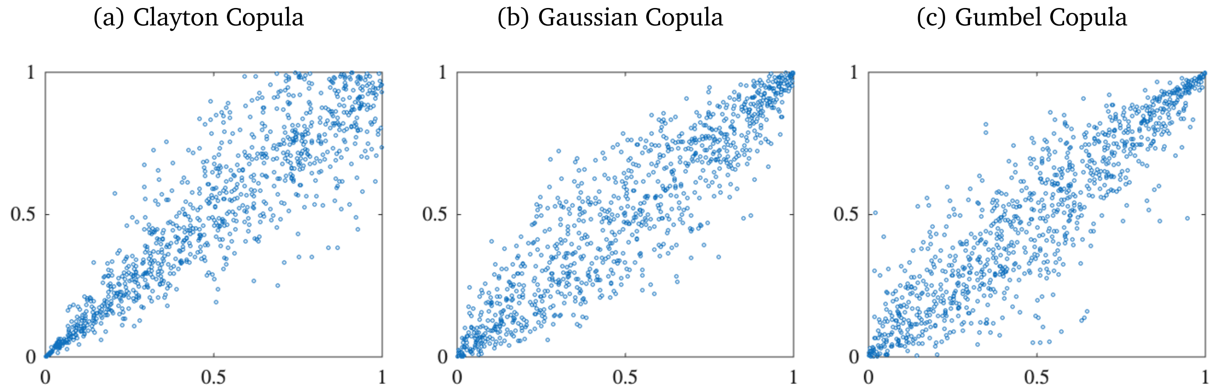
The corresponding wage offer policies in a symmetric duopsony equilibrium are shown in Figure 2a. While always increasing in productivity, wages are quite different across the cases, reflecting how they differ in the dependence structure between efficiencies at the two firms. Relative to the Gaussian copula, wages track productivity more closely if the dependence of efficiencies across firms strengthens with productivity (Gumbel) than if it weakens (Clayton).

Panel b shows how the strength of comparative advantage, $R(y_j) = 1 - \mathbb{E}[y_i/y_j | y_i < y_j]$, varies by productivity. In turn, this directly dictates how wage markdowns respond along the

[11] The independent copula ($\psi(u) = \ln u$) is obtained in the limit as $\theta \to 0$.

[12] The corresponding values are $\rho = \sin(\tau\rho/2) = 0.92$ for the Gaussian copula, $\theta = 2\tau/(1-\tau) = 6$ for the Clayton copula, and $\alpha = 1/(1-\tau) = 4$ for the Gumbel copula.

Figure 1: Dependence Structures with Copulas



Notes: The figure shows the joint distribution of $F_i(y)$ for three different bivariate copulas with a Kendall's τ of 0.75.

efficiency distribution. Markdowns (panel c) increase with productivity under the Gaussian copula, especially at lower levels. The Clayton copula displays a steeper markup trajectory with productivity, reflecting the weakening in cross-firm correlation in efficiencies. Interestingly, markdowns follow a hump-shaped pattern with the Gumbel copula. That reflects the strong correlation of efficiencies at high levels, which makes the market highly competitive for productive workers. Which scenario best describes an economy is essentially an empirical question.

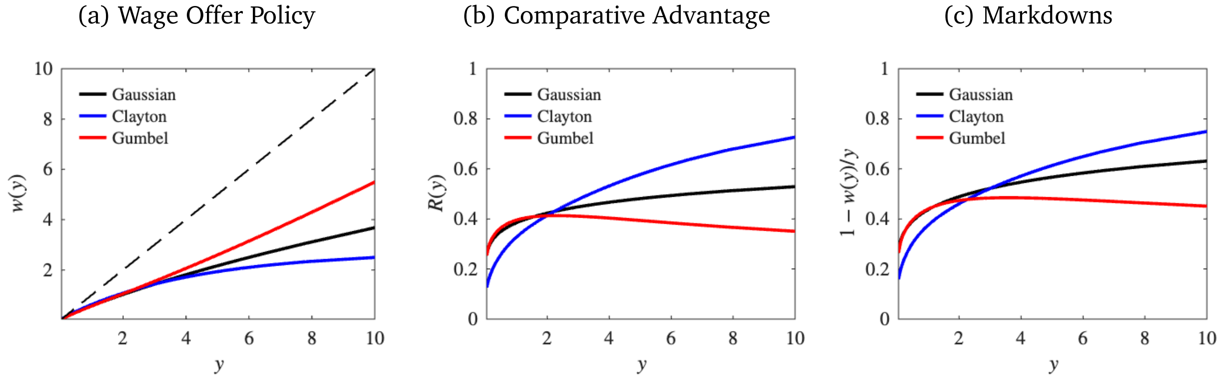
Figure 3 shows how correlation alters the markdown distribution. Generally, a higher correlation of productivity across firms brings pay closer to productivity. But that can affect high- versus low-productivity workers differently. With higher correlation, productive workers at one firm are also productive at competing firms, raising their outside options. Similarly, higher correlation lowers the outside options of less productive workers. As a result, firms mark down the wages of high-productivity workers less, while marking down those of low-productivity workers more. This is visible for the Gaussian and Gumbel copulas among the least productive workers when correlation is low, e.g., independent.[13] By contrast, with the Clayton copula (panel b), higher correlation raises wages uniformly, reducing markdowns at all dependency levels:

Remark 1 *For all $y > 0$, $w^{AC}(y;\theta)$ is increasing in θ and $1-w^{AC}(y)/y$ is decreasing in θ.*

Because correlation is always relatively stronger among low-productivity workers, higher cor-

[13] But those economies imply much larger wage markdowns than typically estimated. An independent copula, for instance, implies an average markdown of about a half, which is two to four times most estimates. At empirically relevant dependency levels, markdowns decline for both copulas as correlation increases. Raising Kendall's τ from 0.5 to 0.75 in Figure 2, markdowns decline uniformly in all panels.

Figure 2: Wage Policies in Duopsony Equilibria



Notes: The figure shows comparative advantage, wage policies, and wage markdowns in duopsony equilibria. Each copula is calibrated to a Kendall's τ of 0.5; marginal productivity distributions are log-normal with $\mathbb{E}[\ln y_i] = 0$ and $\mathbb{V}[\ln y_i] = 1$ for $i \in \{1, 2\}$. Comparative advantage is measured by $R(y_j) = 1 - \mathbb{E}[y_i/y_j | y_i < y_j]/y_j$.

relation via θ mainly affects productive workers on the margin.

Figure 4 shows how more competitors affect the offer markdowns at each y assuming a Clayton or Gumbel copula. For these copulas, as well as for independence, a higher K uniformly increases wage offers at all productivities:

Remark 2 *Let $w(y) \in \{w^I(y), w^{AG}(y), w^{AC}(y)\}$. For all $y > 0$, $w(y)$ is increasing in K and $1 - w(y)/y$ is decreasing in K.*

More competitors increase the odds that a worker is highly productive elsewhere, reducing expected match-specific rents and, thereby, the markdown at each y.[14]

Overall, the simulations show that markdowns within a firm may increase or decrease with wages depending on the underlying distribution of efficiencies across firms. We should expect pay to better align with productivity in economies with more competitors or where worker productivity is highly correlated across firms.

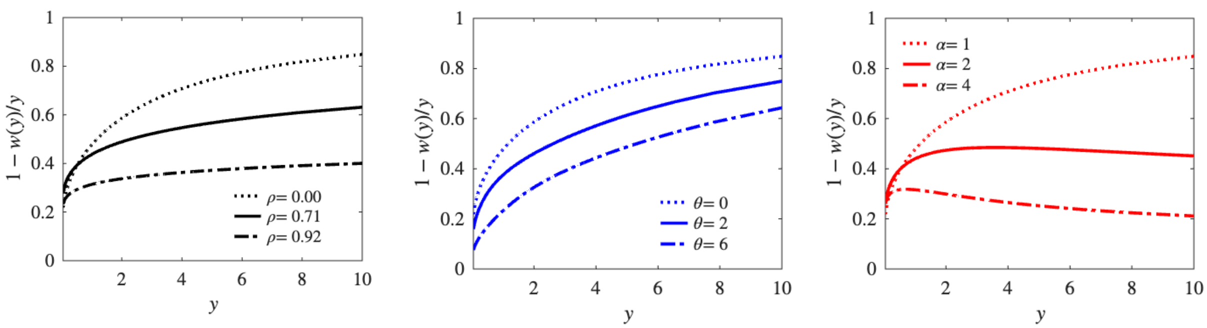
2.4 Asymmetric Equilibria

We make two additional assumptions to study asymmetric equilibria. First, we assume that worker productivity is bounded: $Y_k \leq \bar{y} < \infty$ for all k. Second, because asymmetric equilibria

[14]The equilibrium markdown is the markdown of each worker's chosen firm, where they are most productive. If markdowns are increasing in productivity at each firm, adding more competitors may raise markdowns, by raising a worker's highest productivity, $\max_k\{y_k\}$, possibly offsetting the decrease in markdowns at each y. By contrast, given K, an increase in correlation will reduce the expected highest productivity. So, if markdowns are increasing in y, this reinforces the downward shifts in Figure 3 in reducing the expected equilibrium markdown.

Figure 3: Markdowns in Symmetric Equilibria

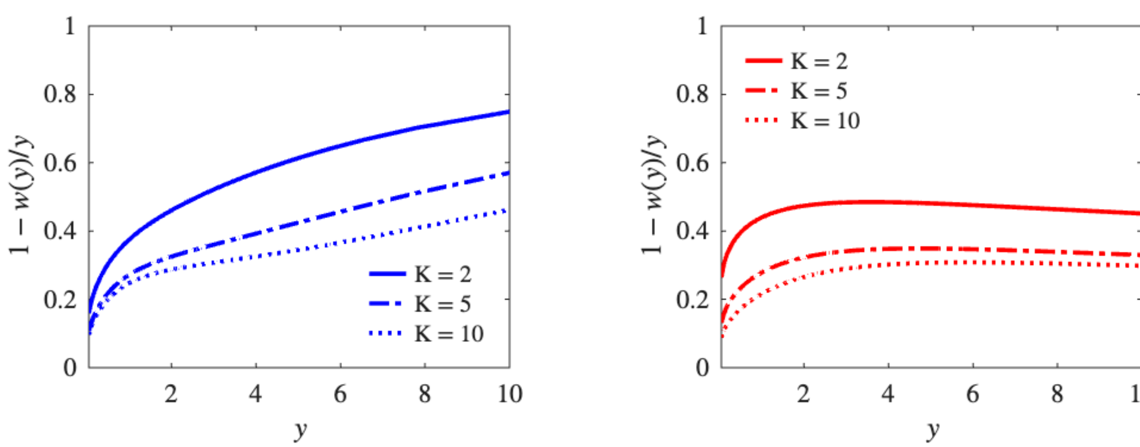
(a) Gaussian Copula (b) Clayton Copula (c) Gumbel Copula



Notes: The figure shows wage markdowns in duopsony equilibria by levels of co-dependence. In each panel, copula parameters respectively correspond to a Kendall's τ of 0 (independence), 0.5, and 0.75. The marginal productivity distribution is log-normal with $\mathbb{E}[\ln y_i] = 0$ and $\mathbb{V}[\ln y_i] = 1$ for $i \in \{1, 2\}$.

Figure 4: Markdowns in Symmetric Equilibria

(a) Clayton Copula (b) Gumbel Copula



Notes: The figure shows wage markdowns in oligopsony equilibria by the number of competitors for the Clayton copula (left) and for the Gumbel copula (right) with $\tau = 0.5$. Solid lines in each panel replicate the corresponding markdowns in Figure 2. The marginal productivity distribution is log-normal with $\mathbb{E}[\ln y_i] = 0$ and $\mathbb{V}[\ln y_i] = 1$ for $i \in \{1, K\}$.

with K firms are inherently complex, we simplify the problem by setting $K = 2$.[15] Generality is not entirely lost by these assumptions because $\bar{y}$ can be arbitrarily large. Furthermore, because the effect of the number of competitors on wage setting is similar to that of co-dependence, the degree of competition can also be captured by raising the correlation of workers' efficiencies between the two firms.

The following proposition characterizes the equilibrium with asymmetric firms.

Proposition 2 (Asymmetric Duopsony Equilibrium) *Let $K = 2$ and $C(u,v)$ denote the joint distribution of worker efficiencies at the two firms with marginal productivity distributions $u = F_1(y)$ and $v = F_2(y)$. The asymmetric duopsony equilibrium is characterized by wage offer policies $\{w_k(y)\}_{k\in\{1,2\}}$ and an employment allocation $\{n_k(y)\}_{k\in\{1,2\}}$ such that:*

1. *Inverse wage offer policies solve the following system of differential equations:*

$$\begin{aligned}(y_1(w)-w)C_{uv}(u,v)F_2'(y_2(w)))dy_2 - C_v(u,v)dw &= 0\\ (y_2(w)-w)C_{vu}(u,v)F_1'(y_1(w)))dy_1 - C_u(u,v)dw &= 0\end{aligned}$$

   where $y_k(w) = w_k^{-1}(y)$.

2. *Employment levels are:*

$$n_1(y) = C_u(F_1(y), F_2(y_2(w_1(y))))F_1'(y) \quad (7)$$

$$n_2(y) = C_v(F_1(y_1(w_2(y))), F_2(y))F_2'(y)\,. \quad (8)$$

We consider two cases of asymmetry. In the first, we allow one firm to be generally more productive than the other. We view this as a firm-specific component of productivity and formalize it with a stochastic dominance relationship between the marginal productivity distributions. In this scenario, the more productive firm generally offers higher wages. In response, the less productive firm marks down wages less aggressively to remain competitive.

For the theoretical results in this section, we limit the joint productivity distribution to the class of Archimedean copulas, which we denote by $\mathscr{C}^A$. Let us now introduce the stochastic dominance concept we use:

Definition 6 (Affiliated Probability Difference Order) *Let F_A and F_B be distribution functions defined over the interval $[0, y^{max}]$ and $C(F_A, F_B) \in \mathscr{C}^A$ and an Archimedean copula with the*

[15] For this reason, much of the auction-theory literature assumes symmetry. Reny and Zamir (2004) and Jackson and Swinkels (2005) analyze first-price auctions with asymmetric bidders, focusing on the existence of equilibrium.

generator function ψ. We say $F_A \succ_{APD} F_B$ if and only if $\forall x < y \in [0, y^{max}]$:

$$\psi(F_A(x)) - \psi(F_B(x)) > \psi(F_A(y)) - \psi(F_B(y)).$$

Affiliated probability difference (APD) order requires that $\psi(F_A(x)) - \psi(F_B(x))$ be decreasing in x, which is a stronger requirement than first-order stochastic dominance that postulates $F_A(x) > F_B(x)$ for all $x \in (0, y^{max})$. When F_A and F_B are independent, APD order is equivalent to the standard monotone probability ratio order, which requires $F_A(z)/F_B(z)$ to be increasing in z, as we prove in the Appendix.

Theorem 5 *Suppose $C(u, v) \in \mathscr{C}^A$ and $F_2 \succ_{APD} F_1$. Then, (i) for all $w > 0$: $F_1(y_1(w)) > F_2(y_2(w))$, and (ii) for all $y > 0$: $w_1(y) > w_2(y)$.*

Part *(i)* of Theorem 5 says that if the productivity distribution at Firm 2 stochastically dominates Firm 1's in the APD sense, a larger fraction of the workforce is offered a wage below w at Firm 1 than at Firm 2. Equivalently, to merit the same wage offer w, a worker has to rank higher in Firm 1's productivity distribution than they would in Firm 2's. Consequently, because Firm 2 generally pays better, in a stochastic sense, it employs a larger share of the workforce.

Part *(ii)* says that Firm 1 offers a higher wage for a given y, implying smaller markdowns at Firm 1 at any y. This generates an inefficient employment allocation where some workers that are more productive at Firm 2 work for Firm 1, because their wages are marked down by more at Firm 2. Consequently, although Firm 2 employs a larger share of the labor force, it is not as large as it would be in a competitive equilibrium (or if markdowns were symmetric).

