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Daniel Monte and Roberto B. Pinheiro

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Costly Information Intermediation: Quality vs. Spillovers*

Daniel Monte Roberto Pinheiro

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Abstract

We analyze information intermediaries in large economies with costly information acquisition. Intermediaries face a trade-off between quality and dissemination speed. Both altruistic policymakers and profit-maximizing monopolists optimally choose to sample limited information, increasing the number of partially informed agents and enhancing spillovers despite slower information accumulation. Altruistic information-sharing bureaus minimize fees by inducing low provider default rates, while monopolist bureaus maximize fees through higher faulty service rates. Information trade resembles a natural monopoly, where competition reduces efficiency through redundant costs and lower information spillovers. These findings inform regulatory design in platforms and information-intensive markets.

Keywords: Costly Information Trade, Market Structure, Natural Monopoly
JEL D47, D83, D85

Conflict of Interest Statement: Nothing to Declare

*Monte: University of Torino, ESOMAS Department, Collegio Carlo Alberto and São Paulo School of Economics - FGV. E-mail: daniel.monte@unito.it. Pinheiro: Federal Reserve Bank of Cleveland. E-mail:roberto.pinheiro@clev.frb.org. We thank Luis Araujo, Joyee Deb, Bruce Fallick, Renato Gomes, Pawel Krolikowski, Igor Livshits, Lucas Maestri, Joao Ramos, and Jan-Peter Siedlarek for many helpful comments as well as seminar participants at Bristol University, University of Miami, 2018 Canadian Economic Association Meetings, IIOC 2018, 2018 Midwest Economic Theory, 2018 RES meetings, and 2018 Summer Workshop on Money, Banking, Payments, and Finance. The views in this article are those of the authors and do not necessarily reflect the views of the Federal Reserve System, the Federal Reserve Bank of Cleveland, or the Board of Governors.

1 Introduction

Market interactions often occur infrequently, making the reputational cost of bad service or small loan defaults seem minimal, especially when legal recourse is costly. While reputation would matter if information spread quickly, word-of-mouth is less effective in large, mobile populations. To address this, markets develop information-sharing systems that allow consumers to report dishonest behavior, incentivizing better service and reducing harm. Understanding the role and effectiveness of these systems is critical to promoting cooperation and ensuring the well-functioning of such markets.

In the United States, information reporting is mainly done through the private sector, with varying structures. In the consumer protection realm, there are non-profit organizations such as the Better Business Bureau and for-profit entities such as Angi (formerly Angie's List), a bureau focused on home improvement services established in 1995. Information-reporting systems can adopt different revenue models, such as explicit membership fees or advertising revenue. For example, the Better Business Bureau charges membership fees to accredited businesses, and Angi, for more than two decades, had its revenue model primarily focused on membership fees charged to customers. In 2016, facing a decline in member growth, Angi introduced a free tier of services alongside its paid subscription model. Other well-known information intermediaries such as TripAdvisor, Yelp, and HomeAdvisor offer free access to reviews and rely upon on-site advertisements for revenue generation.

We study information-reporting systems in large populations. Our economy has three significant features that aim to represent the economies served by the above-mentioned information intermediaries. First, there is moral hazard: service providers have a myopic incentive to shirk after being hired. Second, providing feedback is costly to consumers, creating a double moral hazard problem. Third, the population is large, and consumers are anonymous. Each of these three features hinders cooperation, and a market cannot exist where consumers hire services without providers shirking. In other words, in the absence of a third party, there is an impossibility result: cooperation cannot be sustained.

An information intermediary or bureau[1] might induce cooperation by collecting data from transactions and informing consumers about the past behavior of service providers, thereby disciplining providers' behavior. This information intermediation imposes an important challenge for the bureau: how to balance the incentives on both sides of the market? If consumers do not have enough incentives to report faulty services, then providers will shirk. On the other hand, if incentives are designed in a way that exposes providers to a very high cost of default, then there will be free riding by consumers. Consumers will hire service providers without the need to rely on the information bureau, thereby eroding its profitability and halting cooperation altogether. The challenge lies in finding the institutional design

[1] While in many of the examples we described the information intermediary calls itself a platform, we prefer the term *bureau*, since it describes more clearly the intermediaries in our model. As stated by the Merriam-Webster dictionary, a bureau is an "agency that serves as an intermediary especially for exchanging information or coordinating activities."

that balances these incentives and sustains cooperation.

Our model is a random matching set-up with one large population of consumers and another large population of service providers in an infinite horizon economy.[2] At the start of each period, each consumer is randomly matched to a single service provider and they play a sequential game in which the provider has a short-run incentive to shirk. Agents are assumed to be risk-neutral and forward-looking.[3] All providers are rational, and they consider the loss of future earnings as their only motivation for good behavior. At the end of each period, each consumer may send a public message to inform others about their current encounter. Consumers are anonymous but service providers are not. We assume that the consumers' feedback is costly, so in any equilibrium without an information-sharing bureau, no messages are sent, even when costs are arbitrarily small. Consequently, without a bureau, the market for services collapses and providers are never hired.

The bureau's chosen pricing strategy includes a membership fee that allows a paying customer to access the bureau's information, a compensation paid to customers for providing feedback, and the proportion of feedback bought every period. In our model, the membership fee and compensation paid are explicitly modeled as monetary transfers between the bureau and its customers. More interestingly, they can be thought of as advertising revenues and the costs of acquiring feedback, such as the costs of soliciting feedback or making the website more attractive to induce the provision of feedback. Finally, we focus on stationary rules, implying that pricing cannot be conditioned on time.[4]

The difficulty in finding the bureau's optimal strategy is that for each possible pricing mechanism chosen by the bureau, there is a different induced equilibrium with a corresponding mass of informed and uninformed consumers and an induced quality of the information (or coarseness of the information set) that informed consumers obtain. In other words, each possible pricing scheme determines in equilibrium the number of informed consumers in the economy, the number of service providers that default in equilibrium, and the induced cost of implementing such a pricing policy.

The main trade-off faced by the bureau is between offering a large informational base from the start, which is expensive to obtain but can be sold at a higher membership price, and offering a less extensive informational base, which is cheaper to obtain and can be purchased by a larger mass of consumers. In other words, the counterpart of the differences in information quality is the cost of information acquisition. There are three components to the cost of information. First, there is a direct cost, which is captured by the membership fee. This direct cost is strictly increasing in information quality, since the quality depends on the number of consumers that must be compensated for reporting

[2]The population is large enough that a folk theorem such as the one presented in Kandori (1992) and Ellison (1994) is not possible.

[3]We focus on the case of an economy with risk-neutral agents to avoid issues of insurance against negative shocks that may complicate the analysis.

[4]In an earlier version of this paper (available upon request) we considered different market structures. Specifically, we considered different pricing methods such as cases without membership fees, but in which the bureau charges a fee whenever a consumer or creditor would like to access the bureau's database (pays a compensation fee for reports of services provided).

a transaction with a provider. Hence, any arrangement in which the bureau chooses to buy a large amount of information from consumers in every period has a high direct cost. The direct cost shrinks a bit over time as members learn about defaulting providers and stop buying their services, but it does not fluctuate much. Second, there is an indirect cost due to the expected loss from faulty services that informed consumers may face. Notice that this cost is strictly decreasing in the information's quality. In the case of a hypothetical bureau that acquired all information at once, the indirect cost would be zero. In contrast, in a membership arrangement, the indirect cost is strictly positive and approaches zero as time passes and the bureau learns over time. However, the speed at which the bureau's quality improves varies with the bureau's size. Large bureaus learn faster, so indirect costs converge to zero at a faster rate. Third, there is a cost of a lower service rate, i.e., the cost of having a meeting with a known faulty service provider that results in no service purchase. This cost is proportional to the number of providers that choose to provide faulty service in equilibrium and negatively related to the number of consumers that choose to buy membership.

To balance this trade-off, the bureau optimally chooses its information sampling scheme, that is, the probability with which it buys information from its members. We show that the optimal sampling scheme involves a low probability of buying (inducing) feedback that allows for a small membership fee, which, in turn, maximizes the bureau's size. As a result, a service provider that delivers bad service is more likely to get away with it for a while. However, whenever faulty services are detected, the information is spread across all consumers in the economy. Hence, once a faulty provider is detected, its sales drop to zero. As a result, the larger bureau size reduces the share of providers that choose to deliver faulty services in equilibrium, reducing the cost of lower service rates as well as indirect costs in the early years of the bureau. We show that the optimal information-sampling scheme is the same for altruistic bureaus that seek to maximize social welfare as well as for a bureau that maximizes its profit.

To further understand the trade-off between information quality and the degree of dissemination, we extend our study and consider competition among for-profit bureaus. We show that the equilibrium number of providers that default is the same as in the monopoly case. By studying competition among bureaus, we show that the trade of costly information in a market with atomistic agents has characteristics similar to a natural monopoly. The direct cost of information acquisition can be seen as a high fixed cost that may be duplicated because of competition among information intermediaries. Moreover, when bureaus with partial information compete, the average number of agents that transact with each bureau goes down. Consequently, the number of agents that learn about a previously faulty service at each round is smaller, slowing down information diffusion while increasing indirect costs. Both the monopoly case and the case of competing bureaus result in inefficient allocations.

In the Online Appendix, we present several robustness checks for our results. First, we present the case of a bureau that only buys negative information. Second, we assume that agents incur a positive cost of accessing information. Finally, we consider the case of observable membership. While these robustness checks help us to further understand the presented model, their results do not qualitatively change the messages presented in the body of the paper.

Literature Review

The literature on cooperation in long-term relationships has focused mainly on two mechanisms underlying the notions of trust and reputation.[5] One is a standard repeated game approach based on repeated interactions. The other one is based on an adverse selection argument and reputation is viewed as learning the underlying types. In the former case, agents can use strategies that condition future behavior on current actions and cooperation can be sustained if agents are sufficiently patient. In the latter case, agents may want to build a reputation by either mimicking a particular type or by behaving in ways that distance them from incompetent types. We depart from both cases in this paper. First, since we have a large population, standard game theory tools for cooperation, such as equilibria with grim-trigger strategies, cannot be used. Second, there is no uncertainty about types in our model, so we cannot use the tools of reputation games, such as Fudenberg and Levine (1992), Mailath and Samuelson (2001), and more recently, Tadelis (2016). Indeed, our model can be seen as a different version of Tadelis's model, one in which all sellers are strategic. Consequently, incentives for cooperation must be provided at all periods.

We study how an information intermediary may help in sustaining cooperation in large economies with costly feedback. Therefore, our paper contributes to bringing together the literature on community enforcement under costly feedback and the literature on optimal information intermediation, such as the design of rating systems.

Our underlying model is related to the literature on community enforcement starting with Okuno-Fujiwara and Postlewaite (1995) and other recent papers on community enforcement, such as Takahashi (2010), Bhaskar and Thomas (2019), Ali and Miller (2016) and Deb (2020).

Cooperation in large economies with observation of others: Mainly, our paper contributes to the literature on cooperation in large economies with imperfect observation of others' past play. Some authors have considered the case in which players can only observe outcomes from interactions in which they have been directly involved. In these cases, with finite populations, it is possible to achieve cooperation through contagious equilibria (Ellison (1994), Harrington (1995), and Kandori (1992)). Deb (2020) extends these results and shows the folk theorem for general stage games with cheap talk. Takahashi (2010) considers an environment like ours: a continuum of players randomly matched in each period. However, in Takahashi's model the cost of observing the partner's past play is free.[6] Awaya (2014) shows that such equilibria cannot be sustained when there is a cost for accessing a partner's past play. Instead, Awaya demonstrates that with cheap-talk communication it is possible to construct equilibria that are robust to small observation costs. In our paper, observation of past play is only

[5] See Cabral (2005) and Mailath and Samuelson (2006) for a discussion.

[6] That is, Takahashi's model is such that first-order information is free, whereas higher-order information (information about a partner's partner's past play, for example) is prohibitively high.

possible if: (i) costly feedback was successfully induced, and (ii) some player or institution collected this feedback.

Our paper is also related to the design of optimal pricing schemes in two-sided platforms. Some well-known papers that investigate this include Baye and Morgan (2001), Caillaud and Jullien (2003), and Rochet and Tirole (2003). We extend that literature by studying optimal platform design in dynamic environments, so the dissemination of information over time is a crucial aspect of our work.

Optimal information intermediation in long-term games: There is also a recent literature on the optimal design of information intermediaries in long-term relationships. Ekmekci (2011) constructs a rating system that will enable long-run cooperation between a long-run player and a sequence of short-run players restoring efficiency. Vong (2022) constructs a rating system that induces cooperation in repeated games with moral hazard, and he shows that approximately efficient equilibria might require coarse information disclosure. Lorecchio and Monte (2023a) show how to construct simple rating systems that induce cooperation in bad reputation environments, while Lorecchio and Monte (2023b) construct an information intermediary to elicit feedback from products and persuade consumers to purchase. More generally, by studying optimal information structures from a third party's point of view, our work is also related to the literature on reputation and information design, such as Hörner and Lambert (2021); Smolin (2021); Halac et al. (2017); Kremer et al. (2014) and Che and Hörner (2018). In our paper, the environment is different: there is a large economy with anonymous customers and the platform chooses how much information to purchase as well as the pricing scheme. There is also an extensive literature on the collection and transmission of consumer information. See Taylor (2004) and Calzolari and Pavan (2006) for classic references on the topic as well as Ichihashi (2020), Bergemann et al. (2022), and Acemoglu et al. (2022) for some of the more recent references. See also Goldfarb and Tucker (2019); Goldfarb and Que (2023), and Bergemann and Bonatti (2019) for recent surveys. Also related are papers on the optimal design of platforms in one-sided markets, such as Jacobs et al. (2021) and Xiao and Van Der Schaar (2021).

Finally, our paper is also related to the literature on information sharing in credit markets, as presented by Pagano and Jappelli (1993), Jappelli and Pagano (2002), and Brown et al. (2009). A few papers have studied how markets that depend on permanent reputations are affected by different information-sharing mechanisms. Some important papers in this area that are linked to ours are Vercammen (1995), Ekmekci (2011), Liu and Skrzypacz (2014), Elul and Gottardi (2015), Kaya and Roy (2022) and Kovbasyuk and Spagnolo (2024). In addition, our paper is related to mechanisms that were developed by society throughout history in order to overcome the lack of community enforcement in large societies, as pointed out by Milgrom et al. (1990), Araujo (2004), and Araujo and Minetti (2011), among others.

This paper is divided as follows. Section 2 introduces the environment without an information bureau. Section 3 considers first the case in which a non-profit altruistic bureau is introduced and then the case of a for-profit bureau. Section 4 presents the case where the bureau chooses the optimal information sampling. Section 5 introduces competition among profit-seeking bureaus. Finally, Section 6 concludes by summarizing the paper's results. All proofs are presented in the Appendix.

2 Basic Model

Consider two populations indexed by i, where i lies in $I_i = [0, 1]$, $\forall i \in \{1, 2\}$. We assume that $x \in I_1$ is a consumer and $y \in I_2$ is a provider. Agents are distributed according to a Lebesgue measure. We assume that consumers are anonymous but providers are not. In each period $t = 1, 2, ...$, each consumer is randomly matched to a provider to play a stage game Γ. We assume that the probability distribution over possible matches in each period is uniform regardless of the matching history. Therefore, the probability that a currently matched pair of players will match again is zero.

