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Business Cycles and Low-Frequency Fluctuations in the US Unemployment Rate*

Kurt G. Lunsford†

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Abstract

I show that business cycles can generate most of the low-frequency movements in the unemployment rate. First, I provide evidence that the unemployment rate is stationary, while its flows have unit roots. Then, I model the log unemployment rate as the error correction term of log labor flows in a vector error correction model (VECM) with intercepts that change over the business cycle. Feeding historical expansions and recessions into the VECM generates large low-frequency movements in the unemployment rate. Frequent recessions from the late 1960s to the early 1980s interrupt labor market recoveries and ratchet the unemployment rate upward. Long expansions in the 1980s and 1990s undo this upward ratcheting. Finally, the VECM predicts that the unemployment rate will be near 3.6 percent after a 10-year expansion and that lower unemployment rates are possible with longer expansions.

Keywords: cointegration, common trend, HP trend, labor flows, longer-run unemployment rate

JEL Codes: C32, E24, E32, J64

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†Federal Reserve Bank of Cleveland, P.O. Box 6387, Cleveland, OH 44101; kurt.lunsford@clev.frb.org

1 Introduction

The US unemployment rate is an important business cycle indicator. It is closely monitored by policymakers and widely researched. However, the unemployment rate appears to contain a slow-moving trend. Hence, researchers and policymakers typically adjust the unemployment rate before assessing its business cycle features. Researchers often remove a Hodrick and Prescott (1997) (HP) trend before studying the unemployment rate.[1] Similarly, policymakers often remove a time-varying unemployment rate benchmark before assessing the state of the economy (Crump, Nekarda, and Petrosky-Nadeau, 2020).

In this paper, I study the unemployment rate's low-frequency movements, defined as HP trend movements computed with standard or larger HP parameter values or as movements at frequencies lower than business cycle frequencies. I provide evidence that the US unemployment rate is stationary, implying that it does not have a stochastic trend.[2] However, the unemployment rate does have large low-frequency movements, giving the appearance of a trend. I show that these low-frequency movements can be attributed to business cycles: the unemployment rate's rapid increases in expansions and slow decreases in recessions coupled with the pattern of US business cycles can generate the low-frequency movements. These findings suggest that researchers and policymakers may be removing business cycle features instead of a non-business cycle trend when removing an HP trend or time-varying benchmark from the unemployment rate.

I begin my analysis in Section 2 by testing for unit roots in the unemployment inflow hazard rate (s_t), the unemployment outflow hazard rate (f_t), and the unemployment churn rate ($s_t + f_t$). Collectively, I refer to these rates as "labor flows." While I find evidence of unit roots in the levels and logs of these labor flows, I do not find evidence of a unit root in the level or log of the unemployment rate. As in Hall (2005) and Elsby, Michaels, and Solon (2009), I model the unemployment rate with $u_t = s_t/(s_t + f_t)$ so that taking logs implies $\ln(u_t) = \ln(s_t) - \ln(s_t + f_t)$. Hence, a stationary log unemployment rate implies that the log inflow hazard rate and the log churn rate are cointegrated (Granger, 1981; Engle and Granger, 1987).

This analysis has two implications. First, the log unemployment inflow hazard rate and the

[1] Ferraro (2018), Lepetit (2020), and Gornemann, Kuester, and Nakajima (2021) are recent examples that I cite below. As an alternative to HP filtering, Clark (1989), Kim and Nelson (1999), and Sinclair (2009) separate the unemployment rate's trend and businesses cycle components with unobserved components models.

[2] The stochastic trend is a random walk component of a non-stationary variable that is the "the mid-point of the predictive distribution for the future path" of the variable (Beveridge and Nelson, 1981). This stochastic trend may also be called a "non-stationary trend" as in Clark (1989), a "permanent component" as in Sinclair (2009), or a "secular trend" as in Crump et al. (2019).

log unemployment churn rate being cointegrated means that they share a common stochastic trend (Stock and Watson, 1988). Second, the log unemployment rate being stationary means that it does not have a stochastic trend. The linear combination of the log flows, $\ln(s_t) - \ln(s_t + f_t)$, eliminates the stochastic trend so that it is not passed to the log unemployment rate.

The rest of the paper imposes this cointegration relationship and then answers two questions. First, if the unemployment rate is stationary, then what accounts for its low-frequency movements? Second, if the unemployment rate does not have a stochastic trend, then what should policymakers use as an unemployment rate benchmark?

I answer these questions with a time-series model that I build in Section 3. I model the log unemployment inflow hazard rate and the log unemployment churn rate with a vector error correction model (VECM), yielding the log unemployment rate as the error correction term. Following Hamilton's (2005) model for the unemployment rate, intercepts in the VECM vary over the business cycle, allowing the labor flows and the unemployment rate to display asymmetric business cycle behavior. In particular, unemployment can rise quickly in recessions and fall slowly in expansions as documented in Neftçi (1984), Sichel (1993), Hamilton (2005), McKay and Reis (2008), Ferraro (2018), and Dupraz, Nakamura, and Steinsson (2021).

In Section 4, I show that business cycles generate most of the unemployment rate's low-frequency movements in the VECM. I feed the historical business cycle pattern into the VECM but set the other VECM innovations to zero, producing "business-cycle-only" estimates of the labor flows and the unemployment rate. While the business-cycle-only estimates of the labor flows match the business cycle features of those flows, they do not match the low-frequency movements of those flows. In contrast, the business-cycle-only unemployment rate matches both the business cycle and low-frequency movements of the actual unemployment rate. In particular, the low-frequency pattern of the business-cycle-only unemployment rate aligns closely with the low-frequency pattern of the actual unemployment rate from the late 1960s to the late 1990s. The only material disagreement between the low-frequency patterns occurs in 2008-09 and some subsequent years.

The interaction of unemployment asymmetries and the pattern of US business cycles generates large low-frequency movements. As I discussed in Lunsford (2021), the unemployment rate's rapid rise in a recession followed by a slow fall in an expansion implies that the unemployment rate may not fall to its previous low point if a new recession cuts an expansion short, causing the unemployment rate to begin the new recession at a higher level. Hence, frequent recessions separated by short expansions, such as from 1969 through 1982, lead to an upward ratcheting of the unemployment rate. Conversely, long expansions interrupted by short recessions, such as from 1983 through 1999, undo this upward ratcheting. The resulting low-frequency movements are large. With standard low-frequency parameters, the unemployment rate's low-frequency trends rise about 4 percentage points from 1969 through 1982 and fall by similar amounts from 1983 through 1999.

Crump, Nekarda, and Petrosky-Nadeau (2020) note that policymakers often use a "longer-run unemployment rate" benchmark, defined as the rate that is expected to prevail after adjusting for business cycle shocks, when assessing the economy. Earlier research, such as Tasci (2012) and Crump et al. (2019), estimates this longer-run rate with the unemployment rate's stochastic trend. But if the unemployment rate does not have a stochastic trend, then what should policymakers use as a longer-run benchmark? In Section 5, I interpret "after adjusting for business cycle shocks" to mean where the unemployment rate will go as the economy stays in expansion and the VECM innovations are fixed at zero. That is, I compute a forecast conditional on the economy staying in expansion. Using the unemployment rate peaks in 1982, 1992, 2003, and 2009 as initial conditions, the VECM predicts that the unemployment rate will be between 3.3 and 3.9 percent after 10 years of an uninterrupted expansion, with lower values possible for longer expansions.

I highlight three implications of my findings. First, the unemployment rate is stationary but business cycles can generate large low-frequency movements. Hence, removing low-frequency trends from the unemployment rate, such as the HP trend, may remove business cycle features. Second, the unemployment rate does not have a stochastic trend, so time-varying trends should not be used as longer-run unemployment rate benchmarks. However, my VECM can estimate a longer-run unemployment rate benchmark that could be used by policymakers. Third, the benchmark that I produce is different than what Crump, Nekarda, and Petrosky-Nadeau (2020) call a "stable-price unemployment rate" and inflation does not appear in my model. However, the longer-run and stable-price unemployment rate concepts are related, and researchers may estimate a time-varying longer-run unemployment rate or unemployment rate trend as a step for estimating stable-price unemployment rates.[3] My finding that the unemployment rate is stationary suggests that earlier research that removes a time-varying unemployment rate trend when estimating a stable-price unemployment rate may be mis-specified.

Related Literature: I build on Tasci (2012), Barnichon and Mesters (2018), Crump et al.

[3] Crump et al. (2019) estimate a stable-price unemployment rate in two steps. First, they compute an unemployment rate trend using a time-series model of labor flows. Second, they adjust this trend using Phillips curves. Staiger, Stock, and Watson (2001) previously used these two steps but with different methods for the unemployment rate trend and Phillips curve adjustments.

(2019), and Hornstein and Kudlyak (2020), who model stochastic trends in labor flows and then map those trends into the unemployment rate. Importantly, these papers model stochastic trends with independent processes for different labor flows, yielding a stochastic trend in the unemployment rate. In contrast, I model the log unemployment inflow hazard rate and the log unemployment churn rate as sharing a common stochastic trend that yields a stationary unemployment rate. With independent trends, Crump et al. (2019) and Hornstein and Kudlyak (2020) argue that a low trend inflow rate pushed the unemployment rate trend lower in the late 2010s. However, with a common stochastic trend, this explanation does not work because the inflow hazard rate trend is offset by the unemployment churn rate trend. Rather, the unemployment rate was low in the late 2010s because the economic expansion was very long.

This paper also builds on a time-series literature that models asymmetries in the unemployment rate, including Hansen (1997), Montgomery et al. (1998), Rothman (1998), Kim and Nelson (1999), van Dijk, Franses, and Paap (2002), and Hamilton (2005). The important distinction is that I do not model the unemployment rate directly. Rather, I model the labor flows and then treat the unemployment rate as a stationary function of the flows. Modeling the flows may better fit the dynamics of the labor market and has been shown to be useful for real-time forecasting (Barnichon and Nekarda, 2012; Meyer and Tasci, 2015).

My results are closely related to those in Fatás (2021) and Hall and Kudlyak (2022a,b), who document and discuss the slow but steady unemployment rate declines in US expansions. Mechanically, these slow and steady declines imply that long expansions lead to low unemployment rates and the perception of a downward trend. However, if expansions are regularly interrupted by recessions and left incomplete, the perception of an upward trend can emerge. Fatás (2021) writes that expansions in the US have often been incomplete, implying that the unemployment rate has rarely been close to its potential low.

While I focus on time-series modeling, features of my VECM are consistent with structural models of the labor market. The structural models of Andolfatto (1997), Ferraro (2018), and Dupraz, Nakamura, and Steinsson (2021) generate rapid increases in unemployment with slow decreases. Importantly, Hairault, Langot, and Osotimehin (2010), Jung and Kuester (2011), Benigno, Ricci, and Surico (2015), Lepetit (2020), Dupraz, Nakamura, and Steinsson (2021), and Gornemann, Kuester, and Nakajima (2021) show that the volatility of shocks or the ability of policymakers to stabilize the economy affects the average level, not just the volatility, of the unemployment rate. As in my VECM, these papers indicate that where the unemployment rate will go after removing the effects of shocks is much lower than its historical average. Further, as shown in Dupraz, Nakamura, and Steinsson (2021, Figure 4), clusters of recessions and expansions can generate low-frequency movements in the unemployment rate even when the unemployment rate is stationary.

Finally, my paper is related to a literature that emphasizes demographics as affecting the unemployment rate at low frequencies. Demographics cause all of the movements in the Congressional Budget Office's natural rate of unemployment (Shackleton, 2018, Appendix B). Aaronson et al. (2015), Barnichon and Mesters (2018), Crump et al. (2019), Tüzemen (2019), and Fallick and Foote (2022) also highlight demographics. In contrast, I highlight that business cycles can account for most of the low-frequency movements in the unemployment rate, allowing little role for demographics. However, demographics may still drive the low-frequency movements in labor flows because business cycles account for little of the low-frequency movements in these flows. An implication of my findings is that the demographic effects on the labor flows should offset and not affect unemployment.

2 US Labor Market Data and Tests for Unit Roots

2.1 US Labor Market Data

I jointly study the unemployment rate (u_t), the unemployment inflow hazard rate (s_t), and the unemployment outflow hazard rate (f_t). I follow Shimer (2012) and Elsby, Michaels, and Solon (2009) to compute the inflow and outflow hazard rates, giving details of the source data and computations in Appendix A. Following Hall (2005) and Elsby, Michaels, and Solon (2009), I approximate the unemployment rate with $u_t = s_t/(s_t + f_t)$ to provide a tractable relationship between the hazard rates and the unemployment rate. I then use

$$\ln(u_t) = \ln(s_t) - \ln(s_t + f_t) \tag{1}$$

to model the relationship between the unemployment rate and the hazard rates.

Figure 1 shows the unemployment rate, both hazard rates, and the sum of the hazard rates (the unemployment churn rate) from January 1954 through December 2019. I begin the sample in 1954 to remove the effects of the Korean War draft.[4] I end the sample in 2019 to remove the

[4]The number of people inducted into military service through the US Selective Service System was about 550,000 in 1951, 440,000 in 1952, and 470,000 in 1953 (Selective Service System, 2022). While military induction continued after 1953, it was no higher than about 250,000 people until 1966.

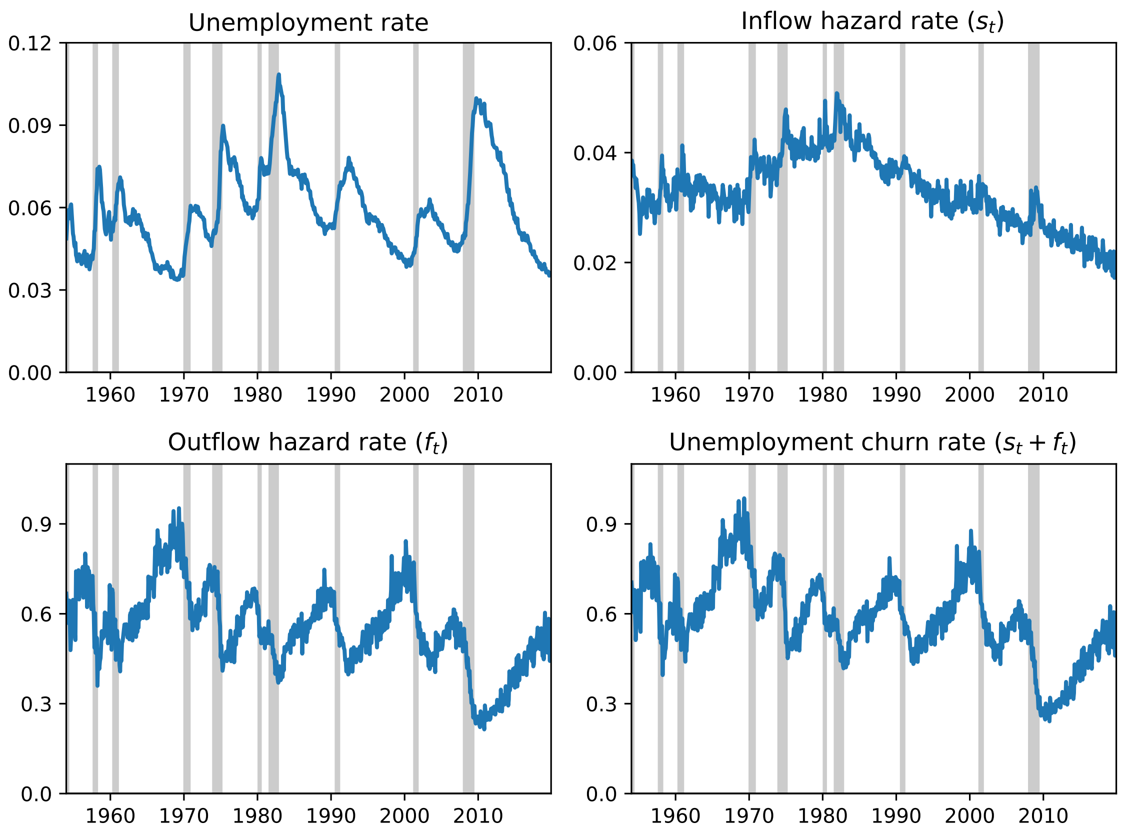

Figure 1: Labor market data from January 1954 through December 2019.
Note: Gray bars show NBER recessions.

unusually large volatility of the COVID-19 pandemic.[5] I highlight three features of the data. First, the inflow hazard rate appears to have a slow-moving trend, drifting up from 1954 until the early 1980s and then drifting back down through 2019. Second, the outflow hazard rate is highly cyclical but also appears to have a slow-moving trend: the peak values at the end of the long expansions in the 1960s, the 1990s and the 2010s become subsequently lower. Third, because the outflow rate is generally between 10 and 30 times larger than the inflow rate, the unemployment churn rate looks very similar to just the outflow rate. I refer to this third feature of the data for several of the results that I discuss below.