Figure 5a illustrates a typical equilibrium in this scenario. In the chart, $\ln Y_1$ and $\ln Y_2$ are normally distributed with common standard deviations, $\sigma_1 = \sigma_2$, but different means: $\mu_2 > \mu_1$. The joint distribution reflects a Clayton copula with positive dependence: $\theta > 0$. The axes show the productivity quantiles at each firm. The solid curve is the locus of worker indifference in an asymmetric equilibrium when Firm 2 is more productive. Workers with efficiency bundles above the curve are offered, and accept, higher wages at Firm 2. The indifference locus lying below the 45-degree line implies Firm 2 is larger.[16]

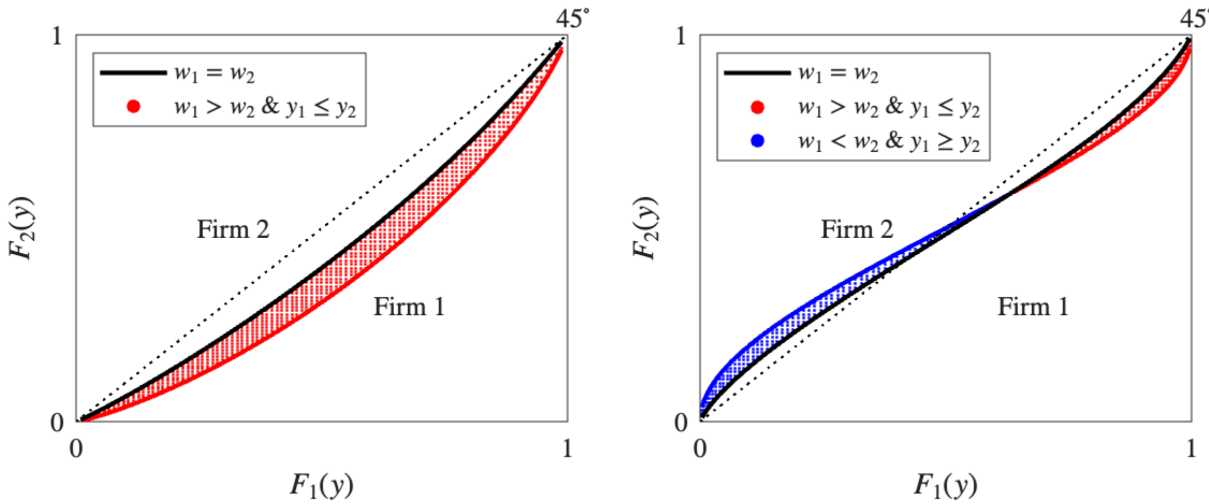
The red line shows the locus of equal productivity: $y_1 = y_2 = y$. Given $F_1(y) > F_2(y)$, this line is strictly below the 45-degree line where $F_1(y_1) = F_2(y_2)$. It is also below the locus of equal pay because pay-to-productivity is higher at Firm 1: $w/y_1(w) > w/y_2(w)$.[17] Therefore, the red shaded area represents workers who are more productive at Firm 2, but work at Firm 1—those offered a higher wage at Firm 1 due only to its lower markdown than Firm 2's.

The second asymmetric case we consider is when the firms' marginal productivity distributions cross once:

[16]By definition, the distributions of $u = F_1(y_1)$ and $v = F_2(y_2)$ are uniform and symmetric around the 45-degree

Figure 5: Employment Allocations in Asymmetric Equilibria

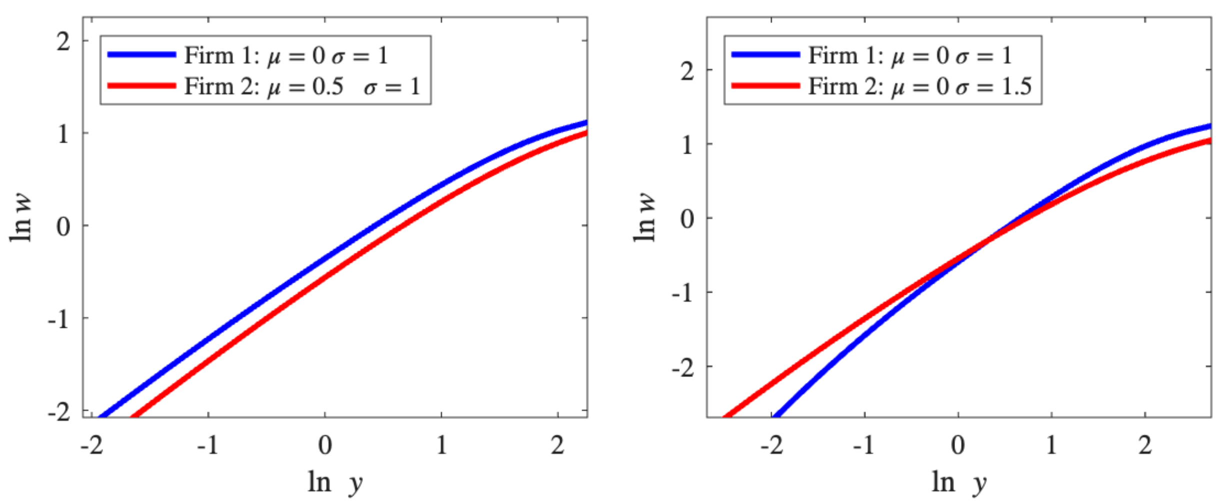
(a) $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$ (b) $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$



Notes: The figure shows the allocation of workers in duopsony equilibria. Workers below the black curve choose Firm 1. The dashed 45-degree line indicates the allocation in symmetric equilibrium. Areas colored in red and blue show inefficiencies in asymmetric equilibria, where the firm-worker allocation deviates from comparative advantage. Productivity levels are log-normal at the firm level with parameters μ_i, σ_i with $i \in \{1, 2\}$ and joint distribution given by a Clayton copula with positive dependence: $\tau = 0.5$.

Figure 6: Wage Offer Policies in Asymmetric Equilibria

(a) $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$ (b) $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$



Notes: The figure shows wage offers in duopsony equilibria. Productivity levels are log-normal at each firm with parameters μ_i, σ_i, $i \in \{1, 2\}$; joint distribution reflects a Clayton Copula with positive dependence: $\tau = 0.5$.

Assumption 1 $\exists\, \tilde{y} \in (0, y^{\max}):\ y > \tilde{y} \implies F_2(y) < F_1(y)$ *and* $y < \tilde{y} \implies F_1(y) < F_2(y)$.

This scenario would arise, for instance, if one firm values labor quality more than the other. The following theorem characterizes sorting of workers between firms in that case, proving a single-crossing result in the interior of the (u, v) space if any crossing exists.[18]

Theorem 6 *If $C(u,v) \in \mathscr{C}^A$ and Assumption 1 holds, then there is a unique $w^* \geq 0$ such that $F_1(y_1(w)) \geq F_2(y_2(w))$ if and only if $w \geq w^*$ in equilibrium.*

Theorem 6 states a single-crossing result for the firms' (inverse) wage offer policies at wage w^*. For higher-productivity workers, who merit a higher wage rate, Firm 2 generally offers better pay, while for lower-productivity workers, Firm 1 offers better pay in a stochastic sense. As a result, Firm 2 hires a larger share of higher-productivity workers, while Firm 1 specializes in lower-productivity workers. This is qualitatively consistent with Roy (1951), who shows, in a competitive equilibrium, that employment follows comparative advantage with skill-elastic firms employing skilled workers more intensively.

To characterize the pattern of markdowns in this environment, we supplement the single-crossing assumption with a local dominance assumption:

Assumption 2 $\exists\, \hat{y} \geq \tilde{y} : y > \hat{y} \implies F_2 \succ_{APD} F_1$ *and* $y < \hat{y} \implies F_1 \succ_{APD} F_2$.

Assumption 2 is a stronger version of Assumption 1, and requires Firm 2 to be dominant in productivity in the APD sense among highly productive workers and Firm 1 to be similarly dominant among lower-productivity workers. It then follows that the markdowns are larger in each firm's area of comparative advantage as the next theorem shows.

Theorem 7 *If $C(u,v) \in \mathscr{C}^A$ and Assumptions 1 and 2 are true, then $\exists y^* = y_1(w) = y_2(w) \in (\tilde{y}, \hat{y})$ for some w, such that $w_1(y) > w_2(y)$ if and only if $y < y^*$.*

Theorem 7 shows the existence of a threshold productivity y^*, above which Firm 1 pays better than Firm 2 at any productivity $y > y^*$, while the reverse is true for $y < y^*$. Because Firm 2 has a comparative advantage among high-productivity workers, Firm 1 offers better pay to remain competitive for highly productive workers. By contrast, Firm 1 has a comparative advantage among lower-productivity workers, which pushes Firm 2 to offer better wages relative to productivity.

line, which therefore represents an equal division of employment in Figure 5.

[17] Because $y_2(w) > y_1(w)$, for all $w > 0$, in equilibrium, we have $F_2(y_2(w))/F_1(y_1(w)) > F_2(y_1(w))/F_1(y_1(w))$, where the left term traces the equal-pay locus and the right term traces the equal-productivity locus.

[18] Single-crossing may not exist when $\tilde{y}$ is sufficiently large, e.g., if $\ln Y_2$ is more dispersed *and* it has a larger mean. Without a crossing, the equilibrium sorting is similar to that characterized by Theorem 5.

Better pay for a given y means lower markdowns. Therefore, Theorem 7 also implies that Firm 2 marks down wages more aggressively for higher-productivity workers and less aggressively for lower-productivity workers. The reverse is true for Firm 1. This leads to incomplete specialization, where some highly-productive workers work for Firm 1 even though they are more productive at Firm 2, and vice versa for some low-productivity workers. As a result, the employment allocation in the monopsonistic equilibrium is inefficient.

An example is shown in Figure 5b, where $\ln Y_j$ is normal with $\mu_1 = \mu_2$ but $\sigma_2 > \sigma_1$. The joint distribution is described by a Clayton copula as in 5a. The black curve shows the locus of points with equal wage offers, $F_1(y_1(w)) = F_2(y_2(w))$, and the red and blue colored curve shows the locus of points with equal productivity: $F_1(y) = F_2(y)$.

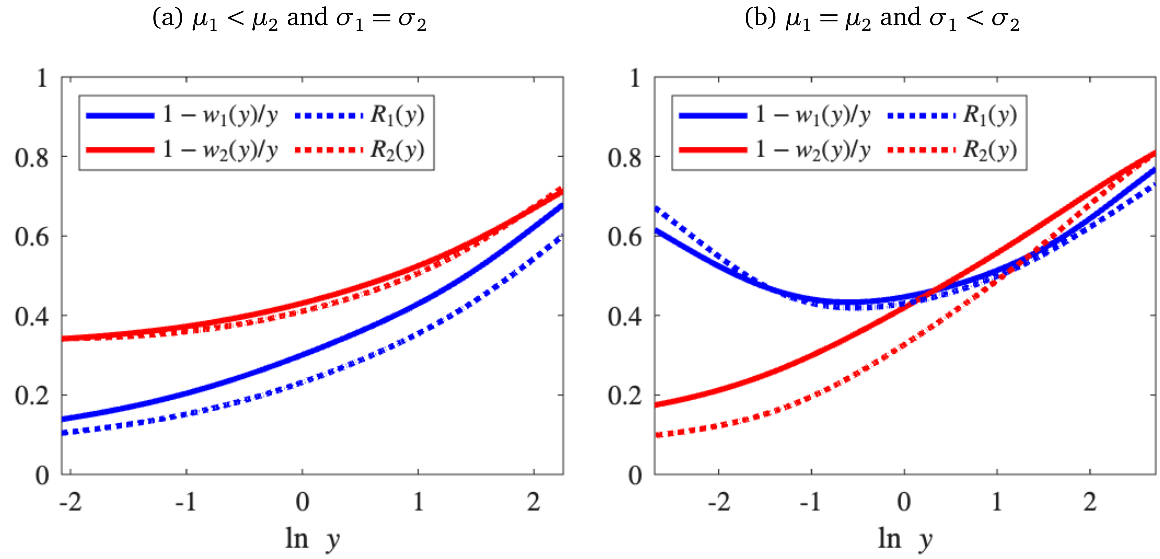
When there is a crossing, firms attract different segments of the workforce. Firm 2 employs a larger share of the more productive workers: the separation curve in Figure 5 lies below the 45-degree line when y is high. Among highly paid workers, where the black equal-pay curve lies above the red equal-productivity curve, $y_2(w) > y_1(w)$. Consequently, both wage offers conditional on productivity and markdowns conditional on the wage rate are larger at Firm 2. Equilibrium in this segment of the market is much like that in Panel a. The situation is reversed at low-productivity levels. Firm 1 employs a larger share of the workforce, offers better pay conditional on productivity, but marks wages down by more at any wage level.

Firm 1 has a comparative advantage among low-productivity (low-wage) workers and Firm 2 has a comparative advantage among high-productivity (high-wage) workers. Yet, because the firm at a disadvantage competes more aggressively by paying closer to productivity, neither firm fully exploits its comparative advantage. This results in an inefficient employment allocation as depicted by shaded zones in Figure 5. Workers in the blue zone are more productive at Firm 2, but are employed by Firm 1. The reverse is true for workers in the red zone. The overall employment shares of the two firms depend on the densities of comparative advantage zones, and are, therefore, generally ambiguous.

Figures 6 and 7 show firms' wage offers and the corresponding markdowns by productivity for our two asymmetric cases. In each figure firm productivities are marginally distributed $\ln Y_j \sim N(\mu_j, \sigma_j)$ for $j \in \{1, 2\}$, with the joint distribution described by a Clayton copula. The left panels show case 1: $\sigma_1 = \sigma_2$ and $\mu_2 > \mu_1$, i.e., firm 1's productivity distribution stochastically dominates. The right panels assume $\sigma_2 > \sigma_1$ and $\mu_2 = \mu_1$. This can be interpreted either as skills specific to Firm 2 are more dispersed, or, equivalently, skills are more valuable at Firm 2.

In the first case Firm 2 offers a higher wage conditional on productivity (Figure 6a)) consistent with Theorem 5. Markdowns (Figure 7a) increase with productivity in both firms, but Firm 1, having a productivity disadvantage, competes by marking wages down relatively less.

Figure 7: Markdowns and Comparative Advantage in Asymmetric Equilibria



Notes: The figure shows the distribution of markdowns (solid) and comparative advantage (dashed) in duopsony equilibria. Productivity levels are log-normal at each firm with parameters μ_i, σ_i with $i \in \{1, 2\}$; the joint distribution reflects a Clayton copula with positive dependence: $\tau = 0.5$.

Figure 7a) also displays workers' expected comparative advantage at their firm. The higher markups at Firm 2 directly reflect its stronger comparative advantage with its workers.

In the second case, $\sigma_2 > \sigma_1$ and $\mu_2 = \mu_1$, Firm 1 has a comparative advantage among low-productivity workers and Firm 2 among high-productivity workers. Consistent with Theorem 6, Firm 2 has a steeper pay-productivity profile (Figure 6b), with Firm 1's wage offer beating that of Firm 2 for low-productivity workers and vice versa for high-productivity workers. In turn, markdowns (7b) track comparative advantage: Firm 1 has stronger comparative advantage with its low-productivity workers than does Firm 2, so it marks down their wages more, while the reverse is true for high-productivity workers.

Interestingly, wage markdowns display different patterns within the firms. Whereas markdowns monotonically increase with productivity at Firm 2, they display a U-shaped pattern at Firm 1. Recall that a Clayton copula always generates increasing markdown-productivity profiles in symmetric equlibria. The declining profile among low-productivity workers at Firm 1 (Figure 7b) stems from the asymmetry between the two firms. With $\sigma_1 < \sigma_2$, workers have greater productivity differences at Firm 2, Firm 1's competitor. From the perspective of Firm 1, marginally better-skilled workers command disproportionately better pay at Firm 2 (in expectation), forcing Firm 1 to offer a larger share of rents to stay competitive for those workers.

We can summarize the asymmetric cases above. i) If an employer is generally more productive, then it pays higher wages, is therefore larger, and at the same time marks down wages more at any productivity level. ii) If efficiency units for an employer are especially responsive to skill, then it disproportionately attracts high-productivity workers—so pays a higher average wage—while exhibiting a steeper profile of markdowns with respect to productivity. But this higher-wage employer is not necessarily larger in terms of employment.

In the empirical work these predictions naturally lead us to stratify employers by both their median wage and their size. Employers could be high-wage because they are generally more productive or because they are skill-elastic. If they are generally more productive, then they should mark down wages more for all workers, even their lower-wage ones. But if they are more skill-elastic, then they should display higher markdowns only for their high-wage workers. Larger employers should exhibit bigger markdowns across their wage distribution, assuming that differences in employer size are driven by general productivity differences.

2.5 Informal Sector and the Legal Minimum Wage

We now consider two prevalent features of the Brazilian labor market: a legal minimum wage and an informal sector. A minimum wage implies that firms do not hire workers whose productivity is below the minimum wage. Workers with productivity levels that equal the minimum wage are offered the minimum wage. Wages for more-productive workers are still marked down, albeit less than in the absence of a minimum wage because the legal minimum wage raises wage offers all along the productivity distribution in our model. The following theorem gives the equilibrium wage policy with a minimum wage in symmetric equilibrium:

Theorem 8 *Consider a symmetric market with K firms and a legal minimum wage $\underline{w}$. For all $y \geq \underline{w}$, the wage offer policy in an oligopsony equilibrium is given by:*

$$w(y) = w^*(y) + [\underline{w} - w^*(\underline{w})]\zeta(y),$$

where $w^*(y)$ denotes the wage offer in the absence of a minimum wage defined in Theorem 1 and $\zeta(y) \leq 1$ is a strictly decreasing function with $\zeta(\underline{w}) = 1$.

With a minimum wage, all workers receive a wage bump because the minimum wage shifts the initial condition for the differential equation that describes the equilibrium wage policy in an oligopsony equilibrium. The additional pay, $\zeta(y)[\underline{w} - w(\underline{w})] > 0$, is a fraction $\zeta(y)$ of the firms' profits in the absence of a legal minimum wage associated with a worker whose productivity equals $\underline{w}$. Minimum wage regulation redistributes that portion from firm to worker, regardless of the worker's productivity, so long as it exceeds $\underline{w}$. Thus it reduces wage markdowns for all workers. This result is consistent with a number of studies that estimate a positive impact of minimum wage legislation on wages well above the legal minimum, e.g., Lee (1999) and Engbom and Moser (2022). Because $\zeta'(y) < 0$, the absolute wage bump is more significant for lower-wage workers, and even more so as a percentage of their wage.

