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Asymmetric Information, Two-Way Learning, and the Fed Information Effect*

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Abstract

How important is the information effect of monetary policy? We first show analytically that the reduced-form method of regressing forecast revisions on monetary policy surprises leads to a biased estimation, due to the correlation between monetary policy surprises and the unobserved shocks. We then develop a New Keynesian model in which asymmetric information originates from a two-way learning mechanism: the central bank learns from lagged aggregate inflation and output, and firms learn from individual marginal costs and the interest rate. We calibrate our model parameters to match macroeconomic dynamics in the US and the forecast accuracy of the Federal Reserve and professional forecasters. Our calibrated model shows that the information effect reduces the output gaps caused by demand shocks and noise shocks, but may lead to a temporary rise in inflation after a contractionary monetary policy shock.

Keywords: information frictions, expectations, monetary policy, central bank communications

JEL classification: E52, E58, D84

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1 Introduction

Monetary policy may have an information effect when information asymmetry exists between the central bank and the private sector. That is, the private sector believes that an interest-rate decision reveals the central bank's private information about the state of the economy, and uses the interest rate as a signal to update beliefs about the state of the economy. However, the literature has yet to agree on whether the information effect of monetary policy is quantitatively important. On the one hand, Nakamura and Steinsson (2018) provide supportive evidence of the information effect, showing that professional forecasters positively revise their output growth forecasts after an unexpected interest-rate hike, contradicting the implications of a traditional New Keynesian model. On the other hand, Bauer and Swanson (2023) argue that the above result might be due to an omitted variable bias. They further show that after controlling for news about the state of the economy, the information effect of monetary policy is not significant.

In this paper, we first construct an analytic model to show that under asymmetric information, monetary policy surprises correlate with the unobserved shock. The correlation leads to the information effect of monetary policy and, at the same time, an omitted variable bias in the reduced-form estimation that regress forecast revisions on monetary policy surprises. We then propose to structurally estimate the information effect using a quantitative New Keynesian model with asymmetric information. The key novelty of our model is the two-way learning mechanism: we assume that the central bank learns from decisions in the private sector (inflation and output) with a one period lag, and the private sector learns from the interest-rate decisions of the central bank. We calibrate model parameters, including the information precision, to match moments of macroeconomic dynamics and survey data in the US economy. Our calibration suggests that exogenous signals about the current shock are noisy, but endogenous information from lagged equilibrium variables is precise. Although the central bank's information is not superior to that of professional forecasters, monetary policy could still alter firms' expectations, as individual firms either pay less attention to or lack the analytical skills to process aggregate economic variables. Our baseline calibration suggests that the information effect largely reduces the output gaps due to demand shocks and noise shocks, but, at the same time, causes temporary inflation fluctuations due to monetary policy shocks. Our model also has implications for the determinants of the size of the information effect and the slope of the Phillips curve.

Our analytic model consists of a central bank and a continuum of professional forecasters. There is only one unobserved fundamental shock, and output is determined by both the fundamental shock and the interest rate. There is a public signal about the fundamental shock and in addition, both the central bank and each professional forecaster get a private signal.

The central bank sets the interest rate following a targeting rule that makes the interest rate respond to the central bank's expected output. The interest rate also contains a policy shock, which is assumed to capture any exogenous variations in the interest rate. Once the interest rate is set, professional forecasters perfectly observe it. Professional forecasters form expectations twice in a given period: once before the interest rate is set (pre-FOMC expectations) and once after (post-FOMC expectations). We define the *monetary policy surprise* as the difference between the actual interest rate and the average of the pre-FOMC expected interest rate across all forecasters.

Our analytic model provides a micro-foundation for monetary policy surprises under asymmetric information. In our framework, the fundamental shock, the noise shocks (both in the central bank's private information and in the public information), and the exogenous policy shock all lead to a monetary policy surprise. After a positive fundamental shock, professional forecasters underestimate the realization of the shock on average, and thereby underestimate the interest-rate response by the central bank. Hence, the actual interest rate is greater than the expected interest rate, leading to a positive monetary policy surprise. Notice that asymmetric information is the key assumption. Without the central bank having private information, the endogenous change in the interest rate are perfectly predictable using public information, and in this case, monetary policy surprises are completely driven by the exogenous policy shocks.

Under asymmetric information, the correlation between monetary policy surprises and the fundamental shock implies that monetary policy contains information about the underlying economy. Our framework allows for a closed-form solution to the size of the information effect. We show that the information effect is stronger when the central bank has more precise private information relative to professional forecasters, and the size of the information effect has a non-linear relationship with the precision of public information and the policy rule coefficient. At the same time, the correlation between monetary policy surprises and the fundamental shock also implies that the reduced-form regression of private forecast revisions on monetary policy surprises produces an upward bias in the estimated size of the information effect. This analytical result is consistent with the empirical evidence in Bauer and Swanson (2023) that the estimated information effect is reduced after controlling for economic news. However, our model also shows that only controlling for economic news cannot completely solve the omitted variable bias problem. This is because the noise shock in the public information is still in the error term and is correlated with the monetary policy surprise.

We then aim to structurally estimate the information effect in a New Keynesian model under asymmetric information. Our model allows rich heterogeneity in expectations. The household, each firm, and the central bank all have different information sets and form their own expectations. By doing so, our model has another advantage over Nakamura and Steinsson (2018) and Bauer and Swanson (2023): instead of measuring the information effect on the expectations of professional forecasters, we measure the information effect on firms, whose expectations are arguably more important than those of professional forecasters, considering firms are the ones making pricing decisions.

We assume that there are two aggregate fundamental shocks: a demand shock and a supply (i.e. cost-push) shock. We assume there are two exogenous signals about the demand shock: one public signal available to all agents and one private signal only available to the central bank. The household is assumed to have perfect information.[1] Each firm observes its own marginal cost, which is assumed to be a linear combination of the equilibrium wage and the firm-specific wage bargaining cost. The firm-specific wage bargaining cost has an aggregate component that maps into a cost-push shock in aggregation, and an idiosyncratic component that cancels out in aggregation. This assumption makes each firm's marginal cost a private signal that provides information about both the aggregate demand shock and the aggregate cost-push shock.

A key novelty in our model is the two-way learning mechanism. In any given period, shocks and exogenous signals are realized in the first stage. In the second stage, the central bank observes exogenous signals up to the current period, and inflation and output up to the previous period. In the last stage, the interest rate becomes an endogenous signal that is available to all private agents.

We calibrate the model parameters to match macroeconomic dynamics in the US economy and forecasting accuracy in the Greenbook[2] and the Survey of Professional Forecasters (SPF). We directly assign values to the parameters commonly used in the New Keynesian literature, including parameters in the household's utility function, the Calvo parameter, and the coefficients of the monetary policy targeting rule. We then internally calibrate the parameters that govern the degree of information frictions. We calibrate the process of the demand shock to match the auto-correlation and the standard deviation of realized inflation in the US. The standard deviation of the exogenous monetary policy shock is calibrated to

[1] This assumption is common in the literature on information frictions. See Adam (2007), Paciello and Wiederholt (2014) and Melosi (2017) for examples. It is also plausible to think that the household knows the demand shock perfectly, since the demand shock directly enters the utility function. In addition, we have also tested that whether or not the household observes the cost-push shock has little impact on the model's quantitative predictions, for reasons explained in Section 3.1.

[2] The Greenbook was produced by the staff at the Board of Governors and provided analysis and forecasts of the US and international economy. The Greenbook and the Bluebook were merged into the Tealbook in June 2010.

match the standard deviation of monetary policy surprises under the high-frequency identification method. We calibrate the standard deviation of individual firms' wage bargaining shock using micro evidence of cross-sectional price changes. We calibrate the precision of the public signal and the central bank's private signal by matching the inflation nowcast accuracy in the Greenbook and in the SPF. A key result of our calibration is that exogenous signals provide noisy information about shocks in the current period, which is suggested by the sizable nowcast error in both the Greenbook and the SPF. In addition, the large cross-sectional price variation suggests that firms' private signals are noisy.

To better illustrate the information effect of monetary policy, we first set up two benchmark cases: the full information benchmark and the no-information-effect benchmark. In the second benchmark, the calibrated information frictions are the same as the ones in the baseline calibrated economy, but the interest rate does not enter firms' information sets. Comparing the two benchmark cases, we show that relative to the full information case, imperfect information widens the output gaps after demand shocks, because the central bank underestimates the realization of demand shocks and therefore does not sufficiently adjust the interest rate. On the other hand, inflation fluctuations are smaller, as firms underestimate the change in the aggregate price level when setting their own prices.

Compared to the no-information-effect benchmark, the information effect of monetary policy reduces output gaps but amplifies inflation fluctuations after demand shocks. This shows that endogenous information is precise. The central bank quickly learns the realization of shocks from lagged inflation and output, and adjusts the interest rate to narrow the output gap. At the same time, the interest rate reveals information to firms, which makes them update their expectations about the aggregate price change and further adjust their own prices, leading to larger inflation fluctuations. After a positive policy shock (an unexpected tightening of monetary policy), the information effect makes firms mistakenly believe there is a positive demand shock, and so they revise up their inflation expectations. Under our baseline calibration, the information effect dominates the direct effect: a contractionary monetary policy shock leads to a temporary rise in inflation.

We study two factors that determine the size of the information effect. First, the size of the information effect depends on the central bank' information advantage over firms. In the context of our model, observation errors about lagged inflation and output reduce the central bank's information advantage and thus reduce the size of the information effect. Second, the information effect weakens if the standard deviation of the exogenous policy shock increases.

Our model also speaks to the "non-linear" Phillips curve in the COVID-19 high-inflation period. Our model suggests that the change in the empirically estimated slope coefficient could be explained by a change in firms' expectation-formation process. Our model simulation shows that compared with the case in which firms are inattentative to interest rates or inflation, the information effect of monetary policy steepens the Phillips curve.

Related Literature

Our paper contributes to the growing literature on the information effect of monetary policy. Romer and Romer (2000) first proposed this concept, by showing how private inflation forecasts respond to changes in the policy rate after FOMC announcements. Some recent studies (Campbell et al. (2012), Nakamura and Steinsson (2018), and Lunsford (2020)) provide further supportive evidence of how policy announcements lead to private-sector forecast revisions. Jarociński and Karadi (2020) use high-frequency co-movement between interest rates and stock prices to identify the information shock from FOMC statements. However, Bauer and Swanson (2023) argue that the empirical evidence of the Fed's information effect is not robust to sample and variable selections and that the estimated result may suffer from an omitted variable bias. The first contribution of our paper is to show why regression-based empirical estimation of the information effect may not yield a conclusive answer.

Motivated by the empirical relevance of the information effect of monetary policy, economists have started to model the information effect of the interest rate in a New Keynesian model. See, for example, Berkelmans (2011), Tang (2015), Melosi (2017), Andrade et al. (2019), and Jia (2023). With a few exceptions, the existing literature on the information effect studies the case of one-way learning: the private sector learns from the central bank's interest-rate decisions. We contribute to this literature by modeling a two-way information flow: the private sector learns from the central bank's interest-rate decisions, and the central bank learns from equilibrium output and inflation in the private sector (with a one-period lag).

More generally, modeling the information effect of monetary policy is related to the broad literature on learning from endogenous signals. Earlier works focus on information revelation in asset prices, including Grossman and Stiglitz (1980), Hellwig (1980), Kasa, Walker, and Whiteman (2006) among others. Paciello and Wiederholt (2014) model endogenous information and endogenous attention in a New Keynesian model and study the implications for optimal monetary policy. Kohlhas (2022) shows how central bank disclosure enhances the efficacy of monetary policy by decreasing higher-order uncertainty in an economy with two-sided learning between the private sector and the central bank. Chahrour and Jurado (2025) revisit the original Townsend (1983) model to characterize how learning from endogenous variables affects equilibrium dynamics, and propose a new numerical solution that operates in the frequency domain.

2 An Analytic Model

In this section, we first set up an analytic model in which information is asymmetric between the private sector and the central bank (Section 2.1). Using this model, we provide a micro-foundation for monetary policy surprises and explain the implications for estimating the information effect of monetary policy (Section 2.2).

2.1 Model Set-up

In our model, both the central bank and the private sector have imperfect information about the state of the economy. We first explain our assumptions on the state variable and the interest-rate rule. Then, we specify the sequence of events and how the central bank's and the private sector's expectations are formed.

