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A DSGE Model Including Trend Information and Regime Switching at the ZLB*

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Abstract

This paper outlines the dynamic stochastic general equilibrium (DSGE) model developed at the Federal Reserve Bank of Cleveland as part of the suite of models used for forecasting and policy analysis by Cleveland Fed researchers, which we have nicknamed CLEMENTINE (CLeveland Equilibrium ModEl iNcluding Trend INformation and the Effective lower bound). This document adopts a practitioner's guide approach, detailing the construction of the model and offering practical guidance on its use as a policy tool designed to support decision-making through forecasting exercises and policy counterfactuals.

Keywords: DSGE model, Labor market frictions, Zero lower bound, Trends, Expectations

JEL classification: E32, E23, E31, E52, D58

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1 Introduction

This paper outlines the dynamic stochastic general equilibrium (DSGE) model developed at the Federal Reserve Bank of Cleveland as part of the suite of models used for forecasting and policy analysis by Cleveland Fed researchers, which we have nicknamed Clementine (CLeveland Equilibrium ModEl iNcluding Trend INformation and the Effective lower bound). Clementine is a practical policy tool, designed to support decision-making through forecasting exercises and policy counterfactuals to complement the existing set of models and data that are routinely consulted. Rather than following the standard style of an academic paper, it adopts a practitioner's guide approach, detailing the construction of the model and offering practical guidance on its use as a policy tool. This document will be subject to regular updates to ensure that it remains an up-to-date and accessible resource.

The model uses a medium-scale New Keynesian framework with search and matching frictions in the labor market, building upon the work of Gertler, Sala, and Trigari (2008) and Furlanetto and Groshenny (2016). The model is estimated using Bayesian techniques to fit it to data on several US macroeconomic variables. Our framework builds upon the Smets and Wouters (2007) model by incorporating search and matching frictions in the labor market to enhance the model's ability to capture observed labor market developments. The Smets and Wouters model is a medium-scale New Keynesian DSGE model without search and matching frictions that has been widely used for macroeconomic forecasting and policy analysis. It features a variety of nominal and real rigidities, such as sticky prices and wages, habit formation in consumption, and investment-adjustment costs. In the model, households maximize utility by choosing consumption, investment, and labor supply, while firms produce goods and services using a combination of labor and capital under monopolistic competition. The models incorporate various shocks to capture fluctuations in the economy, including technology shocks, preference shocks, investment-specific technology shocks, and monetary policy shocks. The central bank follows a Taylor-type rule for setting the nominal interest rate in response to fluctuations in inflation and output growth. These features help capture the persistence observed in macroeconomic data and the transmission mechanisms of monetary policy.

The medium-scale New Keynesian models developed by Gertler et al. (2008) and Furlanetto and Groshenny (2016) build upon the Smets and Wouters (2007) framework by incorporating search and matching friction in the labor market. Search and matching frictions arise from the time and resources required to match job seekers with available job vacancies. In this framework, both workers and firms engage in a costly and time-consuming process to form employment relationships, resulting in a richer depiction of labor market behavior.

By building on these foundational models, the model proposed in this paper aims to enhance the predictive power of medium-scale New Keynesian models by incorporating additional features, notably, a) time-varying job-separation rates, b) low-frequency components that capture trends in the main macroeconomic variables, and c) a zero-lower-bound constraint on the nominal interest rate set by the central bank. These innovations allow the model to account for a broader range of economic fluctuations and long-term trends. First, the incorporation of time-varying job separation allows the model to account for fluctuations in the unemployment rate more comprehensively, as unemployment fluctuations cannot be accounted for only by movements in job-finding rates (Shimer, 2005).

Second, low-frequency components are included to act as attraction points for forecasts at the 5- to 12-year horizon. A low-frequency component in the inflation rate is attributed to a time-varying inflation target in the monetary policy rule followed by the central bank as in Ireland (2007), while slow-moving exogenous trends in the spirit of Ferroni (2011) and Canova (2014) influence growth rates, the unemployment rate, and the real risk-free rate. While, with the exception of the inflation target, these trends have no structural interpretation in the model, they are meant to capture elements outside of the model such as demographic changes and shifts in government policies, in preferences, or in the overall rate of innovation. To discipline these trend terms, the empirical analysis incorporates additional time series data, namely, long-range survey expectations from the Survey of Professional Forecasters (SPF) and Blue Chip (Wolters Kluwer Legal and Regulatory Solutions U.S).

Third, we consider how the model's dynamics change when the zero lower bound on the interest rate set by the central bank is binding. Namely, the model utilizes a piecewise linear solution technique that combines two linearizations corresponding to two monetary policy regimes: one with an unconstrained interest rate and another with the interest rate constrained at the zero lower bound. Unlike the approximation strategy in Guerrieri and Iacoviello (2015), this model allows agents to incorporate the possibility of switching between the two regimes when forming their expectations. These probabilities of switching are endogenously determined and consistent with the approximate solution of the model.

The rest of the paper is organized as follows. Two versions of this model will be presented: Section 2 will present a one-regime version of the model that disregards the zero lower bound; Section 3 will present the baseline version of the model, namely, the two-regime version that is solved by a piecewise linear approximation. Section 4 presents the data used to parameterize the model and presents the estimation results. Section 5 presents the quantitative analysis, illustrates the responses of the two versions of the model to different shocks, and decomposes the observed time series into the relative contribution of the different shocks that hit the economy. By providing the two versions, we illustrate the model's behavior and its implications under different policy regimes.

2 Model: One-Regime Version

2.1 Households

There is a continuum of identical households of mass one. Each household is a large family, made up of a continuum of infinitely lived agents indexed by $i \in (0,1)$ at time t that derive utility from the consumption of market consumption goods C_{it} and home consumption goods H_{it}. Following Merz (1995) and Andolfatto (1996), the family offers perfect consumption insurance to its members.

At each date family members are in one of two states: employed or unemployed. An unemployed household member produces and consumes home consumption goods. Specifically, individual agents have intertemporal utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \omega_t \left[\ln(C_{it} - h_c C_{it-1}) + \chi \ln(H_{it} - h_h H_{it-1})\right]$$

where $0 < \beta < 1$. When $h_c > 0$ and $h_h > 0$, the model allows for internal habit formation in market and home consumption, respectively. The preference shock ω_t follows the autoregressive process

$$\ln \omega_t = \rho_\omega \ln \omega_{t-1} + \varepsilon_{\omega t}, \tag{1}$$

where $\varepsilon_{\omega t} \sim Niid(0, \sigma_\omega^2)$.

Each period N_t family members are employed and $1 - N_t$ members are unemployed and searching for a job. Since perfect consumption insurance implies $C_{it} = C_t$ and $H_{it} = H_t$, where $C_t = \int_0^1 C_{it} di$ and $H_t = \int_0^1 H_{it} di$ are per capita market and home consumption, the household's aggregate utility function is given by

$$\begin{aligned} &E_0 \sum_{t=0}^{\infty} \beta^t \omega_t \int \left[\ln(C_{it} - h_c C_{it-1}) + \chi \ln(H_{it} - h_h H_{it-1})\right] di \\ &= E_0 \sum_{t=0}^{\infty} \beta^t \omega_t \left[\ln(C_t - h_c C_{t-1}) + \chi \ln(H_t - h_h H_{t-1})\right] \end{aligned} \tag{2}$$

At each date t, $N_t(j)$ family members are employed by intermediate goods-producing firm $j \in (0,1)$. Each worker employed at firm j works a fixed amount of hours and earns the nominal wage $W_t(j)$, under the assumption that newly employed workers and all other employed workers in firm j earn the same wage. (By the symmetry of the problem of firms, it will turn out that all employed workers will earn the same wage.) N_t denotes aggregate employment in period t and is given by

$$N_t = \int_0^1 N_t(j)\, dj \tag{3}$$

At the beginning of period t, aggregate unemployment equals the number of household members who are not employed, $U_t = 1 - N_t$, who receive nominal unemployment benefits b_t financed through lump-sum taxes. To ensure that the model is consistent with balanced growth, unemployment benefits b_t are proportional to the value of the nominal wage along the balanced-growth path $b_t = \tau W_{ss,t}$, where τ is the replacement ratio and $W_{ss,t} = wP_tA_t$ is the nominal wage rate on a balanced-growth path, for a steady-state real-wage-to-productivity rate w.

The technology to produce home consumption goods for the ith individual is $A_t 1_{it}$, where 1_{it} is 1 for an individual unemployed at time t and 0 otherwise, and where A_t is the exogenous state of technology, described below, and common to the market production sector. It follows that

$$H_t = \int A_t 1_{it} di = A_t \int_0^{U_t} di = A_t U_t = A_t(1 - N_t) \tag{4}$$

At the end of period $t-1$, workers who were employed in the period separate from their current match with the exogenous probability s_{t-1}. At the beginning of period t, any worker out of those who are unemployed, $U_{t-1} + s_{t-1}N_{t-1}$, searches for a job. The total number of job searchers is

$$S_t = 1 - (1 - s_{t-1})N_{t-1} \tag{5}$$

where

$$\ln s_t = \rho_s \ln s_{t-1} + \varepsilon_{st}, \tag{6}$$

where $\varepsilon_{st} \sim Niid(0, \sigma_s^2)$.

Each period, a fraction p_t of job searchers finds a job. Therefore, employment evolves as

$$N_t = (1 - s_{t-1})N_{t-1} + p_t S_t = (1 - s_{t-1})N_{t-1} + p_t[1 - (1 - s_{t-1})N_{t-1}] \tag{7}$$

By the symmetry of the problem of firms, it will turn out that all employed workers have the same job-finding probability.

Each identical family enters each period t with B_{t-1} bonds and X_{t-1} units of physical capital. At the beginning of each period, bonds mature, providing B_{t-1} units of money. The family uses some of this money to purchase B_t new bonds at nominal cost $e^{-i_t}B_t$, where i_t denotes the net nominal interest rate between period t and $t + 1$.

The family owns capital and chooses the capital utilization rate, υ_t, which transforms physical capital into effective capital according to

$$K_t = \upsilon_t X_{t-1}. \tag{8}$$

The household rents $K_t\,(i)$ units of effective capital to intermediate-goods-producing firm $i \in [0, 1]$ at the real rate R_{kt}. The household's choice of $K_t\,(i)$ must satisfy

$$K_t = \int_0^1 K_t\,(i)\,di. \tag{9}$$

The cost of capital utilization is $a(\upsilon_t)$ per unit of physical capital. We assume the following functional form for the function a,

$$a(\upsilon_t) = \phi_u(\upsilon_t - 1) + \frac{\phi_{uu}}{2}(\upsilon_t - 1)^2, \tag{10}$$

for some parameters ϕ_u and ϕ_{uu}, and that $\upsilon_t = 1$ in the steady state.

During period t, the household receives total nominal factor payments $P_t R_{kt} K_t + W_t N_t + b_t U_t$. In addition, the household also receives nominal profits $D_t(i)$ from each firm $i \in [0,1]$, for a total of

$$D_t = \int_0^1 D_t(i)\, di. \tag{11}$$

In each period t the family uses these resources to purchase finished goods, for both consumption and investment purposes, from the finished goods-producing firms at the nominal price P_t. The law of motion of physical capital is

$$X_t \leq (1-\delta_k) X_{t-1} + \mu_t \left[1 - F\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{12}$$

where δ_k denotes the depreciation rate of physical capital. The function F captures the presence of adjustment costs in investment, as in Christiano, Eichenbaum and Evans (2005). We assume the following functional form for the function F,

$$F\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - z\right)^2, \tag{13}$$

for some adjustment cost parameter ϕ_I, where z is the steady-state growth rate of investment. Hence, along the balanced-growth path, $F(z) = F'(z) = 0$ and $F''(z) = \phi_I > 0$. μ_t is an investment-specific technology shock affecting the efficiency with which consumption goods are transformed into capital. The investment-specific shock follows the exogenous stationary autoregressive process

$$\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_{\mu t}, \tag{14}$$

where $\varepsilon_{\mu t} \sim Niid(0, \sigma_\mu^2)$.

The family's budget constraint is given by

$$P_t C_t + P_t I_t + e^{-i_t} B_t \leq B_{t-1} + W_t N_t + b_t(1 - N_t) + P_t R_{kt} \upsilon_t X_{t-1} - P_t a(\upsilon_t) X_{t-1} + D_t \tag{15}$$

The family chooses C_t, N_t, B_t, υ_t, I_t, and X_t for each t to maximize the expected lifetime utility (2) subject to the constraints (7), (12), and (15), taking the job-finding rate p_t as given, since each family is infinitesimal.

We can set up a Lagrangian for this problem as

$$
\begin{aligned}
E_0 \sum_{t=0}^{\infty} \Bigg(& \beta^t \omega_t \Big(\ln\left(C_t - h_c C_{t-1}\right) + \chi \ln\left[A_t(1-N_t) - h_h A_{t-1}(1-N_{t-1})\right] \Big) \\
& + \beta^t \frac{\Lambda_t}{P_t} \left[B_{t-1} + W_t N_t + b_t\left(1-N_t\right) + P_t R_{kt} \upsilon_t X_{t-1} - P_t a\left(\upsilon_t\right) X_{t-1} - P_t C_t - P_t I_t - e^{-i_t} B_t + D_t\right] \\
& + \beta^t \Lambda_t Q_t \left[\left(1-\delta_k\right) X_{t-1} + \mu_t \left(1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - z\right)^2\right) I_t - X_t\right] \\
& + \beta^t \Lambda_t \Upsilon_t \left[(1-s_{t-1})N_{t-1} + p_t - p_t(1-s_{t-1})N_{t-1} - N_t\right] \Bigg)
\end{aligned}
$$

where it is convenient to express the Lagrange multipliers Q_t and Υ_t in time t consumption units by scaling them by the Lagrange multiplier Λ_t.

This problem implies the first-order conditions for consumption C_t

$$\Lambda_t = \frac{\omega_t}{C_t - h_c C_{t-1}} - \beta h_c E_t \frac{\omega_{t+1}}{C_{t+1} - h_c C_t} \tag{16}$$

for hours worked N_t

$$\Upsilon_t = \frac{W_t - b_t}{P_t} - MRS_t + E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Upsilon_{t+1}(1-s_t)(1-p_{t+1}) \tag{17}$$

where MRS denotes the marginal rate of substitution between consumption and labor,

$$MRS_t = \frac{1}{\Lambda_t} \frac{\omega_t \chi A_t}{H_t - h_h H_{t-1}} - \frac{\beta h_h}{\Lambda_t} E_t \frac{\omega_{t+1} \chi A_t}{H_{t+1} - h_h H_t}$$

for bond holdings B_t

$$\Lambda_t = e^{i_t} \beta E_t \left(\Lambda_{t+1} \frac{P_t}{P_{t+1}}\right) \tag{18}$$

for capital utilization υ_t

$$(\phi_u - \phi_{uu}) + \phi_{uu} \upsilon_t = R_{kt} \tag{19}$$

for investment I_t

$$1 = Q_t \mu_t \left[1 - \frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}} - z\right)^2 - \phi_I \left(\frac{I_t}{I_{t-1}} - z\right)\left(\frac{I_t}{I_{t-1}}\right)\right] + \beta E_t Q_{t+1} \mu_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \phi_I \left(\frac{I_{t+1}}{I_t} - z\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{20}$$

where Q_t is the marginal Tobin's Q, and for capital X_t

$$Q_t = \beta E_t \left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left[\left(1-\delta_k\right) Q_{t+1} + R_{kt+1} \upsilon_{t+1} - a\left(\upsilon_{t+1}\right)\right]\right\} \tag{21}$$

The budget constraint holds with equality as

$$\frac{B_{t-1}+W_tN_t+b_t\left(1-N_t\right)+P_tR_{kt}\upsilon_tX_{t-1}+D_t}{P_t}-a\left(\upsilon_t\right)\overline{K}_{t-1}=C_t+I_t+e^{-i_t}\frac{B_t}{P_t} \tag{22}$$

where Λ_t denotes the Lagrange multiplier on (15), and the law of motion of capital implies

$$X_t=\left(1-\delta_k\right)X_{t-1}+\mu_t\left[1-\frac{\phi_I}{2}\left(\frac{I_t}{I_{t-1}}-z\right)^2\right]I_t \tag{23}$$

2.2 Finished-goods-producing firms

At each date t, identical finished-goods-producing firms use $Y_t\left(j\right)$ units of each intermediate good $j\in(0,1)$, purchased at the nominal price $P_t(j)$, to manufacture Y_t units of the finished good according to the constant-returns-to-scale technology

$$\left[\int_0^1 Y_t\left(j\right)^{(\theta_t-1)/\theta_t}dj\right]^{\theta_t/(\theta_t-1)}\geq Y_t, \tag{24}$$

where θ_t translates into a random shock to the markup of price over marginal cost. This markup shock follows the autoregressive process

$$\ln\theta_t=\left(1-\rho_\theta\right)\ln\theta+\rho_\theta\ln\theta_{t-1}+\varepsilon_{\theta t}, \tag{25}$$

where $\varepsilon_{\theta t}\sim Niid(0,\sigma_\theta^2)$.

Intermediate good j sells at the nominal price $P_t\left(j\right)$, while the finished good sells at the nominal price P_t. Given these prices, the finished-goods-producing firm chooses Y_t and $Y_t(j)$ for all $j\in(0,1)$ to maximize its profits

$$P_tY_t-\int_0^1 P_t(j)Y_t(j)dj, \tag{26}$$

subject to the constraint (17) at each t. The first-order conditions for this problem are (17) with equality and

$$Y_t(j)=\left(\frac{P_t(j)}{P_t}\right)^{-\theta_t}Y_t \tag{27}$$

for all $j\in(0,1)$ and t.

Competition in the market for the finished good drives the finished goods-producing firm's profits to zero in equilibrium. This zero-profit condition determines P_t as

$$P_t=\left(\int_0^1 P_t(j)^{1-\theta_t}dj\right)^{1/(1-\theta_t)} \tag{28}$$

for all t.

2.3 Intermediate-goods-producing firms

Each intermediate-goods-producing firm $j \in (0,1)$ enters in period t with a stock of $N_{t-1}(j)$ employees carried over from the previous period. At the end of period $t-1$, $s_{t-1}N_{t-1}(j)$ jobs are destroyed. The pool of workers $s_{t-1}N_{t-1}$ who have lost their job at the end of period $t-1$ starts searching in period t and can find a match in period t. The number of employees at firm j evolves according to

$$N_t(j) = (1 - s_{t-1})\, N_{t-1}(j) + m_t(j), \tag{29}$$

where $m_t(j)$ denotes the new matches in firm j in period t, and is given by

$$m_t(j) = q_t V_t(j), \tag{30}$$

where $V_t(j)$ denotes the vacancies posted by firm j in period t and q_t is the aggregate probability of filling a vacancy in period t. Workers hired in period t take part in period t production.

Aggregate employment $N_t = \int_0^1 N_t(j)dj$ evolves over time according to (7), or

$$N_t = (1 - s_{t-1})\, N_{t-1} + m_t, \tag{31}$$

where $m_t = \int_0^1 m_t\,(j)\,dj$ denotes aggregate matches in period t. Similarly, the aggregate vacancies are equal to $V_t = \int_0^1 V_t(j)dj$. The pool of job seekers in period t, denoted by S_t, is given by (5). The matching process is described by the following aggregate CRS function

$$m_t = \zeta_t S_t^{\sigma} V_t^{1-\sigma}, \tag{32}$$

where ζ_t is an exogenous disturbance to the efficiency of the matching technology that follows the exogenous stationary stochastic process

$$\ln \zeta_t = (1 - \rho_\zeta) \ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_{\zeta t}, \tag{33}$$

where $\varepsilon_{\zeta t} \sim Niid(0, \sigma_\zeta^2)$.