A legal minimum wage necessarily lowers employment because workers with below-minimum-wage productivity at *all* firms, $\max_k\{y_k\} < \underline{w}$, are not employed, while all are employed in the absence of a minimum wage. Formally, total employment is $1 - C(F(\underline{w}), ..., F(\underline{w}))$, which is decreasing in $\underline{w}$ since both F and C are strictly increasing in their arguments. Furthermore, the minimum wage causes no reallocation of workers between firms in symmetric equilibrium, implying firms have equal employment shares although each has fewer employees.

Next, we add an informal economy. Specifically, we assume workers have the option of working outside the formal sector with pay w_0, which is distributed i.i.d. according to distribution $F_I(w_0)$. We interpret this option as the informal sector in Brazil. More generally, $F_I(w_0)$ represents the distribution of the reservation wage for participating in the (formal) labor market. In this economy, firms have to beat not only their competitors' offers but also the informal sector's offer to attract a worker. The labor supply to a firm is now given as:

$$n_k(w; y) = P(w_j(y_j) \le w, \forall j \neq k | Y_k = y) \cdot P(w_0 \le w) \cdot f_k(y), \tag{9}$$

where $f_k(y) = F_k'(y)$ is the density of workers with $Y_k = y$ and $P(w_0 \le w) = F_I(w)$. Because $F_I(w) < 1$, an informal sector option necessarily reduces total employment in the formal sector.

Because the informal sector intensifies competition for workers, it also raises wages in the formal sector for all workers and, thereby, reduces wage markdowns. An explicit analytical expression for the wage offer policy in oligopsony equilibrium is not feasible in this case. The following theorem characterizes the wage offer policy as a solution to a differential equation.

Theorem 9 *Consider a symmetric market with K formal firms, an informal sector and a legal minimum wage $\underline{w}$. The wage offer policy in an oligopsony equilibrium solves the following differential equation:*

$$\frac{dw}{dy} = \frac{\sum_{j \neq k} C_{kj}(F(y), \cdots, F(y)) F'(y)}{C_k(F(y), \cdots, F(y))} \left[\frac{1}{(y - w)} - \frac{F_I'(w)}{F_I(w)} \right]^{-1}, \tag{10}$$

with the initial condition $w(\underline{w}) = \underline{w}$.

Relative to our benchmark economy, the addition of an informal sector introduces the last term in brackets in equation (10). $F_I'(w)/F_I(w) > 0$ for all w; so dw/dy, the wage productivity slope, is steeper for any (w, y) pair. Given the common initial condition $w(\underline{w}) = \underline{w}$, a steeper profile implies that wage offers are greater at all productivity levels $y > \underline{w}$, and, hence, markdowns are lower. The magnitude of wage reactions to competition from the informal sector varies with productivity in a manner dictated by the distribution of informal wages. Empirically, informal-sector pay is on average lower than formal-sector pay; so the wage gains from informal competition will skew toward low-wage workers. In that sense, introducing an informal sector reinforces our prediction that wage markdowns are largest among high-productivity workers.

These effects are common to all firms in symmetric equilibria. In asymmetric equilibria, competition from an informal sector would be disproportionate across firms, affecting more severely firms that have a comparative advantage among low-wage workers.

3 Data

Our model predicts that monopsony power is most severe for workers in matches that exhibit strong comparative advantage, whom we anticipate to be disproportionately high-wage workers. To test these predictions, in Section 4 we estimate the elasticity of the labor supply facing employers in the Brazilian formal labor marker. But first we discuss our data, how we stratify workers by relative wages at an employer, and how we measure employment and wage changes.

3.1 RAIS Data

Brazil's Annual Social Information Report, RAIS for short, collects establishment-level data on each employee's compensation as well as other characteristics. The RAIS provides the basis for government statistics on Brazil's formal labor market. It is also the basis for government monitoring of employment laws and required payments for benefits.[19]

The RAIS data have important advantages for our purposes. For one, we clearly need micro data at the employer-worker level, as in the RAIS, to establish how both employment and wages change at different segments of an employer's wage distribution. The RAIS panel structure allows us to calculate wage changes for continuing workers at the employer, controlling for compositional changes. The RAIS has several advantages relative to many matched-data for other countries. In addition to annual information, the RAIS provides earnings and hours data specifically for the month of December, allowing us to calculate not just worker earnings, but also a wage rate. The RAIS also provides information on worker characteristics, including age, education, and occupation. Information on employer characteristics is relatively limited; RAIS provides the establishment-level location and industry, but no production information beyond labor (e.g., revenue, capital, inventory, etc.). While the RAIS has comprehensive coverage of

[19]See http://www.rais.gov.br/sitio/index.jsf. Benefits include social security benefits, severance pay, and the legislative one-twelfth annual pay bonus.

Brazil's formal labor market, its primary shortcoming is its omission of Brazil's large informal labor market. We return to this issue in Section 5. Our data cleaning follows Dahis (2024).

3.2 Sample Construction

Our estimates are based on years 2006-2018 of the RAIS.[20] While employers are required to report information on all workers employed during the year, they specifically report how many workers were employed at the end of the year and workers' earnings for the month of December. These variables underlie our measures of employment and wage growth to estimate supply elasticities. Therefore, we begin with a sample of RAIS workers who are still employed at year's end with reported earnings for December. Combining years 2006-2018 yields 553,266,004 worker-year observations.

Table 1 presents restrictions imposed in constructing our sample. Panel A lists those that apply to all statistics (e.g., worker's wage rank, employer's employment, market share, etc.), while Panel B gives those imposed only for estimating wage responses to an employer's employment growth. We restrict our sample to workers aged 18 to 64 whose December pay is at least 95 percent of the legal minimum. We exclude military and government workers, as their wages presumably do not reflect profit maximization. We exclude certain occupations: workers on apprentice contracts, upper management (directors/managers), and science or art professionals. We view management and professional positions as sufficiently idiosyncratic that wage setting could well reflect individual bargaining. The excluded sectors and occupations are 29.1 percent of observations. Below we will define a worker's labor market largely by their metropolitan area. Therefore, we restrict our sample to establishments in one of 74 metropolitan areas. Excluding rural areas drops 28.3 percent of observations. The union of the above restrictions excludes 50.9 percent of the worker-year observations.

Panel B reports the further sample restrictions for our main regressions. Our wage measure is monthly pay for December, reflecting overtime, bonuses, commissions, and tips, but not the bonus "thirteenth-month" pay required in Brazil. To ensure that pay corresponds to a wage, we restrict our regression sample to those paid monthly, were employed the entirety of December, and worked 40 to 44 hours per week. Respectively, 90.8 percent, 96.2 percent, and 91.2 percent of observations meet these constraints.[21] December wages are deflated by the December IPCA

[20] An establishments' 7-digit industry, required for one of our instruments, is first reported in 2006 with the 2nd edition of Brazil's National Classification of Economic Activities (CNAE 2.0). We exploit years of the RAIS back to 1985 to benchmark the age of establishments, required as another instrument.

[21] We only observe a worker's union status in the RAIS data for 2017. For our regression sample, which excludes heavily unionized government jobs, the unionization rate for 2017 is only 12%. Non-union workers in Brazil can legally appeal to apply union negotiated terms for their position. But these terms primarily dictate working conditions, e.g. paid/unpaid leave, rather than wage levels or employment. See Lagos (2024).

Table 1: Sample Selection Criteria

Panel A: Cross-Section Restrictions

Restrictions	percent meeting each†
$18 \leq$ age ≤ 64	98.2
Paid ≥ 0.95 times legal minimum	97.5
Not military, government, apprentice, director/manager, or science/art prof.	70.9
In one of 74 metropolitan areas	71.7
Meeting all Panel A restrictions	49.1

Panel B: Regression Restrictions

Restrictions	percent meeting each‡
Paid monthly	90.8
At establishment all of December	96.2
$40 \leq$ weekly hours ≤ 44	91.2
Employer ≥ 10 workers with stayers above & below median	61.7
Stayer with consecutive wages	52.6
Meeting all Panel B restrictions	29.3

Resulting Sample Size

Worker-year	79,614,942	Employer-year	3,415,115

Notes: † expressed in terms of sample of workers employed at year's end with a December wage. ‡ expressed in terms of sample of workers who met all restrictions in Panel A.

price index.[22]

We examine year-over-year changes in employment and wages at each employer. For instance, for year t we compare an employer's employment at the end of year t to its December 31st employment of year $t-1$. For wage responses, we compare a worker's wage for December of year t to the same worker's wage in December of $t-1$. We focus on wage changes for stayers at an employer to avoid any impact on the wage change from compositional effects.

We stratify workers at an employer into quartiles based on their relative wages.[23] To make meaningful distinctions across an employer's wage distribution, we restrict the sample to employers (establishments by occupation) that average at least 10 employers between the year-ends of t and $t+1$ and who have at least one stayer at both above and below the median wage. This cuts 38.3 percent of observations from our sample. The requirement of having a wage at the same employer for consecutive Decembers, necessary only for our wage-change regression, excludes 47.4 percent of observations. The union of the regression-sample restrictions in Panel B excludes 70.7 percent of workers. The final sample for wage-change regressions is comprised

[22] The IPCA is constructed by the Brazilian Institute of Geography and Statistics (IBGE) to track prices for goods and services with weights reflecting household expenditure shares.

[23] Stayers' relative wages equal the average of their real wages in December of t and $t-1$. For those departing during t, we average their December of $t-1$ real wage with an imputed wage for December t, based on the employers' average wage change for stayers. Similarly, for those arriving in t, we average their wage in December of t and an imputed wage for December of $t-1$. We then weight exiters and entrants by 0.5 and group the workers into quartiles constructed consistently with STATA's xtile formula. Because of ties, the quartiles are not equal-sized; the first to fourth quartiles account respectively for 23 percent, 24 percent, 26 percent, and 27 percent of workers.

Table 2: Rates of Hiring from Same Metro, Occupation, Industry

	Same metro area	Same occupation	Same industry
Percent of hires	81.1	75.5	19.5

Notes: Sample is restricted to job matches that satisfy (i) all restrictions in Table 1, except for the last restriction in Panel B, "stayer with consecutive wages" is replaced by "new hires," (ii) the worker's last job within the past year is observed and satisfies Panel A restrictions in Table 1. Sample size is 22,063,887.

of $79,614,942$ worker-year-level observations at 3,415,115 employer-years.

We judge an employer's employment growth relative to its labor market by including a full set of year-market fixed effects. We define a labor market along two dimensions: (i) the worker's metropolitan area, and (ii) the worker's broad occupation. We distinguish three occupations defined in the RAIS as technicians/repairman, service workers, and production workers. We define an employer by each of the three occupations at an establishment. Thus our estimates ask: when an establishment expands employment of an occupation, say, service workers, how much does it affect the wage they pay to service workers?

Our definition of a market as a metropolitan area/occupation appears roughly consistent with transitions in the RAIS data. The first column of Table 2 reports that 81.1 percent of hires in our sample reflect workers previously employed in the same metro area.[24] From the second column, 75.5 percent are hired from within the same occupation. By contrast, we see from column 3 that only 19.5 percent are hired from the same 7-digit industry. We highlight the low inter-industry hiring rate because one instrument we consider for an employer's growth is the growth rate for its industry nationally relative to that for other employers in its market.

4 Estimated Supply Elasticities and Implied Markdowns

4.1 Empirical Specifications and Estimating Strategy

Identifying the employer-level supply elasticities requires measuring the relative responses of employment and wages to a shift in an employer's labor demand, holding its supply curve fixed. Consider a set of workers whose productivity is y at employer j, but then shifts to $y' = z \cdot y$. The shifter z could reflect a productivity change or a shift in demand for the employer's output. The elasticity of supply for workers of productivity y at employer j is:

$$\eta_j(w;y) = \frac{d\ln n_j(w;y)/d\ln z_j(y)}{d\ln w_j(y)/d\ln z_j(y)}\,. \tag{11}$$

[24]The statistics for hires in Table 2 reflect the formal-sector contract that ended most recently to the start date at the new employer, while ending in the same or prior year as that start date.

Note that, while the elasticity $\eta_j(w;y)$ is specific to y, the labor demand shifter, $z_j(y)$, can be specific to y-workers or more employer-wide, i.e., $z_j(y) = z_j$ for all y. The key is that the shift in labor demand for workers of productivity y should be orthogonal to any shifts in outside opportunities for those workers. With that in mind, in estimating $\eta_j(w;y)$ we consider a variety of instruments for an employer's employment growth.

Specifically, we estimate wage and employment responses at each quartile of an employer's wage distribution to year-over-year growth in its total employment. For worker i at employer j in year t, let q be the worker's wage quartile. We consider the specifications:

$$\Delta \ln \text{wage}_{it} = \beta_q \, \Delta \ln \text{emp}_{jt} + \boldsymbol{x}_{it}\boldsymbol{\mu} + \alpha_{mqt} + \varepsilon_{it} \,, \tag{12}$$

$$\Delta \ln \text{emp}_{jqt} = \gamma_q \, \Delta \ln \text{emp}_{jt} + \kappa_{mqt} + \xi_{jqt} \,. \tag{13}$$

The first equation is estimated at the worker-year level, just for stayers at their employer, reflecting the 79,614,942 worker-year observations. The operator Δ denotes the backward-looking one-year difference: $\Delta \ln \text{wage}_{it}$ denotes stayer i's rate of wage growth from December $t-1$ to December t, while $\Delta \ln \text{emp}_{jt}$ denotes employer j's rate of employment growth between year ends $t-1$ and t. The row vector $\boldsymbol{x}_{it}$ controls for quadratics in the worker's years of schooling and age. The subscript m denotes the market (metropolitan area and occupation), with α_{mqt} market-by-quartile-by-year fixed effects in stayers' wage changes. ε_{it} is residual wage growth for stayer i. The coefficients of interest are the β_q's; differences in β_q's across quartiles capture how wages respond for an employer's higher- versus lower-wage workers.

The second equation is estimated at the employer-quartile-year level. The sample reflects 3,415,115 employer-year observations. For each employer-year, $\Delta \ln \text{emp}_{jqt}$ denotes the year-end rate of employment change from year $t-1$ to t for wage quartile q within the employer. κ_{mqt} is a market-by-quartile-by-year fixed effect for rate of employment growth; ξ_{jqt} is residual employment growth. We weight each observation by the average of the employer's number of employees at year ends $t-1$ and t. The coefficients of interest are the γ_q's; their cross-quartile differences capture, when employment grows for an employer, whether that growth occurs more at the top or bottom of the employer's wage distribution.

The estimates from equations (12) and (13) jointly yield estimates of quartile-specific labor-supply elasticities, $\widehat{\eta}_q = \widehat{\gamma}_q / \widehat{\beta}_q$, with implied quartile-specific markdowns, $1/(1+\widehat{\eta}_q)$. Of course, OLS parameter estimates are biased unless employment changes are unrelated to potential shifts in the employer's labor supply—in particular, the wage response and implied markdown are biased downward if employment is partly responding to supply shocks.

To address this we consider three sets of instruments. We first exploit that younger and smaller establishments predictably grow faster. Specifically, the instrumental variables are quadratics in the employer's age and its log employment in $t-2$.[25] The identifying assumption is that faster growth for these employers reflects productivity gains or product development that expands labor demand, not systematic shifts in their labor supply functions. To highlight this possible challenge to identification, we note that Kim (2024) predicts an employer's labor supply will grow over its life-cycle because, by learning, workers resolve their uncertainty about its productivity. In this scenario, our instrument would yield an upward-biased estimate of employer labor supply elasticity, implying a downward-biased markdown.

As an alternative strategy we treat national employment growth for an industry as an instrument for employment growth for employers in that industry, *holding fixed employment in each employer's labor market*. This strategy exploits that each labor market—defined by metropolitan area and occupation—contains workers from many industries. Specifically, the instrument is the annual growth rate in an employer's 7-digit industry across all labor markets.[26]

The identifying assumptions behind this instrument are two-fold. First, it assumes that an industry's nationwide employment growth reflects shifts to its labor demand. Second, the instrument assumes an employer's labor supply is dictated by its location and the occupation of its workers, not its 7-digit industry. For robustness with respect to the first assumption, in Appendix B.1 we replace industry employment growth as an instrument with an interaction of the Brazilian business cycle with an industry's employment cyclicality. This weakens the assumption to require only that demand factors (e.g., producing durable goods) dictate why some industries are more cyclical than others. For robustness to the second assumption we also exclude those industries for which more than 10 percent of hires, on average, come from the same industry. In both cases, the estimation results are quite similar to the benchmark reported in the text below; but standard errors are somewhat larger.

Lastly, we combine and interact both sets of instruments. While each set exploits the labor demand changes rooted in an employers' life-cycle or industry, the interaction term allows, for instance, the demand-driven employment volatility to change over an employer's life-cycle (see recent evidence in Crouzet and Mehrotra, 2020; Clymo and Rozsypal, 2023). The identifying assumption, once again, requires these three sources of employment variation to be uncorrelated with changes in labor supply.

[25] The panel data reveal establishment births back to 1985. Given our regression sample begins in 2006, for all years we can construct exact age only up to 21 years. For this reason, our exact instruments are a dummy variable indicating age 21 or below plus its interaction with a quadratic in establishment age.