Figure 2 shows the unemployment rate and its approximation, $s_t/(s_t + f_t)$. While the unemployment rate approximation introduces some high-frequency noise, it closely matches the unemployment rate's business cycle and low-frequency movements, making it useful for studying these unemployment rate movements.

[5] In Appendix B, I show all figures in this section with data running through May 2023.

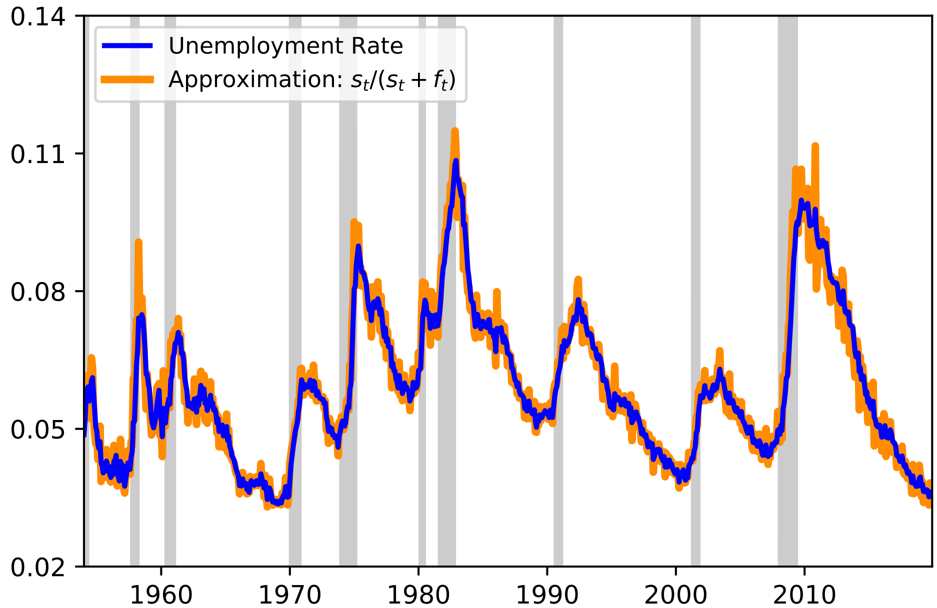

Figure 2: The unemployment rate and its approximation.
Note: Gray bars show NBER recessions.

2.2 Testing for Unit Roots and Cointegration

Earlier papers that model the trend in the unemployment rate assume that the labor flows have unit root components (Tasci, 2012; Barnichon and Mesters, 2018; Crump et al., 2019). This assumption is consistent with the visual evidence in Figure 1.[6] However, these papers also assume that the unit root components are independent across each flow, permitting a unit root in the unemployment rate. In this section, I provide evidence that unit root components are indeed present in the labor flows. However, I fail to find evidence that u_t, $\ln(u_t)$, and $\ln(s_t) - \ln(s_t + f_t)$ have unit root components. Hence, $\ln(s_t)$ and $\ln(s_t + f_t)$ have a cointegrating relationship and a common stochatsic trend, implying that $\ln(u_t)$ does not inherit a stochastic trend from the labor flows.

I follow Müller and Watson (2008, 2013) for unit root testing: I extract low-frequency information from the data by computing a small number of weighted averages and then form tests from those weighted averages. Let x_t be a 1-dimensional variable and $\{x_1, \ldots, x_T\}$ be the observed sample. I compute $T^{-1} \sum_{t=1}^{T} \psi_{j,t} x_t$ for $j = 1, \ldots, q$. The weights are slow-cycling cosine waves, $\psi_{j,t} = \sqrt{2} \cos(\pi j(t - 1/2)/T)$, and so the weighted averages are called "discrete cosine transforms."

[6] Tasci (2012) and I use the same hazard rates. Barnichon and Mesters (2018) and Crump et al. (2019) further decompose the hazard rates into demographic-level hazard rates and impose unit root components at the demographic-level. In addition, Crump et al. (2019) assume that the demographic level inflow rates are integrated of order 2.

Table 1: p-Values for the LFST statistics

Variable	$q = 13$	$q = 14$	$q = 15$	$q = 16$
s_t	0.00	0.00	0.00	0.00
f_t	0.03	0.03	0.02	0.02
$s_t + f_t$	0.02	0.02	0.02	0.01
$\ln(s_t)$	0.00	0.00	0.00	0.00
$\ln(f_t)$	0.03	0.02	0.02	0.02
$\ln(s_t + f_t)$	0.02	0.01	0.01	0.01
Δs_t	0.52	0.47	0.45	0.52
Δf_t	0.96	0.98	0.97	0.96
$\Delta(s_t + f_t)$	0.96	0.98	0.97	0.96
$\Delta \ln(s_t)$	0.39	0.36	0.34	0.42
$\Delta \ln(f_t)$	0.93	0.96	0.94	0.94
$\Delta \ln(s_t + f_t)$	0.92	0.96	0.94	0.94
u_t	0.38	0.39	0.40	0.35
$\ln(u_t)$	0.40	0.39	0.37	0.33
$\ln(s_t) - \ln(s_t + f_t)$	0.40	0.39	0.37	0.32
period of fastest-cycling cosine wave in years	10.2	9.4	8.8	8.3

Notes: The null hypothesis is that the data are stationary. The alternative hypothesis is that the data have a low-variance unit root component. p-values less than 0.10, 0.05, and 0.01 indicate rejection of the null hypothesis at the 10 percent, 5 percent, and 1 percent levels, respectively. The data sample is January 1954 through December 2019.

The jth cosine wave completes one cycle in $2T/j$ periods. With 66 years of data (1954 through 2019) and $q = 16$, the fastest cycling cosine wave completes one cycle in about 8.3 years. Hence, 16 discrete cosine transforms extract information at frequencies corresponding to 8.3 years and longer. I then compute a low-frequency stationarity test (LFST) statistic. The LFST is a likelihood ratio statistic computed from the cosine transforms that tests the null hypothesis that x_t is stationary against the alternative hypothesis that x_t has a low-variance unit root component.

Table 1 shows the p-values for the LFST statistics for different labor market variables and different values of q.[7] I provide details for computing the LFST statistics and the corresponding

[7]In the far right column of Table 1, $q = 16$ picks up frequencies corresponding to more than 8 years, which are the frequencies used in Müller and Watson (2008, 2013). As discussed in Müller and Watson (2015), a trade-off exists

p-values in Appendix C. Small values in Table 1 indicate rejection of the null of stationarity in favor of the alternative that the data have a low-variance unit root component. The sample for Table 1 is January 1954 through December 2019. Results for January 1954 through May 2023 are similar, and I show them in Appendix B.

The first six rows of Table 1 provide statistical evidence of a unit root component in the levels and logs of the labor flows. These results support the visual evidence in Figure 1 that stochastic trends exist in the flows and support earlier papers that model the flows with unit root processes.

The next six rows of Table 1 test for unit roots in the changes of the levels and logs of the labor flows, using the notation $\Delta x_t = x_t - x_{t-1}$ for any variable x_t. The idea is that if the flows have a unit root, then differencing the flows should eliminate the unit root and yield a stationary variable. I find that all of the differenced data fail to reject the null hypothesis of stationarity.

The last three rows of Table 1 test for unit roots in u_t, $\ln(u_t)$, and $\ln(s_t) - \ln(s_t + f_t)$ and fail to reject the null of stationarity. The failure to reject stationarity for $\ln(s_t) - \ln(s_t + f_t)$ provides evidence for a cointergation relationship between $\ln(s_t)$ and $\ln(s_t+f_t)$ based on Müller and Watson's (2013) low-frequency version of Wright (2000). That is, while $\ln(s_t)$ and $\ln(s_t + f_t)$ have unit root components, the linear combination $\ln(s_t) - \ln(s_t + f_t)$ is stationary.

3 A VECM for Labor Flows

Based on Equation (1) and the results in Table 1, I model the log of the unemployment rate as the error correction term $\ln(s_t) - \ln(s_t + f_t)$. In Subsection 3.1, I write down a VECM with intercepts that vary over the business cycle, modeling the time-variation with dummy variables. In Subsection 3.2, I discuss my choice of business cycle dummy variables. In Subsection 3.3, I show and discuss some estimated VECM parameters.

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when choosing q. A lower value uses lower-frequency information when testing and is less subject to mis-specification, while a higher value yields more powerful inference. To establish robustness, I show p-values in Table 1 for some values of q smaller than 16. These p-values are essentially unchanged for different values of q.

3.1 The VECM with Time-Varying Intercepts

I model the log of the unemployment inflow rate and the log of the unemployment churn rate with

$$\begin{aligned}\begin{bmatrix}\Delta \ln(s_t) \\ \Delta \ln(s_t + f_t)\end{bmatrix} &= \nu d_{t-1} + \alpha(\ln(s_{t-1}) - \ln(s_{t-1} + f_{t-1})) \\ &\quad + \Gamma_1 \begin{bmatrix}\Delta \ln(s_{t-1}) \\ \Delta \ln(s_{t-1} + f_{t-1})\end{bmatrix} + \cdots + \Gamma_p \begin{bmatrix}\Delta \ln(s_{t-p}) \\ \Delta \ln(s_{t-p} + f_{t-p})\end{bmatrix} + v_t,\end{aligned} \tag{2}$$

in which ν, α, and $\Gamma_1, \ldots, \Gamma_p$ are matrices that hold the parameters of the model. Based on the results in Table 1, I assume that $\Delta \ln(s_t)$, $\Delta \ln(s_t + f_t)$, and $\ln(s_t) - \ln(s_t + f_t)$ are stationary. I also assume that d_t is stationary and discuss this assumption following Equation (4). Finally, I assume $\mathbb{E}(v_t) = 0$, $\mathbb{E}(v_t v_t') = \Sigma_{vv}$, and $\mathbb{E}(v_t v_{t-j}') = 0$ for $j \neq 0$.

I have written Equation (2) across two rows. The first row includes two important features of the model, while the second row is just a VAR in differences with p lags. The first important feature in the first row is the time-varying intercepts, modeled with a vector of dummy variables d_t. This time-variation is intended to capture the business cycle asymmetries in the labor market that have been previously documented in the literature. In my baseline model, d_t is 3-dimensional and used to model expansions and two types of recessions. The first element of d_t is 1 in expansions and 0 otherwise; the second element is 1 in the first type of recession and 0 otherwise; and the third element is 1 in the second type of recession and 0 otherwise. The approach follows in the spirit of Hamilton (2005), who also uses time-varying intercepts to model expansions and two types of recessions. The main difference here is that I model the unemployment rate indirectly via labor market flows, while Hamilton (2005) models the unemployment rate directly. Hamilton (2005) refers to his recessions as "mild" and "severe" recessions, and my two recessions will end up having the same interpretation. I discuss the recession dummy variables further in the next subsection.

The second important feature in the first row of Equation (2) is the error correction term $\ln(s_{t-1}) - \ln(s_{t-1} + f_{t-1})$. I am treating the cointegrating vector $\beta = [1, -1]'$ as known and do not estimate it. I do this for two reasons. First, the results in Table 1 fail to reject $\beta = [1, -1]'$ as a cointegrating vector. Second, this choice aids interpretability because Equation (1) shows that I can interpret this error correction as $\ln(u_t)$ and understand the error correction mechanism in terms of values of the unemployment rate.

To reduce notation going forward, I use $y_t = [\ln(s_t), \ln(s_t + f_t)]'$ and write Equation (2) as

$$\Delta y_t = \nu d_{t-1} + \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_p \Delta y_{t-p} + v_t. \tag{3}$$

I assume that the business cycle dummy variables follow a Markov chain, which I model with a VAR(1) as in Hamilton (1994, Chapter 22):

$$d_t = \Phi d_{t-1} + w_t, \tag{4}$$

in which Φ is the transition matrix. If Φ has one eigenvalue equal to one with the remaining eigenvalues inside the unit circle, then d_t is covariance stationary (Hamilton, 1994, Chapter 22). This will be the case, and my assumption that d_t is stationary will not be violated. I also assume that w_t is independent of v_t, $\mathbb{E}(w_t) = 0$, $\mathbb{E}(w_t w_t') = \Sigma_{ww}$, and $\mathbb{E}(w_t w_{t-j}') = 0$ for $j \neq 0$. Again, these modeling choices follow in the spirit of Hamilton (2005). The primary difference is that I treat recessions as observed, while Hamilton (2005) treats them as unobserved. I now turn to my discussion of recession dates.

3.2 Recessions and Business Cycle Dummy Variables

In my baseline model, I use three dummy variables to model expansions and two types of recessions. I define an expansion as any period in which one of the recessions does not apply. For my first type of recession, I follow the NBER's dates of business cycle peaks and troughs. I define the first period of a recession as the month following a business cycle peak and the last period of a recession as the month of the business cycle trough (National Bureau of Economic Research, 2022). I refer to these recessions as "NBER recessions."

For my second type of recession, I follow in the spirit of previous macro-labor research. Because the unemployment rate is not a main variable that the NBER considers when determining recession dates, peaks and troughs of the unemployment rate do not generally align with NBER recessions and previous research has not always used NBER recession dates.[8] For example, Elsby, Michaels, and Solon (2009) define the beginning of a recession as the quarter with the lowest unemployment rate preceding an NBER recession. They define the end of a recession as the quarter with the highest unemployment rate following an NBER recession.[9] Tasci and Zevanove (2019) similarly

[8]The NBER provides a list of its main variables at `https://www.nber.org/research/business-cycle-dating/business-cycle-dating-procedure-frequently-asked-questions`.

[9]Elsby, Hobijn, and Şahin (2010), Dupraz, Nakamura, and Steinsson (2021), and Hall and Kudlyak (2022a) also

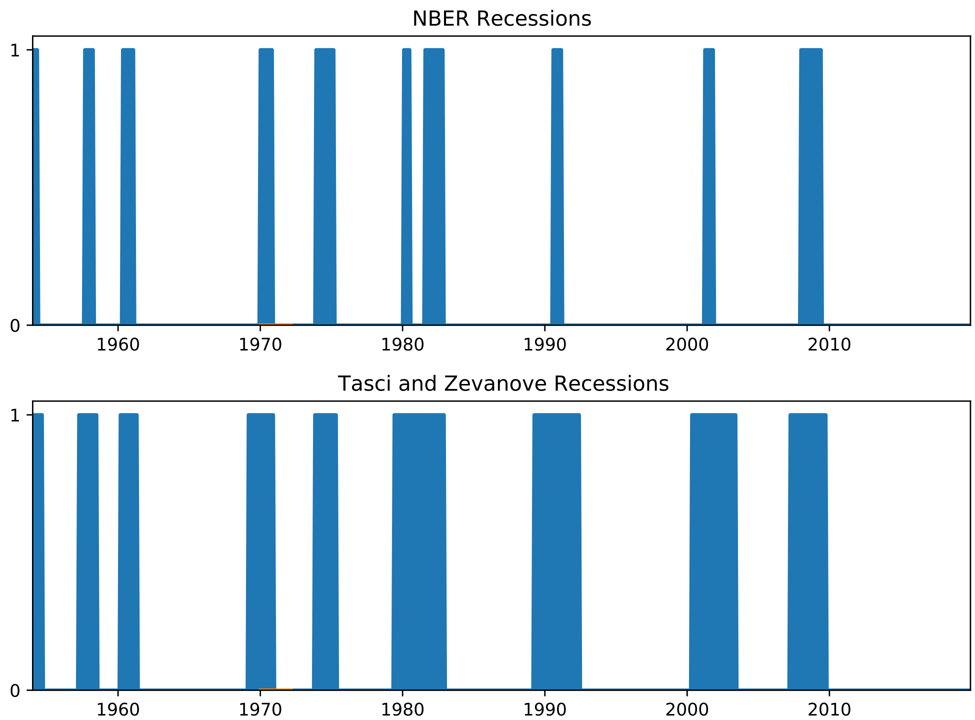

Figure 3: Recession dates from January 1954 through December 2019.

date expansions and recessions with unemployment rate troughs and peaks but with monthly data, and I use their recession dates as my second type of recession.[10]

Figure 3 shows the NBER and Tasci and Zevanove (2019) (TZ) recession dates from 1954 through 2019. I highlight that TZ recessions are all longer than NBER recessions and that every TZ recession contains at least one NBER recession. This feature yields three states of the world: an expansion, a TZ recession without an NBER recession, and a TZ recession with an NBER recession. It is never the case that the economy is in an NBER recession but not a TZ recession.