The job-filling probability q_t in period t is given by

$$q_t = \frac{m_t}{V_t} = \zeta_t \Theta_t^{-\sigma}, \tag{34}$$

where Θ denotes the tightness of the labor market $\Theta_t = V_t/S_t$. The probability p_t that a job seeker finds a job is

$$p_t = \frac{m_t}{S_t} = q_t \Theta_t = \zeta_t \Theta_t^{1-\sigma}. \tag{35}$$

At each date t, each intermediate-goods-producing firm combines $N_t(j)$ homogeneous employees with $K_t(j)$ units of capital to produce $Y_t(j)$ units of intermediate good j according to the CRS technology

$$Y_t(j) = [A_t N_t(j)]^{1-\alpha} K_t(j)^{\alpha} \tag{36}$$

where A_t is an aggregate labor-augmenting technology shock whose growth rate, $z_t \equiv A_t/A_{t-1}$, follows the exogenous stationary stochastic process

$$\ln z_t = (1-\rho_z)\ln z + \rho_z \ln z_{t-1} + \varepsilon_{zt}, \tag{37}$$

where $\varepsilon_{zt} \sim Niid(0, \sigma_z^2)$.

The firm faces costs to hiring workers. As in Yashiv (2000) and Furlanetto and Groshenny (2016), hiring costs are a convex function of the linear combination of the number of vacancies and the number of hires. Hiring costs are measured in terms of aggregate output, which implies they are compatible with balanced growth, and given by

$$\frac{\tilde{\kappa}}{2}\left(\frac{\phi_V V_t(j) + (1-\phi_V)\, q_t V_t(j)}{N_t(j)}\right)^2 Y_t, \tag{38}$$

where the parameters $\tilde{\kappa}$ and ϕ_V govern the magnitude of these costs.

Intermediate goods substitute imperfectly for one another in the production function of finished-goods-producing firms. Hence, each intermediate-goods-producing firm $j \in (0,1)$ sells its output $Y_t(j)$ in a monopolistically competitive market, setting $P_t(j)$, the price of its own product, with the commitment to satisfy the demand for good j at that price. Firms take the nominal wage as given when maximizing the discounted value of expected future profits.

Each intermediate-goods-producing firm faces costs of adjusting its nominal price between periods (Rotemberg, 1982), measured in terms of the finished good and given by

$$\frac{\phi_P}{2}\left(\frac{P_t(j)}{e^{\varsigma\pi_{t-1}+(1-\varsigma)\pi_t^*} P_{t-1}(j)} - 1\right)^2 Y_t. \tag{39}$$

where $\phi_P \geq 0$ governs the magnitude of the price-adjustment cost. $\pi_t = \ln(P_t/P_{t-1})$ denotes the inflation rate in period t. π_t^* denotes a time-varying target for the inflation rate that coincides with the central bank's target in the monetary policy rule, and with steady-state value $\pi^* > 0$. The parameter $0 \leq \varsigma \leq 1$ governs the importance of backward-looking behavior in price setting.

Firms face quadratic wage-adjustment costs that are proportional to the size of their workforce and measured in terms of the finished good

$$\frac{\phi_W}{2}\left(\frac{W_t(j)}{z e^{\varrho\pi_{t-1}+(1-\varrho)\pi_t^*} W_{t-1}(j)} - 1\right)^2 N_t(j) Y_t, \tag{40}$$

where $\phi_W \geq 0$ governs the magnitude of the wage-adjustment cost. The parameter $0 \leq \varrho \leq 1$ governs the importance of backward-looking behavior in wage setting.

Adjustment costs to hiring and to price and wage changes make the problem of the intermediate-goods-producing firm dynamic. The firm chooses $K_t(j)$, $N_t(j)$, $V_t(j)$, $Y_t(j)$, and $P_t(j)$ at each date t to maximize the expected discounted value of profits,

$$E_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t+s} \frac{D_{t+s}(j)}{P_{t+s}} \tag{41}$$

where $\beta^t \Lambda_t / P_t$ measures the marginal utility to the families, who own the firms, of an additional dollar of profits during period t and where

$$D_t(j) = P_t(j)Y_t(j) - \left(1 - \frac{1}{\theta}\right)(W_t(j)N_t(j) + P_t R_{kt} K_t(j)) - \left(1 - \frac{1}{\theta}\right)\frac{\tilde{\kappa}}{2}\left(\frac{\phi_V V_t(j) + (1-\phi_V)\, q_t V_t(j)}{N_t(j)}\right)^2 P_t Y_t$$
$$- \frac{\phi_P}{2}\left(\frac{P_t(j)}{e^{\varsigma \pi_{t-1} + (1-\varsigma)\pi_t^*} P_{t-1}(j)} - 1\right)^2 P_t Y_t - \frac{\phi_W}{2}\left(\frac{W_t(j)}{z e^{\varrho \pi_{t-1} + (1-\varrho)\pi_t^*} W_{t-1}(j)} - 1\right)^2 N_t(j) P_t Y_t - P_t T_t,$$

subject to the constraints

$$Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\theta_t} Y_t \tag{42}$$

$$Y_t(j) \leq [A_t N_t(j)]^{1-\alpha} K_t(j)^{\alpha} \tag{43}$$

$$N_t(j) = (1 - s_{t-1}) N_{t-1}(j) + q_t V_t(j) \tag{44}$$

and taking the job-filling rate q_t as given.

The firm receives from the government an employment, hiring, and capital rental subsidy equal to $1/\theta$ levied by the government in a lump-sum fashion on firms via transfers T_t. This subsidy offsets any steady-state distortions due to monopolistic competition. For convenience, we define the scaled hiring cost parameter $\kappa = (1 - 1/\theta)\tilde{\kappa}$.

This problem is equivalent to the one of choosing $K_t(j)$, $N_t(j)$, $V_t(j)$ and $P_t(j)$ to maximize (41), where

$$\frac{D_t(j)}{P_t} = \left(\frac{P_t(j)}{P_t}\right)^{1-\theta_t} Y_t - \left(1 - \frac{1}{\theta}\right)\frac{W_t(j)N_t(j) + P_t R_{kt} K_t(j)}{P_t} - \frac{\kappa}{2}\left(\frac{\phi_V V_t(j) + (1-\phi_V)\, q_t V_t(j)}{N_t(j)}\right)^2 Y_t$$
$$- \frac{\phi_P}{2}\left(\frac{P_t(j)}{e^{\varsigma \pi_{t-1} + (1-\varsigma)\pi_t^*} P_{t-1}(j)} - 1\right)^2 Y_t - \frac{\phi_W}{2}\left(\frac{W_t(j)}{z e^{\varrho \pi_{t-1} + (1-\varrho)\pi_t^*} W_{t-1}(j)} - 1\right)^2 N_t(j) Y_t - T_t,$$

subject to the constraints

$$\left[\frac{P_t(j)}{P_t}\right]^{-\theta_t} Y_t \leq [A_t N_t(j)]^{1-\alpha} K_t(j)^{\alpha} \tag{45}$$

$$N_t(j) = (1 - s_{t-1}) N_{t-1}(j) + q_t V_t(j), \tag{46}$$

for all t.

We can set up a Lagrangian for this problem as

$$E_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \Bigg(\left(\frac{P_t(j)}{P_t}\right)^{1-\theta_t} Y_t - \left(1 - \frac{1}{\theta}\right) \frac{W_t(j)N_t(j) + P_t R_{kt} K_t(j)}{P_t} - \frac{\kappa}{2} \left(\frac{\phi_V V_t(j) + (1-\phi_V)\, q_t V_t(j)}{N_t(j)}\right)^2 Y_t$$
$$- \frac{\phi_P}{2} \left(\frac{P_t(j)}{e^{\varsigma \pi_{t-1} + (1-\varsigma)\pi_t^*} P_{t-1}(j)} - 1\right)^2 Y_t - \frac{\phi_W}{2} \left(\frac{W_t(j)}{z e^{\varrho \pi_{t-1} + (1-\varrho)\pi_t^*} W_{t-1}(j)} - 1\right)^2 N_t(j) Y_t - T_t$$
$$+ \Psi_t(j) \left[(1 - s_{t-1}) N_{t-1}(j) + q_t V_t(j) - N_t(j)\right] + \xi_t(j) \left[\left[A_t N_t(j)\right]^{1-\alpha} K_t(j)^{\alpha} - \left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} Y_t \right] \Bigg)$$

The multiplier $\Psi_t(j)$ measures the value to firm j in time-t consumption units of an additional job in period t. The multiplier $\xi_t(j)$ measures the value to firm j in time-t consumption units of an additional unit of output in period t, or the jth firm's real marginal cost in period t.

We derive the first-order conditions for this problem for demand of capital services $K_t(j)$

$$\left(1 - \frac{1}{\theta}\right) R_{kt} = \xi_t(j) \alpha \left[A_t N_t(j)\right]^{1-\alpha} K_t(j)^{\alpha - 1} \tag{47}$$

for labor demand $N_t(j)$

$$\Psi_t(j) = \xi_t(j) (1-\alpha) \frac{Y_t(j)}{N_t(j)} - \left(1 - \frac{1}{\theta}\right) \frac{W_t(j)}{P_t} - \frac{\phi_W}{2} \left(\frac{W_t(j)}{z e^{\varrho \pi_{t-1} + (1-\varrho)\pi_t^*} W_{t-1}(j)} - 1\right)^2 Y_t$$
$$+ \frac{\kappa}{N_t(j)} \left[\frac{\phi_V V_t(j) + (1-\phi_V)\, q_t V_t(j)}{N_t(j)}\right]^2 Y_t + \beta (1 - s_t) \frac{\Lambda_{t+1}}{\Lambda_t} \Psi_{t+1}(j) \tag{48}$$

which equates the costs and benefits of hiring an additional worker, of vacancy posting $V_t(j)$

$$\Psi_t(j) = \left(\frac{\phi_V + (1-\phi_V)\, q_t}{N_t(j)}\right)^2 \frac{\kappa Y_t V_t(j)}{q_t} \tag{49}$$

which is the analogue of the familiar free-entry condition, and for newly set prices $P_t(j)$

$$(1-\theta_t) \left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} = \phi_P \left(\frac{P_t(j)}{e^{\varsigma \pi_{t-1} + (1-\varsigma)\pi_t^*} P_{t-1}(j)} - 1\right) \left(\frac{P_t}{e^{\varsigma \pi_{t-1} + (1-\varsigma)\pi_t^*} P_{t-1}(j)}\right) - \theta_t \xi_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-(1+\theta_t)}$$
$$- \beta \phi_P E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{P_{t+1}(j)}{e^{\varsigma \pi_t + (1-\varsigma)\pi_{t+1}^*} P_t(j)} - 1\right) \left(\frac{P_{t+1}(j)}{e^{\varsigma \pi_t + (1-\varsigma)\pi_{t+1}^*} P_t(j)}\right) \frac{Y_{t+1}}{Y_t} \frac{P_t}{P_t(j)} \right] \tag{50}$$

The law of motion of employment is

$$N_t(j) = (1 - s_{t-1}) N_{t-1}(j) + q_t V_t(j) \tag{51}$$

and the production function combined with the product demand curve implies

$$A_t^{1-\alpha} K_t(j)^{\alpha} N_t(j)^{1-\alpha} = \left(\frac{P_t(j)}{P_t}\right)^{-\theta_t} Y_t \tag{52}$$

2.4 Wage setting

Each period, intermediate-goods-producing firm j bargains with each of its employees over the nominal wage $W_t(j)$ to split the match surplus according to Nash bargaining.

The Nash-bargained flow wage solves

$$W_t = \arg\max \ \Upsilon_t^{\eta_t} \Psi_t^{1-\eta_t} \tag{53}$$

where Υ_t denotes the surplus of the worker and Ψ_t denotes the surplus of the firm. Both Υ_t and Ψ_t are expressed in time-t consumption units. η_t denotes the worker's bargaining power, which evolves exogenously according to

$$\ln \eta_t = (1 - \rho_\eta) \ln \eta + \rho_\eta \ln \eta_{t-1} + \varepsilon_{\eta t}, \tag{54}$$

where $\varepsilon_{\eta t} \sim Niid(0, \sigma_\eta^2)$.

Here the family's surplus from employing an additional worker at firm j, expressed in time-t consumption goods, is given by the Lagrange multiplier Υ_t in (17), namely,

$$\Upsilon_t = \frac{W_t}{P_t} - \frac{b_t}{P_t} - MRS_t + \beta E_t (1 - s_t)\,(1 - p_{t+1})\,\frac{\Lambda_{t+1}}{\Lambda_t} \Upsilon_{t+1} \tag{55}$$

and the employer's surplus from the match is given by $\Psi_t(j)$ in (48), namely,

$$\begin{aligned}\Psi_t(j) = {} & \xi_t(j)\,(1-\alpha)\,\frac{Y_t(j)}{N_t(j)} - \left(1 - \frac{1}{\theta}\right)\frac{W_t(j)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(j)}{z e^{\varrho \pi_{t-1} + (1-\varrho)\pi_t^*} W_{t-1}(j)} - 1\right)^2 Y_t \\ & + \frac{\kappa}{N_t(j)}\left[\frac{\phi_V V_t(j) + (1-\phi_V)\, q_t V_t(j)}{N_t(j)}\right]^2 Y_t + \beta (1 - s_t)\frac{\Lambda_{t+1}}{\Lambda_t}\Psi_{t+1}(j)\end{aligned} \tag{56}$$

Nash bargaining over the nominal wage yields the following first-order condition

$$\eta_t \Psi_t(j) \frac{\partial \Upsilon_t(j)}{\partial W_t(j)} = -\,(1 - \eta_t)\,\Upsilon_t(j)\frac{\partial \Psi_t(j)}{\partial W_t(j)}, \tag{57}$$

where

$$\frac{\partial \Upsilon_t(j)}{\partial W_t(j)} = \frac{1}{P_t}, \tag{58}$$

$$-\frac{\partial \Psi_t(j)}{\partial W_t(j)} = \left(1-\frac{1}{\theta}\right)\frac{1}{P_t} + \phi_W Y_t \left(\frac{1}{ze^{\varrho\pi_{t-1}+(1-\varrho)\pi_t^*}W_{t-1}(j)}\right)\left(\frac{W_t(j)}{ze^{\varrho\pi_{t-1}+(1-\varrho)\pi_t^*}W_{t-1}(j)}-1\right) \tag{59}$$

$$-\beta(1-s_t)\phi_W E_t\left[\frac{\Lambda_{t+1}Y_{t+1}}{\Lambda_t W_t(j)}\left(\frac{W_{t+1}(j)}{ze^{\varrho\pi_t+(1-\varrho)\pi_{t+1}^*}W_t(j)}\right)\left(\frac{W_{t+1}(j)}{ze^{\varrho\pi_t+(1-\varrho)\pi_{t+1}^*}W_t(j)}-1\right)\right]$$

or

$$\Upsilon_t(j) = \Gamma_{jt}\Psi_t(j), \tag{60}$$

where

$$\Gamma_{jt} \equiv -\frac{\eta_t}{1-\eta_t}\frac{\partial \Upsilon_t(j)/\partial W_t(j)}{\partial \Psi_t(j)/\partial W_t(j)} \tag{61}$$

Substituting the expressions of the two partial derivatives into the first-order condition, we obtain

$$\begin{aligned}
&\Gamma_{jt}\left[\xi_t(j)\,(1-\alpha)\,\frac{Y_t(j)}{N_t(j)} - \left(1-\frac{1}{\theta}\right)\frac{W_t(j)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(j)}{ze^{\varrho\pi_{t-1}+(1-\varrho)\pi_t^*}W_{t-1}(j)}-1\right)^2 Y_t\right]\\
&+\Gamma_{jt}\left[\frac{\kappa}{N_t(j)}\left(\frac{\phi_V V_t(j)+(1-\phi_V)\,q_t V_t(j)}{N_t(j)}\right)^2 Y_t\right]\\
&+\Gamma_{jt}\beta(1-s_t)E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\Psi_{t+1}(j)\right] = \frac{W_t(j)}{P_t} - \frac{b_t}{P_t} - MRS_t + \beta(1-s_t)E_t\left[(1-p_{t+1})\,\frac{\Lambda_{t+1}}{\Lambda_t}\Upsilon_{t+1}(j)\right]
\end{aligned}$$

Using the fact that $\Upsilon_{t+1}(j) = \Gamma_{jt+1}\Psi_{t+1}(j)$ in the above equation, we obtain

$$\begin{aligned}
&\Gamma_{jt}\left[\xi_t(j)\,(1-\alpha)\,\frac{Y_t(j)}{N_t(j)} - \left(1-\frac{1}{\theta}\right)\frac{W_t(j)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(j)}{ze^{\varrho\pi_{t-1}+(1-\varrho)\pi_t^*}W_{t-1}(j)}-1\right)^2 Y_t\right]\\
&+\Gamma_{jt}\left[\frac{\kappa}{N_t(j)}\left(\frac{\phi_V V_t(j)+(1-\phi_V)\,q_t V_t(j)}{N_t(j)}\right)^2 Y_t\right]\\
&+\Gamma_{jt}\beta(1-s_t)E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\Psi_{t+1}(j)\right] = \frac{W_t(j)}{P_t} - \frac{b_t}{P_t} - MRS_t + \beta(1-s_t)E_t\left[(1-p_{t+1})\,\frac{\Lambda_{t+1}}{\Lambda_t}\Gamma_{jt+1}\Psi_{t+1}(j)\right]
\end{aligned}$$

Now, let us recall the definition of the firm's surplus

$$\Psi_t(j) = \left(\frac{\phi_V+(1-\phi_V)\,q_t}{N_t(j)}\right)^2\frac{\kappa Y_t V_t(j)}{q_t} \tag{62}$$

Using this expression of $\Psi_{t+1}(j)$, the real-wage equation becomes

$$
\begin{aligned}
&\frac{W_t(j)}{P_t} - \Gamma_{jt}\left[\xi_t(j)(1-\alpha)\frac{Y_t(j)}{N_t(j)} - \left(1-\frac{1}{\theta}\right)\frac{W_t(j)}{P_t} - \frac{\phi_W}{2}\left(\frac{W_t(j)}{ze^{\varrho\pi_{t-1}+(1-\varrho)\pi_t^*}W_{t-1}(j)}-1\right)^2 Y_t\right] \\
&-\Gamma_{jt}\left[\frac{\kappa}{N_t(j)}\left(\frac{\phi_V V_t(j) + (1-\phi_V)\,q_t V_t(j)}{N_t(j)}\right)^2 Y_t\right] \\
&=\Gamma_{jt}\beta(1-s_t)E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\phi_V + (1-\phi_V)\,q_{t+1}}{N_{t+1}(j)}\right)^2\frac{\kappa Y_{t+1}V_{t+1}(j)}{q_{t+1}}\right] \\
&+\frac{b_t}{P_t} + MRS_t - \beta(1-s_t)E_t\left[(1-p_{t+1})\frac{\Lambda_{t+1}}{\Lambda_t}\Gamma_{jt+1}\left(\frac{\phi_V + (1-\phi_V)\,q_{t+1}}{N_{t+1}(j)}\right)^2\frac{\kappa Y_{t+1}V_{t+1}(j)}{q_{t+1}}\right]
\end{aligned}
$$

Finally, the equation governing the dynamics of the real wage at firm j is given by

$$
\begin{aligned}
\frac{W_t(j)}{P_t} = {}& \frac{\Gamma_{jt}}{1+\left(1-\frac{1}{\theta}\right)\Gamma_{jt}}\left[\xi_t(j)(1-\alpha)\frac{Y_t(j)}{N_t(j)} - \frac{\phi_W}{2}\left(\frac{W_t(j)}{ze^{\varrho\pi_{t-1}+(1-\varrho)\pi_t^*}W_{t-1}(j)}-1\right)^2 Y_t\right] \\
&+\frac{\Gamma_{jt}}{1+\left(1-\frac{1}{\theta}\right)\Gamma_{jt}}\frac{\kappa}{N_t(j)}\left(\frac{\phi_V V_t(j) + (1-\phi_V)\,q_t V_t(j)}{N_t(j)}\right)^2 Y_t \\
&+\frac{\Gamma_{jt}}{1+\left(1-\frac{1}{\theta}\right)\Gamma_{jt}}\beta(1-s_t)E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\phi_V + (1-\phi_V)\,q_{t+1}}{N_{t+1}(j)}\right)^2\frac{\kappa Y_{t+1}V_{t+1}(j)}{q_{t+1}} \\
&+\frac{1}{1+\left(1-\frac{1}{\theta}\right)\Gamma_{jt}}\left[\frac{b_t}{P_t} + MRS_t - \beta(1-s_t)E_t\Gamma_{jt+1}(1-p_{t+1})\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{\phi_V + (1-\phi_V)\,q_{t+1}}{N_{t+1}(j)}\right)^2\frac{\kappa Y_{t+1}V_{t+1}(j)}{q_{t+1}}\right]
\end{aligned}
\tag{63}
$$

2.5 Government

The central bank adjusts the short-term nominal interest rate i_t according to a Taylor-type rule

$$
\begin{aligned}
i_t &= \rho_r i_{t-1} + (1-\rho_r)\left[r + \pi_t^* + \rho_\pi(\pi_t - \pi_t^*) + \rho_y \Delta y_t\right] + \varepsilon_{mt} \\
&= \rho_r i_{t-1} + (1-\rho_r)\left[r + \pi_t^* + \rho_\pi(\pi_t - \pi_t^*) + \rho_y(\Delta y_t^{obs} - g_t^*)\right] + \varepsilon_{mt}
\end{aligned}
$$

where $\Delta y_t = \ln(Y_t/Y_{t-1})$ denotes the growth rate of output and Δy_t^{obs} the observed growth rate of output, which differs from the growth rate Δy_t by a trend component g_t^*. π_t^* and g_t^* denote the time-varying inflation target and a slow-moving exogenous component in output, respectively, that capture components outside of the model such as demographic forces. The degree of interest-rate smoothing ρ_r and the reaction coefficients ρ_π and ρ_y are positive. The monetary policy shock is distributed as $\varepsilon_{mt} \sim Niid(0, \sigma_m^2)$, while the inflation target and slow-moving growth shocks π_t^* and g_t^* follow AR(1) processes

$$
\pi_t^* = \rho_* \pi_{t-1}^* + \varepsilon_{\pi t}^* \tag{64}
$$

$$
g_t^* = \rho_* g_{t-1}^* + \varepsilon_{gt}^* \tag{65}
$$

where $\varepsilon_{\pi t}^{*} \sim Niid(0, \sigma_{\pi*}^{2})$ and $\varepsilon_{gt}^{*} \sim Niid(0, \sigma_{g*}^{2})$.