2.1.1 Economic Fundamentals and the Policy Rule

We assume that there is only one fundamental shock in the private sector, which is denoted by x_t and is not observable. For simplicity, we assume x_t is *i.i.d.* in each period, and follows

$$x_t \sim N\left(0, \nu_x^{-1}\right).$$

The economic outcome, y_t, is determined by the fundamental shock, x_t, and the interest rate set by the central bank, i_t. Specifically,

$$y_t = x_t - \kappa i_t, \tag{1}$$

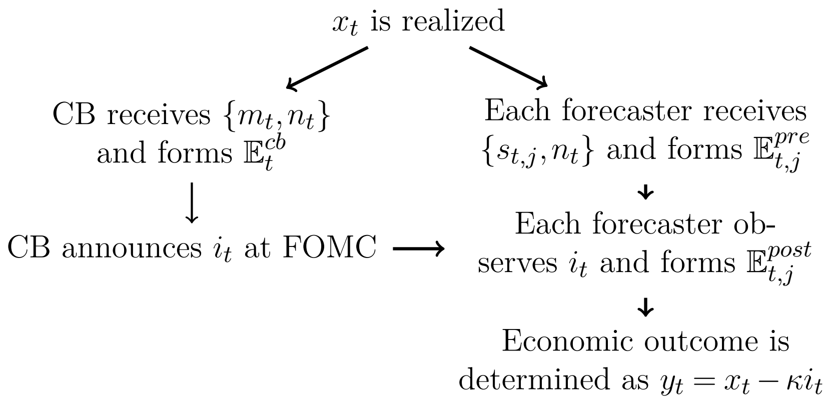
where $\kappa > 0$ captures the direct effect of monetary policy. In the context of the New Keynesian literature, one can regard x_t as a demand shock and y_t as the output level. In this analytic model, we assume that the private sector's expectations are formed by a continuum of professional forecasters who do not make output decisions. In this case, the economic outcome, y_t, is not affected by private sector's expectations and noise shocks do not change the economic outcome, y_t.[3]

The central bank sets the interest rate conditional on its expectations and according to a targeting rule that is given by

$$i_t = \phi_y \mathbb{E}_t^{cb} y_t + e_t, \qquad e_t \sim N\left(0, \nu_e^{-1}\right) \tag{2}$$

[3] We relax this assumption in Sections 3 and 4, and study the case in which equilibrium output and inflation are affected by expectation terms.

Figure 1: Sequence of Events



where $\phi_y > 0$. The value of ϕ_y is known by the private sector. e_t is a "monetary policy shock" capturing the exogenous variation in the interest rate that is not explained by the endogenous response to $\mathbb{E}_t^{cb} y_t$. The existence of e_t makes the relationship between i_t and $\mathbb{E}_t^{cb} y_t$ non-invertible, meaning that the private sector cannot perfectly back out $\mathbb{E}_t^{cb} y_t$ from i_t.

2.1.2 Sequence of Events

To capture monetary policy *surprises*, we assume that in each period the private sector forms expectations twice: once before and once after the policy announcement. Under this assumption, we model monetary policy surprises in the same way that monetary policy shocks are measured in the empirical literature that uses the high-frequency identification method.

Figure 1 illustrates the timing of events. Specifically, each period t can be divided into four stages. At stage 1, x_t is realized. At stage 2, a public signal, n_t, is received by both the central bank and private forecasters. In addition, the central bank receives a private signal, m_t, and each forecaster receives an individual-specific signal, $s_{t,j}$. Conditional on their own information sets, the central bank and the professional forecasters form their expectations. At this point, the private sector's expectations are called pre-FOMC expectations. At stage 3, the central bank sets and announces the interest rate at the FOMC meeting. The private sector observes the interest rate and updates expectations. At this stage, the private sector's expectations are called post-FOMC expectations. At the last stage, the economic outcome is determined as $y_t = x_t - \kappa i_t$.

Our assumption on the sequence of events is also very similar to Bauer and Swanson (2023). The stage at which the central bank and the private sector receive imperfect information corresponds to the stage of "economic news release" in Bauer and Swanson (2023). In addition, the "forecast revisions" in Bauer and Swanson (2023) are equivalent to the post-FOMC expectations in our model. This is because "forecast revisions" in Bauer and Swanson (2023) measure the changes in private-sector expectations from $t-1$ to t, the same as the expectations at t in our static model.

2.1.3 Information Sets and Expectations

We first specify the information sets of each agent and then solve for the expectations of the central bank and professional forecasters.

Central bank's expectations and policy decisions

Before making policy decisions, the central bank receives two signals: a public signal n_t and a private signal m_t. Both signals are assumed to be normally distributed around the true state, x_t. Specifically,

$$n_t = x_t + \varepsilon_t^n, \quad \varepsilon_t^n \sim N\left(0, \nu_n^{-1}\right),$$
$$m_t = x_t + \varepsilon_t^m, \quad \varepsilon_t^m \sim N\left(0, \nu_m^{-1}\right).$$

Conditional on $\{m_t,\ n_t\}$, the central bank's expectation on x_t is given by

$$\mathbb{E}_t^{cb} x_t \equiv \mathbb{E}_t[x_t|\mathcal{I}_t^{cb}] = \frac{\nu_m}{\nu_x + \nu_m + \nu_n} m_t + \frac{\nu_n}{\nu_x + \nu_m + \nu_n} n_t, \tag{3}$$

where $\mathcal{I}_t^{cb} = \{m_t,\ n_t\}$.

To find the interest rate set by the central bank, combine Equations (2) and (1), and notice that $\mathbb{E}_t^{cb} i_t = i_t$. Next, substitute $\mathbb{E}_t^{cb} x_t$ with Equation (3). The interest rate is found as

$$i_t = \frac{\phi_y}{1+\kappa\phi_y}\left[\frac{\nu_m}{\nu_x + \nu_n + \nu_m} m_t + \frac{\nu_n}{\nu_x + \nu_n + \nu_m} n_t\right] + \frac{1}{1+\kappa\phi_y} e_t. \tag{4}$$

Pre-FOMC expectations in the private sector

In the private sector, professional forecasters are indexed by j, and $j \in (0,1)$. Before an FOMC meeting, each forecaster receives two signals: a public signal, the same one received by the central bank (n_t) and a private, individual-specific signal, which is denoted by $s_{t,j}$ and follows

$$s_{t,j} = x_t + \varepsilon_{t,j}^s, \quad \varepsilon_{t,j}^s \sim N\left(0, \nu_s^{-1}\right)$$

The forecaster j's pre-FOMC expectation about the state of the economy is formed as

$$\mathbb{E}_{t,j}^{pre} x_t \equiv \mathbb{E}_t[x_t|\mathcal{I}_{t,j}^{pre}] = \frac{\nu_s}{\nu_x + \nu_n + \nu_s} s_{t,j} + \frac{\nu_n}{\nu_x + \nu_n + \nu_s} n_t. \tag{5}$$

where $\mathcal{I}_{t,j}^{pre} = \{s_{t,j},\ n_t\}$.

In addition to forming expectations about x_t, the professional forecasters also form expectations about the interest rate. With rational expectations, professional forecasters understand that i_t follows Equation (4). They expect that policy shocks are zero ex-ante, $\mathbb{E}_{t,j}^{pre} e_t = 0$, and the noise in the central bank's private signal is also zero ex-ante, so that $\mathbb{E}_{t,j}^{pre} m_t = \mathbb{E}_{t,j}^{pre} x_t$.

Forecaster j's pre-FOMC expected i_t is given by

$$\mathbb{E}_{t,j}^{pre} i_t = \frac{\phi_y}{1+\kappa\phi_y}\left[\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\left(\frac{\nu_s}{\nu_x+\nu_n+\nu_s}s_{t,j}+\frac{\nu_n}{\nu_x+\nu_n+\nu_s}n_t\right)+\frac{\nu_n}{\nu_x+\nu_n+\nu_m}n_t\right] \tag{6}$$

where we make use of $\mathbb{E}_{t,j}^{pre} m_t = \mathbb{E}_{t,j}^{pre} x_t$.

Post-FOMC expectations in the private sector

After an FOMC meeting, the interest rate becomes a public signal. To see how the interest rate serves as another signal of x_t, re-arrange Equation (4) and get

$$\hat{i}_t = \frac{i_t - \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_n}{\nu_x+\nu_n+\nu_m}n_t}{\frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m}} = x_t + \varepsilon_t^m + \frac{1}{\phi_y\frac{\nu_m}{\nu_x+\nu_n+\nu_m}}e_t \tag{7}$$

Equation (7) shows that $\hat{i}_t$ is an unbiased signal of x_t, with the precision of the signal given by

$$\nu_i^{-1} = \nu_m^{-1} + \left(\frac{\nu_x+\nu_n+\nu_m}{\phi_y\nu_m}\right)^2\nu_e^{-1}. \tag{8}$$

Lemma 1 *$\hat{i}_t$ is an unbiased signal of x_t, and its precision, ν_i,*

- *increases with the precision of the central bank's private information (ν_m),*
- *decreases with the precision of the public information (ν_n),*
- *increases with the interest-rate rule coefficient (ϕ_y),*
- *and decreases with the standard deviation of the policy shock (σ_e).*

Forecaster j's post-FOMC expectation about the state of the economy is given by

$$\mathbb{E}_{t,j}^{post} x_t \equiv \mathbb{E}_t[x_t|\mathcal{I}_{t,j}^{post}] = \frac{\nu_s}{V}s_{t,j} + \frac{\nu_n}{V}n_t + \frac{\nu_i}{V}\hat{i}_t, \tag{9}$$

where $\mathcal{I}_{t,j}^{post} = \left\{s_{t,j},\ n_t,\ \hat{i}_t\right\}$ and $V = \nu_x+\nu_n+\nu_s+\nu_i$.

2.2 Model Implications

In this section, we provide a micro-foundation for monetary policy surprises under asymmetric information and a closed-form solution to the information effect of monetary policy.

2.2.1 A Micro-Foundation for Monetary Policy Surprises

The monetary policy surprise, mps_t, or the unexpected change in the interest rate, is defined as the difference between the actual and the average of the pre-FOMC expected interest rate. To calculate mps_t, we first take the difference between i_t (Equation (4)) and each individual forecaster's pre-FOMC expected i_t (Equation (6)), and then integrate over all forecasters. The monetary policy surprise is found to be[4]

$$mps_t = \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\left(\frac{\nu_x}{\nu_x+\nu_n+\nu_s}x_t+\varepsilon_t^m-\frac{\nu_n}{\nu_x+\nu_n+\nu_s}\varepsilon_t^n\right)+\frac{1}{1+\kappa\phi_y}e_t, \tag{10}$$

The following proposition explains how each shock contributes to a monetary policy surprise.

Proposition 1 *There is a positive monetary policy surprise after*

1. *a positive fundamental shock ($x_t > 0$),*
2. *a positive noise shock in the central bank's private information ($\varepsilon_t^m > 0$),*
3. *a negative noise shock in the public information ($\varepsilon_t^n < 0$), or*
4. *a positive exogenous policy shock ($e_t > 0$).*

Equation (4) suggests that a positive mps_t occurs when $m_t > \mathbb{E}_{t,j}^{pre} m_t$, which explains Propositions 1.1 and 1.2. When the central bank's private information contains a positive noise shock, the central bank increases i_t more than the private sector expects. When there is a positive fundamental shock, $x_t > 0$, professional forecasters underestimate x_t on average due to imperfect information, and thereby underestimate the monetary policy's response to the fundamental shock as well. This leads to a positive monetary policy surprise.

To understand the intuition for Proposition 1.3 (the negative relationship between mps_t and the noise shock in the public information), notice that the public signal is over-weighted by professional forecasters when forming expectations about the interest rate. They correctly understand that the interest rate responds to the public signal as Equation (4) describes, but in addition, they also use the public signal when guessing the central bank's private

[4] Detailed derivations are presented in the Appendix.

signal, m_t. In other words, when observing a positive public signal, professional forecasters mistakenly think that the central bank will get a positive private signal on top of the positive public signal, and therefore their expected interest rate is higher than the actual interest rate.

In summary, under asymmetric information, not only a monetary policy shock, but also all other shocks can lead to a monetary policy surprise. The key assumption is that the central bank has private information; otherwise, the endogenous change in the interest rate will be perfectly predictable by professional forecasters, in which case any monetary policy surprise is due to the exogenous policy shock.

Is the assumption that the central bank has some private information plausible? We argue that it is. Even though the central bank and professional forecasters may have the same access to economic data, the two can use different sets of indicators to evaluate the state of the economy and to form expectations.

