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# A Unified Framework to Estimate Macroeconomic Stars*

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**Abstract**

This paper develops a semi-structural model to jointly estimate "stars" — long-run levels of output (its growth rate), the unemployment rate, the real interest rate, productivity growth, price inflation, and wage inflation. It features links between survey expectations and stars, time-variation in macroeconomic relationships, and stochastic volatility. Survey data help discipline stars' estimates and have been crucial in estimating a high-dimensional model since the pandemic. The model has desirable real-time properties, competitive forecasting performance, and superior fit to the data compared to variants without the empirical features mentioned above. The by-products are estimates of various objects of great interest to the broader profession.

**Keywords**: state-space model, Bayesian analysis, time-varying parameters, natural rates, survey expectations, COVID-19 pandemic

**JEL Codes**: C5, E4, E31, E24, O4

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# 1 Introduction

The long-run equilibrium levels of macroeconomic variables (often denoted with the "star" symbol) are central to macroeconomics and policymaking. Their estimates inform counter-cyclical responses and the long-run assessment of the economy.

In practice, determining the values of stars – which are unobserved – is difficult, not least because even some of their determinants are unobserved. The multivariate unobserved components (UC) models, which are statistical models that use economic theory to frame the empirical specification, are the dominant methods for estimating stars (e.g., Kuttner, 1994; Laubach and Williams, 2003; Chan et al., 2016). With few exceptions, the popular UC models focus on estimating up to two or three stars using minimal structure (e.g., Laubach and Williams, 2003 [henceforth LW]). A few studies have incorporated greater information to jointly estimate more stars but have abstracted from important empirical features such as time-varying parameters and stochastic volatility (e.g., Hasenzagl et al., 2022; Del Negro et al., 2017).

This paper takes on the challenge of implementing a semi-structural UC trend-cycle model, with time-varying parameters and stochastic volatility [SV], to jointly estimate seven macroeconomic stars: the level of potential output (gdp-star), the growth rate of potential output (g-star), the long-run equilibrium levels of the unemployment rate (u-star), the real short-term interest rate (r-star), labor productivity growth (p-star), price inflation (pi-star), and the nominal wage inflation (w-star).[1] The model has several other distinctive features, discussed below. By jointly estimating seven long-run equilibrium variables in a single large model, it can exploit numerous cross-equation equilibrium restrictions, allowing more information to estimate each star. The model has a rich structure for each star, whose elements are guided by past research and informed by economic theory. For instance, to estimate r-star,

[1]The subset of these stars–p-star, g-star, u-star, and r-star–reflect the fundamental structural features of the economy, whereas others–pi-star and w-star–are influenced by monetary policy.

the investment-savings (IS) equation provides information about the trend in the long-run real interest rates; the Taylor-type rule equation provides information about the trend in short-run interest rates; g-star and r-star are theoretically linked; and r-star is related to its survey expectations. The rich structure combined with time variation in select parameters and SV should in principle allow the model to better distinguish between cyclical fluctuations and lower-frequency movements in macroeconomic aggregates, helping provide more credible estimates of stars and other model parameters.

Chan et al. (2018) – henceforth CCK – use long-run survey expectations of inflation to improve the estimation of pi-star. I extend their approach to other macroeconomic stars. Specifically, I model the direct relationships between the select model-based stars (g-star, r-star, u-star, and pi-star) and the expectations about those stars contained in the Blue Chip survey of economic forecasters (or as reported by the Congressional Budget Office [CBO] for the pre-1983 period when the survey estimate is not available). Each relationship is estimated and allowed to be flexible and data-dependent. In particular, the data may suggest that survey expectations are equated to stars in some periods or that the two are unrelated or something in between.

The paper is differentiated from those in the existing literature by its novel combination of features: (1) time-varying parameters, (2) stochastic volatility, (3) joint modeling of multiple stars, (4) implementation of a richer structure, (5) allowing for a direct relationship between stars and long-term survey expectations, and (6) the provision of w-star (and its model-based decomposition into its theoretical determinants p-star and u-star). Many popular small-scale UC models could be viewed as special nested cases of my larger UC model, facilitating model comparison. I estimate the feature-rich UC model using Bayesian techniques.

To assess the empirical relevance of features included in the large (baseline) model, such as survey data, time variation in parameters, and SV, several model specifications are estimated that restrict one or more of these features. The full-sample and real-time properties of the stars, the model's fit to the data, the precision of the stars, and the real-time forecast performance are compared across the various model specifications. The estimates of the stars from the baseline model are also compared to parsimonious smaller-scale UC models routinely used by Federal Reserve staff and academic research. Lastly, the usefulness of the stars from the baseline model is evaluated as steady-state values for external models.

The results indicate that including the survey data on expectations- targeted and "direct" measures of stars - is a crucial element of the model, particularly in times of high uncertainty. These data are generally helpful in disciplining model estimates of the stars. But, since the onset of the pandemic, survey data have been critical for estimating the high-dimensional model. The model's rich features, including SV and time-varying parameters, make estimation complex, and feeding in pandemic data results in the failure of the estimation to converge without the survey data. The survey data have the most influence on the estimation of pi-star, followed by r-star and u-star, and is somewhat helpful for g-star.

The results also indicate that SV is highly effective in identifying high-volatility periods, including the extreme data fluctuations associated with the pandemic. This helps limit the influence of these observations on model estimates, yielding more credible and precise parameter inference. Broadly speaking, volatility estimates show more time variation than estimates of the parameters. Therefore, regarding inference about the stars, allowing for SV is more important than time variation in parameters. The baseline model with and without time variation in parameters generally implies similar estimates of stars but improves the precision of the estimates of most stars, the model's fit to the data, and out-of-sample predictions.

It has been widely acknowledged that considerable uncertainty surrounds the estimated stars, complicating reliable inference. My baseline model estimates suggest that the estimated uncertainty around the stars is lower than what we are used to seeing, though it has increased since the onset of the pandemic. The results indicate that each feature incorporated into the Base model, which includes a richer structure (including multivariate information), time-variation in some parameters, SV in cyclical components, and the inclusion of survey expectations, contributes to improved precision of the stars and the output gap. This is an encouraging result, since it increases the appeal of relying on the estimates of the stars for policymaking.

When assessed over long periods, the *broad* contours of estimated stars echo those documented elsewhere, which is not surprising because, for a given star, the main underlying variable is the same across papers (e.g., real GDP for g-star), so stars are expected to track the long-run movements in the respective underlying variables. The differences in modeling choices and assumptions lead to potential differences in the stars' estimates over certain periods. Indeed, my model estimates indicate differences in some stars over extended periods, differences that are significant enough to matter for policymaking. Based on an extensive examination using this paper's unified framework, the stars most sensitive to modeling choices are r-star and u-star, followed by pi-star, w-star, and g-star (and the output gap), with p-star (trend productivity) being the least sensitive.

A key challenge in the literature is that real-time estimates of stars tend to be unstable; that is, there are notable differences between real-time and final estimates. This instability has contributed to their limited use in policy discussions in recent years (see Powell, 2018). This paper has made progress along this dimension as well. Including survey data and SV improves the real-time stability of the stars.

This paper also demonstrates the usefulness of the proposed model's estimated stars as terminal values in other (external) models. Previous research shows that forecasting models, such as steady-state vector autoregressive (VAR) models, often improve their forecast accuracy by using external information about steady states informed by long-run survey expectations (e.g., Wright, 2013). Using a real-time forecasting comparison, I show that if one were to inform steady states in a VAR with the stars from my baseline UC model, one would achieve gains in forecast accuracy for some variables compared to the standard approach relying on survey expectations. An advantage of the proposed framework compared to surveys is that it provides estimates of stars for variables not covered by surveys (e.g., w-star) and offers both point and uncertainty estimates. Research has recently shown the benefits of using survey data to inform medium-run trend estimates in DSGE models (e.g., Rychalovska et al., 2023). Hence, this paper's estimates of stars, which are internally consistent, could be helpful for a broad class of models.

As a second real-time out-of-sample predictive exercise, I show that the baseline model predictions themselves do at least as well as the standard "hard to beat" benchmark models in the literature. Supplementary exercises suggest that including survey data, SV, and parameter time variation all contribute to improved predictions. Hence, this model produces accurate forecasts of key macroeconomic variables of broader interest.

The model's estimates yield insights into several policy questions that have been of high interest since the 2021 inflation surge. Two are highlighted here.

*Permanent vs. transitory inflation*: While the post-pandemic inflation surge was in full swing, there was considerable interest in determining how much was permanent versus transitory. By 2023, however, inflation had moderated quickly from a high of 8% to a rate closer to 3%, indicating that most of the surge was transitory. Somewhat remarkably, the baseline model and its variants, which include survey expectations, during the surge indicated that pi-star had risen only 0.5-0.6 percentage points. Dropping survey data from the model(s) yielded sharply different inferences, attributing most of the pandemic surge to pi-star, similar to the UC model of Stock and Watson (2007) and that of Schmitt-Grohe and Uribe (2024) when those models are estimated with postwar inflation data. Schmitt-Grohe and Uribe, who do not use survey data, show that if one were to estimate their model with data starting in 1900, the model infers the recent surge as mostly transitory. They conjecture that the latter inference is more credible given the recent developments in inflation and hence, they advocate estimating macro models with data that start in 1900. In contrast, my results indicate that if survey data on inflation are included, estimation with postwar data is sufficient to get a similar (credible) inference.

*Strength of the Phillips curve*: Post-COVID inflation dynamics have renewed interest in the Phillips curve (PC relationship). Many models estimate a weakening of the PC relationship after the 2008 financial collapse, and several recent papers (e.g., Hobijn et al., 2023; Cecchetti et al., 2023) find evidence of a steepening in the PC slope since the pandemic's onset. In contrast, my model indicates that the strength of the PC relationship has remained stable from the mid-1980s through 2023 (though the PC parameter estimate over this period, near -0.2, is about half of what it was during the 1970s).

The paper is organized as follows. The next section describes the semi-structural model and its variants in detail. Section 3 describes the data and estimation. Section 4 presents and discusses empirical results based on full-sample estimation. Section 5 reports real-time estimates of stars, compares them to the final estimates, and summarizes the forecasting results. Section 6 concludes. This paper has a supplementary online appendix that discusses the related literature, a detailed model description, Bayesian estimation steps, prior and posterior comparison, and additional results.

## 2 Empirical Macro Model and Variants

The ingredients of the proposed econometric model are guided both by economic theory and by empirical considerations – namely, features that previous research has demonstrated to be empirically relevant. These features include SV and time-varying parameters, implying time-varying predictability. Collectively, these empirical features permit flexible modeling of changing macroeconomic relationships.

Following CCK, Mertens (2016), among others, this paper defines the long-run equilibrium (or star) of a particular macroeconomic series as its infinite-horizon forecast conditional on the current information set. This definition of a star is consistent with the notion of Beveridge-Nelson trend decomposition, and an extensive literature has adopted it to estimate stars.

I represent the empirical baseline model using six blocks of equations. These six blocks, which allow for *contemporaneous interactions* between them, characterize the joint dynamics of the unemployment rate, output growth, labor productivity growth, price inflation, nominal wage inflation, the nominal interest rate, and corresponding stars. To be sure, the model assumes that all innovations are uncorrelated both serially and across equations. However, I emphasize that any assumed current period correlations between the cyclical components and/or between stars are directly modeled via the model equations that define the contemporaneous relationships between the components (e.g., the cyclical output gap at time $t$ with the cyclical unemployment gap at $t$; r-star and g-star).

For space reasons, I describe briefly the elements of the various model blocks, but in Appendix A2, I provide a detailed description of the model blocks, including references motivating modeling choices.

## 2.1 Bringing in survey expectations

An important contribution of this paper is to provide a direct role for long-run survey expectations in refining the stars' estimates. CCK explicitly estimate an equation linking the observed measure of a long-run survey expectation of inflation to unobserved trend inflation (pi-star). I extend their approach to four of the six macroeconomic stars considered in this paper. Specifically, with the exceptions of nominal wage inflation and labor productivity, for each of the remaining four variables, I model a direct but flexible link between long-run survey projections (or the long-run CBO projections in the years for which survey projections are unavailable) and the corresponding star using the following econometric equations:[2]

(1) $$Z_t^j = C_t^j + \beta^j j_t^* + \varepsilon_t^{zj}, \;\; \varepsilon_t^{zj} \sim N(0, \sigma_{zj}^2), \;\; j = \pi, u, g, r$$

(2) $$C_t^j = C_{t-1}^j + \varepsilon_t^{cj}, \;\; \varepsilon_t^{cj} \sim N(0, \sigma_{cj}^2), \;\; j = \pi, u, g, r$$

where $\pi$ refers to price inflation, $u$ refers to the unemployment rate, $g$ refers to real GDP growth, $r$ refers to the real short-term interest rate, $Z_t^j$ refers to the long-run survey forecast

[2]For the long-run inflation forecast, I use the Survey of Professional Forecasters (SPF) (and the PTR series that is available to download from the website of the Federal Reserve Board), and for the long-run forecasts of the other three variables, I use the Blue Chip (BC) survey.

corresponding to the variable $j$, and $j_t^*$ is the unobserved $j$ star.[3]

$C_t^j$ is a time-varying intercept assumed to evolve as an RW process to possibly capture the permanent wedge between the survey estimate and the model-based star. This wedge can arise for several reasons, including the fact that star is assumed to be the infinite-horizon forecast, whereas the survey forecast refers to the average forecast for the five-year period starting seven years into the future in the case of BC and the ten-year-ahead forecast in the case of the SPF (for price inflation).