The government's budget constraint is of the form

$$P_t G_t + b_t (1 - N_t) = e^{-i_t} B_t - B_{t-1}, \tag{66}$$

In the background, note that the subsidies for intermediate-goods-producing firms are exactly offset by their lump-sum taxation; so those terms do not show up in the government's budget constraint. Public spending is an exogenous time-varying fraction of GDP,

$$G_t = \gamma_t Y_t, \tag{67}$$

where γ_t evolves according to

$$\ln \gamma_t = (1 - \rho_\gamma) \ln \gamma + \rho_\gamma \ln \gamma_{t-1} + \varepsilon_{\gamma t}, \tag{68}$$

with $\varepsilon_{\gamma t} \sim Niid(0, \sigma_\gamma^2)$.

2.6 Aggregate resource constraint

In a symmetric equilibrium, all intermediate-goods-producing firms make identical decisions, so that $Y_t(j) = Y_t$, $P_t(i) = P_t$, $N_t(j) = N_t$, $V_t(j) = V_t$, $K_t(j) = K_t$ for all $j \in (0,1)$ and t. Moreover, workers are homogeneous and all workers at a given firm j receive the same nominal wage $W_t(j)$; hence $W_t(j) = W_t$ for all $j \in (0,1)$ and t. The aggregate resource constraint is obtained by aggregating the household budget constraint over all intermediate sectors $j \in (0,1)$,

$$\begin{aligned} &\left[1 - \frac{\kappa}{2} M_t^2 - \frac{\phi_P}{2}\left(\frac{P_t}{e^{\varsigma \pi_{t-1} + (1-\varsigma)\pi_t^*} P_{t-1}} - 1\right)^2 - \frac{\phi_W}{2}\left(\frac{W_t}{z e^{\varrho \pi_{t-1} + (1-\varrho)\pi_t^*} W_{t-1}} - 1\right)^2 N_t\right] Y_t \\ &= C_t + I_t + G_t + \left[\phi_u (\upsilon_t - 1) + \frac{\phi_{uu}}{2} (\upsilon_t - 1)^2\right] X_{t-1} \end{aligned} \tag{69}$$

where

$$M_t = \frac{\phi_V V_t + (1 - \phi_V) m_t}{N_t}$$

2.7 Long-run trends

In the data, there are important slow-moving components in inflation, output growth, unemployment, and the real rate. While we choose to attribute such slow-moving components in inflation to the monetary policy stance, through an exogenous choice for an inflation target, we instead attribute the slow-moving components in output growth, unemployment, and the real rate to factors outside of the model, for example, those related to demographic changes or slow-moving changes in government policies or the overall rate of innovation.

Our model, even in the two-regime version described below, will have a unique steady state to which the (detrended) economy will revert. Such a steady state reflects historical mean values and is typically not a relevant point to anchor forecasts at a 5- to 12-year horizon. It is therefore important to include components that slow down such a mean reversion of the model. In fact, by including such highly persistent components, this mean reversion can be made arbitrarily slow—in practice, it can be made to take several decades—with the slow-moving components acting as attraction points at the forecast horizons relevant for policymaking. Namely, we use long-run survey expectations about the inflation rate, the unemployment rate, the output growth rate, and the real interest rate to discipline such slow-moving components, and hence pin down our 5- to 12-year forecasts to values that are more realistic than the steady state of the model. The inclusion of long-range forecasts to improve the forecasting performance of dynamic models has been documented, for example, by Tallman and Zaman (2020) in a BVAR setup, while Del Negro and Schorfheide (2013) and Del Negro, Giannoni, and Schorfheide (2015) incorporate long-run survey expectations of inflation and output growth in DSGE setups.

In practice, we therefore model a link between long-run survey forecasts and the model variables as follows:

- The 5y/5y inflation rate forecasts from the Survey of Professional Forecasters (SPF) available from the website of the Federal Reserve Bank of Philadelphia,[1] denoted as $\pi^{obs}_{lr,t}$, are linked to the variables in the model as

$$\pi^{obs}_{lr,t} = \sum_{j=20}^{40} E_t \pi_{t+j} \tag{70}$$

  We consider the SPF long-run forecast rather than the similar Blue Chip version because of its more common usage in the BVAR and DSGE literature (e.g., Del Negro et al., 2015; Tallman and Zaman, 2020).

  The corresponding shock to account for such observations is the slow-moving inflation target

$$\pi^*_t = \rho_* \pi^*_{t-1} + \varepsilon^*_{\pi t} \tag{71}$$

  which will turn out to play the most important role in accounting for movements in the long-run inflation expectations of the model.

- The 7y/5y real rate forecasts from Blue Chip,[2] denoted as $r^{obs}_{lr,t}$, are linked to the variables in

[1] The SPF is a quarterly survey of macroeconomic forecasts in the US from several individual economists collected since 1968 and covering short- and long-run forecasts.

[2] The Blue Chip long-range consensus US economic projections are a quarterly survey collecting macroeconomic forecasts related to the US economy by polling several business economists.

the model as

$$r_{lr,t}^{obs} = \sum_{j=28}^{48} E_t(i_{t+j}^{obs} - \pi_{t+j+1}) \tag{72}$$

with $i_t^{obs} = i_t + r_t^*$, where the measurement error r_t^* is the exogenous process

$$r_t^* = \rho_* r_{t-1}^* + \varepsilon_{rt}^* \tag{73}$$

where $\varepsilon_{rt}^* \sim Niid(0, \sigma_{r*}^2)$.

- The 7y/5y GDP growth forecasts from Blue Chip, denoted as $\Delta y_{lr,t}^{obs}$, are linked to the variables in the model as

$$\Delta y_{lr,t}^{obs} = \sum_{j=28}^{48} E_t \Delta y_{t+j}^{obs} \tag{74}$$

with the observed growth rates of output $\Delta y_t^{obs} = \Delta y_t + g_t^*$, consumption $\Delta c_t^{obs} = \Delta c_t + g_t^*$, investment $\Delta inv_t^{obs} = \Delta inv_t + g_t^*$, and real wages $\Delta w_t^{obs} = \Delta w_t + g_t^*$, where the measurement error g_t^* is the exogenous process

$$g_t^* = \rho_* g_{t-1}^* + \varepsilon_{gt}^* \tag{75}$$

where $\varepsilon_{gt}^* \sim Niid(0, \sigma_{g*}^2)$.

Here note that, even though long-run survey expectations of output, consumption, investment and real wages need not coincide in practice, the assumption of a common trend is a parsimonious one and can be motivated, for example, as a trend component in technology that would therefore be common to all real growth rates. Still, an extension that allows for multiple trends in growth rates informed by the respective long-range forecasts could be easily accommodated in our methodology.

- The 7y/5y unemployment rate forecasts from Blue Chip, denoted as $U_{lr,t}^{obs}$, are linked to the variables in the model as

$$U_{lr,t}^{obs} = \sum_{j=28}^{48} E_t U_{t+j}^{obs} \tag{76}$$

with $U_t^{obs} = U_t + u_t^*$, where the measurement error u_t^* is the exogenous process

$$u_t^* = \rho_* u_{t-1}^* + \varepsilon_{ut}^* \tag{77}$$

where $\varepsilon_{ut}^* \sim Niid(0, \sigma_{u*}^2)$.

2.8 Symmetric equilibrium and the balanced-growth path

Since all firms face the same optimization problem, we will look at a symmetric equilibrium such that $Y_t(j) = Y_t$, $P_t(j) = P_t$, $N_t(j) = N_t$, $V_t(j) = V_t$, $K_t(j) = K_t$, $W_t(j) = W_t$ for all $j \in (0, 1)$ and t.

Moreover, output, consumption, investment, capital, and the real wage share the stochastic trend induced by the unit root process of neutral technological progress. We therefore rewrite the model in terms of stationary variables. In the absence of shocks, such a detrended economy converges to a steady-state growth path in which all stationary variables are constant.

We then log-linearize this transformed model economy around its steady state. This approximate model can then be solved using standard methods.

2.9 Natural efficient allocation

We can compute a counterfactual competitive equilibrium that removes the frictions due to the monopolistic competition in the goods market, that removes the nominal rigidities in the goods and labor markets, and that removes inefficiencies in wage setting. Namely, we define an alternative version of our economy in which we set nominal price- and wage-adjustment costs to zero ($\phi_P = \phi_W = 0$); in which we neutralize the markups, including their time variation, due to monopolistic competition ($\sigma_\theta = 0$); and in which we neutralize inefficiencies in the labor market due to Nash bargaining ($\sigma_\eta = 0$).

In the background, note that the presence in the steady state of the subsidy $\tau_f = 1/\theta$ for firms removes the steady-state effect of monopolistic competition, while the Hosios (1990) condition $\sigma = \eta$ removes steady-state inefficiencies in wage setting due to the bargaining protocol. Therefore, by removing the shocks that generate any remaining departures from competition in the goods market (the shocks to the markup θ_t) and from efficient wage setting (the shocks to the workers' bargaining power, so that it equals the curvature of the matching function at all dates t) we remove any distortions caused by these two frictions.

In this sense, this economy defines a flexible-price constrained-efficient allocation. We denote the associated levels of the variables by an n subscript. For example, Y_{nt} is the natural output, R_{nt} the natural real interest rate, and so on.

Accordingly, since these are benchmark variables often discussed in the literature, we can define the output gap as the distance of output from this benchmark level,

$$\tilde{y}_t = y_t - y_{nt}$$

and the natural 10-year real rate, or natural r-star, r^*_{nt}, as

$$r^*_{nt} = \frac{1}{40} \sum_{j=1}^{40} E_t(\tilde{r}_{nt+j})$$

where $\tilde{r}_{nt} = r_{nt} + r_t^*$ is the natural real rate that includes the real rate trend. Note that we include in this variable the trend r_t^*, so that it shares the same long-run expectations as the real rate of the model with frictions; consequently, the real rate gap $i_t^{obs} - E_t\pi_{t+1} - \tilde{r}_{nt}$ is independent of r_t^*.

3 Model: Two-Regime Version

This section extends the model described in the previous section to incorporate how the model's dynamics change when the zero lower bound on the interest rate set by the central bank is binding. Notably, the zero lower bound has been a constraint in practice on monetary policy during the Great Recession and during the COVID period.

To capture such nonlinearities, we develop here a piecewise linear solution technique that combines two linearizations corresponding to two monetary policy regimes: one with an unconstrained interest rate and another with the interest rate constrained at the zero lower bound. Unlike the approximation strategy in Guerrieri and Iacoviello (2015), this model allows agents to incorporate the possibility of switching between the two regimes when forming their expectations. In particular, the novelty of our approximation is that these probabilities of switching are endogenously determined and consistent with the approximate solution of the model.

The key change to the model's equations in the two-regime version of the model is in the monetary policy rule. In particular, we replace the Taylor rule (64) with the following rule

$$i_t = \max(0, \tilde{i}_t) \tag{78}$$

$$\tilde{i}_t = \rho_r \tilde{i}_{t-1} + (1-\rho_r)\left[r + \pi_t^* + \rho_\pi(\pi_t - \pi_t^*) + \rho_y \Delta y_t\right] + \varepsilon_{mt}, \tag{79}$$

where $\tilde{i}_t$ denotes a latent interest rate unconstrained by the zero lower bound on the interest rate. To capture the change in dynamics implied by the zero lower bound, this section describes how we build an approximate piecewise linear solution of the model.

As we introduce the zero-lower-bound constraint in the monetary policy rule, we reinterpret our model as a two-regime model with an endogenous probability of switching between regimes. The two regimes of interest are an unconstrained regime (regime 1) in which the interest rate is unrestricted and a constrained regime (regime 2) in which the interest rate is constrained by the zero lower bound. Specifically, the linearized solution in the unconstrained regime crosses the steady state associated with the strictly positive interest rate, which Aruoba, Cuba-Borda, and Schorfheide (2018) name the targeted-inflation regime. The solution in the constrained regime represents a deviation from that solution that captures in a piecewise linear form the change in policy rules when the interest rate rule becomes constrained.

Therefore, we do not consider the possibility of switching to what Aruoba et al. (2018) call the deflationary steady state and of the (linearized) solutions around it. Doing so would bring about two additional regimes, for a total of four regimes determined by whether the model reverts back to the targeted-inflation or the deflationary regimes and by whether the interest rate is constrained or unconstrained.[3] Whether the piecewise solution around the targeted-inflation regime or the piecewise solution around the deflationary regime holds at a given date is theoretically indeterminate, so an agnostic approach following Aruoba et al. (2018) would be to assume an exogenous Markov chain representing a sunspot that indicates on which steady state people coordinate, and hence which piecewise solution holds at any given date. The data would then be used to determine at each date which regime is most likely, thereby pinning down the sunspot and its transition matrix. Thus, while our solution method can be extended to switch across four regimes, we follow Aruoba et al. (2018), who find that the historical evidence suggests that the US has never shifted to the deflationary steady state. We therefore disregard the regimes around the deflationary steady state.[4]

When looking at linear approximate solutions, two key elements characterize the two regimes and the probabilities of switching between them. First, since regimes 1 and 2 have a common steady state to which they revert, we define a perturbation such that the different sets of equations associated with the two regimes can nonetheless both be satisfied at the expansion point. Second, the switching probabilities between regimes 1 and 2 are endogenous in that they equal the conditional probability that the latent interest rate falls below zero.

3.1 General framework for the endogenous ZLB model

The model can be written as:

$$
\begin{aligned}
0 &= E_{s_t,t} f(x_{t-1}, x_t, x_{t+1}, \varepsilon_t, s_t, s_{t+1}) \\
s_t &= 1_{\{S'x_t \geq 0\}} + (1 - 1_{\{S'x_t \geq 0\}})2
\end{aligned}
\tag{80}
$$

where $S \in \mathbb{R}^{n_x}$ is a selection matrix that selects the latent interest rate $\tilde{i}_t = Sx_t$ out of the vector $x_t \in \mathbb{R}^{n_x}$ that contains all variables except the innovations ε_t and a Markov state s_t described next. $s_t \in \{1, 2\}$ is the state at time t of a 2-state Markov chain with endogenous, time-varying transition matrix

$$
P_t = \begin{bmatrix} P_{11t} & P_{12t} \\ P_{21t} & P_{22t} \end{bmatrix}
$$

and $\varepsilon_t \sim Niid(0, I)$ are exogenous shocks other than shocks to s_t. Function f maps into $\mathbb{R}^{n_x}$. The expectations operator $E_{s_t,t}(\cdot)$ denotes the integral over both the Markov state s_{t+1} and the shock ε_{t+1}, conditional on the current state. Here the current state is $(x_{t-1}, \varepsilon_t, s_t)$.

[3]People's expectations will consistently reflect the possibility of switching between these four regimes. Note that this fact can imply the determinacy in the Blanchard and Kahn (1980) sense of the whole model even though taken separately the deflationary regime, when it is sufficiently persistent, is well-known to be associated with indeterminate dynamics.

[4]See also Aruoba et al. (2021) for an alternative piecewise linear approximation method that discusses how to deal with these four regimes.

Note that here the regime-switching probabilities are endogenous. In particular, the switching is driven entirely by the condition $\tilde{i}_t \geq 0$, which indicates regime 1 ($\tilde{i}_t \geq 0 \Leftrightarrow s_t = 1$), and the condition $\tilde{i}_t < 0$, which indicates regime 2 ($\tilde{i}_t < 0 \Leftrightarrow s_t = 2$). Accordingly, the switching probabilities are the probability of starting in regime 1 and staying in regime 1,

$$P_{11t} = Prob(\tilde{i}_{t+1} \geq 0 | x^t, s_t = 1)$$

and the probability of starting in regime 2 and staying in regime 2,

$$P_{22t} = Prob(\tilde{i}_{t+1} < 0 | x^t, s_t = 2)$$

while the other two switching probabilities satisfy $P_{12t} = 1 - P_{11t}$ and $P_{21t} = 1 - P_{22t}$.

We posit that the solution takes the piecewise form

$$x_{s_t t} = h^{(s_t)}(x_{t-1}, \varepsilon_t), \qquad\qquad x_t = 1_{\{s_t=1\}} x_{1t} + (1 - 1_{\{s_t=1\}}) x_{2t} \tag{81}$$

where $1_{\{s_t=i\}}$ is an indicator function, equal to one when in regime $s_t = i$ at date t, that controls which part of the piecewise solution holds at a given date.

We next introduce the notation

$$f^{(i,j)}(x_{t-1}, x_{it}, x_{jt+1}, \varepsilon_t) \equiv f(x_{t-1}, x_t, x_{t+1}, \varepsilon_t, i, j), \qquad i, j = 1, 2$$

to make the dependence of function f on the Markov state more compact, and accordingly rewrite (80) by writing out the conditional expectation with respect to the Markov state as

$$\begin{aligned} 0 &= \sum_{j=1}^{2} P_{ijt} E_t f^{(i,j)}(x_{t-1}, x_{it}, x_{jt+1}, \varepsilon_t) \\ &= \sum_{j=1}^{2} P_{ijt} E_t f^{(i,j)}\Big(x_{t-1}, h^{(i)}(x_{t-1}, \varepsilon_t), h^{(j)}\big(h^{(i)}(x_{t-1}, \varepsilon_t), \varepsilon_{t+1}\big), \varepsilon_t\Big) \end{aligned} \tag{82}$$

where the operator $E_t(\cdot)$ now denotes the integral only over the innovations ε_{t+1}, and where the last equality used the unknown solution (81).