[26] Industries reflect Brazil's National Classification of Economic Activities (CNAE 2.0). Our sample has 1,393 industries. Across our 222 markets (3 occupations times 74 metropolitan areas) the average number of industries is 847; the bottom 1 percentile has 211 industries, and the top 1 percentile has 1,054 industries.

Table 3: OLS Estimates: Wage and Employment Responses by Wage Quartile

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
all	0.027 (0.001)	1.00 (−)	37.4 (1.3)	2.6 (0.1)
1st	0.013 (0.001)	1.22 (0.01)	93.2 (5.3)	1.1 (0.1)
2nd	0.023 (0.001)	0.92 (0.00)	40.5 (1.5)	2.4 (0.1)
3rd	0.030 (0.001)	0.81 (0.01)	27.3 (1.1)	3.5 (0.1)
4th	0.036 (0.001)	0.76 (0.01)	20.8 (0.8)	4.6 (0.2)

Notes: Wage responses reflect 79,614,942 worker-year observations; employment responses reflect 3,415,115 employer-year observations, with observations weighted by employer's employment. Standard errors (in parentheses) for wage and employment responses are clustered at employer-by-year level. Those for labor supply elasticity and markdown are derived by the Delta Method. Not every employer is represented in all four quartiles.

4.2 Within Employers: Less Elastic Labor Supply for Higher-Wage Workers

We present wage and employment responses, β_q's and γ_q's, to an employer's employment growth, together with the implied labor supply elasticity and markdown. Standard errors are clustered at the employer-by-year level. Table 3 presents the OLS results. The first row reports the supply elasticity without distinguishing wage quartiles.[27] Stayers' wage growth exhibits an elasticity of 0.027 to its employer's employment growth. The associated standard error is only 0.001—so the wage response is clearly statistically different from zero. Nevertheless, it implies an extremely high labor supply elasticity, η, of 37.4 with standard error 1.3. That maps to a markdown of only 2.6 percent, which is quite low compared to estimates in the literature.

The rest of Table 3 presents the OLS results for (12) and (13), i.e., how both wages and employment at each wage quartile respond to its overall employment growth.[28] Wage responses monotonically increase moving up an employer's wage quartiles—the top quartile's response is nearly three times that at the bottom. By sharp contrast, the employment responses systematically decline moving up the wage distribution. This indicates a less elastic labor supply for higher-wage workers. The supply elasticity declines from 93.2 to 20.8 from the bottom to the top quartile, with the implied markdown going from 1.1 percent to 4.6 percent.

The markdown implied by the OLS estimates is likely biased downward unless employment changes are unrelated to potential shifts in supply. To address this, Table 4 reports estimates under each set of instruments for employment growth described in Section 4.1. Each set passes the weak-instrument test.[29] The first panel instruments with the employer's life-cycle via quadratics in its log size and age; being both smaller and younger increases expected em-

[27]When not distinguishing across quartiles, the fixed effect α_{mqt} in equation (12) simplifies to market by year, α_{mt}. This also applies to first-row regressions of each panel in Tables 4 and A1.

[28]Average wages across the quartiles, expressed relative to the lowest quartile, are 1, 1.30, 1.63, and 2.54.

[29]We report the Kleibergen and Paap (2006) Wald rk F-statistic because standard errors are clustered.

ployment growth, with a first-stage F-statistic of 2471.9. The second panel instruments with industry employment growth; its first-stage F-statistic equals 39.8. The third panel, which combines and interacts the previous two instrument sets, yields a first-stage F-statistic of 4742.

We begin again, in the first row of each panel, with the employer-level supply elasticity without distinguishing by quartile. As expected, IV wage responses are much higher than OLS. Across the first two sets of instruments, stayers' wages respond with respective elasticities of 0.19 and 0.12, corresponding to markdowns of 15.7 percent and 10.9 percent. Combining the instruments (Panel 3) yields estimates that are intermediate: an elasticity of wage response of 0.15 (s.e. 0.02) with implied markdown of 12.7 percent (s.e. 1.9 percent). As discussed in the literature review, this markup estimate is comparable to estimates for the US labor market and elsewhere.

The remaining rows in each panel of Table 4 present the IV estimates by workers' wage quartiles. Instrumenting yields the same relative pattern as OLS, but with much larger wage responses and implied markdowns. Furthermore, these patterns are consistent across the choice of instruments. In particular, all three panels show wage responses that increase monotonically across wage quartiles, with a top-quartile response on the order of five times that at the bottom, while showing employment responses that monotonically decline, with a top-quartile response that is less than half that of the bottom's. As a result, regardless of instrument set, the estimates imply a supply elasticity that is at least an order of magnitude larger for the bottom quartile as the top. The upshot is that we estimate that monopsony power is far greater at the top of employers' wage distributions. Focusing on combined instruments in Panel 3, the estimated markdown for top quartiles is 28.1 percent (s.e. 2.9 percent), whereas for bottom quartiles it is extremely low at 2.3 percent (s.e. 1.0 percent).

Section 2.5 showed how a minimum wage affects wage markdowns in our model. We note here that our empirical results are not driven by workers earning near the minimum wage. In Appendix B.1 we exclude workers with wages less than 120 percent of the legal minimum. We continue to see a sharp decline in the labor supply elasticity moving up the wage distribution.

Viewed from the model, our results imply an employer's high-wage workers have greater match comparative advantage. But this is consistent with skills being partially portable across employers—the model captures this with positive dependence of productivities. One arguably portable skill is formal schooling. We partially control for schooling heterogeneity because we stratify markets by occupation. In Appendix B.1, we go further, ranking workers within employers by quartiles of their wage residuals, i.e., wages after removing a common estimate for schooling dummies. We find that the within-employer dispersion in wages controlling for schooling is nearly as large as that for wages. Furthermore, labor supply elasticities show a similar pattern of decline across the distribution of wage residuals similar to that for wages.

Table 4: IV Estimates: Wage and Employment Responses by Wage Quartile

IV1: employer's life-cycle (quadratics in log size and log age)

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
all	0.19 (0.02)	1.00 (–)	5.4 (0.6)	15.7 (1.5)
1st	0.05 (0.02)	1.41 (0.03)	29.1 (13.6)	3.3 (1.5)
2nd	0.16 (0.02)	0.91 (0.02)	5.6 (0.6)	15.2 (1.5)
3rd	0.23 (0.02)	0.64 (0.03)	2.8 (0.3)	26.6 (2.2)
4th	0.31 (0.03)	0.52 (0.02)	1.7 (0.2)	36.8 (2.4)

IV2: industry's employment growth

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
all	0.12 (0.02)	1.00 (–)	8.2 (1.6)	10.9 (1.9)
1st	0.04 (0.03)	1.46 (0.12)	41.0 (30.0)	2.4 (1.7)
2nd	0.10 (0.03)	0.96 (0.07)	9.8 (2.9)	9.2 (2.5)
3rd	0.14 (0.02)	0.77 (0.01)	5.4 (0.9)	15.5 (2.2)
4th	0.18 (0.03)	0.65 (0.06)	3.5 (0.7)	22.0 (3.4)

IV3: employer's life-cycle × industry's employment growth

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
all	0.15 (0.02)	1.00 (–)	6.9 (0.8)	12.7 (1.9)
1st	0.03 (0.01)	1.38 (0.10)	42.2 (18.5)	2.3 (1.0)
2nd	0.13 (0.02)	0.96 (0.05)	7.5 (1.1)	11.7 (1.5)
3rd	0.18 (0.02)	0.71 (0.02)	3.9 (0.6)	20.5 (1.9)
4th	0.23 (0.03)	0.59 (0.03)	2.6 (0.4)	28.1 (2.9)

Notes: IV1 instruments include quadratics for log establishment size and age, where establishment size is measured by 2-period lagged employment; in addition, we interact log age quadratics with an indicator for age below 21 because the birth year of establishment is truncated at 1985 by construction. The IV2 instrument is the establishment's 7-digit CNAE industry employment growth. IV3 instruments are those for IV1 and IV2 and their interactions. The pooled regressions (labeled as "all") control for market-by-year fixed effects; others control for market-by-year-by-quartile fixed effects. Standard errors, in parentheses, are clustered at the employer-by-year level. Standard errors in Columns 3 and 4 are calculated by the Delta Method.

Table 5: Wage Gains (in percentages) for Mass Hires by Wage Quartile

mass hires of	quartile (q) 1st	2nd	3rd	4th
at least 50 percent	5.6	17.4	24.8	37.5
at least 80 percent	4.3	16.9	24.8	39.8

Notes: Samples are restricted to job matches that satisfy (i) all restrictions in Table 1, except for the last restriction in Panel B, "stayer with consecutive wages" is replaced by "new hires," (ii) the worker's last job within the past year is observed and satisfies the Panel A restrictions in Table 1, (iii) the employer experiences a mass hire in the year, and (iv) the last job's separation is initiated by the worker. The resulting sample size is 1,841,134 for at least 50 percent mass hires and 313,284 for at least 80 percent mass hires. We control for market-by-year fixed effects.

From our framework, our results imply that workers' comparative and absolute advantages align at their employers, with the higher-wage workers exhibiting higher match rents and larger monopsony markdowns on those rents. While we cannot directly measure comparative advantage, because workers' productivities at their current and next-best jobs are unobservable, we can still substantiate comparative advantage differences across workers by comparing wage gains of new employees who switched jobs during mass hire events. Mass hires emulate the scenario where a worker's new job option emerges from a new establishment, which leads to arguably exogenous wage gains from switching. To capture workers who still had the option to keep their former jobs, we further restrict attention to workers who initiated the separation according to the former employer.

Table 5 reports the log-wage gains associated with mass hires by workers' wage quartile at their new employer. The first row defines mass hires by employers who hired at least half of their year-end staff within that year; the second row raises the new-hire threshold share to 80 percent, but yields similar results. The first row shows significant wage gains for all hires, but especially large gains for workers higher in the wage distribution at the new employer, e.g., 37.5 percent for workers in the top quartile compared to 5.6 percent in the bottom. These suggest greater match rents from comparative advantage for high-wage workers. In Section 5.1 we find that expected wage differences between a worker's best and second-best options closely approximate markdowns in the calibrated model. Alternatively, we regress a new hire's prior wage on their current wage at the mass-hiring employer (in logs, controlling for employer-by-year fixed effects), which yields a coefficient of 0.54. Thus, high-wage hires show larger opportunity costs, but the elasticity is well below one, implying disproportionately larger gains from the transition.

4.3 Across Employers: Less Elastic Labor Supply for Higher-Wage Employers

We next explore how labor supply elasticities vary across employers in light of Section 2.4's predictions under asymmetric duopsony: i) bigger markdowns at higher-wage/larger employers; ii) steeper profiles of markdowns with respect to productivity for higher-wage employers.

We first group employers according to wages. To be specific, we first rank employers in each market by median wage, then label high-wage employers as the minimal upper part of that distribution required to account for 50 percent of workers in each market. High-wage employers are 46 percent of all employers in our sample. The average median wage among our high-wage employers is 92 percent higher than that among the low-wage employers.

We then divide employers into large versus small based on their relative size in their market. To be comparable with the common definition in the literature (see, e.g., Berger et al., 2022), we define large employers as the top 5 percent in each market in number of workers. These employers account for about 44 percent of workers in our sample, with small employers accounting for 56 percent.

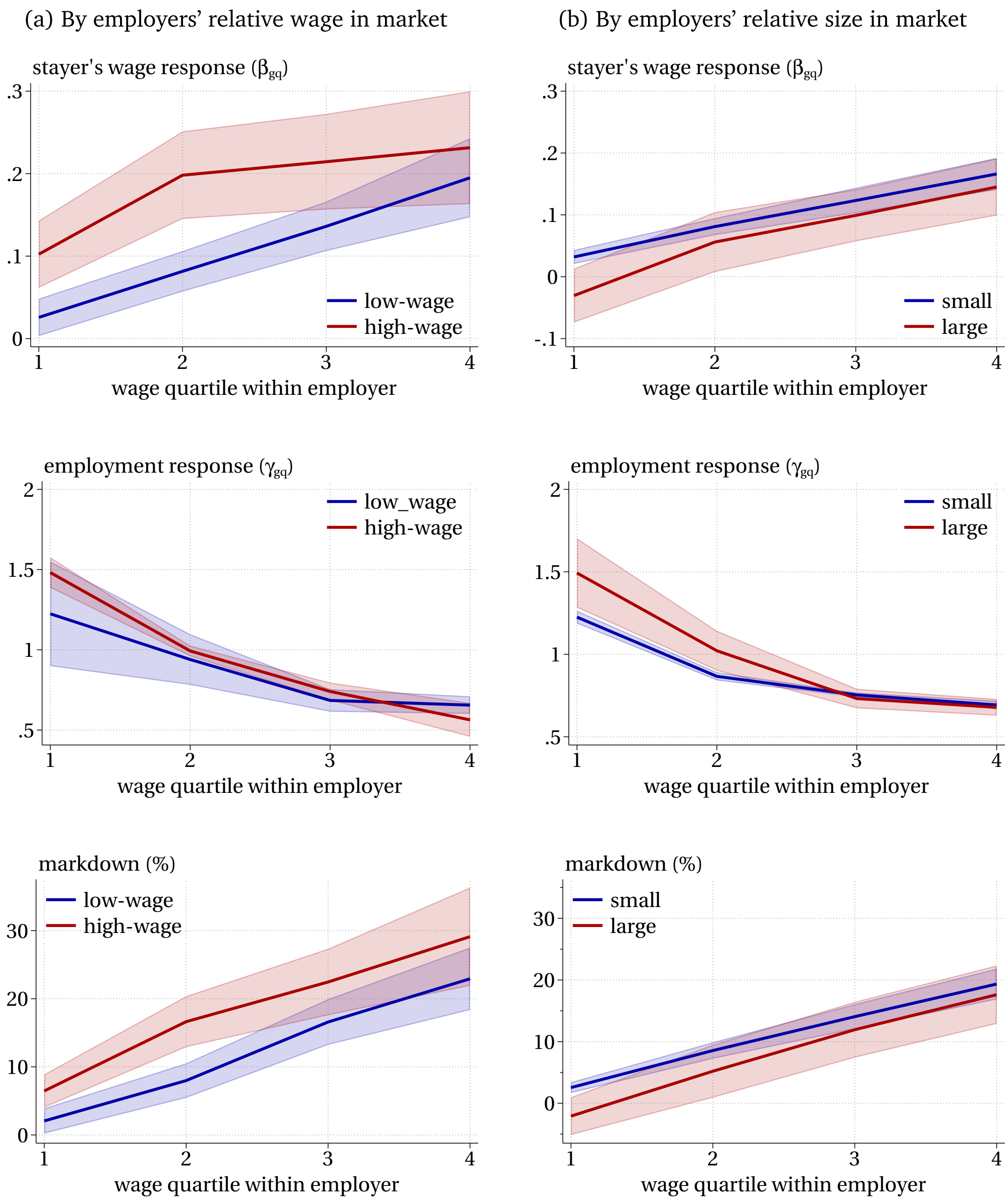
Denote the employer group as $g \in \{\text{high-wage, low-wage}\}$ when grouping by wage, and $g \in \{\text{large, small}\}$ when grouping by size. We modify the regression specifications (12) and (13) to allow for differing wage and employment responses across employer groups (β_q now β_{gq} and γ_q now γ_{gq}) and fixed effects that span g as well as m, q, and t. Because the three instrument sets delivered similar findings in the previous subsection, here we only report estimates using the last set of instruments, i.e., the combination and interaction of life-cycle and industry growth instruments. Standard errors are, again, clustered at the employer-by-year level.

Figure 8 depicts the results: Panel (a) groups employers by relative wage, while Panel (b) does so by relative size. In each panel the top figure plots the estimated stayers' wage responses, $\widehat{\beta}_{gq}$, by within-employer wage quartile q; the middle panel plots the estimates for employment responses, $\widehat{\gamma}_{gq}$, by q; the bottom panel plots the implied markdown, $1/(1+\widehat{\gamma}_{gq}/\widehat{\beta}_{gq})$. For each Panel (a) figure, the red and blue lines depict respective estimates for high-wage and low-wage employers; for Panel (b), the red and blue lines depict respective estimates for large and small employers. The shaded areas represent the estimates' 95 percent confidence intervals.

We highlight three findings. First, the *within-employer* pattern of markdowns by wage quartile, documented above, holds for each employer group: Wages respond more to labor demand for higher-wage workers, while high-wage employment is less responsive, indicating they have less elastic labor supply and, hence, face larger markdowns.

The second finding, Panel (a), is that wages of high-wage employers respond more to demand, indicating they face less elastic supply and impose bigger markdowns. Viewed from our cases of asymmetric duopsony in Section 2, this appears more consistent with high-wage firms having a general productivity advantage (Figure 7a) rather than being specific to high-

Figure 8: Markdown Heterogeneity across Employers



Notes: For each Panel (a) figure, the red and blue lines depict respective estimates for high-wage and low-wage employers; for Panel (b), the red and blue lines depict respective estimates for large and small employers. The shaded areas represent the estimates' 95 percent confidence intervals.

productivity workers (Figure 7b).

Third, from Panel (b), implied markdowns do not differ economically or statistically between large and small employers.[30] Through the lens of our model, large firms, if generally more productive, should exhibit a smaller supply elasticity—which we do not see. Furthermore, employer size and wage should be tightly related. But the elasticity of median employer wage with respect to its employment is only 0.05 in Brazil's formal sector (correlation of 0.09). As a result, our large employers have only 11 percent higher average wages than small employers.[31] One interpretation of these findings is that size differences across employers in the Brazilian labor market may be driven by sources of employer heterogeneity other than productivity.