Using these recession dates, I construct d_t as follows. Expansion dummies equal 1 in any month without an NBER or a TZ recession. NBER dummies equal 1 exactly as in the top panel of Figure 3. TZ dummies equal 1 in months with a TZ recession but no NBER recession. For example, TZ dummies equal 1 following the 1991 and 2001 NBER recessions when the unemployment rate continued to rise. Using this TZ dummy rather than the TZ recession series in Figure 3 yields results

deviate from NBER recession dates when studying aggregate labor variables.

[10] Tasci and Zevanove (2019) define a business cycle peak as a month in which the unemployment rate is lower than in any of the previous or subsequent 24 months. A business cycle trough is a month in which the unemployment rate is higher than in any of the previous or subsequent 24 months. I define the first period of a recession as the month following a business cycle peak and the last period of a recession as the month of the business cycle trough. Dupraz, Nakamura, and Steinsson (2021) use a different algorithm but end up with very similar peak and trough dates.

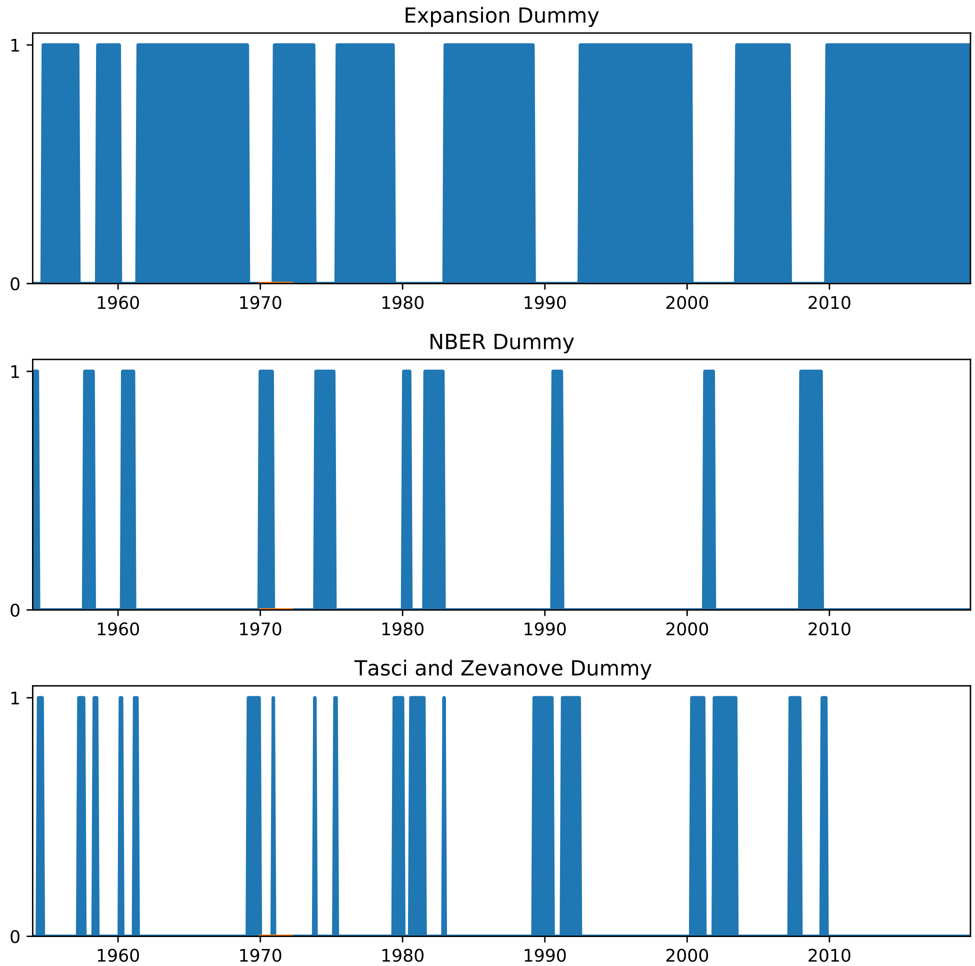

Figure 4: Business cycle dummy variables from January 1954 through December 2019.

similar to Hamilton's (2005) mild and severe recessions and aids interpretation of the regression results in the next subsection. Further, this TZ dummy ensures that the dummy variables sum to 1 in each month and that they are consistent with Markov chain interpretation of Equation (4). I show the dummy variables in Figure 4. In my baseline model, I use $d_t = [d_t^{expansion}, d_t^{NBER}, d_t^{TZ}]'$ with the individual elements corresponding to each panel in Figure 4.

3.3 VECM Estimation

The unknown VECM parameters are the coefficients ν, α and $\Gamma_1, \ldots, \Gamma_p$. I estimate these parameters with ordinary least squares. The estimation sample is January 1954 through December 2019 and I use $p = 12$. I provide a full description of estimation and inference in Appendix D. I also estimate the transition matrix Φ by ordinary least squares. I show the estimate of Φ, note that d_t is stationary, and give the estimate of $\mathbb{E}(d_t)$ in Appendix E.

In addition to estimating my baseline model in which $d_t = [d_t^{expansion}, d_t^{NBER}, d_t^{TZ}]'$ is 3-dimensional, I estimate two additional models for comparison purposes. In one model, d_t is 1-dimensional and equal to 1 (a constant-only model). In the other model, d_t is 2-dimensional with an expansion dummy and an NBER recession dummy (a one-recession model). Before discussing the results, I emphasize that expansions in the one-recession model and the baseline model cover different periods of time. In the one-recession model, $d_t^{expansion} = 1 - d_t^{NBER}$. In the baseline model, $d_t^{expansion} = 1 - d_t^{NBER} - d_t^{TZ}$.

Table 2 shows selected VECM estimates. Panel A of Table 2 shows the estimated values of ν and α for the constant-only, one-recession, and baseline models. Panel B of Table 2 shows differences between the coefficients on the dummy variables. This panel highlights when the coefficients differ across the dummy variables. Panel C of Table 2 shows $\bar{R}^2$, the Hannan and Quinn (1979) and Quinn (1980) criterion (HQ), and the Schwarz (1978) criterion (SC) as measures of fit.[11] Overall, the baseline model has the highest $\bar{R}^2$ and the lowest criterion values, indicating that it fits the data better the other models.[12]

Table 2 shows that the coefficients on the constant for the constant-only model are negative for the $\Delta \ln(s_t)$ row of Equation (2) and positive for the $\Delta \ln(s_t + f_t)$ row of Equation (2). Comparing these coefficients for the constant-only model to the coefficients on the dummy variables for the one-recession model, we see that the constant-only coefficients are driven by expansionary periods, which have non-zero coefficients. In contrast, the coefficients on the NBER recession dummy are not different from zero in either row of Equation (2).[13] Panel B of Table 2 shows that the differences between the coefficients on the NBER recession dummy and the expansion dummy are statistically significant in the one-recession model. Compared to expansions, NBER recessions are periods of time with high inflows into unemployment. Recalling from Figure 1 that the unemployment churn rate is essentially the same as the unemployment outflow rate, NBER recessions are also periods of time when outflows from unemployment are low compared to expansions.

Turning to the baseline model, Table 2 shows that the coefficient on the TZ dummy in the

[11] I compute the criterion values following equations in Lütkepohl (2005, Chapter 4). Let $\hat{v}_t$ be the residual from Equation (3) and $\hat{\Sigma}_{vv} = (T-p-1)^{-1} \sum_{t=p+2}^{T} \hat{v}_t \hat{v}_t'$. Then, $HQ = \ln(|\hat{\Sigma}_{vv}|) + 2\ln(\ln(T-p-1))K_m/(T-p-1)$ and $SC = \ln(|\hat{\Sigma}_{vv}|) + \ln(T-p-1)K_m/(T-p-1)$ with K_m being the number of estimated parameters in model m. With $p = 12$, I have $K_{constant} = 52$, $K_{one-recession} = 54$, and $K_{baseline} = 56$.

[12] In Appendix F, I briefly discuss models in which I interact the dummy variables with some of the right-hand variables in Equation (3). These less parsimonious models do not improve model fit and I do not discuss them here.

[13] The joint Wald statistic is 1.44. The associated χ^2_2 p-value is 0.487.

Table 2: Selected VECM Results for Different Models

	Constant-only model		One-recession model		Baseline model	
Dependent variable:	$\Delta\ln(s_t)$	$\Delta\ln(s_t+f_t)$	$\Delta\ln(s_t)$	$\Delta\ln(s_t+f_t)$	$\Delta\ln(s_t)$	$\Delta\ln(s_t+f_t)$
Panel A: Coefficient Estimates and Associated t-statistics						
constant	-0.043**	0.039				
	(-1.97)	(1.40)				
$d_{t-1}^{expansion}$			-0.052**	0.064**	-0.052**	0.052*
			(-2.38)	(2.34)	(-2.33)	(1.93)
d_{t-1}^{NBER}			-0.026	-0.007	-0.024	-0.039
			(-1.16)	(-0.27)	(-1.06)	(-1.42)
d_{t-1}^{TZ}					-0.050**	0.017
					(-2.13)	(0.59)
$\ln(s_{t-1})-\ln(s_{t-1}+f_{t-1})$	-0.014*	0.014	-0.016**	0.020**	-0.016**	0.013
	(-1.84)	(1.48)	(-2.09)	(2.07)	(-2.01)	(1.34)
Panel B: Differences Between Coefficient Estimates and Associated t-statistics						
NBER dummy slope			0.027***	-0.072***	0.028***	-0.091***
less expansion dummy slope			(3.32)	(-6.78)	(3.24)	(-8.03)
NBER dummy slope					0.026***	-0.055***
less TZ dummy slope					(3.09)	(-5.06)
TZ dummy slope					0.002	-0.036***
less expansion dummy slope					(0.33)	(-5.13)
Panel C: Measures of Model Fit						
$\bar{R}^2$	0.38	0.40	0.39	0.44	0.39	0.45
HQ	-11.31		-11.44		-11.47	
SC	-11.12		-11.24		-11.26	

Notes: Coefficients are estimated by ordinary least squares from January 1954 through December 2019. t-statistics are shown in parentheses and estimated with heteroskedasticity robust standard errors. Stars, *, ** and ***, indicate that parameter estimates are statistically significantly different from zero at the 10, 5, and 1 percent levels. $\bar{R}^2$ is adjusted R^2, HQ is the Hannan and Quinn (1979) and Quinn (1980) criterion, and SC is the Schwarz (1978) criterion.

$\Delta \ln(s_t)$ row of Equation (2) is nearly the same as the coefficient on the expansion dummy. In contrast, the coefficient on the NBER recession dummy is higher than the coefficients on both the TZ and the expansion dummies. In other words, NBER recessions are periods of time when inflows into unemployment are higher than during expansions. However, the TZ dummy picks up periods of time when inflows into unemployment are not high compared to expansions. In the $\Delta \ln(s_t + f_t)$ row of Equation (2), the TZ dummy has a coefficient that is lower than the coefficient on the expansion dummy but higher than the coefficient on the NBER recession dummy. Recalling from Figure 1 that the unemployment churn rate is essentially the same as the unemployment outflow rate, the TZ dummy picks up periods of time when unemployment outflows are lower than in expansions but higher than in NBER recessions.

Overall, NBER recessions look like severe recessions: periods of time when unemployment inflows are high compared to expansions and when unemployment outflows are very low compared to expansions. In contrast, the TZ dummies are picking up periods of time that look like mild recessions: unemployment inflows are similar to those in expansions, while unemployment outflows are lower than in expansions but not as low as in NBER recessions.

For all three models, the coefficients on the error correction term are jointly statistically significantly different from zero.[14] Further, the direction of the slopes is the same across all three models: negative for the $\Delta \ln(s_t)$ row of Equation (2) and positive for the $\Delta \ln(s_t + f_t)$ row. Recall from Equation (1) that the error correction term, $\ln(s_t) - \ln(s_t + f_t)$, can be interpreted as the log unemployment rate, $\ln(u_t)$. Then, error correction works by decreasing unemployment inflows when the unemployment rate is high and increasing unemployment outflows when the unemployment rate is high. These lower inflows and higher outflows then bring the unemployment rate down.

4 Business-Cycle-Only Counterfactuals

In Subsection 4.1, I compute business-cycle-only counterfactuals with the VECM to show that business cycles can generate the low-frequency movements in the unemployment rate. In Subsection 4.2, I study the important features of VECM by computing business-cycle-only counterfactuals using a VAR with the labor flows in differences and a VAR with the labor flows in levels.

[14] For the constant-only model, the Wald statistic is 9.45 with a corresponding χ^2_2 p-value of 0.009. For the one-recession model, the Wald statistic is 15.62 with a corresponding χ^2_2 p-value of essentially zero. For the baseline model, the Wald statistic is 10.59 with a corresponding χ^2_2 p-value of 0.005.

4.1 Counterfactuals with the VECM

I produce business-cycle-only counterfactuals of labor flows and the unemployment rate by feeding the historical values of d_t into the VECM and setting $v_t = 0$ for $t = p+2, \ldots, T$. I denote the estimated parameters with hats and the matrix of estimated parameters with $\hat{\Theta} = [\hat{\nu}, \hat{\alpha}, \hat{\Gamma}_1, \ldots, \hat{\Gamma}_p]$. Following the notation in Equation (3), I denote the counterfactual flow rate variables with $\tilde{y}_t$. I set the initial values to match the data: $\tilde{y}_t = y_t$ for $t = 1, \ldots, p+1$. Then, I compute $\tilde{y}_t$ for $t = p+2, \ldots, T$ recursively. I collect the right-hand side variables with the actual business cycle dummies and the counterfactual labor variables: $\tilde{z}_{t-1} = [d'_{t-1}, \beta'\tilde{y}_{t-1}, \Delta\tilde{y}'_{t-1}, \ldots, \Delta\tilde{y}'_{t-p+1}]'$. Then, I compute $\Delta\tilde{y}_t = \hat{\Theta}\tilde{z}_{t-1}$ and $\tilde{y}_t = \Delta\tilde{y}_t + \tilde{y}_{t-1}$.[15]

Figure 5 shows the business-cycle-only counterfactuals along with the actual data. The top panel shows the inflow hazard rate and the middle panel shows the unemployment churn rate. The bottom panel shows the business-cycle-only counterfactual unemployment rate in percent, which I compute from the counterfactual flows with $100 \times e^{\beta'\tilde{y}_t}$.

Consistent with the estimates in Table 2, Figure 5 shows that the counterfactual unemployment inflow rate increases in NBER recessions. Similar to the data, the counterfactual increase in the inflow rate during recessions is relatively short-lived. The business-cycle-only unemployment churn rate, primarily driven by the outflow rate, falls in NBER recessions. It may also fall in periods covered by the TZ dummy variable, such as the periods immediately following the 1991 and 2001 NBER recessions. In contrast to the inflow rate, the effects of recessions on the churn rate are persistent because the churn rate increases slowly throughout an expansion.