3.1.1 A perturbation consistent with two regimes around a common steady state

We now parameterize the problem by the scalars σ and τ as follows:

$$\begin{aligned}
0 &= P_{i1t}(\tau) E_t f^{(i,1)}\Big(x_{t-1}, h^{(i)}(x_{t-1}, \sigma\varepsilon_t, \sigma, \tau), h^{(1)}\big(h^{(i)}(x_{t-1}, \sigma\varepsilon_t, \sigma, \tau), \sigma\varepsilon_{t+1}, \sigma, \tau\big), \sigma\varepsilon_t\Big) \\
&+ [1 - P_{i1t}(\tau)] E_t f^{(i,2)}\Big(x_{t-1}, h^{(i)}(x_{t-1}, \sigma\varepsilon_t, \sigma, \tau), h^{(2)}\big(h^{(i)}(x_{t-1}, \sigma\varepsilon_t, \sigma, \tau), \sigma\varepsilon_{t+1}, \sigma, \tau\big), \sigma\varepsilon_t\Big) \\
&+ (1 - \sigma)\Big[f^{(1,1)}(x, x, x, 0) - P_{i1t}(\tau) f^{(i,1)}(x, x, x, 0) - [1 - P_{i1t}(\tau)] f^{(i,2)}(x, x, x, 0)\Big], \qquad i = 1, 2 \quad (83)
\end{aligned}$$

where $P_{i1t}(\tau) = \tau P_{i1t} + (1 - \tau)\pi^{(i)}(x_{t-1}, \varepsilon_t)$ and we specify $\pi^{(i)}$ below, and where we posited a solution with form

$$x_{it} = h^{(i)}(x_{t-1}, \sigma\varepsilon_t, \sigma, \tau)$$

By construction, system (83) coincides with (82) when $\sigma = \tau = 1$.

We define in the usual manner the (anchored-expectations) deterministic steady state as the point $(x_{t-1}, \sigma\varepsilon_t, \sigma, \tau) = (x, 0, 0, 0)$ that satisfies

$$f^{(1,1)}(x, x, x, 0) = 0, \qquad x = h^{(1)}(x, 0, 0, 0) \tag{84}$$

and verify that the property that system (83) evaluated at $(x_{t-1}, \sigma\varepsilon_t, \sigma, \tau) = (x, 0, 0, 0)$ is satisfied.

Namely, evaluating system (83) at the deterministic steady state, we have

$$\begin{aligned}
0 &= f^{(1,1)}\Big(x, x, x, 0\Big) + P_{i1}(0)\Big[f^{(i,1)}\Big(x, h^{(i)}(x, 0, 0, 0), h^{(1)}\big(h^{(i)}(x, 0, 0, 0), 0, 0, 0\big), 0\Big) - f^{(i,1)}\Big(x, x, x, 0\Big)\Big] \\
&+ [1 - P_{i1}(0)]\Big[f^{(i,2)}\Big(x, h^{(i)}(x, 0, 0, 0), h^{(2)}\big(h^{(i)}(x, 0, 0, 0), 0, 0, 0\big), 0\Big) - f^{(i,2)}\Big(x, x, x, 0\Big)\Big] \qquad (85)
\end{aligned}$$

which has a solution for $h^{(2)}(x, 0, 0, 0) = x$. The deterministic steady state can therefore be used as an expansion point that satisfies equation (83) for both $i = 1, 2$.

A first-order perturbed solution to system (83) will result in the piecewise linear approximate solution with form

$$x_{it} = h^{(i)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0) = x + h_1^{(i)}\hat{x}_{t-1} + h_2^{(i)}\sigma\varepsilon_t + h_3^{(i)}\sigma \tag{86}$$

with, as in (81), $x_t = 1_{\{s_t=1\}}x_{1t} + (1 - 1_{\{s_t=1\}})x_{2t}$.

3.1.2 Approximate endogenous switching probabilities

Using the definition of the selection matrix S, which selects the latent interest rate out of the vector of variables x, we further assume that $S'x \geq 0$; so the Markov state in the anchored-expectations deterministic steady state is $s = 1$.[5] The transition probabilities are characterized by

$$P_{11t} = Pr(S'x_{1t+1} \geq 0 | x_{t-1}, \varepsilon_t, s_t = 1), \qquad P_{22t} = Pr(S'x_{1t+1} < 0 | x_{t-1}, \varepsilon_t, s_t = 2)$$

We can now use the approximate solution (86) to characterize the transition probabilities. Namely, when $\sigma = 1$, the approximate solution (86) implies

$$\begin{aligned}
& Pr\Big(S'[x + h_1^{(1)}\hat{x}_t + h_2^{(1)}\varepsilon_{t+1} + h_3^{(1)}] \geq 0 \Big| x_{t-1}, \varepsilon_t, s_t = i\Big) && (87)\\
&= Pr\left(\varepsilon_{t+1} \geq -\frac{S'[x + h_3^{(1)} + h_1^{(1)}\hat{x}_{it}]}{\sqrt{S'h_2^{(1)}h_2^{(1)\prime}S}} \Big| x_{t-1}, \varepsilon_t\right) \\
&= 1 - \Phi\left(-\frac{S'[x + h_3^{(1)} + h_1^{(1)}(h_3^{(i)} + h_1^{(i)}\hat{x}_{t-1} + h_2^{(i)}\varepsilon_t)]}{\sqrt{S'h_2^{(1)}h_2^{(1)\prime}S}}\right) \\
&= \Phi\left(\frac{S'[x + h_3^{(1)} + h_1^{(1)}(h_3^{(i)} + h_1^{(i)}\hat{x}_{t-1} + h_2^{(i)}\varepsilon_t)]}{\sqrt{S'h_2^{(1)}h_2^{(1)\prime}S}}\right) \equiv \pi^{(i)}(x_{t-1}, \varepsilon_t) && (88)
\end{aligned}$$

where Φ is the standard normal cumulative distribution function. Through probabilities $\pi^{(i)}(x_{t-1}, \varepsilon_t)$ the endogeneity of the switching probabilities is characterized consistently with the approximate solution.

3.1.3 Approximate piecewise solution

Denoting $f^{(i,j)} \equiv f^{(i,j)}(x, x, x, 0)$ and using $f^{(1,1)}(x, x, x, 0) = 0$, we rewrite (83) evaluated at $\tau = 0$ for $i = 1$:

$$\begin{aligned}
0 &= \pi^{(1)}(x_{t-1}, \sigma\varepsilon_t) E_t \left[f^{(1,1)}\Big(x_{t-1}, h^{(1)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0), h^{(1)}\big(h^{(1)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0), \sigma\varepsilon_{t+1}, \sigma, 0\big), \sigma\varepsilon_t\Big)\right] \\
&+ [1 - \pi^{(1)}(x_{t-1}, \sigma\varepsilon_t)] E_t \left[f^{(1,2)}\Big(x_{t-1}, h^{(1)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0), h^{(2)}\big(h^{(1)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0), \sigma\varepsilon_{t+1}, \sigma, 0\big), \sigma\varepsilon_t\Big) - (1-\sigma) f^{(1,2)}\right]
\end{aligned}$$

and for $i = 2$:

$$\begin{aligned}
0 &= \pi^{(2)}(x_{t-1}, \sigma\varepsilon_t) E_t \left[f^{(2,1)}\Big(x_{t-1}, h^{(2)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0), h^{(1)}\big(h^{(2)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0), \sigma\varepsilon_{t+1}, \sigma, 0\big), \sigma\varepsilon_t\Big) - (1-\sigma) f^{(2,1)}\right] \\
&+ [1 - \pi^{(2)}(x_{t-1}, \sigma\varepsilon_t)] E_t \left[f^{(2,2)}\Big(x_{t-1}, h^{(2)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0), h^{(2)}\big(h^{(2)}(x_{t-1}, \sigma\varepsilon_t, \sigma, 0), \sigma\varepsilon_{t+1}, \sigma, 0\big), \sigma\varepsilon_t\Big) - (1-\sigma) f^{(2,2)}\right]
\end{aligned}$$

[5] Here note that the deflationary deterministic steady state, which is the solution x_d such that $f^{(2,2)}(x_d, x_d, x_d, 0) = 0$, is not a solution to system (83) because of the last term in (83).

and then expand both equations around the deterministic steady state $(x_{t-1}, \sigma\varepsilon_t, \sigma) = (x, 0, 0)$. The expansion of the equation for $i = 1$ results in

$$
\begin{aligned}
0 &= \pi^{(1)} \left(f_1^{(1,1)} \hat{x}_{t-1} + f_2^{(1,1)} [h_1^{(1)} \hat{x}_{t-1} + h_2^{(1)} \sigma\varepsilon_t + h_3^{(1)} \sigma] + f_3^{(1,1)} h_1^{(1)} [h_1^{(1)} \hat{x}_{t-1} + h_2^{(1)} \sigma\varepsilon_t + h_3^{(1)} \sigma] + f_3^{(1,1)} h_3^{(1)} \sigma + f_4^{(1,1)} \sigma\varepsilon_t \right) \\
&+ (1 - \pi^{(1)}) \left(f_1^{(1,2)} \hat{x}_{t-1} + f_2^{(1,2)} [h_1^{(1)} \hat{x}_{t-1} + h_2^{(1)} \sigma\varepsilon_t + h_3^{(1)} \sigma] + f_3^{(1,2)} h_1^{(2)} [h_1^{(1)} \hat{x}_{t-1} + h_2^{(1)} \sigma\varepsilon_t + h_3^{(1)} \sigma] + f_3^{(1,2)} h_3^{(2)} \sigma + f_4^{(1,2)} \sigma\varepsilon_t \right) \\
&+ (1 - \pi^{(1)}) f^{(1,2)} \sigma
\end{aligned}
\tag{89}
$$

while the expansion of the equation for $i = 2$ yields:

$$
\begin{aligned}
0 &= \pi^{(2)} \left(f_1^{(2,1)} \hat{x}_{t-1} + f_2^{(2,1)} [h_1^{(2)} \hat{x}_{t-1} + h_2^{(2)} \sigma\varepsilon_t + h_3^{(2)} \sigma] + f_3^{(2,1)} h_1^{(1)} [h_1^{(2)} \hat{x}_{t-1} + h_2^{(2)} \sigma\varepsilon_t + h_3^{(2)} \sigma] + f_3^{(2,1)} h_3^{(1)} \sigma + f_4^{(2,1)} \sigma\varepsilon_t \right) \\
&+ (1 - \pi^{(2)}) \left(f_1^{(2,2)} \hat{x}_{t-1} + f_2^{(2,2)} [h_1^{(2)} \hat{x}_{t-1} + h_2^{(2)} \sigma\varepsilon_t + h_3^{(2)} \sigma] + f_3^{(2,2)} h_1^{(2)} [h_1^{(2)} \hat{x}_{t-1} + h_2^{(2)} \sigma\varepsilon_t + h_3^{(2)} \sigma] + f_3^{(2,2)} h_3^{(2)} \sigma + f_4^{(2,2)} \sigma\varepsilon_t \right) \\
&+ \pi^{(2)} f^{(2,1)} \sigma + (1 - \pi^{(2)}) f^{(2,2)} \sigma
\end{aligned}
\tag{90}
$$

We can now match the coefficients and find the system of equations that identifies coefficient $h_1^{(i)}$:

$$
\begin{aligned}
0 &= \pi^{(1)} \left[f_1^{(1,1)} + f_2^{(1,1)} h_1^{(1)} + f_3^{(1,1)} h_1^{(1)} h_1^{(1)} \right] + (1 - \pi^{(1)}) \left[f_1^{(1,2)} + f_2^{(1,2)} h_1^{(1)} + f_3^{(1,2)} h_1^{(2)} h_1^{(1)} \right] \\
0 &= \pi^{(2)} \left[f_1^{(2,1)} + f_2^{(2,1)} h_1^{(2)} + f_3^{(2,1)} h_1^{(1)} h_1^{(2)} \right] + (1 - \pi^{(2)}) \left[f_1^{(2,2)} + f_2^{(2,2)} h_1^{(2)} + f_3^{(2,2)} h_1^{(2)} h_1^{(2)} \right]
\end{aligned}
\tag{91}
$$

coefficient $h_2^{(i)}$:

$$
\begin{aligned}
0 &= \pi^{(1)} \left[f_2^{(1,1)} h_2^{(1)} + f_3^{(1,1)} h_1^{(1)} h_2^{(1)} + f_4^{(1,1)} \right] + (1 - \pi^{(1)}) \left[f_2^{(1,2)} h_2^{(1)} + f_3^{(1,2)} h_1^{(2)} h_2^{(1)} + f_4^{(1,2)} \right] \\
0 &= \pi^{(2)} \left[f_2^{(2,1)} h_2^{(2)} + f_3^{(2,1)} h_1^{(1)} h_2^{(2)} + f_4^{(2,1)} \right] + (1 - \pi^{(2)}) \left[f_2^{(2,2)} h_2^{(2)} + f_3^{(2,2)} h_1^{(2)} h_2^{(2)} + f_4^{(2,2)} \right]
\end{aligned}
\tag{92}
$$

and coefficient $h_3^{(i)}$:

$$
\begin{aligned}
0 &= \pi^{(1)} \left[f_2^{(1,1)} h_3^{(1)} + f_3^{(1,1)} h_1^{(1)} h_3^{(1)} + f_3^{(1,1)} h_3^{(1)} \right] + (1 - \pi^{(1)}) \left[f_2^{(1,2)} h_3^{(1)} + f_3^{(1,2)} h_1^{(2)} h_3^{(1)} + f_3^{(1,2)} h_3^{(2)} + f^{(1,2)} \right] \\
0 &= \pi^{(2)} \left[f_2^{(2,1)} h_3^{(2)} + f_3^{(2,1)} h_1^{(1)} h_3^{(2)} + f_3^{(2,1)} h_3^{(1)} + f^{(2,1)} \right] + (1 - \pi^{(2)}) \left[f_2^{(2,2)} h_3^{(2)} + f_3^{(2,2)} h_1^{(2)} h_3^{(2)} + f_3^{(2,2)} h_3^{(2)} + f^{(2,2)} \right]
\end{aligned}
\tag{93}
$$

thereby completely characterizing the unknown solution (86).

Here note that there is a constant correction of the solution in that, generically, $h_3^{(i)} = 0$ is not a solution to (93). Note also that the time variation in probabilities does not affect the solution to first order. Still, the average values of $\pi^{(1)}$ and $\pi^{(2)}$ are consistent with the endogenous switching probabilities when the state x is at the deterministic steady state.

Finally, note that in our application to the zero lower bound, $f^{(1,1)} = f^{(1,2)}$ and $f^{(2,1)} = f^{(2,2)}$, and $f^{(2,1)}$ differs from $f^{(1,1)}$ in only one equation, the one associated with the Taylor rule—constrained at $i_t = 0$ in the second regime.

3.1.4 Stationary distribution

We can define the stationary distribution of the variables in the model. Namely, the stationary distribution

$$p_i \equiv Prob(s = i), \quad i = 1, 2$$

of our 2-state Markov chain with transition matrix P is

$$p_1 = P_{11}p_1 + (1 - P_{22})(1 - p_1) = \frac{1 - P_{22}}{2 - P_{11} - P_{22}}$$

and $p_2 = 1 - p_1$.

We then define the variable

$$x_t = (p_1 + \hat{s}_t)x_{1t} + (1 - p_1 - \hat{s}_t)x_{2t}$$

where $\hat{s}_t \equiv 1_{\{s_t=1\}} - p_1$ is a Bernoulli variable that equals $1 - p_1$ when $s_t = 1$ and $-p_1$ when $s_t = 2$. The stationary distribution of $\hat{s}_t$ is such that it equals $1 - p_1$ with probability p_1 and $-p_1$ with probability $1 - p_1$. The variable has therefore mean 0 and variance $p_1(1 - p_1)$. Indeed, in the implementation, we will approximate $\hat{s}_t \sim Niid(0, p_1(1 - p_1))$. Therefore, the ergodic steady-state value of variable x_t, which holds when $\hat{s} = 0$, can be computed as

$$x = p_1x_1 + (1 - p_1)x_2$$

which intuitively acts as an unconditional value to be targeted using moments from the data.

When calibrating the model, this ergodic steady-state value of the variables is the natural value to be matched to moments of the model.

3.2 Filtering and forecasts

Filtering of the latent Markov state can be computed by a simple guess-and-verify strategy. Forecasts and impulse responses can be computed by simulations.

3.2.1 Filtering

The two-regime structure adds the Markov state s_t to the filtering problem. In practice, this filtering can be done as follows. At each given date t and given the history of shocks up to time $t - 1$ backed out to account for the observations up until date $t - 1$, we start from the guess that $s_t = 1$ and proceed to reconstruct the shocks that account for the observables at date t. At this stage we can compute the latent interest rate at date t, $\tilde{i}_t$, and if it turns out that $\tilde{i}_t \geq 0$, then the guess is confirmed. If instead $\tilde{i}_t < 0$, we update our guess to $s_t = 2$ and repeat the construction of the shocks at time t.

3.2.2 Impulse responses

The nonlinearities introduced by the piecewise linear solution imply that the responses of the variables to a shock will not be independent of the state of the economy when the shock hits the economy and will not simply scale with the size of the shock.

In this context, impulse responses can be computed as generalized impulse responses by simulations. Namely, for a given initial state at time 0, $x_0 = x$, for example, the ergodic steady state described above, we can draw T vectors of shocks from the distribution of the shock vector, $\{\varepsilon_t\}_{t=0}^T$, and simulate the model forward. As we do so, we proceed by making a guess similar to that described in the filtering procedure: at the simulated date t, we guess that $s_t = 1$ and compute all variables at date t accordingly, including the latent interest rate $\tilde{i}_t$; if $\tilde{i}_t \geq 0$, the guess is verified and we can proceed to the next date; otherwise, we update the guess to $s_t = 2$. This T-period simulation will be repeated a large number of times and averaged out to find a forecast $E_0(x_t|\varepsilon_0, ..., \varepsilon_t, x_0 = x)$.

We then repeat the same procedure by replacing at date 1 the simulated innovations ε_1 with $\varepsilon_1 + \Delta$ for some perturbation Δ, representing the shock that hits the economy at date 1. By averaging the simulations as before, we now find the forecast $E_0(x_t|\varepsilon_0, \varepsilon_1 + \Delta, \varepsilon_2, ..., \varepsilon_t, x_0 = x)$.

The impulse responses are then computed as the difference

$$E_0(x_t|\varepsilon_0, \varepsilon_1 + \Delta, \varepsilon_2, ..., \varepsilon_t, x_0 = x) - E_0(x_t|\varepsilon_0, ..., \varepsilon_t, x_0 = x)$$

which represents the predicted effect of the perturbation Δ on the future variables.

Analogously, we can compute confidence intervals around these impulse responses by computing the quantiles of the variable x_t across simulations, reflecting uncertainty around the realization of the shocks.

3.2.3 Forecasts

Forecasts at date $t + h$ can be computed similarly to how we compute impulse responses by simulations. Namely, for filtered shocks and backed out variables $\hat{x}_t$ up until date t, we construct a forecast at date $t + h$ as

$$E_t(x_{t+h}|\varepsilon_t, ..., \varepsilon_{t+h}, x_t = \hat{x}_t)$$

by the same procedure as described above for impulse responses.[6]

Using the same procedure as for impulse responses, we can compute confidence intervals around these forecasts by computing the quantiles x_{t+h} across the simulated paths. This dispersion reflects

[6] Note that, while the presence of the measurement error component r_t^* to account for the observed interest rate $i_t^{obs} = i_t + r_t^*$ poses no issue when it comes to filtering the historical shocks, in that the observed i_t^{obs} is above the zero lower bound and always explicitly accounted for by the model, nothing guarantees that our forecast for i_{t+h}^{obs} is always positive, since the model only ensures that i_{t+h} is. Accordingly, when simulating future observed interest rates, we compute them as $i_{j,t+h}^{obs} = i_{j,t+h} + \max(-i_{j,t+h}, r_{j,t+h}^*)$, where j denotes the jth simulation.

uncertainty around the realization of the future shocks.