5 Implications for Minimum Wage Policies

We now return to the model to see what discipline our estimates impose in describing monopsony power in the Brazilian labor market. We first calibrate our model under symmetric duopsony allowing for an informal sector and a minimum wage. We then use our calibrated model to gauge the extent to which one should expect minimum wages to counter monopsony power in Brazil.

5.1 Model Calibration

We consider a symmetric duopsony with log-normal marginal productivity distributions. We entertain three model versions corresponding to joint productivities that reflect a Gaussian, Clayton, or Gumbel copula. For comparison, we first consider the simpler case of no informal market and no minimum wage. Normalizing mean log-productivity to zero, only two parameters remain: the standard deviation (σ) of the marginal distribution and the dependence parameter (ρ, θ, or α) corresponding to each copula. We calibrate these to match the average within-employer standard deviation of wages in our RAIS sample of 0.31 (weighting employers by their employments) and an average markdown of 16.1 percent, reflecting the average of markdown estimates for the four quartiles in Table 4 using the third set of instruments.

We then extend the model for a legal minimum wage, $\underline{w}$, and a competitive informal labor market (see Section 2.5.) We assume the informal wage, w_0, is distributed log-normally

[30] In Figure A1 of Appendix B we stratify employers instead by the level of employment concentration, measured by the Herfindahl-Hirschman Index, in their market. We see no difference in supply elasticities in more- versus less-concentrated markets.

[31] The estimated elasticity controls for market-by-year fixed effects. We also note that employment is not so concentrated in Brazil—the average market employment share of the employers we classify as large is only 0.3 percent.

Table 6: Targeted Moments for Calibration with Minimum Wage and Informal Sector

Targeted Moments	Data Source	Data
Within-formal-firm std dev of log wage	Authors' calculation	0.31
Formal sector average markdown (%)	Authors' estimation	16.1
Minimum-to-formal wage ratio	Authors' calculation	0.33
Formal sector employment share (%)	Engbom et al. (2022)	66
Informal-to-formal wage ratio	Meghir et al. (2015)	0.69

Notes: The within-employer std. dev. of log wage and the minimum-to-formal wage ratio are calculated by the authors from our RAIS sample that satisfies Table 1 restrictions except for being stayers (i.e., the last restriction in Panel B). The formal sector average markdown reflects IV3 estimates in Table 4. Under our baseline calibration (Clayton copula with informal sector and minimum wage), the within-formal-firm std of log productivity is 0.41.

with mean μ_0 and standard deviation σ_0, independent of formal-sector productivities. Three additional parameters are required: μ_0, σ_0, and the minimum wage $\underline{w}$. To do so, we target an informal sector that is 34 percent of total employment with an average wage that is 0.69 that in the formal sector. We base the first moment on Engbom et al.'s (2022) comparison of informal and formal employments from survey data for Brazil's six largest cities. We base the latter on Meghir et al.'s (2015) comparison of wages for formal and informal workers for Brazil's Sao Paulo and Salvordor regions.[32] We target a minimum wage that is 33 percent of the average formal-sector wage, a figure we calculate by comparing a time series for the Brazilian minimum wage to wages in our RAIS sample. Table 6 summarizes the calibration targets.

The calibrated parameters for each model version are given in Appendix Table A5. All versions are able to match the corresponding calibrated moments quite precisely.[33] All require high dependence to hit an average wage markdown of 16.1 percent. In particular, allowing for an informal sector and a mininum wage, the calibrated correlation coefficient for the Gaussian copula is 0.90. Kendall's tau for the calibrated Gaussian, Clayton, and Gumbel copulas is respectively 0.71, 0.67, and 0.68.

[32] Meghir et al. (2015) report wages separately by region and gender. Weighting the informal-to-formal differential for each group by its relative size yields the informal ratio of 0.69. The ratio of average informal to average formal wage for the six cities in Engbom and Moser (2022) is lower; we calculate it at 0.55 from the earnings and hours they report for each sector. But this does not control for differences between formal and informal workers' markets, genders, et cetera. Meghir et al. (2015) control for schooling partially by including those with less than 9 years of schooling. Engbom and Moser (2022) report a formal wage premium of 26.3 percent controlling for region, worker characteristics, and some job characteristics (industry, tenure), implying an informal-to-formal wage ratio of 0.77. Our results are quite similar if we calibrate to a 0.77 ratio.

[33] For the extended model, we first compute formal firms' equilibrium wage function for a given set of parameters using a forward shooting algorithm (with MATLAB's `ode15s` solver) to solve the differential equation specified in Theorem 9 of Section 2.5. We then simulate workers' draws from the productivity and informal wage distributions, calculate wages and job choices for each worker, and use the simulated moments as a numerical approximation to the model moments. For models with no informal sector we can analytically compute the equilibrium wage function using the formulas in Examples 2, 3, and 4 of Section 2.

Table 7: Implied Markdowns

		No informal or $\underline{w}$			With informal and $\underline{w}$		
quartile (q)	Data	Gaussian	Clayton	Gumbel	Gaussian	Clayton	Gumbel
1st	2.3	13.4	7.9	15.2	10.4	6.9	10.4
2nd	11.7	15.1	12.0	16.6	14.6	11.4	16.2
3rd	20.5	16.5	16.7	16.9	17.7	16.6	19.2
4th	28.1	18.9	26.3	15.7	20.9	27.4	17.9

Notes: For all six models parameters are calibrated to match the first two moments in Table 6; while parameters for models with an informal sector and minimum wage are calibrated to match all moments in the table.

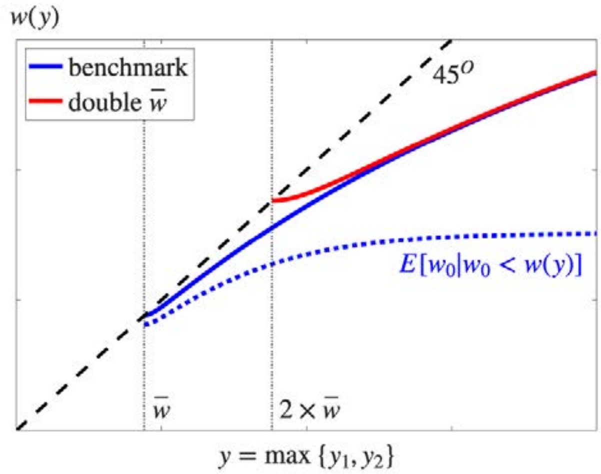
While we only target an average markdown of 16.1 percent, Table 7 reports the pattern for markdowns by wage quartile across the calibrated models. With no minimum wage or informal sector, Columns 3-5, the results are well anticipated by model Figure 2. For the Gaussian copula, markdowns increase moving up the wage distribution, but the model fails to match the sharpness at which the estimated markdowns increase across the wage quartiles, shown in Column 1. It performs much better under a Clayton copula, which allows for higher correlation at low productivities. In particular, the model captures nearly three-quarters of the large differential in markdowns between the 4^{th} and 1^{st} quartiles. The model under a Gumbel copula is directly at odds with the estimates—with markdowns quite flat across the wage quartiles.

Markdown patterns for the extended model are presented in columns 6-8 of Table 7. Across all copulas, the bottom wage quartile now displays lower markdowns, consistent with the model predictions that an informal sector and a minimum wage primarily reduce markdowns for low-wage workers. This effect is most striking under the Gumbel copula—now markdowns rise with wages, though not monotonically. But the calibrated model under a Clayton copula still clearly matches the estimated markdowns across wage quartiles most closely. Therefore, we focus on the extended model under a Clayton copula going forward.

Before considering minimum wage policy, we highlight two other results from the calibrated model. One is that the larger markdowns for high-wage workers imply that monopsony power reduces wage inequality in the formal sector. In the extended model under Clayton, the standard deviation of log-productivity in the formal sector is 0.41 versus 0.31 for log-wages. Therefore, compared to competition with wages equal to productivities, monopsony power reduces (within-firm) wage dispersion in the formal sector by 24 percent.

Second, wage differences between workers' first and second-best formal-sector matches are closely related to markdowns in the calibrated models. For the extended model under Clayton, that difference averages 6, 8, 12, and 19 percent, respectively, for workers in firms' 1^{st} to 4^{th}

Figure 9: Wage Function: Baseline vs. Double Minimum Wage



Notes: The baseline model is calibrated to match moments reported in Table A5. The counterfactual doubles the minimum wage, $\underline{w}$, holding other parameters constant.

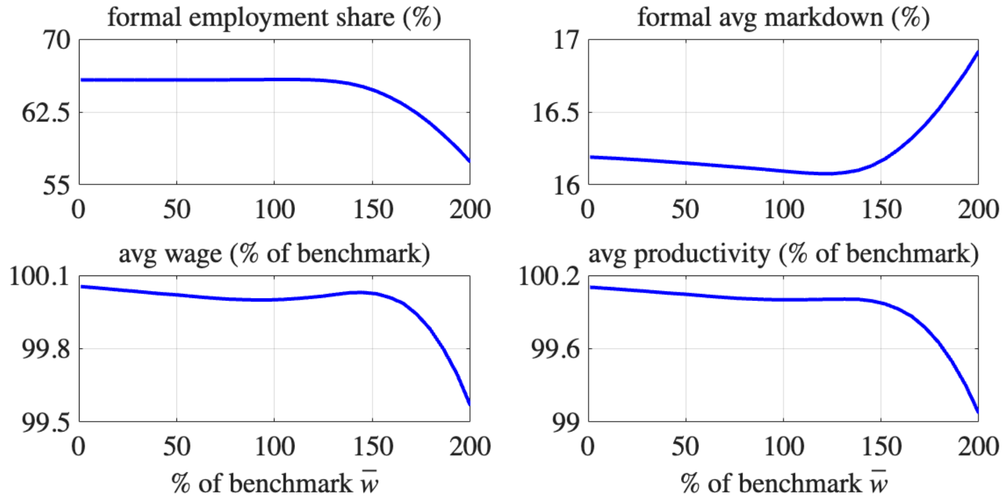
wage quartiles compared to a markdown profile of 7, 11, 17, and 27 percent. (Results for all calibrated models are given in Appendix Table A6.) At the end of Section 4.2 we showed that workers' wage gains from mass hires rose even more sharply by empirical wage quartile. Thus we view that finding as complementary support that comparative advantage and monopsony power are both stronger for employers' high-wage workers.

5.2 The Effect of Minimum Wage Policies

To illustrate the effect of the minimum wage on the labor market, we consider doubling that minimum in our calibrated economy with an informal sector under a Clayton copula. Figure 9 compares formal-sector wages by productivity under minimum wages of $\underline{w}$ and $2\underline{w}$. Workers whose productivities are eclipsed by the minimum wage are forced to the informal sector. The dotted line in Figure 9 shows the counterfactual wage a formal-sector worker would receive if forced to be informal at each productivity $y \geq \underline{w}$. For workers with $\underline{w} \leq max(y_1, y_2) \leq 2\underline{w}$, the minimum wage increase makes this counterfactual wage become factual. The average wage loss for those displaced is given by the distance between the solid and dotted blue lines. The average productivity loss for displaced workers is even larger, reflecting the distance between the 45^o line and the dotted line. Conditional on displacement, wage and productivity losses grow with the minimum wage level.

At the same time, a higher minimum wage raises pay for all formal-sector workers not displaced, especially for productivity levels near the new minimum. That increase is shown by the distance between the solid red and blue lines in Figure 9. Workers with informal wages between the two lines are drawn into the formal sector by the decline in formal-sector mark-

Figure 10: Model Implications: Rising Minimum Wage



Notes: The baseline model is calibrated to match moments reported in Table A5. The counterfactuals vary the minimum wage, $\underline{w}$, from 0 to 200 percent of its benchmark value, holding other parameters constant.

downs. They also exhibit productivity gains transiting to formal that exceed their wage gains.

More generally, Figure 10 presents model outcomes varying the minimum wage from zero to twice its calibrated value. The top left panel shows that the formal share of employment is remarkably stable as long as the minimum wage is below 150 percent of its calibrated value. In that range, the minimum wage is below most workers' informal sector offers and therefore has little bearing on workers' sectoral choices. The competing allocative effects of a minimum wage, which displace low-productivity workers but reinstate productive ones by lowering markdowns, largely offset each other. At higher minimums, the displacement effect dominates.

The upper right panel shows how the average formal-sector markdown responds to the minimum wage. Here again there are two offsetting effects. A higher minimum reduces markdowns for all workers not displaced; but, since displaced workers are subject to lower markdowns, average markdown rises among stayers. The two effects nearly offset each other until the minimum wage rises 25 percent above its calibrated value. Beyond that, the displacement effect dominates, with the average markdown modestly rising.

Finally, the bottom panels show how average wages and productivity vary with the minimum wage if we aggregate workers across the formal and informal sectors. Both are stable for minimum wages up to 150 percent of its calibrated value. But doubling the minimum wage, thereby dropping the formal employment share by about 8 percentage points, reduces average wages by about half a percent and average productivity by about 1 percent.

6 Concluding Remarks

By reframing monopsony power as arising from workers' comparative advantage in their jobs at a specific employer, we offer a new perspective on how employers may influence wages. That shift in perspective leads to a key insight—employers perceive comparative advantage as workers' rents that they can tax via wage markdowns. If higher-wage workers are the ones generating larger rents for the firm, they will face steeper markdowns. We find strong empirical support from Brazil supporting this prediction: within firms, the labor supply is less elastic for higher-wage workers, leading to larger implied markdowns. In fact, we estimate that monopsony has little impact on wages among lower-wage workers. This challenges the notion, sometimes emphasized in the literature, that monopsony effects are predominantly a concern for low-wage workers.

The empirical literature on monopsony power is mushrooming. It will be helpful in reading that literature to have a well-informed prior on where to expect monopsony power. We believe a focus on comparative advantage can help inform such a prior. If one cannot envision an employment match that exhibits comparative advantage, then one should not anticipate that the firm will wield much monopsony power.

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Appendix

A Mathematical Appendix

Theorem 1 (Symmetric Equilibrium) *Consider a symmetric market with K firms. The symmetric oligopsony equilibrium is characterized by the following wage offer policy and employment allocation.*

1. *Wage offer policy*

$$w(y) = \int_0^y x d\tilde{F}(x|y) = \mathbb{E}_{\tilde{F}}(x|x \leq y),$$

where $\tilde{F}(x|y) = \exp[-\int_x^y \sum_{j \neq k} \frac{C_{kj}(F(z),...,F(z))}{C_k(F(z),...,F(z))} F'(z)dz]$ and C_{kj} is the cross-partial derivative of the C w.r.t. $u_k = F_k(y)$ and $u_j = F_j(y)$ for all $j \neq k \in \{1,...,K\}$.

2. *For any $\mathbf{y} = \{y_1,...,y_K\} \in \mathbb{R}^K_+$, $\mathbf{y} \in \pi_k$ if and only if $k = argmax_k\{y_1,...,y_K\}$, and employment at firm k is given by:*

$$n_k(\pi_k) = C_k\left[F(y),\cdots,F(y)\right]F'(y) \quad \forall k = \{1,...,K\}.$$

Proof. Suppose that the wage strategy is strictly increasing in productivity: $w_k'(y_k) > 0$. We will verify that this is the case at the end. Define the inverse wage function $a_k(w) = w_k^{-1}(w)$ for each firm k.