The stage game Γ is represented by the game tree in Figure 1.[7] Consumers initially decide whether or not they should hire the provider. Not hiring the services generates a payoff of zero to both parts. Hiring the provider's services implies that the consumer must pay w to the provider, irrespective of the quality of the service. If the provider is hired, then the provider must decide whether or not he will put effort into the service. If the provider puts effort, the service is of high quality, inducing a payoff of $P > w$ to the consumer. If no effort is exerted, in which case we say that the provider defaulted, the service has low quality, generating no benefit to the consumer. Putting effort into a task generates a disutility e to the provider. We assume that $0 < e < w$, but $P - e > 0$, so hiring the service and exerting effort are socially optimal. Effort is verifiable but the expected cost of a lawsuit is too high to be used as a credible threat. After the service is provided, the consumer must decide whether or not she will send a message to others about the quality of the service received. These messages consist of saying whether or not the provider made an effort. We must be aware that this is not related to any intrinsic quality of the provider, but only to his immediate previous action (all providers are *ex ante* identical, acting rationally to maximize their payoffs). The consumer incurs a fixed cost $c > 0$ for sending each message. For this basic model, we may assume that these messages are publicly available to the consumers.[8]

This is an infinite horizon problem in which each agent discounts future periods by the same rate $\delta \in (0, 1)$. We assume a minimum patience level throughout the paper. Precisely, we assume that $\delta > e/w$. Most of our results, unless explicitly mentioned, do not depend on a high discount factor, except for this minimum threshold.[9] Histories are private and a history observed by an agent i at time $\tau > 1$, denoted by $h_{i,\tau}$, is a sequence of private interactions that agent i observed from periods 1,2,...,$\tau - 1$. The set of all (private histories) at time t is denoted by $\mathcal{H}_t$ and the set of all histories by

[7]Figure 1 is at the end of the paper.

[8]Once we introduce information intermediaries, we will assume that messages can be made public only through these intermediaries.

[9]When this threshold is not met, the provider would prefer to default even in a world in which all consumers would immediately become informed about the default. When we look at the competitive bureau case, it will sometimes be convenient to assume that $\delta > \sqrt{e/w}$.

$\mathcal{H} = \cup_{t=1}^{\infty} \mathcal{H}_t$. Let σ denote the behavior strategy profile. In each stage game, the consumer's behavior strategy encompasses the actions the consumer must take, conditional on all the possible histories. In particular, it includes her initial decision of whether or not to hire the provider, conditional on the information set at that period t. It also includes her next decision in the stage game, where she must decide whether or not to send a costly message to inform others about the received service.[10] Similarly, a behavior strategy for each provider i is denoted by $\sigma_i : \mathcal{H} \times \{hire, don't\ hire\} \rightarrow \{effort,\ default\}$. A strategy profile σ^* is a *perfect Bayesian equilibrium* if, for every $t \geq 1$, every $h \in \mathcal{H}$, every pair (i, j), every play of the stage game and every σ it holds that: $U_t(\sigma^*|h_t) \geq U_t\left(\sigma_i, \sigma_j^*|h_t\right)$, for all σ_i, and where $U_t(\cdot)$ represents the expected continuation payoff of the repeated game.

Given that there is a continuum of agents, there is a zero probability of rematching with a former partner. Thus, there is no incentive to either insure oneself against former deviations or obtain gains punishing former deviators by sending messages. Therefore, since messages are costly, no consumer would send a message. Hence, there is no way to punish a former deviator. Therefore, the only equilibrium would be the infinite repetition of the stage game Nash equilibrium. We state the result in the following proposition.

Proposition 1 (No Trade) *The only equilibrium in this game is one in which there is no trade: Providers are never hired on the equilibrium path and providers shirk and consumers do not send messages off the equilibrium path.*

In summary, the market collapses in the absence of an information-sharing bureau, regardless of how small the cost of providing information is. In this sense, the introduction of an information-sharing bureau is likely to improve social welfare. The question becomes: can a profit-seeking bureau improve welfare? We consider a bureau that operates with a membership system: members must pay a membership fee, and then they can access information at no additional costs. Moreover, members are compensated for sending information to their bureaus. To carry out our analysis, we assume that the bureau can keep track of individual providers and their past behaviors.[11]

3 Information-Sharing Bureau: Benchmark

In this section we introduce the information-sharing bureau. Throughout the paper we focus on equilibria that are stationary in the sense that a given agent i always takes the same action when she is indifferent.[12]

[10]Her behavior strategy can be described by the following pair $s_i = (s_{h,i}, s_{m,i})$, where $s_{h,i} : \mathcal{H} \rightarrow \{hire, don't\ hire\}$ and $s_{m,i} : \mathcal{H} \times \{hire, don't\ hire\} \times \{effort,\ default\} \rightarrow \{good, bad, \emptyset\}$.

[11]Formally, this assumption means that a bureau can distinguish between two distributions even if they differ by measure zero. It is often assumed that two random variables are equal if they are equal almost everywhere (a.e.), using some notion of measure zero. Here we assume that two random variables are equal only if they are equal everywhere. A similar assumption is present in Kocherlakota (1998).

[12]Note that this restriction rules out certain equilibria that are not robust to small shocks, such as belief-free equilibria.

Further, we focus on stationary equilibria in which uninformed players choose to hire on the equilibrium path.

Before we formally define a stationary equilibrium, it is convenient to define the set $A(h)$ to be the set of actions available to player i in the information set following history h. Additionally, define $I_j(h)$ to be the summary statistic information about the provider j's past behavior at history h. I.e., $I_j(h) \in \{-1, 1\}, \forall j$, where -1 represents the case in which the provider has ever chosen to provide faulty service and 1 if it has never been caught providing faulty service.

Definition 1 (Stationary Equilibrium) *A behavior strategy σ_i is a* stationary strategy *if for any two histories h^t and h^τ (of equal or different lengths) with $A(h^t) = A(h^\tau)$: provider i chooses the same action, that is, $\sigma_i(h^t) = \sigma_i(h^\tau)$; and if, in addition, $I_j(h^t) = I_k(h^\tau)$, $\forall j, k$, consumer i chooses the same actions, that is, $s_i(h^t) = s_i(h^\tau)$. The strategy profile σ is a* stationary equilibrium *if σ is a stationary strategy profile and a perfect Bayesian equilibrium.*

In these equilibria, stationary strategies are best responses when all other agents are playing stationary strategies. In this class of equilibria, there is always a trivial equilibrium in which every provider defaults if hired, but none are hired on the equilibrium path. We focus on the non-trivial equilibria in stationary strategies.

The bureau chooses a pricing scheme (f_m, f_s) in which f_m is the membership fee paid by consumers once and for all. Once the consumer pays the membership fee, she is able to freely access the bureau's information database and is able to sell information to the bureau at price f_s whenever she experiences a transaction. In this section, we assume that the bureau buys information from all members who hired services. In the next section, we consider the optimal information sampling.

We assume that there is a time period $t = 0$ in which the bureau commits to a pricing scheme and customers must decide whether or not to become members; this period is before the first interaction between customers and sellers happens.[13]

Bureau members always receive compensation for giving information, just enough to cover their costs of sending the information. *Only members can receive or report information.* Assume that there are two technological constraints in the environment: (1) it is impossible for the bureau to credibly reveal to the service provider who is a member and who is not.[14] Indeed, think of this as a rating system where the service provider cannot really tell where the customer got her information from; and (2) it is not possible to make information public market-wide unless it is through the bureau and only to members.

Let $Y_{A,member}$ be the fraction of providers who default in equilibrium and $X_{A,member}$ be the fraction of consumers who buy a membership. If a consumer is a member, her period payoff depends on the fraction of providers who put in effort $(1 - Y_{A,member})$, the fraction of providers who default every period and have served a bureau's member at least once, and the fraction of providers who default, but have

[13]For convenience, we work with per period membership fees denoted by f^A_{ee}.

[14]In the Online Appendix we discuss an extension that considers the case of observable membership.

never previously served a bureau's member. The latter measure is the source of an indirect cost for informed consumers. Bureau members have just partial information, facing the possibility of default even after acquiring information. A reduction in information quality induces a lower membership fee. Moreover, the likelihood of members facing default decreases over time. Since providers that default eventually meet a bureau's member, the information eventually becomes available to all other members, reducing the likelihood of a member facing default in the future.

Consumers We initially focus on a stationary equilibrium in which all consumers that join the membership do so in period $t = 0$.[15] Thus, in period $t = 0$, there is a $1 - Y_{A,member}$ chance that the consumer faces a provider who does not default and a $Y_{A,member}$ chance that the matched provider defaults. Given that this is the first period, no consumer knows which provider she's facing. In the second period, assuming that a fraction $X_{A,member}$ has bought the membership, there is a $1 - Y_{A,member}$ chance of facing a provider that does not default; a $X_{A,member}Y_{A,member}$ chance of facing a provider who is known to default – therefore, a bureau's member does not hire his services; and a $(1 - X_{A,member})\,Y_{A,member}$ chance of facing a provider who defaults, but was not caught in the previous period. Summing up for all periods, this means that the payoff of the consumer joining the bureau in period $t = 0$ is:

$$\begin{aligned}(1 - Y_{A,member})(P - w) - f_{ee}^{A} + (1-\delta)\textstyle\sum_{t=0}^{\infty}\delta^{t}(1 - X_{A,member})^{t}\,Y_{A,member}(-w) \\ = (1 - Y_{A,member})(P - w) - f_{ee}^{A} - (1-\delta)\frac{Y_{A,member}w}{1-\delta\left(1-X_{A,member}\right)}\end{aligned} \tag{1}$$

If a consumer does not buy a membership, her payoff when hiring is:

$$(1 - Y_{A,member})(P - w) + Y_{A,member}(-w) = (1 - Y_{A,member})\,P - w. \tag{2}$$

Therefore, if the consumer is indifferent between joining the bureau or not in period $t = 0$, equations (1) and (2) give us the following condition:

$$Y_{A,member}\,w\frac{\delta X_{A,member}}{1-\delta\,(1 - X_{A,member})} = f_{ee}^{A}. \tag{3}$$

We now restrict our analysis to the case in which non-members as well as members purchase the service in the period before the membership kicks in. Notice that the expected payoff to hiring a service with no record on the provider is $(1 - Y_{A,member})\,P - w$. Thus, in the equilibria that we are considering, $(1 - Y_{A,member})\,P - w \geq 0$, i.e.:

$$Y_{A,member} \leq 1 - \frac{w}{P}. \tag{4}$$

Providers Each provider chooses whether or not to default. If he chooses not to default, he gets a payoff of $w - e$ for every period (recall that in the equilibrium we focus on, non-members also hire every

[15] In a previous version of the paper, we show that we can replicate the results even after considering that consumers could join the bureau at any period.

period). If the provider decides to default in any period t, he may default against either a member or a non-member. If he defaults against a member, he will never be hired by members again. Consequently, the payoff to defaulting can be obtained using the following recursive equation:

$$U_{Default} = w + \delta \left\{ X_{A,member} \frac{(1 - X_{A,member})w}{1-\delta} + (1 - X_{A,member})\, U_{Default} \right\}$$

therefore, the provider is indifferent between defaulting and not defaulting if:

$$\frac{(1-\delta)\, w}{1 - \delta\,(1 - X_{A,member})} + \delta \frac{X_{A,member}\,(1 - X_{A,member})\, w}{(1 - \delta\,(1 - X_{A,member}))} = w - e.$$

Simplifying it, we get:

$$\frac{\delta X_{A,member}^2 w}{1 - \delta(1 - X_{A,member})} = e \tag{5}$$

Solving the equation for $X_{A,member}$ and keeping in mind that $X_{A,member} \in [0, 1]$, we have

$$X_{A,member} = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta\,(1-\delta)\, we}}{2\delta w}. \tag{6}$$

Note that given the parameters e, δ, w, there is only one value of $X_{A,member}$ that is consistent with a stationary equilibrium. Moreover, given that $\delta > \frac{e}{w}$, from equation (6) we confirm that $X_{A,member} \in (0, 1)$. Finally, in a stationary equilibrium, conditions (3) and (4) must also hold. Combining both conditions gives us:

$$f_{ee}^A \leq \left(1 - \frac{w}{P}\right) \frac{w\delta X_{A,member}}{1 - \delta\,(1 - X_{A,member})},$$

with equality when $Y_{A,member} = \left(1 - \frac{w}{P}\right)$.

3.1 Altruistic Bureau's Problem

The altruistic bureau's problem is to maximize an egalitarian social welfare function that equally weights consumers' and providers' utilities, conditional on some restrictions that include a break-even condition, i.e., that the bureau must be self-funded. Consequently, let's start by looking at the bureau's profit function:

$$\Pi_{A,member} = X_{A,member} \left\{ \tfrac{f_{ee}^A}{1-\delta} - c - \tfrac{\delta(1-Y_{member})c}{1-\delta} - Y_{A,member}\, c \textstyle\sum_{t=1}^{\infty} \delta^t \left(1 - X_{A,member}\right)^t \right\}$$

from equation (3), we have that, after a few simplifications:

$$\Pi_{A,member} = \frac{X_{A,member}}{1-\delta} \left\{ f_{ee}^A \frac{c+w}{w} - c \right\}$$

where $X_{A,member}$ in equilibrium does not depend on f_{ee}^{A}, so in order to keep the expression simple, we are not going to substitute it here.

Then, moving to the social welfare function, we have that the per period social welfare function is given by

$$SW_{A,member}(t) = \left\{ \begin{array}{c} \frac{1}{2}\left[\begin{array}{c} (1-Y_{A,member})(P-w) \\ +Y_{A,member}(-w) \end{array} \right] \\ +\frac{1}{2}(w-e) \end{array} \right\} = \frac{1}{2}\left[(1-Y_{A,member})P-e\right] \tag{7}$$

Consequently, the altruistic bureau's problem in the case of membership is given by:

$$\mathbf{SW_{A,member}} \equiv \max_{f_{ee}^{1}} \frac{1}{2(1-\delta)}\left[(1-Y_{A,member})P-e\right] \tag{8}$$

subject to:

$$\begin{array}{ll} \frac{X_{A,member}}{1-\delta}\left\{f_{ee}^{A}\frac{c+w}{w}-c\right\} \geq 0 & \text{(C.1)} \\ 0 \leq Y_{A,member} \leq \frac{P-w}{P} & \text{(C.2)} \\ Y_{A,member} = \frac{\left[1-\delta(1-X_{A,member})\right]}{\delta X_{A,member}w} f_{ee}^{A} & \text{(C.3)} \end{array}$$

where again (C.1) is the break-even constraint and the second constraint is obtained by a combination of $Y_{A,member} \in [0,1]$ and equation (4). Restriction (C.3) is given by equation (3). Substituting (C.3) into (C.2) and the objective function, we can see that the objective function is linearly decreasing in f_{ee}^{A}. Therefore, at the optimum (C.1) must be binding:

$$f_{ee}^{A} = \frac{cw}{w+c}$$

Finally, in order to create a sustainable bureau, (C.2) must be satisfied.[16] Using (C.3), the restriction presented by (C.2) is given by:

$$\frac{\left[1-\delta(1-X_{A,member})\right]}{\delta X_{A,member}w}\frac{cw}{w+c} \leq \frac{P-w}{P} \tag{9}$$

substituting equation (5) and manipulating, we have:

$$\frac{cw}{w+c} \leq \frac{P-w}{P}\frac{e}{X_{A,member}}$$

where $X_{A,member}$ is given by equation (6) and it is a function of e, w, and δ.