4 Data and Parameterization

4.1 Data

The estimation uses quarterly data on 13 macroeconomic variables over the period 1959Q1-2020Q1. All data except the survey expectations data are public data downloadable from FRED. We choose a relatively long sample period, which includes, for example, periods of persistently high inflation in the 1970s and persistently low inflation during the Great Moderation period, precisely because of our inclusion of low-frequency trends, which will arguably capture such slow movements in the structure of the economy.

Here note that we explicitly include in the estimation stage the period following the Great Recession, in which the policy rate was constrained by the zero lower bound, which the model is suited to capture owing to the regime-switching structure. However, we exclude from the estimation stage the COVID pandemic period starting in 2020Q1, which was characterized by extreme fluctuations in the real variables, especially over 2020Q2-2020Q3. We do, however, include those data points when running our historical decompositions and forecasting exercises. In other words, we assume that the deep parameters of the economy can be estimated with less noise on a sample that excludes the COVID pandemic period but that they have remained stable after the COVID period; so the parameters and shocks of the model remain appropriate to account for the observed data during the pandemic period and the subsequent recovery.

We use the following observations from FRED:

- Real GDP growth per capita (GDPC1 on FRED), transformed to per capita using the civilian noninstitutional population (CNP16OV);

- Real investment growth per capita, measured as the growth rate of the sum of PCE durable consumption goods (PDCG) and of gross private domestic investment (GPDI), transformed to per capita using the civilian noninstitutional population and transformed into real terms using the GDP price deflator (GDPDEF);

- Real consumption growth per capita, measured as the growth rate of the sum of PCE nondurable consumption goods (PCND) and PCE service goods (PCESV), transformed to per capita using the civilian noninstitutional population and transformed into real terms using the GDP price deflator;

- Core PCE inflation, measured as the growth rate of the PCE price index less food and energy (JCXFE);

- Real wage growth, measured as the growth rate of the nonfarm business sector compensation per hour (COMPNFB) transformed into real terms using the GDP price deflator;
- Effective federal funds rate (DFF);
- Civilian unemployment rate (UNRATE);
- Vacancy rate, measured as total nonfarm job openings (JTSJOR) since 2001 and using the series constructed by Barnichon (2010) before 2001, all expressed as the ratio of vacancies to the sum of vacancies and the number of employed people;
- Job-finding rate, measured as in Shimer (2005) by combining the civilian unemployment rate and a measure of short-term unemployment, namely, the number of unemployed for less than 5 weeks (UEMPLT5). Following Elsby, Michaels and Solon (2009), we scale the short-term unemployment rate by 1.16 after 1994 to account for the redesign in the survey that occurred at that date. The monthly job-finding rate can then be constructed as

$$p_{mt} = 1 - \frac{u_{t+1} - u_{s,t+1}}{u_t}$$

where u_s denotes the measure of short-term unemployment, and the associated monthly measure of the job-separation rate can be constructed, under the assumption of a uniform separation probability over the month, as

$$s_{mt} = \frac{u_{s,t+1}}{(1-u_t)(1-0.5p_{mt})}$$

The quarterly job-finding rate is then constructed as the probability that a worker unemployed at the start of a given month t is employed three months later, which can be computed as

$$\begin{aligned} p_t = {} & p_{mt}s_{m,t+1/3}p_{m,t+2/3} + p_{mt}(1-s_{m,t+1/3})(1-s_{m,t+2/3})+ \\ & + (1-p_{mt})p_{m,t+1/3}(1-s_{m,t+2/3}) + (1-p_{mt})(1-p_{m,t+1/3})p_{m,t+1/3} \end{aligned}$$

Note that the presence of both job-finding and vacancy rates as observables, along with the unemployment rate, allows us to pin down the match-efficiency and job-separation shocks separately. In fact, the law of motion of unemployment can be written as

$$U_t = s_{t-1}(1-U_{t-1}) + U_{t-1} - p_t S_t \tag{94}$$

$$= s_{t-1}N_{t-1} + U_{t-1} - \zeta_t \left(\frac{V_t}{S_t}\right)^{1-\sigma} S_t \tag{95}$$

with job searchers $S_t = s_{t-1} + (1 - s_{t-1})U_{t-1}$. Therefore, by observing U_t and p_t we pin down the exogenous processes s_{t-1} by solving (94) rewritten as

$$U_t = s_{t-1}(1 - U_{t-1}) + U_{t-1} - p_t[s_{t-1} + (1 - s_{t-1})U_{t-1}]$$

for s_{t-1}; and by observing U_t and V_t, using the backed-out s_{t-1} to pin down S_t, we pin down ζ_t from (95).

Moreover, we use long-term survey expectations to discipline the low-frequency components of the model that act as attraction points for the forecasts at the 5- to 12-year horizon, as discussed above. In particular, we use the following data:

- 5-year average inflation expectations starting 5 years in the future, measured from 5-year and 10-year median core PCE survey expectations from the Survey of Professional Forecasters after 2007, and proxied as the PTR average inflation forecasts constructed using the FRB/US model before 2007;

- 5-year average real GDP growth expectations starting 7 years in the future, measured as the quarterly long-range Blue Chip forecasts for real GDP growth;

- 5-year average unemployment rate expectations starting 7 years in the future, measured as the quarterly long-range Blue Chip forecasts for the unemployment rate;

- 5-year average real short rate expectations starting 7 years in the future, measured as in Zaman (2022) as the quarterly long-range Blue Chip forecasts for the 3-month Treasury bill nominal rate minus inflation in the GDP chained price index plus 0.3 percentage points to account for the historical difference between the fed funds rate and the 3-month T-bill rate.

These observed variables are associated with an equal number of shocks in the model. Namely, the model includes 13 exogenous states: the preference shock ω_t, the job-separation shock s_t, the investment-specific shock μ_t, the markup shock θ_t, the match-efficiency shock ζ_t, the productivity shock z_t, the bargaining-power shock η_t, the government spending shock γ_t, the monetary policy shock ε_{mt}, the inflation target shock π_t^*, and the slow-moving measurement error shocks g_t^* in the real growth rate, u_t^* in the unemployment rate, and r_t^* in the real interest rate.

4.2 Calibrated parameters

Table 1 reports all calibrated parameter values. The steady-state values of output growth, inflation, the interest rate, and the unemployment rate are set equal to their respective sample average over the period 1959Q1-2020Q1—equal an annualized rate of 1.6 percent, 3.2 percent, 4.8 percent, and 5.9 percent, respectively.

Calibrated parameters		
z	Steady-state output growth	0.0041
π	Steady-state inflation	0.0079
i	Steady-state interest rate	0.0124
U	Steady-state unemployment rate	0.0595
p	Job-finding rate	0.7900
δ_k	Capital depreciation rate	0.0250
α	Capital share of value added	0.3300
θ	Elasticity of substitution between intermediate goods	6.0000
G/Y	Government spending-to-output ratio	0.2000
$\tilde{\kappa}$	Hiring cost	0.4890
ρ_*	Persistence of the slow-moving components	0.9950
γ	Steady-state government spending shock	1.2500
Parameters implied by steady-state restrictions		
s	Job-separation rate	0.2380
κ	Scaled hiring cost	0.5867
β	Discount rate	0.9997
ζ	Steady-state match-efficiency shock	0.7900
η	Steady-state workers' bargaining-power shock	0.3780

Table 1: Calibrated parameters

Note: The implied parameters are computed by setting the estimated parameters at the posterior modes of their estimated posterior distributions.

We calibrate the average job-destruction rate to equal the average job-destruction rate at a quarterly frequency we construct in the data. Namely, in our construction of the job-finding and job-separation rates at a monthly frequency described above, we find an average monthly job-finding rate of $p_m = 0.43$ and an average monthly job-separation rate of $s_m = 0.032$. These monthly numbers imply a job-finding rate at a quarterly frequency equal to $p = s_m p_m^2 + p_m(1-s_m)^2 + p_m(1-p_m)(1-s_m) + p_m(1-p_m)^2 = 0.79$. Given the steady-state value of unemployment U, we derive the quarterly steady-state job-separation rate residually as $s = pU/(1-p)(1-U) = 0.24$.

We impose the condition that the elasticity of the matching function with respect to unemployment σ equals the steady-state workers' bargaining power η, consistent with the Hosios (1990) condition holding at the steady state, and we will estimate the common parameter below. We normalize the value of market tightness by choosing the steady-state value of match efficiency to imply a steady-state job-filling rate equal to the steady-state job-finding rate.

We choose a standard value for the capital depreciation rate of 0.025 at a quarterly frequency. The capital share of value added parameter $\alpha = 0.33$ is chosen to hit a steady-state labor share of 0.56, as observed over the sample period. The elasticity of substitution between intermediate goods is set equal to 6, implying a steady-state markup of 20 percent as in Rotemberg and Woodford (1995). The home consumption habit parameter is set equal to the market consumption habit parameter, $h_c = h_h = h$, which will in turn be estimated below. The steady-state government-spending-to-output ratio is set equal to 0.20, as in Furlanetto and Groshenny (2016). Finally, we specify hiring costs to equal 3.6 percent of the labor share, consistent with the evidence in Silva and

Toledo (2009) on turnover costs. Together with the replacement rate, which will also be estimated, since its dispersion in the literature is large, this number will pin down the utility parameter χ.

We also calibrate the persistence of the slow-moving components $\rho_* = 0.995$ to have processes close to random walks that can act as the relevant attraction points for forecasts at the 5- to 12-year horizon. Slightly different choices such as $\rho_* = 0.99$ or $\rho_* = 0.999$ yield similar forecasts at the 5- to 12-year horizon.

4.3 Estimated parameters

The rest of the parameters of the model are estimated using standard Bayesian techniques. Our priors for the exogenous shock processes are similar to the ones routinely used in the literature (Smets and Wouters 2007; Gertler et al. 2008; Furlanetto and Groshenny, 2016). The priors for the remaining deep parameters cover typical parameters found in the literature. The priors for the parameters of the policy function, for the habit parameter, and for the adjustment-cost and indexation parameters are standard. We adopt a relatively wide prior for the weight of pre-match hiring costs in the matching function, and, similar to Shimer (2005), we center the curvature of the matching function around 0.6 and the replacement rate around 0.4.

We use a random walk Metropolis-Hastings algorithm to generate 1,000,000 draws from the posterior distribution and discard the first 250,000 draws. The algorithm is tuned to achieve the benchmark acceptance ratio of 25 to 30 percent. Table 2 summarizes the prior and posterior distributions.

5 Empirical Illustration

This section illustrates the role of some of the main ingredients of the model. We describe the decomposition of the observed time series into the relative contribution of the different shocks that hit the economy and discuss the main drivers of the observables and their transmission mechanism. We also discuss here the roles of the two-regime structure and of the long-range survey forecasts.

5.1 Historical decompositions and transmission mechanism

Figures 1 to 9 plot the historical decompositions of the main observable variables used to estimate the model. Additionally, Figures 10 and 11 report the historical decompositions of two benchmark variables defined above: the output gap and the natural r-star.

Table 3 reports the average contribution of each shock in the historical decomposition of Figures 1 to 9. We compute such contributions as follows: at each date, we compute the movement (in absolute value) in a variable associated with a shock as a fraction of the total movement in the

			Prior			Posterior		
Deep and policy parameters		Distr.	Mean	Std.	5%	Mean	95%	Mode
ρ_r	Interest smoothing parameter	Beta	0.7	0.2	0.7584	0.7875	0.8181	0.7832
ρ_π	Monetary policy response to inflation gap	IG	1.5	0.1	1.6388	1.8379	2.0349	1.8269
ρ_y	Monetary policy response to output growth	IG	0.5	0.1	0.2950	0.3669	0.4352	0.3535
$h_c = h_h$	Habit formation	Beta	0.7	0.2	0.7471	0.7874	0.8303	0.7893
ϕ_V	Weight of pre-match hiring cost	Beta	0.6	0.2	0.0646	0.0943	0.1247	0.0870
$\sigma = \eta$	Curvature of matching function	Beta	0.6	0.1	0.4429	0.4979	0.5506	0.5115
τ	Replacement rate	Beta	0.4	0.05	0.3474	0.4224	0.4966	0.4135
ϕ_I	Investment adjustment costs	IG	5.0	1.0	2.5879	3.1580	3.7219	2.9887
ϕ_{uu}	Capital utilization costs	IG	0.5	0.1	0.5865	0.9242	1.2477	0.8239
ϕ_P	Price-adjustment costs	IG	50	20	82.797	115.20	146.19	109.44
ς	Price indexation	Beta	0.5	0.2	0.0255	0.1090	0.1894	0.0922
ϕ_W	Wage-adjustment costs	IG	50	20	248.66	373.47	488.17	353.46
ϱ	Wage indexation	Beta	0.5	0.2	0.7895	0.8855	0.9864	0.9233
Exogenous shock parameters								
σ_ω	Preference shock volatility	IG	0.03	0.1	0.0219	0.0264	0.0309	0.0265
σ_s	Separation shock volatility	IG	0.03	0.1	0.0883	0.0969	0.1058	0.0953
σ_μ	Investment-specific shock volatility	IG	0.03	0.1	0.0470	0.0585	0.0697	0.0561
σ_θ	Markup shock volatility	IG	0.03	0.1	0.0752	0.1047	0.1310	0.0994
σ_ζ	Match efficiency volatility	IG	0.03	0.1	0.1183	0.1352	0.1519	0.1311
σ_z	Productivity volatility	IG	0.03	0.1	0.0096	0.0104	0.0113	0.0104
σ_η	Bargaining shock volatility	IG	0.03	0.1	1.4817	2.4605	3.3992	2.2474
σ_γ	Government spending volatility	IG	0.03	0.1	0.0076	0.0082	0.0089	0.0081
σ_m	Monetary policy shock volatility	IG	0.03	0.1	0.0025	0.0028	0.0030	0.0028
$\sigma_{\pi*}$	Inflation target volatility	IG	0.03	0.1	0.0004	0.0005	0.0006	0.0005
σ_{g*}	Growth measurement shock volatility	IG	0.03	0.1	0.0017	0.0019	0.0021	0.0019
σ_{u*}	Unemp. measurement shock volatility	IG	0.03	0.1	0.0303	0.0338	0.0373	0.0340
σ_{r*}	Real rate measurement shock volatility	IG	0.03	0.1	0.0018	0.0020	0.0021	0.0020
ρ_ω	Productivity persistence	Beta	0.6	0.1	0.5675	0.6885	0.8164	0.7114
ρ_s	Separation shock persistence	Beta	0.6	0.1	0.8809	0.8995	0.9183	0.9000
ρ_μ	Investment-specific shock persistence	Beta	0.6	0.1	0.4741	0.5513	0.6274	0.5465
ρ_θ	Markup shock persistence	Beta	0.6	0.1	0.7905	0.8635	0.9750	0.8447
ρ_ζ	Match efficiency persistence	Beta	0.6	0.1	0.9848	0.9916	0.9985	0.9940
ρ_z	Productivity persistence	Beta	0.3	0.1	0.0909	0.1625	0.2380	0.1616
ρ_η	Bargaining shock persistence	Beta	0.6	0.1	0.0314	0.1089	0.1823	0.0944
ρ_γ	Government spending persistence	Beta	0.6	0.1	0.8999	0.9189	0.9381	0.9224

Table 2: Priors and posteriors of estimated structural parameters

Note: In the 'Distribution' column, Beta denotes a beta distribution and IG an inverted Gamma distribution.

	z	ε_m	μ	ω	θ	γ	s	ζ	η	π^*	g^*, u^*, r^*	Regime
GDP growth	13.5	4.3	19.3	6.4	7.0	18.2	16.3	2.3	5.0	1.0	5.7	1.2
Consumption growth	21.8	3.1	7.3	23.6	2.5	11.0	7.3	3.4	4.7	0.7	14.1	0.6
Investment growth	15.1	6.1	26.9	5.8	10.6	11.8	8.3	1.3	7.2	1.2	4.2	1.5
Real wage growth	13.3	3.7	14.6	4.6	11.8	13.9	11.6	1.9	15.8	0.5	7.2	1.2
Inflation rate	11.9	11.9	14.5	8.9	7.9	10.7	7.3	1.6	5.6	19.8	0	0.0
Fed funds rate	5.9	7.9	13.6	8.5	3.1	10.3	5.8	1.9	2.9	13.9	11.9	14.4
Unemployment rate	13.3	5.9	11.6	3.2	10.2	10.2	9.5	12.6	13.3	1.2	8.5	0.5
Vacancy rate	9.6	3.9	10.1	2.9	6.9	8.3	22.0	27.0	8.1	0.8	0	0.5
Job-finding rate	14.6	6.4	13.0	3.7	11.0	11.3	10.7	13.1	14.3	1.3	0	0.6

Table 3: Summary of historical decompositions

Note: Average contribution of each shock in the historical decomposition, in percentage points. z: productivity; ε_m: monetary policy; μ: investment-specific technology; ω: preference; θ: markup; γ: government spending; s: separation; ζ: match efficiency; η: workers' bargaining power; π^*: inflation target; g^*: growth measurement; u^*: unemployment measurement; r^*: real rate measurement; 'Regime': shock to 2-state regime.

variable, defined as the sum of movements associated with each individual shock. We then average these movement shares through time over our sample for each variable.

Technology shocks, including labor-augmenting and investment-specific technology shocks, are the main drivers of our model. They account for 25 to 40 percent of the fluctuations in the growth rates of output, consumption, investment, and real wages over the sample and for 20-25 percent of the fluctuations in the inflation, unemployment, vacancy, job-finding, and interest rates. Figure 12 plots impulse responses to all shocks for the main variables of interest in the one-regime version of the model. Figures 12a and 12b plot impulse responses to these shocks for the main variables of interest in the one-regime version of the model, which is unaffected by the nonlinearities and by the initial state of the economy. (We will later illustrate how the zero-lower-bound constraint distorts these impulse responses.) Labor-augmenting technology shocks produce positive comovement in output, consumption, and investment but the inflation rate and, at least initially, employment move in the opposite direction by standard forces. In contrast, investment-specific shocks produce positive comovement in output, investment, employment, and the inflation rate, but consumption moves in the opposite direction, as the opportunity cost of consumption becomes larger.

Monetary policy shocks play a minor role in overall fluctuations, in line with the literature (e.g., Smets and Wouters, 2007; Furlanetto and Groshenny, 2016). Still, they account for more than 10 percent of fluctuations in inflation. Figure 12c shows the response of the main variables to a monetary policy shock, ignoring the zero lower bound on the interest rate. A surprise interest-rate increase of 90 basis points is associated with peak drops in GDP of 30 basis points, in consumption of nearly 10 basis points, in investment of 60 basis points, and in inflation of 40 basis points; at the same time, the peak increase in the unemployment rate is around 40 basis points. These numbers and their front-loaded effects are in line with the literature (for example, the evidence in Miranda-Agrippino and Ricco, 2021, as well as the models in Christiano, Eichenbaum, and

Trabandt, 2016).

Preference shocks play a major role in consumption growth, accounting for nearly a quarter of its fluctuations, and a sizable role, close to 10 percent, in the inflation and interest rates. Figure 12d shows the response of the main variables to a preference shock. GDP, consumption, employment, and inflation comove positively after a preference shock, while investment moves in the opposite direction, as its opportunity cost of devoting consumption goods to investment increases.

Markup shocks play a sizable role in this model for inflation, accounting for around 8 percent of its fluctuations. Note that this number is lower than the number in Smets and Wouters (2007) but is in line with the model in Furlanetto and Groshenny (2016) with labor market frictions. Also as in Furlanetto and Groshenny (2016), since markup shocks drive profitability, in this setup with search and matching frictions they are likewise a notable driver of labor market variables and investment. Figure 12e shows how markup shocks produce positive comovement in output, consumption, investment, and employment but send inflation in the opposite direction, as firms' prices comove with their markups.

Government spending shocks play a similar role across our observables, accounting for 10 to 15 percent of fluctuations. Figure 12f shows how a positive government spending shock reduces consumption, investment, and employment as it increases output and causes inflationary pressures.