Step 1 – Firm k's problem: The probability that firm k's wage offer beats competition for a worker is equivalent to that worker not being sufficiently efficient to warrant a better wage offer at other firms: $\forall j \neq k,\ w_k(y_k) > w_j(y_j) \Leftrightarrow y_j < a_j(w_k(y_k))$. Using that fact, the conditional probability in equation 2 can be represented as:

$$P(w_j(y_j) \leq w, \forall j \neq k | Y_k = y) = P(y_j \leq a_j(w); \forall j \neq k | Y_k = y)$$

Using the copula notation to represent the probability above, the labor supply faced by firm k for workers with efficiency $Y_k = y$ is:

$$n_k(w;y) = C_k(u_1,\cdots,F_k(y),\cdots,u_K)F_k'(y),$$

where $u_j = F_j(a_j(w))$ for all $j \neq k$ and $C_k()$ is the partial derivative of C copula with respect to u_k. The slope of the labor supply curve is:

$$n_w(w;y) = \sum_{j \neq k} C_{kj}(F_1(a_1(w)),\cdots,F_k(y),\cdots,F_K(a_K(w)))F_j'(a_j(w))a_j'(w)F_k'(y),$$

where C_{kj} denotes the cross-partial derivative of C w.r.t. u_k and u_j. Substituting into the optimality condition for wages (equation (4)) and canceling $F_k'(y)$ gives:

$$(y-w)\sum_{j\neq k} C_{kj}(u_1,\cdots,F_k(y),\cdots,u_K)F_j'(a_j(w))a_j'(w) = C_k(u_1,\cdots,F_k(y),\cdots,u_K), \tag{A1}$$

with $u_j = F_j(a_j(w))$ for all $j \neq k$. Equation (A1) defines a K-dimensional system of first-order, non-linear differential equations. Note that the solution to each equation gives $w_k(y)$, or, equivalently, $y = a_k(w)$. Substituting $F_k(y) = a_k(w)$, and then imposing symmetry, by setting $a_j(w) = a(w)$ and $F_j(y) = F(y)$ for all j, reduces it to a single differential equation, stated in standard notation (which sets $a'(w) = da/dw$) below:

$$(a-w)\sum_{j\neq k} C_{kj}(F(a),\cdots,F(a))F'(a)da - C_k(F(a),\cdots,F(a))dw = 0, \tag{A2}$$

Step 2 – Solving the Differential Equation:

To solve the differential equation above, consider the integrating factor below

$$I(a) = \exp\left(-\int_0^a F'(x)\frac{C_{kk}(F(x),..F(x))}{C_k(F(x),..F(x))}dx\right) \equiv e^{-\gamma(a)},$$

where we introduce $\gamma(a)$ for notational convenience. Multiplying equation (A2) by $I(a)$ gives an exact ODE of the form $\Psi_a(a,w)da + \Psi_w(a,w)dw = 0$ with a generic solution $\Psi(a,w) = d_0$ for any constant of integration $d_0 \in \mathbb{R}$:

$$\underbrace{e^{-\gamma(a)}(a-w)\sum_{j\neq k} C_{kj}(F(a),\cdots,F(a))F'(a)}_{\Psi_a(a,w)}da \overbrace{-e^{-\gamma(a)}C_k(F(a),\cdots,F(a))}^{\Psi_w(a,w)}dw = 0,$$

Exactness can be established by verifying $\Psi_{wa}(a,w) = \Psi_{aw}(a,w)$:

$$\begin{aligned}\Psi_{wa}(a,w) &= e^{-\gamma(a)}\left[\gamma'(a)C_k(F(a),...,F(a)) - F'(a)\sum_{j=1}^{K} C_{kj}(F(a),...,F(a))\right] \\ &= -e^{-\gamma(a)}F'(a)\sum_{j\neq k} C_{kj}(F(a),...,F(a)) = \Psi_{aw}(a,w),\end{aligned}$$

where the second equality sets $\gamma'(a) = F'(a)C_{kk}(F(a),...,F(a))/C_k(F(a),...,F(a))$ using the definition of $\gamma(a)$ above.

The solution can be obtained in two steps. First, integrate $\Psi_w(a,w)$ along w:

$$\Psi(a,w) = \int \Psi_w(a,w)dw = -e^{-\gamma(a)}C_k(F(a),...,F(a))w + d(a), \tag{A3}$$

where $d(a)$ is the constant of integration, which may depend on a. To determine $d(a)$, differentiate the term above w.r.t. a and equate to $\Psi_a(a,w)$ from the ODE:

$$\Psi_a(a,w) = -we^{-\gamma(a)}F'(a)\sum_{j\neq k} C_{kj}(.) + d'(a) = e^{-\gamma(a)}(a-w)\sum_{j\neq k} C_{kj}(.)F'(a)$$

implying:

$$d'(a) = ae^{-\gamma(a)}\sum_{j\neq k} C_{kj}(.)F'(a)$$

Integrating $d'(a)$ and substituting back in equation (A3) gives the solution for $\Psi(a,w)$:

$$\Psi(a,w) = -we^{-\gamma(a)}C_k(F(a),...,F(a)) + \int_0^a xe^{-\gamma(x)}\sum_{j\neq k} C_{kj}(F(x),...,F(x))F'(x)dx = d_0. \tag{A4}$$

To determine d_0, note that it is optimal to offer $w(0) = 0$ when productivity is nil. Therefore, $d_0 = \Psi(0,0) = 0$. Finally, solving for w and setting $a = y$ gives the optimal wage offer policy:

$$w(y) = \int_0^y \frac{\sum_{j\neq k} e^{-\gamma(x)}C_{kj}(F(x),...,F(x))F'(x)}{e^{-\gamma(y)}C_k(F(y),...,F(y))} x dx \tag{A5}$$

Notice that the term in the numerator is the derivative of the term in the denominator, implying:

$$\int_0^y \sum_{j\neq k} e^{-\gamma(x)}C_{kj}(F(x),...,F(x))F'(x)dx = e^{-\gamma(y)}C_k(F(y),...,F(y)).$$

Therefore setting

$$\tilde{F}(x|y) = \frac{e^{-\gamma(x)}C_k(F(x),...,F(x))}{e^{-\gamma(y)}C_k(F(y),...,F(y))}$$

gives the following representation for the optimal wage function:

$$w(y) = \int_0^y x d\tilde{F}(x|y) = \mathbb{E}_{\tilde{F}}[x|x \leq y], \tag{A6}$$

where $\tilde{F}'(x)$ are weights assigned to $x \leq y$ since $\tilde{F}'(x) \geq 0$ for all $x \in [0,y]$ and $\int_0^y \tilde{F}'(x) = 1$.

For the alternative representation of $\tilde{F}(x)$ stated in the proposition, consider the following:

$$
\begin{aligned}
\ln C_k = \int \frac{d}{dy}\ln C_k &= \int \frac{C_{kk}}{C_k}F' + \int \sum_{j\neq k}\frac{C_{kj}}{C_k}F' \\
-\int \frac{C_{kk}}{C_k}F' &= -\ln C_k + \int \sum_{j\neq k}\frac{C_{kj}}{C_k}F' \\
\underbrace{e^{-\int \frac{C_{kk}}{C_k}F'}}_{e^{-\gamma(y)}} &= \frac{e^{\int \sum_{j\neq k}\frac{C_{kj}}{C_k}F'}}{C_k} \\
e^{-\gamma(y)}C_k &= e^{\int \sum_{j\neq k}\frac{C_{kj}}{C_k}F'}
\end{aligned}
$$

Substituting this back in the definition of $\tilde{F}$ gives:

$$
\tilde{F}(x|y) = \frac{e^{\int_0^x \sum_{j\neq k}\frac{C_{kj}(F(z),\ldots,F(z)}{C_k(F(z),\ldots,F(z))}F'(z)dz}}{e^{\int_0^y \sum_{j\neq k}\frac{C_{kj}(F(z),\ldots,F(z)}{C_k(F(z),\ldots,F(z))}F'(z)dz}} = e^{-\int_x^y \sum_{j\neq k}\frac{C_{kj}(F(z),\ldots,F(z)}{C_k(F(z),\ldots,F(z))}F'(z)dz} \tag{A7}
$$

Step 3 – Verify $w'(y) > 0$: To conclude the proof, we next verify that $w'(y) > 0$, which ensures the existence of $y = a(w) = w^{-1}(w)$ with $a'(w) > 0$ (continuity of $w(y)$ is ensured by the continuity of the density function $h(y_1,\ldots,y_k)$.

In equilibrium, $a'(w(y)) = 1/w'(y)$. From the optimality condition, we have:

$$
w'(y) = (y-w)\left(\sum_{j\neq k}\frac{C_{kj}(F(y),\cdots,F(y))}{C_k(F(y),\cdots,F(y))}F'(y)\right).
$$

From equation (A6), $y - w \geq 0$, with strict inequality when $y > 0$. $F'(y) > 0$ by assumption. Also recall that for any $k \in \{1,\ldots,K\}$, $C_k(F(y),\cdots,F(y)) = P(Y_j \leq y, \forall j \neq k | Y_k = y) > 0$ and, for any $j,k \in \{1,\ldots,K\}$ with $j \neq k$, $C_{kj}(F(y),\cdots,F(y)) = P(Y_j = y, Y_l \leq y,\ \forall l \notin \{j,k\} \text{ and } j \neq k | Y_k = y) > 0$, which are both positive. Therefore the derivative, $w'(y)$, is strictly positive for $y > 0$ (and $w'(0) = 0$).

■

Lemma 1 *The joint distribution of wage offers $w(y_k)$ is described by the copula $C^w = C$, and the common marginal distribution $F^w(x) = F(w^{-1}(x))$.*

Proof. Proof follows from the stability of copulas under monotonic transformations and the strict monotonicity of the wage offer function. Formally, suppose X_k are K random variables whose joint distribution is described by copula C^x and marginals $F_k(X_k)$. Then for any set of continuous and strictly monotonic functions $H_k : \mathbb{R} \to \mathbb{R}$,

$$
P(H_k(X_k) \leq h_k) = P(x_k \leq H_k^{-1}(h_k)) = C^x(F_k(H_k^{-1}(h_k)), \quad \forall k = 1,\ldots,K.
$$

Noting that the equilibrium wage offer function $w(y)$ is continuous and strictly increasing implies the lemma. ■

Theorem 3 *The equilibrium distribution of wages in a symmetric oligopsony equilibrium is:*

$$P(w^* \leq \omega) = C(F^w(\omega), \cdots, F^w(\omega)). \tag{5}$$

Proof. For any K dimensional copula C that describes the joint distribution of random variables X_k for $k = 1, ..., K$, the distribution of $\max\{X_k : k = 1, ..., K\}$ is given by its diagonal section. Hence:

$$P(\max_j\{w(y_j)\} \leq \omega) = P(w(y_1) \leq \omega, ...w(y_k) \leq \omega) = C(F^w(\omega), ..., F^w(\omega))$$

■

Example 1 (Independent Copula) *If Y_k is independent across employers, then the optimal wage offer policy is $w(y) = \int_0^y x d\tilde{F}^I(x|y)$ with*

$$\tilde{F}^I(x|y) = \left[\frac{F(x)}{F(y)}\right]^{K-1}.$$

Proof. The independence copula is $C^I = \Pi_k u_k$. At $Y_k = z$ for all k, $u_k = F(z)$ for all k, implying, $C_k = F^{K-1}(z)$ and $C_{kj} = F^{K-2}(z)$ for all k. But then:

$$\int_x^y \sum_{j \neq k} \frac{C_{kj}(F(z), ..., F(z)}{C_k(F(z), ..., F(z))} F'(z) dz = (K-1) \int_x^y \frac{F'(z)}{F(z)} dz = (K-1)[\ln F(y) - \ln F(x)]$$

Substituting this in the definition of $\tilde{F}$ in Theorem 1 gives the result:

$$\tilde{F}(x|y) = e^{-(K-1)[\ln F(y) - \ln F(x)]} = \frac{F^{K-1}(x)}{F^{K-1}(y)} \tag{A8}$$

■

Proposition 1 (Archimedean Copulas) *Let $C^A = \psi^{-1}(\sum_j \psi(u_j))$ be an Archimedean copula. The optimal wage offer policy is $w^A(y) = \int_0^y x d\tilde{F}(x|y)$ with*

$$\tilde{F}(x|y) = \left[\frac{\psi'(C^A(F(x), ..., F(x))}{\psi'(C^A(F(y), ..., F(y))}\right]^{-\frac{K-1}{K}}.$$

Proof. Taking the appropriate derivatives of the equation $\psi(C^A) = \sum_j \psi(u_j)$, the following can be established for Archimedean copulas for any $j \neq k$:

$$\frac{C^A_{kj}}{C^A_k} = \frac{\psi''(C^A)}{\psi'(C^A)} C^A_j,$$

with $C_j^A = \psi'(u_j)/\psi'(C^A)$. Substituting back in the definition of $\tilde{F}$ gives the result:

$$\begin{aligned}
\tilde{F}(x|y) &= \exp\left(-\int_x^y \sum_{j\neq k} \frac{C_{jk}^A(F(z),...,F(z))}{C_k^A(F(z),...,F(z))} F'(z)dz\right) \\
&= \exp\left(-\int_x^y (K-1)\frac{\psi''(C^A(.)}{[\psi'(C^A(.)]^2}\psi'(F(z))dz)\right) \\
&= \exp\left(-\frac{K-1}{K}\ln\psi'(C^A(F(z),...,F(z))|_x^y\right) \\
&= \left[\frac{\psi'(C^A(F(y),...,F(y))}{\psi'(C^A(F(x),...,F(x))}\right]^{\frac{K-1}{K}} \\
&= \left[\frac{\psi'(\psi^{-1}(K\psi(F(y))))}{\psi'(\psi^{-1}(K\psi(F(x))))}\right]^{\frac{K-1}{K}}.
\end{aligned}$$

The last equality follows from the definition of C^A under symmetry. ∎

Example 2 (Gaussian Copula) *Let $K = 2$ and $C = C^G(\rho)$ be the Gaussian copula with correlation $\rho \in [-1,1]$ and normal marginals $F_k(\ln x) = \Phi(\frac{\ln x - \mu}{\sigma})$ for $k \in \{1,2\}$. The wage offer policy in a symmetric duopsony equilibrium is $w^G(y,\rho) = \int_0^y x d\tilde{F}^G(x|y;\rho)$ with*

$$\tilde{F}(x|y;\rho) = \left[\frac{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\ln x-\mu)}{\sigma}\right)}{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\ln y-\mu)}{\sigma}\right)}\right]^{\frac{1}{1-\rho}}.$$

Proof. *Step 1*: Consider the inverse wage functions $y = a_j(w)$ for $j \in \{1,2\}$. The labor supply function is:

$$n_j(w; y_j, a_i(w)) = P(y_i < a_i(w)|y_j)\cdot f(y_j) = \Phi\left(\frac{\log a_i(w) - \mu_{i|j}(y_j)}{\sigma_{i|j}}\right)\cdot f(y_j)$$

where $\mu_{i|j}(y_j) = \mu_i + \beta_{i|j}(\log y_j - \mu_j)$ with $\beta_{i|j} = \sigma_{ij}/\sigma_j^2$, and $f(y_j)$ is the marginal density of y_j. Semi-elasticity of the labor supply is:

$$\frac{n_j'(w; y_j, a_i(w))}{n_j(w; y_j, a_i(w))} = \frac{\phi\left(\frac{\log a_i(w)-\mu_{i|j}(y_j)}{\sigma_{i|j}}\right)}{\Phi\left(\frac{\log a_i(w)-\mu_{i|j}(y_j)}{\sigma_{i|j}}\right)}\frac{a_i'(w)}{\sigma_{i|j}a_i(w)}$$

Recall the first-order condition for firm j:

$$(y_j - w)\frac{n_j'(w; y_j, a_i(w))}{n_j(w; y_j, a_i(w))} = 1$$

Substituting from above gives the following:

$$(y_j - w)\frac{\phi\left(\frac{\log a_i(w) - \mu_{i|j}(y_j)}{\sigma_{i|j}}\right)}{\Phi\left(\frac{\log a_i(w) - \mu_{i|j}(y_j)}{\sigma_{i|j}}\right)}\frac{a_i'(w)}{\sigma_{i|j}a_i(w)} = 1$$

Step 2: The condition above implicitly defines $a_j(w)$, firm j's inverse wage function. Impose equilibrium by setting $y_j = a_j(w)$

$$(a_j(w) - w)\frac{\phi\left(\frac{\log a_i(w) - \mu_{i|j}(a_j(w))}{\sigma_{i|j}}\right)}{\Phi\left(\frac{\log a_i(w) - \mu_{i|j}(a_j(w))}{\sigma_{i|j}}\right)}\frac{a_i'(w)}{\sigma_{i|j}a_i(w)} = 1$$

This is a 2-equation system of ODE's for inverse functions $a_i(w)$ and $a_j(w)$.

Step 3: Impose symmetry. Set $\mu_i = \mu$ and $\sigma_i = \sigma$, and, thereby, $a_i(w) = a(w)$ for $i \in \{1, 2\}$.

Note that with symmetry, the arguments in ϕ and Φ become:

$$\frac{\log a_i(w) - \mu_{i|j}(a_j(w))}{\sigma_{i|j}} = \frac{\log a(w) - \mu - \beta_{i|j}(\log a(w) - \mu)}{\sigma_{i|j}} = \frac{1 - \beta_{i|j}}{\sigma_{i|j}}(\log a(w) - \mu)$$

Substituting in the ODE, denoting $a'(w) = da/dw$ and $a(w) = a$, and rearranging gives a single equation ODE in standard form:

$$(a - w)\phi\left(\frac{1 - \beta_{i|j}}{\sigma_{i|j}}(\log a - \mu)\right)da - \Phi\left(\frac{1 - \beta_{i|j}}{\sigma_{i|j}}(\log a - \mu)\right)\sigma_{i|j}a\,dw = 0 \tag{A9}$$

Multiplying the ODE with $I(a) = \frac{\Phi(.)^{\frac{\beta}{1-\beta}}}{a}$ gives an exact ODE defined by $d\Psi(a, w) = 0$. Ignoring the arguments of ϕ and Φ, and denoting $\beta_{i|j} = \beta$ for simplicity:

$$\underbrace{\frac{(a - w)}{a}\phi\ \Phi^{\frac{\beta}{1-\beta}}}_{\Psi_a}\ da \underbrace{-\Phi^{\frac{1}{1-\beta}}\sigma_{i|j}}_{\Psi_w}\ dw = 0$$

The exactness can be verified by showing $\Psi_{wa} = \Psi_{aw} = -\phi\Phi^{\frac{\beta}{1-\beta}}/a$

Step 4: Solve the ODE.

First, note that:

$$\Psi(w, a) = \int \Psi_w dw = w\Psi_w + h(a)$$

since Ψ_w does not depend on w. $h(a)$ is the constant of integration, which generally depends on a.