As will be shown later, the size of the information effect of monetary policy depends on the correlation between mps_t and x_t, which we summarize in the following corollary.

Corollary 1.1 *A positive innovation in the unobserved state variable (x_t) results in a positive monetary policy surprise. In addition, for a given change in x_t, the size of mps_t*

- *increases with the precision of the central bank's private information (ν_m),*
- *decreases with the precision of the forecasters' private information (ν_s),*
- *decreases with the precision of the public information (ν_n),*
- *and increases with policy rule coefficient (ϕ_y),*

The intuition for Corollary 1.1 is the following. First, when the central bank's information is more precise, the actual interest rate is more responsive to x_t, and when professional forecasters' private information is more precise, the expected interest rate is less responsive to x_t. In both cases, the size of the monetary policy surprise is greater. Second, the public signal is over-weighted by professional forecasters when forming $\mathbb{E}_{t,j}^{pre}\, i_t$; so when the public signal becomes more precise, the expected interest rate becomes more sensitive to x_t than the actual interest rate does. This reduces the size of mps_t. In addition, conditional on the differences between the central bank's and the forecasters' expected state of the economy, a larger ϕ_y means a larger change in i_t, and thus a greater mps_t.

2.2.2 Economic News Predicts Monetary Policy Surprises

Bauer and Swanson (2023) argue that there might be an omitted variable bias if one regresses the private sector's forecast revisions only on monetary policy surprises. The reason is that the monetary policy surprise is not exogenous and correlates with other variables that also explain forecast revisions in the private sector. To support this argument, Bauer and Swanson (2023) show that monetary policy surprises are predictable by economic news, where economic news is defined as the difference between the realized and the expected economic indicators. Specifically, Bauer and Swanson (2023) estimate the predictability of mps_t by the following regression

$$mps_t = \boldsymbol{\alpha} + \boldsymbol{\beta' news_t} + \boldsymbol{\varepsilon_t}, \tag{11}$$

where $\boldsymbol{news_t}$ denotes a vector of macroeconomic news, such as the surprise components in unemployment, and financial news, such as changes in the $S\&P$ 500 index. Their estimation shows that news about higher output or inflation predicts a positive policy surprise.

Our model allows for an analytic solution to the predictability of monetary policy surprises by economic news. To make our results comparable to those in Bauer and Swanson (2023), we define $news_t$ in our model as the difference between the actual and the pre-FOMC expected state of the economy, i.e., $news_t = x_t - \int \mathbb{E}_{t,j}^{pre} x_t$, where $\mathbb{E}_{t,j}^{pre} x_t$ is given by Equation (5). It yields

$$news_t = \frac{\nu_x}{\nu_x + \nu_n + \nu_s} x_t - \frac{\nu_n}{\nu_x + \nu_n + \nu_s} \varepsilon_t^n. \tag{12}$$

Re-arrange Equation (10) to express mps_t as a function of $news_t$:

$$mps_t = \frac{\phi_y}{1 + \kappa\phi_y} \frac{\nu_m}{\nu_x + \nu_n + \nu_m} news_t + \frac{\phi_y}{1 + \kappa\phi_y} \frac{\nu_m}{\nu_x + \nu_n + \nu_m} \varepsilon_t^m + + \frac{1}{1 + \kappa\phi_y} e_t \tag{13}$$

Comparing Equation (13) with Equation (11) immediately yields

$$\boldsymbol{\alpha} = 0, \quad \boldsymbol{\beta} = \frac{\phi_y}{1 + \kappa\phi_y} \frac{\nu_m}{\nu_x + \nu_n + \nu_m},$$
$$\boldsymbol{\varepsilon_t} = \frac{\phi_y}{1 + \kappa\phi_y} \frac{\nu_m}{\nu_x + \nu_n + \nu_m} \varepsilon_t^m + \frac{1}{1 + \kappa\phi_y} e_t.$$

The first thing to notice is that $\beta > 0$. It means that mps_t can be predicted by economic news, and the positive sign reflects the fact that the central bank tightens monetary policy in response to a positive fundamental shock. This is consistent with the empirical results in Bauer and Swanson (2023).

In addition, the size of β depends on the degree of information friction. Specifically, β increases with ν_m and decreases with ν_n. The intuition is that $news_t$ measures the fraction of x_t that is underestimated by the private sector (less the overestimated fraction of x_t due to the noise in the public signal), which leads to a difference between the actual and the expected i_t. A larger mps_t can result from either a larger change in the actual i_t or a smaller change in the expected i_t. The former requires the central bank to have more precise information, Furthermore, the latter requires the private sector to have precise information.

Monetary policy surprises are predictable by economic news both in our model and in Bauer and Swanson (2023) but for different reasons. In Bauer and Swanson (2023), the underlying assumption is that the private sector has perfect information about the state of the economy but imperfect information about the policy rule (i.e., the value of ϕ_y in our model). In contrast, we assume that the private sector has perfect information about the policy rule, but has imperfect information about the state of the economy, which gives rise to both the correlation between monetary policy surprises and the state of the economy and the predictability of monetary policy by economic news.

2.2.3 The Information Effect of Monetary Policy

Because monetary policy surprises correlate with the unobserved shock to the state of the economy, agents with rational expectations will find it useful to extract information about the unobserved shock from monetary policy surprises. In this section, we solve for the closed-form solution to the size of the information effect of monetary policy, and analyze why a reduced-form regression may result in a biased estimation.

To derive the information effect of monetary policy, first substitute $\hat{i}_t$ with mps_t in $\mathbb{E}_{t,j}^{post} x_t$ (Equation (9), and then apply $\mathbb{E}_{t,j}^{post} y_t = \mathbb{E}_{t,j}^{post} x_t \kappa i_t$. The post-FOMC average expected output can be written as

$$\overline{\mathbb{E}}_t^{post} y_t = \underbrace{\left[\frac{1+\kappa\phi_y}{\phi_y}\frac{\nu_i}{V}\frac{\nu_x+\nu_n+\nu_m}{\nu_m} - \kappa\right]}_{\Phi} mps_t + \underbrace{\frac{\nu_s}{\nu_x+\nu_n+\nu_s}\left(1-\kappa\frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\right)}_{\Psi_1} x_t$$
$$+ \underbrace{\left[\frac{\nu_n}{\nu_x+\nu_n+\nu_s}\left(1-\kappa\frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\right) - \kappa\frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_n}{\nu_x+\nu_n+\nu_m}\right]}_{\Psi_2} n_t \quad (14)$$

The value Φ in Equation (14) measures how private agents revise their forecasts about the economy with respect to monetary policy surprises, which corresponds to the estimated information effect in Nakamura and Steinsson (2018) and Bauer and Swanson (2023). We first show that Φ measures the combination of the information effect and the direct effect. The direct effect of monetary policy is $-\kappa$, and the information effect is given by the first term in Φ, $\frac{1+\kappa\phi_y}{\phi_y}\frac{\nu_i}{V}\frac{\nu_x+\nu_n+\nu_m}{\nu_m}$. The information effect is positive, meaning that after an unexpected interest-rate hike, private agents positively revise their forecasts of output. The information effect counteracts the direct effect of monetary policy.

Proposition 2 *The size of of the information effect of monetary policy surprises*

- *increases with the precision of the central bank's private information* (ν_m)*,*
- *decreases with the precision of the forecasters' private information* (ν_s)*,*
- *has a non-linear relationship with the precision of the public information* (ν_n) *and the policy rule coefficient* (ϕ_y)*,*
- *and decreases with the standard deviation of the policy error,* σ_e*.*

We illustrate how Φ changes with different parameters by the solid blue line in Figure 2. First, when the central bank has more precise private information or professional forecasters have less precise private information, the central bank has an information advantage over the private sector; in this case, the information effect is stronger.

Second, the precision of the public information has a non-linear effect on the size of Φ. This is because when ν_n increases, two competing forces jointly determine the size of the information effect. First, under a larger ν_n, mps_t is smaller after a given change in x_t (Corollary 1.1), or equivalently, the private sector expects a larger change in x_t for a given mps_t. However, a larger ν_n also makes $\hat{i}_t$ a less precise signal of x_t, (Lemma 1) for which the private sector updates expected x_t by a smaller amount. The effect of ϕ_y also has a non-linear effect on the size of Φ for a similar reason. Lastly, a smaller σ_e makes $\hat{i}_t$ a more precise signal of x_t, strenghening the information effect of monetary policy.

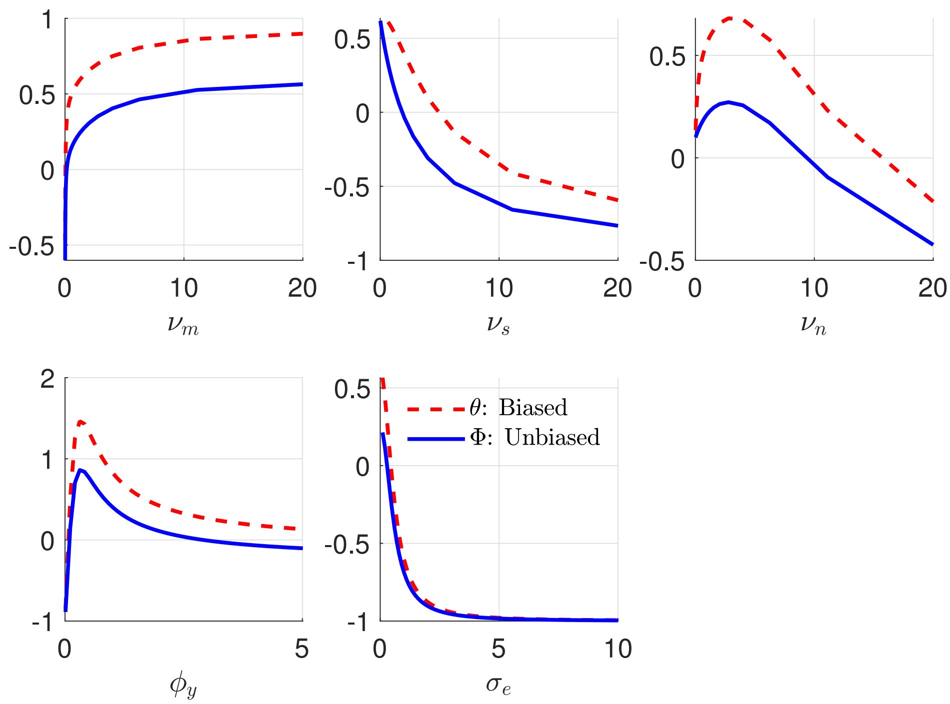
Omitted Variable Bias. Equation (14) also explains why a reduced-form regression of private forecast revisions on monetary policy surprises may lead to a biased estimation. If one regresses $\overline{\mathbb{E}}_t^{post} y_t$ only on mps_t, the omitted variable bias occurs because both x_t and n_t would be left in the regression residuals, but both x_t and n_t correlate with mps_t and affect $\mathbb{E}_t^{post} y_t$.

To calculate the size of the omitted variable bias in our model, we re-write Equation (14) to express the regression equation in Nakamura and Steinsson (2018). Specifically, we single out mps_t as the only independent variable and leave other variables as error terms in explaining post-FOMC expectations, i.e.,

$$\overline{\mathbb{E}}_t^{post} y_t = \boldsymbol{\phi} + \boldsymbol{\theta} mps_t + \boldsymbol{\eta_t}. \tag{15}$$

$\boldsymbol{\theta}$ in Equation (15) is estimated by the correlation between $\overline{\mathbb{E}}_t^{post} y_t$ and mps_t. Some

Figure 2: Private Forecast Revisions w.r.t. Monetary Policy Surprises



Notes: The solid blue line plots the value of Φ in Equation (14), and the dashed red line plots the value of θ in Equation (15). Benchmark parameters are set as: $\sigma_x = 1$, $\sigma_m = 1$, $\sigma_n = 1$, $\sigma_s = 1$, $\sigma_e = 0.1$, $\kappa = 1$, and $\phi_y = 1.5$.

additional algebra shows that

$$\begin{aligned}\theta &= \frac{cov(mps_t, \overline{\mathbb{E}}_t^{post} y_t)}{var(mps_t)} \\ &= \frac{(\Phi A + \Psi_1 + \Psi_2)A/\nu_x + \Phi B^2/\nu_m + (\Phi C + \Psi_2)C\nu_n + \Phi D^2/\nu_e}{A^2/\nu_x + B^2/\nu_m + C^2/\nu_n + D^2/\nu_e}.\end{aligned} \tag{16}$$

where A, B, C, and D are coefficients when mps_t in Equation (10) is re-written in terms of $mps_t = Ax_t + B\varepsilon_t^m + C\varepsilon_t^n + De_t$.