Labor market shocks, including shocks to job separation, to match efficiency, and to workers' bargaining power, are the main drivers of labor market variables—nearly 40 percent of the fluctuations in the unemployment and job-finding rates and 60 percent of fluctuations in the vacancy rate—and account for up to a quarter of fluctuations in output. Figures 12g, 12h, and 12i show the effects of these shocks on the main variables. They produce comovement in output and employment and have different effects on the labor market. A positive job-separation shock has a contractionary effect on impact, but subsequently increases vacancy creation, job-finding rates, and employment, as necessary to maintain employment when labor market churn is higher. A positive match-efficiency shock reduces vacancies, since fewer openings are needed to produce a given number of matches; after an initial negative effect on employment, it therefore increases, along with the job-finding rate. Finally, a positive shock to workers' bargaining power, which is a major driver of real wages, increases wages and inflation, and depresses employment and output.

Finally, we note the contribution of the slow-moving components, represented by the inflation target shock π^* and the measurement shocks, which include g^*, u^*, and r^*, and the contribution of the regime-switching shock, which in the shock decomposition contribute a level shift in the variables that depends on the regime at each date. Changes in the central bank's inflation target account for nearly 20 percent of fluctuations in the inflation rate. In particular, a shock to the central bank's inflation target increases inflation persistently and has an accommodative effect on economic activity. Still, this shock does not operate at business cycle frequencies and has instead a low-frequency effect on the variables. Changes in the growth-rate trend account for 5 to 15 percent of the fluctuations in output, consumption, investment, and real wage growth rates. Changes in dynamics due to changes in the policy regime account for more than 10 percent of the dynamics of the federal funds rate, but play a minor role for all other variables.

5.2 The role of the zero lower bound in the interest rate

We further illustrate the mechanism behind the historical decomposition by looking at the response of the observable variables to their main drivers. Recall that the nonlinearities imposed by the zero lower bound imply that the responses of the variables to a shock will not be independent of the state of the economy when the shock hits and will not simply scale with the size of the shock. Therefore, while the impulse response functions in the unconstrained version of the model reported above were unaffected by the nonlinearities, it is no longer the case in the two-regime version of the model.

We illustrate how our piecewise linear approximation captures the effects of the presence of the zero-lower-bound constraint by considering the effects of shocks when the state of the economy is near the zero lower bound. In practice, we will consider an economy that starts at the steady state and is hit by a large, persistent monetary policy shock that brings the federal funds rate close to but strictly above zero. We then hit that economy with additional shocks to send the interest rate against the zero-lower-bound constraint.

Accordingly, Figure 13 plots such impulse response functions to the main drivers of the nominal interest rate. The system starts at the steady state and is sent close to the zero lower bound as just described. The additional shocks that send the fed funds rate against the zero lower bound then allow us to illustrate the solution of the two-regime model and compare it to the one-regime version. The blue lines describe the effects of shocks in the two-regime structure that cause a switch to a zero-lower-bound regime on impact relative to the effects of those same shocks in the one-regime version of the model.

The piecewise approximation strategy captures a well-documented property of the New Keynesian model at the zero lower bound, namely, deflationary forces and more depressed economic activity relative to an unconstrained model, in which the interest rate can fall below zero, with the associated accommodative effect on the economy. Indeed, across all shocks, the economy constrained by the zero lower bound displays a slightly lower inflation rate, markedly lower growth rates, and higher unemployment rates. The strength of the deflationary and contractionary forces varies across parameterizations, but it is qualitatively robust across them. In particular, at the estimated parameters, the deflationary forces are relatively minor and the contractionary forces are sizable.

In the background, we note that at the parameter estimates, the model implies an average switching probability of staying in the unconstrained regime of 0.99 and an average switching probability of staying in the constrained regime of 0.31. Therefore, people expect that spells at the zero-lower-bound regime will be short-lived.

5.3 The role of long-range expectations

Long-range survey expectations discipline the slow-moving components that now act as attraction points at the 5- to 12-year horizon. Figure 14 reports the forecasts of the model over a period of 100 years for the inflation rate, the unemployment rate, the fed funds rate, and GDP growth rates (solid black lines). The figure also reports the projected long-run forecasts predicted by the model that would exist at each date (dashed blue lines), namely, the 5y/5y inflation rate and the 7y/5y unemployment, fed funds, and GDP growth rates, respectively.

These long-range forecasts show how the slow-moving components act as the relevant attraction points for the forecasts at the 5- to 12-year horizon and slow down the reversion of the variables to their historical sample means—equal to 1.6 percent per annum for output growth per capita, 3.2 percent per annum for the inflation rate, 4.8 percent per annum for the interest rate, and 5.9 percent for the unemployment rate, respectively. In fact, such a reversion to the mean takes several decades—only 100 years out, around 2120, would the variables of interest be forecasted to equal their historical sample means. Similarly, the long-run expectations implied by the model would be attracted by those same points, and their reversion is likewise very slow.

Thus, the steady state of the model remains an attraction point for forecasts by the stationarity of the model. Still, it is not a point that affects the forecasts at the business-cycle frequency—the typical forecast horizon of interest for policymakers. The long run over which the reversion to the historical sample mean is complete can take up to 100 years to manifest.

6 Concluding Remarks

This paper has presented a practitioner's guide to the DSGE model developed at the Federal Reserve Bank of Cleveland as part of the suite of models used for forecasting and policy analysis by Cleveland Fed researchers, which we have nicknamed Clementine. This model is a practical policy tool designed for forecasting exercises and policy counterfactuals to help support decision-making and complement the existing set of models and data that are routinely consulted. Departing from the standard academic style, this document details the construction of the model and offers practical guidance for using the model.

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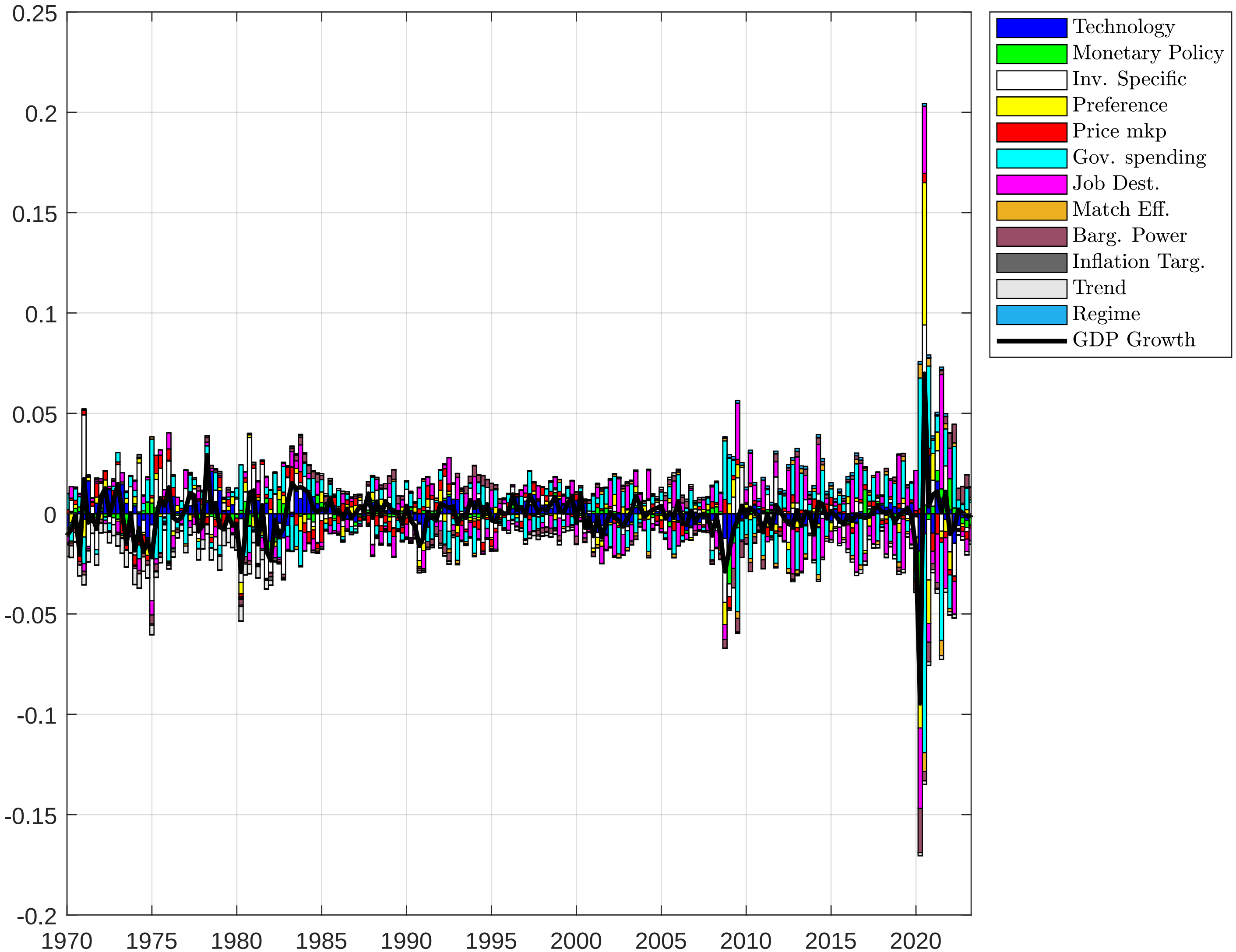