## 2.2 Unemployment block

Following Chan et al. (2016) [henceforth CKP], the observed unemployment rate is decomposed into a (bounded) RW trend component (u-star) and a stationary cyclical component.

$$U_t = U_t^* + U_t^c \tag{3}$$

The cyclical component is modeled as an AR(2) process. Because I also model the output gap, I depart from CKP by adding the output gap (denoted ogap) to the equation.

$$U_t - U_t^* = \rho_1^u(U_{t-1} - U_{t-1}^*) + \rho_2^u(U_{t-2} - U_{t-2}^*) + \phi^u ogap_t + \varepsilon_t^u, \;\; \varepsilon_t^u \sim N(0, e^{h_t^u}) \tag{4}$$

where $\rho_1^u + \rho_2^u < 1$, $\rho_2^u - \rho_1^u < 1$, and $|\rho_2^u| < 1$. The estimate, $\frac{1-\rho_1^u-\rho_2^u}{\phi^u}$, could be interpreted as the Okun's law coefficient. U-star is modeled as a bounded RW, where the bounds' values are fixed at 3.5% (lower bound) and 7.5% (upper bound).[4]

$$U_t^* = U_{t-1}^* + \varepsilon_t^{u*}, \;\; \varepsilon_t^{u*} \sim TN(a_u - U_{t-1}^*, b_u - U_{t-1}^*; 0, \sigma_{u*}^2) \tag{5}$$

[3]Because survey expectations of g-star, i.e., $Z^g$, are reported as annualized rates, the precise formulation of the equation linking survey expectations to g-star is $Z_t^g = C_t^g + \beta^g * 4 * g_t^* + \varepsilon_t^{zg}$. I note that r-star survey projections are not direct estimates; instead, they are inferred from the Blue Chip survey's long-run projections of the GDP deflator and short-term interest rates using the long-run Fisher equation. The inferred estimates of survey expectations for r-star go back to 1983.Q1. Please refer to Appendix A4 for details on the procedure to back-cast estimates all the way back to 1959.

[4]These values are informed by estimating the CKP model over the estimation sample and are close to values reported in CKP based on their estimation sample. As a further check, most estimates of u-star reported in the commonly cited literature fall within the bounds used in this paper.

where the notation $TN(a, b; \mu, \sigma^2)$ refers to normal distribution with mean $\mu$ and variance $\sigma^2$ but truncated in the interval $(a, b)$.

### 2.3 Output block

To feasibly estimate both the potential output (i.e., $gdp^*$) and the growth rate in potential output (i.e., $g^*$), I follow the commonly adopted approach, which decomposes the level of aggregate output into the level of potential output and a cyclical component (output gap).

$$gdp_t = gdp_t^* + ogap_t \tag{6}$$

where $gdp \equiv log(GDP)$ and $gdp^*$ refers to potential output, which is unobserved.

As in Grant and Chan (2017), $gdp^*$ is assumed to follow a second-order Markov process.

$$gdp_t^* = 2gdp_{t-1}^* - gdp_{t-2}^* + \varepsilon_t^{gdp*}, \;\; \varepsilon_t^{gdp*} \sim N(0, \sigma_{gdp*}^2) \tag{7}$$

Assuming, $g_t^* \equiv \triangle gdp_t^*$ , where $\triangle$ is the first difference operator, then,

$$g_t^* = g_{t-1}^* + \varepsilon_t^{gdp*} \tag{8}$$

The cyclical component, ogap, is assumed to be a stationary AR2 process augmented with additional variables: the real interest rate gap and the unemployment gap,[5],

$$ogap_t = \rho_1^g(ogap_{t-1}) + \rho_2^g(ogap_{t-2}) + a^r(r_t^L - r_t^* - tp_t^*) + \lambda^g(U_t - U_t^*) + \varepsilon_t^{ogap} \tag{9}$$

where, $\varepsilon_t^{ogap} \sim N(0, e^{h_t^o})$, $\rho_1^g + \rho_2^g < 1$, $\rho_2^g - \rho_1^g < 1$, and $|\rho_2^g| < 1$

Equation (9) could be interpreted as defining an IS-curve (as in LW and subsequent papers modeling r-star) that allows feedback (via parameter $a^r$) from the real interest rate gap to the output gap. The long-term real interest rate, $r^L$, is treated as an exogenous (observable) variable and is constructed as the difference between the nominal yield on a 10-year Treasury bond and the 10-year inflation expectations (i.e., the PTR series for PCE inflation). The long-run value of the term premium, $tp^*$, is also treated as an exogenous variable and is constructed as the average of the differential between the long-term interest rate (i.e., the

[5]Morley and Wong (2020), Grant and Chan (2017), and Fleischman and Roberts (2011), among many, have shown the usefulness of the unemployment rate for the output gap estimation.

10-year Treasury bond) and the federal funds rate, similar to Johannsen and Mertens (2021).

### 2.4 Productivity block

The productivity gap, defined as (nonfarm) labor productivity growth (quarterly annualized) less p-star, is modeled as a function of a one-quarter lag in the productivity gap and the contemporaneous cyclical unemployment gap.

$$P_t - P_t^* = \rho^p(P_{t-1} - P_{t-1}^*) + \lambda_t^p(U_t - U_t^*) + \varepsilon_t^p, \ \ \varepsilon_t^p \sim N(0, e^{h_t^p}) \tag{10}$$

where $|\rho^p| < 1$. Including the cyclical unemployment gap helps tease out movements in productivity associated with the business cycle. Galí and van Rens (2021) find a weakening correlation between labor productivity and the cyclical indicator, which motivates time variation in the coefficient $\lambda^p$.

$$\lambda_t^p = \lambda_{t-1}^p + \varepsilon_t^{\lambda p}, \ \ \varepsilon_t^{\lambda p} \sim N(0, \sigma_{\lambda p}^2) \tag{11}$$

P-star is modeled as a driftless random walk component.

$$P_t^* = P_{t-1}^* + \varepsilon_t^{p*}, \ \ \varepsilon_t^{p*} \sim N(0, \sigma_{p*}^2) \tag{12}$$

### 2.5 Price inflation block

The formulation for the price inflation block closely follows CKP and CCK, combining elements from both papers: a link to survey data, time-varying parameters to capture evolving inflation persistence, and the Phillips curve relationship. Specifically, the inflation gap (defined as the deviation of inflation from pi-star) is modeled as a function of the one-quarter lagged inflation gap and the unemployment gap.

$$\pi_t - \pi_t^* = \rho_t^\pi(\pi_{t-1} - \pi_{t-1}^*) + \lambda_t^\pi(U_t - U_t^*) + \varepsilon_t^\pi, \ \ \varepsilon_t^\pi \sim N(0, e^{h_t^\pi}) \tag{13}$$

$$\rho_t^\pi = \rho_{t-1}^\pi + \varepsilon_t^{\rho\pi}, \ \ \varepsilon_t^{\rho\pi} \sim TN(0 - \rho_{t-1}^\pi, 1 - \rho_{t-1}^\pi; 0, \sigma_{\rho\pi}^2) \tag{14}$$

The innovations to the AR(1) coefficient, $\rho^\pi$ are truncated so that $0 < \rho_t^\pi < 1$.

$$\lambda_t^\pi = \lambda_{t-1}^\pi + \varepsilon_t^{\lambda\pi}, \ \ \varepsilon_t^{\lambda\pi} \sim TN(-1 - \lambda_{t-1}^\pi, 0 - \lambda_{t-1}^\pi; 0, \sigma_{\lambda\pi}^2) \tag{15}$$

$\lambda^{\pi}$, the slope of the price Phillips curve, is constrained in the interval (-1,0). Pi-star is modeled as a driftless random walk component (as in CKP).

$$\pi_t^* = \pi_{t-1}^* + \varepsilon_t^{\pi *}, \;\; \varepsilon_t^{\pi *} \sim N(0, \sigma_{\pi *}^2) \tag{16}$$

## 2.6 Wage inflation block

In the long run, economic theory posits that nominal wage inflation equals the sum of its fundamentals—the long-run growth rate of labor productivity and the long-run level of price inflation. I impose this relationship to define w-star.

$$W_t^* = \pi_t^* + P_t^* + Wedge_t + \varepsilon_t^{w*}, \;\; \varepsilon_t^{w*} \sim N(0, \sigma_{w*}^2) \tag{17}$$

$$Wedge_t = Wedge_{t-1} + \varepsilon_t^{wlr}, \;\; \varepsilon_t^{wlr} \sim N(0, \sigma_{wlr}^2) \tag{18}$$

Because the data on all three–nominal wage inflation, price inflation, and labor productivity growth–come from different sources, they differ in scope and coverage; hence, a time-varying wedge, which is assumed to evolve as an RW process, is added to the above equation. The above equation implies that $W^*$ adjusted for the wedge is approximately equal to the sum of $\pi_t^* + P_t^*$.

Equation (19) relates the nominal wage inflation gap – defined as the difference between the nominal wage inflation and w-star – to its one-quarter lagged gap, the cyclical unemployment gap, and the price inflation gap.

$$W_t - W_t^* = \rho_t^w (W_{t-1} - W_{t-1}^*) + \lambda_t^w (U_t - U_t^*) + \kappa_t^w (\pi_t - \pi_t^*) + \varepsilon_t^w, \;\; \varepsilon_t^w \sim N(0, e^{h_t^w}) \tag{19}$$

The findings in previous research motivate time-variation in the parameters, $\rho^w$, which quantifies the persistence in wage inflation dynamics; $\lambda^w$, which measures the strength of the cyclical relationship between the nominal wage gap and labor market slack (aka the slope of the wage Phillips curve); and $\kappa^w$, which measures the link between price inflation and nominal wage inflation – the expression $\frac{\kappa_t^w}{1-\rho_t^w}$ could be interpreted as an estimate of the short-run pass-through from price inflation to wage inflation.

$$\rho_t^w = \rho_{t-1}^w + \varepsilon_t^{\rho w}, \;\; \varepsilon_t^{\rho w} \sim TN(0 - \rho_{t-1}^w, 1 - \rho_{t-1}^w; 0, \sigma_{\rho w}^2) \tag{20}$$

The innovations to the AR(1) coefficient, $\rho^w$, are truncated so that $0 < \rho_t^w < 1$.

$$\lambda_t^w = \lambda_{t-1}^w + \varepsilon_t^{\lambda w}, \;\; \varepsilon_t^{\lambda w} \sim TN(-1 - \lambda_{t-1}^w, 0 - \lambda_{t-1}^w; 0, \sigma_{\lambda w}^2) \tag{21}$$

$\lambda^w$ in equation (19), the slope of the wage Phillips curve is constrained in the interval (-1,0).

$$\kappa_t^w = \kappa_{t-1}^w + \varepsilon_t^{\kappa w}, \;\; \varepsilon_t^{\kappa w} \sim N(0, \sigma_{\kappa w}^2) \tag{22}$$

## 2.7 Interest rate block

I close the model with the interest rate block characterizing the interest rate dynamics and the law of motion for r-star (the long-run equilibrium real short-term interest rate).

The first equation of the block relates the nominal interest rate gap (based on the shadow federal funds rate) to its one-period lag interest rate gap, the current quarter inflation gap, and the unemployment rate gap. This equation roughly characterizes the monetary policy reaction function. To capture both conventional and unconventional monetary policy effects when the (observed) nominal federal funds rate is constrained at the effective lower bound (ELB), I use the shadow interest rate measure of Wu and Xia (2016).

$$i_t - \pi_t^* - r_t^* = \rho^i(i_{t-1} - \pi_{t-1}^* - r_{t-1}^*) + \lambda^i(U_t - U_t^*) + \kappa^i(\pi_t - \pi_t^*) + \varepsilon_t^i, \;\; \varepsilon_t^i \sim N(0, e^{h_t^i}) \tag{23}$$

where $\rho^i$ is truncated so that $0 < \rho^i < 1$. The second equation, motivated by economic theory (and LW), expresses r-star as a linear function of g-star and a "catch-all" component D, which follows a random walk process.

$$r_t^* = \zeta g_t^* + D_t. \tag{24}$$

$$D_t = D_{t-1} + \varepsilon_t^d, \;\; \varepsilon_t^d \sim N(0, \sigma_d^2) \tag{25}$$

## 2.8 Stochastic volatility in "gap" equations

As discussed earlier, the variance of the error terms, $\varepsilon_t^u$, $\varepsilon_t^{ogap}$, $\varepsilon_t^p$, $\varepsilon_t^\pi$, $\varepsilon_t^w$, and $\varepsilon_t^i$ is allowed to vary over time. This implies that the model permits sudden and/or persistent increases and reductions in the volatility of the shocks to processes defining the gap components of unemployment, output, productivity, price inflation, nominal wage inflation, and the nominal interest rate. The choice not to incorporate SV into shocks to equations defining stars is made for two reasons: one, to keep the estimation manageable, and two, the literature documenting weak empirical support for SV in stars' equations when using smaller models, except for pi-star, for which there is mixed evidence (see footnote 12 in Appendix A2). The SV process is modeled as a driftless random walk in the log-variance.

$$(26)\quad h_t^{id} = h_{t-1}^{id} + \varepsilon_t^j,\ \ \varepsilon_t^j \sim N(0, \sigma_j^2) \quad id = \{u, ogap, p, \pi, w, i\},\ \ j = \{hu, ho, hp, h\pi, hw, hi\}$$

## 2.9 Model discussion

In this subsection, I highlight at a high level the unique features of my model, the assumptions, and differences relative to prior work. For a more detailed discussion, including references to prior work motivating my modeling choices, please see Appendix A2.