Figure 2 illustrates the size of the information effect and the size of the bias due to omitted variables. The solid blue line is Φ in Equation (14), which is the true size of forecast revisions after a positive monetary policy surprise. The dashed red line is $\boldsymbol{\theta}$ in Equation (16), which is the biased estimation. All five panels suggest that the omitted variable bias leads to an overestimation of the size of the information effect, which is consistent with the conclusion in Bauer and Swanson (2023).

Bauer and Swanson (2023) also suggest that controlling for economic news is a way to correct for the omitted variable bias. However, can the bias be eliminated entirely? Our model suggests no. To see this, substitute x_t in $\mathbb{E}_t^{post} y_t$ with $news_t$ using Equation (12), and rewrite $\mathbb{E}_t^{post} y_t$ as

$$\mathbb{E}_t^{post} y_t = \Phi mps_t + (\Psi_1 + \Psi_2) \frac{1}{P} news_t - \left((\Psi_1 + \Psi_2) \frac{Q}{P} + \Psi \right) \epsilon_t^n \tag{17}$$

where P and Q are coefficients when $news_t$ in Equation (12) is written in terms of $news_t = Px_t + Q\epsilon_t^n$.

Equation (17) suggests that adding $news_t$ to the regression can be regarded as controlling for the state of the economy, but still leaves out the noise component of the public information in the error term. Since the noise shock correlates with mps_t and affects $\mathbb{E}_t^{post} y_t$, the estimation of the coefficient on mps_t will still be biased.

Summary. There are two main takeaways from the analytic model. First, asymmetric information between the central bank and private agents leads to the correlation between monetary policy surprises and the unobserved shock, which generates the information effect of monetary policy. Second, a reduced-form regression of private forecast revisions on monetary policy surprises leads to a biased estimation of the information effect, due to the omitted variable bias problem. Therefore, we turn to a structural model to quantify the information effect of monetary policy.

3 A New Keynesian Model

To quantify the information effect of monetary policy in the real world, we develop a New Keynesian model under information frictions, building on Nimark (2008) and Melosi (2017). The key innovation of our model is to assume that signals used by the central bank and the private sector also include endogenous variables in the equilibrium.

Our quantitative model has two extensions of our analytic model in Section 2. First, the private sector consists of different agents, including households, firms, and professional forecasters, each forming expectations conditional on their own information sets. Therefore, in contrast to Nakamura and Steinsson (2018) or Bauer and Swanson (2023), where the information effect is measured on the expectations of professional forecasters, our model captures different information effects on different agents. Second, the central bank internalizes the information effect of its policy decisions on equilibrium output and inflation.

3.1 Model Set-up

The economy consists of a representative household that demands consumption goods and supplies labor, a continuum of firms that produce intermediate goods and set prices, and a central bank that sets the interest rate according to a targeting rule. We first describe the optimization problems of the household and the firms, as well as the interest-rate rule. We then specify the information structures of all agents, and explain how we solve the equilibrium with heterogeneous expectations.

3.1.1 Household

The representative household chooses aggregate consumption, C_t, relative to an external habit stock hC_{t-1}, and labor supply, N_t, to maximize its lifetime utility.

$$\mathbb{E}_0^{HH}\sum_{t=0}^{\infty}\beta^t U(C_t,N_t)=\mathbb{E}_0^{HH}\sum_{t=0}^{\infty}\beta^t\exp(d_t)\left[\log(C_t-hC_{t-1})-\frac{N_t^{1+\psi}}{1+\psi}\right], \tag{18}$$

where $\beta\in(0,1)$ is the deterministic discount factor, $h\in(0,1)$ governs the strength of habits, and $1/\psi>0$ is the Frisch elasticity of labor supply. $\mathbb{E}_t^{HH}(\cdot)\equiv\mathbb{E}(\cdot|\mathcal{I}_t^{HH})$ is the expectation operator conditional on the household's information set, $\mathcal{I}_t^{HH}$, which will be specified later. The aggregate good, C_t, consists of a continuum of intermediate goods in the form of $C_t=\left(\int_{[0,1]}C_{t,j}^{(\nu-1)/\nu}dj\right)^{\frac{\nu}{\nu-1}}$, where ν is the elasticity of substitution between different intermediate goods $C_{t,j}$.

The preference of the household is subject to a stochastic demand shock d_t, which is assumed to follow an AR(1) process:

$$d_t=\rho d_{t-1}+\varepsilon_t,\quad \varepsilon_t\sim N(0,\sigma_d^2). \tag{19}$$

The intra-temporal labor supply decision sets the marginal rate of substitution between leisure and consumption equal to the real wage, which is given by[5]

$$w_t=\frac{1}{1-h}(c_t-hc_{t-1})+\psi n_t. \tag{20}$$

Using the intertemporal consumption decision and the goods market clearing condition,

[5] We use the lowercase variables to denote the logarithmic deviations from their deterministic steady states.

we derive the following expression for aggregate output,

$$y_t = \frac{1}{1+h}\left(\mathbb{E}_t^{HH} y_{t+1} + h y_{t-1}\right) - \frac{1-h}{1+h}\left(i_t - \mathbb{E}_t^{HH}\pi_{t+1}\right) + \frac{1-h}{1+h}\left(d_t - \mathbb{E}_t^{HH} d_{t+1}\right), \tag{21}$$

where i_t and π_t denote the nominal interest rate and the inflation rate.

3.1.2 Firms

A continuum of competitive monopolistic firms is indexed by $j \in [0,1]$ and produces intermediate goods under the Calvo type of price rigidity (Calvo (1983)) and information frictions. Production technology is homogeneous, and each firm's production function is linear, given by $Y_{t,j} = N_{t,j}$. When firm j gets the opportunity to reset the price, it chooses $P_t^*(j)$ to maximize its expectation of the sum of all discounted profits while $P_t^*(j)$ remains effective. The profit-optimization problem is given by

$$max_{P_t^*(j)} \Sigma_{k=0}^{\infty} \theta^k \mathbb{E}_{t,j}^{Firm}\left\{Q_{t,t+k}\left[P_t^*(j)Y_{t+k}(j) - MC_{t+k}(j)N_t(j)\right]\right\}, \tag{22}$$

where θ is the Calvo parameter, the probability that the current price does not change from this period to the next. The individual firm's expectations operator $\mathbb{E}_{t,j}^{Firm}$ is conditional on its information set, $I_{t,j}^{Firm}$. $Q_{t,t+k}$ is the stochastic discount factor given by: $Q_{t,t+k} = \beta^k \frac{U'(C_{t+k})}{U'(C_t)} \frac{P_t}{P_{t+k}}$. $MC_{t+k}(j)$ denotes the firm-specific nominal marginal cost.

The optimal price for a firm j that can re-optimize at time t is determined by the discounted sum of its current and expected future nominal marginal costs

$$p_{t,j}^* = (1-\beta\theta)\mathbb{E}_{t,j}^{Firm} \sum_{k=0}^{\infty} (\beta\theta)^k (p_{t+k} + mc_{t+k,j}), \tag{23}$$

where $mc_{t,j}$ is firm j's real marginal cost at time t. Following Nimark (2008), we assume $mc_{t,j}$ is a linear combination of the equilibrium real wage and some supply-side shock. The supply-side shocks include an *i.i.d.* aggregate cost-push shock $u_t \sim N(0,\sigma_u^2)$, and an idiosyncratic wage bargaining shock $\eta_{t,j} \sim N(0,\sigma_\eta^2)$, such that

$$mc_{t,j} = w_t + u_t + \eta_{t,j}. \tag{24}$$

The idiosyncratic components of $mc_{t,j}$ are canceled out in aggregation, i.e., $\int_{[0,1]} \eta_{t,j} dj = 0$.

Firms observe their own marginal costs perfectly, but cannot separately observe the economy-wide shocks $\{d_t, u_t\}$ and the firm-specific bargaining shock $\eta_{t,j}$. When a firm observes a higher real marginal cost, it cannot perfectly tell whether it is due to a higher real demand that drives up the equilibrium wage, a positive aggregate cost-push shock, or a positive firm-specific shock to the bargaining power of its workers.

3.1.3 Central Bank

We assume the central bank sets the nominal interest rate i_t following a Taylor-type targeting rule. Due to imperfect information, the central bank sets the interest rate in response to the central bank's expected, not actual, inflation and output.[6] Specifically, the interest rate follows

$$i_t = \phi_\pi \mathbb{E}_t^{CB} \pi_t + \phi_y \mathbb{E}_t^{CB} y_t + e_t. \tag{25}$$

$\mathbb{E}_t^{CB}$ denotes the central bank's expectation operator, and $e_t \sim N(0, \sigma_e^2)$ stands for the exogenous monetary policy shock. The policy coefficients satisfy $\phi_\pi > 1, \phi_y > 0$.

3.1.4 Exogenous Signals

There are three aggregate shocks in the economy: a persistent demand shock d_t, a transitory cost-push shock u_t, and a monetary policy shock e_t. We assume all agents in the economy receive a public signal of the demand shock, which follows

$$n_t = d_t + \varepsilon_t^n, \quad \varepsilon_t^n \sim N(0, \sigma_n^2). \tag{26}$$

In addition to n_t, the central bank also receives a private signal, m_t, which follows

$$m_t = d_t + \varepsilon_t^m, \quad \varepsilon_t^m \sim N(0, \sigma_m^2). \tag{27}$$

To avoid unnecessary computation complexity, we assume there are no signals about the cost-push shock.

3.1.5 Timing Protocol and Information Sets

The timing protocol follows the convention in the literature on the information effect of monetary policy. Most importantly, we assume that in each period, the central bank sets the interest rate before the private sector forms expectations and makes consumption and pricing decisions. Any period t is divided into three stages. At stage 1, all shocks are realized, and all exogenous signals are observed by corresponding agents. At stage 2, the central bank

[6] Since we do not model technology shocks, there are no fluctuations in the natural level of output. Hence, equilibrium output equals the output gap in our model.

sets the interest rate. At stage 3, agents in the private sector form expectations and make economic decisions.

We now describe the information sets for each agent. Since the demand shock directly enters the household's utility function, we assume that the household has perfect information about the entire history of the demand shocks. In addition, it turns out that the model's quantitative predictions are not sensitive to whether or not the household observes the temporary cost-push shock. This is because as the central bank does not have information about the cost-push shock, the interest rate does not respond to the cost-push shock. Consequently, a temporary cost-push shock drives a one-time jump in inflation on impact, without causing any future deviations in either output or inflation. Therefore, without loss of generality, we assume that the household has perfect information on all shocks.

We assume that the central bank's information set includes the exogenous signals of the demand shock and the monetary policy shock up till t. It also includes endogenous signals of lagged inflation and output.

$$\mathcal{I}_t^{CB} = \{n_{t-k}, m_{t-k}, e_{t-k}, \pi_{t-1-k}, y_{t-1-k} | k \geq 0\}. \tag{28}$$

This assumption is motivated by the fact that, in practice, most central banks, including the Federal Reserve and the European Central Bank, closely monitor statistics about the aggregate economy, which often come with a time lag.

Due to the scarcity of surveys of firm expectations, it is difficult to gauge how firms form expectations. For example, while Melosi (2017) and Nakamura and Steinsson (2018) allow all firms to learn from nominal interest rates, Candia, Coibion, and Gorodnichenko (2024) find that inattention to monetary policy is pervasive among firms. Similarly, Coibion, Gorodnichenko, and Kumar (2018) point to firms being inattentive to equilibrium outcomes (i.e., GDP and inflation), especially during normal times. We therefore consider three types of firms, with three nested information sets, $\mathcal{I}_{t,j}^{Firm}$.

$$\textbf{Type I:} \quad \mathcal{I}_{t,j}^{Firm} = \{mc_{t-k,j}, n_{t-k} | k \geq 0\}; \tag{29}$$

$$\textbf{Type II:} \quad \mathcal{I}_{t,j}^{Firm} = \{mc_{t-k,j}, n_{t-k}, i_{t-k} | k \geq 0\}; \tag{30}$$

$$\textbf{Type III:} \quad \mathcal{I}_{t,j}^{Firm} = \{mc_{t-k,j}, n_{t-k}, i_{t-k}, \pi_{t-1-k} | k \geq 0\}. \tag{31}$$

Of all $\mathcal{I}_{t,j}^{Firm}$ types considered, firms always learn from the histories of their real marginal cost and the public signal. Each firm's real marginal cost serves as a private signal about both the aggregate demand shock and the cost-push shock.