By creating a bureau based on membership, the policymaker avoids spending too much money by purchasing the information of non-members. While the bureau's information set is not as fine as it would be if it bought information from all consumers, the fact that it is cheaper implies that in equilibrium

[16] Keep in mind that if $Y_{A,member} > \frac{P-w}{P}$, no consumer buys the service in period 0 and no information is available to the bureau to start building its database. As a consequence, the only equilibrium is no trade.

more consumers will become informed and the fraction of times that providers that default will not be hired actually goes up. However, since not all information is aggregated by the bureau, even members face default in equilibrium. Since we have only one large bureau, the information propagates relatively fast, as we can observe in Figure 2.

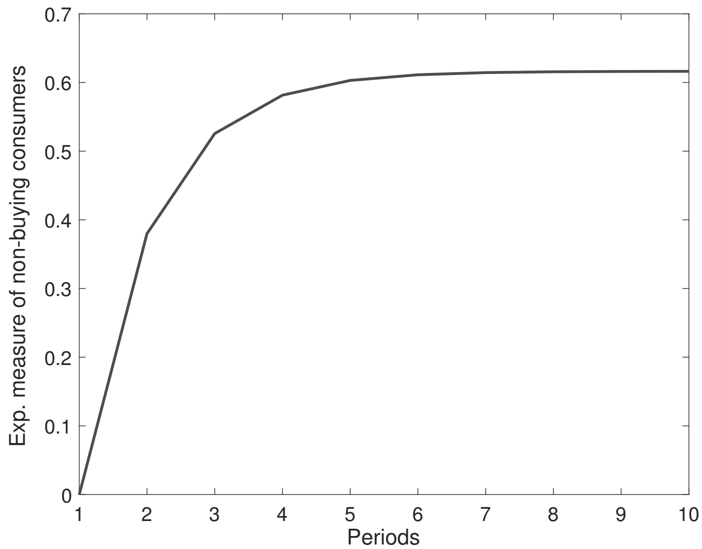

Figure 2: Speed of learning

Note: This graph plots the expected share of meeting with a defaulting provider in which the provider is not hired. We assume $\delta = 0.75$, $e = 2$, $P = 10$, and $c = 1.5$.

3.2 Bureau's Profit Maximization

The bureau's profit maximization problem is given by:

$$\Pi_{M,member} \equiv \max_{f_{ee}^M} X_{M,member} \left[\frac{f_{ee}^M}{1-\delta} - c - \sum_{t=1}^{\infty} \delta^t \left\{ (1 - Y_{M,member}) + Y_{M,member}(1 - X_{M,member})^t \right\} c \right]$$

subject to:

$$f_{ee}^M \leq \left(1 - \tfrac{w}{P}\right) \frac{w\delta X_{M,member}}{1-\delta\left(1-X_{M,member}\right)} \quad \text{(D.1)}$$

The profit function takes into account the fact that, while all members must pay the membership fee, only the ones that hire the provider's service must be compensated for sending information to the bureau. Consequently, the only members that send information to the bureau at time t are the ones matched to providers with no registered history of default. There are two types of providers with a clean history at time t: providers that always offer good services $(1 - Y_{M,member})$ and providers that provide bad services but have not matched with bureau members before, i.e., $Y_{M,member}(1 - X_{M,member})^t$. Moreover, the constraint (D.1) is just a combined version of constraints (C.2) and (C.3) for the altruistic bureau's problem presented in equation (8). Simplifying and substituting equation (3), the bureau's problem becomes:

$$\Pi_{M,member} \equiv \max_{f_{ee}^M} \frac{X_{M,member}}{1-\delta} \left\{ \left(\frac{w+c}{w} \right) f_{ee}^M - c \right\} \tag{10}$$

subject to:

$$f_{ee}^M \leq \left(1 - \frac{w}{P}\right) \frac{w\delta X_{M,member}}{1-\delta(1-X_{member})} \quad \text{(C.1)}$$

Since $X_{M,member}$ does not depend on f_{ee}^M, we can see that $\Pi_{M,member}$ is linearly increasing in f_{ee}^M. Consequently, the restriction is binding and we have that:

$$f_{ee}^M = \left(1 - \frac{w}{P}\right) \frac{w\delta X_{M,member}}{1-\delta\left(1-X_{M,member}\right)}$$

Substituting equation (5) and manipulating it, we obtain:

$$f_{ee}^M = \left(1 - \frac{w}{P}\right) \frac{e}{X_{M,member}}$$

and the profit of the monopolistic bureau that provides membership is then given by:

$$\Pi_{M,member} = \frac{1}{1-\delta} \left\{ \left[\frac{(w+c)(P-w)}{Pw} \right] e - cX_{M,member} \right\}$$

where $X_{M,member} = \frac{e\delta + \sqrt{e^2\delta^2 + 4\delta(1-\delta)we}}{2\delta w}$. As before, notice that since $\delta > \frac{w}{e}$, $X_{M,member} \in (0,1)$.

Social Welfare

Let's now consider the social welfare function. Apart from the measure of providers that default in equilibrium, the social welfare function in the monopoly case with membership is the same as the one presented in equation (7) for the altruistic case. Consequently, we have that:

$$\mathbf{SW_{M,member}} = \frac{1}{2(1-\delta)}[(1-Y_{M,member})P - e] = \frac{1}{2(1-\delta)}(w-e)$$

Moreover, from equation (10), we have that $\Pi_{M,member}$ is linearly increasing in f_{ee}^M and $\Pi_{M,member} = 0$ if $f_{ee}^M = \frac{cw}{c+w}$. Therefore, unless the constraint in (D.1) is exactly binding at $f_{ee}^M = \frac{cw}{c+w} \equiv f_{ee}^A$, we have that $f_{ee}^M > \frac{cw}{c+w}$. Then, from (C.3), we have that, if $f_{ee}^M > \frac{cw}{c+w}$, $Y_{A,member} < Y_{M,member}$. As a result, using (9) we have that $\mathbf{SW_{A,member}} > \mathbf{SW_{M,member}}$. We collect these results in the following proposition.

Proposition 2 *If establishing a bureau is strictly welfare improving, we must have that* $\mathbf{SW_{A,member}} > \mathbf{SW_{M,member}}$.

4 Optimal Information Sampling

We now consider the case in which the bureau is allowed to choose a sample of consumers from whom to buy information, instead of constantly buying information from its entire membership base. The advantage of doing so is that information acquisition becomes less costly; thus, a stationary equilibrium with a lower default rate might be possible.

Since information acquisition is costly, buying too much information can be suboptimal. In particular, buying too much information may have a deleterious effect by inducing a high membership fee and a smaller bureau size in equilibrium. As a result, the number of providers choosing to default in equilibrium may be larger, even though informed consumers have more knowledge of past deviations. Therefore, a policymaker establishing a bureau may decide to pin down the optimal sampling scheme in order to maximize social welfare. In particular, let's assume that, in every period, the bureau buys any given member's information with probability $(1-q) \in (0,1)$. Where we denote by $X_{A,mq}$ the mass of consumers buying from the altruistic bureau that samples information buying with probability $(1-q)$.

Provider's Problem

In this general case, we have that a provider will be indifferent between making an effort or defaulting if:

$$(1-\delta)\, w + (1 - X_{A,mq})\, \delta w + X_{A,mq}\, (1-\delta) \textstyle\sum_{t=1}^{\infty} \delta^t w \sum_{t_1=0}^{t} \binom{t}{t_1} \left(X_{A,mq} q\right)^{t-t_1} \left(1 - X_{A,mq}\right)^{t_1} = w - e \tag{11}$$

Solving this equation for $X_{A,mq}$, we obtain:

$$X_{A,mq} = \frac{\delta e + \sqrt{\delta^2 e^2 + \frac{4\delta(1-\delta)we}{(1-q)}}}{2\delta w} \tag{12}$$

First, note that if $q = 0$, we are in the previous case. Moreover, notice that $\frac{\partial X_{A,mq}}{\partial q} > 0$, i.e., the bureau's size increases as the likelihood of acquiring information – and consequently the direct cost of membership – declines.

Consumer's Problem

Now, let's consider the consumer's decision. First, the consumer's payoff to becoming a member in period 1 is:

$$(1 - Y_{A,mq})\,(P - w) - f_{ee} - Y_{A,mq}\,(1-\delta) \sum_{t=1}^{\infty} \delta^t w \sum_{t_1=0}^{t} \binom{t}{t_1} \left(X_{A,mq} q\right)^{t-t_1} \left(1 - X_{A,mq}\right)^{t_1}$$

In equilibrium, we must have that consumers are indifferent between applying for membership or not. Consequently, in equilibrium we must have:

$$f_{ee} = \frac{\delta Y_{A,mq} w (1-q) X_{A,mq}}{1-\delta [1-(1-q) X_{A,mq}]} \tag{13}$$

Importantly, conditions (12) and (13) hold in both the case of the altruistic bureau and the case of a profit-maximizer bureau.

4.1 Altruistic Bureau's Problem

The altruistic bureau's problem is to maximize an egalitarian social welfare function that equally weights consumers' and providers' utilities, conditional on some restrictions that include a break-even condition, i.e., that the bureau must be self-funded. Consequently, the altruistic bureau's problem in the case of membership and information sampling is given by:

$$\mathbf{SW_{A,mq}} \equiv \max_{f_{ee},q} \frac{1}{2} \{(1-Y_{A,mq}) P - e\}$$

subject to:

$$\begin{array}{ll} f_{ee} \geq c(1-q) \left\{ 1 - \frac{\delta Y_{A,mq} X_{A,mq}(1-q)}{1-\delta[1-X_{A,mq}(1-q)]} \right\} & (C.1) \\ 0 \leq Y_{A,mq} \leq \frac{P-w}{P} & (C.2) \\ Y_{A,mq} = \frac{f_{ee}\left\{1-\delta\left[1-(1-q)X_{A,mq}\right]\right\}}{\delta w(1-q)X_{A,mq}} & (C.3) \\ X_{A,mq} = \frac{\delta e + \sqrt{\delta^2 e^2 + \frac{4\delta(1-\delta)we}{(1-q)}}}{2\delta w} \in [0,1] & (C.4) \end{array}$$

where $(C.1)$ is the break-even condition. Notice that now the bureau can choose not only the fee but also the sampling frequency. We are now able to show the following auxiliary results.

Lemma 1 *At the optimum, $(C.1)$ must be binding.*

In summary, Lemma 1 shows that $(C.1)$ is binding at the optimum, pinning down f_{ee}. Therefore, we just need to pin down q.

Then, Lemma 2 shows that $Y_{A,mq}$ is strictly decreasing in q. This is a feature of the equilibrium: as the bureau buys more information, it needs to raise more money, meaning that the product it is selling (information) must be more valuable, in order to induce buyers to buy more. But becoming more valuable means that more providers must be defaulting in equilibrium (a higher $Y_{A,mq}$).

Lemma 2 *$Y_{A,mq}$ is decreasing in q.*

Finally, $\mathbf{SW_{A,mq}}$ is linearly decreasing in $Y_{A,mq}$. Consequently, in order to maximize $\mathbf{SW_{A,mq}}$, the bureau must choose the highest value of q. Therefore, at the optimum we must have:

$$q^* = \frac{\delta w - e}{\delta(w - e)} \tag{14}$$

Importantly, this implies that $X_{A,mq} = 1$. Hence, all consumers become bureau members. The next theorem collects all of these results:

Theorem 1 (Optimal Sampling Theorem: Full Membership) *If the credit bureau can choose the sampling frequency in order to maximize social welfare in a membership pricing mechanism, we have that all consumers become members. Moreover, the membership fee and the fraction of defaulting providers are given by:*

$$f_{ee} = \frac{c(1-\delta)ew}{\delta w(w-e) + c(1-\delta)e} \quad \text{and} \quad Y_{A,mq} = \frac{(1-\delta)cw}{w\delta(w-e) + c(1-\delta)e}$$

Finally, we can show the following corollary:

Corollary 1 *The speed of learning – determined by the decline in the fraction of unknown bad providers – declines with q.*

Consequently, the policymaker optimally chooses to minimize the costs of information acquisition by reducing the sampling frequency in order to maximize the equilibrium bureau's size. In other words, in terms of welfare effects, it is best to have a large bureau, maximizing information spillovers (dissemination) even though information accumulation may occur more slowly. As a result, we may infer that the social welfare loss in the case of competitive bureaus with membership occurs mainly through the reduction in the bureau's size, instead of through the slower accumulation of information.

Figure 3 highlights the trade-off of speed of learning, bureau size, and indirect costs for different levels of information sampling. High information sampling (low q) implies a faster learning speed (Figure 3a). In contrast, by increasing the membership cost and consequently reducing total bureau membership, high sampling implies a larger share of defaulting providers and high expected indirect costs in the first years of the bureau (Figure 3b). While learning happens faster in the case of higher sampling, the higher share of defaulting providers also introduces the cost of a lower service rate, since over time there are more meetings on average in which service is not acquired. Hence, the optimal sampling $q = q^*$ implies slower learning speed, lower share of defaulting providers, lower indirect costs in the early periods, but a higher indirect cost over the long run, while also increasing the share of meetings that induce successful service.

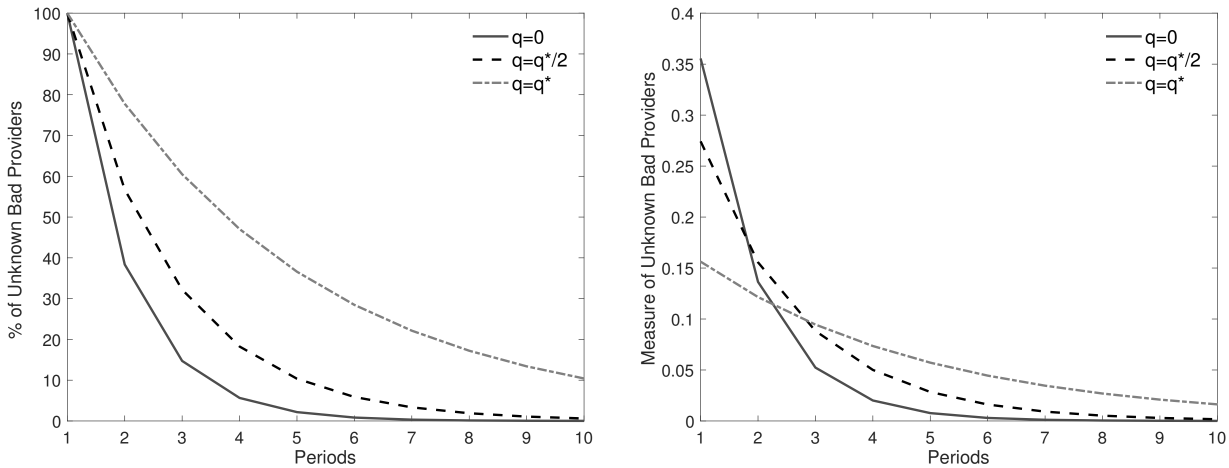

Figure 3a: Speed of learning

Figure 3b: Normalized indirect costs

Figure 3: Learning Speed and Indirect Costs – Impact of Information Sampling
Note: Panel a plots the percentage of defaulting providers ($Y_{A,member}$) that have not yet been detected by the altruistic bureau at a given period for different levels of information sampling (q). Panel b plots the measure of defaulting providers that have not been detected by the altruistic bureau at a given period for different levels of information sampling. We assume $\delta = 0.75$, $e = 2$, $P = 10$, and $c = 1.5$.