As indicated in the introduction, in contrast to the previous research on estimating stars using UC models, I jointly estimate a greater number of blocks and, within a block, implement a richer structure. CCK use long-run survey expectations of inflation to improve the estimation of pi-star. I extend their approach to other macroeconomic stars. As a result, four of the six macroeconomic stars considered in this paper are directly informed by survey data. Following a long list of papers, in my model setup, the persistence of the output and the unemployment gaps is modeled using a time-invariant AR2 process and of other variables using an AR1 process, with time-variation permitted in the case of price inflation and nominal wage inflation; in the case of the interest rate gap and the labor productivity gap, the data preferred a time-invariant AR1 process over a time-varying AR1. (Recall the equation for the interest-rate gap mirrors the monetary policy reaction function, and the literature has commonly adopted a fixed value of 0.85 for the inertia parameter.)

In estimating u-star, with few exceptions (e.g., Hasenzagl et al. (2022)), most of the previous literature has relied on bivariate UC models focused on joint modeling of the unemployment rate combined with either price inflation (price PC), real GDP growth (Okun's law relationship), or nominal wages (wage PC). Recently, research using small-scale UC models has shown the empirical relevance of allowing for TVP in parameters (capturing inflation persistence, price, and wage PC relationships) and support for SV in the cyclical components of price inflation, the unemployment rate, and real GDP. CKP extend the bivariate Phillips curve UC model by proposing a bounded RW for u-star. In my model, I combine elements from this research and go further by using information from interest rates, productivity, and survey data on u-star to estimate u-star.

Similarly, for g-star (and gdp-star), researchers have generally relied on univariate and bivariate UC models (Okun's law or IS curve equations), mainly without SV. I combine elements from this previous research (including Okun's law and IS curve), allow SV in equations defining cyclical components of output and unemployment, and use survey data on g-star. Some of the previous research permits the contemporaneous interactions between the output and the unemployment gaps via the error covariance structure; I instead allow for direct contemporaneous interactions between the two gaps. The IS (output gap) equation is inspired by LW but with two modifications. First, I use the long-term real interest rate instead of the short-term real rate. Second, information about the unemployment gap is included.

The estimation of r-star is informed by various channels: information from long-term interest rates (via the IS eq.), from short-term interest rates (TR eq.), an equation linking r-star to survey expectations (unique to this paper), and an equation relating r-star to g-star (popularized by LW). The previous research has entertained only a subset of these channels in estimating r-star. SV is included in both the IS and TR equations, but other parameters are assumed to be time-invariant because the empirical evidence against it was weak.

The formulation for the price inflation block combines elements from CKP and CCK. The bivariate models in CKP and CCK are extensions of the univariate Stock and Watson (2007). In my model, the equation linking pi-star and survey expectations is motivated by

CCK, time-variation in the price Phillips curve by CKP, and time-variation in the persistence of the inflation gap by CKP and CCK. Following much of the literature, pi-star is modeled as a driftless random walk component, and the variance of the shocks to this component is assumed to be constant (as in CKP, among others), in contrast to Stock and Watson (2007), and select others. My reason for not incorporating SV into shocks to pi-star is to keep the estimation manageable and maintain consistency with the modeling assumptions for the other stars, for which evidence in favor of SV has been shown to be weak.

In estimating p-star, some researchers (e.g., Kahn and Rich, 2007) have proposed using additional variables alongside labor productivity to tease out movements in productivity associated with the business cycle. Many studies apply the univariate UC (without SV) model to productivity data to estimate p-star. Accordingly, I model the productivity gap as a function of a one-quarter lag in the productivity gap and the contemporaneous unemployment gap. In contrast to the existing literature, I allow time variation in the parameter $\lambda^p$ that captures the strength of the cyclical relationship and SV in the equation defining the productivity gap. P-star is modeled as a driftless random walk component.

Lastly, to motivate the elements of the wage block, I take guidance from several theoretical and empirical studies (reported in Appendix A2). Specifically, in the nominal wage gap equation, I allow for a time-varying wage PC, short-run passthrough from prices to wages, and SV. The law of motion for w-star is governed by the theoretical restriction that labor productivity growth is the only fundamental driver of real wages, i.e., w-star = pi-star + p-star + wedge. (The time-varying wedge is added for the reasons mentioned earlier.) This theoretical restriction acts as an additional channel influencing pi-star and p-star. Because modeling w-star is one of the novelties of this paper, this restriction is unique to this paper, and I find that it improves the overall model fit to the data.

## 2.10 Base model and its variants

The equations (1), (2)... (26) define our baseline model formulation (denoted *Base*). Section A5.a. of the appendix lists all of the equations for the Base model for easy reference. To assess the usefulness of various empirical features incorporated in the Base model, additional model specifications are estimated. The five main specifications are listed below, and several additional variants are detailed in Appendix A3.

**Base-NoSV**. To assess the empirical support of SV in shock variances, I estimate a variant of the baseline model with no SV in any of the measurement (gap) equations.

**Base-NoTVP**. To assess the empirical support of time variation in important macroeconomic relationships, I estimate a variant of the baseline model that shuts down time variation in the parameters, $\lambda^p$ (eq.10), $\lambda^\pi$ and $\rho^\pi$ (eq. 13), $Wedge$ (eq. 17), $rho^w$, $\lambda^w$, and $\kappa^w$ (eq. 19). I still allow for SV in the measurement (gap) equations.

**Base-NoSV-NoTVP**. To jointly assess the empirical support of SV in shock variances and time variation in important macroeconomic relationships, I estimate a variant of the baseline model with no SV in any of the measurement (gap) equations and no time variation in the parameters capturing macroeconomic relationships (similarly to *Base-NoTVP*).

**Base-NoSurvey**. To assess the usefulness of survey expectations, I estimate a variant of the baseline model that excludes all the equations linking surveys to stars.

**Base-NoSV-NoTVP-NoSurvey**. Because the model variant *Base-NoSurvey* fails to estimate with COVID-19 pandemic data, I estimate this specification, which shuts down SV and time-variation in the parameters, in addition to excluding equations linking surveys to stars. Comparing this model specification's estimates with *Base-NoSV-NoTVP* provides a sense of the usefulness of the survey data.

# 3 Data and Bayesian Estimation

## 3.1 Data

I estimate the empirical model using the following quarterly data: (1) the unemployment rate; (2) real GDP growth; (3) nonfarm labor productivity growth; (4) the inflation rate in the personal consumption expenditures (PCE) price index; (5) average hourly earnings (AHE) of production and nonsupervisory workers (total private industries);[6] (6) the federal funds rate; (7) the nominal yield on the 10-year Treasury bond; (8) the shadow federal funds rate from Wu and Xia (2016); (9) Blue Chip[7] (real-time) long-run projections of three-month Treasury bill, real output growth, the unemployment rate, and GDP deflator inflation; and (10) long-run inflation expectations of PCE inflation (PTR series). I also collect the real-time long-run CBO projections of real output growth, the level of real potential output, and the natural rate of unemployment. I use data starting in 1959Q4 through 2023Q3 from the 2023Q4 vintage (downloaded from Haver Analytics) as a featured sample for this paper. For production of real-time estimates of stars and forecast evaluation exercises, the real-time data vintages of real GDP growth, PCE inflation, the unemployment rate, AHE, and nonfarm labor productivity spanning 1998Q1 through 2023Q3 are downloaded from the ALFRED database maintained by the St. Louis Fed and the real-time database maintained by the Federal Reserve Bank of Philadelphia.

## 3.2 Bayesian estimation

I use Bayesian methods to estimate the Base model and its variants. The use of inequality restrictions on latent parameters in the model(s) setup leads to a nonlinear state-space model, which renders estimation using standard Kalman filter methods infeasible. Accordingly, I im-

[6] AHE of production and nonsupervisory workers in total private industries goes back to 1964Q1. From 1959Q4 through 1963Q4, I use the AHE of workers in goods-producing industries. I splice them together.

[7] Blue Chip Economic Indicators data published by Wolters Kluwer Legal and Regulatory Solutions U.S.

plement the Markov chain Monte Carlo (MCMC) posterior sampler based on computational methods developed in CKP, who use the band and sparse matrix algorithms detailed in Chan and Jeliazkov (2009). The CKP posterior sample developed for a relatively smaller-scale nonlinear state-space model is carefully extended to accommodate the additional structure and numerous features of my model(s). Since the computational methods I use are an extension of those based on CKP, I relegate the specific details of the sampler to Appendix A5.c.[8]

Bayesian model comparison is based on the marginal likelihood (ML) metric. In computing ML for various models, I use the approach proposed by CCK (also used by Carriero et al., 2022), which decomposes the marginal density of the data (e.g., inflation) into the product of predictive likelihoods; see Appendix A5.d for details. An advantage of the CCK approach is that it permits the computation of marginal data density for each variable of interest. The variable-specific marginal densities prove useful because they allow for deeper insights into the source of the deficiencies, which helps differentiate models at a more granular level.

As discussed in CCK, UC models with several unobserved variables, such as the one developed in this paper, require informative priors. Accordingly, the prior values for most parameters are only slightly informative and are similar to those used in CKP, CCK, and González-Astudillo and Laforte (2020) for the common parameters. The use of inequality restrictions on some parameters, such as the Phillips curve, persistence, and bounds on u-star, could be viewed as additional sources of information that eliminate the need for tight priors. I use relatively tight priors for the parameters on which there is agreement in the empirical literature on their values, such as the Taylor-rule equation parameters. The priors are kept the same for the common parameters across models in model comparison exercises. In Appendix A9, I provide figures plotting prior and posterior distributions for each parameter. The plots in those figures indicate that for most parameters, the data influence the posterior

[8]For each model specification, I simulate 1 million posterior draws from the MCMC posterior sampler. I then discard the first 500,000 draws, and of the remaining, I keep every 100th draw. Accordingly, all of the reported results for the Base model and its variants are based on 5000 retained draws.

estimates as evidenced by well-peaked posterior distributions and/or movements in posterior distributions away from prior distributions. (Also, see Appendix A10 for additional analysis.) I also performed a simple prior sensitivity analysis, which I report in Appendix A7.

# 4 Full Sample Estimation Results

This section discusses the results obtained by estimating the models using the entire sample from 1959Q4 through 2023Q3.

## 4.1 Stars and the output gap from the Base model

Figure 1 presents estimates of the six stars (pi-star, p-star, w-star, u-star, g-star, and r-star) and the output gap (in place of gdp-star) from the Base model. Also plotted are the survey estimates of stars that enter the Base model to provide a visual sense of the influence of survey data for model-based stars. Complementing the visual evidence, Figure 2 plots the posterior estimates of the coefficients capturing the estimated relationships between survey expectations and model-based stars.

Figure 1, upon careful visual inspection, reveals periods of a tight association between stars, as dictated by economic theory, in that periods of higher p-star are generally associated with higher g-star, r-star, w-star, and lower u-star; and vice versa. For example, from the mid-1990s to the late-1990s, a period of strong economic growth fueled by the technology boom, the Base model estimates paths of rapidly increasing p-star, g-star, and r-star and steadily falling u-star and modestly rising w-star. Similarly, a coherent narrative is evident during and immediately following the Great Recession, with p-star, g-star, r-star, w-star falling, and u-star remaining elevated. This consistency lends credibility to the Base model and its estimates of various objects.

Panel (a) of Figure 1 presents the estimates of pi-star. The model indicates that pi-star was low in the 1960s, high in the 1970s (model inferred significant increases in inflation as mostly permanent), fell sharply in the 1980s, continued a steady deceleration in the 1990s, fluctuated in a narrow range between 2.0% and 2.5% in the 2000s, held steady at 1.8-2.0% from 2012 through 2021Q1, increased gradually during the post-pandemic inflation surge to reach 2.3% by 2022, and falling only by a tenth, to 2.2%, in 2023.