Type I firms are the least informed ones among the three types, as their information sets do not have additional aggregate equilibrium variables. We consider the case in which firms are Type I to be the no-information-effect benchmark. Our main analysis is the economy in which firms are Type II firms. Since Type II firms observe the interest rate, monetary policy has an information effect. Type III firms are the most informed ones, because on top of the signals observed by Type II firms, Type III firms also learn from the history of realized inflation.[7]

Besides households, firms, and the central bank, there is also a professional forecaster, allowing us to connect our model to the SPF data and the literature on the high-frequency identification of monetary policy surprises. A professional forecaster forms expectations twice in a given period: once before and once after the FOMC meeting. We specify the pre-FOMC and post-FOMC information sets of the professional forecaster as:

$$\mathcal{I}_t^{PF,pre} = \{n_{t-k}, i_{t-k-1}, \pi_{t-1-k}, y_{t-1-k} | k \geq 0\}. \tag{32}$$

$$\mathcal{I}_t^{PF,post} = \{n_{t-k}, i_{t-k}, \pi_{t-1-k}, y_{t-1-k} | k \geq 0\}. \tag{33}$$

The professional forecaster has the same information about lagged aggregate equilibrium variables as the central bank, but does not know the central bank's private signals, m_t, or the exogenous monetary policy shocks e_t. The Professional forecaster keeps track of the history of interest rates, but cannot distinguish the policy shock. The only difference between $\mathcal{I}_t^{PF,pre}$ and $\mathcal{I}_t^{PF,post}$ is the inclusion of the latest interest-rate decision i_t in the latter.

3.1.6 Solution to the Aggregate Equilibrium

The inflation rate is defined as the change in the aggregate price level, $\pi_t = p_t - p_{t-1}$. To solve for the aggregate price level, one needs to integrate prices of all firms. Individual prices are functions of firm-specific expectations of the aggregate price, as specified in Equation (23). However, expectations of the aggregate price are heterogeneous due to private information.

To address the issue, we first follow Han, Ma, and Mao (2023) and reformulate the individual firm's optimal pricing problem with an auxiliary variable, $z_{t,j}$. Define $z_{t,j} = (1-\theta)(p^*_{t,j} - p_{t-1})$, so that the price aggregation implies $\pi_t = \int_{[0,1]} z_{t,j} dj$. Rewriting equation (23) recursively in terms of $z_{t,j}$ leads to

$$z_{t,j} = (1-\beta\theta)(1-\theta)mc_{t,j} + (1-\theta)\mathbb{E}_{t,j}^{Firm}\pi_t + \beta\theta\mathbb{E}_{t,j}^{Firm} z_{t+1,j}, \tag{34}$$

[7]We omit the case where $\mathcal{I}_{t,j}^{Firm} = \{mc_{t-k,j}, n_{t-k}, i_{t-k}, \pi_{t-1-k}, y_{t-1-k} | k \geq 0\}$, because we will show later that the economy with Type II firms and the economy with Type III firms are very similar, and thus the alternative specification of adding both lagged inflation and lagged output to firms' information set would yield similar results.

Equation (34) suggests that firms' expectations have a contemporaneous effect on actual inflation. If there is a public noise shock that drives up firms' expectations of current inflation, $\mathbb{E}_{t,j}^{Firm}\pi_t$, firms increase their prices in addition to their actual real marginal costs $mc_{t,j}$, which leads to higher actual inflation.

The equilibrium conditions (21), (34) and (25) contain three types of non-nested conditional expectations $\mathbb{E}_t^{HH}$, $\mathbb{E}_{t,j}^{Firm}$, and $\mathbb{E}_t^{CB}$, resulting in a breakdown of the law of iterated expectations and leading to the infinite regress problem of "forecasting the forecast of others" (see Townsend (1983)). Solving the equilibrium in the time domain requires including many higher-order expectations to form a suitable state space. Frequency-domain methodologies (Kasa (2000), Huo and Takayama (2025), Rondina and Walker (2021), Jurado (2023), and Chahrour and Jurado (2025)) effectively circumvent the infinite regress problem by transforming the equilibrium to only one state variable (i.e., the frequency z in the spectral density). We apply the analytic policy function iteration (APFI) method and the "z-Tran" toolbox developed in Han, Tan, and Wu (2022), which is built upon policy function iteration in the frequency domain and is particularly suitable for solving equilibria with asymmetric information sets and endogenous signals.

Technically, given a candidate model solution, Han, Tan, and Wu (2022) shows how to apply the inverse discrete-time Fourier transform to calculate conditional expectations in the frequency domain when applying the Wiener-Hopf formula. APFI then utilizes the equilibrium conditions to construct an updated solution candidate given the existing candidate and its implied conditional expectations. APFI stops when the relative distance between the updated and existing solution candidates is smaller than a prespecified criterion.[8]

3.2 Calibration

We calibrate the model to the US economy at a quarterly frequency. First, for parameters that are commonly used in the New Keynesian literature, we directly assign them values that are standard in the literature. Specifically, the discount factor, β, is set to 0.99. The inverse Frisch elasticity of labor supply, $1/\psi$, is set to 0.5. We set the Calvo parameter $\theta = 2/3$, consistent with an average price duration of three quarters. The habit formation parameter, h, is set to 0.75. The Taylor rule coefficients are $\phi_\pi = 1.5$ and $\phi_y = 0.5$. The AR(1) parameter of the demand shock is set to be $\rho = 0.9$.

Our calibration focuses on the parameters that determine the degree of the information frictions and thereby, determine the size of the information effect of monetary policy. These parameters govern the precision of the priors, the private and public signals, and the standard

[8]See Section 4 of Han, Tan, and Wu (2022) for a detailed description of the algorithm.

Table 1: Data and Model-Simulated Moments

Moment	Data	Type-I firms	Type-II firms	Type-III firms
$\sigma(\pi_t)$	0.59	0.59	0.59	0.59
$\sigma(mps_t)$	0.05	0.05	0.05	0.05
avg. $\lvert\Delta p_j\rvert$	9.7%	9.7%	9.7%	9.7%
$\sigma(\pi_t - \mathbb{E}_t^{CB}\pi_t)$	0.23	0.23	0.22	0.22
$\sigma(\pi_t - \mathbb{E}_{t-1}^{CB}\pi_t)$	0.28	0.31	0.32	0.32
$\sigma(\pi_t - \mathbb{E}_t^{PF}\pi_t)$	0.24	0.23	0.22	0.22
$\sigma(\pi_t - \mathbb{E}_{t-1}^{PF}\pi_t)$	0.31	0.31	0.33	0.33

deviation of the policy shock, i.e.,

$$\mathcal{P} = \{\sigma_d, \sigma_u, \sigma_\eta, \sigma_n, \sigma_m,\ \sigma_e\}.$$

Since we consider that firms' information sets may be either Type I, Type II, or Type III, we perform three rounds of calibration. In each round, parameters are jointly calibrated to match the standard deviation of inflation, cross-sectional price dispersion, the standard deviation of monetary policy surprises,[9] and the nowcast and forecast accuracies of inflation in the Greenbook and the Survey of Professional Forecasters (SPF).

The targeted moments, together with their empirical and model-predicted values, are displayed in Table 1. Although targeted moments are determined simultaneously, in what follows, we discuss each moment in relation to the parameters for which the moment yields the highest identification power. First, we choose the standard deviation of the demand shock, σ_d, to match the standard deviation of quarterly inflation.[10] Second, we calibrate σ_e to target the standard deviation of the high-frequency monetary policy surprises in the empirical literature.[11] Third, we calibrate the standard deviation of the individual firm's wage bargaining shock $\eta_{t,j}$ to match the micro evidence on firms' pricing behaviors. Specifically, the target moment is the average absolute size of price changes for the price-changing firms, which is found to be 9.7 percent in Klenow and Kryvtsov (2008).

The rest of the parameters are calibrated to match the nowcast and forecast accuracies in

[9] In our model, the monetary policy surprise (mps_t) is defined as the difference between the actual and the professional forecaster's pre-FOMC expected interest rate: $mps_t = i_t - \mathbb{E}_t^{PF,pre} i_t$. This definition corresponds to the empirical measure of monetary policy surprises using the high-frequency identification method.

[10] We use the GDP deflator to measure the inflation rate for both the actual and the forecast data. Data on the actual GDP deflator are retrieved from the FRED database for the sample period 1970 Q1 – 2024 Q2.

[11] Acosta, Brennan, and Jacobson (2024) constructed a time series of monetary policy surprises using approaches similar to those in Gürkaynak, Sack, and Swansonc (2005) and in Nakamura and Steinsson (2018) for an updated sample period. Depending on different definitions, the standard deviations of the monetary policy surprises range from 0.032 to 0.098. We use 0.05 as a rough average.

Table 2: Calibrated Parameters $\mathcal{P}$

Shock description	std	Type-I firms	Type-II firms	Type-III firms
Demand shock d_t	σ_d	2.18	1.88	1.88
Cost-push shock u_t	σ_u	1.85	1.85	1.85
Monetary policy shock e_t	σ_e	0.05	0.01	0.01
Firm j's wage bargaining shock $\eta_{t,j}$	σ_η	33.8	29.1	29.1
Public noise ε_t^n	σ_n	6.56	6.02	6.03
CB's private noise ε_t^m	σ_m	10.5	11.3	11.3

Note: the unit is percent.

the Greenbook and the SPF. First, the aggregate cost-push shock drives temporary inflation fluctuations that are unpredictable by the central bank. We thus calibrate the standard deviation of the cost-push shock by targeting the standard deviation of the forecast errors in the Greenbook. Second, the standard deviations of the public and private signals, $\{\sigma_n, \sigma_m\}$, jointly determine how far away the central bank's and the professional forecaster's expectations of the current economy are from the actual data. We calibrate them to match the standard deviations of the inflation nowcast (inflation forecast of the current quarter) errors in the Greenbook and the SPF.

The calibrated values are presented in Table 2. First, the calibrated standard deviations of demand and cost-push shocks are similar across the three specifications. Second, similar to Mackowiak and Wiederholt (2009) and Andrade et al. (2022), we also find sizable variation in firms' idiosyncratic conditions. The calibrated standard deviation of an individual firm's wage bargaining shock, σ_η, is more than ten times larger than the standard deviations of the aggregate demand and cost-push shocks. This can explain the empirical fact that individual price changes can be significant while aggregate inflation remains mild.

The calibrated standard deviation of the policy shock is much smaller in the model of Type II (0.01) and Type III firms (0.01) than in the model of Type I firms (0.05), and is also much smaller than the values commonly used in the traditional New Keynesian literature, $\sigma_e \in [0.2, 1]$. This implies that under the information effect of monetary policy, monetary policy surprises are driven by other components in addition to the policy shock. A small value of σ_e is more aligned with the general intuition that in recent decades, the Federal Reserve has had reasonably good control of the policy rate.

Finally, we find that, across all three cases, the public and the central bank's private signals are both noisy, as suggested by the large values of σ_n and σ_m relative to σ_d. The standard deviation of the central bank's inflation nowcast errors from the Greenbook is 0.23, which is over one-third of the standard deviation of actual inflation (0.59). If either the public or the private signal is precise, the central bank would have close-to-zero nowcast errors. We find that the central bank's private information is even less precise than the public information, as $\sigma_m > \sigma_n$. This is consistent with the fact that the nowcast errors of professional forecasters (SPF) are very similar to the ones of the central bank (Greenbook). A very noisy central bank private signal indicates that the central bank does not have information advantage over professional forecasters in real time, which is consistent with Bauer and Swanson (2023).

4 Quantitative Analysis

In this section, we analyze the information effect of monetary policy by comparing model predictions under different information structures. We consider the economy in which all firms are Type II firms (see Equation (30)) to be our baseline case, and define two benchmark cases for comparison. The first benchmark is the full-information benchmark. The second benchmark is the "no-information-effect" benchmark, in which case firms are all Type I firms that do not use the interest rate as a signal (see Equation (29)).[12] We first highlight the role of imperfect information in Section 4.1, and then study the information effect of monetary policy in Section 4.2. Finally, we illustrate how the size of the information effect depends on the precision of signals (Section 4.3) and the implications of the information effect on the Phillips curve (Section 4.4).