4.2 Profit-Maximizing Bureau

We consider now the case of a bureau that maximizes expected profit. The bureau buys information from each member with probability (1-q) and its profit for any given chosen pair $\{f_{ee}, q\}$ is denoted by $\mathbf{\Pi_{M,mq}}$ and given by:

$$X_{M,mq}\left[\frac{f_{ee}}{1-\delta} - (1-q)c(1+\sum_{t=1}^{\infty}\delta^t\left\{(1-Y_{M,mq}) + Y_{M,mq}(1-X_{M,mq}+X_{M,mq}q)^t)\right\})\right]$$

Note that in any stationary equilibria, it must be the case that conditions (12) and (13) hold. Similarly to the altruistic bureau case, we again restrict our analysis to the case in which non-members find it profitable to hire. This has the advantage of avoiding having to deal with a cold start problem. Therefore, we have the same constraint as (4), namely

$$Y_{M,member} \leq 1 - \frac{w}{P}.$$

The profit $\mathbf{\Pi_{M,mq}}$ is maximized when the sampling frequency q is at its maximum. This result is stated in the theorem below (the proof is in the Appendix). This optimal sampling theorem is analogous to the case of the altruistic bureau but with a very different implication. There, the altruistic bureau was buying information at the lowest possible frequency with the purpose of making the bureau cheap, so that more agents would join, thus increasing social welfare. The fee was the lowest possible that would satisfy the break-even condition. In the monopolist case, the bureau buys less information in order to (i.) increase the mass of members, but also to (ii.) save on costs. The optimal fee is the highest possible that would still induce agents to become members.

Theorem 2 (Monopolist's Optimal Sampling: Full Membership) *If the credit bureau that maximizes profit can choose the sampling frequency, the mechanism is chosen so that all consumers become members.*

5 Competitive Profit-Maximizing Bureaus

In this section, we consider the case of for-profit bureaus that face competition in equilibrium. We develop a model of competition between bureaus with a membership pricing mechanism. Specifically, we consider two bureaus A and B, where bureau i charges f_i for the membership. In the stationary equilibria we consider, each bureau has a consumer base X_i ($X_A + X_B \leq 1$) and there is a fraction $Y_{C,member}$ of providers that choose to default every period. The timing of this game is the following: first, the bureaus post their membership fees simultaneously, then the consumers and providers play an infinitely repeated game with private histories given the fees that were posted.[17] We look for the subgame perfect equilibria of this repeated game.

Providers Let us now take a closer look at the providers' incentives. We introduce the following notation: U^0 is the expected continuation utility of a provider that decides to default and: (i.) either has never defaulted before or (ii.) has defaulted but has never been reported to an information bureau; U^i is a provider that has been reported by at least one member of bureau i, but has not interacted with members of bureau $j \neq i$; and U^{AB} is the expected continuation payoff of a provider that has been reported by at least one member of A and by at least one member of B. These expected continuation payoffs can be written in recursive form as follows:

$$\begin{aligned}
U^0 &= w + \delta\left(X_A U^A + X_B U^B + (1 - X_A - X_B) U^0\right) \\
U^A &= X_A \delta U^A + X_B\left(w + \delta U^{AB}\right) + (1 - X_A - X_B)\left(w + \delta U^A\right) \\
U^B &= X_A\left(w + \delta U^{AB}\right) + X_B \delta U^B + (1 - X_A - X_B)\left(w + \delta U^B\right) \\
U^{AB} &= X_A \delta U^{AB} + X_B \delta U^{AB} + (1 - X_A - X_B)\left(w + \delta U^{AB}\right)
\end{aligned}$$

We are able to recursively solve this system of linear equations in order to obtain U^A, U^B, and U^{AB}. Taking into account that providers are indifferent between defaulting and exerting effort if

[17] For simplicity, we assume here that all membership affiliations are decided at this initial time.

$U^0 = w - e$, our first stationary equilibrium condition is:

$$\frac{w}{1-\delta\left(1-X_A-X_B\right)}\left\{1+\delta\left(\begin{array}{c}\frac{X_A(1-X_A)}{1-\delta(1-X_B)}+X_AX_B\delta\frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_B))}+\\ \frac{X_B(1-X_B)}{1-\delta(1-X_A)}+X_BX_A\delta\frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_A))}\end{array}\right)\right\}=\frac{w-e}{1-\delta} \tag{15}$$

The following result relates the consumer basis of the two bureaus in the stationary equilibria.

Lemma 3 *Equation (15) defines a strictly decreasing relationship between X_A and X_B.*

Consumers In this section, for convenience, we focus on the case where $\delta > \sqrt{e/w}$.[18] Whenever there are two bureaus operating in equilibrium, that is, with positive consumer bases, the utility of the consumers who buy from bureau A must be the same as the utility of those who buy from bureau B and the same as not buying at all. This leads us to the following indifference conditions:

$$\left(1-Y_{C,member}\right)\left(P-w\right)-f_i+\left(1-\delta\right)\sum_{t=0}^{\infty}\delta^t\left(1-X_i\right)^t Y_{C,member}\left(-w\right)=\left(1-Y_{C,member}\right)P-w,$$

where the LHS is the consumer's payoff from joining bureau i and the RHS is the payoff of not joining any bureau. Given that the indifference must hold for both bureaus, we have the following two equations:

$$f_i = Y_{C,member}\frac{w\delta X_i}{1-\delta\left(1-X_i\right)},\ \forall i = A, B \tag{16}$$

The ratio of fees is given by:

$$\frac{f_A}{f_B}=\frac{X_A\left(1-\delta\right)+\delta X_AX_B}{X_B\left(1-\delta\right)+\delta X_AX_B} \tag{17}$$

Proposition 3 *For any pair of fees (f_A, f_B), there is, at most, one stationary equilibrium with two operating bureaus ($X_A > 0$ and $X_B > 0$) in the continuation game.*

Implicit in the statement of the proposition above is the fact that there might be equilibria in the continuation game in which either only one bureau operates (that is, $X_i > 0$ and $X_j = 0$) or in which there is no bureau operating, so essentially there is no market ($X_A = X_B = 0$).

In any stationary equilibrium, we can partition the set of consumers into two subsets: consumers who join at least one bureau and consumers who do not join any bureau. The last condition that we need to construct a stationary equilibrium with two operating bureaus is the condition that the consumers who do not join any bureau also find it profitable to hire providers, despite the fact that they have no information on the provider's past behavior. This gives us our

[18] If $\sqrt{e/w} > \delta > e/w$, there might be an equilibrium in which all consumers join bureaus and some providers default twice: the first time the provider meets a member of each bureau.

last equilibrium condition:

$$Y_{C,member} \leq \frac{P-w}{P} \tag{18}$$

It is convenient to define a feasible set for the providers' fees. Let us note that there is an upper bound for a fee of a bureau that operates in equilibrium. We can compute this upper bound using (16) and (18) and the fact that X_i has an upper bound, which is given by $X_i \leq \frac{e\delta+\sqrt{e^2\delta^2+4\delta(1-\delta)we}}{2\delta w}$. A fee f_i is *feasible* if it is below this upper bound, that is, if there exists a stationary equilibrium in which consumers buy from firm i at this fee f_i:

$$f_i \leq \frac{P-w}{P} w \frac{\delta \frac{e\delta+\sqrt{e^2\delta^2+4\delta(1-\delta)we}}{2\delta w}}{1-\delta\left(1-\frac{e\delta+\sqrt{e^2\delta^2+4\delta(1-\delta)we}}{2\delta w}\right)}. \tag{19}$$

A very important issue in this section is the role played by beliefs off the equilibrium path. The timing of the duopoly game is such that first the bureaus simultaneously choose fees and then consumers decide whether or not to join. From Proposition 3, we know that for any given pair of fees, there might be a multiplicity of continuation equilibria. Nevertheless, at most one continuation equilibrium has two bureaus operating in equilibrium. We assume an equilibrium refinement in which in every continuation equilibrium two bureaus operate whenever possible. Precisely, this means that starting from a given pair (f_i, f_j), a deviation by one of the two bureaus, say, bureau i, leads to a new pair (f_i', f_j) in which it is possible to construct a stationary equilibrium of the continuation game in which (i.) both bureaus still operate; (ii.) only bureau i operates; (iii.) only bureau j operates; or (iv.) neither of the two bureaus operates. Our refinement is to consider equilibria in which the equilibrium after the deviation is the one in which both bureaus operate, if such an equilibrium exists. If, after a deviation, there is no equilibrium with two operating bureaus, then we consider only the equilibrium in which only the cheapest bureau operates.[19]

Given that each bureau's profit is a direct function of $fee \times X$, any deviation that increases f might seem like a profitable deviation. But can the bureau increase its fee without bounds? Note that if $f_A > Y_{C,member} \frac{w\delta X_A}{1-\delta(1-X_A)}$, then nobody buys from bureau A. Therefore, certainly (19) imposes an upper bound on a fee of an operating bureau. This imposes a non-tight bound.

[19] Suppose, instead, that we consider a refinement in which whenever there is no equilibrium with two bureaus, the consumers buy only from the most expensive bureau. Then, there are two equilibria only: one in which bureau i is the monopolist and one in which j is the monopolist. For the i monopolist, the fee is given by $f_i = \frac{P-w}{P} w \frac{\delta X_i}{1-\delta(1-X_i)}$, and the fractions of members and defaulters are, respectively, $X_i = \frac{e\delta+\sqrt{e^2\delta^2+4\delta(1-\delta)we}}{2\delta w}$ and $Y_{C,member} = \frac{P-w}{P}$, with $f_j \geq f_i$ and $X_j = 0$.

Consider a case in which both bureaus set the same fee and suppose that

$$f_A = f_B = \frac{P-w}{P} \frac{w\delta X}{1-\delta(1-X)},$$

where X solves

$$\frac{(1-\delta)w}{1-\delta(1-2X)} + \delta 2 \frac{wX}{(1-\delta(1-2X))(1-\delta(1-X))}((1-\delta)(1-X) + X\delta(1-2X)) = w - e$$

Denote the X that solves the above equation by X^{sym}. Thus, consider a situation in which both firms charge

$$\bar{f} = \frac{P-w}{P} \frac{w\delta X^{sym}}{1-\delta(1-X^{sym})}$$

Before we prove the main result in this section, the next lemma will be useful.

Lemma 4 *For any given pair (f_i, f_j) with $f_i \leq f_j \leq \frac{P-w}{P}\frac{w\delta X^{sym}}{1-\delta(1-X^{sym})}$ there exists an equilibrium with two operating bureaus (unique in this class) in the continuation game.*

In an economy in which the two bureaus operate, consumers join at most one of them. The intuition for this result comes from the fact that the marginal benefit of a second bureau is smaller than it is for the first bureau. Since consumers are indifferent between joining and not joining the bureau, the cost is equal to the benefit of joining, but joining a second bureau has negative expected cost. We state this result formally below.

Proposition 4 *Suppose that $\delta > \sqrt{\frac{e}{w}}$. Then, in a stationary equilibrium with two bureaus, each consumer joins at most one bureau.*

We are now ready to prove the main result of this section.

Proposition 5 (Unique Stationary Competitive Equilibrium) *There is a unique stationary equilibrium in which both bureaus operate. In this equilibrium, $f_i = f_j = \frac{P-w}{P}\frac{w\delta X^{sym}}{1-\delta(1-X^{sym})}$, with symmetric consumer bases $X_i = X_j = X^{sym}$ and $Y_{C,member} = \frac{P-w}{P}$.*

Therefore, there is a unique stationary equilibrium in the duopoly competition game where the firms set fees simultaneously and the consumers and providers play an infinitely repeated game following the chosen fees. In this equilibrium, the fraction of providers who default is $Y_{C,member} = \frac{P-w}{P}$, which is the lowest possible fraction that sustains a stationary equilibrium.

5.1 Social Welfare

We are focusing on equilibria in which consumers are indifferent between buying a membership from bureau i, j, or not buying at all, but hiring providers nonetheless. Thus, we have that:

$$U_{consumer} = (1 - Y_{C,member})P - w \quad \text{and} \quad U_{provider} = w - e$$

where $Y_{C,member} = \frac{P-w}{P}$, so that $U_{consumer} = (1 - \frac{P-w}{P})P - w = 0$. The social welfare becomes $\mathbf{SW_{C,member}} = \frac{1}{2(1-\delta)} \{w - e\}$. Given that social welfare in the case of a monopoly is $\mathbf{SW_{M,member}} = \frac{1}{2(1-\delta)}(w - e)$, we have:

$$\mathbf{SW_{A,member}} > \mathbf{SW_{M,member}} = \mathbf{SW_{C,member}}$$

Consequently, competition is strictly worse than the benchmark case of the altruistic bureau. The reason for this is that the indirect costs of competition become significantly higher, not only because bureaus are smaller, but also because the measure of defaulting providers must be higher in equilibrium in order to sustain multiple bureaus. In summary, not only is learning slowed down but also the indirect costs that arise from facing default while informed are consistently higher throughout. We can see the negative effects of slow learning and higher expected indirect costs in Figures 4a and 4b, respectively. Overall, information trade in many ways presents the same characteristics as natural monopolies, where, in order to avoid the duplication of costs and harvest the benefits of economies of scale, the optimal number of producers (or, in this case, information brokers) is one, provided it is regulated in order to avoid a concentration of market power.