The two periods, 2012-2019 and post-COVID, were of great interest from the perspective of pi-star. In the first period from 2012 to 2019, when inflation remained stubbornly below 2%, pi-star is estimated to be stable at a touch below 2%. The model explains low inflation realizations partly by slack in the labor market (positive unemployment rate gap, as u-star is estimated to be above the unemployment rate; see the figure in Appendix A12; or equivalently, a negative output gap, see panel g in Figure 1) over most of this period and partly by the increasing volatility of the (mostly negative) shocks to the inflation gap. During the post-COVID inflation surge, pi-star is estimated to rise only modestly, about 0.4 ppts. The model inferred the sharp spike in inflation as mostly transitory, evidenced by the increasing volatility of the (positive) shocks to the inflation gap (see Figure 3, panel d), and the remainder of the spike is explained by a tight labor market, a negative unemployment gap, with the unemployment rate averaging 0.6 ppts below the estimated u-star.

The survey's influence on pi-star estimation is strong, as seen in Figure 1 and confirmed by estimates reported in panel (b) of Figure 2: posterior mean of 0.99 for $\beta^{\pi}$, with 90% credible intervals spanning 0.91 to 1.07 (also reported in Appendix A6). The estimates of the time-varying intercept $C^{\pi}$ (panel a) indicate evidence of time variation in the estimated relationship between the survey pi-star and model pi-star. Specifically, in the first half of the sample, there are sizeable deviations between the model and survey pi-star; however, since 2000, the gap between the two has diminished.

Panel (b) of Figure 1 presents the estimates of p-star. After averaging between 2% and 3% in the 1960s, the estimates indicate that p-star experienced a sharp deceleration in the 1970s through the mid-1980s, mirroring the dramatic fall in productivity growth (not shown). The mean estimates show p-star trending lower from 2.8% in early 1970 to 1.1% by the mid-1980s;

from there on through the late 1990s, p-star increased sharply, at a pace roughly equivalent to its deceleration in prior periods, to reach a level of 2.5% by 1999. The literature attributes part of this acceleration in the latter half of the 1990s to the information technology (IT) boom. In the 2000s, p-star gradually drifts lower to a level close to 1.0% by 2012. It remained close to that level until 2016, after which it steadily increased to 1.6% by early 2020, right before the pandemic started. Since then, it is estimated to have remained stable at 1.6% with 68% credible intervals ranging from 1.1% to 2.2%.

Panel (c) of Figure 1 presents the posterior estimates of w-star. Movements in pi-star and p-star mainly govern the dynamics of w-star. (Appendix A13 provides a more elaborate decomposition of w-star into its drivers, pi-star and p-star, and the time-varying wedge.) W-star increased steadily in the 1970s and peaked at 6.8% in the early 1980s. This increasing w-star reflected an upward drift in pi-star that offset the downward drift in p-star. Thereafter, w-star decelerated, slowing to 2.7% by the end of 2017. From then on, it gradually increased, with the pace increasing sharply during the post-COVID inflation surge.

From 2020 through the end of 2022, w-star is estimated to have increased by 0.5 ppts (from 3.4% to 3.9%), reflecting a 0.4 ppts increase in pi-star and 0.1 ppts increase in p-star. Over this same period, nominal wage growth fluctuated wildly from an annualized quarterly growth of +16.0% (in 2020Q2) to -3.2% (in 2020Q3). The model attributes these extreme swings to the spike in the volatility of the transitory shocks to the wage inflation gap (see panel e, Figure 3). Since then, the increased volatility of the shocks has mostly reversed, but as of 2023Q3, it is still estimated to be higher than in normal periods.

Panel (d) of Figure 1 presents the estimates of u-star. From 1960 through the early 1980s, u-star gradually increased from 5.5% to 6.6%. It held stable at or close to 6.6% until the mid-1980s. From thereon, it steadily drifted lower, reaching 5.1% by the late 1990s, likely reflecting the declining reallocation in the labor market. From the early 2000s until the onset of the Great Recession, u-star fluctuated in a narrow range between 5.1 to 5.3%. During the Great Recession and the subsequent recovery, u-star is estimated to have increased to 5.9%, about the same as the survey estimate. In the literature on u-star, the increase of this magnitude during that period is attributed to structural factors such as extended unemployment benefits. It is instructive to highlight that during this period, the unemployment rate increased by 5.6 percentage points, from 4.4% to 10.0%. With u-star estimated to have increased by only 0.9 percentage points, the Base model attributes the bulk of the rise in the unemployment rate to the cyclical component (see Appendix A12).

Since the onset of the COVID pandemic, which included a brief period of unprecedented swings in the unemployment rate, u-star is estimated to have remained remarkably stable at 4.5%, about 0.5 ppts higher than the survey u-star of 4.0-4.1%. During this period, the sudden and unprecedented simultaneous movements in the unemployment rate and real GDP (as discussed below) but stable survey expectations of u-star and g-star led the model to infer these movements in the data as large transitory shocks, reflected in the spikes in the volatility of the transitory shocks to these variables' cyclical components.

As the plots show, u-star from the survey displays more pronounced shifts than the model-based u-star estimates. However, due to a strong estimated relationship between the survey u-star and Base u-star (posterior mean of $\beta^u = 0.95$, Figure 2), the Base estimate of u-star reflects the contours in survey u-star. During the pandemic, survey data were critical in guiding the model-based estimates of u-star to sensible values.

Panel (e) of Figure 1 presents the estimates of g-star, and panel (g) the output gap estimates. According to the posterior mean of g-star, the growth rate of potential output has steadily drifted lower from an annualized rate of close to 4.5% in early 1960 to a pessimistic rate of 1.2% in 2014, except for a temporary rise in the late 1990s, likely reflecting the internet technology boom. This latter increase in g-star is consistent with the rise in p-star shown in panel (b) and the fall in u-star to its lowest level at the time, shown in panel (d).[9]

Similarly, a coherent narrative (consistent with the characterization of "secular stagna-

[9]The Base model variant that defines g-star as a function of p-star attributes most of the increase in g-star to p-star over this boom period.

tion") emerged following the Great Recession. The plots suggest an economic environment characterized by low p-star, low and falling g-star, high u-star, and steadily declining r-star until 2014. The fact that g-star continued to decline during and after the Great Recession indicates that the recession contributed to lowering g-star. From 2014 onward, an increasingly positive economic climate emerged that drove g-star steadily higher, reaching 1.9% just before the COVID-19 pandemic and 2.1% by 2023Q3. This gradual acceleration in g-star has been accompanied by rising p-star and falling u-star.

More notably, the plots of g-star reveal a large and persistent deviation of the Base model's g-star from the survey estimate from 1990 to 2019. This is also indicative in the posterior estimates of parameters $\beta^g$ and $C^g$, which define the relationship between g-star and the survey data. The estimates suggest statistically significant deviations of $C^g$ from zero over the period 1990 through 2019. Survey participants were very slow in recognizing the growth slowdown following the Great Recession compared to the model's smoothed estimate (panel e) and real-time estimate (Appendix A19). In contrast, during the post-pandemic period, the survey expectations of g-star have been less optimistic than the model, since they have hovered close to the model's lower bound of the 68% intervals.

The relatively modest uptick in the model estimates of g-star during the pandemic and afterward suggests that according to the model, most of the fluctuations in real GDP are attributed to the cyclical component, as evidenced by the widening output gap (panel g) and increasing volatility of the transitory shocks to the output gap (Figure 3, panel b). The posterior mean estimates of parameters $\rho_1^g$ and $\rho_2^g$ indicate a high degree of persistence ($\rho_1^g + \rho_2^g = 0.73$; see Table 2 in Appendix A6) and suggest a hump-shaped response of the output gap to transitory shocks (as $\rho_1^g > 1$). These parameters are precisely estimated as evidenced by tight posterior credible intervals. The output gap estimate from the Base model aligns well with the NBER recession dates. It supports the business cycle asymmetry in that recessions are shorter in duration but deeper than expansions in the US.

The parameter estimates indicate a strong Okun's law relationship in the data. The implied mean of the Okun's law coefficient, $\frac{(1-\rho_1^u-\rho_2^u)}{\phi_u}$, is $-2.1$, with 90% credible intervals spanning $-2.4$ to $-1.9$ and is identical to the conventional estimate reported in macroeconomic textbooks. Therefore, the estimated output and unemployment gaps (Appendix A12) reveal similar cyclical dynamics. For instance, according to both cyclical measures, the 1981-82 recession is estimated to have been more profound than the Great Recession. The more negative output gap ($-8.7\%$; posterior mean) during the 1981-82 recession compared to the Great Recession ($-6.4\%$) occurred because, during the Great Recession, g-star (and gdp-star not shown) fell significantly more, resulting in a smaller negative output gap.

Panel (f) of Figure 1 presents the estimates of r-star. The posterior mean estimate from the Base model shows r-star staying relatively flat at 3.4% in the 1960s and then slowly trending down through the 1970s, reaching 2.5% by early 1980. Thereafter, it fluctuates between 2.0% and 3.0% until the beginning of 2000. From there on, it steadily declines, reaching 0.7% by mid-2021. Since then, it has slowly increased to 1.0% in 2023Q3. To understand the evolution of r-star, looking at the dynamics of g-star (panel e) and the nongrowth component D, plotted in Appendix A14 (panel (c) of the figure), is helpful. The estimated link between r-star and g-star is of moderate strength (posterior mean of parameter $m = \frac{\zeta}{4} = 0.65$; see Table 2); therefore, movements in g-star play an influential role in driving r-star. The estimated dynamics of component D are shaped by the survey data on r-star and information from the Taylor rule and IS equations.

According to the posterior mean of component D, which is imprecisely estimated, in the 1960s, it exerts a slight upward force on r-star that is mostly offset by a downward pull coming from g-star (via equation 27), helping to keep r-star relatively flat. From 1970 through 1999, with D remaining flat, developments in g-star shape the trajectory of r-star. From 2000 through 2012, all economic forces (as captured through the model structure) worked in the same direction to drive r-star steadily downward. Besides g-star, these forces likely include rising premiums for convenience yield (Del Negro et al., 2017) and excess global savings (Pescatori and Turunen, 2016), captured in the model via component D. From 2012 through

2021, with g-star moving higher, the upward force from g-star is more than offset by the downward pull from D, resulting in continuing decline in r-star.

As can be seen, the contours of r-star from the Base model track the survey estimate. This suggests that survey data influence the model's assessment of r-star via its D; confirmed by the joint assessment of the parameters $\beta^r$ and $C^r$ (last row in Figure 2).

## 4.2 Estimates of SV and TVP in the Base model

Figure 3 plots the posterior mean estimates of the time-varying standard deviation of the transitory shocks to the Base model's gap components: the unemployment rate gap, output gap, labor productivity gap, price inflation gap, nominal wage inflation gap, and the nominal interest rate gap. Also plotted are the corresponding 90% credible intervals. The plots indicate statistically significant evidence of time variation in the volatility of the shocks in all of the gap equations. Importantly, the estimates suggest that SV is highly effective in identifying high-volatility periods in the postwar data, including the extreme fluctuations in nominal wages, the unemployment rate, and real output associated with the pandemic in 2020. For example, the standard deviation of the shocks to the unemployment gap in 2020Q2 and 2020Q3 is estimated to be about 9 times larger than in 2009Q1 (Great Recession) and 35 times larger than that of a typical period.

The volatility spikes identified during the pandemic are characterized as short-lived for unemployment, wages, and output, effectively categorizing the observations during that period as outliers. However, for labor productivity, price inflation, and interest rates, the increase in volatility was less extreme and moderated only partially. The volatility estimates also highlight the flexibility of the assumed SV process, since it can capture sudden and/or persistent increases and reductions in the volatility of the shocks.

Figure 4 presents the time-varying posterior mean estimates (and the 68% credible intervals) of the parameters from the Base model describing the persistence in the price and nominal wage inflation gaps, the price and wage Phillips curves, the short-run pass-through from prices to wages, and the cyclical dynamics of labor productivity. Also plotted are the estimates from the Base model variant that shuts down time-variation in the parameters (i.e., Base-NoTVP). A comparison between the estimates from the two models provides a sense of the importance of time variation in the parameters.

*Price inflation gap persistence (Panel a, Figure 4).* There is strong evidence of time variation in parameter $\rho^{\pi}$, inflation gap persistence. For example, gap persistence was low (0.25) in the early 1960s, high in the 1970s and early 1980s, declined steadily in the 1990s, remained stable in the 2000s, and increased sharply during the post-COVID inflation surge.

*Nominal wage gap persistence (Panel b, Figure 4).* The posterior mean estimate of the parameter $\rho^{w}$, capturing the persistence in the nominal wage inflation gap, indicates weak evidence of time variation in this parameter. However, the estimate, which has varied between 0.15 and 0.30, mostly falls outside of the 68% interval of the Base-NoTVP model, suggesting it may be worthwhile to allow time-variation in this parameter, since doing so improves the precision of w-star and forecast accuracy of nominal wage inflation (discussed later).

*Price Phillips curve (Panel c, Figure 4).* The plot indicates time variation in the slope of the Phillips curve ($\lambda^{\pi}$). The estimated relationship between prices and unemployment at business cycle frequency was strong in the 1970s, subsequently weakened by the mid-1980s, and has remained stable since then, including through the post-pandemic inflation surge. This latter result contrasts with the findings of others who estimate a weakening of the PC relationship after the 2008 financial collapse and evidence of a steepening in the PC slope since the pandemic's onset. (e.g., Hobijn et al., 2023; Cecchetti et al., 2023). See Appendix A15 for the reasoning behind this contrasting finding.