4.1 Two Benchmark Cases: Full Information vs. Imperfect Information

Figure 3 plots the impulse responses of output y_t, inflation π_t, and the nominal interest rate i_t after a demand shock, a noise shock in the public signal, a noise shock in the central bank's signal, a policy shock, and a cost-push shock. The differences between the full-information benchmark (dot-dashed red line) and the no-information-effect benchmark (dashed blue line) capture the effect of imperfect information on macroeconomic dynamics.

After a positive demand shock, output increases by a larger amount under the no-information-effect case (dashed blue line) than under the full-information case (dot-dashed red line). This is because with imperfect information, the central bank knows little about the demand shock on impact, and therefore the central bank fails to tighten the policy rate enough to offset the positive demand shock, leading to a larger output gap than the one under full information.

[12] To make sure that the differences in model predictions are explained by the difference in information structure only, we set the parameter values across all models to be the same ones that are calibrated for Type II firms in Section 3.2.

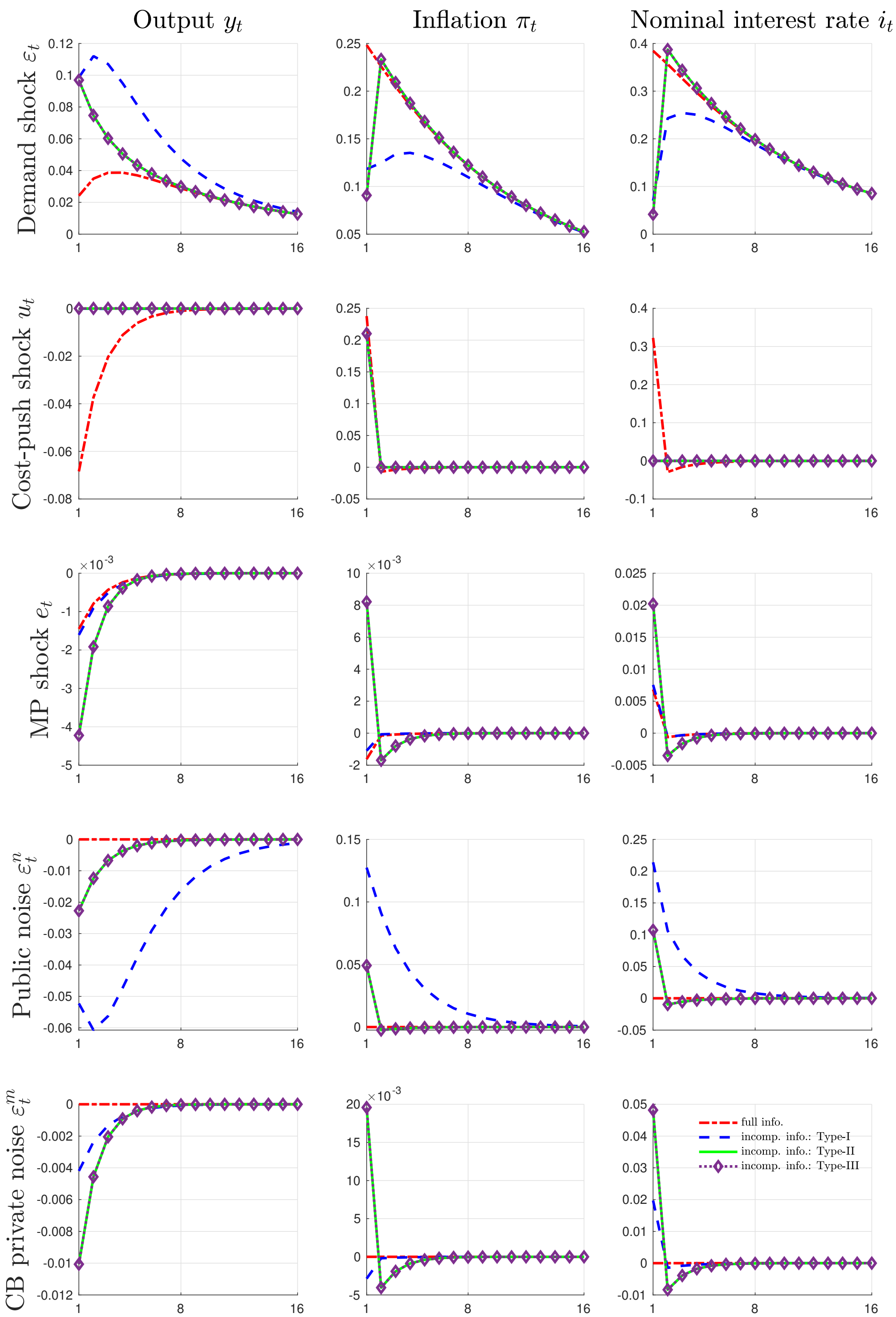


Figure 3: Impulse Responses

Note: In each row, the size of the shock equals one standard deviation of the corresponding shock.

Although the output gap is larger under the no-information-effect benchmark case, it does not feed into higher inflation. On the contrary, inflation is lower in the no-information-effect case than in the full-information case. This is because firms underestimate the actual size of the rise in output due to imperfect information about the persistent demand shock. Therefore, firms increase prices by a lesser amount than they would under full information. Inflation under imperfect information is persistently lower than that under full information.

The second row of Figure 3 shows the dynamics after a positive cost-push shock. Under full information, the central bank responds by tightening monetary policy, leading to a negative output gap and positive inflation on impact. Under imperfect information, the central bank does not have any information about the cost-push shock on impact, and therefore does not adjust the interest rate. The household is also assumed to have no information about the cost-push shock or the rising inflation. In the absence of interest-rate changes or information about price changes, output stays unchanged. Inflation rises on impact, because the cost-push shock increases the production costs of all firms, and firms that win the Calvo lottery reset prices in response to the changes in their production costs. Inflation is lower under the no-information-effect case than in the full-information case. This is because price-resetting firms change prices only in response to the change in their firm-specific production costs while failing to optimize its price in response to the rising aggregate price level. In other words, the strategic complementarity in price-setting is greatly reduced due to imperfect information.

The third row of Figure 3 shows that after a monetary policy shock, the full-information benchmark economy and the no-information-effect benchmark economy evolve similarly. The slight difference between the dot-dashed red line and the dashed blue line occurs because the interest rate is not directly observable under the no-information-effect case, but rather, firms know the direct effect of the interest rate through observing their marginal cost.

The last two rows show the impulse responses after a noise shock in the public information and in the central bank's private information. By assumption, the full-information benchmark is free from noise shocks. With imperfect information, the public noise shock works like a supply shock, driving positive inflation and negative output gaps. In this case, all firms and the central bank mistakenly believe that output would be higher, and therefore, firms increase prices and the central bank raises the interest rate. Both the higher aggregate price level and the higher interest rate lower demand in the absence of an actual demand shock. The economy after a noise shock to the central bank's private information is similar to the one after a monetary policy shock. In both cases, the higher interest rate lowers output and inflation at the same time.

The role of imperfect information can be summarized in the following proposition.

Proposition 3 *Compared with the full-information benchmark case, imperfect information*

- *reduces inflation fluctuations and widens output gaps after demand shocks,*

- *reduces both inflation fluctuations and output gaps after cost-push shocks, and*
- *does not significantly change the effect of monetary policy shocks.*

4.2 The Information Effect of Monetary Policy

We analyze the information effect of monetary policy conditional on the assumption of imperfect information. To this end, we compare our baseline economy, in which case all firms are Type II, with the no-information benchmark, in which case all firms are Type I.

The first row of Figure 3 shows that after a positive demand shock, the economy with the information effect of monetary policy (solid green line) and the no-information-effect benchmark economy (dashed blue line) are the same at $t = 1$ when the demand shock hits the economy. However, over time, the information effect of monetary policy brings the economy closer to the full-information benchmark. This is because after the first period, the central bank learns the realization of the demand shock from lagged inflation and output, and this information is transmitted to firms as the interest rate enters the firms' information sets, making firms' pricing decisions more similar to the ones under full information. In turn, as inflation and output become more reflective of the demand shock, the central bank learns more precise information over time. Although inflation and the interest rate quickly converges to that under perfect information, the convergence of output takes about two years due to households' habit formation in consumption.

The second row shows that the economy after a transitory cost-push shock does not differ much with or without the information effect of monetary policy. This is because when the cost-push shock hits the economy at $t = 1$, the output gap is zero and inflation is positive. In the following period, when the central bank learns from inflation and output with a one-period lag, it understands that the shock was a transitory cost-push shock and, therefore, does not change the interest rate at $t = 2$. As the interest rate stays unchanged, no information is revealed.

The third row shows that after a positive monetary policy shock, inflation rises and output decreases on impact under the information effect of monetary policy, which is a key difference from the no-information-effect benchmark. The reason is that with the information effect, firms mistakenly believe the increase in the interest rate reflects the central bank's private information about a positive demand shock, and therefore increase prices. Both positive inflation and the positive interest rate dampen demand, leading to a negative output gap.

The fourth row shows the responses to a positive noise shock to the public information. On impact, inflation and output move in the same direction with or without the information effect of monetary policy. However, the economy under the information effect of monetary policy quickly converges to the full-information benchmark after $t = 2$. The reason is that by observing a negative output gap and a positive inflation in the last period, the central bank learns that the public information reflects a noise shock instead of an actual demand shock. It then swiftly lowers the interest rate at $t = 2$, and subsequently, firms update their expectations after observing the interest-rate change. The information effect of monetary policy helps stabilize inflation and output gap fluctuations that are caused by public noise shocks.

The last row exhibits the economy after a positive noise shock to the central bank's private information. Contrary to the case of a public noise shock, the economy under the information effect of monetary policy experiences greater fluctuations in both inflation and output gaps. In this case, the central bank mistakenly believes that output will be higher and thus raises the interest rate. The information effect of monetary policy makes firms positively update their inflation expectations, and hence raise prices, leading to higher actual inflation. Both a higher interest rate and higher prices lead to a negative output gap. At $t = 2$, the central bank observes the lagged output gap and realizes its previous private information was noise. It then lowers the interest rate immediately, but it takes several quarters to stabilize the output gap due to habit formation in consumption.

We summarize the information effect of monetary policy in the following result:

Proposition 4 *Compared to the no-information-effect benchmark, the information effect of monetary policy*

- *increases inflation fluctuations and reduces output gaps after a demand shock,*
- *reduces fluctuations in both inflation and output gaps after a public noise shock, and*
- *leads to positive inflation after a tightening policy shock.*

Does the information effect change if firms' information sets also contain lagged equilibrium variables? That is, what if firms are Type III? Figure 3 shows that impulse responses of output, inflation, and the interest rate are the same whether firms' information sets include lagged inflation or not. This indicates that as the current interest rate contains the central bank's knowledge of lagged inflation, firms do not need to pay additional attention to lagged inflation. In other words, lagged inflation provides redundant information to firms on top of the interest rate.

4.3 Determinants of the Size of the Information Effect

Proposition 2 argues that a quantitatively important information effect relies on two conditions: first, the central bank's private information needs to be superior to the public information, and second, the exogenous variation in the interest rate needs to be small. In this section, we study how the size of the information effect may be reduced when the central bank has less precise private information and when the interest rate has a larger exogenous variation.

In the context of our New Keynesian model, the central bank's information advantage does not come from its private exogenous signal, ϵ_t^m. In fact, our calibration suggests that the central bank's private exogenous signal provides very imprecise information. Rather, the source of the central bank's information advantage is a result of the assumption that the central bank learns from lagged equilibrium variables, whereas Type II firms do not. To test how the size of the information effect depends on the central bank's ability to learn from lagged equilibrium variables, we simulate a counter factual economy, in which the central bank observes past equilibrium variables with some observation errors. Specifically, the central bank's information set is given by,

$$\mathcal{I}_t^{CB} = \{n_{t-k}, m_{t-k}, e_{t-k}, \hat{\pi}_{t-1-k}, \hat{y}_{t-1-k} | k \geq 0\}. \tag{35}$$

where $\hat{\pi}_t = \pi_t + \epsilon_t^\pi$ and $\hat{y}_t = y_t + \epsilon_t^y$, and the observation errors follow $\varepsilon_t^\pi \sim N(0, \sigma_\pi^2)$, $\varepsilon_t^\pi \sim N(0, \sigma_\pi^2)$.