While the business-cycle-only labor flows match the business cycle patterns of the data, the top two panels of Figure 5 show that these business-cycle-only flows do not match the low-frequency movements in the data. Feeding the historical recession dummies into the VECM while setting $v_t = 0$ gives a counterfactual inflow rate that misses the persistent rise in the actual inflow rate beginning in the 1970s. The business-cycle-only inflow rate is below the actual inflow rate from 1970 until the late 2010s. The business-cycle-only counterfactual for the unemployment churn rate is also persistently different from the actual data. Because the two business-cycle-only rates in Figure 5 share a common trend, the business-cycle-only churn rate is also below the actual churn

[15] This counterfactual can also be understood through the joint dynamics of the variables in Equations (3) and (4). I want to know how the historical innovations in business cycles, w_t, affect the data. I denote the estimated values with $\hat{w}_t$ and set the counterfactual values to be $\tilde{w}_t = \hat{w}_t$ for $t = p+2, \ldots, T$. Because w_t is independent of v_t, $\tilde{w}_t$ has no information for the counterfactual values of v_t. Hence, I set counterfactual values of v_t equal to their unconditional mean: $\tilde{v}_t = \mathbb{E}(v_t) = 0$ for $t = p+2, \ldots, T$. Then, I jointly feed $[\tilde{v}'_t, \tilde{w}'_t]'$ into Equations (3) and (4), using $\tilde{y}_t = y_t$ for $t = 1, \ldots, p+1$ and $\tilde{d}_{p+1} = d_{p+1}$ as the initial conditions.

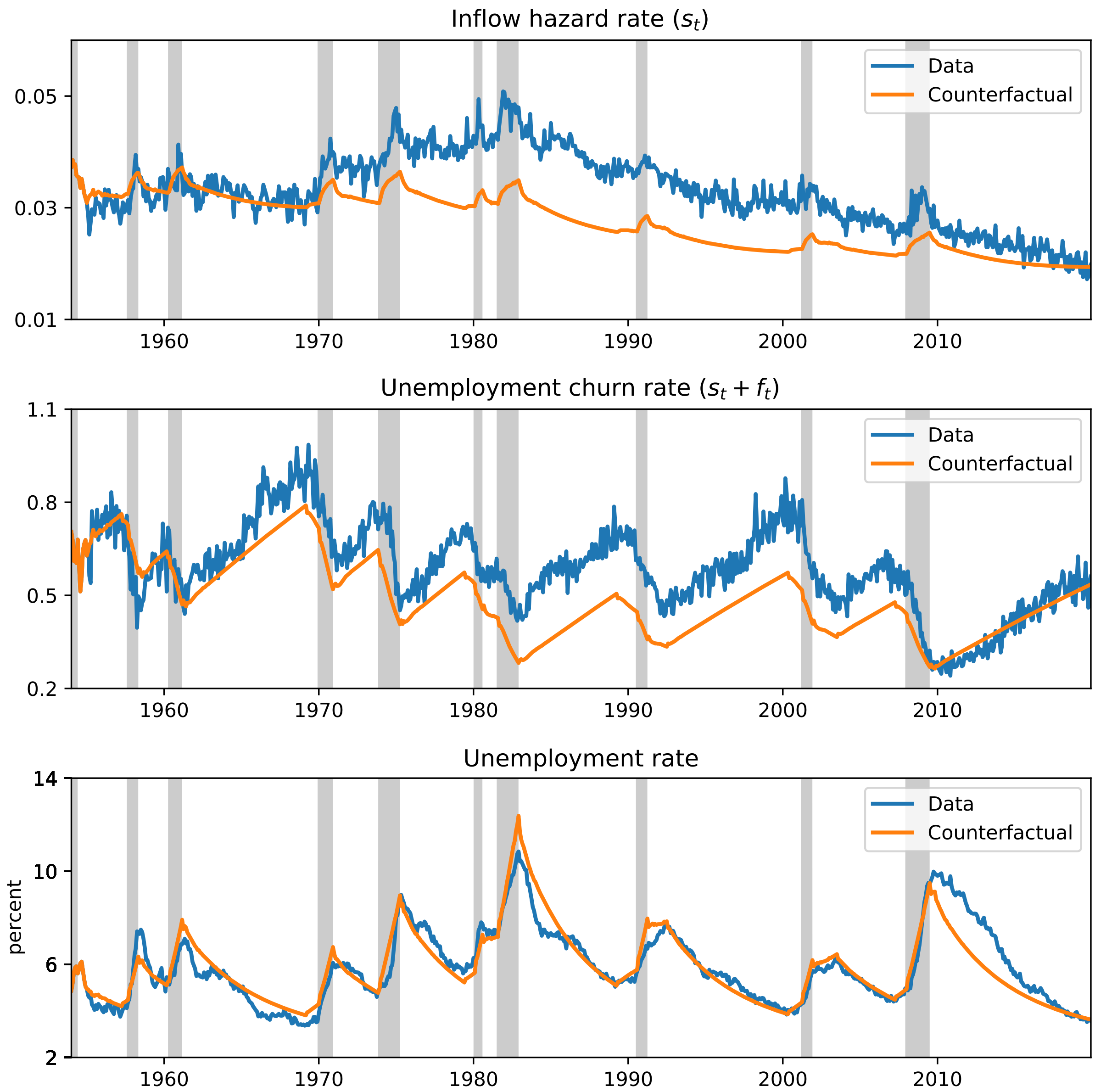

Figure 5: Labor data and business-cycle-only counterfactuals computed with the VECM. Note: Gray bars show NBER recessions

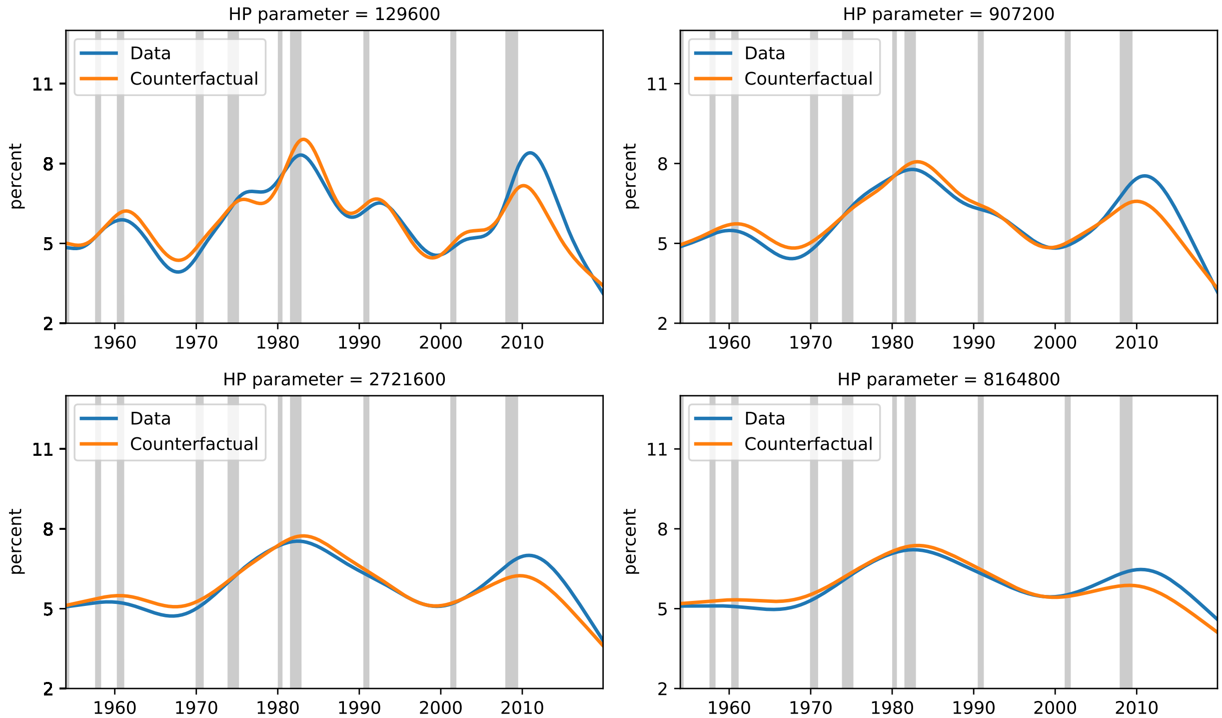

Figure 6: HP trends of the unemployment rate and its business-cycle-only counterfactual. Note: Gray bars show NBER recessions

rate for most of the sample.

The bottom panel of Figure 5 shows that the business-cycle-only unemployment rate is similar to the actual unemployment rate in terms of both its business cycle movements and its low-frequency movements. The only material period of disagreement comes after the 2008-09 recession when the business-cycle-only counterfactual does not match the slow recovery following the 2008-09 recession. However, this disagreement is confined to one recovery and is much less persistent than the disagreement between the business-cycle-only labor flows and the actual labor flows.

To further study the low-frequency movements in the unemployment rate and its business-cycle-only counterfactual, I compute low-frequency trends for these variables. First, I compute HP trends, shown in Figure 6, with different HP parameters. The top left panel shows the HP trend with a parameter of 129,600, which scales the standard 1,600 for quarterly data up by 81 to convert to a monthly frequency (Ravn and Uhlig, 2002). However, following Shimer (2005), researchers often compute HP trends for labor data with higher parameters and the other panels in Figure 6 scale the 129,600 parameter up by 7, 21, and 63.[16]

[16]Shimer (2005) uses an HP parameter of 100,000 for quarterly data, which converts to a monthly parameter of

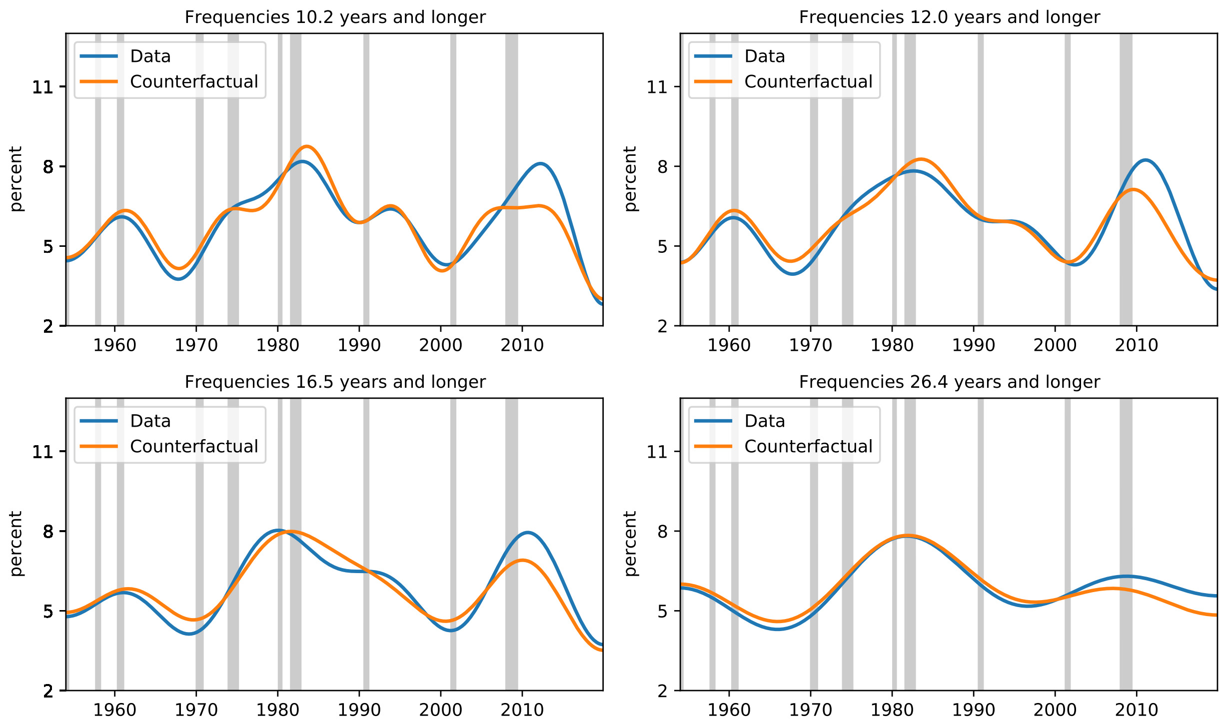

Figure 7: MW trends of the unemployment rate and its business-cycle-only counterfactual. Note: Gray bars show NBER recessions

Second, I compute low-frequency filter trends from Müller and Watson (2015), henceforth MW trends, with different low-frequency cut-offs.[17] I show these MW trends in Figure 7. In the top-left panel, I show frequencies corresponding to 10.2 years and longer, which are lower than the 10-year business cycle frequencies put forward in Beaudry, Galizia, and Portier (2020). To parallel the HP trends in Figure 6 and further study the low-frequency movements in the data, I use three additional low-frequency cut-offs for the MW trends in Figure 7.

I highlight two results in Figures 6 and 7. First, the actual unemployment rate has large low-frequency movements. In the top left panels of both figures, which correspond to the conventional HP parameter and business cycle frequency cut-off, the unemployment rate's low-frequency trends rise a little more than 4 percentage points from 1969 through 1982 and then fall a little less than

8,100,000. I choose to scale 129,600 up by 63 to approximately match the monthly parameter of 8,100,000.

[17]While HP trends are more commonly used than MW trends, previous research has noted some shortcomings of the HP filter (Cogley and Nason, 1995; Hamilton, 2018). One appeal of the MW trends is that they are interpreted in terms of frequencies in years. In addition, MW trends allow for low-frequency regression, which I use below. MW trends are computed from the discrete cosine transforms that I used for hypothesis testing in Section 2.2. For Figure 7, I use 13, 11, 8, and 5 cosine transforms, picking up frequencies equal to and longer than 10.2 years, 12.0 years, 16.5 years, and 26.4 years.

4 percentage points from 1983 through 1999. These fluctuations are dampened in the other panels but remain substantial. In the bottom right panel of Figure 6, the HP trend rises and falls by about 2 percentage points from 1969 through 1999. In the bottom right panel of Figure 7, the MW trend rises a little more than 3 percentage points from 1969 through 1982 and falls a little more than 2 percentage points from 1983 through 1999.

The second result in Figures 6 and 7 is that the unemployment rate's business-cycle-only counterfactual has essentially the same low-frequency movements as the actual unemployment rate. That is, the VECM indicates that business cycles can generate the unemployment rate's large low-frequency movements, even at very low frequencies. The only period of disagreement between the actual trends and the business-cycle-only trends begins near the 2008-09 recession. From the bottom panel of Figure 5, this disagreement occurs because the business-cycle-only counterfactual does not match the slow recovery following the 2008-09 recession.

To quantify the relationship between the actual unemployment rate and the business-cycle-only counterfactual, I use the low-frequency regression in Müller and Watson (2015, Section 4.1).[18] At frequencies of 10.2 years and longer (corresponding to the top left panel in Figure 7), regressing the unemployment rate's MW trend on the business-cycle-only counterfactual's MW trend yields a regression slope of 0.99. That is, the business-cycle-only counterfactual requires essentially no scaling to best predict fluctuations in the actual unemployment rate at low frequencies. The associated low-frequency R^2 is 0.84, indicating that the business-cycle-only counterfactual explains 84 percent of the low-frequency fluctuation in the actual unemployment rate.[19]

Intuitively, the unemployment rate's business cycle asymmetries allow business cycles to generate large low-frequency movements. Both in the data and in the VECM, the unemployment rate rises quickly in NBER recessions but falls slowly in expansions. Hence, the unemployment rate may not fall to its previous low point if an expansion is too short. As shown in the bottom panel of Figure 5, the repeated short expansions from January 1969 through December 1982 caused both the actual and the business-cycle-only unemployment rates to start subsequent recessions at higher levels. This ratcheting up of the unemployment rate over multiple business cycles yields a low-frequency increase in the unemployment rate. In contrast, a long expansion gives the unemployment rate time to fall below its previous low point in both the data and the VECM. Hence,

[18] Müller and Watson's (2015) low-frequency regression is akin to Engle's (1974) band spectral regression, but uses discrete cosine transforms instead of discrete Fourier transforms. Yamada (2020) recommends discrete cosine transforms rather than discrete Fourier transforms for economic time series.