*Wage Phillips curve (Panel d, Figure 4).* The plot provides evidence supporting the existence of the wage Phillips curve in the postwar data but weak evidence of time variation in the slope of the wage PC ($\lambda^{w}$). According to the posterior mean estimate, the strength of the wage Phillips curve has ranged in a narrow band between -0.2 and -0.4, with the most negative (strongest) during the period of the mid-1980s through the mid-2000s and the weakest during the post-Great Recession period. The mean estimate of the slope of the wage PC from the Base-NoTVP model is -0.26, which is close to the Base model's time-varying estimate through most sample periods except for the mid-1980s to the mid-2000s. Interestingly, similar to the price PC, the strength of the estimated wage PC has remained stable over the post-pandemic period.

*Short-run pass-through from prices to wages (Panel e, Figure 4).* The posterior estimate of pass-through ($\frac{\kappa_t^w}{1-\rho_t^w}$) indicates a weakening relationship between wages and prices at business cycle frequency over the estimation sample up through the COVID-19 pandemic. The relationship between the two was strong from the 1970s to the mid-1980s; thereafter, it gradually weakened to near zero in the decade before the COVID-19 pandemic. Since the post-pandemic inflation surge, the pass-through is estimated to have increased to a positive (though moderate) level, and the increase is statistically significant.

*Cyclical dynamics of labor productivity (Panel f, Figure 4).* The plot of the estimate of the parameter $\lambda^p$ indicates a countercyclical behavior of labor productivity, with this relationship exhibiting weak evidence of time variation.

## 4.3 Base vs. Variants of Base: Role of SV, TVP, and Survey data

To formally assess the importance of SV, TVP, and survey data in the Base model, Table 1 reports the Bayesian model comparison metric of marginal data densities (MDD) for the Base and its variants. The top panel reports the models' fit to the data over the entire sample, and the bottom panel over the pre-COVID pandemic sample. The MDDs indicate that compared to its variants, the Base model provides the best data fit for all postwar data. Including survey data, allowing for time-varying parameters in some relationships, and modeling SV in the cyclical components all contribute to the Base model's improved fit to the data.

Including SV substantially enhances the model's fit to the data, as evidenced by the significantly inferior fit of the Base-NoSV model to the data compared to the Base and model variants that allow for SV. Without SV, the pandemic outlier observations drastically affect some of the model's parameter estimates, as illustrated in Appendix A18, and evidenced by a significant deterioration in the model fit of the Base-NoSV when comparing the top and bottom panels of Table 1. Further improvements in model fit are achieved by adding time variation in the parameters. Regarding survey data, even though bringing in additional information from surveys leads to more reasonable and precise estimates of stars, the model comparison between Base vs. Base-NoSurvey and between Base-NoSV-NoTVP vs. Base-NoSV-NoTVP-NoSurvey reveals mixed results. Adding survey data to a model with SV and TVP overall neither hurts nor improves the fit to the data (Base=−1587.3 vs. Base-NoSurvey=−1587.1). However, in the model that does not have SV and TVP, adding survey data to it worsens the fit to the data: Base-NoSV-NoTVP-NoSurvey=−1941.6 (pre-COVID sample), −2622.2 (full) vs. Base-NoSV-NoTVP=−1969.7 (pre-COVID), −2653.2 (full).

To get a visual sense of the empirical relevance of the various features incorporated into the Base model, Figure 5 plots the estimates of stars from the Base and its variants: Base-NoSV (shuts down SV), Base-NoSV-NoTVP (shuts down both SV and time-variation in parameters capturing macro relationships), and Base-NoSV-NoTVP-NoSurvey (shuts down SV, time-variation in parameters, and excludes survey data); the latter variant is featured because the Base-NoSurvey model fails estimation with pandemic data. The figure in Appendix A16 presents the corresponding precision estimates of the stars from the Base and its variants (where precision is measured as the width of the 90% intervals). The precision estimates indicate that all three features, including survey data, SV, and time-variation in parameters, generally help improve the precision of the stars, as evidenced by the precision plots corresponding to the Base model lying below the other plots with few exceptions.

Figure 5 shows that stars' estimates are sensitive to modeling choices (SV, TVP, and survey data), with those sensitivities varying with the sample period. These modeling choices matter somewhat less for the estimation of g-star, as can be seen by the trivial differences in the estimates across models except for two periods: the late 1990s (IT boom) and the recovery years following the Great Recession. In those two periods, Base model estimates of g-star indicate notable divergence compared to other models. Whereas variants of the Base model estimated g-star to be stable at or close to 3.2% during the second half of the 1990s, the Base model had g-star accelerating to 3.7%. Similarly, in the years following the Great Recession through mid-2013, the Base model had g-star falling more than its variants. More recently, the Base has g-star very slowly moving up to 2.1% by 2023Q3, whereas other models have g-star remaining stable at 1.7%. These differences may appear small, but they are important enough to affect the output gap estimates. Furthermore, the differences are more significant in real-time (as seen in the figure in Appendix A19), with SV playing an essential role in helping with the real-time stability of the output gap.

The Base model and its variants with survey data generate estimates of stars whose broader contours are similar. However, there are sizeable and important differences in the stars' estimates due to the inclusion of SV and/or time-variation in parameters. The model, Base-NoSV, produces estimates of u-star that are well above the estimates from the Base and other variants, and its estimates of pi-star, w-star, and r-star are consistently below the estimates from other models. For example, during the Great Inflation and the post-pandemic inflation surge, Base-NoSV had the lowest pi-star. At first pass, this seems surprising, as without SV, one would expect to see a higher pi-star during inflation spikes due to the model's inability to attribute realizations of high inflation to the increasing volatility of shocks to the inflation gap. However, because this model includes survey data, which are estimated to have a tight link with the model pi-star, pi-star is estimated to stay close to the survey estimate. Instead, the model mainly attributes high inflation to tighter labor markets (a more negative unemployment gap) via estimating a higher u-star, as can be seen in panel (d) of Figure 5.

In contrast, the model variant that excludes both SV and survey data in addition to TVP (Base-NoSV-NoTVP-NoSurvey) produces a pi-star that responds strongly to inflation spikes. Accordingly, this model variant had the highest pi-star during the Great Inflation of the 1970s and the recent post-pandemic inflation surge. In the latter period, the Base-NoSV-NoTVP-NoSurvey model had pi-star increasing to 4.0% in 2022 compared to 2.3% estimated in the Base. Schmitt-Grohe and Uribe (2024) emphasize that differences of this magnitude can have policy implications. The story is similar for w-star, partly because pi-star informs the movements in w-star. Specifically, not bringing in survey data and shutting down both SV and TVP (including the time-varying wedge in w-star) have notable implications for the estimate of w-star, its precision, and the model's fit to the wage data. The resulting estimate is more volatile as it responds strongly to movements in nominal wages; during the Great Inflation period of the 1970s and the post-COVID inflation surge, when nominal wages accelerated sharply, the model attributes most of the increase to w-star.

The inclusion of SV in model equations contributes to the sizeable but constant differential in the r-star estimates from the mid-1990s onward, as is evident when comparing the Base and Base-NoSV models. The r-star estimates from the Base model are roughly 100 basis points higher than those from the Base-NoSV model over this later sample period. Given the slight difference in the estimates of g-star between the Base and Base-NoSV models, the source of the differential in r-star estimates comes from the estimates of component D. Without SV in the Taylor rule and IS equations, the estimated trajectory of D is significantly lower than in the Base model, resulting in lower r-star (as can be seen in Appendix A14). During the pandemic, the r-star from the Base-NoSV model became slightly negative compared to the Base model, whose mean estimates only fell modestly in the range between 0.7-1.0 percent, and the lower bound of the 68% credible intervals remained positive. Allowing for time-variation in the parameters has a modest effect on r-star, as is evident when comparing r-star estimates between Base-NoSV and Base-NoSV-NoTVP.

The survey data play a vital role in influencing the trajectory of r-star. This is because including survey data in the models leads to a stronger role of g-star driving r-star. (The posterior mean of $m = \frac{\zeta}{4}$ in Base is 0.65, in Base-NoSV is 0.69, and in Base-NoSV-NoTVP is 0.68.) Without survey data, the estimated link between g-star and r-star is substantially weaker, i.e., in the model Base-NoSV-NoTVP-NoSurvey, the parameter $m = \frac{\varsigma}{4}$ is 0.30.

The uncertainty around the r-star estimate from the Base-NoSV-NoTVP-NoSurvey model is substantially higher than that in the Base and its variants that include survey data, as seen in panel f of the figure in Appendix A16. The increased uncertainty in r-star comes from D, which is imprecisely estimated without the survey data. It is evident that including all three – survey data, SV, and TVP- contributes to the improved precision of r-star. Based on the model comparison metric reported in Table 1, the Base model ranks as the best-fitting model for the interest rate data.

### 4.4 Stars: Base vs. External models

Figure 6 compares the smoothed estimates of pi-star (panel a), p-star (panel b), w-star (panel c), u-star (panel d), g-star (panel e), r-star (panel f), and the output gap (panel g) from the Base model and outside sources, which include small-scale UC models routinely used by Federal Reserve staff and featured in academic research. To better highlight the differences and similarities in the estimates across models, the plots are shown to span the sample period from 1990 onward. These plots reveal interesting similarities and differences between the Base model estimates and those from outside models. As can be seen, at times, the differences in the estimates are sizeable enough to have policy implications. For space reasons, the detailed discussion of the estimates' comparison is relegated to Appendix A17, which also includes an analogous figure showing a comparison over the full sample.

## 5 Real-Time Estimates and Forecasting

*Real-time versus final estimates.* Up to this point, I have presented the smoothed estimates of the stars inferred using all the sample data, i.e., from 1959Q4 through 2023Q3, which I denote here as final estimates. As discussed in CKP and Clark and Kozicki (2005), the examination of final estimates is beneficial for "historical analysis," such as the evaluation of past policy. However, for real-time analysis, such as forecasting and policymaking, real-time estimates at time t – estimates based on the data and model estimation through time t (instead of through 1:T) – are the relevant measures. In estimating the stars, a voluminous number of papers have documented the typical pattern of notable differences between real-time and final estimates; e.g., see Clark and Kozicki (2005) for r-star. These differences have contributed to past policy mistakes (e.g., see Powell (2018) and references therein). Hence, there is a preference for methods that provide credible inferences about stars in real-time.

Figure 7 plots the real-time posterior estimates of r-star and u-star and figures in Appendix A19 for other stars and the output gap from 1999Q1 to 2023Q3. Also plotted are the corresponding final or smoothed posterior estimates and the 68% and 90% credible bands, respectively. In all figures, the top row plots the estimates from the Base model, the middle row from Base-NoSV, and the bottom row from Base-NoSV-NoTVP-NoSurvey. The estimates from the latter two models are presented to highlight the role of SV and survey data in helping improve real-time stability. The estimates from the Base-NoTVP are not shown because they are similar to those from the Base model. The real-time estimates are the end-sample posterior mean (of the smoothed) estimates at any given period.

As can be seen by comparing the rows in each figure, it is generally the case for the Base model that real-time and final estimates track each other closely, suggesting real-time stability of the inference about the stars. Shutting down SV makes the inference less reliable, and excluding information from surveys adversely impacts the inference. These results indicate that in the Base model, survey data and SV contribute to the real-time stability of the stars and the output gap. It is also evident that including survey data leads to more precise inference, since the uncertainty bands are much narrower for the Base and Base-NoSV models than for the Base-NoSV-NoTVP-NoSurvey model. For example, in the case of rstar, the width of the estimated 90% intervals from the Base model has ranged between 1.7 and 2.3% in the last 25 years compared to the Base-NoSV-NoTVP-NoSurvey model's range of 3.0 to 3.8%. For reference, the typical estimates of 90% bands from popular models such as LW and Lubik and Matthes (2015) have a width averaging more than 3.5%.

***Forecasting evaluation.*** This subsection summarizes the results of three real-time out-of-sample forecasting exercises. The first two exercises examine the forecasting prowess of the Base model to its main variants and "hard to beat" benchmarks, and the third exercise examines the usefulness of the Base model's stars for external models. The evaluation sample spans 1999Q1 through 2019Q4. For purposes of brevity, I relegate the presentation and detailed discussion of forecasting results to Appendices A21 and A22, respectively.

Specifically, in the first exercise, I compare the forecasting performance of the Base model to its variants Base-NoSV, Base-NoTVP, and Base-NoSV-NoTVP-NoSurvey. I evaluate the accuracy of the point and density forecast for real GDP growth, PCE inflation, the unemployment rate, nominal wage inflation, labor productivity growth, and the shadow federal funds rate. The results, which are shown in Appendix A20, indicate that the Base model is generally more accurate on average compared to its variants for all variables considered. Forecasting results also indicate that variants of the Base model that include survey expectations data are more accurate than the variants that exclude survey information. In the second exercise, I compare the Base model's point and density accuracy to standard benchmark models, including small-scale UC models. The results of this exercise support the Base's comparable and, in some cases, superior forecasting properties relative to benchmarks.