In Figure 4, we plot the economic dynamics when $\sigma_y = \sigma_\pi = 0.5$. Comparing the counterfactual economy (dot-dashed black line) and our baseline economy (solid green line), we find that when the central bank has less precise information about the past economy, the information effect is reduced: after demand shocks, it takes longer for the economy to converge to the full-information benchmark. On the other hand, the output and inflation deviations caused by the exogenous policy shock are also reduced. The last row in Figure 4 shows that fluctuations caused by the noise in the central bank's private exogenous information increase. After a positive ϵ_t^m shock, the size of the output gap and the inflation rate both become larger when the central bank has observation errors. This is because as the central bank's endogenous information becomes less precise, its private exogenous signal gets a larger weight and thus has a larger impact on equilibrium inflation and output.

Another important determinant of the size of the information effect of monetary policy is the standard deviation of the policy shock. A greater standard deviation of the policy shock makes the interest rate a nosier signal. In Figure 4, we plot the impulse responses in a counter factual economy in which the standard deviation of the policy shock increases to 0.25 (dashed pink line). It shows that a larger σ_e decreases the size of the information effect of monetary policy. After a demand shock, it takes longer for the economy to converge to the full-information benchmark. On the other hand, the fluctuations caused by either a

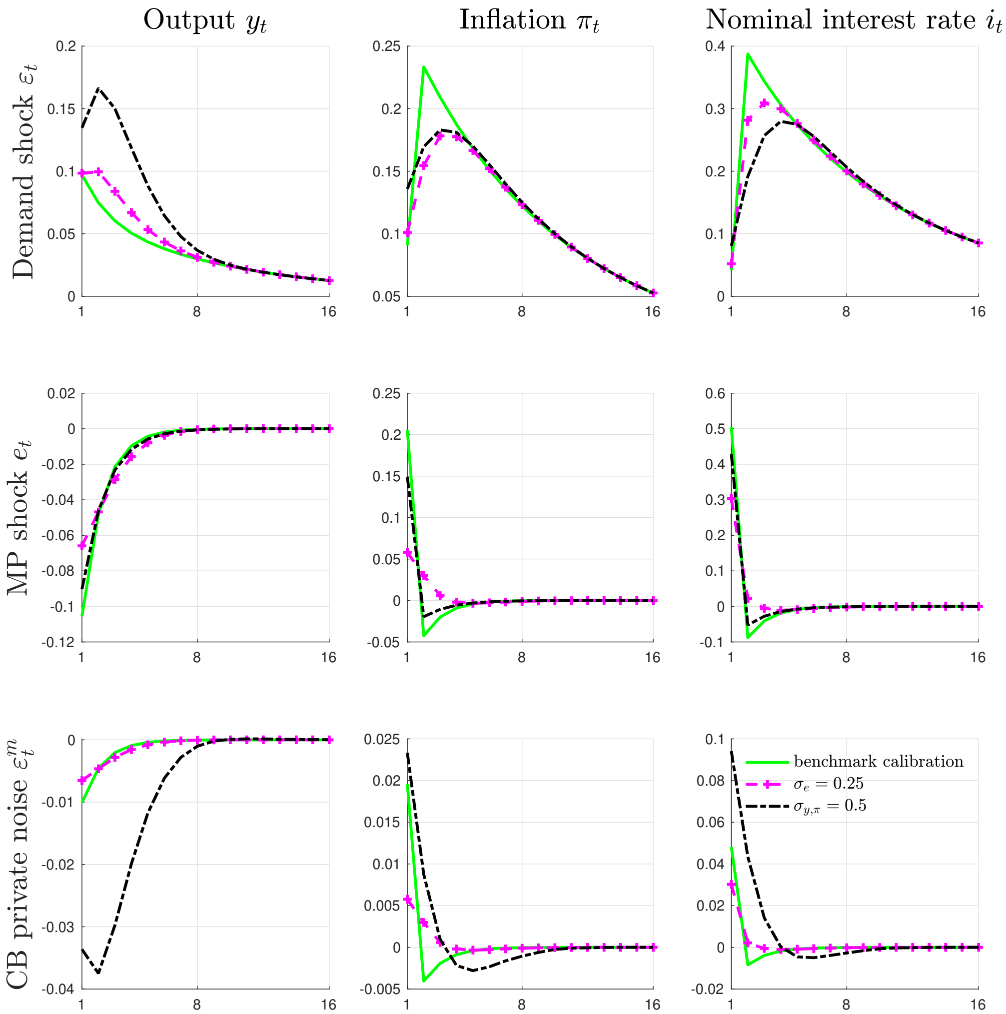


Figure 4: Impulse Responses

Note: In the first and the last rows, the size of the shock equals one standard deviation of the corresponding shock. In the second row, to make the impulse responses comparable when σ_e changes, we fix $\sigma_e = 0.25$ for all three cases.

policy shock or a noise shock from the central bank's private exogenous information are also reduced when the information effect weakens.

The comparison between our baseline case and the two counter factual economies highlights the important mechanism of two-way learning: the central bank learns from the private sector's demand and pricing decisions (with a one-period lag), and the private sector learns from the central bank's interest-rate decisions. If the effectiveness of one side of the learning process weakens, the information transmission of the other side is undermined, too. For example, after a positive demand shock, when the cental bank has observation errors, the central bank learns less from the private sector's decisions, and hence, the interest rate adjusts to the demand shocks to a lesser extent than it would in the baseline economy. In turn, because the interest rate responds by less, the information transmission from the central bank to the private sector becomes less precise.

4.4 A Non-Linear Phillips Curve

After a decade of low and stable inflation following the Great Recession, the surge of inflation in the early 2020s sparked a discussion of a "non-linear" Phillips curve. Most of the previous studies attribute the steepening of the Phillips curve in the post-COVID period to labor market frictions or to supply-chain disruptions (Benigno and Eggertsson (2023) and Ball, Leigh, and Mishra (2022)). Our model suggests that a change in the estimated slope of the Phillips curve could be driven by a change in firms' expectation-formation process.

The impulse responses in Figure 3 offer the intuition of why the information effect may change the empirical relationship between output and inflation. When the information effect provides correct information about the demand shock that drives up output, inflation is higher for a given output level. In this case, the information effect strengthens the correlation between output and inflation. On the other hand, when the interest rate responds to a noise shock or to a policy shock, and the information effect makes firms mistakenly believe there is a demand shock, inflation and output move in the opposite direction. In this case, the information effect weakens the correlation between inflation and output.

To see how the information effect alters the empirically estimated slope of the Phillips curve, we perform the following ordinary least squares regression using simulated moments from our model:

$$\pi_t = \beta_c + \beta_y y_t + + \beta_u u_t + \beta_{\mathbb{E}\pi} \mathbb{E}_t \pi_{t+1}^{pf} + \upsilon_t \tag{36}$$

where $\pi_t, y_t, u_t, \mathbb{E}_t \pi_{t+1}^{pf}$ are model-simulated inflation, output, the cost-push shock, and the one-period-ahead inflation expectations of professional forecasters. $\beta_c, \beta_y, \beta_u, \beta_{\mathbb{E}\pi}$ are parameters to be estimated, and υ_t is a zero-mean normally distributed error.

Table 3: The Slope of the Phillips Curve

	Full Information	No Information Effect	With Information Effect
output	2.229***	0.293***	1.529***
	(0.013)	(0.002)	(0.004)
cost-push shock	0.212***	0.114***	0.114***
	(0.001)	(0.000)	(0.000)
expected inflation	0.531***	0.891***	0.609***
	(0.003)	(0.002)	(0.001)
constant	0.001*	0.001	0.001
	(0.001)	(0.001)	(0.000)
R^2 adjusted	0.878	0.871	0.930

Notes: The table presents the estimated coefficients of the regression given by Equation (36). Data points are model simulated.

To mimic the methods used in the empirical literature, we control for the cost-push shock because it is widely known that the endogenous policy response to cost-push shocks leads to a downward bias in the estimation of the slope coefficient. We do not control for noise shocks because noise shocks are hard to measure empirically and are typically not controlled for. Although inflation should be directly determined by firms', not professional forecasters', expected inflation, we control for professional forecasters' expected inflation. This is because surveys of professional forecasters are more widely used to control for inflation expectations in the empirical literature.

Table 3 shows the regression results. The estimated slope of the Phillips curve, β_y, is the largest in the full-information benchmark and is the smallest in the no-information-effect benchmark. It is plausible to consider that the US economy from 2008 to 2020 was similar to the no-information-effect benchmark, in which case firms pay little attention to aggregate inflation or monetary policy decisions. Candia, Coibion, and Gorodnichenko (2024) find evidence that US firms are largely uninformed about both inflation and monetary policy in a low-inflation environment. In contrast, a rational expectations model predicts that firms pay more attention to information on aggregate inflation when actual inflation is high or volatile, which is likely to describe the high-inflation period after the COVID-19 recession. Chahrour, Shapiro, and Wilson (2024) find that exposure to high-inflation news changes inflation expectations more than exposure to low-inflation news, and that COVID-era media coverage of inflation significantly contributed to the increase in households' inflation expectations. Our regression exercise shows that when firms start to pay more attention to the aggregate economy and learn from the interest rate, the estimated slope coefficient of the Phillips curve increases.

The goal of this regression exercise is not to produce an unbiased estimation, but to show that the information effect of monetary policy may change the empirically estimated slope of the Phillips curve. In fact, a key variable that we do not control for is the expectation nowcast, $\mathbb{E}_t^s \pi_t$. Our model suggests that monetary policy affects both y_t (the direct effect) and $\mathbb{E}_t^s \pi_t$ (the information effect), and thus the error term in Equation (36) correlates with y, leading to a biased estimation. Some scholars point out that a key distinction between the COVID-19 high-inflation period and the 1970s high-inflation period is that unlike in the 1970s, *long-term inflation expectations* remained mostly stable during the COVID-19 period. Our model points out a key role of *inflation nowcasts (or near-term inflation forecasts)* in explaining the estimated slope of the Phillips curve.

5 Conclusion

In this paper, we first analytically show that the reduced-form estimation of the information effect of monetary policy is likely to be biased even after controlling for economic news. We then develop a New Keynesian model under asymmetric information and two-way learning: the central bank learns from the demand and pricing decisions of the private sector, and the private sector learns from the central bank's interest-rate decisions.

The main takeaways from our calibrated model are: first, both the central bank and professional forecasters have noisy exogenous information about current shocks. However, endogenous information from lagged equilibrium variables, including output and inflation, provides precise information. The two-way learning mechanism helps the information transmission between the central bank and the private sector, reducing the output gaps caused by demand shocks or noise shocks. At the same time, the information effect also leads to positive inflation after a policy tightening shock. We also show that the size of the information effect may be reduced if the central bank has observation errors or if the policy shock has a larger standard deviation. Finally, we show that the information effect may explain the non-linear Phillips curve in the COVID-19 high-inflation period.

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Appendices

A Detailed Derivations to the Analytic Model in Section 2

This appendix presents the derivations of the analytic model in Section 2. We briefly reiterate the model environment for completeness.

A.1 The Central Bank's Expectations and the Interest-Rate Rule

The state of the economy (i.e., the unobserved shock) follows $x_t \sim N(0,\sigma_x^2)$. The economic outcome y_t follows $y_t = x_t - \kappa i_t$, where i_t is the nominal interest rate and $\kappa > 0$ captures the direct effect of monetary policy. Monetary policy follows a targeting rule, given by

$$i_t = \phi_y \mathbb{E}_t^{cb} y_t + e_t, \quad e_t \sim N(0,\sigma_e^2), \tag{A.1}$$

with $\phi_y > 0$ and $\mathbb{E}_t^{cb}(\cdot)$ stands for the central bank's expectations operator.