Second, differentiate $\Psi(w, a)$ above w.r.t. a and equate it to Ψ_a from the ODE to solve for $h'(a)$:

$$\Psi_a = w\Psi_{wa} + h'(a) = -\frac{w}{a}\phi\ \Phi^{\frac{\beta}{1-\beta}} + h'(a) = \frac{(a - w)}{a}\phi\ \Phi^{\frac{\beta}{1-\beta}}$$

This gives:

$$h'(a) = \phi\ \Phi^{\frac{\beta}{1-\beta}},$$

which implies

$$h(a) = \int_0^a \phi\left(\frac{1-\beta}{\sigma_{i|j}}(\log x - \mu)\right)\Phi^{\frac{\beta}{1-\beta}}\left(\frac{1-\beta}{\sigma_{i|j}}(\log x - \mu)\right)dx + h_0,$$

where h_0 is the constant of integration.

Substituting $h(a)$ back in $\Psi(a,w)$ along with Ψ_w from the ODE gives:

$$-\Phi^{\frac{1}{1-\beta}}\sigma_{i|j}w + \int_0^a \phi\left(\frac{1-\beta}{\sigma_{i|j}}(\log x - \mu)\right)\Phi^{\frac{\beta}{1-\beta}}\left(\frac{1-\beta}{\sigma_{i|j}}(\log x - \mu)\right)dx + h_0 = 0$$

Now, we can isolate the wage above:

$$w(a) = \frac{\int_0^a \phi\left(\frac{1-\beta}{\sigma_{i|j}}(\log x - \mu)\right)\Phi^{\frac{\beta}{1-\beta}}\left(\frac{1-\beta}{\sigma_{i|j}}(\log x - \mu)\right)dx + h_0}{\Phi^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{\sigma_{i|j}}(\log a - \mu)\right)\sigma_{i|j}} \tag{A10}$$

$$= \frac{\int_0^a x d\Phi^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{\sigma_{i|j}}(\log x - \mu)\right) + h_0}{\Phi^{\frac{1}{1-\beta}}\left(\frac{1-\beta}{\sigma_{i|j}}(\log a - \mu)\right)} \tag{A11}$$

Note that $w(0) = 0$ implies $h_0 = 0$, $\beta_{1|2} = \rho\sigma_1/\sigma_2 = \rho$ by symmetry, and

$$\frac{1-\beta_{1|2}}{\sigma_{1|2}} = \frac{1-\rho\sigma_1/\sigma_2}{\sigma_1\sqrt{1-\rho^2}} = \frac{1-\rho}{\sigma\sqrt{1-\rho^2}} = \frac{1}{\sigma}\sqrt{\frac{1-\rho}{1+\rho}}.$$

Substituting back into the wage function gives:

$$w(a) = \int_0^a x d\left[\frac{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\log x - \mu)}{\sigma}\right)}{\Phi\left(\sqrt{\frac{1-\rho}{1+\rho}}\frac{(\log a - \mu)}{\sigma}\right)}\right]^{\frac{1}{1-\rho}} = \int_0^y x d\tilde{F}(x|y),$$

where we have set $a = y$ by definition. ■

Theorem 4 *Let $G(y) = \int_0^y \tilde{F}(x|y)dx$. Then for any $y > 0$, $1 - w(y)/y$ is increasing in y if and only if the elasticity of $G(y)$ with respect to y exceeds 1.*

Proof. Applying integration by parts to the definition of equilibrium wage policy gives:

$$w(y) = \int_0^y x d\tilde{F}(x|y) = y - \int_0^y \tilde{F}(x|y)dx,$$

since $\tilde{F}(y|y) = 1$. Solving for markdown, we have:

$$1 - \frac{w(y)}{y} = \frac{\int_0^y \tilde{F}(x|y)dx}{y} = \frac{G(y)}{y},$$

which increases with y if and only if $G'(y)y - G(y) > 0$, or, equivalently:

$$\frac{G'(y)y}{G(y)} > 1$$

■

Before we begin the proofs for Theorems 5 - 7, it is worth noting a few implications of APD dominance. First, note that APD dominance implies first-order stochastic dominance.

Remark 3 $F_A \succ_{APD} F_B \implies F_A \succ_{FOSD} F_B$

This can be seen by setting $y = y^{max}$ in the definition of APD dominance, which gives $F_A \succ_{APD} F_B \implies \psi(F_A(x)) > \psi(F_B(x))$ for all $x < y^{max}$, which then implies $F_A(x) < F_B(x)$ for all $x < y^{max}$ since ψ is strictly decreasing in its argument. It is easy to see that FOSD does not necessarily imply APD, which makes APD a stronger condition.

Another useful implication is the following:

Remark 4 $F_A \succ_{APD} F_B \iff \psi'(F_A(x))F_A'(x) < \psi'(F_B(x))F_B'(x)$

By definition, APD implies $\psi(F_A(x)) - \psi(F_B(x))$ is decreasing in x. The result is obtained by differentiation.

When the two distributions are independent, APD is equivalent to a monotone probability ratio order, also known as an increasing reverse hazard ratio.

Remark 5 *If F_A and F_B are independent, then $F_A \succ_{APD} F_B \iff F_A(x)/F_B(x)$ is increasing in x.*

For the independent copula, $\psi(x) = -\ln x$. Substituting that in the definition of APD gives the result.

Theorem 5 *Suppose $C(u,v) \in \mathscr{C}^A$ and $F_2 \succ_{APD} F_1$. Then, (i) for all $w > 0$: $F_1(y_1(w)) > F_2(y_2(w))$, and (ii) for all $y > 0$: $w_1(y) > w_2(y)$.*

Proof.

Part *(i)*:

The proof is established by contradiction. Denoting $u(w) = F_1(y_1(w))$ and $v(w) = F_2(y_2(w))$, the ratio of the optimality conditions for the two firms gives the following:

$$\frac{\psi'(v)}{\psi'(u)}\frac{dv}{du} = \frac{y_2(w) - w}{y_1(w) - w},$$

This is a differential system with boundary values $v(0) = u(0) = 0$ and $v(\bar{w}) = u(\bar{w}) = 1$ at some $\bar{w} > 0$ (Lebrun, 1999).

Suppose $u(w) \leq v(w)$ were true for some $w \in (0, \bar{w})$ in equilibrium. This implies $y_1(w) < y_2(w)$, because $F_1(y) > F_2(y)$ and $F_1'(y) > 0$ by assumption. Furthermore, by convexity of ψ, and the fact that $\psi' < 0$, we have $0 > \psi'(v) \geq \psi'(u)$, implying $0 < \psi'(v)/\psi'(u) \leq 1$.

The relations above together imply $\frac{dv}{du} = \frac{y_2(w)-w}{y_1(w)-w} \cdot \frac{\psi'(u)}{\psi'(v)} > 1$ whenever $v(w) \geq u(w)$ from the equation above. But that reinforces $v > u$ for all wage levels above w, which contradicts $v = u = 1$ at $\bar{w}$. Therefore, $u(w) > v(w)$ must hold in equilibrium for all $w \in (0, \bar{w})$. This implies employment at Firm 2 is larger than that at Firm 1 at any $w \in (0, \bar{w})$. Visually, this is implied by the equilibrium locus lying below the 45-degree line and the symmetry of Archimedean copulas.

Part *(ii)*

The proof is established by contradiction, but first note that if ever $y_1(w) = y_2(w)$, i.e. the two wage strategies cross, then:

$$\frac{\psi'(F_2(y))F_2'(y)y_2'(w)}{\psi'(F_1(y))F_1'(y)y_1'(w)} = 1$$

from the ratio of FOC's of the two firms. Rearranging implies:

$$\frac{y_2'(w)}{y_1'(w)} = \frac{\psi'(F_1(y))F_1'(y)}{\psi'(F_2(y))F_2'(y)} < 1$$

by assumption of the proposition (recall $\psi' < 0$). Because $y_j'(w) = 1/w_j'(y)$, we have $w_2'(y) > w_1'(y)$ whenever $y_1(w) = y_2(w)$.

Suppose now, contrary to the claim, that $w_2(z) \geq w_1(z)$ for some $z \in (w_0, y^{max})$. Then, for all $z' > z$, we have $w_2(z') > w_1(z')$, especially for $z \approx y^{max}$, where $\approx$ means arbitrarily close, because $w_2'(z) > w_1'(z)$ as we established above.

In terms of inverse wage strategies, $w_2(z') > w_1(z')$ is equivalent to $y_2(w) < y_1(w)$ for $w \approx \bar{w}$. But then we have:

$$v(w) = F_2(y_2(w)) < F_2(y_1(w)) < F_1(y_1(w)) = u(w)$$

where the first inequality follows from $F_2'(y) > 0$ by definition, and the second inequality from FOSD, $F_2(y) < F_1(y)$, which is implied by the assumption of the proposition.

Because $u(\bar{w}) = v(\bar{w}) = 1$, we must have $v'(w) > u'(w)$ for $w \approx \bar{w}$ if $v(w) < u(w)$. In other words $v(w)$ has to rise faster to catch up with $u(w)$.

But if $v'(w) > u'(w)$ and $u(w) > v(w)$, then from the ratio of FOC's, we must have:

$$\frac{y_2(w) - w}{y_1(w) - w} = \frac{\psi'(v(w))v'(w)}{\psi'(u(w))u'(w)} > 1,$$

implying $y_2(w) > y_1(w)$, or, equivalently, $w_1(y) > w_2(y)$ which contradicts the supposition that $w_2(z) \geq w_1(z)$.

■

Theorem 6 *If $C(u,v) \in \mathscr{C}^A$ and Assumption 1 holds, then there is a unique $w^* \geq 0$ such that $F_1(y_1(w)) \geq F_2(y_2(w))$ if and only if $w \geq w^*$ in equilibrium.*

Proof. Define $\tilde{w}$ where $y_2(\tilde{w}) = \tilde{y}$ in equilibrium. Following the same steps as in the proof of Proposition 5, it can be shown that $u(w) > v(w)$ for all $w \in (\tilde{w}, \bar{w})$, because $F_1(y) > F_2(y)$ for all $y > \tilde{y}$. Therefore, if there is a w^* such that $u(w^*) = v(w^*)$, $w^* \leq \tilde{w}$ must hold.

It is possible that $w^* = 0$ is the only crossing point with $u(w) > v(w)$ for all $w > 0$. In that case, the equilibrium is similar to that described by Theorem 5. Next, we show that there is a single *interior* crossing point: that if there is a $w^* > 0$, it must be the only such point. We will do so by showing that for all $w \in [w^*, \tilde{w}]$, $dv/du < 1$. That is, the slope at the crossing point is less than one and for all wage levels above that, the differential system reinforces $v(w) < u(w)$ until $\tilde{w}$. Since we already established $u(w) > v(w)$ for all $w \in (\tilde{w}, \bar{w})$, w^* must be the only interior crossing point.

For any $w < \tilde{w}$, suppose $u(w) \geq v(w)$. Then $0 > \psi'(u) > \psi'(v)$ by strict convexity of ψ. Because $y_2(w) < \tilde{y}$ when $w < \tilde{w}$ by definition, $F_2(y) > F_1(y)$ when $y < \tilde{y}$ by assumption, and $u(w) \geq v(w)$ by supposition, $F_1(y_1(w)) > F_2(y_2(w)) > F_1(y_2(w))$ must hold for all $w < \tilde{w}$, which implies $y_1(w) > y_2(w)$. These inequalities imply:

$$\frac{dv}{du} = \underbrace{\frac{\psi'(u)}{\psi'(v)}}_{<1} \underbrace{\frac{y_2(w) - w}{y_1(w) - w}}_{<1} < 1.$$

The inequality above implies that $v(w)$ remains below $u(w)$ in equilibrium, for all wage levels $w \geq w^* > 0$. Equivalently, if the equilibrium equal-pay locus $(v(w), u(w))$ ever touches the 45-degree line, it remains strictly below that line and does not cross it again (that is, of course, until the terminal point: $v(\bar{w}) = u(\bar{w}) = 1$). ■

Theorem 7 *If $C(u,v) \in \mathscr{C}^A$ and Assumptions 1 and 2 are true, then $\exists y^* = y_1(w) = y_2(w) \in (\tilde{y}, \hat{y})$ for some w, such that $w_1(y) > w_2(y)$ if and only if $y < y^*$.*

Proof. *Step 1*: We first show that if ever $y_1(w) = y_2(w) = y$ for some w, then $y_2'(w) > y_1'(w)$ if $y < \hat{y}$ and $y_1'(w) > y_2'(w)$ otherwise.

Since $y_1(w) = y_2(w) = y$, we have, from the ratio of FOC's for the two firms,

$$\frac{\psi'(F_2(y))F_2'(y)y_2'(w)}{\psi'(F_1(y))F_1'(y)y_1'(w)} = \frac{y-w}{y-w} = 1.$$

Rearranging terms gives:

$$\frac{y_2'(w)}{y_1'(w)} = \frac{\psi'(F_1(y))F_1'(y)}{\psi'(F_2(y))F_2'(y)} < 1,$$

which is strictly less than 1 if and only if $y > \hat{y}$, and exceeds 1 if $y < \hat{y}$ by Assumption 2 and Remark 4.

Step 2: $\forall y > \hat{y},\ w_1(y) > w_2(y)$.

Because $y_j'(w) = 1/w_j'(y)$, we have $w_2'(y) > w_1'(y)$ whenever $y_1(w) = y_2(w) = y > \hat{y}$. Suppose, contrary to the claim, that $w_2(z) \geq w_1(z)$ for some $z \in (\hat{y}, y^{max})$. Then, for all $z' > z$, we have $w_2(z') > w_1(z')$, especially for $z \approx y^{max}$, where $\approx$ means arbitrarily close, because $w_2'(z) > w_1'(z)$ as we established above.

In terms of inverse wage strategies, $w_2(z') > w_1(z')$ is equivalent to $y_2(w) < y_1(w)$ for $w \approx \bar{w}$. But then we have:

$$v(w) = F_2(y_2(w)) < F_2(y_1(w)) < F_1(y_1(w)) = u(w)$$

where the first inequality follows from $F_2'(y) > 0$ by definition, and the second inequality from Assumption 1. Because $u(\bar{w}) = v(\bar{w}) = 1$, we must have $v'(w) > u'(w)$ for $w \approx \bar{w}$ if $v(w) < u(w)$. In other words $v(w)$ has to rise faster to catch up with $u(w)$.

But if $v'(w) > u'(w)$ and $u(w) > v(w)$, then from the ratio of FOC's, we must have:

$$\frac{y_2(w) - w}{y_1(w) - w} = \frac{\psi'(v(w))v'(w)}{\psi'(u(w))u'(w)} > 1,$$

implying $y_2(w) > y_1(w)$, or, equivalently, $w_1(y) > w_2(y)$ which contradicts the supposition that $w_2(z) \geq w_1(z)$.

This implies $y < \hat{y}$ and $y_2'(w) > y_1'(w)$ must hold whenever inverse wage strategies cross. That implies they cross at most once and, if they do, $y_2(w)$ crosses $y_1(w)$ from below, i.e., $y_2(w) < y_1(w)$ for all $z < w$ and $y_2(w) \geq y_1(w)$ otherwise.

Step 3: $y > \tilde{y}$.

Recall from Theorem 7 that $F_1(y_1(w) > F_2(y_2(w))$ for all $w > w^*$ with $y_2(w^*) < \tilde{y}$. Because $F_1(y) \leq F_2(y)$ for all $y \leq \tilde{y}$ by Assumption 1, we must have $y_1(w) > y_2(w)$ for all w such that $\tilde{y} \geq y_1(w)$. Because $y_2(w)$ crosses $y_1(w)$ once from below, that crossing must happen above $\tilde{y}$. ■

Theorem 8 *Consider a symmetric market with K firms and a legal minimum wage $\underline{w}$. For all $y \geq \underline{w}$, the wage offer policy in an oligopsony equilibrium is given by:*

$$w(y) = w^*(y) + [\underline{w} - w^*(\underline{w})]\zeta(y),$$

where $w^*(y)$ denotes the wage offer in the absence of a minimum wage defined in Theorem 1 and $\zeta(y) \leq 1$ is a strictly decreasing function with $\zeta(\underline{w}) = 1$.