Figure 1: Historical decomposition of the real GDP growth rate

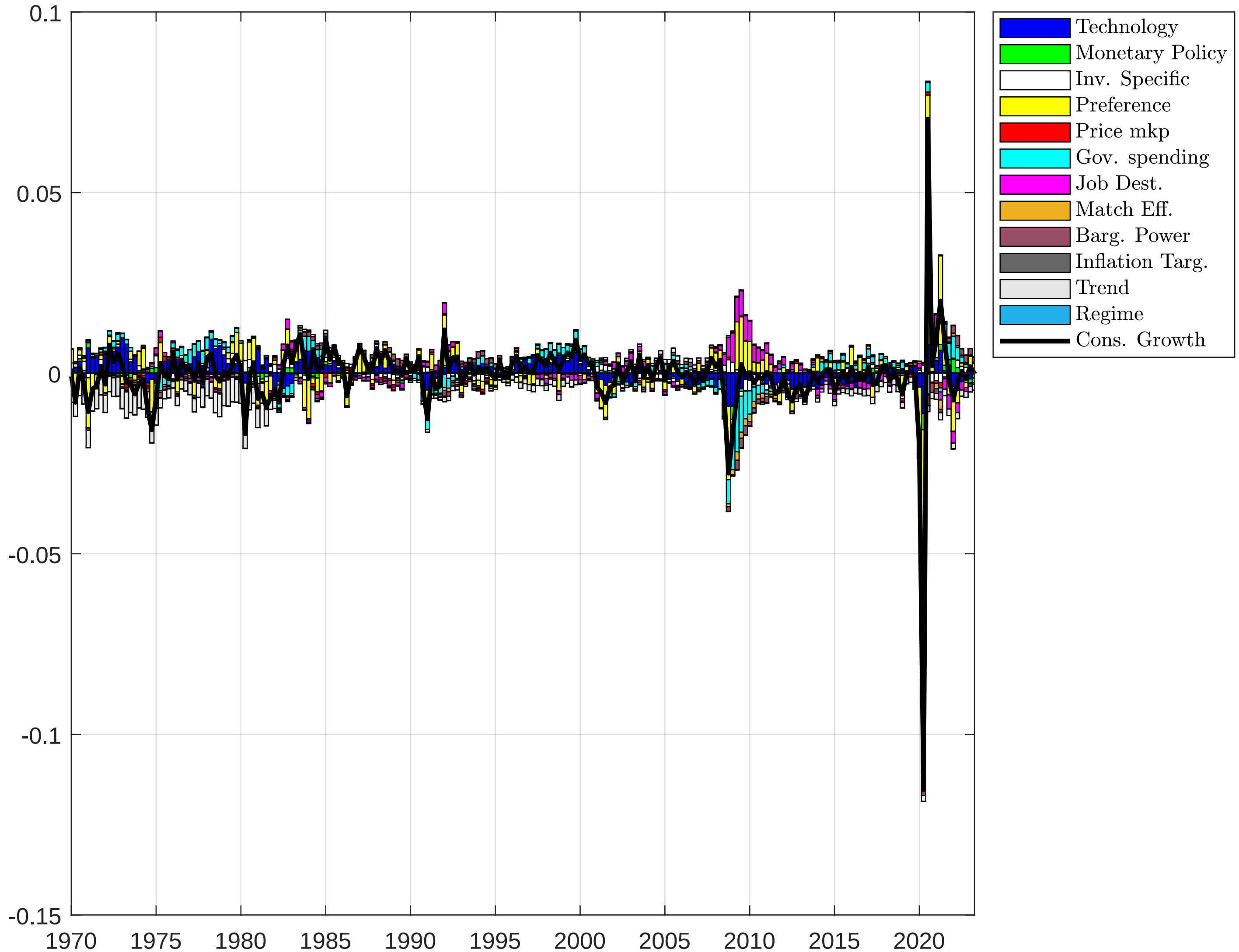


Figure 2: Historical decomposition of the real consumption growth rate

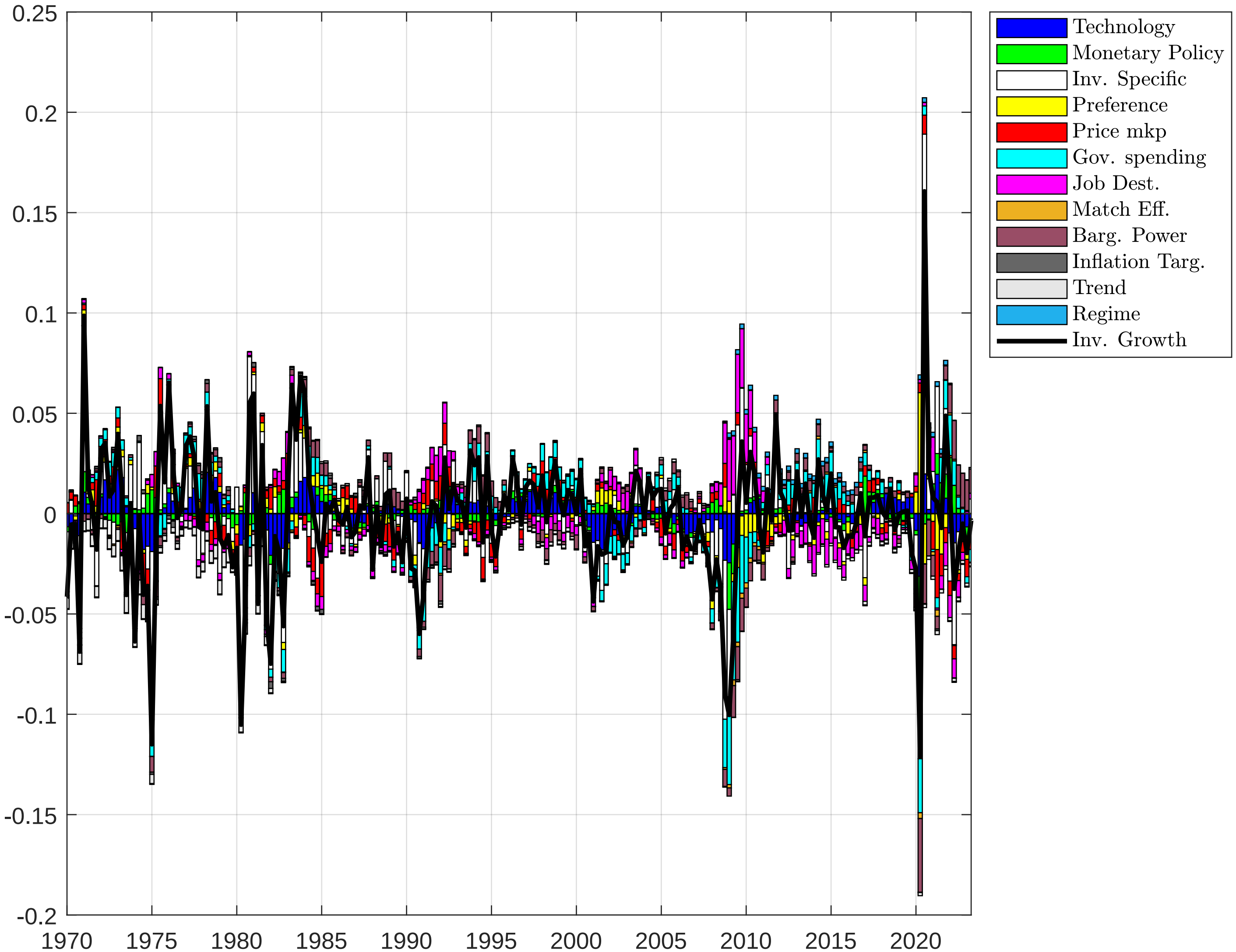


Figure 3: Historical decomposition of the real investment growth rate

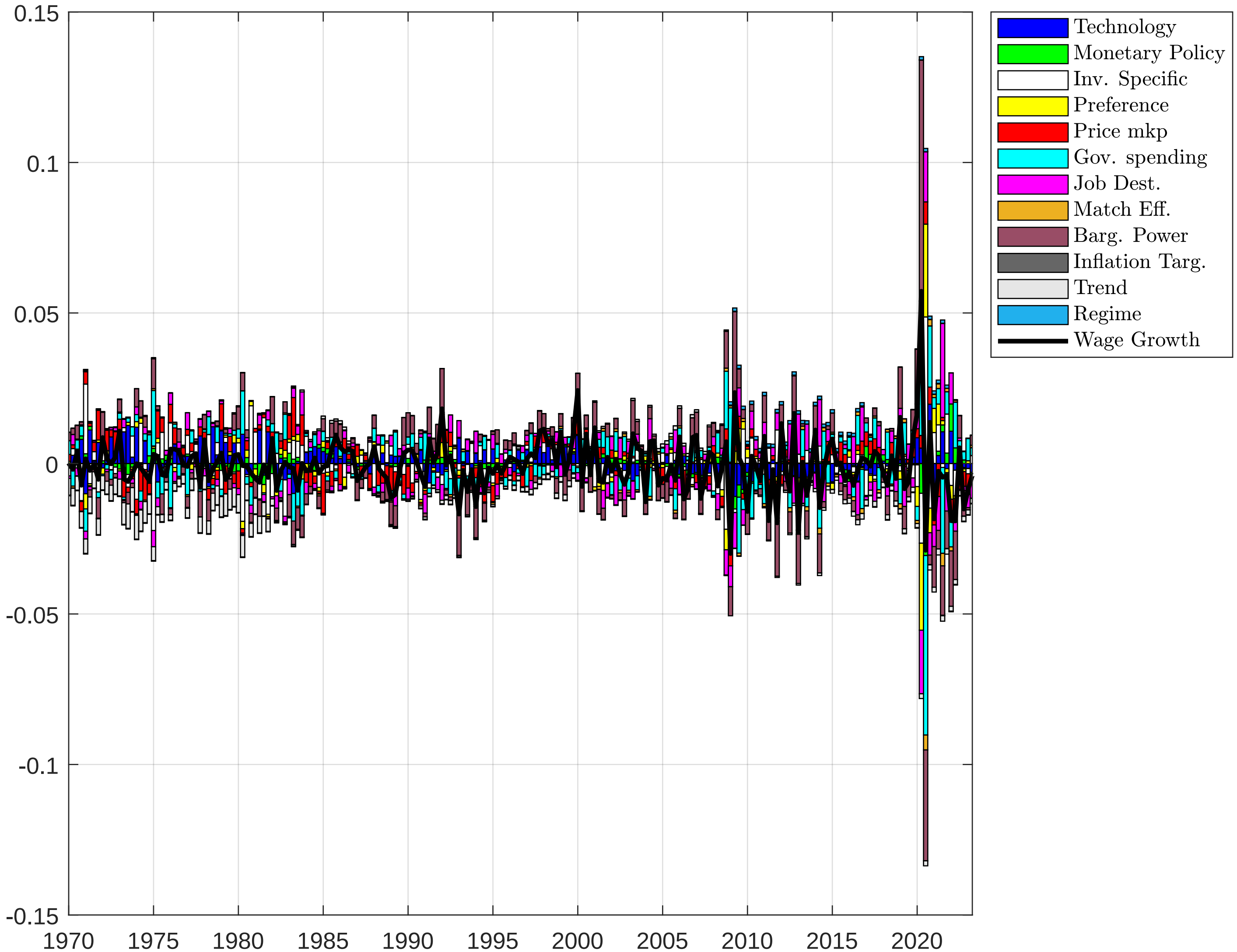


Figure 4: Historical decomposition of the (demeaned) real wage growth rate

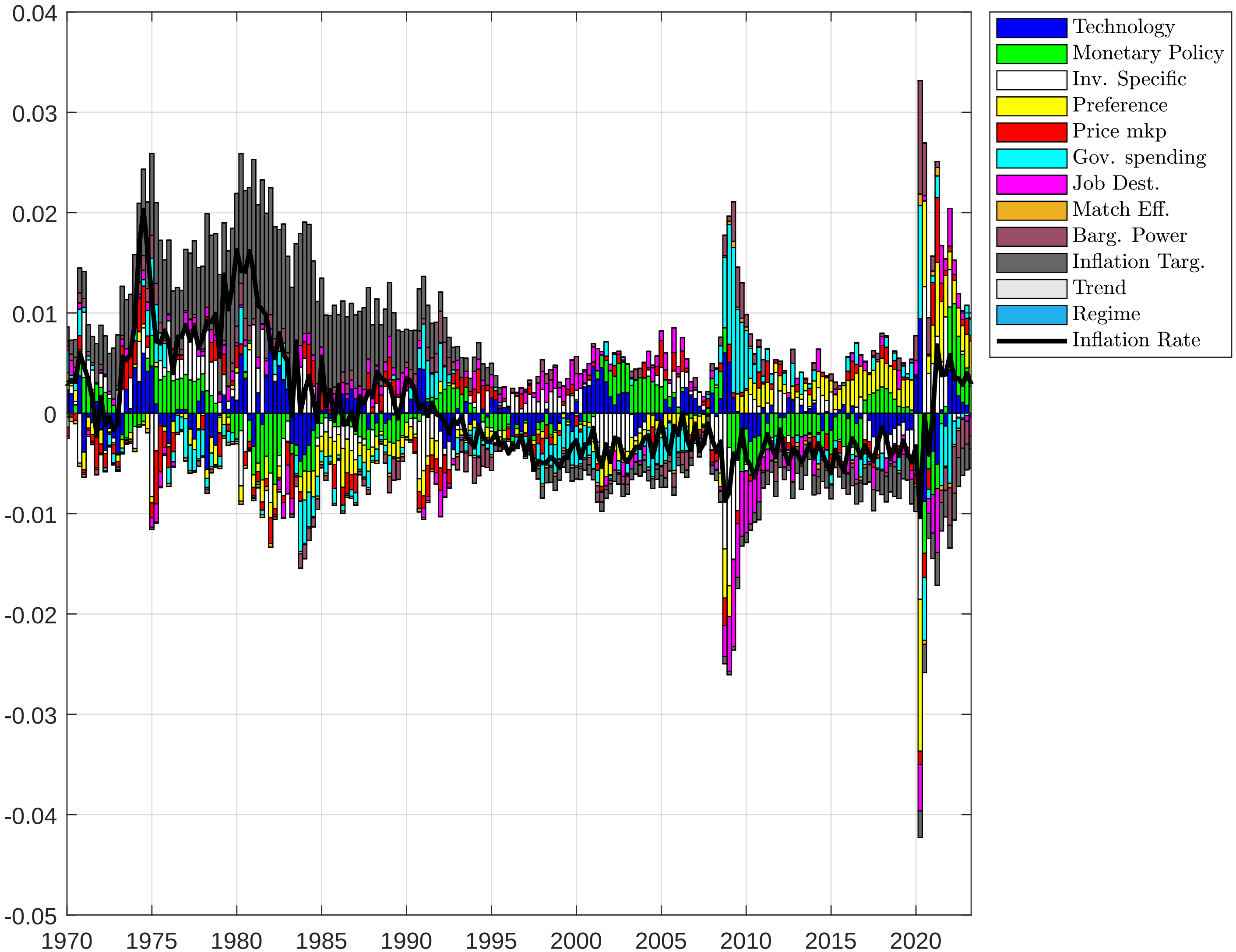


Figure 5: Historical decomposition of the (demeaned) core PCE inflation rate

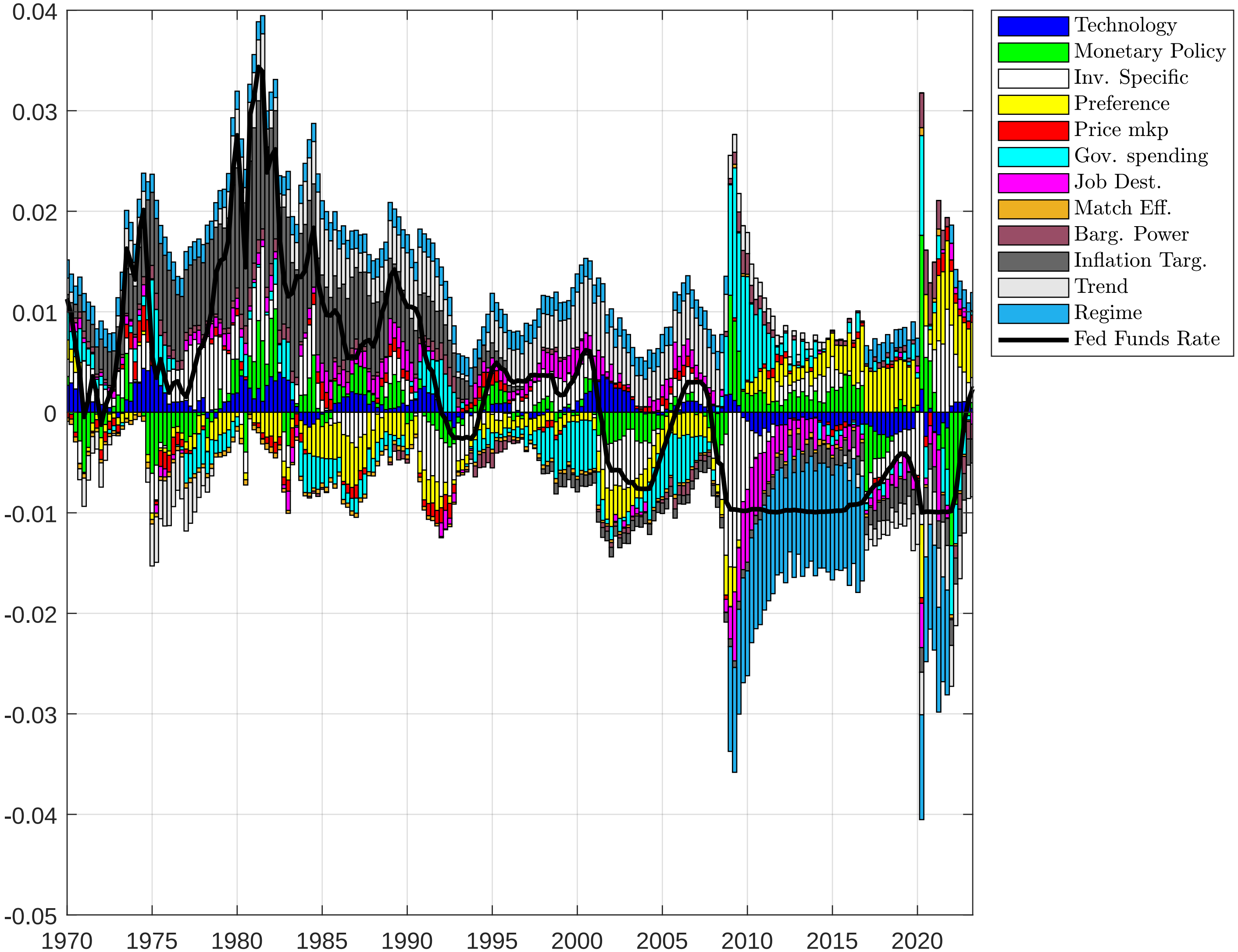


Figure 6: Historical decomposition of the (demeaned) fed funds rate

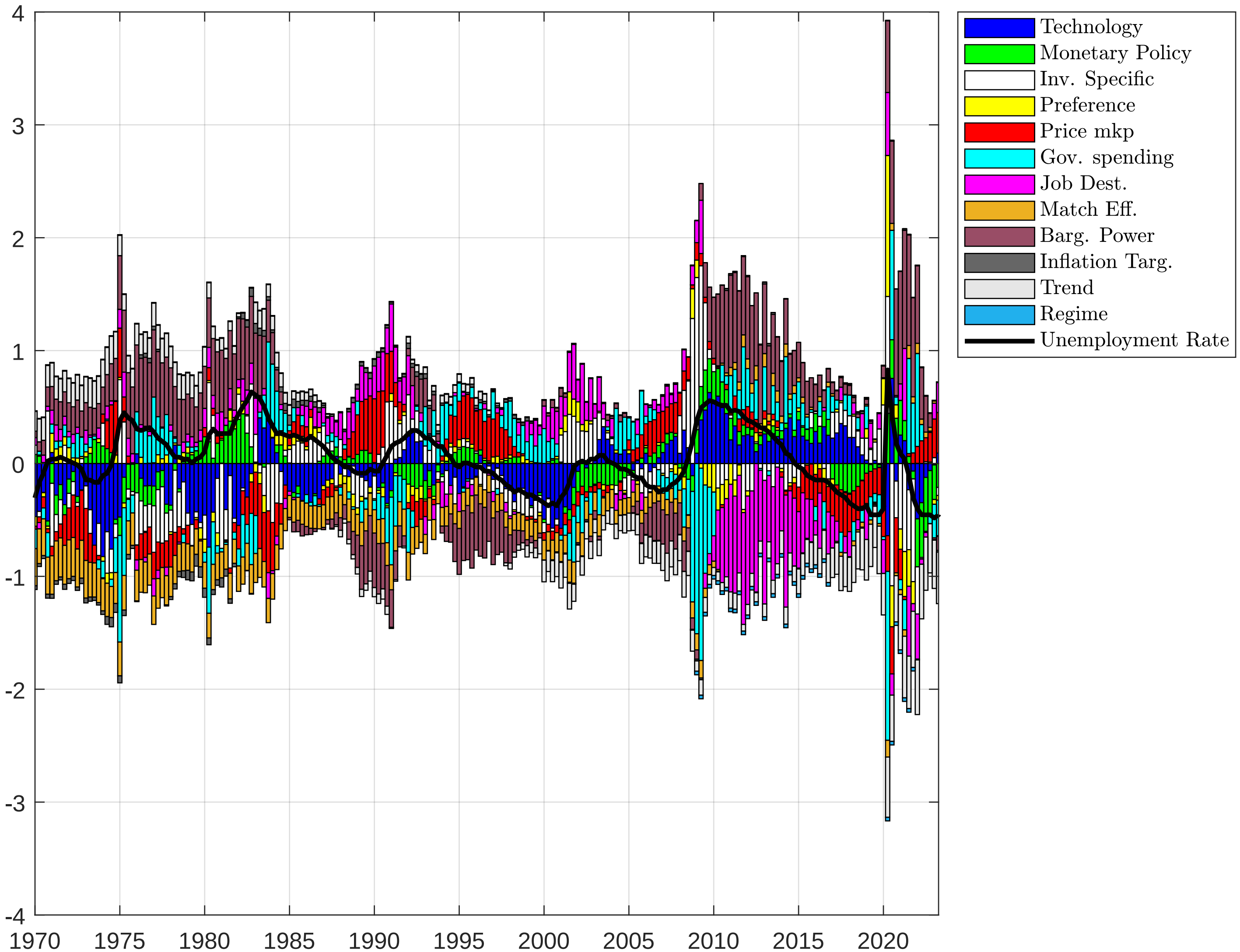


Figure 7: Historical decomposition of the (demeaned) unemployment rate

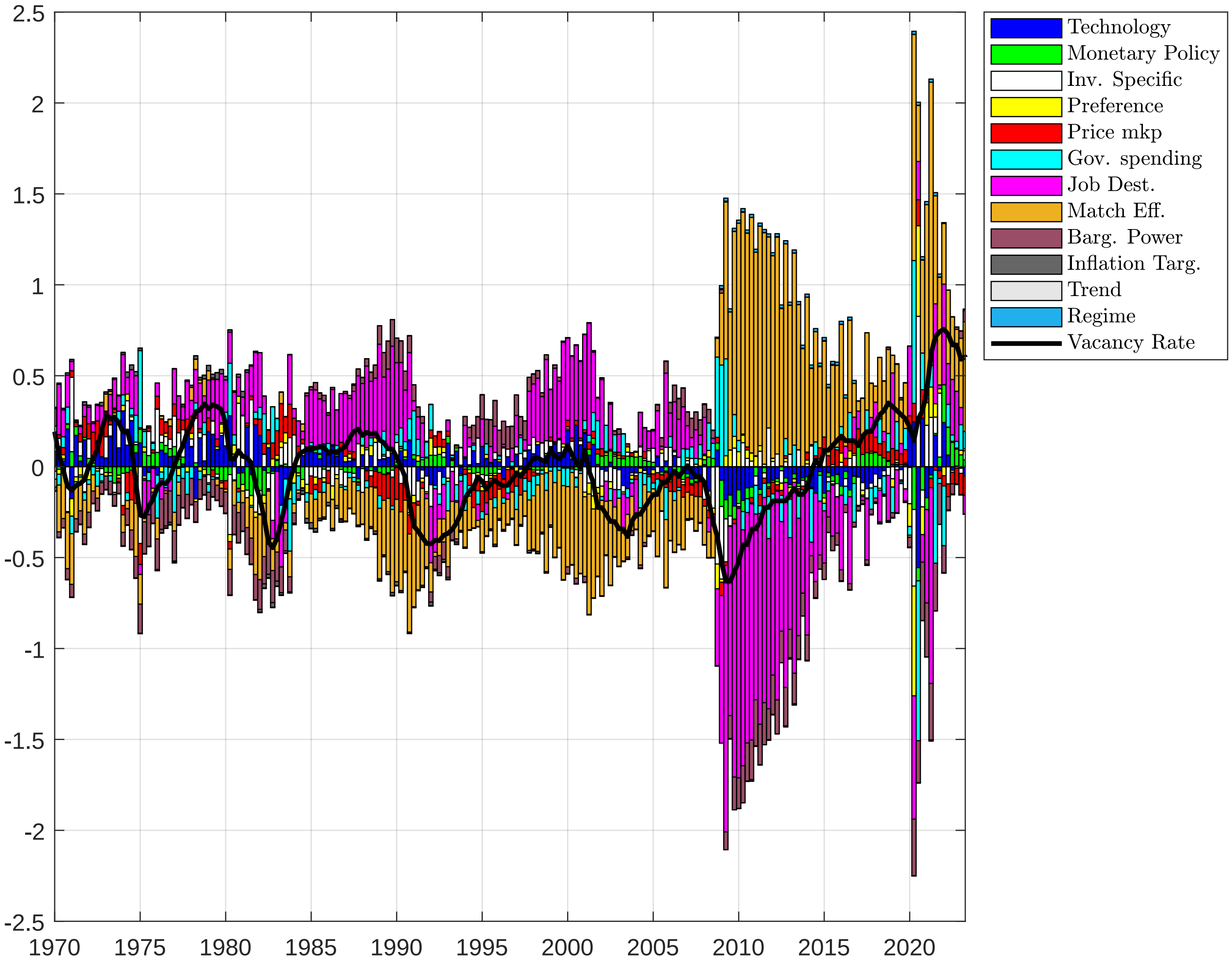


Figure 8: Historical decomposition of the (demeaned) vacancy rate

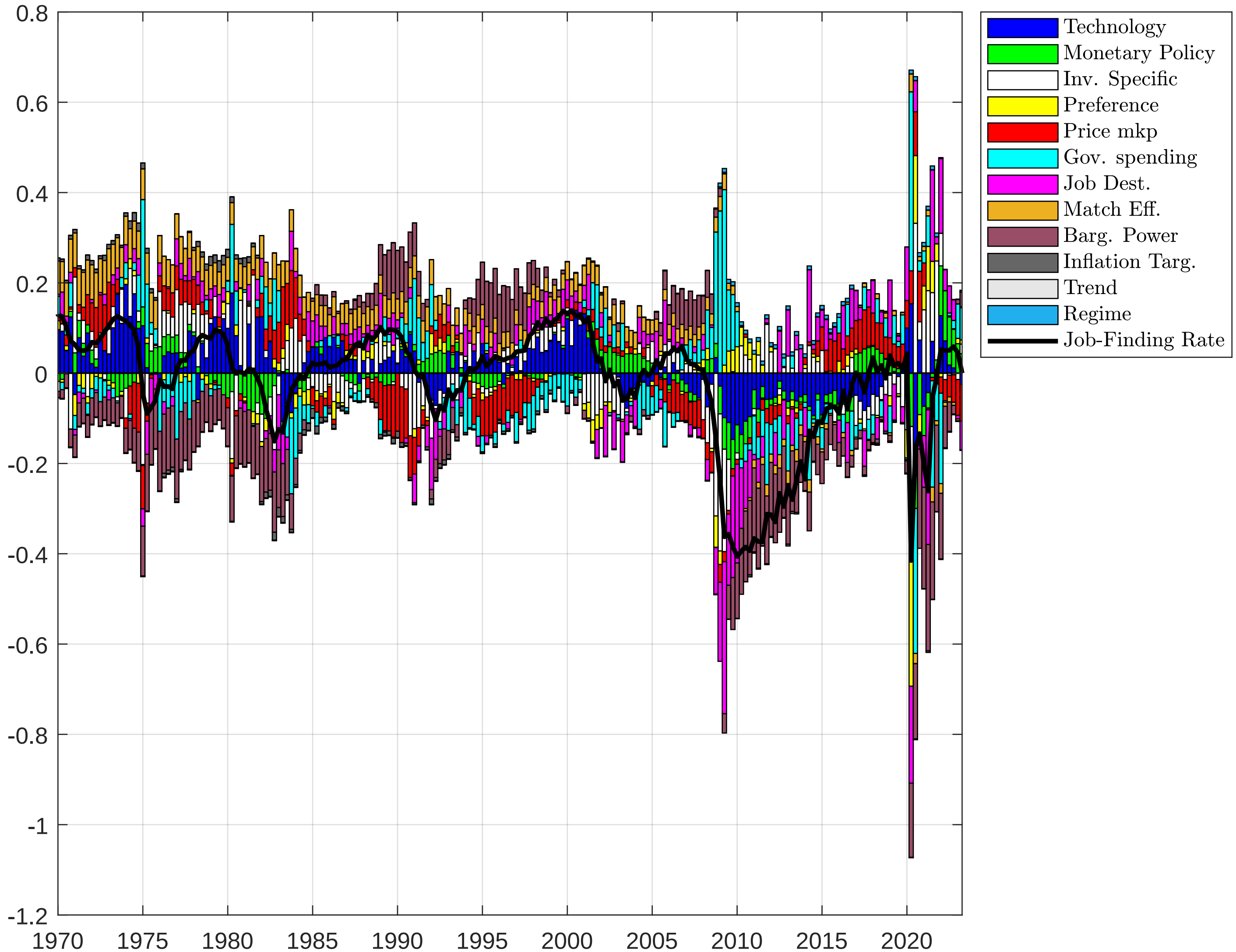


Figure 9: Historical decomposition of the (demeaned) job-finding rate

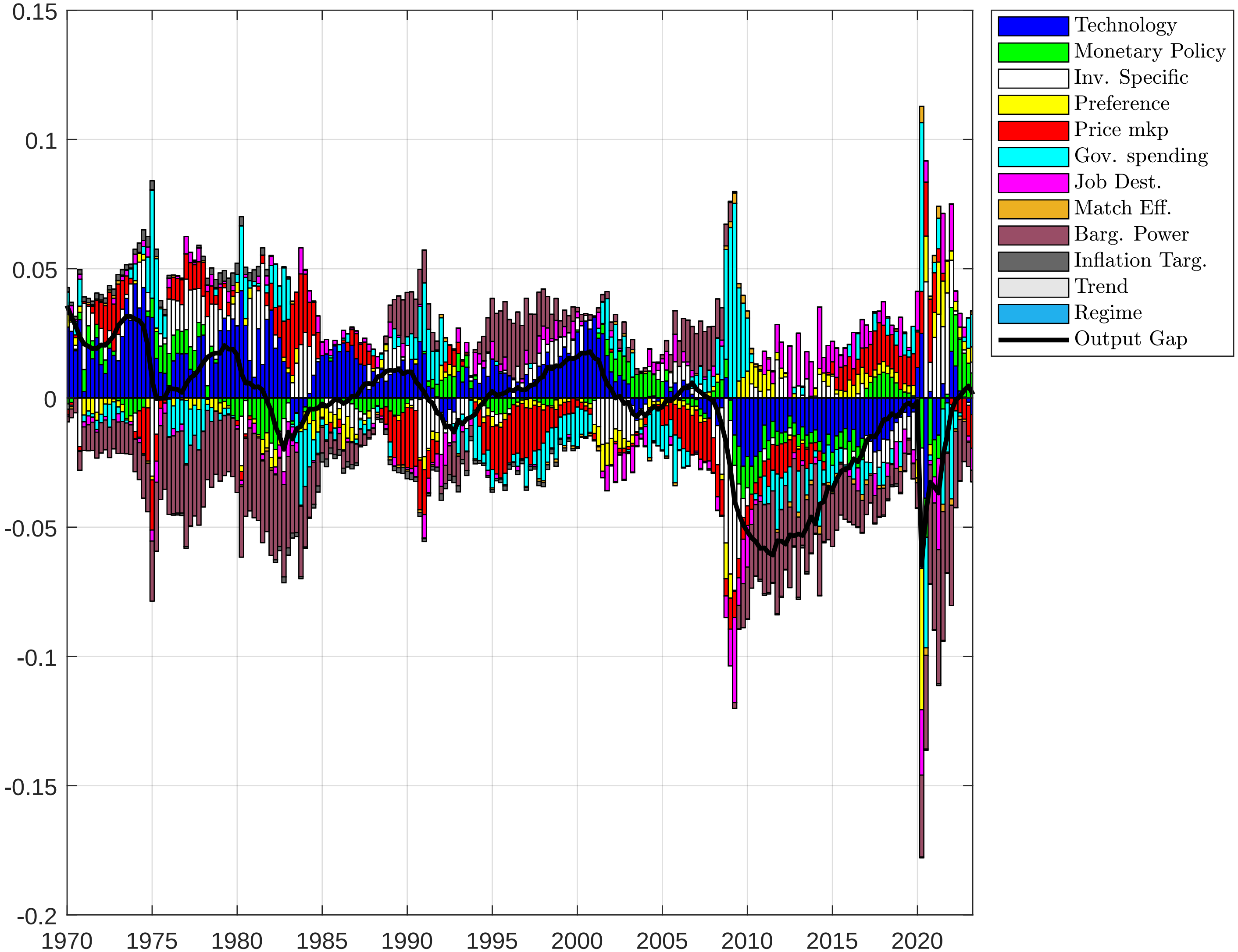


Figure 10: Historical decomposition of the (demeaned) output gap

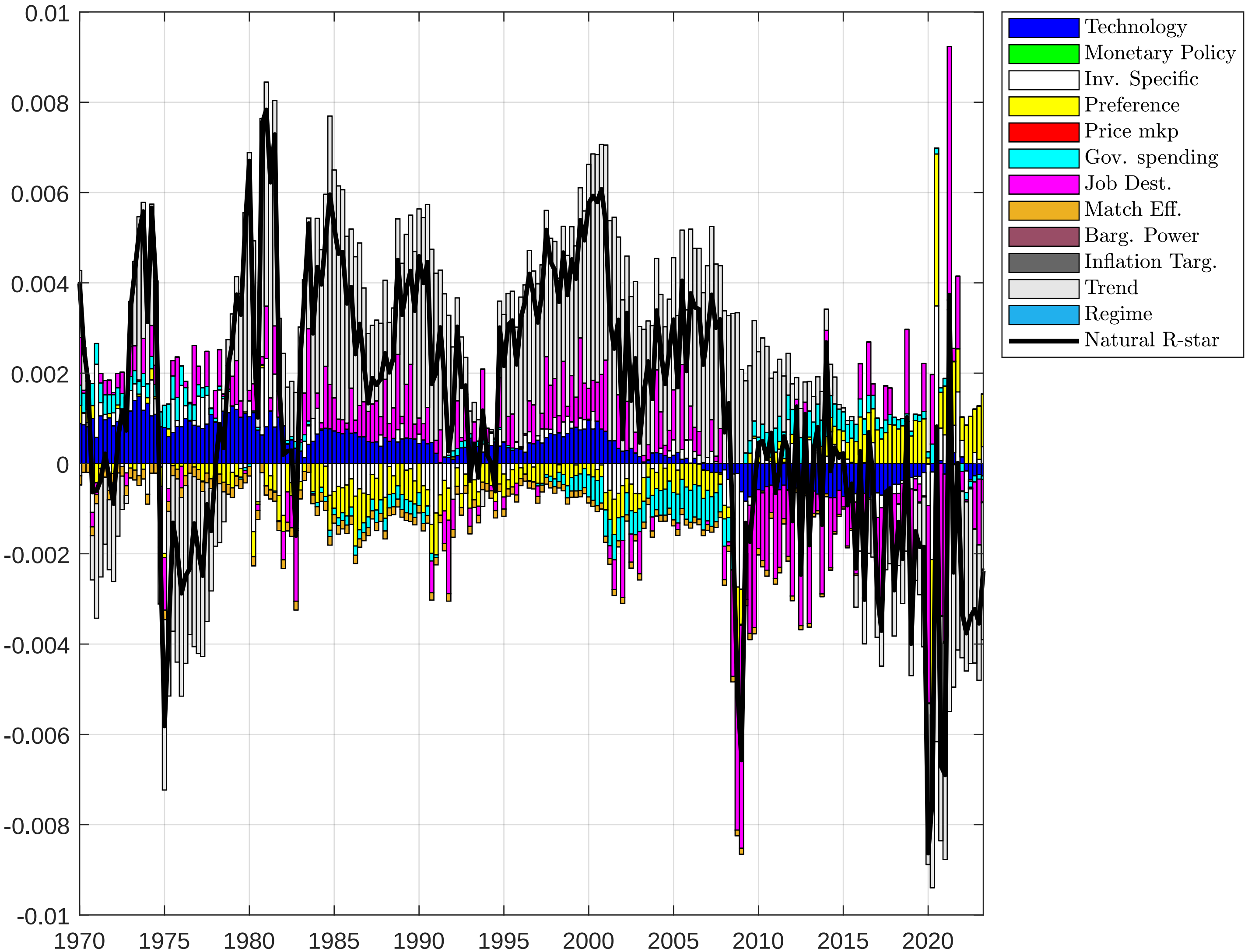
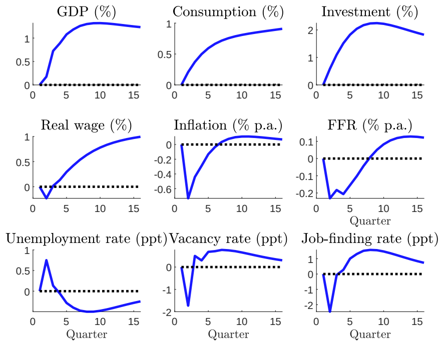
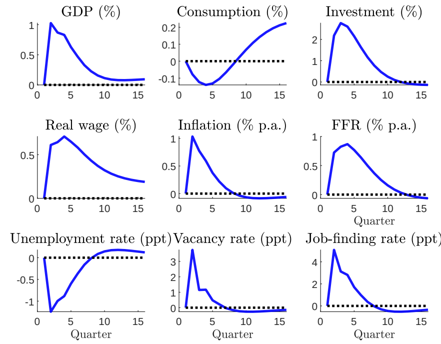


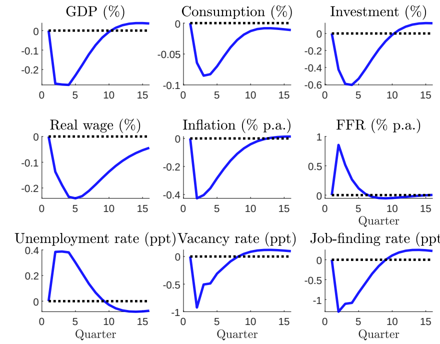
Figure 11: Historical decomposition of the (demeaned) natural r-star



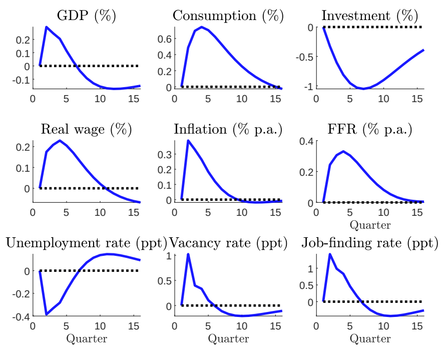
(a) *Technology shock.*



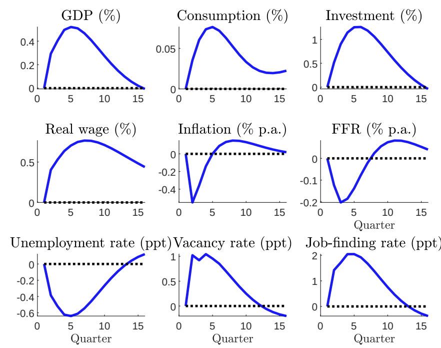
(b) *Investment-specific shock.*



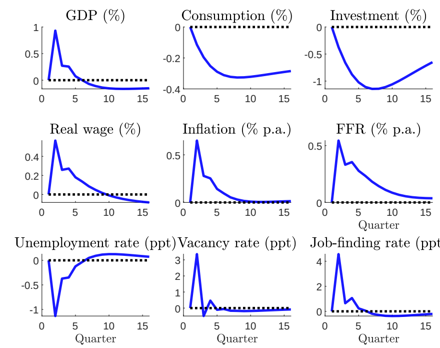
(c) *Monetary policy shock.*



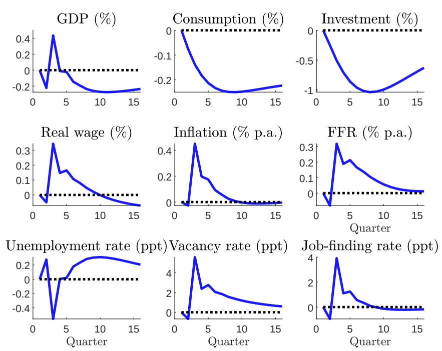
(d) *Preference shock.*



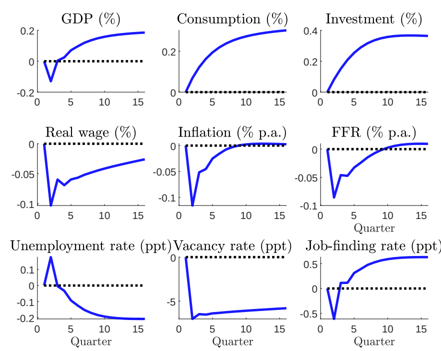
(e) *Markup shock.*



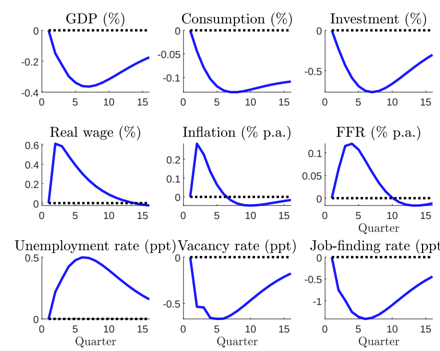