To write i_t in terms of the central bank's expectations on the state of the economy, first substitute y_t with $y_t = x_t - \kappa i_t$ in Equation (A.1). Rational expectations dictate that $\mathbb{E}_t^{CB} i_t = i_t$, since i_t is the central bank's decision variable. Re-arranging the monetary policy rule (A.1) yields

$$i_t = \frac{\phi_y}{1+\kappa\phi_y}\mathbb{E}_t^{cb} x_t + \frac{1}{1+\kappa\phi_y} e_t.$$

Next, we express $\mathbb{E}_t^{cb} x_t$ with respect to signals. The public signal n_t is defined as

$$n_t = x_t + \varepsilon_t^n, \quad \varepsilon_t^n \sim N(0,\sigma_n^2). \tag{A.2}$$

The central bank also receives a private signal, given by

$$m_t = x_t + \varepsilon_t^m, \quad \varepsilon_t^m \sim N(0,\sigma_m^2).$$

The central bank's information set is $\mathcal{I}_t^{cb} = \{n_t, m_t\}$. Denote $\nu_x = 1/\sigma_x^2, \nu_e = 1/\sigma_e^2, \nu_n = 1/\sigma_n^2$, and $\nu_m = 1/\sigma_m^2$. It follows

$$\mathbb{E}_t^{cb} x_t = \mathbb{E}_t\left[x_t | \mathcal{I}_t^{cb}\right] = \frac{\nu_m}{\nu_x+\nu_m+\nu_n} m_t + \frac{\nu_n}{\nu_x+\nu_m+\nu_n} n_t. \tag{A.3}$$

Substitute $\mathbb{E}_t^{cb} x_t$ with Equation (A.3), and get

$$i_t = \frac{\phi_y}{1+\kappa\phi_y}\left[\frac{\nu_m}{\nu_x+\nu_n+\nu_m} m_t + \frac{\nu_n}{\nu_x+\nu_n+\nu_m} n_t\right] + \frac{1}{1+\kappa\phi_y} e_t. \tag{A.4}$$

A.2 The Private Sector's Pre-FOMC Expectations

There is a continuum of professional forecasters $j \in (0,1)$. Before the FOMC meeting, each forecaster j receives a noisy private signal, given by

$$s_{t,j} = x_t + \varepsilon^s_{t,j}, \quad \varepsilon^s_{t,j} \sim N(0, \sigma^2_s). \tag{A.5}$$

Each professional forecaster also observes the public signal n_t.

The pre-FOMC information set of professional forecaster j is $\mathcal{I}^{pre}_{t,j} = \{n_t, s_{t,j}\}$, and the signals' precisions are: $\nu_n = 1/\sigma^2_n$ and $\nu_s = 1/\sigma^2_s$. Individual j's pre-FOMC expectations of the state of the economy is:

$$\mathbb{E}^{pre}_{t,j} x_t = \mathbb{E}_{t,j}\left[x_t | \mathcal{I}^{pre}_{t,j}\right] = \frac{\nu_s}{\nu_x + \nu_n + \nu_s} s_{t,j} + \frac{\nu_n}{\nu_x + \nu_n + \nu_s} n_t.$$

Individual j's pre-FOMC expectations of the interest rate is

$$\mathbb{E}^{pre}_{t,j} i_t = \frac{\phi_y}{1+\kappa\phi_y}\left[\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\mathbb{E}^{pre}_{t,j} m_t + \frac{\nu_n}{\nu_x+\nu_n+\nu_m} n_t\right] = \frac{\phi_y}{1+\kappa\phi_y}\left[\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\mathbb{E}^{pre}_{t,j} x_t + \frac{\nu_n}{\nu_x+\nu_n+\nu_m} n_t\right] = \tag{A.6}$$

The *individual* monetary policy surprise is defined as the difference between the actual and the individual's expected interest rate. ($mps_{t,j} \equiv i_t - \mathbb{E}^{pre}_{t,j} i_t$.) Substitute $\mathbb{E}^{pre}_{t,j} i_t$ with Equation (A.6) and get

$$mps_{t,j} = \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\left(m_t - \frac{\nu_s}{\nu_x+\nu_n+\nu_s}s_{t,j} - \frac{\nu_n}{\nu_x+\nu_n+\nu_s}n_t\right) + \frac{1}{1+\kappa\phi_y}e_t. \tag{A.7}$$

The *average* monetary policy surprise is calculated by averaging across all individuals ($mps_t \equiv \int_{j\in(0,1)} mps_{t,j} dj$), which is given by

$$mps_t = \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\left(\frac{\nu_x}{\nu_x+\nu_n+\nu_s}x_t + \varepsilon^m_t - \frac{\nu_n}{\nu_x+\nu_n+\nu_s}\varepsilon^n_t\right) + \frac{1}{1+\kappa\phi_y}e_t. \tag{A.8}$$

A.3 The Private Sector's Post-FOMC Expectations

The realized nominal interest rate i_t becomes public information after the FOMC meeting. The post-FOMC information set of forecaster j is $\mathcal{I}^{post}_{t,j} = \{n_t, s_{t,j}, i_t\}$. Re-arranging the monetary policy rule (A.4) as

$$\hat{i}_t = \frac{i_t - \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_n}{\nu_x+\nu_n+\nu_m}n_t}{\frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m}} = m_t + \frac{1}{\phi_y\frac{\nu_m}{\nu_x+\nu_n+\nu_m}}e_t \tag{A.9}$$

Equation (A.9) indicates that $\hat{i}_t$ is an unbiased signal of x_t. The precision of the signal $\hat{i}_t$, denoted by ν_i, is given by

$$\nu_i^{-1} = \nu_m^{-1} + \left(\frac{\nu_x + \nu_n + \nu_m}{\phi_y \nu_m} \right)^2 \nu_e^{-1}.$$

The individual's post-FOMC expectation about the state of the economy is given by

$$\mathbb{E}_{t,j}^{post} x_t = \frac{\nu_s}{V} s_{t,j} + \frac{\nu_n}{V} n_t + \frac{\nu_i}{V} \hat{i}_t, \tag{A.10}$$

where $V = \nu_x + \nu_n + \nu_s + \nu_i$.

Rewriting the $\mathbb{E}_{t,j}^{post} x_t$ in (A.10) using (A.9) gives

$$\mathbb{E}_{t,j}^{post} x_t = \frac{\nu_s}{V} s_{t,j} + \frac{\nu_n}{V} n_t + \frac{\nu_i}{V} m_t + \frac{\nu_i}{V} \frac{\nu_x + \nu_n + \nu_m}{\phi_y \nu_m} e_t.$$

Re-arranging this equation to express the post-FOMC expectation as a function of an individual monetary policy surprise leads to

$$\begin{aligned}
\mathbb{E}_{t,j}^{post} x_t &= \frac{\nu_s}{V} s_{t,j} + \frac{\nu_n}{V} n_t + \frac{\nu_i}{V} m_t + \frac{\nu_i}{V} \frac{\nu_x + \nu_n + \nu_m}{\phi_y \nu_m} \frac{1+\kappa\phi_y}{1+\kappa\phi_y} e_t \qquad \text{(A.11)} \\
&= \frac{\nu_s}{V} s_{t,j} + \frac{\nu_n}{V} n_t + \frac{\nu_i}{V} m_t + (1+\kappa\phi_y) \frac{\nu_i}{V} \frac{\nu_x + \nu_n + \nu_m}{\phi_y \nu_m} \left[\frac{1}{1+\kappa\phi_y} e_t \right] \\
&= \frac{\nu_s}{V} s_{t,j} + \frac{\nu_n}{V} n_t + \frac{\nu_i}{V} m_t + (1+\kappa\phi_y) \frac{\nu_i}{V} \frac{\nu_x + \nu_n + \nu_m}{\phi_y \nu_m} \\
&\cdot \left[mps_{t,j} - \frac{\phi_y}{1+\kappa\phi_y} \frac{\nu_m}{\nu_x + \nu_n + \nu_m} \left(m_t - \frac{\nu_s}{\nu_x + \nu_n + \nu_s} s_{t,j} - \frac{\nu_n}{\nu_x + \nu_n + \nu_s} n_t \right) \right] \\
&= \frac{\nu_s}{V} s_{t,j} + \frac{\nu_n}{V} n_t + \frac{\nu_i}{V} m_t + \frac{1+\kappa\phi_y}{\phi_y} \frac{\nu_i}{V} \frac{\nu_x + \nu_n + \nu_m}{\nu_m} mps_{t,j} \\
&- \frac{\nu_i}{V} \left(m_t - \frac{\nu_s}{\nu_x + \nu_n + \nu_s} s_{t,j} - \frac{\nu_n}{\nu_x + \nu_n + \nu_s} n_t \right) \\
&= \frac{1+\kappa\phi_y}{\phi_y} \frac{\nu_i}{V} \frac{\nu_x + \nu_n + \nu_m}{\nu_m} mps_{t,j} + \underbrace{\frac{\nu_s}{\nu_x + \nu_n + \nu_s} s_{t,j} + \frac{\nu_n}{\nu_x + \nu_n + \nu_s} n_t}_{\mathbb{E}_{t,j}^{pre} x_t}
\end{aligned}$$

Apply the post-FOMC expectation operator to both sides of $y_t = x_t - \kappa i_t$, and get

$\mathbb{E}^{post}_{t,j} y_t = \mathbb{E}^{post}_{t,j} x_t - \kappa i_t$. Substituting $\mathbb{E}^{post}_{t,j} x_t$ by Equation (A.11) yields

$$
\begin{aligned}
\mathbb{E}^{post}_{t,j} y_t &= \frac{1+\kappa\phi_y}{\phi_y}\frac{\nu_i}{V}\frac{\nu_x+\nu_n+\nu_m}{\nu_m} mps_{t,j} + \frac{\nu_s}{\nu_x+\nu_n+\nu_s} s_{t,j} + \frac{\nu_n}{\nu_x+\nu_n+\nu_s} n_t - \kappa i_t \\
&= \frac{1+\kappa\phi_y}{\phi_y}\frac{\nu_i}{V}\frac{\nu_x+\nu_n+\nu_m}{\nu_m} mps_{t,j} + \frac{\nu_s}{\nu_x+\nu_n+\nu_s} s_{t,j} + \frac{\nu_n}{\nu_x+\nu_n+\nu_s} n_t - \kappa \left[mps_{t,j} + \mathbb{E}^{pre}_{t,j} i_t \right] \\
&= \frac{1+\kappa\phi_y}{\phi_y}\frac{\nu_i}{V}\frac{\nu_x+\nu_n+\nu_m}{\nu_m} mps_{t,j} + \frac{\nu_s}{\nu_x+\nu_n+\nu_s} s_{t,j} + \frac{\nu_n}{\nu_x+\nu_n+\nu_s} n_t \\
&\quad - \kappa \left\{ mps_{t,j} + \frac{\phi_y}{1+\kappa\phi_y}\left[\frac{\nu_m}{\nu_x+\nu_n+\nu_m}\left(\frac{\nu_s}{\nu_x+\nu_n+\nu_s} s_{t,j} + \frac{\nu_n}{\nu_x+\nu_n+\nu_s} n_t \right) + \frac{\nu_n}{\nu_x+\nu_n+\nu_m} n_t \right] \right\} \\
&= \underbrace{\left[\frac{1+\kappa\phi_y}{\phi_y}\frac{\nu_i}{V}\frac{\nu_x+\nu_n+\nu_m}{\nu_m} - \kappa \right] mps_{t,j}}_{\textit{information effect of monetary policy}} + \frac{\nu_s}{\nu_x+\nu_n+\nu_s}\left(1 - \kappa \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m} \right) s_{t,j} \\
&\quad + \left[\frac{\nu_n}{\nu_x+\nu_n+\nu_s}\left(1 - \kappa \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m} \right) - \kappa \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_n}{\nu_x+\nu_n+\nu_m} \right] n_t
\end{aligned}
$$

The *average* post-FOMC expectation is defined as the integral of all professional forecasters' expectations, i.e.,

$$
\overline{\mathbb{E}}^{post}_t(\cdot) = \int_{j\in(0,1)} \mathbb{E}^{post}_{t,j}(\cdot) dj.
$$

The post-FOMC average expectation of the economic outcome is

$$
\begin{aligned}
\overline{\mathbb{E}}^{post}_t y_t &= \underbrace{\left[\frac{1+\kappa\phi_y}{\phi_y}\frac{\nu_i}{V}\frac{\nu_x+\nu_n+\nu_m}{\nu_m} - \kappa \right]}_{\Phi} mps_t + \underbrace{\frac{\nu_s}{\nu_x+\nu_n+\nu_s}\left(1 - \kappa \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m} \right)}_{\Psi_1} x_t \\
&\quad + \underbrace{\left[\frac{\nu_n}{\nu_x+\nu_n+\nu_s}\left(1 - \kappa \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_m}{\nu_x+\nu_n+\nu_m} \right) - \kappa \frac{\phi_y}{1+\kappa\phi_y}\frac{\nu_n}{\nu_x+\nu_n+\nu_m} \right]}_{\Psi_2} n_t
\end{aligned}
\tag{A.12}
$$