[19] These quantitative results are essentially the same for the other frequency cut-offs in Figure 7.

the two long expansions from January 1983 through December 1999 undid the upward ratcheting of both the actual and business-cycle-only unemployment rates, yielding a low-frequency decrease in the unemployment rate. The business-cycle-only counterfactual highlights that business cycle asymmetries and the pattern of US business cycles are sufficient to generate the unemployment rate's low-frequency movements – a stochastic trend is not needed.

4.2 The Role of Error Correction

An important difference between my VECM and the models used in previous research is that the VECM has error correction or a common trend in the labor flows. To understand the role that error correction plays for the results in the previous subsection, I consider two changes to the VECM.

The first change to the VECM is to set $\alpha = [0, 0]'$, which eliminates error correction and imposes independent trends in the flows. With $\alpha = [0, 0]'$, the VECM in Equation (3) can be written as a VAR with the flows in differences:

$$\Delta y_t = \nu d_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_p \Delta y_{t-p} + v_t. \tag{5}$$

I will refer to the model in Equation (5) as a "differences VAR."

The second change to the VECM is to relax the assumption that $\beta = [1, -1]'$. I write the VECM as a VAR with the flows in levels:

$$y_t = \nu d_{t-1} + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_{p+1} y_{t-p-1} + v_t, \tag{6}$$

in which $A_1 = I_2 + \alpha\beta' + \Gamma_1$, $A_j = \Gamma_j - \Gamma_{j-1}$ for $j = 2, \ldots, p$, and $A_{p+1} = -\Gamma_p$. I will not impose any restrictions on A_1 during estimation. Hence, this "levels VAR" permits but does not impose $\alpha = [0, 0]'$ and $\beta = [1, -1]'$, making it is less restricted than either the VECM or the differences VAR.[20] To be clear, the levels VAR permits a common trend in the flows, but does not require that the common trend yield a stationary unemployment rate.[21] Hence, if one is skeptical of basing a time-series model on the hypothesis tests in Section 2, as I do with the VECM, the levels VAR provides results without reliance on pre-testing.

I estimate the differences and levels VARs by ordinary least squares from January 1954 through

[20] Gospodinov, Herrera, and Pesavento (2013) provide evidence that levels VARs are more robust than VECMs or differences VARs when there is uncertainty about the magnitude of the largest roots in the data.

[21] Suppose that the log labor flows are cointegrated with a cointegrating vector given by $\beta = [1, \beta_2]'$ with $\beta_2 \neq -1$. Then, $\ln(s_t) + \beta_2 \ln(s_t + f_t)$ is the stationary error correction term instead of $\ln(u_t) = \ln(s_t) - \ln(s_t + f_t)$.

December 2019. I compute business-cycle-only counterfactual labor flows from these models in a manner parallel to that in the VECM before then using the counterfactual flows to construct business-cycle-only unemployment rates.

Figure 8 shows the unemployment rate and the business-cycle-only counterfactual from the differences VAR in the top panel. Its middle and bottom panels show the corresponding HP and MW trends, using the conventional HP parameter and business cycle frequency cut-off. Figure 8 shows that the business-cycle-only unemployment rate from the differences VAR has large low-frequency movements but that those movements do not align with the actual unemployment rate. The business-cycle-only unemployment rate is too low in the 1970s, with an HP trend that is 1.6 percentage points below the actual HP trend in 1976 and an MW trend that is 1.7 percentage points below the actual MW trend in 1977. The business-cycle-only unemployment rate is too high for much of the 1980s and in the 1990s, with an HP trend that is 1.4 percentage points above the actual HP trend in 1992 and an MW trend that is 1.5 percentage points above the actual MW trend in 1993.

I make two remarks comparing the results of the counterfactuals from the VECM and the differences VAR. First, because the TZ recessions are constructed using peak and trough months for the unemployment rate, the timing of the fluctuations for the business-cycle-only unemployment rates from both the VECM and the differences VAR aligns with the timing of the actual unemployment rate fluctuations. Second, matching the timing of the actual unemployment rate fluctuations is not sufficient to match the low-frequency movements of the unemployment rate. When labor flows are not allowed to share a common stochastic trend, as in the differences VAR, then the business-cycle-only counterfactual does not align with the actual low-frequency movements in the unemployment rate. Hence, the common stochastic trend in the VECM is important for generating the results in Section 4.1.

Figure 9 shows the same objects as in Figure 8 but from the levels VAR. Comparing Figure 8 to Figure 9 shows that the business-cycle-only unemployment rate from the levels VAR is materially different than the business-cycle-only unemployment rate from the differences VAR. Hence, assuming $\alpha = [0, 0]'$ is not innocuous. This assumption materially changes the joint dynamics of the labor flows from the models. In particular, ruling out a common stochastic trend in the labor flows, as Tasci (2012), Barnichon and Mesters (2018), and Crump et al. (2019) do, accentuates the need for a stochastic trend in the unemployment rate in order to match the unemployment rate's

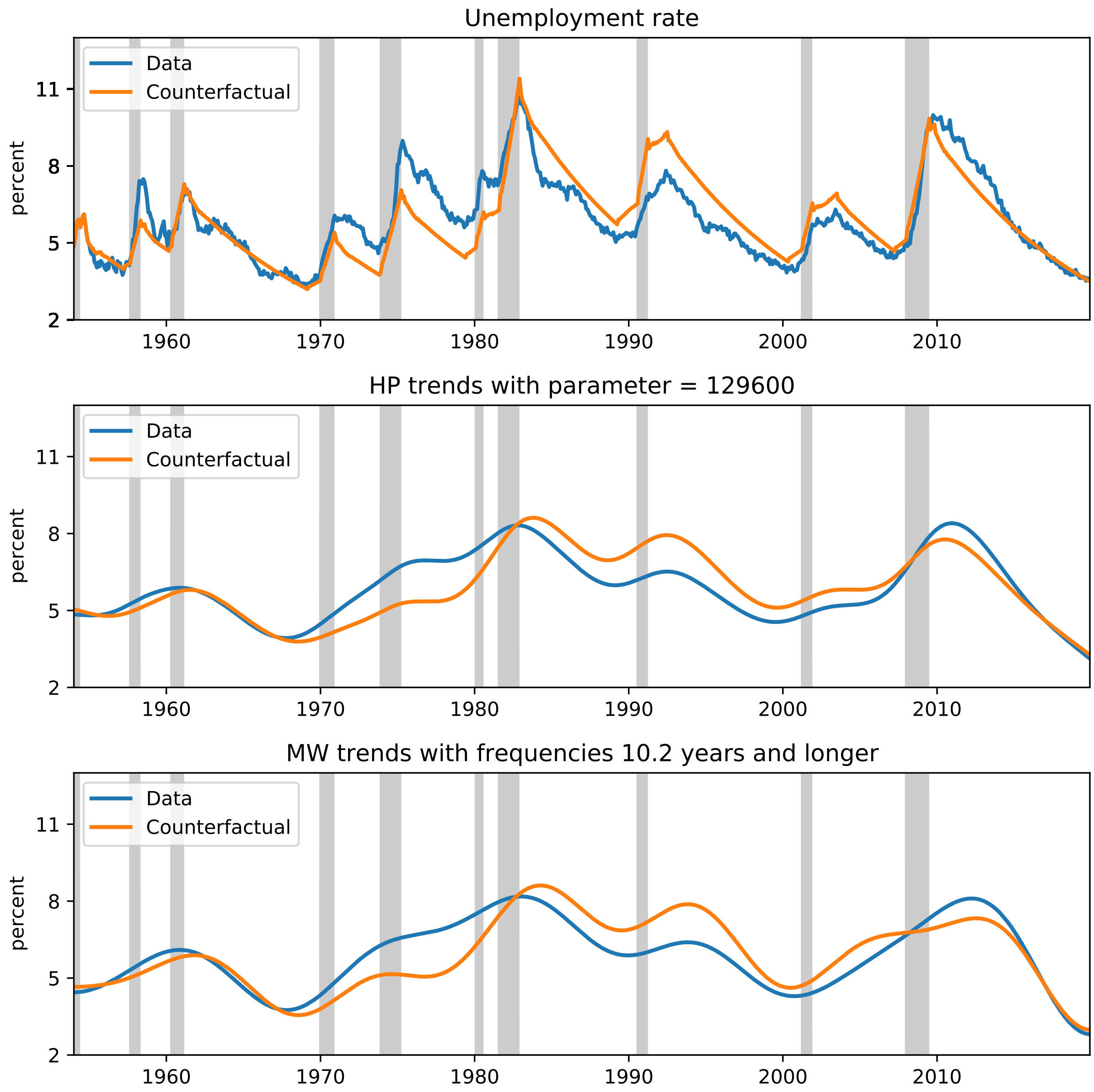

Figure 8: Unemployment rate and business-cycle-only counterfactual from the differences VAR. Note: Gray bars show NBER recessions

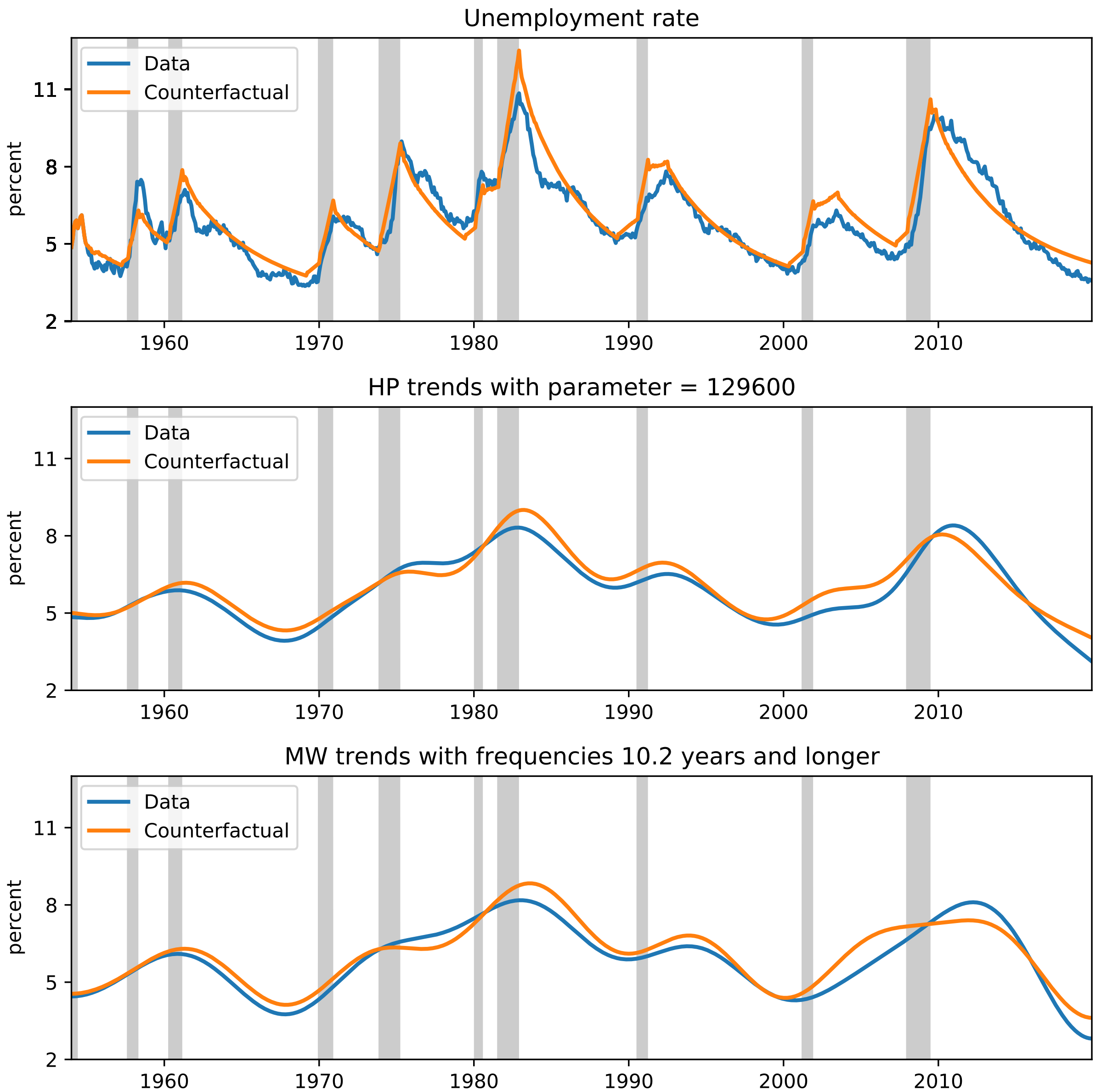

Figure 9: Unemployment rate and business-cycle-only counterfactual from the levels VAR. Note: Gray bars show NBER recessions

low-frequency movements.[22]

Comparing Figure 9 and the top left panels of Figures 6 and 7 shows that business cycles are similarly important for generating low-frequency movements in the unemployment rate in both the VECM and the levels VAR for conventional low-frequency parameters. Using the levels VAR, the average absolute difference between the actual and business-cycle-only HP trends is 0.36 percentage points. Using the VECM, this average absolute difference is 0.35 percentage points.[23] For the MW trends, these average absolute differences are 0.37 percentage points using the levels VAR and 0.38 percentage points using the VECM.[24] Hence, relaxing the VECM assumption that $\beta = [1, -1]$ does not materially change the finding that business cycles can generate the low-frequency movements in the unemployment rate. On the other hand, using $\beta = [1, -1]$ does not impose this finding on the VECM. Rather, the appeal of using the VECM with $\beta = [1, -1]'$ is that $\ln(u_t) = \beta' y_t$ is stationary, consistent with the results in Section 2, and the error correction mechanism can be interpreted in terms of the unemployment rate. I use the stationarity feature in the next section to study where the unemployment rate will go after adjusting for business cycle shocks.

5 Longer-Run Unemployment Rate Estimates

In this section, I estimate a longer-run unemployment rate, defined as the rate expected to prevail after adjusting for business cycle shocks (Crump, Nekarda, and Petrosky-Nadeau, 2020). I begin by interpreting "after adjusting for business cycle shocks" to mean where the unemployment rate will go as the economy stays in expansion. This interpretation follows the spirit of a plucking model where business cycles are caused by recessions that are downward plucks of the economy away from potential (Friedman, 1964, 1993). After a pluck, the unemployment rate will move toward its longer-run benchmark in the absence of further plucks.[25]

To compute the longer-run unemployment rate in the VECM, I set $d_t^{expansion} = 1$ and $v_t = 0$ for all t. Next, I use the notation $\nu = [\nu^{expansion}, \nu^{NBER}, \nu^{TZ}]$, in which $\nu^{expansion}$ is the column of ν that corresponds to $d_t^{expansion}$, ν^{NBER} is the column of ν that corresponds to d_t^{NBER}, and ν^{TZ} is the column of ν that corresponds to d_t^{TZ}. Then, the VECM dynamics after adjusting for business

[22] Gospodinov, Maynard, and Pesavento (2011) show that ruling out low-frequency relationships can also distort how technology shocks are estimated to affect the labor market.

[23] This average absolute difference is 0.62 percentage points for the differences VAR.

[24] This average absolute difference is 0.65 percentage points for the differences VAR.

[25] Kim and Nelson (1999), Tasci and Zevanove (2019), and Dupraz, Nakamura, and Steinsson (2021) provide support for a plucking model of unemployment rate fluctuations.