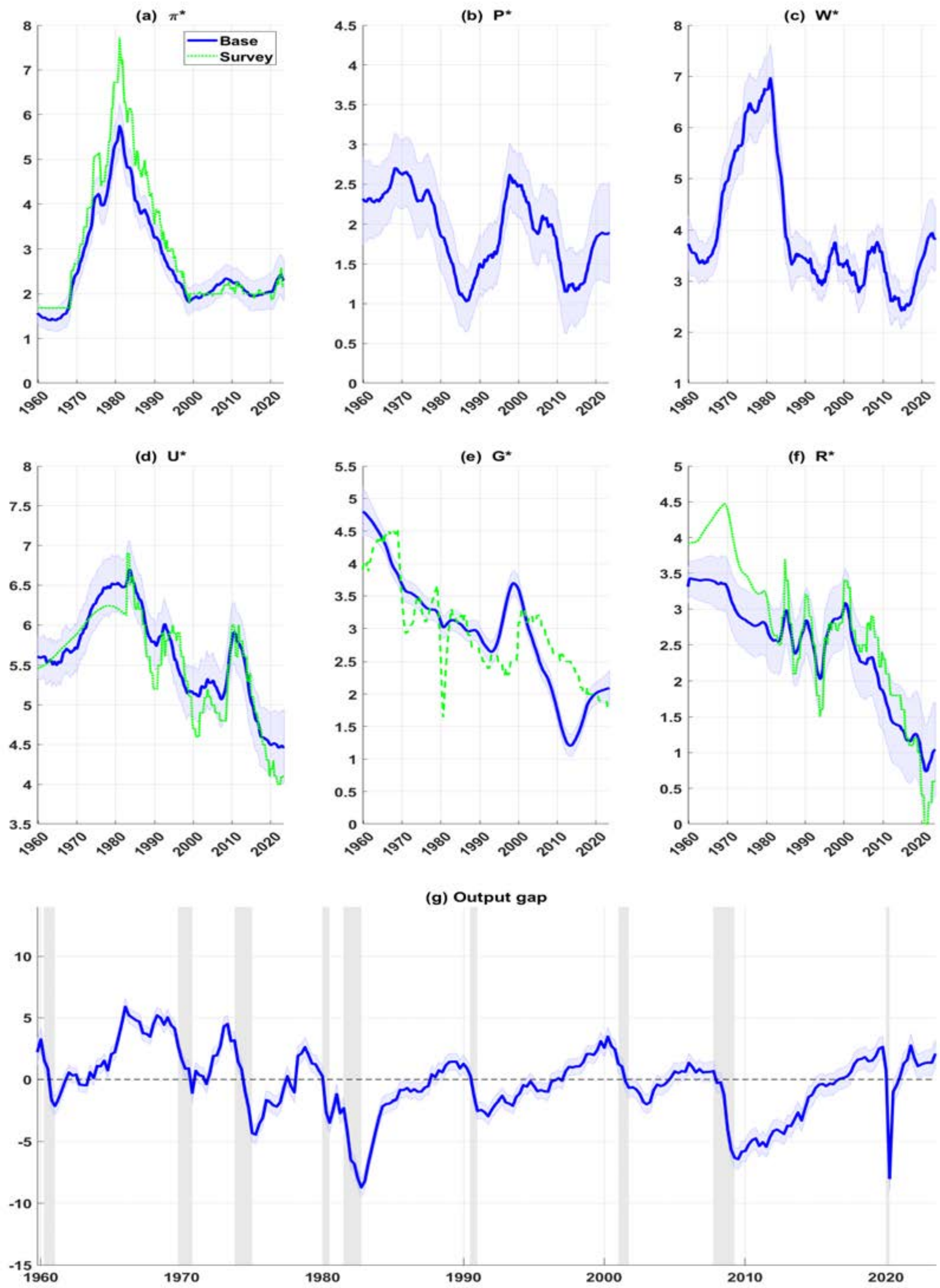
In the third forecasting exercise, using the Base model's real-time stars' estimates obtained, as discussed in the previous section, I illustrate their usefulness in forecasting with external models (e.g., steady-state VARs). Forecasters often use survey estimates to inform the steady-state values in VAR forecasting models, as doing so has been shown to improve forecasts. In this exercise, I show that if one were to inform the steady state values of the VAR model using the Base model's estimates of stars, then one would achieve further improvements in the forecast accuracy of the VAR variables (see Appendix A21). An additional advantage of stars from my framework is the availability of w-star, since surveys do not illicit responses about nominal wages. Overall, the real-time estimates of stars and forecast evaluation based on the past 25 years of data provide empirical evidence supporting the competitive forecasting and stable real-time properties of the Base model.

# 6 Conclusion

This paper implements a semi-structural model to jointly estimate the dynamics of inflation, nominal wages, labor productivity, the unemployment rate, real GDP, and interest rates to back out estimates of the long-run counterparts of these variables, i.e., stars. In contrast to the existing literature, the model features a novel combination of (1) time-varying parameters, (2) stochastic volatility, (3) joint modeling of multiple stars, (4) implementation of a richer structure, (5) allowing for a direct relationship between stars and long-term survey expectations, and (6) the provision of w-star (and its model-based decomposition into its theoretical determinants p-star and u-star). Doing so permits joint inference about stars and other macroeconomic objects of interest.

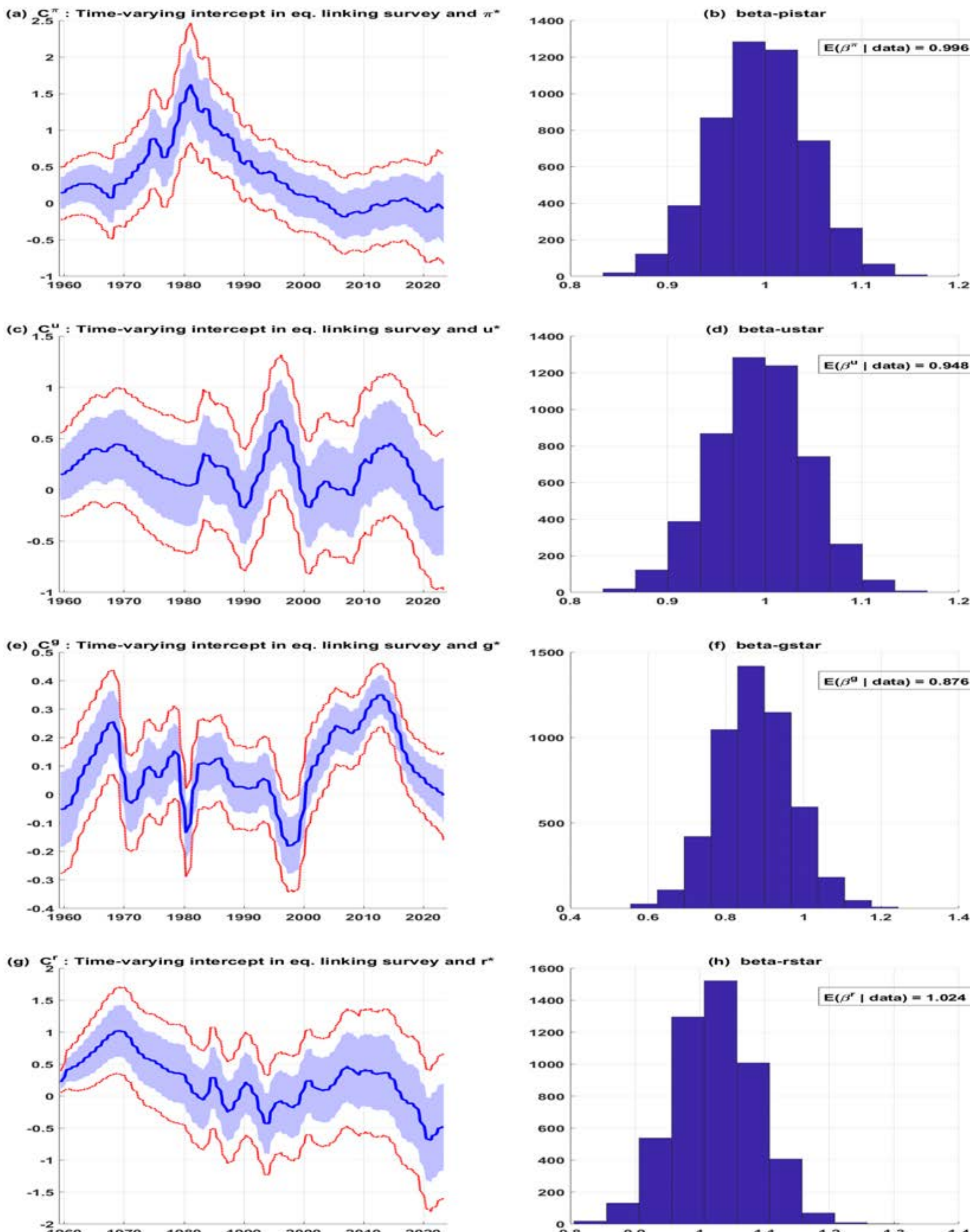
Several variants of the baseline model are estimated to assess the usefulness of survey data, time variation in parameters, SV, and expanded structure. The model comparison metric of marginal likelihood, real-time stability properties, precision metric, and real-time forecasting exercises favor the baseline model, suggesting support for including the various features. The results indicate that SV is highly effective in identifying high-volatility periods, including the extreme data fluctuations associated with the pandemic in 2020. This helps limit the influence of pandemic outliers on model estimates and, in turn, parametric inference. The use of survey data has helped discipline the estimates of the stars and improve their precision. More importantly, since the onset of the pandemic, the survey data have been the critical ingredient in permitting the estimation of the high-dimensional model and its variants. Lastly, the paper documents the competitive real-time forecasting properties of both the baseline model and, separately, the estimates of the stars if they were to be used as steady-state values in external macroeconomic models.

**Figure 1:** Full Sample Estimates of Stars and Output Gap from the Base model



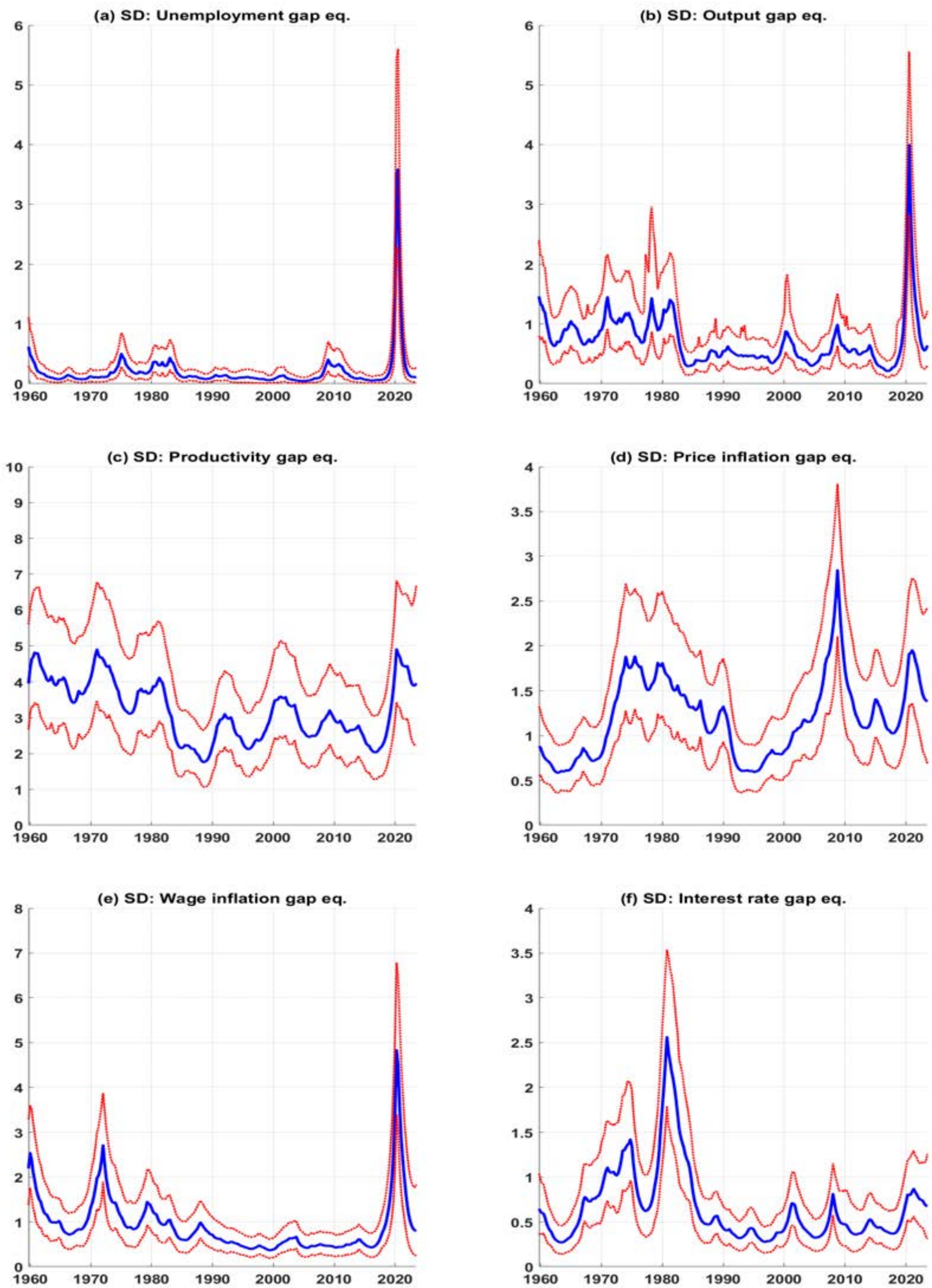

Note: The posterior estimates are based on the full sample (from 1959Q4 through 2023Q3). Shaded area represents the 68% credible intervals