6 Conclusion

In this paper, we show that not only does the availability of information matter for a well-functioning market, but also how information is negotiated. The pricing and selling mechanisms, as well as the number of information brokers in the market, are important to determine not only how many agents choose to become informed, but also the quality of information available to them. At the end, these features pin down how much discipline the information trade imposes on both sides of the market, affecting service providers' incentives and ultimately the social welfare in the economy. These results are true even in an environment in which we disregard insurance issues. We consider the bureau's chosen pricing mechanism and information sampling under different market structures: non-profit, monopoly, and competitive environments. We show that both dimensions affect direct and indirect costs, represented by fees and expected loss due to

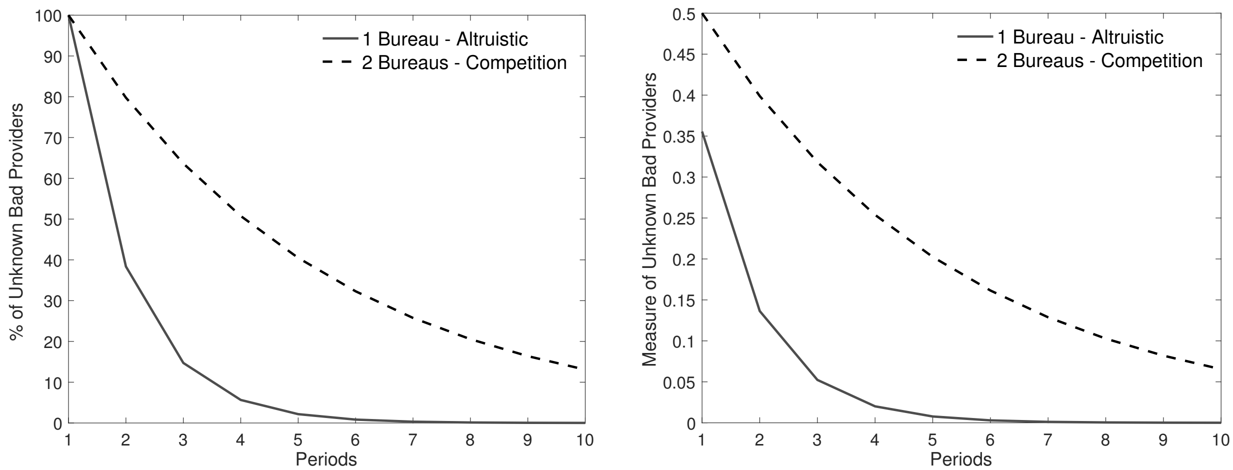

Figure 4a: Speed of learning **Figure 4b:** Normalized indirect costs

Figure 4: Learning Speed and Indirect Costs – Impact of Competition

Note: Panel a plots the percentage of defaulting providers (Y) that have not yet been detected by a bureau at a given period for the cases of an altruistic bureau and two for-profit bureaus in competition. Panel b plots the measure of defaulting providers that have not been detected by a bureau at a given period for the cases of an altruistic bureau and two for-profit bureaus in competition. We assume $\delta = 0.75$, $e = 2$, $P = 10$, and $c = 1.5$.

default while informed, respectively. We also show that information trade has characteristics similar to a natural monopoly, where competition may be hurt by duplicate costs and slower information aggregation by each individual information broker. Moreover, we show that there is a trade-off between information quality and cost. In a world with only one non-profit information bureau, the bureau will choose to have lower information quality, inducing low enough direct costs through fees that more than compensate for the initially high indirect costs. However, this is true only because the bureau is large enough to quickly disseminate information and reduce indirect costs.

Finally, we would like to emphasize that risk aversion may significantly change our results since the bureau may be able to provide insurance against losses through default by paying more for the reported information. However, risk aversion introduces an additional trade-off between insurance and the incentive to buy information, potentially influencing providers' incentives to exercise effort. More research is needed to disentangle these additional complications.

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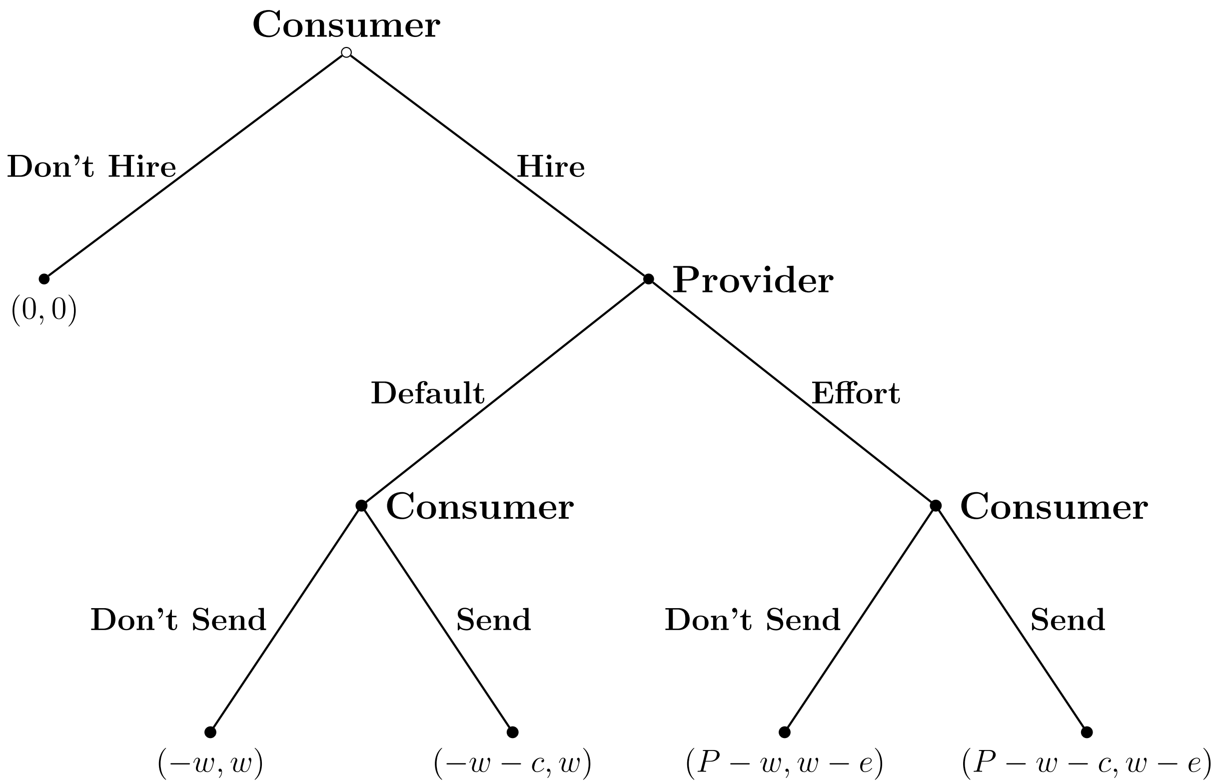

Figure 1: Game Tree – No Bureau Case

Appendix - Proofs

Proof of Proposition 1. Suppose, by contradiction, that there exists a history in which a provider is hired with positive probability. This means that the consumer expects that the provider will exert effort with positive probability. At the stage game reached at this history, the provider has a higher current payoff if he defaults rather than if he exerts effort (by avoiding the cost of effort). Moreover, his expected continuation payoff is given by the payoff that he expects to get by matching to each consumer in each future period. Given that actions are private to the interaction, the maximum number of players that have been exposed to the defection after t periods is at most 2^{t-1}. In particular, there is a countable number of such agents, and the measure of the union of these agents is zero. Thus, if f is the probability density function (henceforth p.d.f.) over all possible consumers and f_t^* is the p.d.f. over consumers who have not been exposed by the original defector at time t, then f and f_t^* are equal almost everywhere (a.e.) for any t. Simply put, the chance that one such consumer will meet the original defector again is zero for any time t, so that there is no incentive to cooperate. In particular, if we tried to apply contagious equilibrium arguments such as by Kandori (1992), it would fail because of the continuum of agents assumption. ■

Proof of Lemma 1. Suppose, by contradiction, that $(C.1)$ is non-binding. Then, from $(C.3)$, we have that:

$$\frac{\partial Y_{A,mq}}{\partial f_{ee}} > 0 \quad \text{and} \quad \frac{\partial Y_{A,mq}}{\partial q} = \frac{(1-\delta)^2 e f_{ee}}{\delta^2 w[(1-q)X_{A,mq}]^2 (X_{A,mq}w - e)^2} \frac{\partial X_{A,mq}}{\partial q} > 0$$

Consequently, we can reduce $Y_{A,mq}$ by either reducing f_{ee} or q without violating $(C.1)$. Since $\frac{\partial \mathbf{SW_{A,mq}}}{\partial Y_{A,mq}} < 0$, we conclude that at the optimum $(C.1)$ must be binding. ■

Proof of Lemma 2. After some manipulations, taking a total derivative of equation (11) with respect to q and manipulating it, we have:

$$\frac{dX_{A,mq}}{dq} = \frac{X_{A,mq}(wX_{A,mq} - e)}{(1-q)\left\{2wX_{A,mq} - e\right\}} \tag{20}$$

Again, from the provider's problem equation (11), after rearranging, we obtain:

$$\frac{1 - \delta[1-(1-q)X_{A,mq}]}{\delta X_{A,mq}} = \frac{(1-\delta)w}{\delta(wX_{A,mq} - e)}$$

Once we assume that $(C.1)$ is binding, from $(C.3)$, we have:

$$Y_{A,mq} = \frac{c}{w + c(1-q)} \left\{ \frac{1 - \delta[1 - (1-q)X_{A,mq}]}{\delta X_{A,mq}} \right\} \tag{21}$$

Substituting (20) into (21), we have:

$$Y_{A,mq} = \frac{(1-\delta)cw}{\delta(wX_{A,mq} - e)} \times \frac{1}{w + c(1-q)} \tag{22}$$

Then, taking the derivative with respect to q, we have:

$$\frac{\partial Y_{A,mq}}{\partial q} = \begin{array}{l} -\frac{(1-\delta)cw}{\delta(wX_{A,mq}-e)^2(w+c(1-q))^2} \times \\ \times \left\{ w\frac{dX_{A,mq}}{dq}(w + c(1-q)) - c(wX_{A,mq} - e) \right\} \end{array}$$

Therefore, $\frac{\partial Y_{A,mq}}{\partial q} < 0$ if:

$$w\frac{dX_{A,mq}}{dq}(w + c(1-q)) - c(wX_{A,mq} - e) > 0$$

Rearranging it and substituting (20), we have:

$$c(1-q) < \frac{[w + c(1-q)]wX_{A,mq}}{2wX_{A,mq} - e}$$

Rearranging it:

$$c(1-q) < \frac{w^2 X_{A,mq}}{wX_{A,mq} - e}$$

Note that $\frac{w^2 X_{A,mq}}{wX_{A,mq}-e} > \frac{w^2}{w-e}$, since $\frac{\partial\left\{\frac{w^2 X_{A,mq}}{wX_{A,mq}-e}\right\}}{\partial X_{A,mq}} < 0$. Thus, if we show that $c(1-q) < \frac{w^2}{w-e}$ we are done. But then, notice that:

$$c(1-q) < c < \frac{we}{w-e} < \frac{w^2}{w-e}$$

where $c < \frac{we}{w-e}$ is trivially satisfied by the parameter restriction established in Lemma 3. ■

Proof of Corollary 1. Keep in mind that the fraction of unknown bad providers in period t is given by $[1 - X(1-q)]^t$. Then, the rate at which the fraction of unknown bad providers declines over time is given by:

$$\frac{[1 - X(1-q)]^{t+1} - [1 - X(1-q)]^t}{[1 - X(1-q)]^t} = -X(1-q), \ \forall t \in \mathbb{N}$$

Consequently, the decline in the fraction of unknown bad providers is constant at $X(1-q)$ for all t. But then, totally differentiating equation (11), we have:

$$\frac{dX(1-q)}{dq} = -\frac{(1-\delta)e}{\delta(Xw-e)^2} \times \frac{dX}{dq} w < 0.$$

Consequently, the speed of learning declines with q. ■

Proof of Theorem 2. We consider now the case of a bureau that maximizes expected profit. Note that in any stationary equilibria it must be the case that conditions (12) and (13) hold. The bureau's profit for any given chosen pair $\{f_{ee}, q\}$ is given by:

$$X_{M,mq}\left[\frac{f_{ee}}{1-\delta} - (1-q)c\left(1 + \sum_{t=1}^{\infty} \delta^t \left\{(1-Y_{M,mq}) + Y_{M,mq}(1 - X_{M,mq} + X_{M,mq}q)^t)\right\}\right)\right]$$

The term in brackets simplifies to

$$\left[\frac{f_{ee}}{1-\delta} - (1-q)c\left\{1 + \delta\frac{1-Y_{M,mq}}{1-\delta} + \delta Y_{M,mq}\frac{1-X_{M,mq}(1-q)}{1-\delta(1-X_{M,mq}(1-q))}\right\}\right] \tag{23}$$

but recall equation (13):

$$f_{ee} = \frac{\delta Y_{A,mq} w\,(1-q)\,X_{A,mq}}{1-\delta\,[1-(1-q)\,X_{A,mq}]}$$

which we rewrite for the context of the monopolist as:

$$Y_{M,mq} = \frac{f_{ee}(1-\delta\,[1-(1-q)\,X_{M,mq}])}{\delta w\,(1-q)\,X_{M,mq}} \tag{24}$$

Substituting in (23):

$$\left[\frac{f_{ee}}{1-\delta} - (1-q)c\left\{1 + \delta\frac{1-Y_{M,mq}}{1-\delta} + \delta f_{ee}\frac{(1-\delta\,[1-X_{M,mq}(1-q)])}{\delta w\,(1-q)\,X_{M,mq}}\frac{1-X_{M,mq}(1-q)}{1-\delta(1-X_{M,mq}(1-q))}\right\}\right]$$

Which simplifies to:

$$\left[\frac{f_{ee}}{1-\delta} - \frac{(1-q)c}{1-\delta} + \delta\frac{(1-q)cY_{M,mq}}{1-\delta} - cf_{ee}\frac{1-X_{M,mq}(1-q)}{wX_{M,mq}}\right]$$

using (24) and substituting $Y_{M,mq}$, we obtain:

$$\left[\frac{f_{ee}}{1-\delta} - \frac{(1-q)c}{1-\delta} + \frac{f_{ee}c}{wX_{M,mq}}\left\{\frac{(1-\delta\,[1-(1-q)\,X_{M,mq}])}{1-\delta} - (1-X_{M,mq}(1-q))\right\}\right]$$

Note that the last two terms (in curly brackets) can be combined. Hence, after some algebra, we obtain:

$$\left[\frac{f_{ee}}{1-\delta}\frac{w+(1-q)c}{w}-\frac{(1-q)c}{1-\delta}\right]$$

Therefore, the bureau's problem is:

$$\Pi_{M,mq} \equiv \max_{f_{ee},q} X_{M,mq}\left[\frac{f_{ee}}{1-\delta}\frac{w+(1-q)c}{w}-\frac{(1-q)c}{1-\delta}\right] \tag{25}$$

Recall that $X_{M,mq}$ can be written as:

$$\frac{\delta e+\sqrt{\delta^2 e^2+\frac{4\delta(1-\delta)we}{(1-q)}}}{2\delta w}$$

Or equivalently,

$$\frac{\delta X_{M,mq}^2 w(1-q)}{1-\delta(1-X_{M,mq}(1-q))}=e$$

Thus,

$$\frac{e}{X_{M,mq}}=\frac{\delta X_{M,mq}(1-q)}{1-\delta(1-X_{M,mq}(1-q))}$$

but recall equation (13):

$$f_{ee}=Y_{M,mq}\frac{\delta w\,(1-q)\,X_{M,mq}}{1-\delta\,[1-(1-q)\,X_{M,mq}]}$$

Then,

$$f_{ee}=Y_{M,mq}\frac{e}{X_{M,mq}}$$

$Y_{M,mq}$ must be maximum at the optimum, thus,

$$f_{ee}=(1-\frac{w}{P})\frac{e}{X_{M,mq}} \tag{26}$$

The profit function can then be written substituting f_{ee} to get:

$$\max_{f_{ee},q}\frac{X_{M,mq}}{1-\delta}\left[\frac{P-w}{P}\frac{e}{X_{M,mq}}\frac{w+(1-q)c}{w}-(1-q)c\right]$$

which is equivalent to maximizing:

$$\frac{1}{1-\delta}\max_{f_{ee},q}\frac{P-w}{P}\frac{e(w+(1-q)c}{w}-X_{M,mq}(1-q)c$$

Substituting $X_{M,mq}$:

$$\max_q \frac{P-w}{P}\frac{e(w+(1-q)c)}{w} - \frac{e+\sqrt{e^2+\frac{4(1-\delta)we}{\delta(1-q)}}}{2w}(1-q)c$$

further simplifying to:

$$\frac{1}{2w}\max_q 2(P-w)e(w+(1-q)c) - P\left(e+\sqrt{e^2+\frac{4\left(1-\delta\right)we}{\delta\left(1-q\right)}}\right)(1-q)c$$

equivalent to maximizing:

$$\max_q (1-q)\left\{2(P-w)e - P\left(e+\sqrt{e^2+\frac{4\left(1-\delta\right)we}{\delta(1-q)}}\right)\right\}$$

To determine the optimal q for $q \in (0,1)$, we analyze the derivative of the function with respect to q. Let's calculate the derivative and examine its sign.