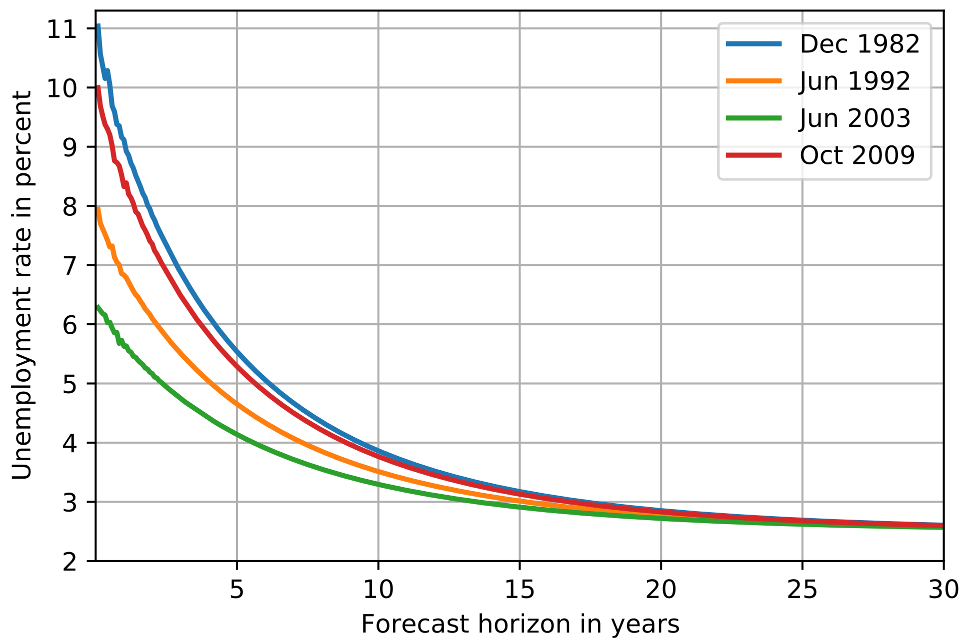

Figure 10: Unemployment rate forecasts conditional on the economy staying in expansion. Note: The legend gives dates of the initial conditions for the forecasts.

cycle shocks are given by

$$\Delta y_t = \nu^{expansion} + \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_p \Delta y_{t-p}. \tag{7}$$

Given some initial conditions, $[y'_{t-1}, \ldots, y'_{t-p-1}]'$ and estimated VECM parameters, I can then iterate Equation (7) forward and use $100 \times u_t = 100 \times e^{\beta' y_t}$ to trace out the unemployment rate in percent. In short, I compute forecasts of the unemployment rate via the flows conditional on the economy staying in expansion.

Figure 10 shows these conditional forecasts for four different initial conditions, setting period $t-1$ to correspond to December 1982, June 1992, June 2003, and October 2009. Each of these months corresponds to an unemployment rate peak. While these initial conditions are different, the unemployment rate will converge to the same value if an expansion is long enough. At the 10-year horizon, the unemployment rates are in a range between 3.3 and 3.9 percent. At the 15-year horizon, the unemployment rates are in a range between 2.9 and 3.2 percent. At the 30-year horizon, all forecasts are 2.6 percent.

Analytically, I can compute where the unemployment rate will go at an infinite horizon. Let

$\Delta y_t \to \Delta y$ and $\beta' y_t \to \beta y$ as $t \to \infty$. Then, Equation (7) can be written as

$$\Delta y = (I_2 - \Gamma_1 - \cdots - \Gamma_p)^{-1}(\nu^{expansion} + \alpha\beta' y)$$

for $t \to \infty$. Note that $\beta' \Delta y_t = \beta' y_t - \beta' y_{t-1}$, which then converges to $\beta' y - \beta' y = 0$ as $t \to \infty$. Then, pre-multiplying Equation (7) by β' yields

$$0 = \beta' \nu^{expansion} + \beta' \alpha \beta' y + \beta'(\Gamma_1 + \cdots + \Gamma_p)\Delta y$$

for $t \to \infty$. Substituting out Δy and solving for $\beta' y$ then yields

$$\beta' y = -\frac{\beta'[I_2 + (\Gamma_1 + \cdots + \Gamma_p)(I_2 - \Gamma_1 - \cdots - \Gamma_p)^{-1}]\nu^{expnsion}}{\beta'[I_2 + (\Gamma_1 + \cdots + \Gamma_p)(I_2 - \Gamma_1 - \cdots - \Gamma_p)^{-1}]\alpha}. \tag{8}$$

Let $\widehat{\beta' y}$ be the estimated value. Then, I estimate the infinite-horizon unemployment rate to be 2.5 percent, which is given by $100 \times e^{\widehat{\beta' y}}$.

Together, the infinite-horizon unemployment rate estimate of 2.5 percent and the forecasts in Figure 10 show that an expansion of roughly 30 years is needed for the unemployment rate to fall near its infinite-horizon forecast conditional on the economy saying in expansion. This is a much longer expansion horizon than what exists in my sample. The longest expansion in my sample is 122 months (10.2 years) from November 2009 through December 2019.[26] The next longest expansions are 94 months (7.8 years) from July 1992 through April 2000 and 93 months (7.8 years) from June 1961 through February 1969. Hence, the 2.5 percent estimate requires the VECM to extrapolate well beyond what is in the data, and I view the 2.5 percent estimate as speculative.

The VECM does not need to extrapolate expansions well beyond what is in the data to predict that the unemployment rate will fall to historically low values in the absence of shocks. As I already noted, Figure 10 shows that the unemployment rate goes to a range 3.3 to 3.9 percent conditional on a 10-year expansion. These values are consistent with what has been observed at the end of long expansions. The unemployment rate fell as low as 3.4 percent in 1968 and 1969, as low as 3.9 percent in 2000, and as low as 3.5 percent in 2019. The VECM's forecasts in Figure 10 indicate that these low values need not be the result of some stochastic trend. Rather, these low values can just be the result of long expansions.

If I allow the VECM to extrapolate beyond 10 years, roughly coinciding with the longest

[26] I am defining expansions here as periods of time without NBER or TZ recessions. See the top panel of Figure 4.

expansion in my sample, and out to 15 years, then the unemployment rates in Figure 10 range from 2.9 to 3.2 percent. Unemployment rates this low have not been observed since the early 1950s, coinciding with the Korean War and the associated draft. However, such values of the unemployment rate are not implausible for sufficiently long expansions. The unemployment rate generally fell throughout 2019 and reached its lowest value of the expansion (when rounded to two digits) in February 2020 — the month before the COVID-19 pandemic disrupted the US labor market. Without this COVID-19 disruption, the unemployment rate falling into the 2.9 to 3.2 percent range is entirely plausible.

I conclude this section by generalizing Equation (8). My purpose is to study how a reduction in the frequency of recessions, but not necessarily an absence of recessions, will affect the expected unemployment rate. Because I model Δy_t, $\beta' y_t$, and d_t as being stationary, $\mathbb{E}(\Delta y_t)$, $\mathbb{E}(\beta' y_t)$, and $\mathbb{E}(d_t)$ are constant for all t. Then, I can derive

$$\mathbb{E}(\ln(u_t)) = \mathbb{E}(\beta' y_t) = -\frac{\beta'[I_2 + (\Gamma_1 + \cdots + \Gamma_p)(I_2 - \Gamma_1 - \cdots - \Gamma_p)^{-1}]\nu\mathbb{E}(d_t)}{\beta'[I_2 + (\Gamma_1 + \cdots + \Gamma_p)(I_2 - \Gamma_1 - \cdots - \Gamma_p)^{-1}]\alpha} \tag{9}$$

in a manner parallel to that for deriving Equation (8). Equation (9) shows that the expected log of the unemployment rate is a function of $\mathbb{E}(d_t)$. Because $d_t = [d_t^{expansion}, d_t^{NBER}, d_t^{TZ}]'$ and $d_t^{expansion} = 1 - d_t^{NBER} - d_t^{TZ}$, I can then say that the expected log of the unemployment rate depends on the expected fraction of months that the economy spends in recession. I estimate $\mathbb{E}(d_t)$ from the estimate of the transition matrix Φ. The expected fraction of months in an expansion is 0.71, the expected fraction of months in an NBER recession is 0.13, and the expected number of months in a TZ recession but not in an NBER recession is 0.16. Then, as a heuristic, I use $e^{\mathbb{E}(\ln(u_t))}$ to approximate $\mathbb{E}(u_t)$ and compute an estimated expected unemployment rate of 5.5 percent.

As a thought experiment, I consider what the expected unemployment rate would be with lower rates of recessions. Suppose $\mathbb{E}(d_t^{expansion})$ increases to 0.80, $\mathbb{E}(d_t^{NBER})$ reduces to 0.04, and $\mathbb{E}(d_t^{TZ})$ is unchanged at 0.16. These values are very similar to the fraction of months spent in recession from January 1983 through December 1999.[27] Then, the expected unemployment rate would be 3.7 percent. That is, reducing the frequency of NBER recessions to rates observed from 1983 through 1999 brings the expected unemployment rate down by almost 2 percentage points. Alternatively, if $\mathbb{E}(d_t^{expansion})$ increases to 0.80, $\mathbb{E}(d_t^{NBER})$ is unchanged at 0.13, and $\mathbb{E}(d_t^{TZ})$ reduces to 0.07, then the expected unemployment rate would be 4.9 percent. Both of these exercises show that reducing

[27] From January 1983 through December 1999, 4 percent of months were in an NBER recession and 19 percent of months were in a TZ recession, implying that the TZ dummy equaled 1 in 15 percent of months.

the frequency of recessions reduces the expected level of the unemployment rate, consistent with the structural models of Hairault, Langot, and Osotimehin (2010), Jung and Kuester (2011), Benigno, Ricci, and Surico (2015), Lepetit (2020), Dupraz, Nakamura, and Steinsson (2021), and Gornemann, Kuester, and Nakajima (2021). Further, reducing the frequency of NBER recessions yields a larger reduction in the expected unemployment rate than the same size reduction in the frequency of TZ dummies, consistent with the interpretation that NBER recessions correspond to severe recessions while the TZ dummies correspond to mild recessions.

6 Conclusions

I provide evidence that the unemployment rate is stationary, while its underlying flows have unit roots. Hence, I model the log unemployment rate as the error correction term of log labor flows in a VECM. To incorporate the business cycle asymmetries documented in previous research, I permit the intercepts in the VECM to vary over the business cycle.

I use this VECM with time-varying intercepts to show that business cycles can generate most of the low-frequency movement in the unemployment rate. The intuition for this result is as follows. Because the unemployment rate falls slowly in expansions, frequent recessions interrupt labor market recoveries and ratchet the unemployment rate upward. Long expansions undo this upward ratcheting by giving the labor market time to fully recover. The historical pattern of US business cycles then generates low-frequency movements: frequent recessions from the late 1960s until the early 1980s caused the unemployment rate to ratchet up, while long expansions in the 1980s and 1990s undid this ratcheting. An implication of this result is that removing low-frequency movements from the unemployment rate may remove business cycle features.

I also use the VECM to estimate a longer-run unemployment rate benchmark by computing forecasts conditional on the economy staying in expansion. Using the unemployment rate peaks in 1982, 1992, 2003, and 2009 as initial conditions, the VECM predicts that the unemployment rate will be between 3.3 and 3.9 percent after a 10-year expansion. Lower unemployment rates are possible with longer expansions, but require the VECM to extrapolate beyond what is in the data. Hence, a longer-run unemployment rate between 3.3 and 3.9 percent could be a reasonable benchmark for policymakers.

References

Aaronson, Daniel, Luojia Hu, Arian Seifoddini, and Daniel G. Sullivan. 2015. "Changing Labor Force Composition and the Natural Rate of Unemployment." *Chicago Fed Letter* No. 338. URL https://www.chicagofed.org/publications/chicago-fed-letter/2015/338.

Andolfatto, David. 1997. "Evidence and Theory on the Cyclical Asymmetry in Unemployment Rate Fluctuations." *Canadian Journal of Economics* 30 (3):709–721. URL https://doi.org/10.2307/136240.

Barnichon, Regis and Geert Mesters. 2018. "On the Demographic Adjustment of Unemployment." *Review of Economics and Statistics* 100 (2):219–231. URL https://doi.org/10.1162/REST_a_00688.

Barnichon, Regis and Christopher J. Nekarda. 2012. "The Ins and Outs of Forecasting Unemployment: Using Labor Force Flows to Forecast the Labor Market." *Brookings Papers on Economic Activity* Fall:83–117. URL https://doi.org/10.1353/eca.2012.0018.

Beaudry, Paul, Dana Galizia, and Franck Portier. 2020. "Putting the Cycle Back into Business Cycle Analysis." *American Economic Review* 110 (1):1–47. URL https://doi.org/10.1257/aer.20190789.

Benigno, Pierpaolo, Luca Antonio Ricci, and Paolo Surico. 2015. "Unemployment and Productivity in the Long Run: The Role of Macroeconomic Volatility." *Review of Economics and Statistics* 97 (3):698–709. URL https://doi.org/10.1162/REST_a_00508.

Beveridge, Stephen and Charles R. Nelson. 1981. "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'." *Journal of Monetary Economics* 7 (2):151–174. URL https://doi.org/10.1016/0304-3932(81)90040-4.

Clark, Peter K. 1989. "Trend Reversion in Real Output and Unemployment." *Journal of Econometrics* 40 (1):15–32. URL https://doi.org/10.1016/0304-4076(89)90027-4.

Cogley, Timothy and James M. Nason. 1995. "Effects of the Hodrick-Prescott Filter on Trend and Difference Stationary Time Series: Implications for Business Cycle Research." *Journal of Economic Dynamics and Control* 19 (1):253–278. URL https://doi.org/10.1016/0165-1889(93)00781-X.

Crump, Richard K., Marc Giannoni, Stefano Eusepi, and Ayşegül Şahin. 2019. "A Unified Approach to Measuring u^*." *Brookings Papers on Economic Activity* Spring:143–214. URL https://doi.org/10.1353/eca.2019.0002.

Crump, Richard K., Christopher J. Nekarda, and Nicolas Petrosky-Nadeau. 2020. "Unemployment Rate Benchmarks." Board of Governors of the Federal Reserve System Finance and Economics Discussion Series 2020-072. URL https://doi.org/10.17016/FEDS.2020.072.

Dupraz, Stéphane, Emi Nakamura, and Jón Steinsson. 2021. "A Plucking Model of Business Cycles." NBER Working Paper 26351. URL https://www.doi.org/10.3386/w26351.

Elsby, Michael W. L., Bart Hobijn, and Ayşegül Şahin. 2010. "The Labor Market in the Great Recession." *Brookings Papers on Economic Activity* Spring:1–48. URL https://doi.org/10.1353/eca.2010.0001.

Elsby, Michael W. L., Ryan Michaels, and Gary Solon. 2009. "The Ins and Outs of Cyclical Unemployment." *American Economic Journal: Macroeconomics* 1 (1):84–110. URL https://doi.org/10.1257/mac.1.1.84.

Engle, Robert F. 1974. "Band Spectrum Regression." *International Economic Review* 15 (1):1–11. URL https://doi.org/10.2307/2526084.

Engle, Robert F. and Clive W. J. Granger. 1987. "Co-Integration and Error Correction: Representation, Estimation, and Testing." *Econometrica* 55 (2):251–276. URL https://doi.org/10.2307/1913236.

Fallick, Bruce C. and Christopher L. Foote. 2022. "The Impact of the Age Distribution on Unemployment: Evidence from US States." Federal Reserve Bank of Cleveland Working Paper No. 22-27. URL https://doi.org/10.26509/frbc-wp-202227.

Fatás, Antonio. 2021. "The Elusive State of Full Employment." CEPR Press Discussion Paper No. 16535. URL https://cepr.org/publications/dp16535.

Ferraro, Domenico. 2018. "The Asymmetric Cyclical Behavior of the US Labor Market." *Review of Economic Dynamics* 30:145–162. URL https://doi.org/10.1016/j.red.2018.05.005.

Friedman, Milton. 1964. "The Monetary Studies of the National Bureau." In *The National Bureau Enters Its Forty-fifth Year*. URL https://www.nber.org/system/files/chapters/c4453/c4453.pdf.

———. 1993. "The "Plucking Model" of Business Fluctuations Revisited." *Economic Inquiry* 31:171–177. URL https://doi.org/10.1111/j.1465-7295.1993.tb00874.x.

Gornemann, Nils, Keith Kuester, and Makoto Nakajima. 2021. "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Systematic Monetary Policy." Opportunity and Inclusive Growth Institute Working Paper No. 50. URL https://doi.org/10.21034/iwp.50.

Gospodinov, Nikolay, Ana María Herrera, and Elena Pesavento. 2013. "Unit Roots, Cointegration and Pre-Testing in VAR Models." *Advances in Econometrics: VAR Models in Macroeconomics - New Developments and Applications: Essays in Honor of Christopher A. Sims*, Volume 32, 81-115. URL https://doi.org/10.1108/S0731-9053(2013)0000031003.