**Figure 2:** Link between Survey Expectations and Base Stars



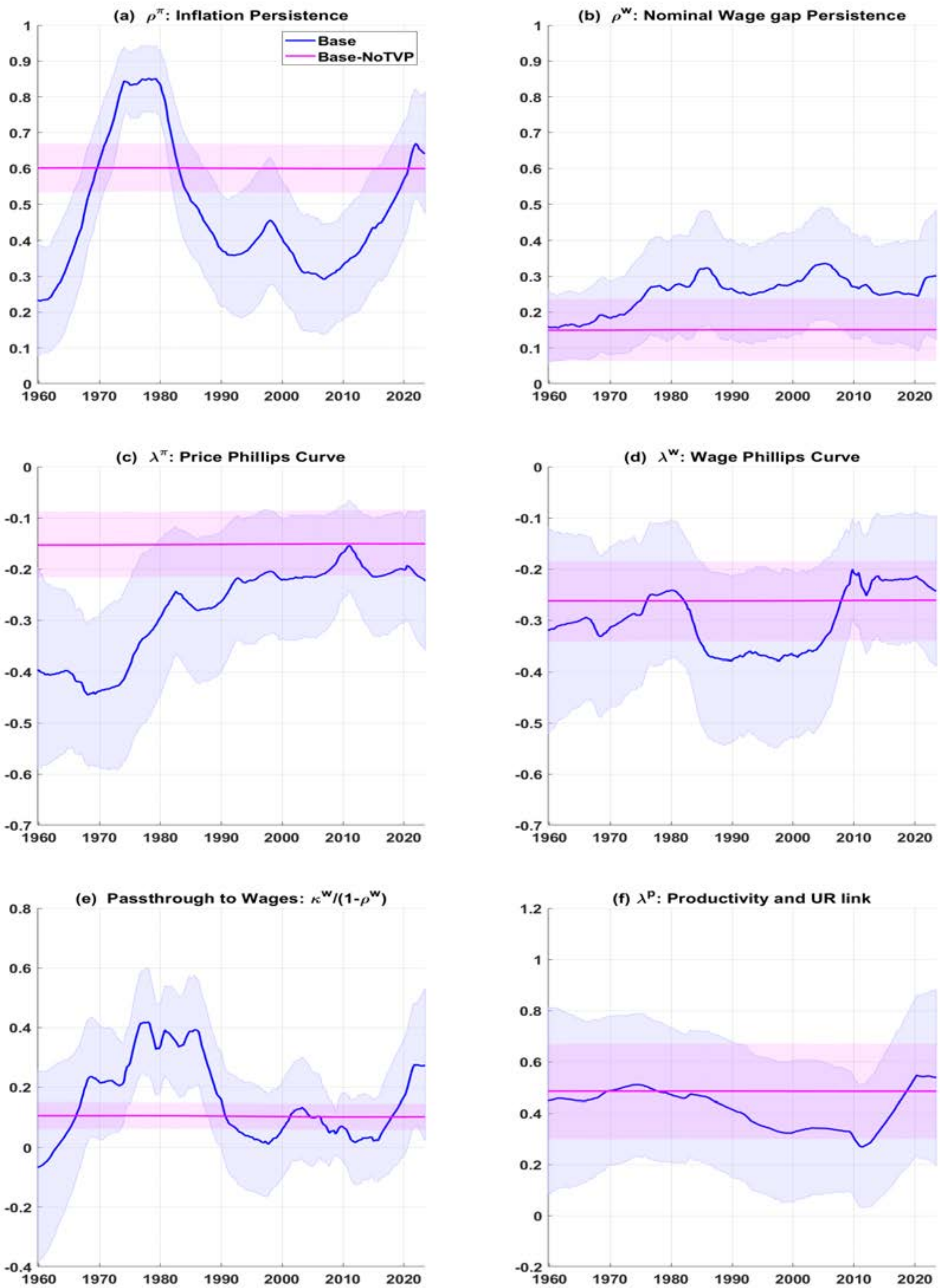

Note: Plotted are the posterior estimates using the full sample (from 1959Q4 through 2023Q3). In panels on the left column, the shaded area represents the 68% credible intervals and the dotted red lines represent 90% credible intervals. The panels on the right plot the posterior distribution of the Beta parameters.

**Figure 3:** Full Sample Estimates of Stochastic Volatility



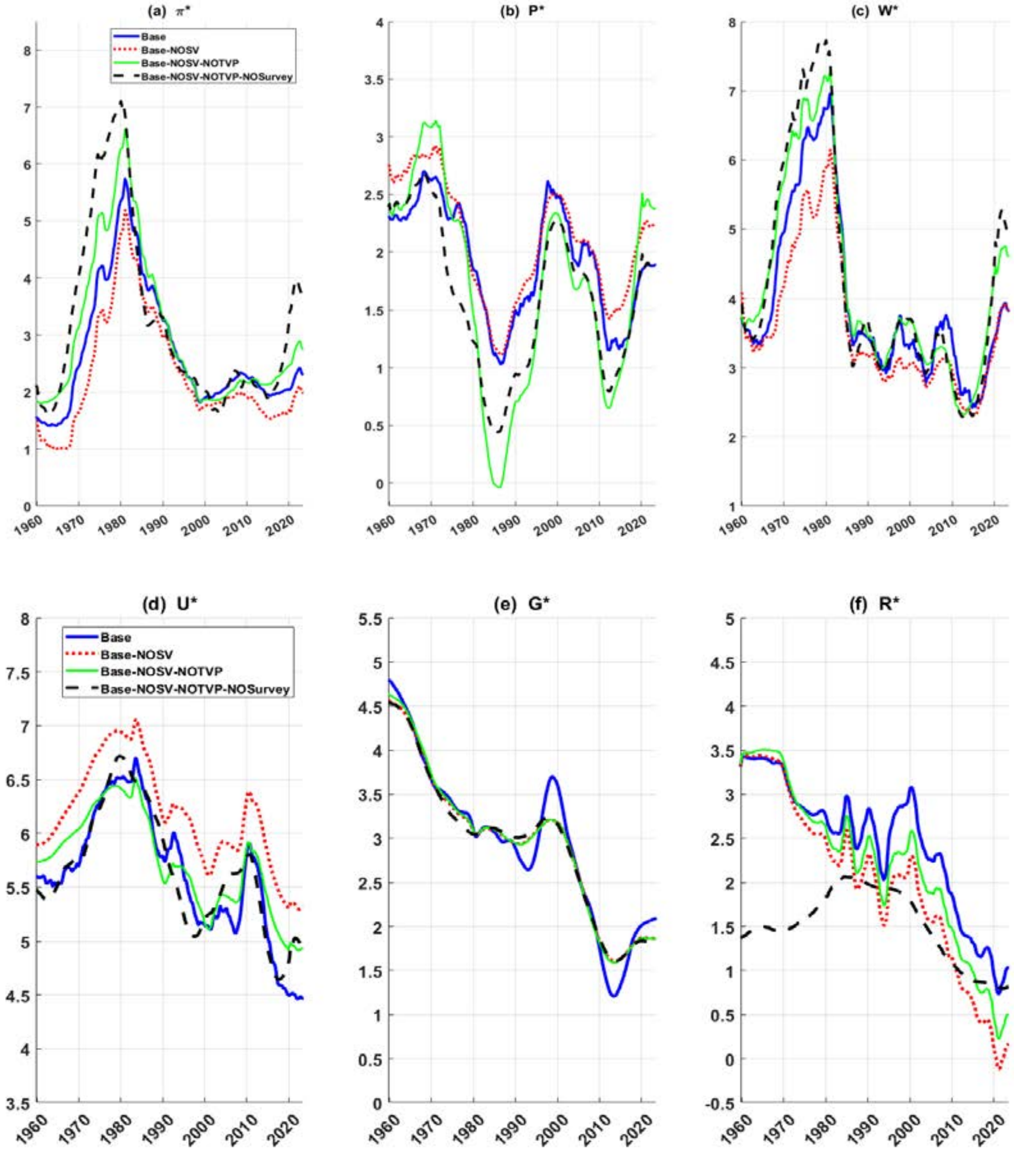

Note: Plotted are the posterior estimates that are computed using the full sample (from 1959Q4 through 2023Q3). The dotted red lines represent 90% credible intervals.

**Figure 4:** Full Sample Estimates of Time Varying Parameters



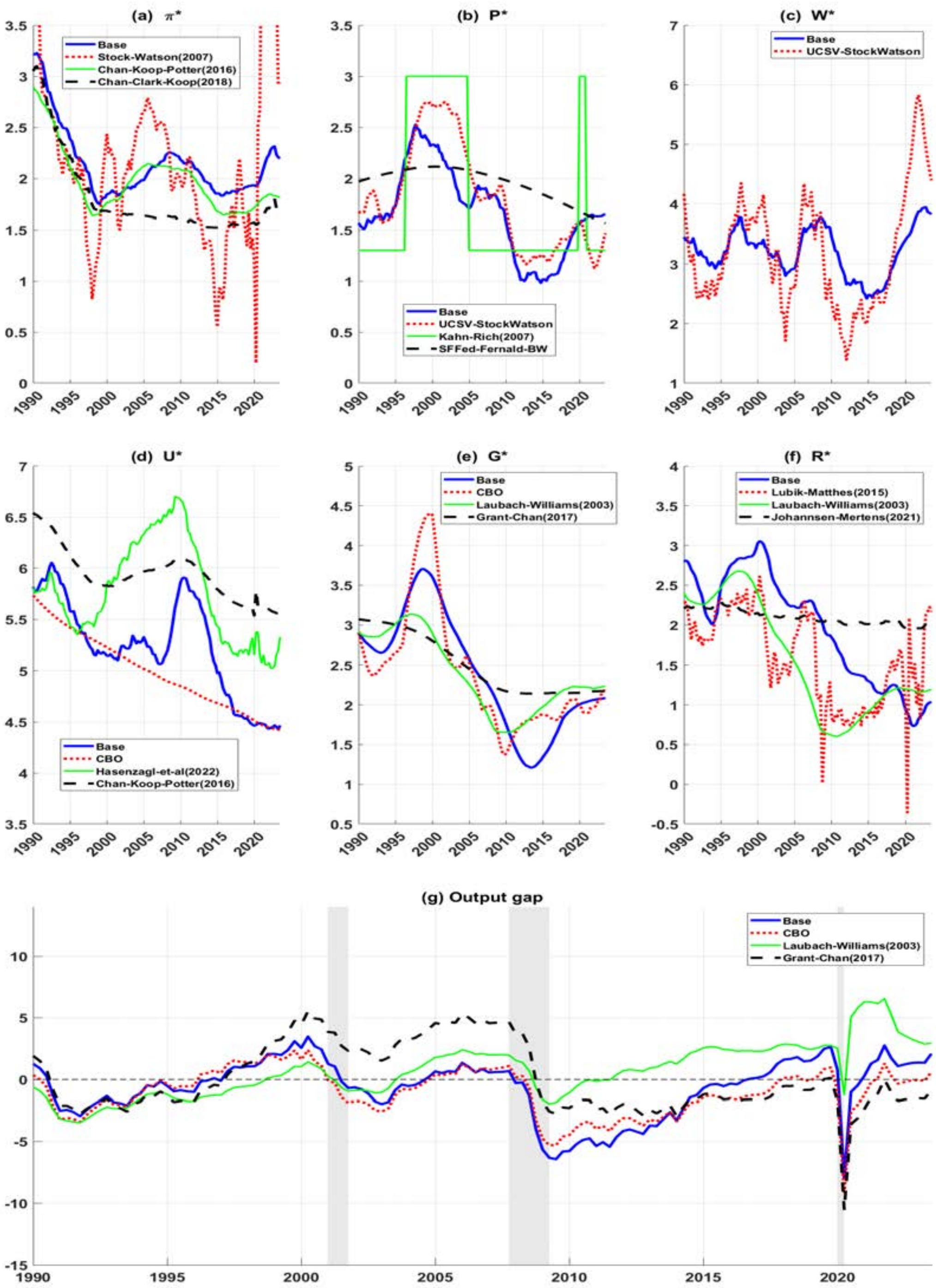

Note: Plotted are the posterior estimates that are computed using the full sample (from 1959Q4 through 2023Q3). Shaded area represents the 68% credible intervals.