Define $(1-q) \equiv t$ and define the function $\phi(t)$:

$$\phi(t) = t(2(P-w)e - Pe) - tP\sqrt{e^2+\frac{4(1-\delta)we}{\delta t}}$$

Let's calculate the derivative step by step. It will be convenient to define a function $f(t)$ by

$$f(t) \equiv -tP\sqrt{e^2+\frac{4(1-\delta)we}{t\delta}}$$

Take the derivative of $\phi(t)$ with respect to t:

$$\frac{d\phi(t)}{dt} = e(P-2w) + \frac{df(t)}{dt} \tag{27}$$

To calculate $\frac{df(t)}{dt}$, we apply the chain rule to obtain:

$$\frac{df}{dt} = -P\sqrt{e^2+\frac{4(1-\delta)we}{t\delta}} - tP\frac{d}{dt}\left[\left(e^2+\frac{4(1-\delta)we}{t\delta}\right)^{\frac{1}{2}}\right] \tag{28}$$

Using the power rule, we can differentiate the function inside the square root:

$$\frac{d}{dt}\left(e^2+\frac{4(1-\delta)we}{t\delta}\right)^{\frac{1}{2}} = \frac{1}{2}\left(e^2+\frac{4(1-\delta)we}{t\delta}\right)^{-\frac{1}{2}} \times \frac{d}{dt}\left(e^2+\frac{4(1-\delta)we}{t\delta}\right) \tag{29}$$

Now, we need to calculate the derivative of the expression inside the parentheses. Applying the sum rule and the quotient rule, we get:

$$\frac{d}{dt}\left(e^2+\frac{4(1-\delta)we}{t\delta}\right)=-\frac{4(1-\delta)we}{t^2\delta}$$

Now, we can substitute the derivatives back into the main expression of (29):

$$\frac{d}{dt}\sqrt{e^2+\frac{4(1-\delta)we}{t\delta}}=\frac{1}{2}\left(e^2+\frac{4(1-\delta)we}{t\delta}\right)^{-\frac{1}{2}}\times\left(-\frac{4(1-\delta)we}{t^2\delta}\right)$$

Simplifying and rearranging equation (28), we get the derivative of f(t) with respect to t.

$$\frac{df}{dt}=-P\sqrt{e^2+\frac{4(1-\delta)we}{t\delta}}+tP\left\{\frac{1}{2}\left(e^2+\frac{4(1-\delta)we}{t\delta}\right)^{-\frac{1}{2}}\times\left(\frac{4(1-\delta)we}{t^2\delta}\right)\right\}$$

This can be rewritten as:

$$\frac{df}{dt}=\frac{P}{\sqrt{(e^2+\frac{4(1-\delta)we}{t\delta})}}\left\{-\frac{4(1-\delta)we}{2t\delta}-e^2\right\}<0 \tag{30}$$

Therefore, $f(1-q)$ is decreasing in $1-q$ (and, consequently, increasing in q). Returning to the derivative of $\phi(t)$ with respect to t (equation (27)), if $P<2w$, then we can directly see that $\frac{d\phi(t)}{dt}<0$. Now suppose that $P>2w$, then let us investigate the sign of the derivative of (27), using (30):

$$\begin{aligned}\frac{d\phi(t)}{dt}&=e(P-2w)+\frac{df(t)}{dt}\\&=-2ew+\frac{P}{\sqrt{(e^2+\frac{4(1-\delta)we}{t\delta})}}\left(e\sqrt{e^2+\frac{4(1-\delta)we}{t\delta}}-\left\{e^2+\frac{4(1-\delta)we}{2t\delta}\right\}\right)\end{aligned}$$

Let us investigate whether the last term is negative or positive. It will be non-positive if and only if:

$$\begin{aligned}e\sqrt{e^2+\frac{4(1-\delta)we}{t\delta}}&\leq e^2+\frac{4(1-\delta)we}{2t\delta}\\\frac{4(1-\delta)we}{t\delta}&\leq\frac{4(1-\delta)we}{t\delta}+\frac{16(1-\delta)^2w^2}{4t^2\delta^2}\end{aligned}$$

Thus,

$$\frac{d\phi(t)}{dt}<0.$$

That is, profit is decreasing in $1-q$. The optimal q will be such that $X_{M,mq}=1$, that is,

$$q^{*}=\frac{\delta w-e}{\delta(w-e)}.$$

■

Proof of Lemma 3.

Let us define the differentiable function $F:[0,1]\times[0,1]\to\mathbb{R}$ as

$$F\left(X_A,X_B\right)=\frac{w}{1-\delta\left(1-X_A-X_B\right)}\left\{1+\delta\left(\begin{array}{c}\frac{X_A(1-X_A)}{1-\delta(1-X_B)}+X_AX_B\delta\frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_B))}+\\ \frac{X_B(1-X_B)}{1-\delta(1-X_A)}+X_BX_A\delta\frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_A))}\end{array}\right)\right\}-\frac{w-e}{1-\delta}$$

and denote the level set of F corresponding to 0 by the set $\{(X_A,X_B):X_A+X_B\leq 1 \text{ and } F\left(X_A,X_B\right)=0\}$. In words, these are the possible combinations of each pair of consumer bases that makes the providers indifferent between exerting effort or defaulting.

$$\frac{w}{1-\delta\left(1-X_A-X_B\right)}+\delta\frac{w}{1-\delta\left(1-X_A-X_B\right)}\left(\begin{array}{c}\frac{X_A(1-X_A)}{1-\delta(1-X_B)}+X_AX_B\delta\frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_B))}+\\ \frac{X_B(1-X_B)}{1-\delta(1-X_A)}+X_BX_A\delta\frac{(1-X_A-X_B)}{(1-\delta)(1-\delta(1-X_A))}\end{array}\right)=\frac{w-e}{1-\delta}$$

Note that $F\left(X_A,0\right)=F\left(0,X_B\right)$ and x that solves $F\left(x,0\right)=F\left(0,x\right)=0$ is $X_A=\frac{e\delta+\sqrt{e^2\delta^2+4\delta(1-\delta)we}}{2\delta w}<1$.

Rearranging it, we have:

$$\begin{array}{c}\frac{w}{1-\delta(1-X_A-X_B)}\times\frac{1}{(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B)}\times\\ \times\left\{\begin{array}{c}(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B)+\\ \delta(1-\delta)X_A(1-X_A)(1-\delta+\delta X_A)+\\ +\delta^2X_AX_B(1-X_A-X_B)(1-\delta+\delta X_A)+\\ \delta(1-\delta)X_B(1-X_B)(1-\delta+\delta X_B)+\\ \delta^2X_BX_A(1-X_A-X_B)(1-\delta+\delta X_B)\end{array}\right\}=\frac{w-e}{1-\delta}\end{array}$$

Notice then that:

$$\left\{\begin{array}{c}(1-\delta)(1-\delta+\delta X_A)(1-\delta+\delta X_B)+\\ \delta(1-\delta)X_A(1-X_A)(1-\delta+\delta X_A)+\\ +\delta^2 X_A X_B(1-X_A-X_B)(1-\delta+\delta X_A)+\\ \delta(1-\delta)X_B(1-X_B)(1-\delta+\delta X_B)+\\ \delta^2 X_B X_A(1-X_A-X_B)(1-\delta+\delta X_B)\end{array}\right\}=$$

$$=(1-\delta(1-X_A-X_B))\left\{\begin{array}{c}-\delta^2 X_A^2 X_B-\delta(1-\delta)X_A^2-\delta^2 X_A X_B^2+\\ +\delta^2 X_A X_B+\delta(1-\delta)X_A-\delta(1-\delta)X_B^2\\ +\delta(1-\delta)X_B+(1-\delta)^2\end{array}\right\}$$

Substituting back and rearranging, we have:

$$\frac{1}{(1-\delta+\delta X_A)(1-\delta+\delta X_B)}\left\{\begin{array}{c}-\delta^2 X_A^2 X_B-\delta(1-\delta)X_A^2-\delta^2 X_A X_B^2+\\ +\delta^2 X_A X_B+\delta(1-\delta)X_A-\delta(1-\delta)X_B^2\\ +\delta(1-\delta)X_B+(1-\delta)^2\end{array}\right\}=\frac{w-e}{w}$$

Then, notice that:

$$\left\{\begin{array}{c}-\delta^2 X_A^2 X_B-\delta(1-\delta)X_A^2-\delta^2 X_A X_B^2+\\ +\delta^2 X_A X_B+\delta(1-\delta)X_A-\delta(1-\delta)X_B^2\\ +\delta(1-\delta)X_B+(1-\delta)^2\end{array}\right\}=\left\{\begin{array}{c}-\delta X_A^2(1-\delta+\delta X_B)\\ -\delta X_B^2(1-\delta+\delta X_A)\\ +(1-\delta+\delta X_A)(1-\delta+\delta X_B)\end{array}\right\}$$

Substituting it back and rearranging, we have:

$$\frac{e}{w}-\frac{\delta X_A^2}{1-\delta+\delta X_A}-\frac{\delta X_B^2}{1-\delta+\delta X_B}=0 \qquad (\star)$$

Then $(\star)$ defines a functional F. Notice that:

$$F_A=-\frac{\delta X_A(2(1-\delta)+\delta X_A)}{(1-\delta+\delta X_A)^2}<0$$

and

$$F_B=-\frac{\delta X_B(2(1-\delta)+\delta X_B)}{(1-\delta+\delta X_B)^2}<0$$

Since $(\star)$ implicitly defines X_B as a function of X_A, from the implicit function theorem, we have:

$$\frac{dX_B}{dX_A}=-\frac{F_A}{F_B}=-\left(\frac{-\frac{\delta X_A(2(1-\delta)+\delta X_A)}{(1-\delta+\delta X_A)^2}}{-\frac{\delta X_B(2(1-\delta)+\delta X_B)}{(1-\delta+\delta X_B)^2}}\right)$$

Simplifying it, we have:

$$\frac{dX_B}{dX_A} = -\frac{(1-\delta+\delta X_B)^2 X_A(2(1-\delta)+\delta X_A)}{(1-\delta+\delta X_A)^2 X_B(2(1-\delta)+\delta X_B)} < 0$$

■

Proof of Proposition 3. We will show that for each given pair of fees f_A and f_B, there is a unique pair (X_A, X_B) that simultaneously solves equations (15) and (17). First, let us show that for each given pair of fees f_A and f_B, equation (17) above defines a strictly increasing function $X_B(X_A)$. For convenience, let us define $\frac{f_A}{f_B} = \frac{1}{l}$.

$$\frac{X_A(1-\delta)+\delta X_A X_B}{X_B(1-\delta)+\delta X_A X_B} = \frac{1}{l}$$

Therefore, for each given pair of fees, the indifference condition of the consumers defines a relation between the consumer bases of the bureaus, and we can write X_B as an explicit function of X_A:

$$X_B = \frac{lX_A(1-\delta)}{(1-\delta)+\delta X_A(1-l)} \tag{31}$$

Thus, the function is increasing in X, but it is discontinuous:

$$\frac{\partial X_B(X_A)}{\partial X_A} = \frac{l(1-\delta)^2}{((1-\delta)+\delta X_A(1-l))^2} > 0, \forall l > 0$$

The shape of this function depends on l. Let us look at this function for each of the possible three cases.

(1) If $f_A > f_B$ $(l < 1)$, then $X_A > X_B > 0$ and $X_B(X_A)$ is a continuous and concave function;

(2) If $l = 1$, then $X_B = X_A$;

(3) Finally, if $f_A < f_B$ $(l > 1)$ then $X_A < X_B$. Moreover, the function is discontinuous at $(1-\delta)+\delta X_A(1-l) = 0$, that is, there is a value $\hat{X}_A > 0$, given by

$$\hat{X}_A = \frac{(1-\delta)}{\delta(l-1)},$$

where if $X_A < \hat{X}_A$, then X_B that solves (31) is an increasing function from zero and increasing asymptotically to ∞ as X_A approaches $\hat{X}_A$.

Lemma (3) completes the proof, i.e., the pair (X_A, X_B) that solves (15) is a strictly decreasing function with both X_A and X_B positives, so there is a unique point at which this decreasing function crosses the curve $X_B(X_A)$ defined by (31). ■

Proof of Lemma 4. First, note that (f_i, f_j) with $f_i = f_j = \frac{P-w}{P}\frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$ with $X_i = X_j = X^{sym}$ and $Y = \frac{P-w}{P}$ is an equilibrium. Second, let us look at the case where $f_i < f_j = \frac{P-w}{P}\frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. There is a unique pair (X_i, X_j) that solves both conditions (15) and (17). This pair is such that $X_i < X^{sym} < X_j$. Also, let

$$Y_{C,member} = \bar{f}\frac{1-\delta(1-X_j)}{\delta w X_j},$$

where $Y_{C,member} = \frac{P-w}{P}\frac{\delta X^{sym}}{1-\delta(1-X^{sym})}\frac{1-\delta(1-X_j)}{\delta X_j} < \frac{P-w}{P}$, since $\frac{\delta X^{sym}}{1-\delta(1-X^{sym})} < \frac{\delta X_j}{1-\delta(1-X_j)}$, so condition (18) is also satisfied.

Now suppose that $f_i = f_j < \frac{P-w}{P}\frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. Then, let $X_i = X_j = X^{sym}$ and Y be given by

$$\begin{aligned} Y_{C,member} &= f_i\frac{1-\delta(1-X^{sym})}{\delta w X^{sym}} \\ &< \frac{P-w}{P}\frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}\frac{1-\delta(1-X^{sym})}{\delta w X^{sym}} \\ &= \frac{P-w}{P}, \end{aligned}$$

so, again, condition (18) is satisfied. Finally, let $f_i < f_j < \frac{P-w}{P}\frac{\delta w X^{sym}}{1-\delta(1-X^{sym})}$. Again, there is a unique pair (X_i, X_j) that solves both conditions (15) and (17). This pair is such that $X_i < X^{sym} < X_j$. Also, let

$$\begin{aligned} Y_{C,member} &= f_j\frac{1-\delta(1-X_j)}{\delta w X_j} \\ &< \frac{P-w}{P}\frac{\delta X^{sym}}{1-\delta(1-X^{sym})}\frac{1-\delta(1-X_j)}{\delta X_j} \\ &< \frac{P-w}{P}. \end{aligned}$$

■

Proof of Proposition 5. Suppose that this is not an equilibrium. First, assume that there is a profitable deviation for A in which A increases its fee. Thus, $f_A > f_B = \bar{f}$. In this case, to satisfy both conditions (15) and (16), we need a higher X_A and smaller X_B. However, note that for firm B to operate in a market with such fees, we need condition (16) to be satisfied. Given that $f_B = \bar{f}$ and that we require a smaller X_B, we need the new mass of providers buying in equilibrium to be higher, that is, $Y' > Y = \frac{P-w}{P}$, but this cannot be an equilibrium, since it violates (18). There are only two equilibria following such a deviation: one in which A is a monopolist and one in which B is a monopolist. Given our refinement, we will assume that following such a deviation, only bureau B (because it has a lower fee) will operate, a contradiction.