Gospodinov, Nikolay, Alex Maynard, and Elena Pesavento. 2011. "Sensitivity of Impulse Responses to Small Low-Frequency Comovements: Reconciling the Evidence on the Effects of Technology Shocks." *Journal of Business & Economic Statistics* 29 (4):455–467. URL https://doi.org/10.1198/jbes.2011.10042.

Granger, Clive W. J. 1981. "Some Properties of Time Series Data and Their Use in Econometric Model Specification." *Journal of Econometrics* 16 (1):121–130. URL https://doi.org/10.1016/0304-4076(81)90079-8.

Hairault, Jean-Olivier, François Langot, and Sophie Osotimehin. 2010. "Matching Frictions, Unemployment Dynamics and the Cost of Business Cycles." *Review of Economic Dynamics* 13 (4):759–779. URL https://doi.org/10.1016/j.red.2010.05.001.

Hall, Robert E. 2005. "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market." *Review of Economics and Statistics* 87 (3):397–407. URL https://doi.org/10.1162/0034653054638346.

Hall, Robert E. and Marianna Kudlyak. 2022a. "The Inexorable Recoveries of Unemployment." *Journal of Monetary Economics* 131:15–25. URL https://doi.org/10.1016/j.jmoneco.2022.06.003.

———. 2022b. "Why Has the US Economy Recovered So Consistently from Every Recession in the Past 70 Years?" *NBER Macroeconomics Annual* 36:1–55. URL https://doi.org/10.1086/718588.

Hamilton, James D. 1994. *Time Series Analysis*. Princeton University Press. URL https://doi.org/10.1515/9780691218632.

———. 2005. "What's Real About the Business Cycle?" *Federal Reserve Bank of St. Louis Review* 87 (4):435–452. URL https://doi.org/10.20955/r.87.435-452.

———. 2018. "Why You Should Never Use the Hodrick-Prescott Filter." *Review of Economics and Statistics* 100 (5):831–843. URL https://doi.org/10.1162/rest_a_00706.

Hannan, Edward J. and Barry G. Quinn. 1979. "The Determination of the Order of an Autoregression." *Journal of the Royal Statistical Society: Series B (Methodological)* 41 (2):190–195. URL https://doi.org/10.1111/j.2517-6161.1979.tb01072.x.

Hansen, Bruce E. 1997. "Inference in TAR Models." *Studies in Nonlinear Dynamics and Econometrics* 2 (1):1–14. URL https://doi.org/10.2202/1558-3708.1024.

Hodrick, Robert J. and Edward C. Prescott. 1997. "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit and Banking* 29 (1):1–16. URL https://doi.org/10.2307/2953682.

Hornstein, Andreas and Marianna Kudlyak. 2020. "Why Is Current Unemployment So Low?" Federal Reserve Bank of San Francisco Working Paper 2020-05. URL https://doi.org/10.24148/wp2020-05.

Jung, Philip and Keith Kuester. 2011. "The (Un)Importance of Unemployment Fluctuations for the Welfare Cost of Business Cycles." *Journal of Economic Dynamics and Control* 35 (10):1744–1768. URL https://doi.org/10.1016/j.jedc.2011.05.008.

Kim, Chang-Jin and Charles R. Nelson. 1999. "Friedman's Plucking Model of Business Fluctuations: Tests and Estimates of Permanent and Transitory Components." *Journal of Money, Credit and Banking* 31 (3):317–334. URL https://doi.org/10.2307/2601114.

Lepetit, Antoine. 2020. "Asymmetric Unemployment Fluctuations and Monetary Policy Trade-Offs." *Review of Economic Dynamics* 36:29–45. URL https://doi.org/10.1016/j.red.2019.07.005.

Lunsford, Kurt G. 2021. "Recessions and the Trend in the US Unemployment Rate." Federal Reserve Bank of Cleveland *Economic Commentary* 2021-01. URL https://doi.org/10.26509/frbc-ec-202101.

Lütkepohl, Helmut. 2005. *New Introduction to Multiple Time Series*. Springer-Verlag. URL https://doi.org/10.1007/978-3-540-27752-1.

McKay, Alisdair and Ricardo Reis. 2008. "The Brevity and Violence of Contractions and Expansions." *Journal of Monetary Economics* 55 (4):738–751. URL https://doi.org/10.1016/j.jmoneco.2008.05.009.

Meyer, Brent and Murat Tasci. 2015. "Lessons for Forecasting Unemployment in the U.S.: Use Flow Rates, Mind the Trend." Federal Reserve Bank of Cleveland, Working Paper No. 15-02. URL https://doi.org/10.26509/frbc-wp-201502.

Montgomery, Alan L., Victor Zarnowitz, Ruey S. Tsay, and George C. Tiao. 1998. "Forecasting the U.S. Unemployment Rate." *Journal of the American Statistical Association* 93 (442):478–493. URL https://doi.org/10.1080/01621459.1998.10473696.

Müller, Ulrich K. and Mark W. Watson. 2008. "Testing Models of Low-Frequency Variability." *Econometrica* 76 (5):979–1016. URL https://doi.org/10.3982/ECTA6814.

———. 2013. "Low-Frequency Robust Cointegration Testing." *Journal of Econometrics* 174 (2):66–81. URL https://doi.org/10.1016/j.jeconom.2012.09.006.

———. 2015. "Low-Frequency Econometrics." NBER Working Paper 21564. URL https://doi.org/10.3386/w21564.

National Bureau of Economic Research. 2022. "US Business Cycle Expansions and Contractions." Retrieved from FRED, Federal Reserve Bank of St. Louis. URL https://fred.stlouisfed.org/series/USREC (accessed on June 16, 2023).

Neftçi, Salih N. 1984. "Are Economic Time Series Asymmetric over the Business Cycle?" *Journal of Political Economy* 92 (2):307–328. URL https://doi.org/10.1086/261226.

Quinn, Barry G. 1980. "Order Determination for a Multivariate Autoregression." *Journal of the Royal Statistical Society: Series B (Methodological)* 42 (2):182–185. URL https://doi.org/10.1111/j.2517-6161.1980.tb01116.x.

Ravn, Morten O. and Harald Uhlig. 2002. "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations." *Review of Economics and Statistics* 84 (2):371–376. URL https://doi.org/10.1162/003465302317411604.

Rothman, Philip. 1998. "Forecasting Asymmetric Unemployment Rates." *Review of Economics and Statistics* 80 (1):164–168. URL https://doi.org/10.1162/003465398557276.

Schwarz, Gideon. 1978. "Estimating the Dimension of a Model." *Annals of Statistics* 6 (2):461–464. URL https://doi.org/10.1214/aos/1176344136.

Selective Service System. 2022. "Induction Statistics." URL https://www.sss.gov/history-and-records/induction-statistics/ (accessed on November 29, 2022).

Shackleton, Robert. 2018. "Estimating and Projecting Potential Output Using CBO's Forecasting Growth Model." Congressional Budget Office Working Paper 2018-03. URL https://www.cbo.gov/system/files/115th-congress-2017-2018/workingpaper/53558-cbosforecastinggrowthmodel-workingpaper.pdf.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95 (1):25–49. URL https://doi.org/10.1257/0002828053828572.

———. 2012. "Reassessing the Ins and Outs of Unemployment." *Review of Economic Dynamics* 15 (2):127–148. URL https://doi.org/10.1016/j.red.2012.02.001.

Sichel, Daniel E. 1993. "Business Cycle Asymmetry: A Deeper Look." *Economic Inquiry* 31 (2):224–236. URL https://doi.org/10.1111/j.1465-7295.1993.tb00879.x.

Sinclair, Tara M. 2009. "The Relationships between Permanent and Transitory Movements in U.S. Output and the Unemployment Rate." *Journal of Money, Credit and Banking* 41 (2-3):529–542. URL https://doi.org/10.1111/j.1538-4616.2009.00220.x.

Staiger, Douglas, James H. Stock, and Mark W. Watson. 2001. "Prices, Wages and the U.S. NAIRU in the 1990s." NBER Working Paper 8320. URL https://doi.org/10.3386/w8320.

Stock, James H. and Mark W. Watson. 1988. "Testing for Common Trends." *Journal of the American Statistical Association* 83 (404):1097–1107. URL https://doi.org/10.1080/01621459.1988.10478707.

Tasci, Murat. 2012. "Ins and Outs of Unemployment in the Long Run: Unemployment Flows and the Natural Rate." Federal Reserve Bank of Cleveland Working Paper No. 12-24. URL https://doi.org/10.26509/frbc-wp-201224.

Tasci, Murat and Nicholas Zevanove. 2019. "Do Longer Expansions Lead to More Severe Recessions?" Federal Reserve Bank of Cleveland *Economic Commentary* 2019-02. URL https://doi.org/10.26509/frbc-ec-201902.

Tüzemen, Didem. 2019. "Job Polarization and the Natural Rate of Unemployment in the United States." *Economics Letters* 175:97–100. URL https://doi.org/10.1016/j.econlet.2018.12.011.

van Dijk, Dick, Philip Hans Franses, and Richard Paap. 2002. "A Nonlinear Long Memory Model, with an Application to US Unemployment." *Journal of Econometrics* 110 (2):135–165. URL https://doi.org/10.1016/S0304-4076(02)00090-8.

Wright, Jonathan H. 2000. "Confidence Sets for Cointegrating Coefficients Based on Stationarity Tests." *Journal of Business & Economic Statistics* 18 (2):211–222. URL https://doi.org/10.1080/07350015.2000.10524863.

Yamada, Hiroshi. 2020. "A Smoothing Method that Looks Like the Hodrick-Prescott Filter." *Econometric Theory* 36:961–981. URL https://doi.org/10.1017/S0266466619000379.

A Computing Unemployment Inflow and Outflow Hazard Rates

In this appendix, I describe how I measure the unemployment inflow and outflow hazard rates, s_t and f_t. My measurement approach follows Shimer (2012) and Elsby, Michaels, and Solon (2009). I begin by defining $S_t \in [0,1]$ as the probability of flowing into unemployment and $F_t \in [0,1]$ as the probability of flowing out of unemployment. I then compute

$$F_t = 1 - \frac{u_t^{level} - u_t^{level,s}}{u_{t-1}^{level}}, \tag{A.1}$$

in which u_t^{level} is the level of unemployment (Bureau of Labor Statistics, 2022c) and $u_t^{level,s}$ is the number of people unemployed for less than 5 weeks (Bureau of Labor Statistics, 2022b). I follow Elsby, Michaels, and Solon (2009) and multiply the Bureau of Labor Statistics' measure of the number of people unemployed for less than 5 weeks by 1.1549 from February 1994 through the end of the sample. Then, the unemployment outflow hazard rate is $f_t = -\ln(1 - F_t)$.

Next, I compute the unemployment inflow hazard rate, defined as $s_t = -\ln(1 - S_t)$. I use

$$u_t^{level} = \frac{(1 - e^{-f_t - s_t})s_t}{f_t + s_t} l_{t-1} + e^{-f_t - s_t} u_{t-1}^{level}, \tag{A.2}$$

in which $l_t = u_t^{level} + e_t^{level}$ is the labor force with e_t^{level} being the level of employment (Bureau of Labor Statistics, 2022a). I solve Equation (A.2) for s_t, given u_t^{level}, u_{t-1}^{level}, l_{t-1}, and the values of F_t from (A.1) and $f_t = -\ln(1 - F_t)$. I solve for s_t using the method of bisection. For each month, my initial lower bound guess for S_t is 0 and my initial upper bound guess for S_t is 1.

Because $t-1$ values of u_{t-1}^{level} and l_{t-1} are needed to compute s_t and f_t, I use data from December 1953 through May 2023 to compute the hazard rates from January 1954 through May 2023.

B Data and LFST Results Using Data through May 2023

In this appendix, I show the data in Figures 1 and 2 through May 2023. These longer samples are in Figures B.1 and B.2. I note that the approximation $s_t/(s_t + f_t)$ takes a value of 0.58 in April 2020 but I truncate the vertical axis of Figure B.2 at 0.22.

I also re-compute the p-values for the LFST statistic using the January 1954 through May 2023 sample. Table B.1 shows the results with this sample. As in the body of the paper, I choose q in the far right column of the table to pick up frequencies corresponding to 8 years and longer. I also

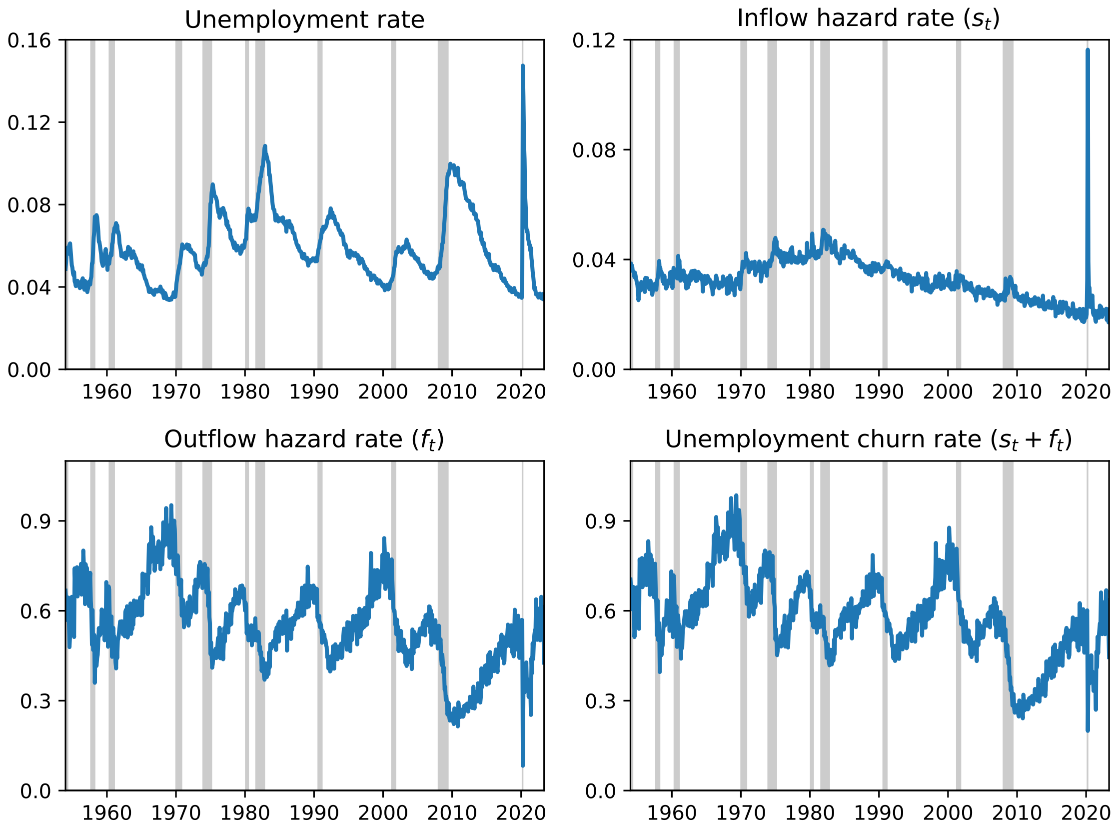

Figure B.1: Labor market data from January 1954 through May 2023.
Note: Gray bars show NBER recessions.

show results for some smaller values of q to establish robustness. Overall, the results in Table B.1 are very similar to those in the body of the paper. The general pattern of results are that labor flows have a unit root, changes in labor flows are stationary, and u_t, $\ln(u_t)$, and $\ln(s_t) - \ln(s_t + f_t)$ are stationary.