**Figure 5:** Full Sample Estimates of Stars: Base model vs. Alternative Variants



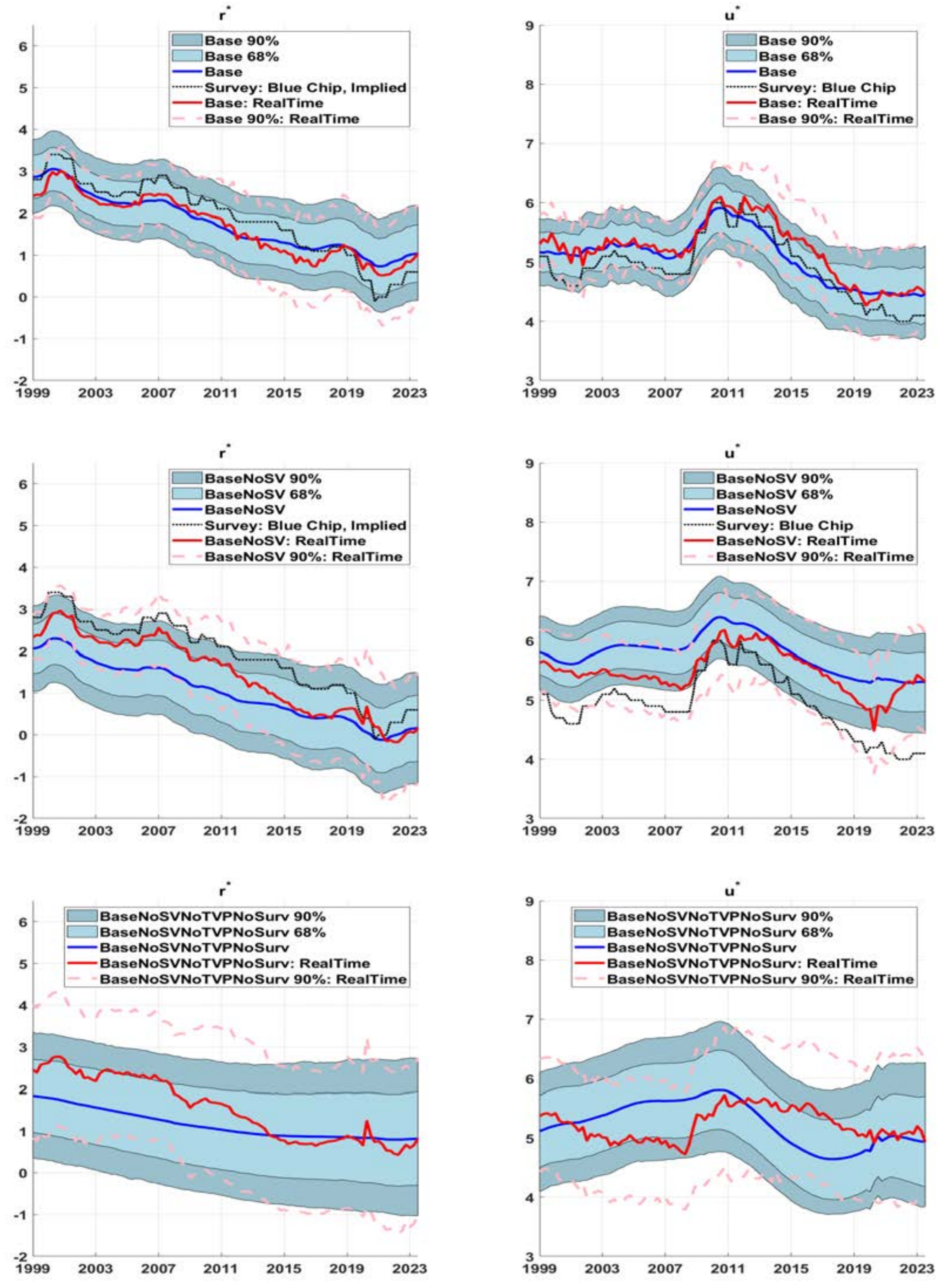

Note: Plotted are the posterior mean estimates, which are based on the full sample (from 1959Q4 through 2023Q3).

**Figure 6:** Estimates of Stars and Output Gap: Base model vs. Outside models




Note: Plotted are the (posterior) mean estimates that are computed using the full sample (from 1959Q4 through 2023Q3).

**Figure 7:** Real-time Recursive Estimates of Stars vs. Smoothed Stars




Note: The first row plots r-star and u-star estimates from the Base model. The second and third rows plot the corresponding estimates from the models Base-NoSV and Base-NoSV-NoTVP-NoSurvey, respectively.

**Table 1:** Bayesian Model Comparison: Base Model and Main Variants

Panel A: Based on estimation through 2023Q3, full-sample

| | **Base** | Base-NoSV | Base-NoTVP | Base-NoSV NoTVP | Base-NoSV NoTVP NoSurvey | Base-NoSurvey |
|---|---|---|---|---|---|---|
| MDD of Inflation | -407.1 | -456.7 | -422.3 | -473.6 | -464.7 | — |
| MDD of Productivity | -663.2 | -686.1 | -671.1 | -689.8 | -688.2 | — |
| MDD of Nominal Wage | -319.2 | -369.5 | -321.2 | -475.7 | -471.2 | — |
| MDD of Unemployment | 64.7 | -276.0 | 70.8 | -280.9 | -268.7 | — |
| MDD of Interest rate | -231.6 | -343.8 | -236.3 | -346.3 | -347.7 | — |
| MDD of GDP | -255.3 | -386.3 | -253.9 | -387.0 | -381.7 | — |
| MDD | -1811.6 | -2518.5 | -1834.0 | -2653.2 | -2622.2 | — |

Panel B: Based on estimation through 2019Q4, pre-COVID

| | **Base** | Base-NoSV | Base-NoTVP | Base-NoSV NoTVP | Base-NoSV NoTVP NoSurvey | Base-NoSurvey |
|---|---|---|---|---|---|---|
| MDD of Inflation | -365.2 | -412.5 | -379.8 | -427.7 | -420.7 | -365.6 |
| MDD of Productivity | -604.3 | -630.1 | -612.0 | -635.1 | -631.7 | -603.7 |
| MDD of Nominal Wage | -275.3 | -341.6 | -273.3 | -348.3 | -343.6 | -274.1 |
| MDD of Unemployment | 89.4 | 40.9 | 89.3 | 40.7 | 53.4 | 93.1 |
| MDD of Interest rate | -211.7 | -323.3 | -215.9 | -326.2 | -326.7 | -215.5 |
| MDD of GDP | -220.1 | -273.0 | -220.1 | -273.1 | -272.4 | -221.4 |
| MDD | -1587.3 | -1939.7 | -1611.8 | -1969.7 | -1941.6 | -1587.1 |

## References


Carriero, Andrea, Todd E. Clark, Massimiliano Marcellino, and Elmar Mertens (2022). "Addressing COVID-19 Outliers in BVARs with Stochastic Volatility." *The Review of Economics and Statistics*, pp. 1–38. doi:10.1162/rest_a_01213.

Cecchetti, S., M. Feroli, P. Hooper, F. S. Mishkin, and K. L. Schoenholtz (2023). "Managing disinflations." Discussion Paper DP18068, CEPR.

Chan, Joshua C. C., Todd E. Clark, and Gary Koop (2018). "A new model of inflation, trend inflation, and long-run inflation expectations." *Journal of Money, Credit and Banking*, 50(1), pp. 5–53. doi:10.1111/jmcb.12452.

Chan, Joshua C. C. and Ivan Jeliazkov (2009). "Efficient simulation and integrated likelihood estimation in state space models." *International Journal of Mathematical Modelling and Numerical Optimisation*, 1(1/2), p. 101. doi:10.1504/IJMMNO.2009.030090.

Chan, Joshua C. C., Gary Koop, and Simon M. Potter (2016). "A bounded model of time variation in trend inflation, Nairu and the Phillips curve." *Journal of Applied Econometrics*, 31(3), pp. 551–565. doi:10.1002/jae.2442.

Clark, Todd E. and Sharon Kozicki (2005). "Estimating equilibrium real interest rates in real time." *The North American Journal of Economics and Finance*, 16(3), pp. 395–413.

Del Negro, Marco, Domenico Giannone, Marc P. Giannoni, and Andrea Tambalotti (2017). "Safety, liquidity, and the natural rate of interest." *Brookings Papers on Economic Activity*, 2017(1), pp. 235–316. doi:10.1353/eca.2017.0003.

Fleischman, Charles A. and John M. Roberts (2011). "From Many Series, One Cycle: Improved Estimates of the Business Cycle from a Multivariate Unobserved Components Model." Technical Report 2011-46, Board of Governors of the Federal Reserve System.

Galí, Jordi and Thijs van Rens (2021). "The vanishing procyclicality of labour productivity." *The Economic Journal*, 131(633), pp. 302–326. doi:10.1093/ej/ueaa065.

González-Astudillo, Manuel and Jean-Philippe Laforte (2020). "Estimates of r* Consistent with a Supply-Side Structure and a Monetary Policy Rule for the U.S. Economy." Technical Report 2020-085. doi:10.17016/FEDS.2020.085.

Grant, Angelia L. and Joshua C. C. Chan (2017). "Reconciling output gaps: Unobserved components model and Hodrick–Prescott filter." *Journal of Economic Dynamics and Control*, 75, pp. 114–121. doi:10.1016/j.jedc.2016.12.004.

Hasenzagl, Thomas, Filippo Pellegrino, Lucrezia Reichlin, and Giovanni Ricco (2022). "A Model of the Fed's View on Inflation." *The Review of Economics and Statistics*, 104(4), pp. 686–704. doi:10.1162/rest_a_00974.

Hobijn, Bart, Russell Miles, James Royal, and Jing Zhang (2023). "The recent steepening of phillips curves." Economic Letter 475, Chicago Fed Letter.

Johannsen, Benjamin K. and Elmar Mertens (2021). "A time-series model of interest rates with the effective lower bound." *Journal of Money, Credit and Banking*, 53(5), pp. 1005–1046. doi:10.1111/jmcb.12771.

Kahn, James A. and Robert W. Rich (2007). "Tracking the new economy: Using growth theory to detect changes in trend productivity." *Journal of Monetary Economics*, 54(6), pp. 1670–1701. doi:10.1016/j.jmoneco.2006.07.008.

Kuttner, Kenneth N. (1994). "Estimating potential output as a latent variable." *Journal of Business & Economic Statistics*, 12(3), pp. 361–368. doi:10.1080/07350015.1994.10524551.

Laubach, Thomas and John C. Williams (2003). "Measuring the Natural Rate of Interest." *Review of Economics and Statistics*, 85(4), pp. 1063–1070. doi:10.1162/003465303772815934.

Lubik, Thomas A. and Christian Matthes (2015). "Calculating the natural rate of interest: A comparison of two alternative approaches." *Richmond Fed Economic Brief*, (Oct).

Mertens, Elmar (2016). "Measuring the Level and Uncertainty of Trend Inflation." *The Review of Economics and Statistics*, 98(5), pp. 950–967. doi:10.1162/REST_a_00549.

Morley, James C. and Benjamin Wong (2020). "Estimating and accounting for the output gap with large Bayesian vector autoregressions." *Journal of Applied Econometrics*, 35(1), pp. 1–18. doi:10.1002/jae.2733.

Pescatori, Andrea and Jarkko Turunen (2016). "Lower for Longer: Neutral Rate in the U.S." *IMF Economic Review*, 64(4), pp. 708–731. doi:10.1057/s41308-016-0017-x.

Powell, Jerome H. (2018). "Monetary policy in a changing economy." *In Changing Market Structure and Implications for Monetary Policy, volume 2018 Federal Reserve Bank of Kansas City, Jackson Hole, WY.*

Rychalovska, Yuliya, Sergey Slobodyan, and Rafael Wouters (2023). "Professional survey forecasts and expectations in DSGE models." Working Paper wp766, CERGE-EI.

Schmitt-Grohe, Stephanie and Martin Uribe (2024). "What do long data tell us about the permanent component of inflation?" Working Paper 30357, NBER.

Stock, James H. and Mark W. Watson (2007). "Why Has U.S. Inflation Become Harder to Forecast?" *Journal of Money, Credit and Banking*, 39, pp. 3–33. doi:10.1111/j.1538-4616.2007.00014.x.

Wright, Jonathan H. (2013). "Evaluating real-time VAR forecasts with an informative democratic prior." *Journal of Applied Econometrics*, 28(5), pp. 762–776. doi:10.1002/jae.2268.

Wu, Jing Cynthia and Fan Dora Xia (2016). "Measuring the macroeconomic impact of monetary policy at the zero lower bound." *Journal of Money, Credit and Banking*, 48(2-3), pp. 253–291. doi:10.1111/jmcb.12300.