Suppose that the profitable deviation is one in which A decreases its fee. Thus, $f_A < f_B = \bar{f}$. From Lemma (4) we know that there exists a unique equilibrium with two operating bureaus. Now, to satisfy both conditions (15) and (16), we need a lower X_A and a higher X_B. A lower X_A, together with a lower f_A, implies that A has decreased its profit, so this is not a profitable deviation either. This proves that the proposed candidate is indeed a stationary competitive equilibrium. Below, we prove that it is unique.

Suppose that bureau j is a monopolist and is charging a feasible fee f_j. Then, any fee $0 < f_i < f_j$ is a profitable deviation for firm i, since it either accommodates two bureaus in the continuation game or it shifts the monopoly to firm i; in either case, firm i will make positive profits.

Now suppose that two firms are operating and $f_i < f_j$. If i increases its fee (but such that it is still lower than f_j) the new pair (f_i', f_j) will increase the consumer basis of firm i, which, together with the higher fee, increases its profit. This proves that a stationary competitive equilibrium must be symmetric. Finally, suppose that $f_i = f_j < \bar{f}$. Then, $X_i = X_j = X^{sym,}$ and $Y < \frac{P-w}{P}$. Consider a deviation in which firm i increases f_i such that both firms still operate (otherwise, given our refinement, only j will operate). We know from Lemma (4) that such a deviation in which the equilibrium in the continuation game has two operating bureaus exists. Then, the new consumer basis of firm i must increase $X_i' > X^{sym} > X_j'$. Given that this is a profitable deviation for firm i, such an equilibrium cannot exist either. Therefore, the symmetric equilibrium with $f_i = f_j = \bar{f}$ is the unique equilibrium in which two bureaus operate. ■

Online Appendix

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A Bureau Buys Only Negative Information

Buying and Selling Information

In this section, we consider the case in which the bureau is able to only buy negative information, i.e., information from consumers that faced default. For simplicity, let us assume that the bureau pays consumers $f_1 = c$ for each reported feedback. Let f_2 be the price asked by the bureau for consumers to be able to access information. Apart from these new assumptions, we keep all the other features of our framework. As presented in Figure OA-1 (at the end of the Online Appendix), the extended game tree has an additional decision node at the beginning of the tree, in which the consumer decides whether or not she will purchase information from the bureau. After this node, all of the remaining tree is identical to the one presented in Figure 1, apart from the payoffs in the terminal nodes, where we must include the paid and received fees. Therefore, if the consumer decides to purchase information from the bureau, we subtract f_2 from her final payoff. Similarly, if the consumer decides to sell information to the bureau, we must add the received fee f_1 to her payoff. No changes are needed for the provider's payoffs or decisions.[1]

Providers' Problem

Assume that a fraction $X_{A,buy}$ of consumers buy the information from the bureau once it is established. Informed consumers only buy services from providers with no history of default. Let's also assume that uninformed consumers hire any provider with whom they match, free-riding on

[1]The game presented in Figure OA-1 has some abuse of notation, considering that we assumed that the bureau would not charge the consumer if there was no information about the match provided. As we will see, the bureau has information about every provider in the case of buying and selling, so the abuse of notation is without loss of generality.

the discipline imposed by informed consumers. We must show that in equilibrium it is optimal for uninformed consumers to hire the service.

Let's consider the decision problem of a provider that has never defaulted before. His only possible stage game action is $\eta = \{\textit{effort, default}\}$. As previously mentioned, consumers that buy information never hire the service of providers that have previously defaulted and all customers sell information. Consequently, providers know that after defaulting once, no informed consumer will hire their services henceforth. As a result, we can focus on the once and for all decision of effort or default. A provider prefers defaulting if:

$$(1-\delta)\,w + \delta\,[(1 - X_{A,buy}) \times w + X_{A,buy} \times 0] > w - e \tag{A.1}$$

where the left-hand side (henceforth, LHS) of equation (A.1) is the payoff to always defaulting, while the expression on the right-hand side (henceforth, RHS) is the payoff to always choosing effort and therefore delivering high-quality service. Simplifying the expression in equation (A.1), we have:

$$\delta < \frac{e}{wX_{A,buy}} \tag{A.2}$$

If the fraction of informed consumers $X_{A,buy}$ is high enough, a provider always puts in effort. However, this would kill the incentive to buy information in the first place. Therefore, given that providers are *ex ante* identical, there is no equilibrium in which all providers follow the same pure strategy and a positive fraction of consumers buy information. Consequently, if in equilibrium a fraction of providers defaults, while the remainder deliver high-quality services, we must have that all providers are indifferent between putting in effort or defaulting. Therefore, from equation (A.2), the measure of informed consumers in equilibrium is:

$$X_{A,buy} = \frac{e}{\delta w} \tag{A.3}$$

Notice from equation (A.3) that the more costly the effort, the higher the measure of informed consumers must be in order to keep providers indifferent between delivering high-quality service or not. In contrast, the more costly it is to lose business – the higher the w – and the more patient providers are – the higher the δ – the smaller is the needed fraction of informed consumers.

Consumers' Problem

We now consider the consumer's decision. Keep in mind that in the stage game, the consumer has three decision nodes. First, she decides whether or not she will buy information from the bureau, paying a fee f_2. Then, based on the information in hand, the consumer must decide whether or not to hire the provider. Finally, if the consumer hires the provider, she must decide whether or not to sell the information to the bureau.

We focus on equilibria in which a bureau is sustained in equilibrium. Consequently, in the equilibria we look at, consumers sell information and uninformed consumers hire providers on the equilibrium path. Let's start with the decision to sell information. As we mentioned before, we assume that all consumers sell information if they are indifferent between selling information or not. Consequently, a consumer sells information as long as $f_1 \geq c$. Let's then consider the decision of an uninformed consumer in purchasing the provider's services. Let $Y_{A,buy}$ denote the fraction of providers that default in equilibrium once the bureau is installed. A consumer that buys no information still prefers hiring a provider if:

$$Y_{A,buy} \leq \frac{P - w + f_1 - c}{P} \tag{A.4}$$

Consequently, as long as the fraction of providers that default is below the threshold presented in equation (A.4), uninformed consumers hire the matched providers. If this restriction is not satisfied, the market unravels. First, because even after the announcement that a bureau will be installed, no agent would buy services in the period prior to the bureau's establishment. Consequently, no information is aggregated by the bureau. Second, if in equilibrium uninformed consumers decide not to hire the service and informed ones only purchase services from providers that always put in effort, there is no incentive for providers to default. Unfortunately, this pattern also eliminates the consumers' incentive to buy information in the first place.

Then, assuming that equation (A.4) and $f_1 \geq c$ are satisfied, we move toward the decision of whether or not to buy information. Since there is no punishment for not buying information in a given period, we just need to compare the consumer's payoff to buying and selling information with the payoff to just selling it. So, the payoff to buying and selling information is given by:

$$\left[-f_2 Y_{A,buy} + (P - w + f_1 - f_2 - c)\,(1 - Y_{A,buy})\right] \tag{A.5}$$

while the payoff to just selling information is given by:

$$(-w + f_1 - c)\,Y_{A,buy} + (P - w + f_1 - c)\,(1 - Y_{A,buy}) \tag{A.6}$$

Therefore, the consumer is indifferent between buying information or not if $Y_{A,buy} = \frac{f_2}{w - f_1 + c}$.

Altruistic Bureau's Problem

The bureau's objective is to maximize the social welfare of consumers and providers. In particular, we consider an egalitarian social welfare function that weights equally consumers and providers. The two populations are equally weighted and normalized to 1 (i.e., $I_1 = I_2 \equiv 1$). Consequently, the social welfare in period t is given by:

$$SW_t = \frac{1}{2}\left\{U_{consumer}(t) + U_{providers}(t)\right\} \tag{A.7}$$

Once we are focusing on the set of equilibria that has a functioning bureau, we consider the equilibria in which providers are indifferent between putting in effort or defaulting and consumers are indifferent between buying information or not. Consequently, we have that:

$$U_{consumer}(t) = (1 - Y_{A,buy})P - w + f_1 - c \quad \text{and} \quad U_{provider}(t) = w - e, \ \forall t > 0$$

where we set $U_{consumer}$ equal to the uninformed consumer's utility, while $U_{provider}$ is set to equal the utility earned by a provider that puts in effort in equilibrium. Substituting back into equation (A.7) and rearranging:

$$SW_t = \frac{1}{2}\left\{(1 - Y_{A,buy})P + f_1 - c - e\right\}$$

Then, the balanced budget condition becomes:

$$\frac{\delta f_2 X_{A,buy}}{1-\delta} - cY_{A,buy} - \sum_{t=1}^{\infty} \delta^t (1 - X_{A,buy}) Y_{A,buy} c \geq 0$$

Substituting the values for $X_{A,buy}$ and $Y_{A,buy} = \frac{f_2}{w}$ obtained above, we have:

$$\frac{\frac{f_2}{w}\left[e - (1 - \frac{e}{w})c\right]}{1-\delta} \geq 0$$

We obtain an equilibrium in which, by setting $f_2 = 0$, the bureau not only establishes a costless bureau, but also maximizes the overall social welfare. The following proposition summarizes these results:

Proposition A.1 *Assume the bureau purchases only negative information and there are no payoff-types among providers. An altruistic bureau is able to obtain the first-best by committing to buying information from consumers that faced default and sell it at no cost to other consumers.*

This result is not robust to small changes in the environment. The main issue is that, off the equilibrium path, the bureau would not have money to purchase information in the case of a deviation. There are a few ways in which we can avoid this result. For example, we implicitly imposed an asymmetry between the costs of reporting and accessing information. We assumed that reporting the information is costly, but accessing the information is costless apart from the fee. We may ease this constraint and assume that accessing the information has an intrinsic cost of c_1 apart from the fee. As we show in the next section, this minor extension rules out this equilibrium, once consumers would skip accessing information if they expect no provider to default.

B Costly Information Access

First of all, let's keep in mind that the utility for consumers is given by:

$$U = (1-\delta)\sum_{t=0}^{\infty}\delta^t c_t$$

where c_t is the consumption of the numeraire with price normalized to 1. In this sense, we have risk neutral agents, but we normalize the utility function such that consuming one unit of the numeraire every period gives a utility of 1 (see Mailath and Samuelson (2006)).

Consumers' Problem

Consumers pay a cost c_1 to access the information gathered by the bureau beyond their membership fees. As before, we assume that the bureau compensates its members for the cost of reporting defaults. Consequently, a consumer is indifferent between becoming a member or not if:

$$(1-Y_{A,member})(P-w) - f_{ee}^1 - \delta c_1 - \frac{(1-\delta)Y_{A,member}}{1-\delta(1-X_{A,member})}w = (1-Y_{A,member})P - w$$

which leads to the following condition:

$$f_{ee}^1 = \delta\left[\frac{X_{A,member}Y_{A,member}w}{1-\delta(1-X_{A,member})} - c_1\right] \tag{A.8}$$

Apart from that, the problem is identical to the one presented in Section 3. Similarly, the provider's problem is the same as the one presented in Section 3.

Altruistic Bureau's Problem

The altruistic bureau's problem is similar to the one presented in Section 3, apart from a few details. First, the bureau's profit function must take into account that only negative information is purchased, i.e., the profit function is now given by:

$$\Pi_{A,member} = X_{A,member}\left\{\frac{f_{ee}^1}{1-\delta} - Y_{A,member}c\sum_{t=0}^{\infty}\delta^t(1-X_{A,member})^t\right\}$$

which impacts the break-even condition. Second, the consumer's problem implies that we must take into account the new fee value presented in (A.8). Consequently, the altruistic bureau's problem is given by:

$$\mathbf{SW_{A,member}} \equiv \max_{f_{ee}^1} \frac{1}{2(1-\delta)}\left[(1-Y_{A,member})P - e\right]$$

subject to:

$$\frac{f_{ee}^1 X_{A,member}}{1-\delta} - \left(\frac{f_{ee}^1+\delta c_1}{\delta w}\right) c \geq 0 \tag{C.1}$$

$$0 \leq Y_{A,member} \leq \frac{P-w}{P} \tag{C.2}$$

$$Y_{A,member} = \frac{\left(f_{ee}^1+\delta c_1\right)\left[1-\delta(1-X_{A,member})\right]}{\delta X_{A,member} w} \tag{C.3}$$

$$X_{A,member} = \frac{e\delta+\sqrt{e^2\delta^2+4\delta(1-\delta)we}}{2\delta w} \tag{C.4}$$

Substituting (C.3) into (C.2) and the objective function, we can see that the objective function is linearly decreasing in f_{ee}^1. Therefore, at the optimum, (C.1) must be binding:

$$f_{ee}^1 = \frac{(1-\delta)\delta c_1 c}{\delta X_{A,member} w - (1-\delta)c}$$

Substituting it back into (C.3), we have:

$$Y_{A,member} = \delta c_1 \frac{\left[1-\delta(1-X_{A,member})\right]}{\delta X_{A,member} w - (1-\delta)c} \tag{A.9}$$

Competitive Bureaus

In Section 5, we showed that there was a unique stationary equilibrium in which two profit-maximizing bureaus offering membership operate in the market. Here, we extend this analysis to the case where bureaus can buy only negative information and there is a cost c_1 to access the bureau's information, even if the consumer is a member. To obtain the uniqueness result, we showed two auxiliary results: 1. the providers' indifference condition generated a decreasing relation between the competing bureaus' sizes, and 2. for any given pair of fees, the consumers' indifference condition generated an increasing relation between the bureaus' sizes. These two results together implied that there was at most one stationary equilibrium with two bureaus.

Here, even though we have the costly access to information and the possibility of the bureaus restricting themselves to buying only negative information, it is still the case that these auxiliary results hold: the providers' indifference condition is unchanged and in any equilibrium with two operating bureaus, we can use (A.8) and with some algebra obtain that the ratio of fees is now given by:

$$\frac{f_A + \delta c_1}{f_B + \delta c_1} = \frac{X_A\,(1-\delta) + \delta X_A X_B}{X_B\,(1-\delta) + \delta X_A X_B} \tag{A.10}$$

Thus, given any costs c_1 and a ratio of fees, it is still the case that we must have the same relation between the relative sizes of the bureaus X_A and X_B. Therefore, we must have a unique stationary equilibrium and it is the same as the one presented in Section 5.

C Membership Status Is Observable

In this case, we assume that bureau membership status can be credibly communicated to providers. Following the case presented in Section A, we also assume that the bureau can purchase only negative information. Moreover, we focus on the case of a non-profit altruistic bureau whose goal is to maximize social welfare.

First of all, we can easily show that, in this case, there is no incentive for providers to exert effort when facing a consumer that is not a bureau member.

Lemma A.1 *There is no equilibrium in which the provider puts in effort with positive probability when facing a non-member.*

Consequently, providers never put in effort when facing a non-member. As a result, it is optimal for non-members not to buy the providers' services, as we summarize in the following corollary.