C Details for the LFST Statistic

In this appendix, I provide details for computing the LFST statistic and the associated p-values. Let x_t be a 1-dimensional random variable. Given a data sample, $\{x_1, \ldots, x_T\}$, the Müller and Watson (2008) (MW) testing approach begins by computing the discrete cosine transforms $\hat{X}_{T,j} = T^{-1} \sum_{t=1}^{T} \sqrt{2} \cos(\pi j (t - 1/2)/T) x_t$ for $j = 1, \ldots, q$, in which q is much smaller than T. Write $\hat{X}_{T,1:q} = [\hat{X}_{T,1}, \ldots, \hat{X}_{T,q}]'$ as a $(q \times 1)$ vector. Then, $T^{1-\kappa} \hat{X}_{T,1:q} \Rightarrow X \sim N(0, \sigma^2 \Sigma)$, in which σ^2 is the long-run variance of the stationary components of x_t and κ is a scaling factor that depends on

Table B.1: p-Values for the LFST statistics

Variable	$q = 13$	$q = 14$	$q = 15$	$q = 16$	$q = 17$
s_t	0.00	0.00	0.00	0.00	0.00
f_t	0.02	0.03	0.02	0.01	0.01
$s_t + f_t$	0.01	0.02	0.01	0.01	0.01
$\ln(s_t)$	0.00	0.00	0.00	0.00	0.00
$\ln(f_t)$	0.02	0.02	0.01	0.01	0.01
$\ln(s_t + f_t)$	0.01	0.01	0.01	0.01	0.01
Δs_t	0.35	0.73	0.70	0.64	0.62
Δf_t	0.99	0.99	1.00	1.00	0.99
$\Delta(s_t + f_t)$	0.99	0.99	1.00	1.00	0.99
$\Delta \ln(s_t)$	0.20	0.60	0.56	0.50	0.48
$\Delta \ln(f_t)$	0.96	0.98	1.00	1.00	0.99
$\Delta \ln(s_t + f_t)$	0.97	0.97	1.00	1.00	0.99
u_t	0.34	0.37	0.33	0.34	0.32
$\ln(u_t)$	0.31	0.36	0.31	0.29	0.27
$\ln(s_t) - \ln(s_t + f_t)$	0.31	0.34	0.29	0.28	0.26
period of fastest-cycling cosine wave in years	10.7	9.9	9.3	8.7	8.2

Notes: The null hypothesis is that the data are stationary. The alternative hypothesis is that the data have a low-variance unit root component. p-values less than 0.10, 0.05, and 0.01 indicate rejection of the null hypothesis at the 10 percent, 5 percent, and 1 percent levels, respectively. The data sample is January 1954 through May 2023.

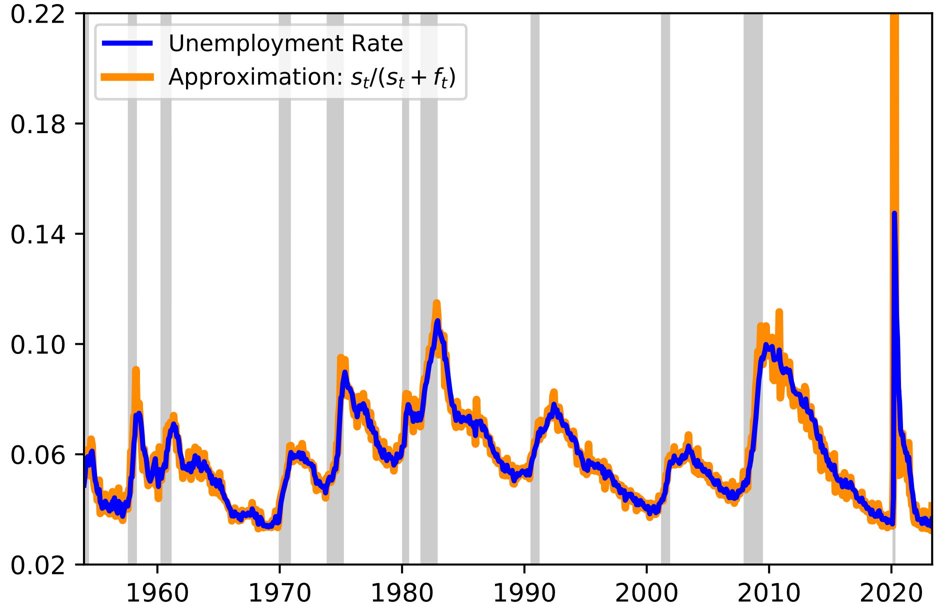

Figure B.2: The unemployment rate and its approximation.
Note: Gray bars show NBER recessions.

the relevant data generating process (DGP) for x_t. If x_t is an $I(0)$ process, then σ^2 is the long-run variance of x_t and $\kappa = 1/2$. If x_t is an $I(1)$ process, then σ^2 is the long-run variance of $x_t - x_{t-1}$ and $\kappa = 3/2$. It will also be the case that Σ depends on the DGP.

MW also consider a DGP that bridges the $I(0)$ and $I(1)$ DGPs, which they call the "local level" DGP. It is $x_t = \mu + \xi_{1,t} + (g/T)\sum_{\tau=1}^{t} \xi_{2,\tau}$ with $\xi_{1,t}$ and $\xi_{2,t}$ both being $I(0)$ processes with mean zero and a joint long-run covariance matrix $\sigma^2 I_2$. The parameter g governs the variance of the unit root component, $\sum_{\tau=1}^{t} \xi_{2,\tau}$. For this DGP, the scale factor is $\kappa = 1/2$. In short, this local level DGP can be interpreted as a process with a low-variance unit root component. The LFST statistic then tests the null hypothesis that a variable is stationary against the alternative hypothesis that the variable follows the local level DGP.

Because σ^2 is not known, MW base their tests on the distribution of $\hat{X}_{T,1:q}/\sqrt{\hat{X}'_{T,1:q}\hat{X}_{T,1:q}}$. By the continuous mapping theorem, $\hat{X}_{T,1:q}/\sqrt{\hat{X}'_{T,1:q}\hat{X}_{T,1:q}} \Rightarrow X/\sqrt{X'X}$. The density of $\eta = X/\sqrt{X'X}$ is

$$f_\eta = (1/2)\Gamma(q/2)\pi^{-q/2}|\Sigma|^{-1/2}(\eta\Sigma^{-1}\eta)^{-q/2}, \tag{C.1}$$

in which $\Gamma(\cdot)$ is the gamma function. Given this density, MW then form the LFST statistic, which is a likelihood ratio statistic. The null hypothesis is that x_t is an $I(0)$ process. With this null, $\Sigma = I_q$.

The alternative hypothesis is that x_t has a local level DGP. With this alternative, $\Sigma = I_q + g^2 D$, in which D is a diagonal matrix with the jth diagonal element being $1/(j\pi)^2$. Then, the likelihood ratio statistic rejects the null hypothesis for large values of

$$\text{LFST} = \frac{\sum_{j=1}^{q} \hat{X}_{T,j}^2}{\sum_{j=1}^{q} \hat{X}_{T,j}^2/(1+(g/(j\pi))^2)}. \tag{C.2}$$

Following MW, I use a value of $g = 10$.

After computing the LFST statistic, I compute the p-value with simulation. Using $X \sim N(0, \sigma^2\Sigma)$, I simulate values of X from a multivariate normal distribution. Under the null, $\Sigma = I_q$. Further, I set $\sigma^2 = 1$ because the value of σ^2 does not affect the LFST statistic. Hence, I draw 200,000 simulations of X from $N(0, I_q)$, compute the LFST statistic for each draw, and compute the p-value as the fraction of the simulated LFST statistics that are greater than the value in (C.2) that I compute from the data.

D VECM Estimation and Inference

Ordinary Least Squares Estimation: I estimate Equations (3) and (4) separately by ordinary least squares. I collect the parameters, $\Theta = [\nu, \alpha, \Gamma_1, \ldots, \Gamma_p]$, and define

$$Y = [\Delta y_{p+2}, \ldots, \Delta y_T], \quad z_t = [d_t', \beta' y_t, \Delta y_t', \ldots, \Delta y_{t-p+1}']', \quad Z_{-1} = [z_{p+1}, \ldots, z_{T-1}]$$
$$D = [d_{p+2}, \ldots, d_T], \quad D_{-1} = [d_{p+1}, \ldots, d_{T-1}], \quad V = [v_{p+2}, \ldots, v_T], \quad W = [w_{p+2}, \ldots, w_T].$$

The parameter estimates are

$$\hat{\Theta} = Y Z_{-1}'(Z_{-1}Z_{-1}')^{-1}$$
$$\hat{\Phi} = D D_{-1}'(D_{-1}D_{-1}')^{-1}.$$

Inference: Let $\theta = \text{vec}(\Theta)$, $\hat{\theta} = \text{vec}(\hat{\Theta})$, $\phi = \text{vec}(\Phi)$, and $\hat{\phi} = \text{vec}(\hat{\Phi})$. For inference, I treat the ordinary least squares estimates as jointly asymptotically normally distributed so that

$$\sqrt{T-p-1}\begin{bmatrix} (\hat{\theta} - \theta) \\ (\hat{\phi} - \phi) \end{bmatrix} \sim N(0, \Sigma).$$

I estimate Σ as follows. I define $\hat{Q}_Z = (T-p-1)^{-1} Z_{-1}Z_{-1}'$ and $\hat{Q}_D = (T-p-1)^{-1} D_{-1}D_{-1}'$.

Then, I have

$$\sqrt{T-p-1}(\hat{\theta}-\theta) = (\hat{Q}_Z^{-1} \otimes I_2)\text{vec}((T-p-1)^{-1/2} V Z'_{-1})$$
$$\sqrt{T-p-1}(\hat{\phi}-\phi) = (\hat{Q}_D^{-1} \otimes I_3)\text{vec}((T-p-1)^{-1/2} W D'_{-1}).$$

Let $\hat{V} = [\hat{v}_{p+2}, \ldots, \hat{v}_T] = Y - \hat{\Theta} Z_{-1}$ and $\hat{W} = [\hat{w}_{p+2}, \ldots, \hat{w}_T] = D - \hat{\Phi} D_{-1}$ be the ordinary least squares residuals. I then compute

$$\hat{\Omega} = (T-p-1)^{-1} \sum_{t=p+2}^{T} \hat{\xi}_t \hat{\xi}'_t,$$

in which

$$\hat{\xi}_t = \begin{bmatrix} (z_{t-1} \otimes I_2)\hat{v}_t \\ (d_{t-1} \otimes I_3)\hat{w}_t \end{bmatrix}.$$

Then, the estimate of the joint asymptotic covariance matrix is

$$\hat{\Sigma} = \begin{bmatrix} (\hat{Q}_Z^{-1} \otimes I_2) & 0_{8+4p \times 9} \\ 0_{9 \times 8+4p} & (\hat{Q}_D^{-1} \otimes I_3) \end{bmatrix} \hat{\Omega} \begin{bmatrix} (\hat{Q}_Z^{-1} \otimes I_2) & 0_{8+4p \times 9} \\ 0_{9 \times 8+4p} & (\hat{Q}_D^{-1} \otimes I_3) \end{bmatrix}'.$$

For inference in Subsection 3.3, I use $[\hat{\phi}', \hat{\theta}']$ divided by the square root of the diagonal elements of $\hat{\Sigma}/(T-p-1)$ as my t-statistics. I use submatrices of $\hat{\Sigma}$ to compute Wald statistics and t-statistics for the differences between slope estimates.

E Estimates of the Dummy Markov Chain

Recall that $d_t = [d_t^{expansion}, d_t^{NBER}, d_t^{TZ}]'$. I estimate Φ in Equation (4) by ordinary least squares as described in Appendix D. The estimation sample is January 1954 through December 2019, and the estimate is

$$\hat{\Phi} = \begin{bmatrix} 0.986 & 0 & 0.066 \\ 0 & 0.911 & 0.074 \\ 0.014 & 0.089 & 0.861 \end{bmatrix}.$$

The eigenvalues of this estimated transition matrix are 1, 0.96, and 0.80. Hence, the Markov chain is ergodic and d_t is covariance stationary (Hamilton, 1994, Chapter 22).

Following Hamilton (1994, Chapter 22), I compute the expectation of d_t as the eigenvalue of $\hat{\Phi}$

associated with the unit eigenvalue. This yields

$$\mathbb{E}(d_t) = \begin{bmatrix} \mathbb{E}(d_t^{expansion}) \\ \mathbb{E}(d_t^{NBER}) \\ \mathbb{E}(d_t^{TZ}) \end{bmatrix} = \begin{bmatrix} 0.71 \\ 0.13 \\ 0.16 \end{bmatrix}.$$

F Models with Variables Interacted with Dummies

In this appendix, I consider two additional models that interact the business cycle dummies with some of the variables on the right-hand side of Equation (3). The first model interacts the recession dummies with the error correction term

$$\begin{aligned} \Delta y_t = \nu d_{t-1} + \alpha\beta' y_{t-1} + \delta_1 \beta' y_{t-1} d_{t-1}^{NBER} + \delta_2 \beta' y_{t-1} d_{t-1}^{TZ} \\ + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_p \Delta y_{t-p} + v_t. \end{aligned} \tag{F.1}$$

The second model interacts the recession dummies with the first lag of Δy_t

$$\begin{aligned} \Delta y_t = \nu d_{t-1} + \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \delta_1 \Delta y_{t-1} d_{t-1}^{NBER} + \delta_2 \Delta y_{t-1} d_{t-1}^{TZ} \\ + \Gamma_2 \Delta y_{t-2} + \cdots + \Gamma_p \Delta y_{t-p} + v_t. \end{aligned} \tag{F.2}$$

I abuse notation and allow δ_1 and δ_2 to be different coefficients and have different dimensions in each equation. With that said, δ_1 and δ_2 are the parameters of interest in this appendix.

I estimate these models with ordinary least squares and do inference in parallel fashion to what is in Appendix D. The main results are as follows:

- For the baseline model in the body of the paper, the HQ value is -11.47 and SC value is -11.26.

- In Equation (F.1), the Wald statistic for the elements of δ_1 and δ_2 being jointly different from zero is 9.459, with an associated p-value of 0.051 (from a χ^2 distribution with 4 degrees of freedom). The HQ value is -11.46 and SC value is -11.24.

- In Equation (F.2), the Wald statistic for the elements of δ_1 and δ_2 being jointly different from zero is 10.276, with an associated p-value of 0.246 (from a χ^2 distribution with 8 degrees of freedom). The HQ value is -11.44 and SC value is -11.21.

Overall, the baseline model in the body of the paper has the best fit according to the HQ and SC values. Further, the elements of δ_1 and δ_2 are not jointly different from zero at a 5 percent level of statistical significance in either Equation (F.1) or (F.2). I do note that the elements of δ_1 and δ_2 are jointly different from zero at the 10 percent level of significance in Equation (F.1). Hence, there is some evidence that the error correction slopes vary over the business cycle. However, this evidence is not overwhelming and I prefer my baseline model in the body of the paper in the interest of parsimony.

Appendix references

Bureau of Labor Statistics. 2022a. "Current Population Survey, Labor Force Statistics: Employment Level, Seasonally Adjusted - LNS12000000." Retrieved from FRED, Federal Reserve Bank of St. Louis. URL https://fred.stlouisfed.org/series/CE16OV (accessed on June 16, 2023).

———. 2022b. "Current Population Survey, Labor Force Statistics: Number Unemployed for Less than 5 Weeks, Seasonally Adjusted - LNS13008396." Retrieved from FRED, Federal Reserve Bank of St. Louis. URL https://fred.stlouisfed.org/series/UEMPLT5 (accessed on June 16, 2023).

———. 2022c. "Current Population Survey, Labor Force Statistics: Unemployment Level, Seasonally Adjusted - LNS13000000." Retrieved from FRED, Federal Reserve Bank of St. Louis. URL https://fred.stlouisfed.org/series/UNEMPLOY (accessed on June 16, 2023).

Elsby, Michael W. L., Ryan Michaels, and Gary Solon. 2009. "The Ins and Outs of Cyclical Unemployment." *American Economic Journal: Macroeconomics* 1 (1):84–110. URL https://doi.org/10.1257/mac.1.1.84.

Hamilton, James D. 1994. *Time Series Analysis*. Princeton University Press.

Müller, Ulrich K. and Mark W. Watson. 2008. "Testing Models of Low-Frequency Variability." *Econometrica* 76 (5):979–1016. URL https://doi.org/10.3982/ECTA6814.

Shimer, Robert. 2012. "Reassessing the Ins and Outs of Unemployment." *Review of Economic Dynamics* 12 (2):127–148. URL https://doi.org/10.1016/j.red.2012.02.001.