Proof. Recall from equation (A4) that the differential equation that describes wage offers in a symmetric oligopsony equilibrium is:

$$\Psi(a, w) = -we^{-\gamma(a)}C_k(F(a), ..., F(a)) + \int_0^a xe^{-\gamma(x)} \sum_{j \neq k} C_{kj}(F(x), ..., F(x))F'(x)dx = d_0.$$

with the initial condition $w(\underline{w}) = \underline{w}$ when there is a minimum wage.[A1] Using that condition to solve the constant of integration, d_0, we have:

$$
\begin{aligned}
\Psi(\underline{w},\underline{w}) = -\underline{w}e^{-\gamma(\underline{w})}C_k(F(\underline{w}),...,F(\underline{w})) + \int_0^{\underline{w}} xe^{-\gamma(x)}\sum_{j\neq k} C_{kj}(F(x),...,F(x))F'(x)dx &= d_0\\
e^{-\gamma(\underline{w})}C_k(F(\underline{w}),...,F(\underline{w}))\left[-\underline{w} + \frac{\int_0^{\underline{w}} xe^{-\gamma(x)}\sum_{j\neq k} C_{kj}(F(x),...,F(x))F'(x)dx}{e^{-\gamma(\underline{w})}C_k(F(\underline{w}),...,F(\underline{w}))}\right] &= d_0\\
e^{-\gamma(\underline{w})}C_k(F(\underline{w}),...,F(\underline{w}))\left[-\underline{w} + \mathbb{E}_{\tilde{F}}[x|x\le \underline{w}]\right] &= d_0.
\end{aligned}
$$

Solving for the wage function from equation (A4) and substituting for d_0 from above gives the solution.

$$
\begin{aligned}
w(y) &= \mathbb{E}_{\tilde{F}}[x|x\le y] - \frac{d_0}{e^{-\gamma(y)}C_k(F(y),...,F(y))}\\
w(y) &= \mathbb{E}_{\tilde{F}}[x|x\le y] + \left[\underline{w} - \mathbb{E}_{\tilde{F}}[x|x\le \underline{w}]\right]\underbrace{\frac{e^{-\gamma(\underline{w})}C_k(F(\underline{w}),...,F(\underline{w}))}{e^{-\gamma(y)}C_k(F(y),...,F(y))}}_{\equiv \zeta(y)}\\
w(y) &= w^*(y) + [\underline{w} - w^*(\underline{w})]\zeta(y),
\end{aligned}
$$

for all $y \ge \underline{w}$, where $w^*(y)$ is the equilibrium wage offer in the absence of minimum wage, i.e. when $\underline{w} = 0$, from Theorem 1. It is easy to see $\zeta(\underline{w}) = 1$. Note also that $\zeta'(y) < 0$, because the numerator term in $\zeta(y)$ is fixed and the denominator is strictly increasing in y:

$$
\begin{aligned}
\frac{de^{-\gamma(y)}C_k(F(y),...,F(y))}{dy} &= e^{-\gamma(y)}C_k(F(y),...,F(y))\left[-\gamma'(y) + \frac{\sum_j C_{kj}(F(y),...,F(y))F'(y)}{C_k(F(y),...,F(y))}\right]\\
&= e^{-\gamma(y)}C_k(F(y),...,F(y))\left[-\frac{C_{kk}(F(y),...,F(y))F'(y)}{C_k(F(y),...,F(y))}\right.\\
&\qquad \left. + \frac{\sum_j C_{kj}(F(y),...,F(y))F'(y)}{C_k(F(y),...,F(y))}\right]\\
&= e^{-\gamma(y)}\left[\sum_{j\neq k} C_{kj}(F(y),...,F(y))F'(y)\right] > 0,
\end{aligned}
$$

since $C_{kj} > 0$ for all $j \neq k$. ∎

[A1] There is no bunching at $\underline{w}$ in equilibrium, because competitors can bid away infra-marginal workers, those with $y > \underline{w}$ and $w(y) = \underline{w}$, by offering marginally more than $\underline{w}$.

Theorem 9 *Consider a symmetric market with K formal firms, an informal sector and a legal minimum wage* $\underline{w}$*. The wage offer policy in an oligopsony equilibrium solves the following differential equation:*

$$\frac{dw}{dy} = \frac{\sum_{j\neq k} C_{kj}(F(y),\cdots,F(y))F'(y)}{C_k(F(y),\cdots,F(y))}\left[\frac{1}{(y-w)} - \frac{F_I'(w)}{F_I(w)}\right]^{-1}, \tag{10}$$

with the initial condition $w(\underline{w}) = \underline{w}$*.*

Proof. Recall from equation (4) that the optimality condition for firm k's wage offer is

$$(y-w)\frac{\partial n_k(w;y)}{\partial w} = n_k(w;y),$$

and from equation (9) that the labor supply to a firm is given by:

$$\begin{aligned}[n_k(w;y) &= P(w_j(y_j) \leq w, \forall j \neq k | Y_k = y) \cdot P(w_0 \leq w) \cdot f_k(y) \\ &= C_k(u_1(w),\cdots,F_k(y),\cdots,u_K(w)) \cdot F_I(w) \cdot F_k'(y),\end{aligned}$$

where $u_j(w) = F_j(y_j(w))$ and $y_j(w)$ are inverse wage offer functions for $j \neq k$. Differentiating with respect to w and dividing by labor supply gives:

$$\frac{\partial n_k(w;y)}{\partial w}\frac{1}{n_k(w;y)} = \frac{\sum_{j\neq k} C_{kj}(u_1(w),\cdots,F_k(y),\cdots,u_K(w))F_j'(y_j(w))}{C_k(u_1(w),\cdots,F_k(y),\cdots,u_K(w))}\frac{dy_j(w)}{dw} + \frac{F_I'(w)}{F_I(w)}$$

Setting $y_j(w) = y(w)$ by symmetry and substituting in the optimality condition gives:

$$(y-w)\left(\frac{\sum_{j\neq k} C_{kj}(F(y),\cdots,F(y))F'(y)}{C_k(F(y),\cdots,F(y))}\frac{dy}{dw} + \frac{F_I'(w)}{F_I(w)}\right) = 1.$$

Solving for dy/dw, we have:

$$\frac{dy}{dw} = \frac{C_k(F(y),\cdots,F(y))}{\sum_{j\neq k} C_{kj}(F(y),\cdots,F(y))F'(y)}\left[\frac{1}{(y-w)} - \frac{F_I'(w)}{F_I(w)}\right].$$

Inverting the ODE gives:

$$\frac{dw}{dy} = \frac{\sum_{j\neq k} C_{kj}(F(y),\cdots,F(y))F'(y)}{C_k(F(y),\cdots,F(y))}\left[\frac{1}{(y-w)} - \frac{F_I'(w)}{F_I(w)}\right]^{-1}.$$

■

B Empirical Appendix

B.1 Regression Robustness

Robustness Check for Instrumenting with Industry Growth. Table A1 reports two robustness regressions for the industry growth instrument (IV2) in Table 4. First, we exclude those industries where more than 10 percent of hires are from within the same industry. This is to avoid conflating the demand effect from industry-wide growth with more competition to find workers, which may result in a backward shift of supply facing an employer. To calculate industries' internal hiring rates, we start from matches that satisfy all restrictions in Table 1 except the last, "stayer with consecutive wages," while restricting to workers hired that year. Hires are classified as internal if the worker held a formal-sector job (satisfying Panel A restrictions in Table 1) ending in the same or prior year as hired, with the prior employer of the same 7-digit industry as the new. Those with no such prior formal-sector employment are implicitly defined as external-industry hires. (This contrasts with the sample behind Table 2, which is restricted to hires from the formal sector.) Excluding industries with more than 10 percent internal hiring rates drops about 16 percent of the industries, accounting for about 50 percent of the regression sample.

Second, to avoid confounding changes in industry-level labor supply, we replace the instrumental variable industry employment growth with its growth predicted by business cycles and the industry's average cyclicality. To calculate cyclicality, we start from an industry's total December employment for each year of the 2006-2018 RAIS data, then project each industry's annual employment growth rate on Brazil's nationwide annual real GDP growth rate (from the World Bank (https://data360.worldbank.org/en/indicator/WB_WDI_NY_GDP_MKTP_KD_ZG). We take an industry's slope estimate as its cyclicality; we use the predicted cyclical value for each industry's employment as an instrumental variable.

Our main estimation results are robust in both cases. In particular, the estimated supply elasticity remains much lower for employers' higher-wage workers, while within-employer wage dispersion remains large. The standard errors are somewhat larger, potentially reflecting weaker instruments—the Kleibergen and Paap (2006) Wald rk F-statistics are 39.8 and 20.6, respectively.

Robustness to Dropping Workers near Minimum Wage. Table A2 shows robustness of our benchmark regression, IV3 in Table 4, to further restricting the sample to workers with wages that are at least 120 percent of the legal minimum. This should largely eliminate any impact of minimum wages on the estimated labor supply elasticity at the bottom of employers' wage distributions. (Note that the cutoffs for the wage quartiles are all now higher.) We continue to

Table A1: Robustness Check for Instrumenting with Industry Growth

IV: industry's growth, excluding industries with > 10 percent internal hires

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
1^{st}	0.07 (0.02)	1.26 (0.15)	16.9 (5.6)	5.6 (1.7)
2^{nd}	0.10 (0.03)	1.00 (0.11)	10.2 (3.2)	8.9 (2.6)
3^{rd}	0.13 (0.03)	0.82 (0.02)	6.2 (1.3)	13.8 (2.4)
4^{th}	0.17 (0.03)	0.72 (0.03)	4.3 (0.9)	18.8 (3.0)

IV: industry's exposure to business cycle due to cyclicality

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
1^{st}	0.04 (0.05)	1.90 (0.27)	45.1 (53.2)	2.2 (2.5)
2^{nd}	0.17 (0.09)	0.70 (0.15)	4.1 (2.4)	19.6 (9.3)
3^{rd}	0.26 (0.09)	0.63 (0.07)	2.4 (0.9)	29.3 (7.8)
4^{th}	0.36 (0.12)	0.52 (0.14)	1.4 (0.6)	40.9 (10.4)

Notes: Standard errors in parentheses, clustered at the employer-by-year level.

Table A2: Robustness Check for Wage $\geq 1.2\times$ Minimum Wage

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
1^{st}	0.04 (0.01)	1.49 (0.04)	36.9 (13.0)	2.6 (0.9)
2^{nd}	0.14 (0.02)	0.88 (0.02)	6.3 (0.7)	13.6 (1.3)
3^{rd}	0.19 (0.02)	0.68 (0.03)	3.6 (0.4)	21.5 (2.0)
4^{th}	0.23 (0.03)	0.58 (0.03)	2.5 (0.3)	28.3 (2.6)

Notes: Standard errors in parentheses, clustered at the employer-by-year level.

Table A3: Robustness Check for Residualized Wages

quartile (q)	wage response ($\widehat{\beta}_q$)	emp response ($\widehat{\gamma}_q$)	elasticity ($\widehat{\eta}_q$)	markdown (%)
1^{st}	0.06 (0.02)	1.38 (0.04)	21.3 (5.2)	4.5 (1.0)
2^{nd}	0.13 (0.02)	0.94 (0.02)	7.0 (1.1)	12.5 (1.7)
3^{rd}	0.18 (0.02)	0.77 (0.03)	4.2 (0.5)	19.1 (1.8)
4^{th}	0.22 (0.03)	0.62 (0.03)	2.8 (0.4)	26.4 (2.6)

Notes: Standard errors in parentheses, clustered at the employer-by-year level.

Table A4: Wages vs. Residualized Wages by Wage Quartiles

group by wages			group by res. wages	
quartile	avg. wage	avg. res. wage	quartile	avg. res. wage
1^{st}	1.00	1.00	1^{st}	1.00
2^{nd}	1.30	1.26	2^{nd}	1.29
3^{rd}	1.63	1.54	3^{rd}	1.59
4^{th}	2.54	2.25	4^{th}	2.40

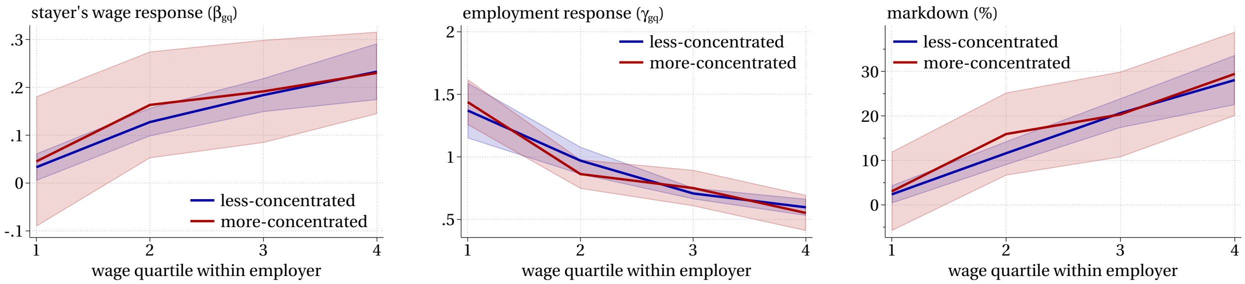
Notes: The correlation coefficient between the original and residualized wages is 0.899.

see much higher implied markdowns for firms' higher wage workers, with that slope as steep as the slope for the fuller sample.

Robustness Check for Residualized Wages. Table A3 repeats our benchmark regression but re-constructs the wage and its quartiles for each employer after first residualizing the log wage with respect to workers' education. One might view controlling for those characteristics as acting to better isolate the match comparative advantage. However, comparative advantage can also be associated with education, so the residualization also removes variations in comparative advantage. Table A3 shows that high-wage-residual workers within a firm face a much less elastic supply, implying they face higher markdowns. The markdown variation across wage quartiles is similar to the benchmark. This mainly reflects that, in our sample, education only accounts for a small amount of wage variation within employers. Table A4 reports the wage gaps between wage quartiles. The first three columns show that the wage gaps among our benchmark wage quartiles only shrink by about 10 percent after residualizing by education. Consequently, the residualized-wage gap is similar to the benchmark when re-grouping the wage quartiles by residualized wages, as shown in the last two columns.

Markdown Heterogeneity by Market's Concentration Ratio To supplement Section 4.3's analysis, we examine the variation in labor supply elasticities across more- vs. less-concentrated labor markets. Specifically, we consider the same specification as there, but now divide employers by their markets' Herfindahl-Hirschman Index in employment shares. We define the more-concentrated markets as those with the highest Herfindahl-Hirchman Index, with its cutoff chosen so that 5 percent of total employment is classified as more-concentrated. We separately present the estimated wage responses, employment responses, and the implied markdown in the three panels of Figure A1. Similar to our findings for the large vs. small employers in Panel (a) of Figure 8, we do not find significant differences along the dimension of market concentration in the Brazilian data.

Figure A1: Markdown Heterogeneity by Market's Concentration Ratio



Notes: In each panel, the red and blue lines depict respective estimates for employers in the more- and less concentrated markets. The shaded areas represent the estimates' 95 percent confidence intervals.

B.2 Additional Calibration Tables

Calibrated Parameters. Table A5 reports the calibrated parameters for Section 5.1. The first three columns consider a simple symmetric duopsony model with no informal sector and no minimum wage; the last three columns extend the model for a legal minimum wage, $\underline{w}$, and an informal sector. In each case, we entertain three versions corresponding to joint productivities that reflect a Gaussian, Clayton, or Gumbel copula, each with log-normal marginal distributions. For the simple model, we calibrate the standard deviation for the log-normal marginal distribution, σ, and the copula's dependence parameter; parameters are calibrated to match the first two moments in Table 6: the average standard deviation of log wages within a firm and the average markdown in the formal sector. For the extended model, we additionally calibrate the minimum wage, $\underline{w}$, and the log-normal parameters for the informal sector productivity distribution, μ_0 and σ_0. To do so, we match three additional moments in Table 6: the ratio of minimum wage relative to the average formal-sector wage, the employment share of the formal sector, and the wage gap between the informal and formal sector.

Implied Wage Comparative Advantages. Table A6 reports the model counterparts to the workers' wage-measured comparative advantage, empirically approximated by the wage gains for mass hires reported in Table 5 at the end of Section 4.2. For each calibrated model, we calculate the average $\ln w_1 - \ln w_2$ for each wage quartile of Firm 1 workers whose best outside option is w_2 instead of w_0; the last condition reflects that we only observe wage gains for those hired from the formal sector in the data. We find that, under the simple model without the informal sector and minimum wage, the Gumbel copula is unable to fit the qualitative pattern that workers' wage comparative advantage aligns with their absolute advantage. While the

Table A5: Calibrated Parameters

parameter and description		No informal or $\underline{w}$			With informal and $\underline{w}$		
		Gaussian	Clayton	Gumbel	Gaussian	Clayton	Gumbel
σ	formal distr. std. dev.	0.36	0.39	0.35	0.53	0.55	0.52
$\rho/\theta/\alpha$	formal distr. dependence	0.90	4.02	3.03	0.94	8.72	3.09
	(Kendall's τ)	(0.71)	(0.67)	(0.67)	(0.78)	(0.81)	(0.68)
$\underline{w}$	minimum wage				0.44	0.44	0.44
μ_0	informal distr. mean				-0.28	-0.31	-0.23
σ_0	informal distr. std. dev.				0.19	0.23	0.15

Notes: For each copula under the case without informal sector or minimum-wage ($\underline{w}$), parameters are calibrated to match the first two moments in Table 6. We match all five moments in the table for each copula under the case with the informal sector and minimum wage.

Table A6: Implied Wage Comparative Advantages (%)

quartile (q)	Data: mass-hire		No informal or $\underline{w}$			With informal and $\underline{w}$		
	50 percent	80 percent	Gaussian	Clayton	Gumbel	Gaussian	Clayton	Gumbel
1^{st}	5.6	4.3	10.7	5.8	12.9	8.4	5.5	7.6
2^{nd}	17.4	16.9	11.2	8.7	14.3	10.4	8.2	10.2
3^{rd}	24.8	24.8	12.2	11.6	14.2	12.5	11.9	14.1
4^{th}	37.5	39.8	13.2	17.6	13.0	15.0	19.3	17.1

Notes: The data numbers (first two columns) are the same as those reported in Table 5. For each copula under the case without informal sector or minimum-wage ($\underline{w}$), parameters are calibrated to match the first two moments in Table 6. We match all five moments in the table for each copula under the case with the informal sector and minimum wage.

other cases are qualitatively consistent with the data, the model's wage gain differences across quartiles are much smaller than in the data from mass hires. Using the Clayton copula gives the largest differences, and hence those closest to the data. We want to emphasize that the wage gain upon mass hire is likely a biased proxy for wage comparative advantage. We do not use it as a quantitative target for estimation, but as a qualitative support for our theory.