Corollary A.1 *Consumers that choose not to become bureau members will optimally choose not to buy the providers' service.*

We consider three sub-cases in terms of the cost of accessing information: 1. costless information access; 2. costly and enforceable information access; and 3. costly and unenforceable information access. We present each sub-case in detail in the following sub-sections.

1. Costless access of information: We establish this as the initial benchmark case, by assuming that $c_1 = 0$.

2. Costly and enforceable information access: In this case, while $c_1 > 0$, we assume that the bureau can punish members that do not access information by stripping them of their membership status;

3. Costly and unenforceable information access: In this case, not only $c_1 > 0$, but the bureau cannot punish members that do not access information. As a result, the choice of accessing information must be optimal by itself.

C.1 Costless Information Access

In this case, $c_1 = 0$ and the consumer can costlessly access the bureau's information, conditional on becoming a member and paying the membership fee. In this case, members will optimally access the information whenever available. Therefore, the consumer's problem is twofold: 1. the decision to become a member or not, and 2. the decision to hire the provider's service or not, conditional on the available information about the provider's past behavior.

In terms of the latter decision, let's focus on the case in which the member hires only the services of providers that have never defaulted in the past, since it induces the strongest incentive for providers to put in effort. Let's assume that the bureau collects all the information about deviations against members (no optimal sampling). In this case, it's optimal for the provider to put in effort if:

$$w - e + \delta \frac{X_{A,k-m}(w-e)}{1-\delta} \geq w \;\Rightarrow\; X_{A,k-m} \geq \frac{(1-\delta)e}{\delta(w-e)} \tag{A.11}$$

Therefore, there is a threshold size of the bureau that would induce providers to put in effort. In particular, if $\delta > \frac{e}{w}$, the RHS of (A.11) is strictly less than one and the threshold problem is well-defined. Let's assume that (A.11) is satisfied and that the bureau pays any incurred cost of reporting default. In this case, a consumer decides to become a member and pay the membership fee if:

$$\frac{P-w}{1-\delta} - \frac{\delta f_{ee}^{k-m}}{1-\delta} \geq 0 \;\Rightarrow\; f_{ee}^{k-m} \leq \frac{P-w}{\delta} \tag{A.12}$$

As a result, as long as the membership fee is less than the flow benefit of buying high-quality services, the consumer joins the bureau. Therefore if (A.12) is satisfied with inequality, $X_{A,k-m} = 1$.

Finally, notice that if inequalities (A.11) and (A.12) are satisfied and non-binding, all consumers become members and all providers put in effort whenever facing a member. As a result, in order to maximize social welfare, a bureau can set a zero membership fee and still satisfy the break-even condition, as in the case of Proposition A.1 in Section A. The following proposition summarizes the results:

Proposition A.2 *Assume that membership status is observable, the bureau is able to buy only negative information, and there is no cost of accessing information available to the bureau. An altruistic bureau is able to obtain the first-best by setting a zero-cost membership fee and committing to repaying members any incurred cost of reporting default.*

C.1.1 Costly and Enforceable Information Access

In this case, while there is a cost of accessing information $c_1 > 0$, if members choose not to access information, the bureau can punish them by expelling them from the bureau. Notice that, while the provider's problem is the same as the one defined by (A.11), the consumer's problem has changed. In particular, assuming that $X_{A,k-m}$ satisfies the inequality (A.11) and providers exert effort when facing a member, the consumer now decides to join the bureau if:

$$\frac{P-w}{1-\delta} - \frac{\delta(c_1 + f_{ee}^{k-m})}{1-\delta} \geq 0 \ \Rightarrow \ f_{ee}^{k-m} \leq \frac{P-w}{\delta} - c_1 \tag{A.13}$$

As in the case with costless information access, notice that if inequalities (A.11) and (A.13) are satisfied and non-binding, all consumers become members and all providers put in effort whenever facing a member. As a result, in order to maximize social welfare a bureau can again set a zero fee and still satisfy a break-even condition, in a manner similar to the one presented in Proposition A.1 in Section A.

Therefore, apart from a reduction in social welfare due to the incurred cost of information access, the equilibrium outcome is the same as the one obtained in the case of costless information access. In particular, the social welfare becomes:

$$\mathbf{SW_{A,k-m}} = \frac{1}{2(1-\delta)} \{P - e - c_1\}$$

C.1.2 Costly and Unenforceable Information Access

In this case, some consumers that became members may prefer to skip information access in order to save information access costs. It is clear that there is no equilibrium in which either all consumers skip accessing information or all consumers access information. Therefore, we should expect that at least a fraction of members access the information, incurring the cost c_1. However, in order for this strategy to be optimal, we must have some providers defaulting against members in equilibrium. As a result, we reestablish a mixed strategy equilibrium. Therefore, in equilibrium, while all providers default against non-members, some would also default against members. Define ε as the fraction of members that decide not to access information. Then, the provider's problem becomes:

$$w - e + \delta\frac{X_{A,k-m}(w-e)}{1-\delta} = w + \delta\frac{X_{A,k-m}\varepsilon w}{1-\delta} + \sum_{t=1}^{\infty} \delta^t (1 - X_{A,k-m})^t X_{A,k-m}(1-\varepsilon)w$$

Keep in mind that we are already assuming here that non-members do not purchase the provider's services. Rearranging it, we have:

$$(w-e)\left[1+\frac{\delta X_{A,k-m}}{1-\delta}\right]=w\left[1+\frac{\delta\varepsilon X_{A,k-m}}{1-\delta}+\frac{\delta X_{A,k-m}(1-X_{A,k-m})}{1-\delta(1-X_{A,k-m})}(1-\varepsilon)\right] \tag{A.14}$$

Let's consider the consumer's problem now. First, the consumer must decide whether or not to become a bureau member. Furthermore, if a consumer becomes a member, she must decide whether or not to access information. However, even members that did not access information will report poor service, since they are reimbursed for the cost of reporting. The consumer's utility of becoming a member and regularly accessing the information is:

$$(1-Y_{A,k-m})\tfrac{(P-w)}{1-\delta}-\tfrac{f_{ee}^{k-m}}{1-\delta}-\delta\tfrac{c_1}{1-\delta}+\textstyle\sum_{t=0}^{\infty}\delta^t(1-X_{A,k-m})^t Y_{A,k-m}(-w)=$$
$$\tfrac{1}{1-\delta}\left\{(1-Y_{A,k-m})(P-w)-f_{ee}^{k-m}-\delta c_1-\tfrac{(1-\delta)Y_{A,k-m}w}{1-\delta(1-X_{A,k-m})}\right\}$$

While the utility of becoming a member and not accessing the information is:

$$(1-Y_{A,k-m})\frac{(P-w)}{1-\delta}-\frac{f_{ee}^{k-m}}{1-\delta}-\frac{Y_{A,k-m}w}{1-\delta}$$

Then, the consumer is indifferent between accessing the information or not if:

$$c_1=\frac{X_{A,k-m}Yw}{1-\delta(1-X_{A,k-m})}\Rightarrow Y_{A,k-m}=\frac{[1-\delta(1-X_{A,k-m})]\,c_1}{wX_{A,k-m}} \tag{A.15}$$

Let's assume that (A.15) is satisfied. Moreover, notice that the RHS(A.15) is strictly decreasing in $X_{A,k-m}$. Then, the consumer prefers to become a member of the bureau if:

$$(1-Y_{A,k-m})\frac{(P-w)}{1-\delta}-\frac{f_{ee}^{k-m}}{1-\delta}-\frac{Y_{A,k-m}w}{1-\delta}\geq 0\ \Rightarrow\ f_{ee}^{k-m}\leq(1-Y)P-w \tag{A.16}$$

Let's assume that (A.16) is satisfied with inequality. In this case, all consumers prefer becoming members and we have $X_{A,k-m}=1$. Then, from (A.15), we have $Y_{A,k-m}=\frac{c_1}{w}$. Similarly, from (A.14), we have:

$$(w-e)=w\,[1-\delta(1-\varepsilon)]\ \Rightarrow\ \varepsilon=1-\frac{e}{\delta w} \tag{A.17}$$

since we have maintained the assumption that $\delta>\frac{e}{\delta w}$, (A.17) implies that $0<\varepsilon<1$.

Finally, let's look at the bureau's problem. First, let's look into the break-even condition. As previously, let's assume the case in which the bureau only buys negative information. Then, the bureau's profit function is given by:

$$\Pi_{A,k-m} = X_{A,k-m} \left\{ \frac{f_{ee}^{k-m} - Y_{A,k-m} c\varepsilon}{1-\delta} - \frac{Y_{A,k-m} c(1-\varepsilon)}{1-\delta(1-X_{A,k-m})} \right\}$$

So the break-even condition implies:

$$f_{ee}^{k-m} \geq Y_{A,k-m} c\varepsilon + \frac{Y_{A,k-m} c(1-\delta)(1-\varepsilon)}{1-\delta(1-X_{A,k-m})} \tag{A.18}$$

From equation (A.18), we can see that increasing $X_{A,k-m}$ eases the break-even constraint, allowing the bureau to reduce its membership fee.

Finally, let's consider the bureau's problem. First of all, it's easy to see that there is no equilibrium in which all members access information. Similarly, there is no subgame perfect equilibrium in which non-members purchase the services. In both cases, there are optimal one-shot deviations by consumers and providers, respectively. Consequently, in equilibrium we expect non-members to choose not to buy the service and a fraction ε of members will skip accessing the information. As a result, we will focus on the bureau's choices in this case. Then, the bureau's problem becomes:

$$\mathbf{SW_{A,k-m}} \equiv \max_{f_{ee}^{k-m}} \frac{1}{2(1-\delta)} \left\{ (w-e) \left[1-\delta(1-X_{A,k-m})\right] + (1-Y_{A,k-m})P - f_{ee}^{k-m} - w \right\}$$

subject to:

$$\begin{aligned}
& f_{ee}^{k-m} \geq Y_{A,k-m} c\varepsilon + \tfrac{Y_{A,k-m} c(1-\delta)(1-\varepsilon)}{1-\delta(1-X_{A,k-m})} && (C.1)\\
& Y_{A,k-m} = \tfrac{\left[1-\delta(1-X_{A,k-m})\right]c_1}{wX_{A,k-m}} && (C.2)\\
& (w-e)\left[1+\tfrac{\delta X_{A,k-m}}{1-\delta}\right] = w\left[1+\tfrac{\delta\varepsilon X_{A,k-m}}{1-\delta}+\tfrac{\delta X_{A,k-m}(1-X_{A,k-m})(1-\varepsilon)}{1-\delta(1-X_{A,k-m})}\right] && (C.3)
\end{aligned}$$

But then, notice that:

$$\frac{\partial \mathbf{SW_{A,k-m}}}{\partial X_{A,k-m}} = \delta(w-e) - \frac{\partial Y_{A,k-m}}{\partial X_{A,k-m}} P$$

Since $\frac{\partial Y_{A,k-m}}{\partial X_{A,k-m}} = -\frac{(1-\delta)c_1}{wX_{A,k-m}^2} < 0$, we have that $\frac{\partial \mathbf{SW_{A,k-m}}}{\partial X_{A,k-m}} > 0$. Therefore, a bureau that is trying to maximize social welfare would like to set $X_{A,k-m} = 1$. In order to obtain that, we must satisfy (A.16) with inequality. Therefore, the bureau's problem becomes:

$$\mathbf{SW_{A,k-m}} \equiv \max_{f_{ee}^{k-m}} \frac{1}{2(1-\delta)} \left\{ (w-e) \left[1-\delta(1-X_{A,k-m})\right] + (1-Y)P - f_{ee}^{k-m} - w \right\}$$

subject to:

$$
\begin{array}{ll}
f_{ee}^{k-m} \geq Y_{A,k-m}c\varepsilon + \frac{Y_{A,k-m}c(1-\delta)(1-\varepsilon)}{1-\delta(1-X_{A,k-m})} & (C.1) \\
Y_{A,k-m} = \frac{\left[1-\delta(1-X_{A,k-m})\right]c_1}{wX_{A,k-m}} & (C.2) \\
(w-e)\left[1+\frac{\delta X_{A,k-m}}{1-\delta}\right] = w\left[1+\frac{\delta\varepsilon X_{A,k-m}}{1-\delta}+\frac{\delta X_{A,k-m}(1-X_{A,k-m})(1-\varepsilon)}{1-\delta(1-X_{A,k-m})}\right] & (C.3) \\
f_{ee}^{k-m} < (1-Y)P - w & (C.4) \\
X_{A,k-m} = 1 & (C.5)
\end{array}
$$

Simplifying the problem by substituting (C.5) and (C.3), we have:

$$
\mathbf{SW_{A,k-m}} \equiv \max_{f_{ee}^{k-m}} \frac{1}{2(1-\delta)} \left\{(1-\frac{c_1}{w})P - f_{ee}^{k-m} - e\right\}
$$

subject to:

$$
\begin{array}{ll}
\frac{c_1}{w}c\left[1-\delta+\delta\varepsilon\right] \leq f_{ee}^{k-m} < (1-\frac{c_1}{w})P - w & (C.1') \\
\varepsilon = 1 - \frac{e}{\delta w} & (C.3')
\end{array}
$$

Finally, since $\mathbf{SW_{A,k-m}}$ is strictly decreasing in f_{ee}^{k-m}, (C.1')'s lower bound restriction must be satisfied with equality at the optimum. Therefore, we have that:

$$
f_{ee}^{k-m} = \frac{c_1}{w}c\left[1-\delta+\delta\varepsilon\right] = \frac{c_1}{w}c\left[1-\frac{e}{w}\right] \tag{A.19}
$$

where the second equality in (A.19) is obtained by substituting (C.3'). Finally, from previous calculations, we obtained $X_{A,k-m} = 1$ and $Y_{A,k-m} = \frac{c_1}{w}$.

Let's now present a few auxiliary results in terms of welfare:

Lemma A.2 *The fraction of providers that choose to default is lower in the case of known membership.*

Corollary A.2 *As $\delta \to 1$ we have that $Y_{A,u-m} \to \frac{c_1}{w}$.*

Therefore, notice that $Y_{A,u-m} \to Y_{A,k-m}$ as $\delta \to 1$. Consequently, we obtain the following result.

Proposition A.3 *In the case in which information access is costly and unenforceable, we have that $\mathbf{SW_{A,k-m}} - \mathbf{SW_{A,u-m}} \to -\infty$ as $\delta \to 1$.*

Consequently, in the case in which information access is costly but unenforceable, known membership becomes counterproductive as agents become patient. The fact that even effectively "uninformed" consumers feel compelled to become members and consequently report deviations over time induces an increase in the overall cost for consumers that reduces social welfare. Making membership unknown allows uninformed consumers to free-ride on the incentives to providers delivered by informed consumers without paying a membership fee. Similarly, it allows bureaus to save on the direct costs of acquiring information from uninformed consumers, relying on the learning process over time in order to induce effort by providers. Finally, notice that as agents – both consumers and providers – become more patient, the gap between the incentives for providers' efforts induced by known and unknown membership narrows, being equal at the limit. In contrast, notice that $X_{A,u-m} \to \frac{e}{w}$ and in fact, $\frac{\partial X_{A,u-m}}{\partial \delta} < 0$ for $\delta > \frac{1}{2}$. Therefore, as agents become increasingly patient, fewer and fewer informed consumers are necessary in order to induce effort.

Figure OA-1: Extended Game Tree – Bureau That Buys and Sells Information