(f) *Government spending shock.*



(g) *Job separation shock.*

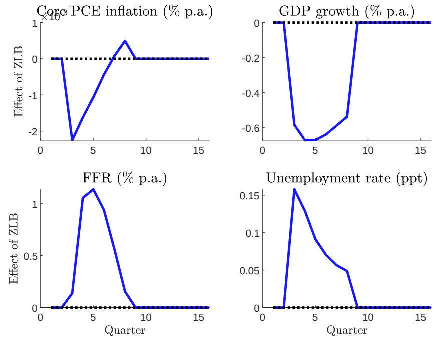


(h) *Match efficiency shock.*

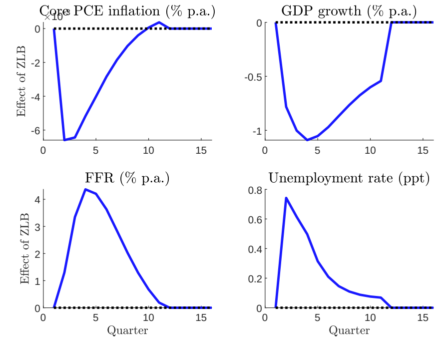


(i) *Bargaining power shock.*

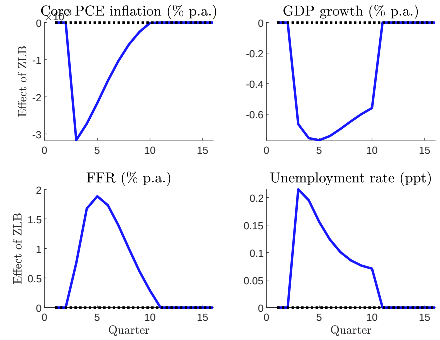
Figure 12: Impulse responses in the one-regime version of the model to 1-standard deviation shocks



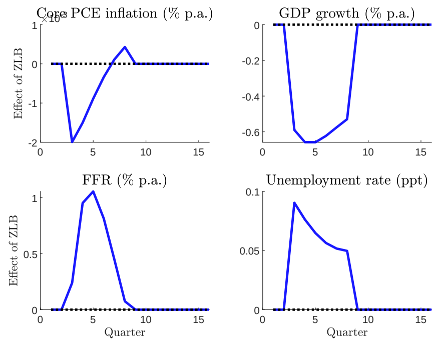
(a) *Technology shock.*



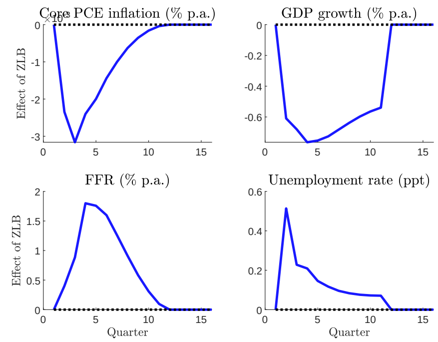
(b) *Investment specific shock.*



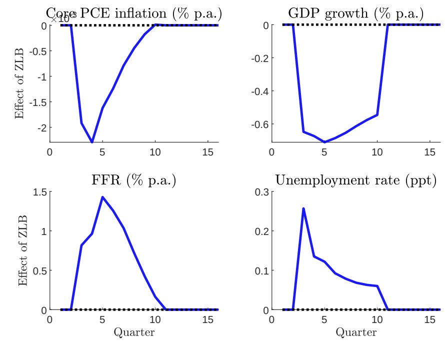
(c) *Preference shock.*



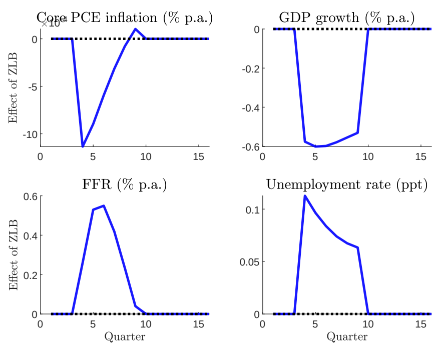
(d) *Markup shock.*



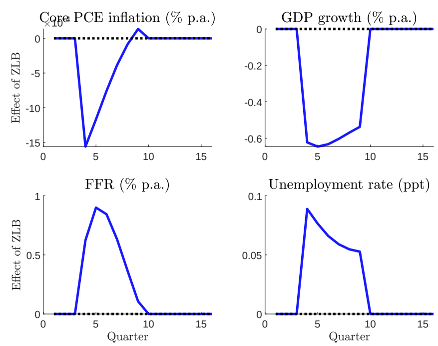
(e) *Government spending shock.*



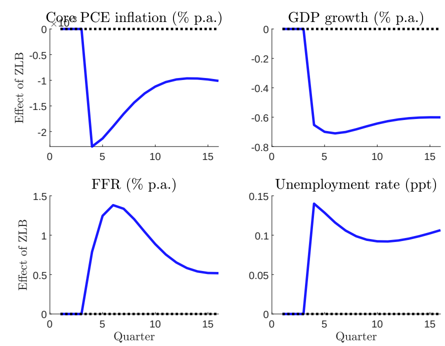
(f) *Job separation shock.*



(g) *Match efficiency shock.*



(h) *Bargaining power shock.*



(i) *Inflation target shock.*

Figure 13: Effect of the zero-lower-bound constraint on the response of the economy to different shocks

Note: Impulse responses in the two-regime model minus impulse responses in the one-regime model. The economy starts at the steady state and is hit in period 1 by a persistent, three-standard-deviation monetary policy shock that brings the federal funds rate to a level close to zero but still strictly positive. On top of that scenario, we add the indicated shocks, which are single five-standard-deviation shocks that hit the economy in period 1 and trigger the zero-lower-bound constraint.

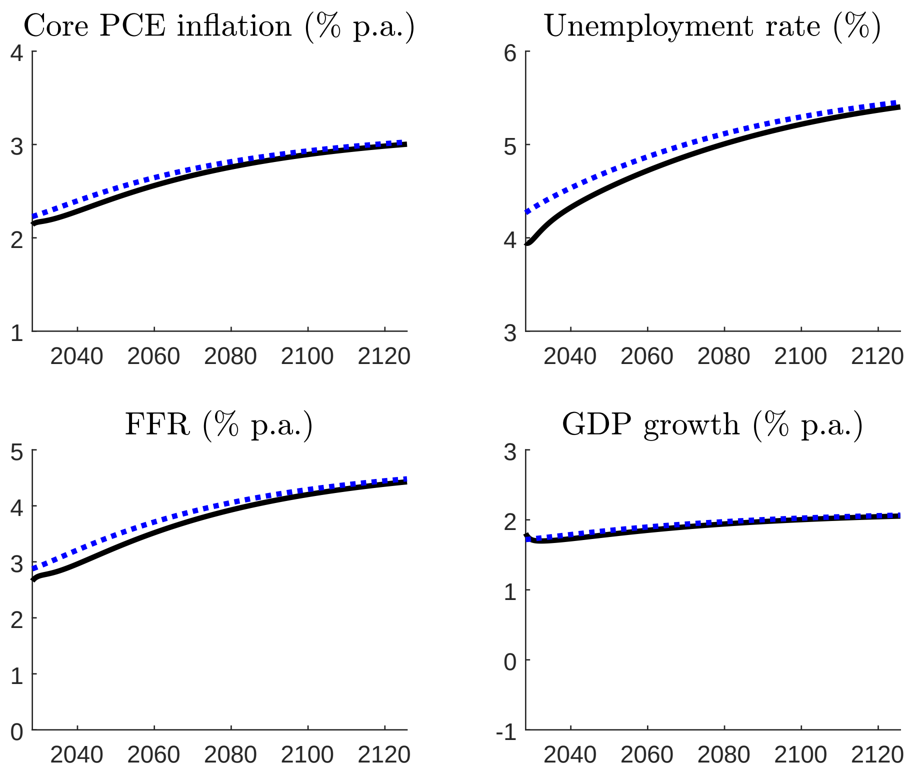


Figure 14: Long-run forecasts

Note: Long-run forecasts starting from observations up to 2023q1 (black solid lines) and corresponding long-run expectations predicted by the model (dashed blue lines).